Diffractive Vector Meson Production: Dipole Model vs. pQCD

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Motivation

- The exclusive vector meson diffractive cross-section is proportional to $g^2(x, Q^2)$ and
- Channel for observing gluon saturation effects

Several successful descriptions (for HERA data):

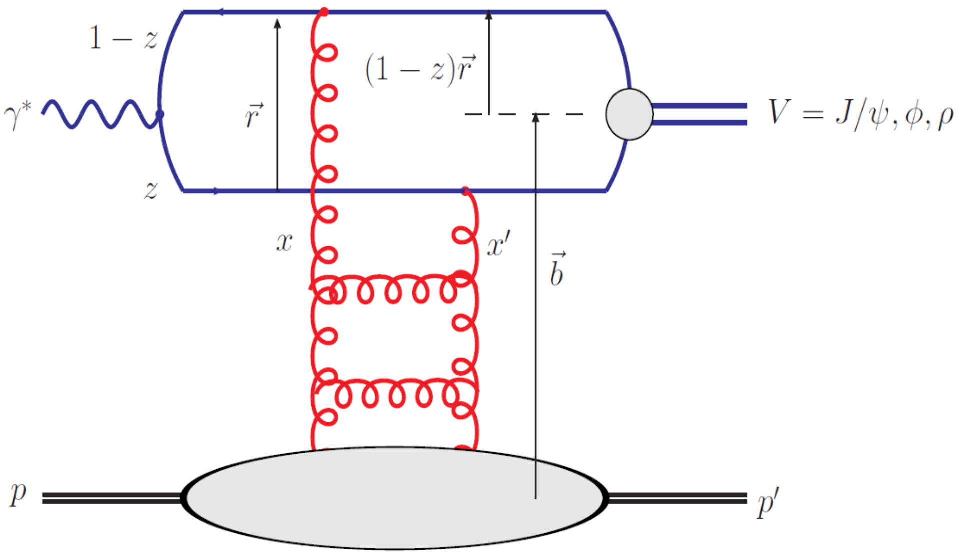
- Color Dipole Models
- pQCD
- Vector Meson Dominance (not discussed here)

- What VM parameters are important and how do we measure them at eRHIC?
- What needs to be done differently for e+A?

Dipole Model & KWM

Based on:

Exclusive diffractive processes at HERA within the dipole picture, H. Kowalski, L. Motyka, G. Watt, PhysRev D74, 074016, arXiv:hep-ph/0606272v2



Dipole model:

1. γ* fluctuates into qq pair
2. qq scatters elastically on p(A)
3. qq pair recombines into γ*
4. γ* decays into VM

Cross-section for production of final state VM:

$$\frac{d\sigma_{T,L}^{\gamma^* p \to Ep}}{dt} = \frac{1}{16\pi}\left|\mathcal{A}_{T,L}^{\gamma^* p \to Ep}\right|^2 = \frac{1}{16\pi}\left|\int d^2\boldsymbol{r}\int_0^1 \frac{dz}{4\pi}\int d^2\boldsymbol{b}\,(\Psi_E^*\Psi)_{T,L}\, e^{-i[\boldsymbol{b}-(1-z)\boldsymbol{r}]\cdot\boldsymbol{\Delta}}\,\frac{d\sigma_{q\bar{q}}}{d^2\boldsymbol{b}}\right|^2$$

Amplitude

Overlap between photon and VM wave function

Dipole Cross-Section

Dipole Cross-Section: b-Sat Model

$$\frac{d\sigma_{q\bar{q}}}{d^2b} = 2\left(1 - e^{-\frac{\pi^2}{2N_c}r^2\alpha_s(\mu^2)xg(x,\mu^2)T(b)}\right)$$

scale μ^2 is related to the dipole size r by $\mu^2 = 4/r^2 + \mu_0^2$

The initial gluon density at the scale μ_0^2 is taken in the form

$$xg(x,\mu_0^2) = A_g\, x^{-\lambda_g}\,(1-x)^{5.6}$$

Gluon density for VM production is evaluated at scale:

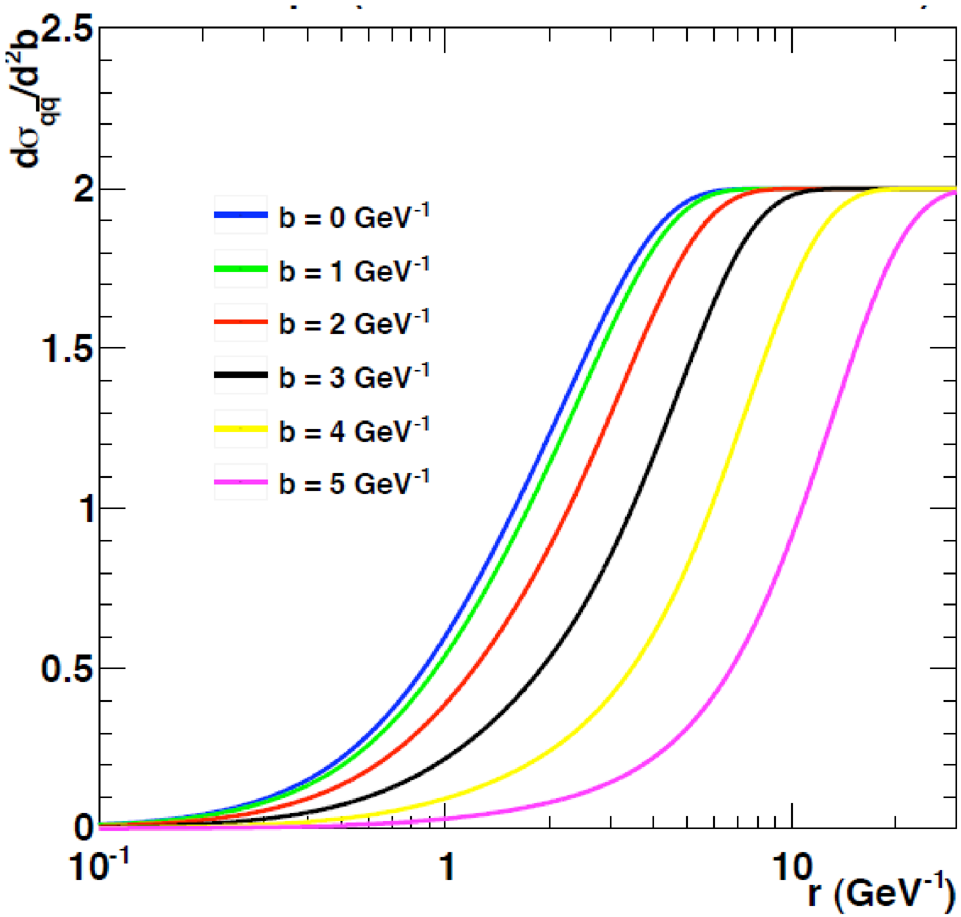
$$x = x_B(1 + M_V^2/Q^2)$$

Proton shape: Gaussian or step function, here only former is used:

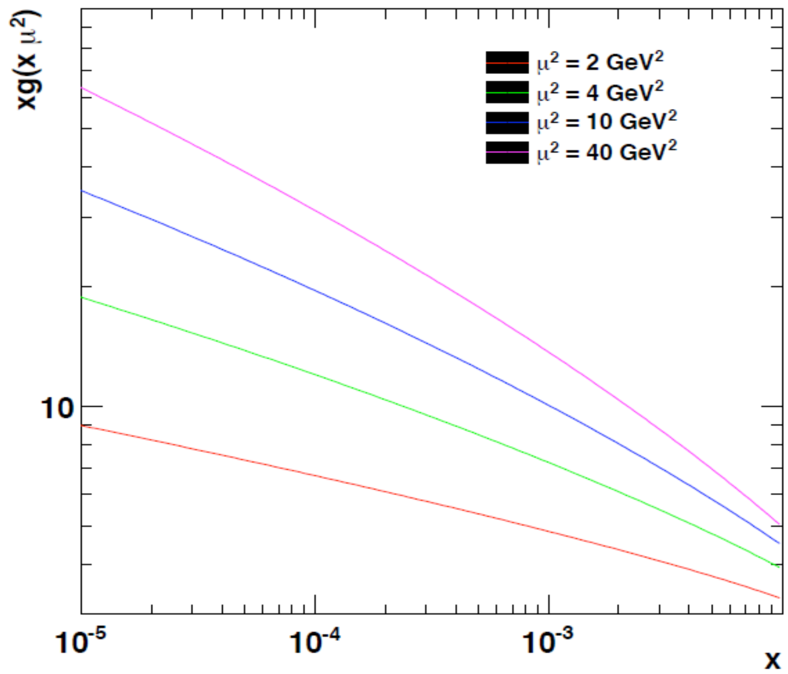
$$T_G(b) = \frac{1}{2\pi B_G}e^{-\frac{b^2}{2B_G}}$$

$B_G = 4\ \mathrm{GeV}^{-2}$ from fits to HERA data

Dipole Cross-Section: b-Sat Model



DGLAP evolved gluon density with gluon splittings only. Initial density determined by fit to HERA data



Overlap Function

$$(\Psi_V^*\Psi)_T = \hat{e}_f e \frac{N_c}{\pi z(1-z)} \left\{ m_f^2 K_0(\epsilon r)\phi_T(r,z) - \left[z^2+(1-z)^2\right] \epsilon K_1(\epsilon r)\partial_r \phi_T(r,z) \right\},$$

$$(\Psi_V^*\Psi)_L = \hat{e}_f e \frac{N_c}{\pi} 2Q z(1-z)\, K_0(\epsilon r) \left[M_V \phi_L(r,z) + \delta \frac{m_f^2 - \nabla_r^2}{M_V z(1-z)} \phi_L(r,z) \right]$$

Follows: Forshaw, Sandepen, Shaw description

where:

$$\hat{e}_f = 2/3,\ 1/3,\ \text{or}\ 1/\sqrt{2},\ \text{for}\ J/\psi,\ \phi,\ \text{or}\ \rho\ \text{mesons}$$

$$\nabla_r^2 = (1/r)\partial_r + \partial_r^2$$

$$e = \sqrt{4\pi\alpha_{em}}$$

$$\epsilon^2 = z(1-z)Q^2 + m_f^2$$

z = fraction of photon's light cone momentum carried by quark

r = dipole size

m_f = quark mass

M_V = vector meson mass

N_c = 3 colors

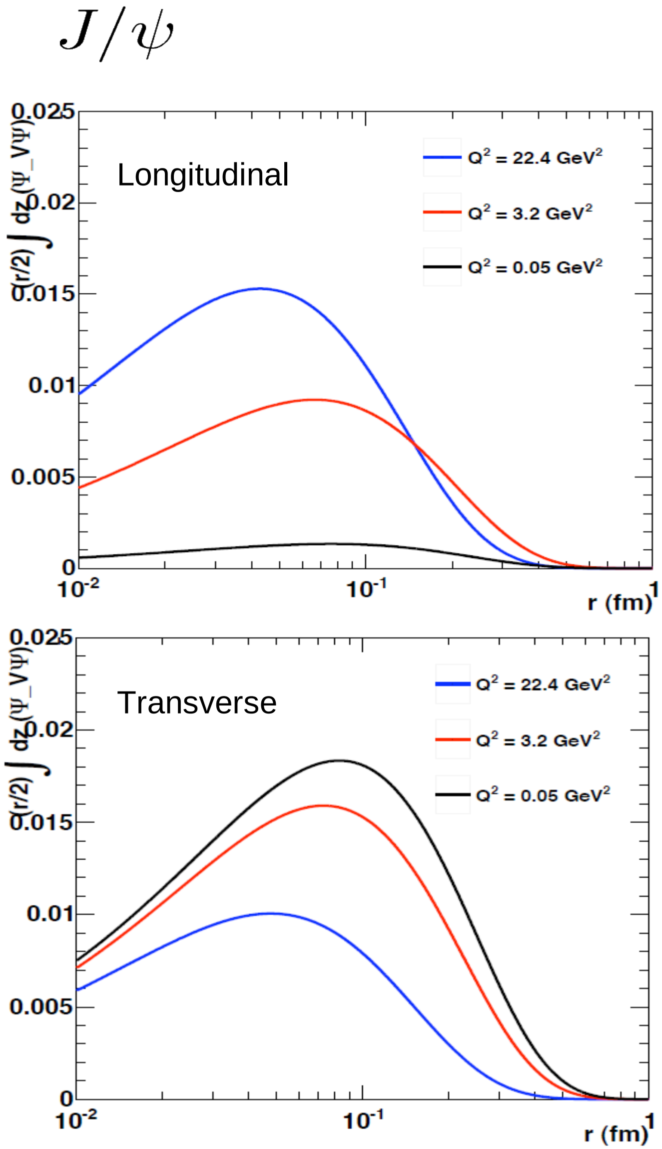
$K_{0,1}$: Bessel functions

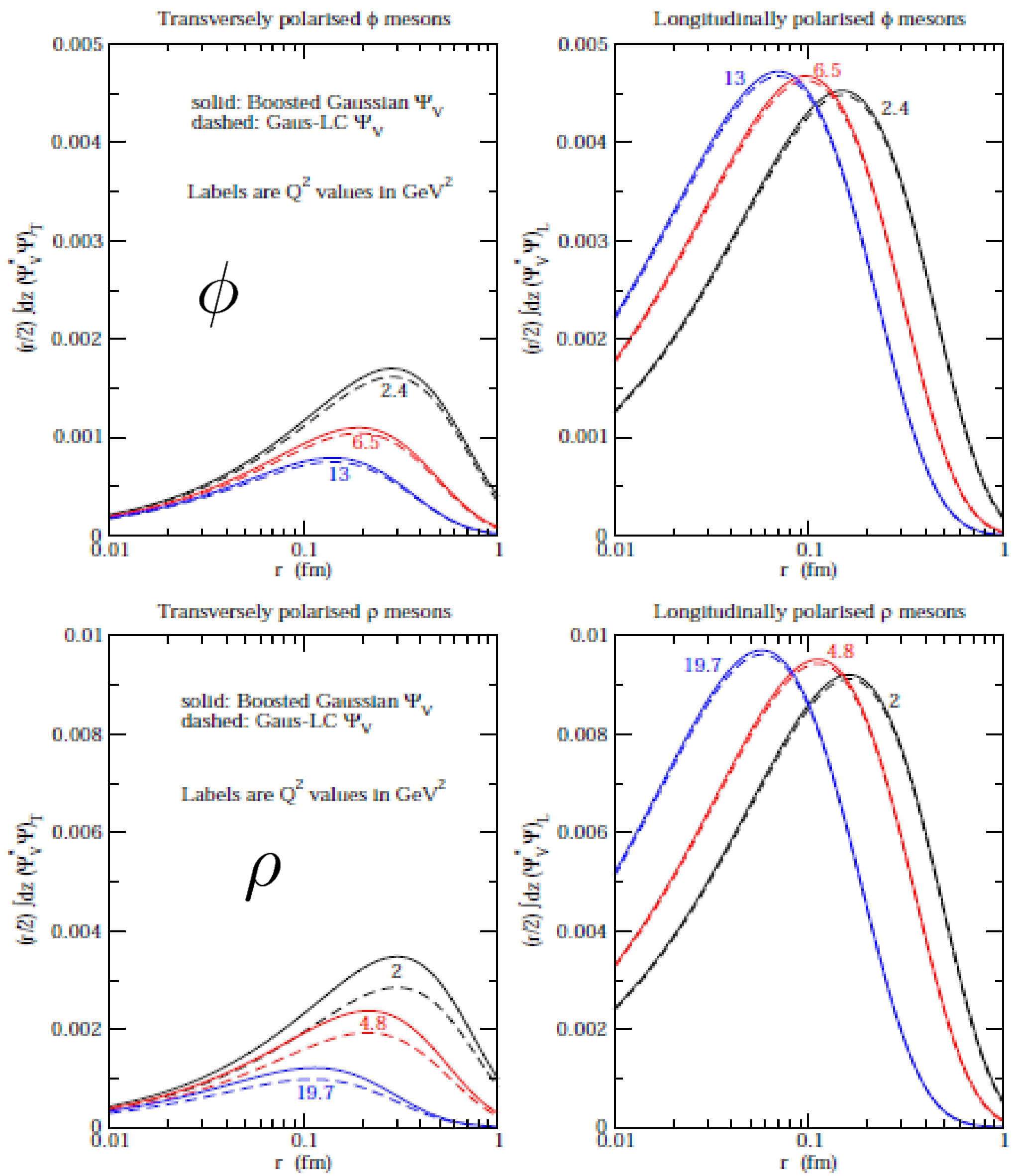
δ = 0 or 1 (model/author dependent - here always 1)

$\varphi_{T,L}(r,z)$ = VM wave function

$$\phi_T(r,z) = N_T[z(1-z)]^2 \exp(-r^2/2R_T^2),$$
$$\phi_L(r,z) = N_L z(1-z) \exp(-r^2/2R_L^2).$$

Overlap Function

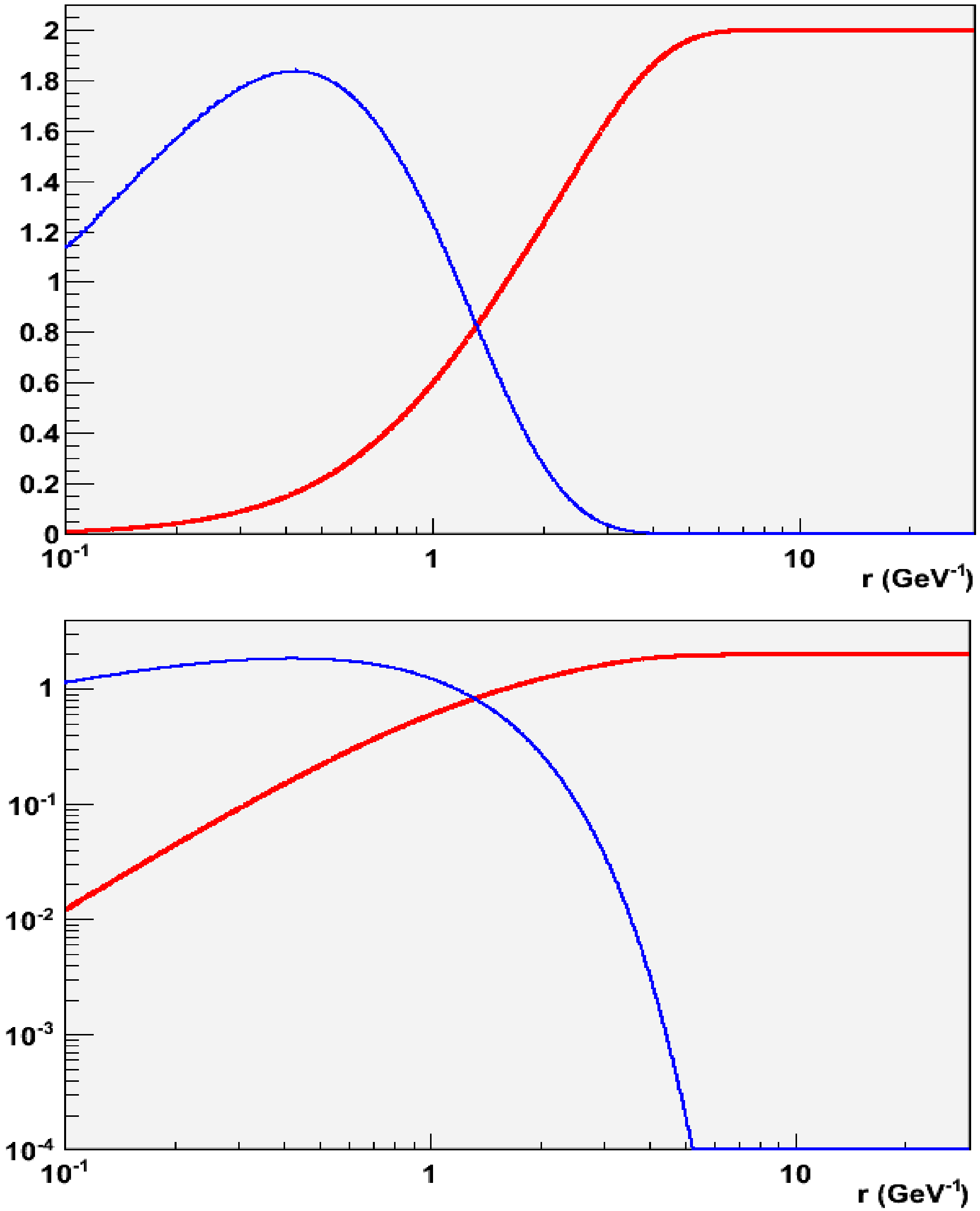




Suppression of the Dipole XSection

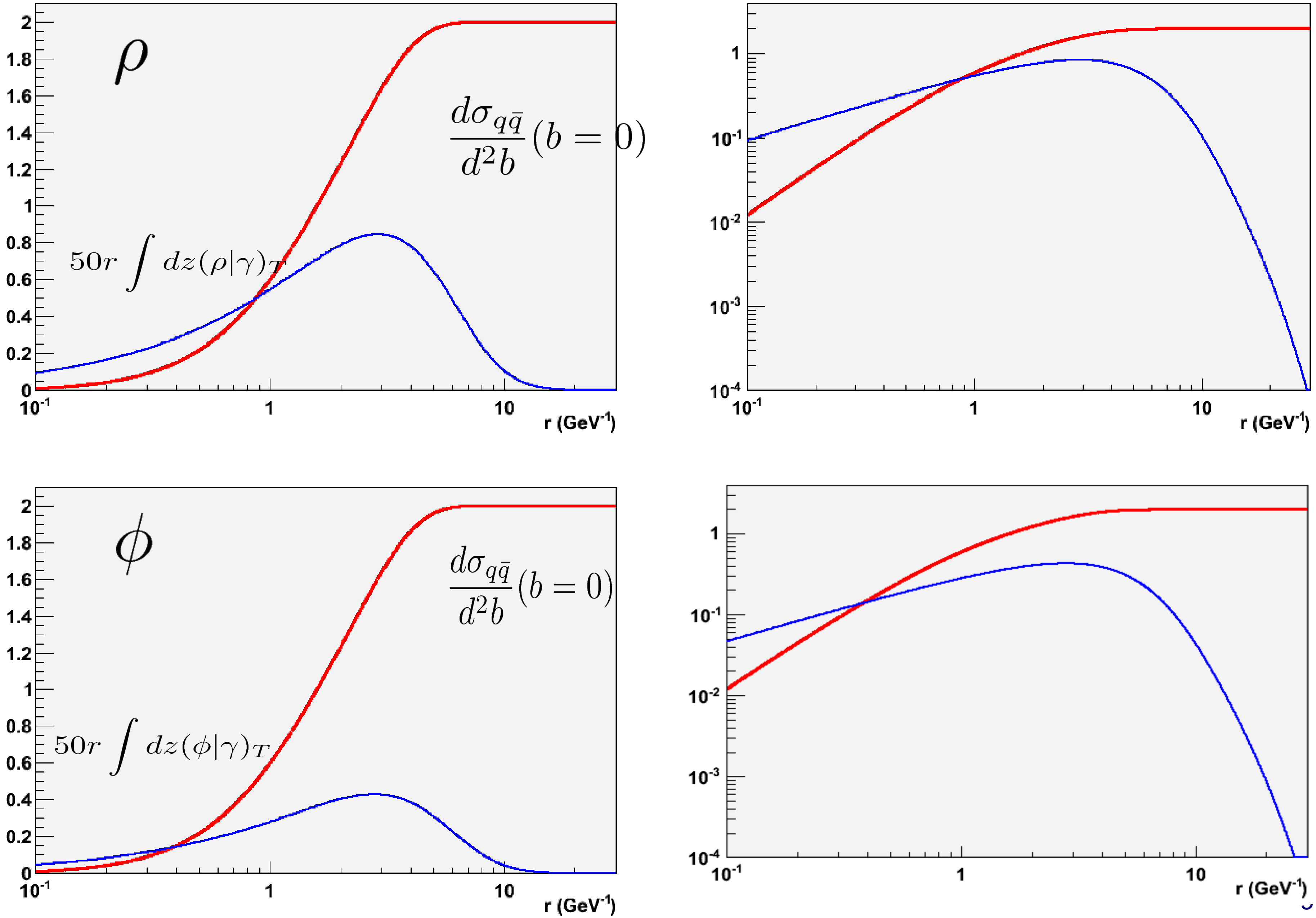
$50r \int dz (J/\psi|\gamma)_T$

$\frac{d\sigma_{q\bar{q}}}{d^2b}(b=0)$



J/ψ

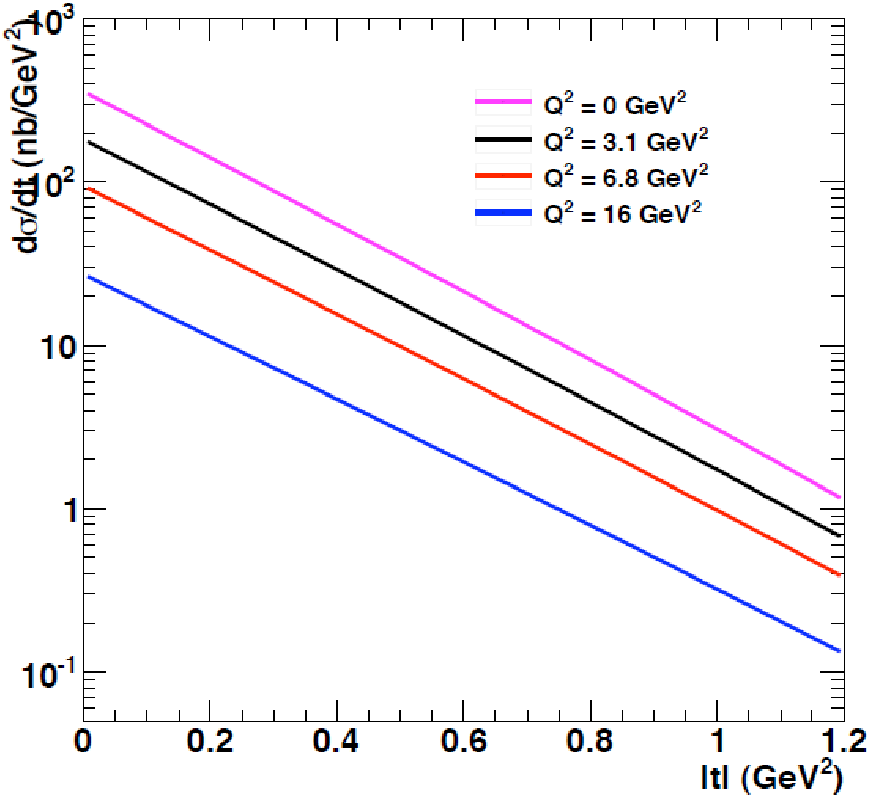
Suppression of the Dipole XSection

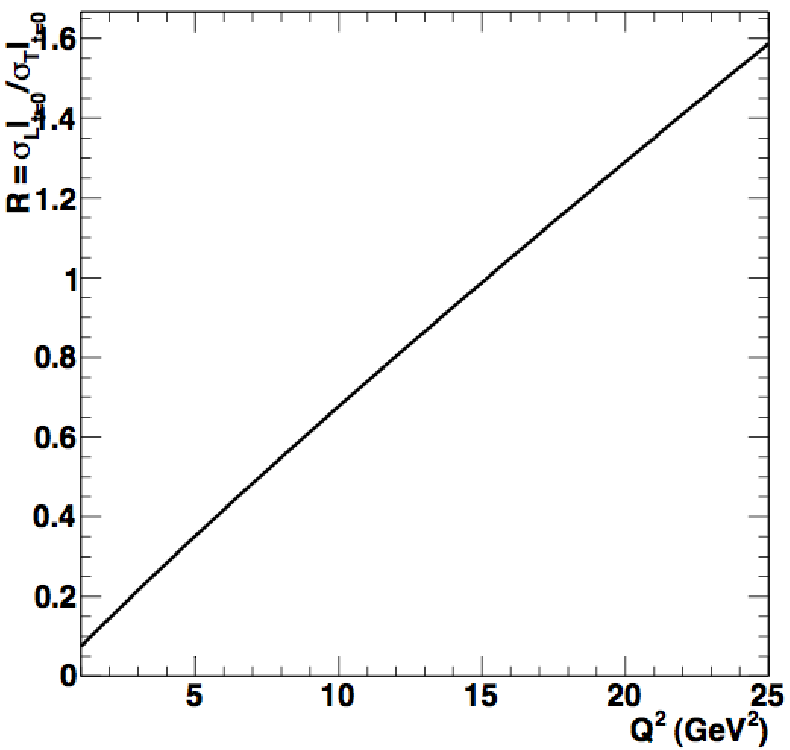


Putting it Together

$$\frac{d\sigma^{\gamma^* p\to pV}_{T,L}}{dt} = \frac{1}{16\pi}\left|\int dr(2\pi r)\int_0^1 \frac{dz}{4\pi}\int db(2\pi b)(\Psi_V^*\Psi)_{T,L} J_0(b\Delta)J_0([1-z]r\Delta)\;\frac{d\sigma_{q\bar q}}{d^2\vec{b}}\right|^2$$

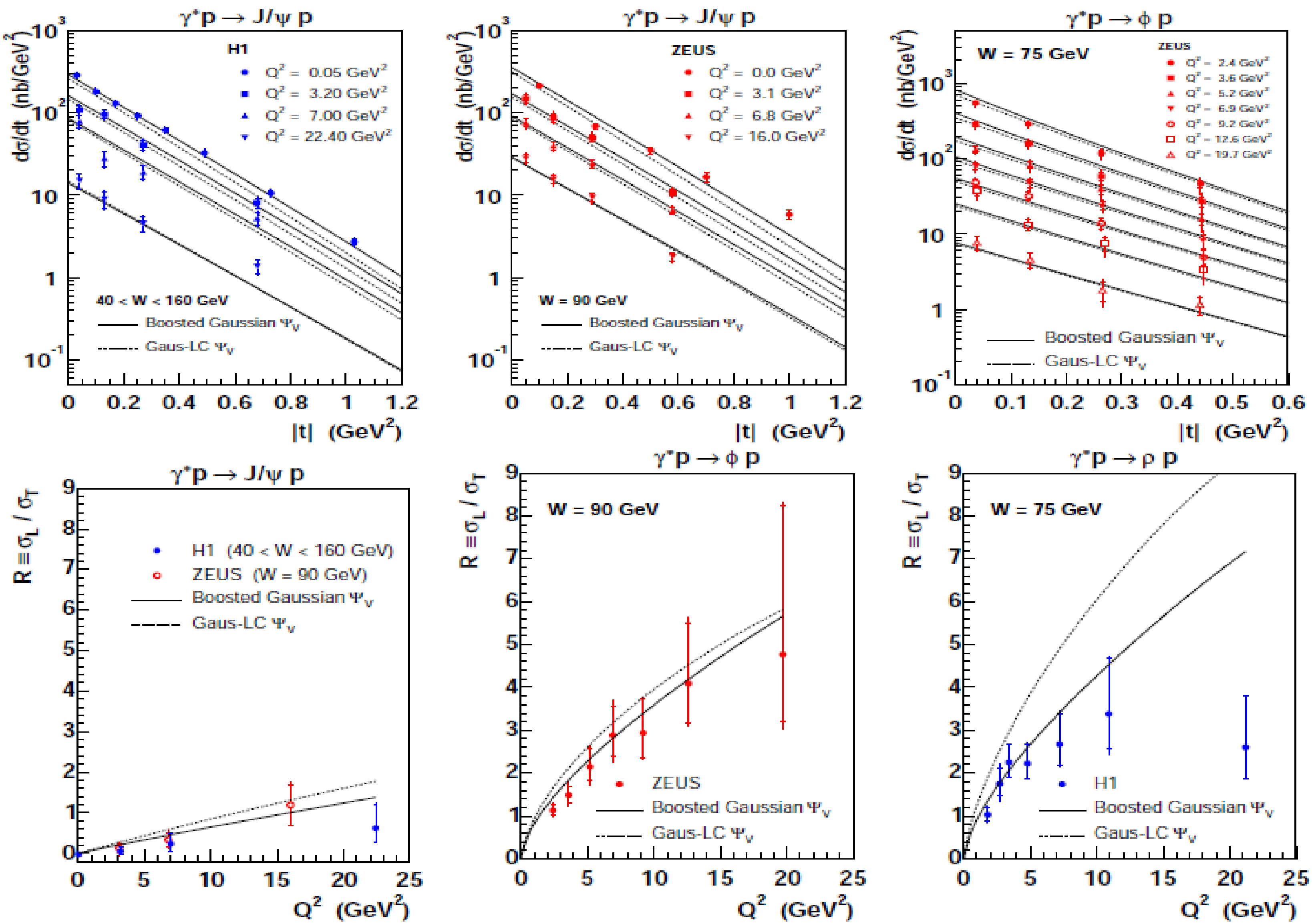
After angular integrations



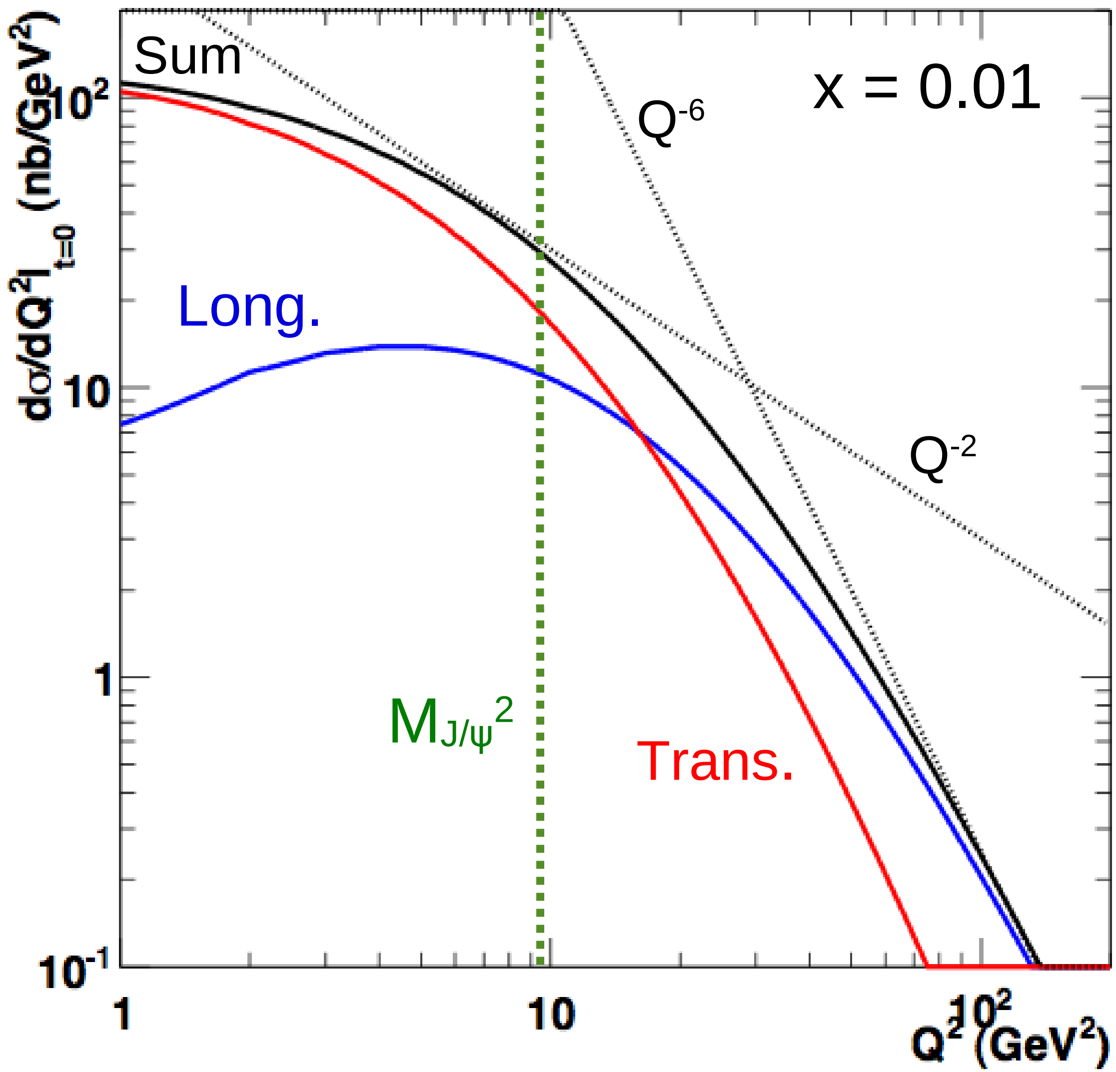


Putting it Together

[7] Kowalski, Motyka, Watt, hep-ph/0606272



e+p: J/ψ production, b-Sat model



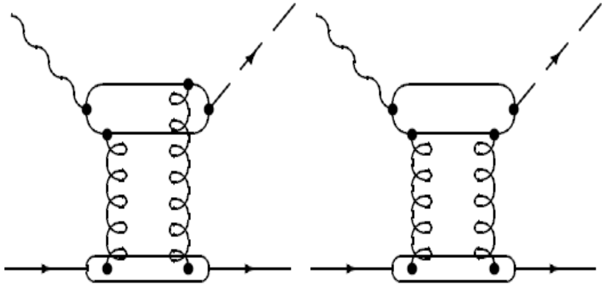
Diffractive VM in pQCD

[1] S. Brodsky et al., Phys.Rev.D50:3134,1994, e-Print: hep-ph/9402283

[2] L. Frankfurt et al., Phys. Rev. D 54, 3194 - 3215 (1996) (corrects above)

$$\left.\frac{d\sigma_L^{\gamma^* N \to VN}}{dt}\right|_{t=0} = \frac{12\pi^3 \Gamma_V m_V \eta_V^2 T(Q^2)}{\alpha_{em} N_c^2} \cdot \frac{\alpha_s^2(Q) \left| \left[1 + i\frac{\pi}{2}\frac{d}{d\ln x}\right] xG(x,Q^2) \right|^2}{Q^6}$$

$T(Q^2)$: Introduced in [2].

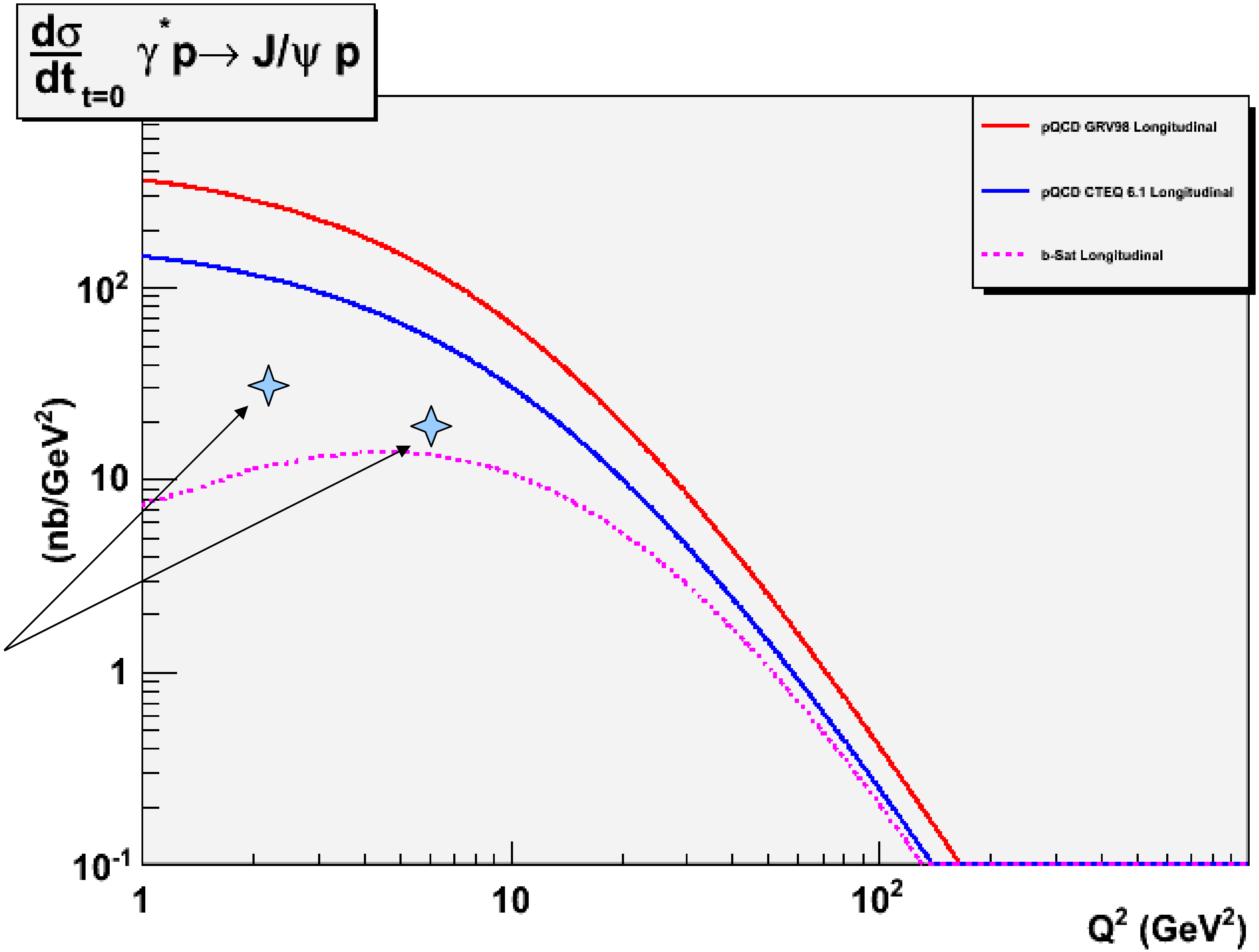


$T(Q^2)$: Accounts for "preasymptotic" effects

$T(Q^2 \to \infty) = 1$

Formula (w/o T) is only valid when transverse momenta in q̄q dipole (Fermi-motion) are neglected, i.e., at sufficiently large Q^2. Otherwise corrections are needed.

Comparison with Dipole Model



Approximate corrections to CTEQ cross-section at this point from T(Q)

e+A

Based on
KT: Henri Kowalski , Derek Teaney, PRD68:114005, hep-ph/0304189
KLV: H. Kowalski, T. Lappi, R. Venugopalan, PRL100:022303, arXiv:0705.3047 [hep-ph]

$$\frac{d\sigma^A_{dip}}{d^2b} = 2\left[1 - e^{-r^2 F(x,r^2)\sum_{i=1}^{A} T_p(b-b_i)}\right]$$ for e+A

$$F(x,r^2) = \frac{\pi^2}{2N_c}\alpha_s(\mu^2)xg(x,\mu^2)$$

The average differential dipole cross-section can be approximated by:

$$\left\langle \frac{d\sigma^A_{dip}}{d^2b} \right\rangle \approx 2\left[1 - \left(1 - \frac{T_A(b)}{2}\sigma^p_{dip}\right)^A\right]$$

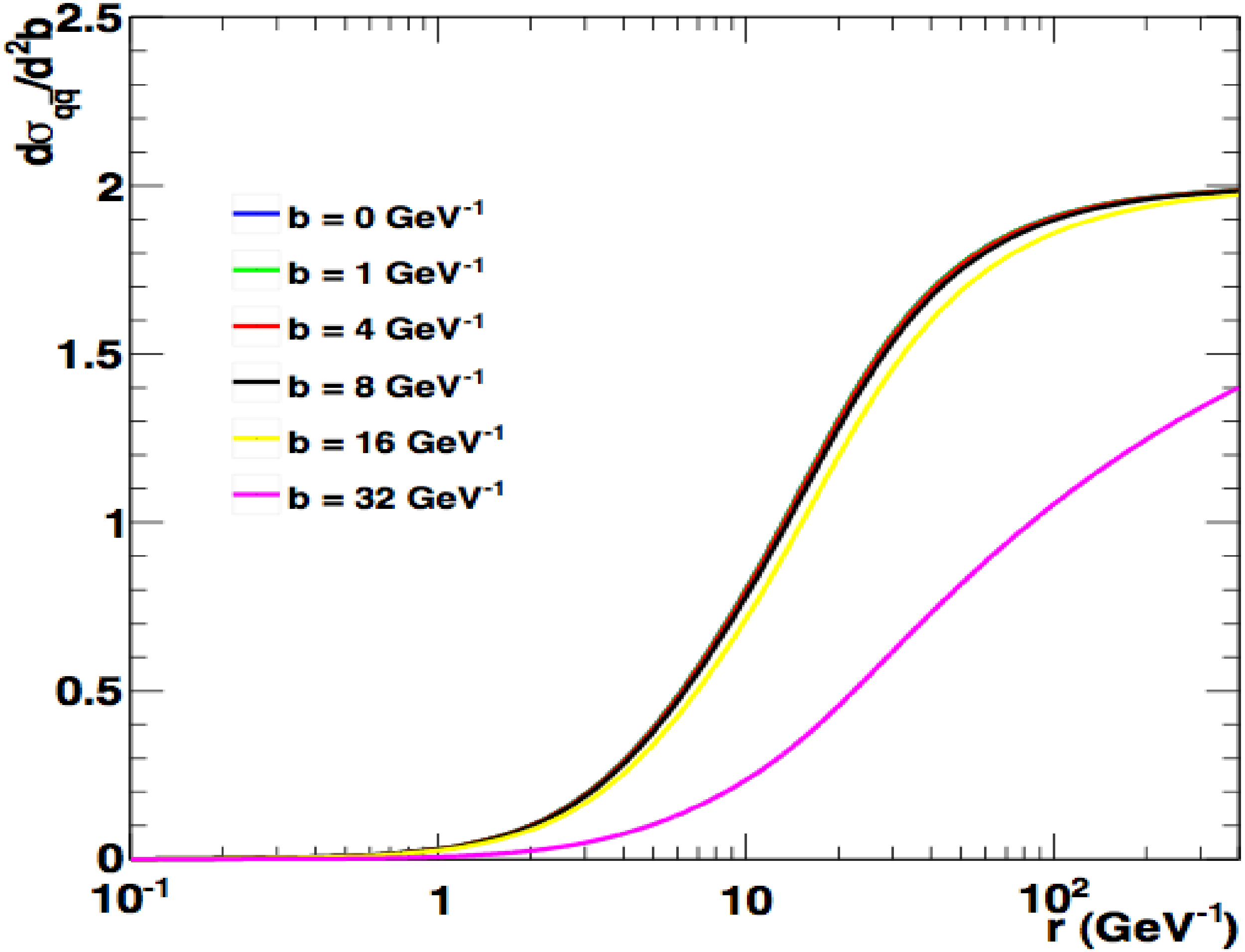
where

$$\sigma^p_{dip} = \int d^2b\, \frac{d\sigma^p_{dip}}{d^2b}$$

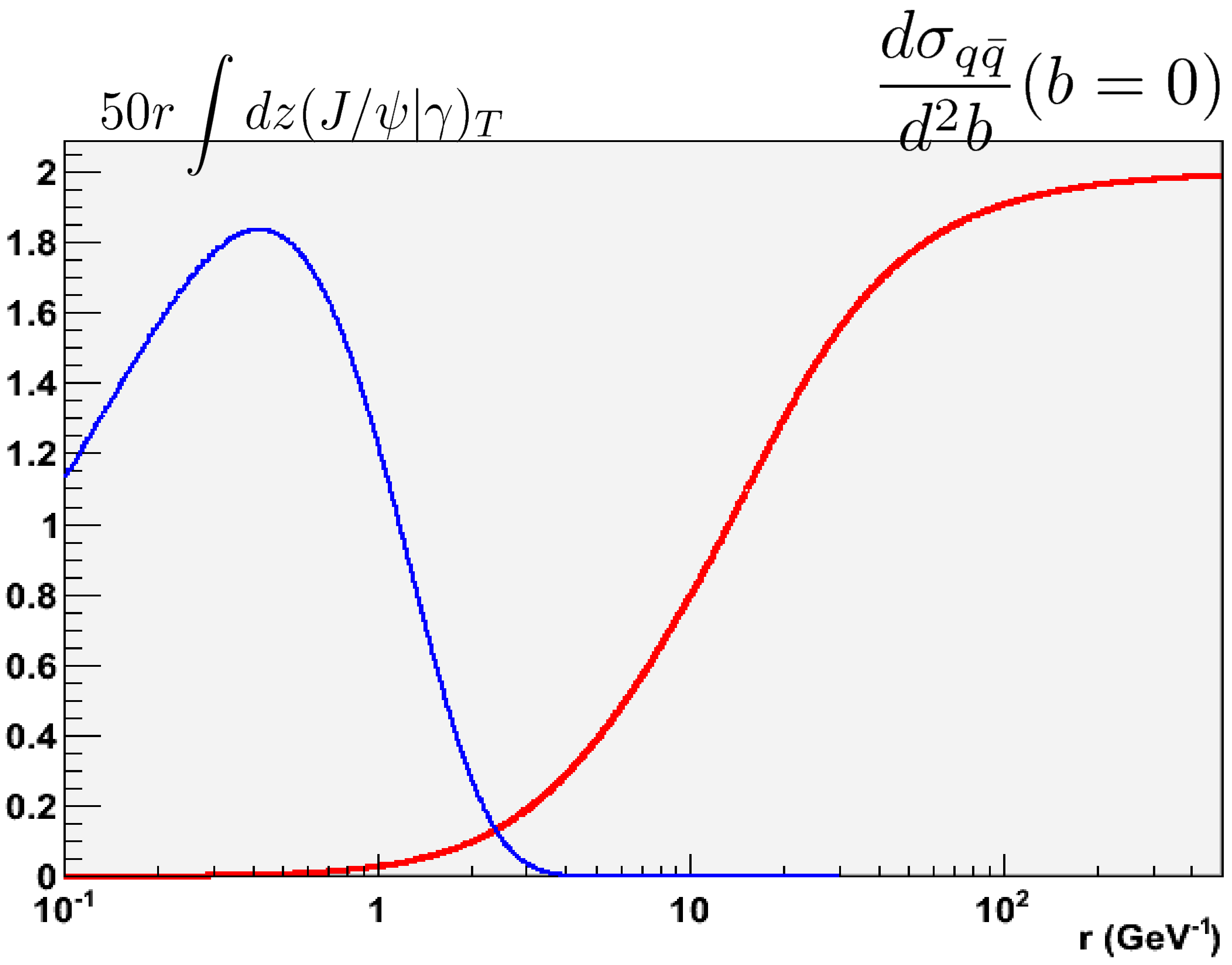
which can be calculated from the b-Sat in e+p

e+A: J/ψ production, b-Sat model

$$\langle \frac{d\sigma^A_{dip}}{d^2 b_\perp} \rangle \approx 2 \left[1 - \left(1 - \frac{T_A(b_\perp)}{2} \sigma^p_{dip} \right)^A \right]$$

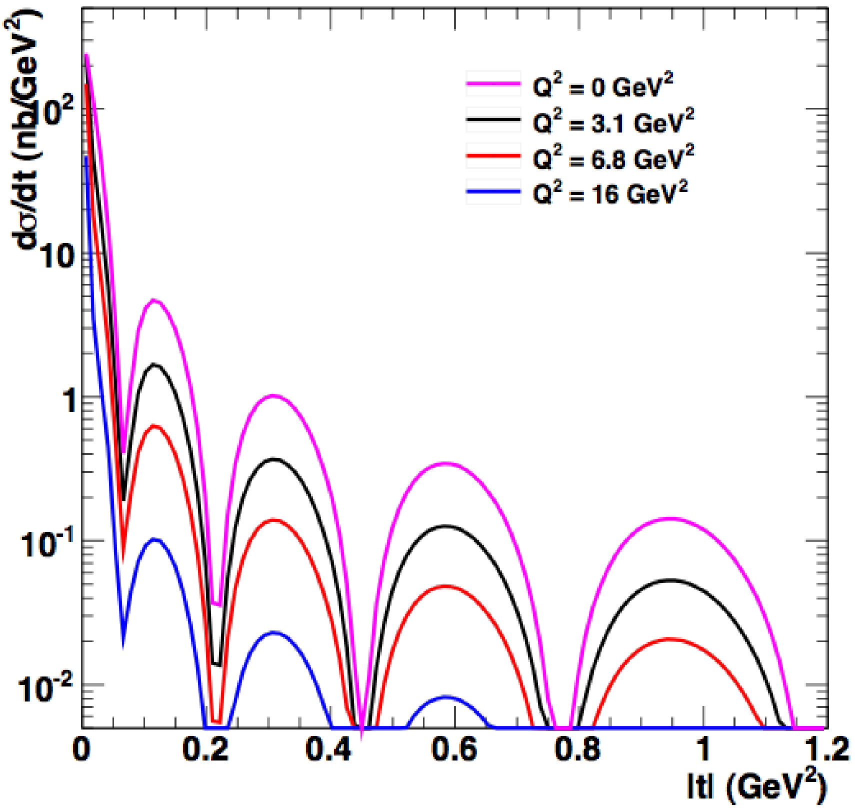


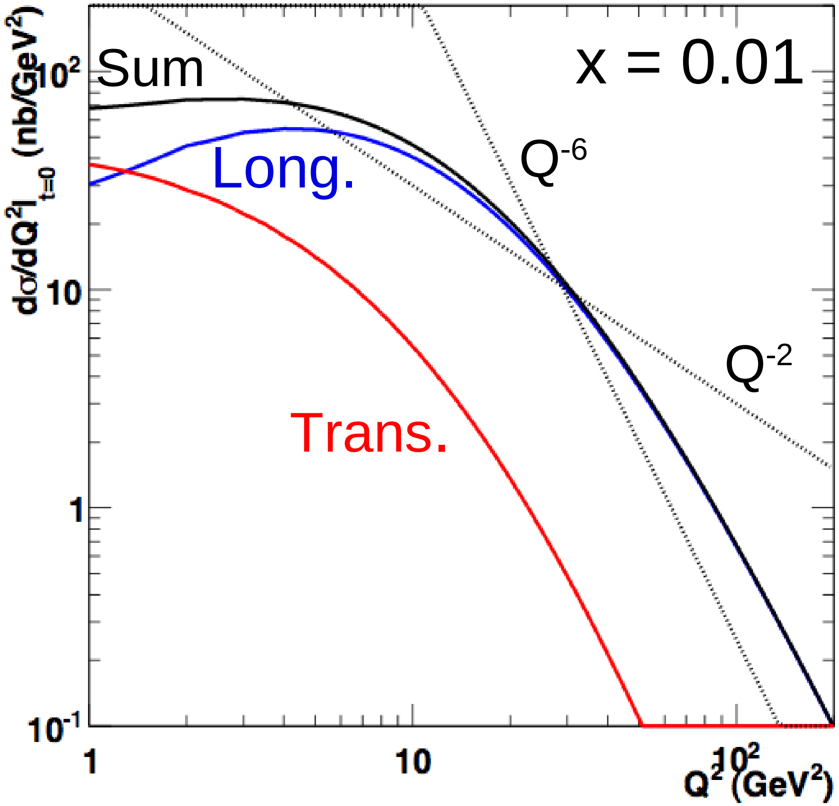
Suppression of the Dipole XSection



e+A: J/ψ production, b-Sat model

(coherent only)





Conclusions/Questions

- Dipole model and pQCD look consistent except at very low Q^2 as claimed by the authors
- $\rho,\ \phi$ May be better for measuring saturation effects as J/ψ wavefunction suppresses the saturation part of the dipole xsection
- e+A: saturation part of dipole cross-section extends to larger r
- t Dependence challenging to measure in e+A, limiting observable parameters

- Can the effects of saturation be effectively measured using diffractive VM production at eRHIC?
- What are the difficulties of obtaining the dipole amplitude from production? ρ, ϕ

Extra Slides

Theory (III): Understanding the formula

$$\left.\frac{d\sigma_L^{\gamma^* N \to VN}}{dt}\right|_{t=0} = \frac{12\pi^3 \Gamma_V m_V \eta_V^2 T(Q^2)}{\alpha_{em} N_c^2} \cdot \frac{\alpha_s^2(Q)\left|\left[1 + i\frac{\pi}{2}\frac{d}{d\ln x}\right] xG(x,Q^2)\right|^2}{Q^6}$$

η_V: effective inverse momentum of the vector meson distribution amplitude that controls the *leading twist* contribution to the lepto-production amplitude.

$$\eta_V \equiv \frac{1}{2} \frac{\int dz d^2k_T [z(1-z)]^{-1} \Phi_V(z,k_T)}{\int dz d^2k_T \Phi_V(z,k_T)}$$

$\Phi_V(z)$: wave function of longitudinal polarized vector meson

Roughly: Describes the distribution amplitudes of the longitudinal momentum fraction z of the quark in the meson.

Light mesons (ρ,ϕ): $\Phi_V(z) \sim 6\, z(1-z)$

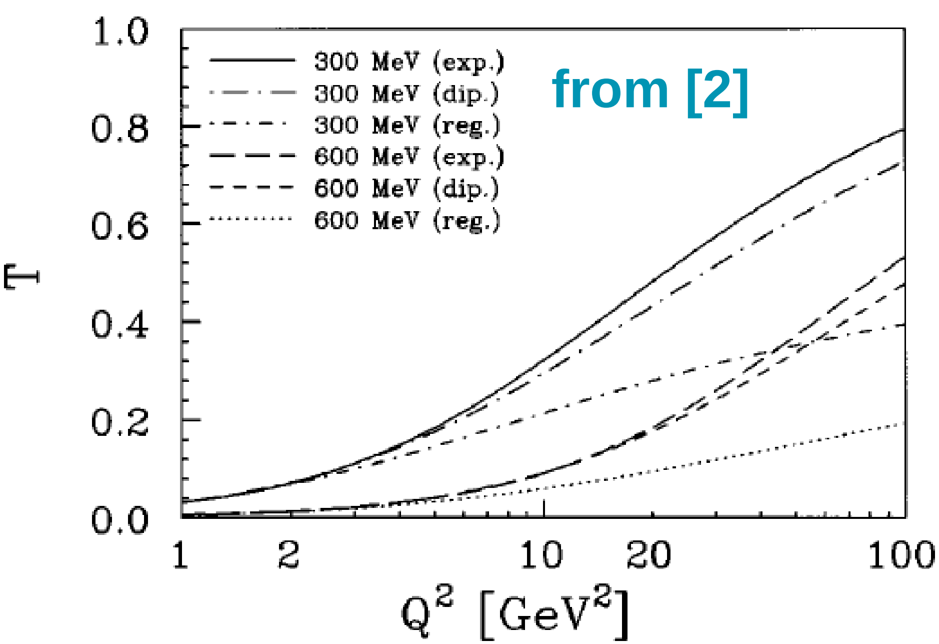
Heavy mesons (J/ψ, Υ): $\Phi_V(z) \sim \delta(z-1/2)$ (non-rel. picture)

Typical values used: $\eta_\rho \approx 2 - 5$ $\eta_{J/\psi} \approx 2$ (model dep.)

Theory (V): Understanding the formula

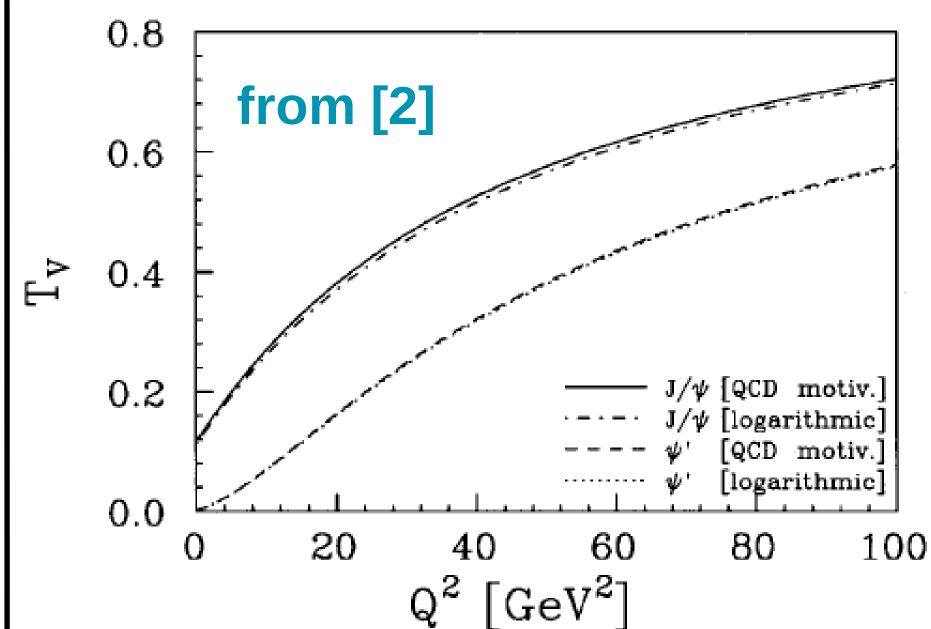
Light quark vector mesons

$$T(Q^2) = \left(\frac{\int_0^1 dz \int_0^{Q^2} d^2k_t \psi_V(z,k_t)(-\frac{1}{4}\Delta_t)\left[\frac{Q^4}{Q^2+\frac{k_t^2+m^2}{z(1-z)}}\right]}{\int_0^1 \frac{dz}{z(1-z)} \int_0^{Q^2} d^2k_t \psi_V(z,k_t)} \right)^2 .$$



Heavy quark vector mesons

$$T_V(Q^2) = \left(\frac{\int d^3k\, \psi_V(k)\left(-\frac{1}{16}\Delta_t\right)\left[\frac{(Q^2+4m_c^2)^2}{Q^2+\frac{k_t^2+m_c^2}{z(1-z)}}\right]}{\int d^3k\, \psi_V(k)\, \frac{k^2+m_c^2}{k_t^2+m_c^2}} \right)^2$$



Theory (VIII): transversely polarized case

[3] L. Frankfurt et al., Phys.Rev.D57:512,1998, hep-ph/9702216

$$\left.\frac{d\sigma^{\gamma^* N\to VN}}{dt}\right|_{t=0} = \frac{12\pi^3\Gamma_V m_V^3}{\alpha_{em}} \cdot \frac{\alpha_s^2(Q)|[1+i\frac{\pi}{2}\frac{d}{d\ln x}]xG(x,Q^2)|^2}{(Q^2+4m^2)^4} \cdot \left(1+\epsilon\frac{Q^2}{m_V^2}\right)\mathcal{C}(Q^2)$$

where

$$\mathcal{C}(Q^2) = \left(\frac{\eta_V}{3}\right)^2 \left(\frac{Q^2+4m^2}{Q^2+4m_{run}^2}\right)^4 T(Q^2)\frac{R(Q^2)+\epsilon(Q^2/m_V^2)}{1+\epsilon(Q^2/m_V^2)}$$

ε=0 purely transverse polarized (real photons $Q^2 = 0$)
ε=1 equal mix

Experimental Side (I)

[4] ZEUS, Eur.Phys.J.C6:603,1999, hep-ex/9808020

[5] ZEUS, Nucl.Phys.B695:3,2004, hep-ex/0404008

[6] H1 Eur.Phys.J.C13:371,2000, hep-ex/9902019

Experiments measure ep (eA) cross-section not virtual photo-production cross-sections

theory

In Born approximation:

Measured

$$\frac{d^2\sigma^{ep}}{dydQ^2} = \Gamma_T(y,Q^2)(\sigma_T^{\gamma^*p} + \epsilon\sigma_L^{\gamma^*p})$$

Flux of *transverse* virtual photons

transverse and longitudinal virtual photoproduction cross-section

Recall: $Q^2 = sxy$

Experimental Side (II)

$$\frac{d^2\sigma^{ep}}{dy dQ^2} = \Gamma_T(y, Q^2)(\sigma_T^{\gamma * p} + \epsilon \sigma_L^{\gamma * p})$$

ε is the ratio of long. and transv. virtual photon flux

$$\epsilon = \frac{2(1-y)}{1+(1-y)^2}$$

typically 0.8 - 1

and the transverse photon flux is:

$$\Gamma_T = \frac{\alpha_{em}}{2\pi}\frac{1+(1-y)^2}{yQ^2}$$

together:

$$\frac{d^2\sigma^{ep}}{dy dQ^2} = \frac{\alpha_{em}}{\pi Q^2 y}[(1-y+\frac{y^2}{2})\sigma_T^{\gamma * p} + (1-y)\sigma_L^{\gamma * p}]$$

Experimental Side (III)

The virtual photon cross-section

$$\sigma^{\gamma^* p} \equiv \sigma_T^{\gamma^* p} + \epsilon \sigma_L^{\gamma^* p}$$

can be used to evaluate the total exclusive cross-section

$$\sigma_{tot}^{\gamma^* p} \equiv \sigma_T^{\gamma^* p} + \sigma_L^{\gamma^* p}$$

through:

$$\sigma_{tot}^{\gamma^* p} = \frac{1+R}{1+\epsilon R} \sigma^{\gamma^* p}$$

where

$$R = \frac{\sigma_L^{\gamma^* p}}{\sigma_T^{\gamma^* p}}$$

Experimental Side (IV)

In our case:

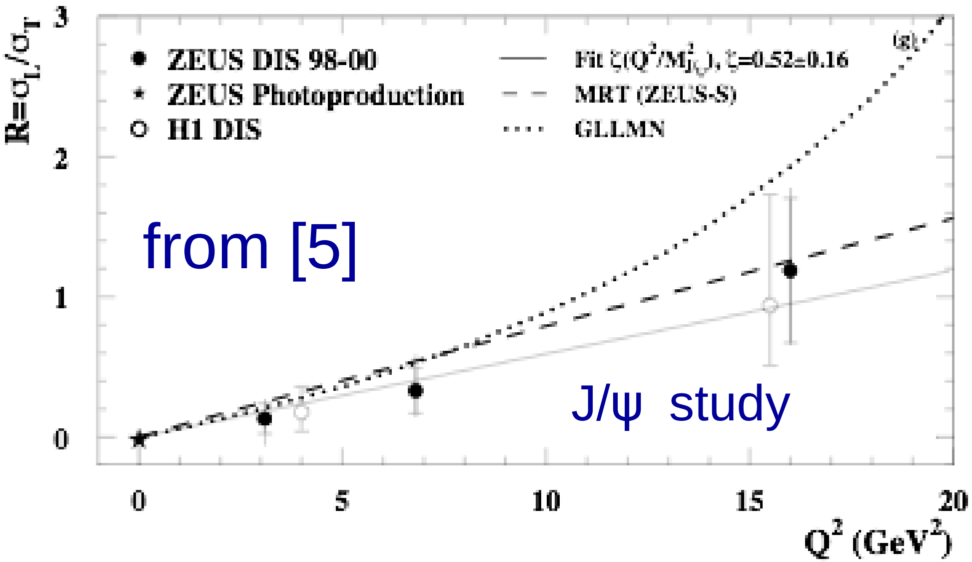
$$\sigma^{\gamma^* p \to pJ/\psi} \equiv \sigma_T^{\gamma^* p} + \epsilon \sigma_L^{\gamma^* p}$$

can be used to obtain:

$$\begin{aligned} \sigma_{tot}^{\gamma^* p \to pJ/\psi} &\equiv \sigma_T^{\gamma^* p \to pJ/\psi} + \sigma_L^{\gamma^* p \to pJ/\psi} \\ &= \frac{1+R}{1+\epsilon R} \sigma^{\gamma^* p \to pJ/\psi} \end{aligned}$$

Experimental Side (V)

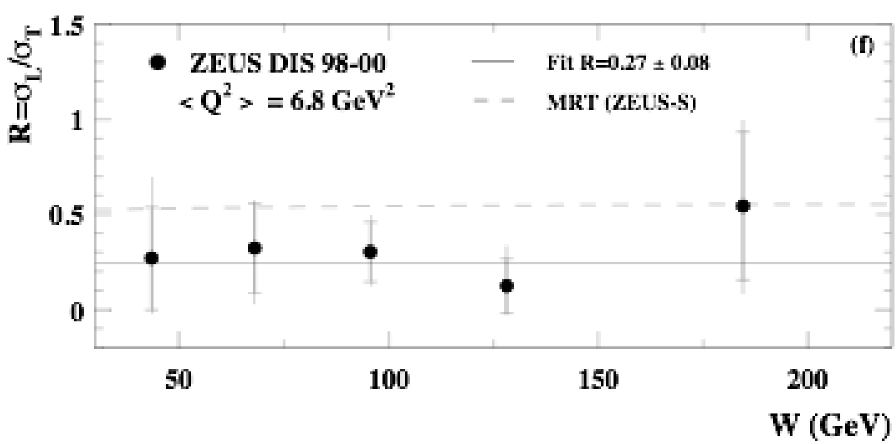
- Model predictions: e.g. $R = 0.5 \cdot (Q^2/M_{J/\psi})$
- Helicity structure of VM production can be used to get R

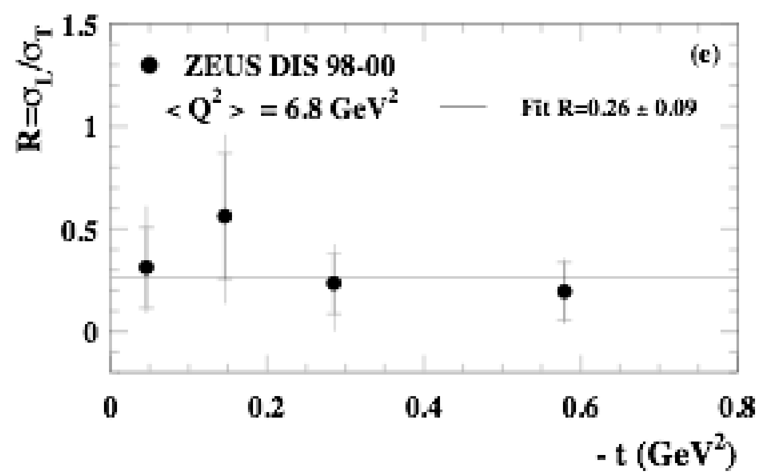


R from polar angle distributions:

$R = 0.52 \pm 0.16\ (Q^2/M_{J/\psi})$

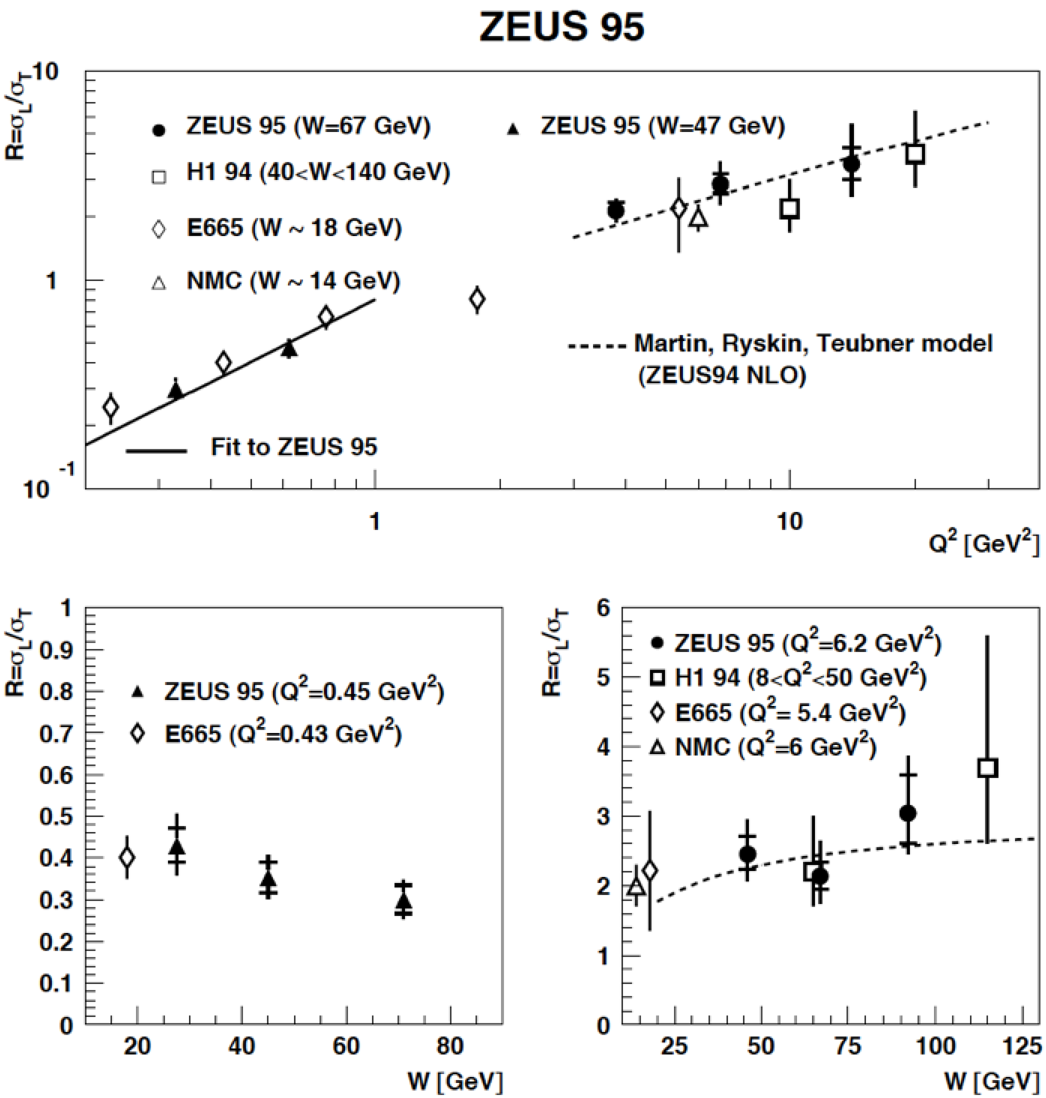
No W, t dependence?!

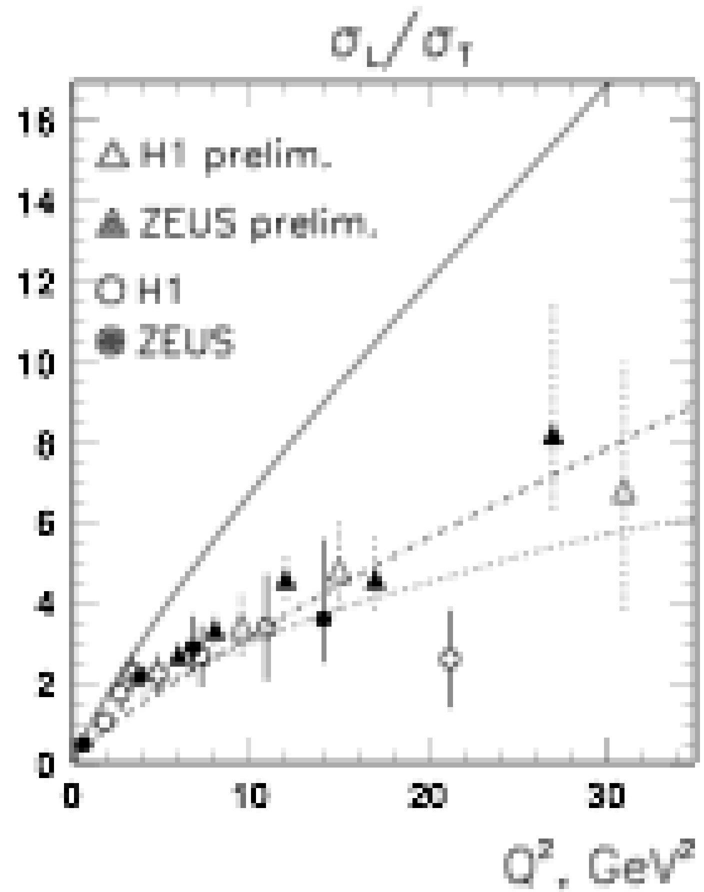




Experimental Side (VI)

Much bigger (and more uncertain) for ρ





Model prediction deviate big time

Comparing Theory with Experiment

In order to compare results with calculations and thus relate measured value with $G(x,Q^2)$ we need:

$$\left.\frac{d\sigma_L^{\gamma^* p}}{d|t|}\right|_{t=0} = \frac{R}{1+R} \cdot \frac{b}{1-e^{-b|t|_{max}}} \cdot \sigma_{tot}^{\gamma^* p}$$

since $\frac{d\sigma}{d|t|} \propto e^{-b|t|}$

In e+A at eRHIC we are not going to measure any *t*-dependence

So what is b? What is t_{max}?

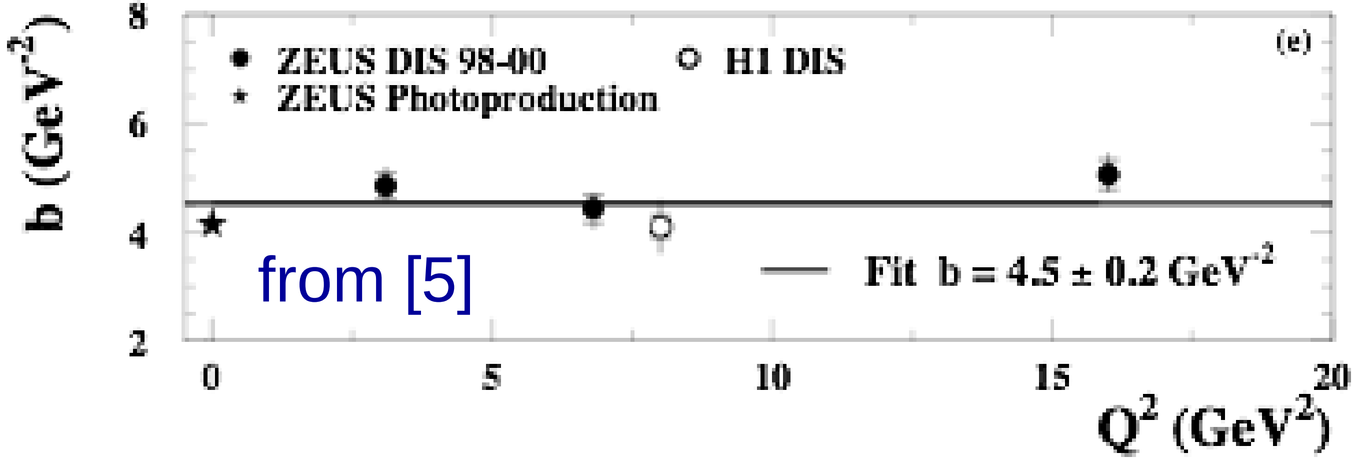
Guess t_{max} will be related to the point where incoherent sets in?

We can get an estimate from e+p - is that good enough?

More on b

J/ψ: no significant Q dependence

$b = 4.5 \pm 0.2\ GeV^{-2}$



from [5]

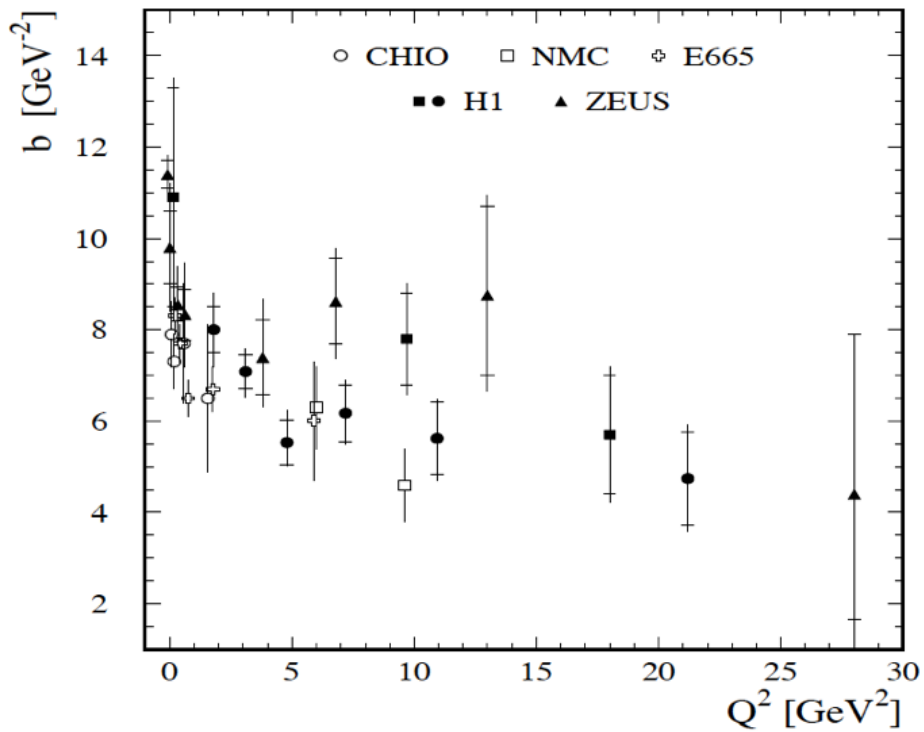
ρ: carful about what is said here:

while for the J/ψ photo and electroproduction give the same b this is not true for the ρ

At times authors are not careful in their statements

Even more on b for the ρ

from [6] elastic ρ productior



b = 2.5± 1.0 GeV^{-2}

diffractive component with proton dissociation and the non-resonant two-pion background. The elastic component is fitted with a free slope parameter b, whereas the contribution of diffractive ρ events with proton dissociation, which amounts to $11 \pm 5\%$ of the elastic signal, has a fixed slope parameter $b_{pd} = 2.5 \pm 1.0\ \mathrm{GeV}^{-2}$ (see section 3.2.2).[11] The non-resonant background,

Mail from Markus Diehl (translated)

As an upshot I should say right away that in my opinion the exclusive channels (vector mesons or DVCS)

are NOT usable to determine the conventional gluon density g(x). In the factorization approach one always have the Gluon-GPD. Only at normal gluon densities and in a rough approximation (leading log x) they become equivalent.

Beyond that there is no model independent connection between GPD(x,xi,t=0) and PDF(x).

The Durham group (Martin Ryskin Teubner) has promoted that over the years but had to retract some of their statements recently after critique by Dieter Mueller and myself.

From excl. J/psi production one can hopefully learn a lot about the gluon GPD, or, if saturation effects become important, about the dipole cross-sections. (The t-dependence of the Gluon-GPD is for example best determined by the J/psi data since theory works better than for the rho. To promote these channels as “golden channel” for determening the normal gluon density is misleading.

Q: Why is J/psi not suitable to measure saturation?

A: the reason for this is not the J/psi but more precise the wave function of a heavy quark in the photon:

there distances r >> 1/m are exponentially suppressed(*). One is dominated by small

Mail from Raju

I don't understand Markus' critique as stated if it is based on the overlap of wavefunctions. The overlap K_0 is first of all only for longitudinal polarization-for K_1 even smaller dipole

sizes are enhanced. (This is even more true for F_2 by the way.) But the enhancement of small dipole sizes by kinematics is what we want (K_1 is good!) because then if there is a large cross-section

for small dipoles, we know it is saturation not confinement. Perhaps Markus' point is the opposite-which would then make sense-namely, K_0 is too sensitive to LARGE transverse sizes, so if we

see a large effect, it could instead be due to confinement not saturation. But the easy way to resolve the two would be the x dependence of the result. One would predict a very different x dependence from the

other.

Dipole Model (I)

[7] Kowalski, Motyka, Watt, hep-ph/0606272

$$\frac{d\sigma^{\gamma^* p\to pV}_{T,L}}{dt} = \frac{1}{16\pi}\left|\int dr(2\pi r)\int_0^1 \frac{dz}{4\pi}\int db(2\pi b)(\Psi_V^*\Psi)_{T,L} J_0(b\Delta)J_0([1-z]r\Delta)\ \frac{d\sigma_{q\bar{q}}}{d^2\vec{b}}\right|^2$$

after angular integrations

Overlaps between photon and VM wave fct.:

$$\begin{aligned}
(\Psi_V^*\Psi)_T &= \hat{e}_f e\,\frac{N_c}{\pi z(1-z)}\,\left\{m_f^2 K_0(\epsilon r)\phi_T(r,z) - \left[z^2+(1-z)^2\right]\epsilon K_1(\epsilon r)\partial_r\phi_T(r,z)\right\},\\
(\Psi_V^*\Psi)_L &= \hat{e}_f e\,\frac{N_c}{\pi}\,2Qz(1-z)\,K_0(\epsilon r)\,\left[M_V\phi_L(r,z)+\delta\,\frac{m_f^2-\nabla_r^2}{M_V z(1-z)}\phi_L(r,z)\right]
\end{aligned}$$

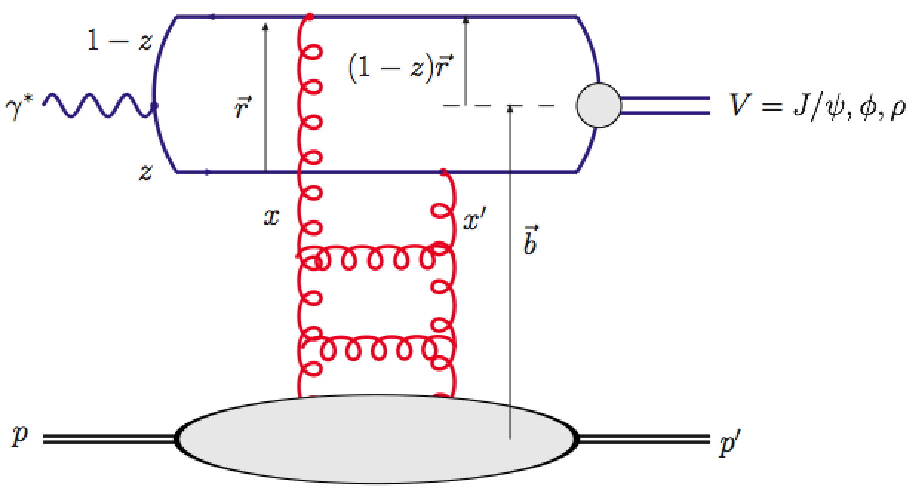
where

$$\begin{aligned}
\phi_T(r,z) &= N_T[z(1-z)]^2\exp(-r^2/2R_T^2),\\
\phi_L(r,z) &= N_L z(1-z)\exp(-r^2/2R_L^2).
\end{aligned}$$

Dipole Model (II)

What is what:

- r: dipole radius
- z: fraction of photon's light-cone momentum carried by quark
- b: impact parameter
- Δ: transverse momentum lost by outgoing proton
- J_0: mod. Bessel 1st kind
- K_0: mod. Bessel 2nd kind
- $N_c = 3$



$$\begin{aligned} \epsilon^2 &\equiv z(1-z)Q^2 + m_f^2 \\ e &= \sqrt{4\pi\alpha_{\rm em}} \\ \hat{e}_f &= 2/3,\ 1/3,\ \text{or}\ 1/\sqrt{2},\ \text{for}\ J/\psi,\ \phi,\ \text{or}\ \rho \end{aligned}$$

Dipole Model (III)

- N_T, N_L, R_T, and R_L: parameters of Gaus-LC VM wave fct.

Meson	M_V/GeV	f_V	m_f/GeV	N_T	R_T^2/GeV^{-2}	N_L	R_L^2/GeV^{-2}
J/ψ	3.097	0.274	1.4	1.23	6.5	0.83	3.0
ϕ	1.019	0.076	0.14	4.75	16.0	1.41	9.7
ρ	0.776	0.156	0.14	4.47	21.9	1.79	10.4

- or of "boosted Gaussian"

Meson	M_V/GeV	f_V	m_f/GeV	$\mathcal{N}_T$	$\mathcal{N}_L$	$\mathcal{R}^2/\text{GeV}^{-2}$	$f_{V,T}$
J/ψ	3.097	0.274	1.4	0.578	0.575	2.3	0.307
ϕ	1.019	0.076	0.14	0.919	0.825	11.2	0.075
ρ	0.776	0.156	0.14	0.911	0.853	12.9	0.182

Dipole Model (III)

Dipole cross-section (Glauber-Mueller)

$$\frac{d\sigma_{q\bar{q}}}{d^2\vec{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_S(\mu^2) x g(x, \mu^2) T(b)\right)\right].$$

For simplicity (flat gluon distribution):

$$T(b) = \frac{1}{\pi b_S^2} \Theta\left(b_S - b\right),$$