

# Flow in small systems: a few recent results

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## Утро в сосновом лесу





# A very brief history of recent heavy ion physics

1980s and 1990s—AGS and SPS... QGP at SPS!

Early 2000s—QGP at RHIC! No QGP at SPS. d+Au as control.

Mid-late 2000s—Detailed, quantitative studies of strongly coupled QGP. d+Au as control.

2010—Ridge in high multiplicity p+p (LHC)! Probably CGC!

Early 2010s—QGP in p+Pb!

Early 2010s—QGP in d+Au!

Mid 2010s and now-ish—QGP in high multiplicity p+p? QGP in mid-multiplicity p+p??

QGP in d+Au even at low energies???

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QGP in d+Au even at low energies???

“Twenty years ago, the challenge in heavy ion physics was to find the QGP. Now, the challenge is to not find it.” —Jürgen Schukraft, QM17

Particle production in small systems

- Final state effects are observed
- Photon modification consistent with QGP formation

Small systems geometry scan

- Observation that correlations are geometrical in origin
- Data well-reproduced by hydro

Small systems energy scan

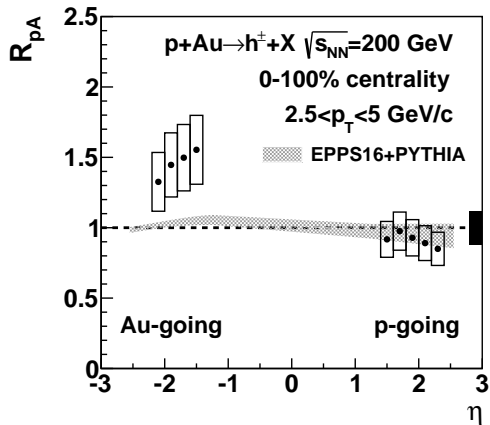
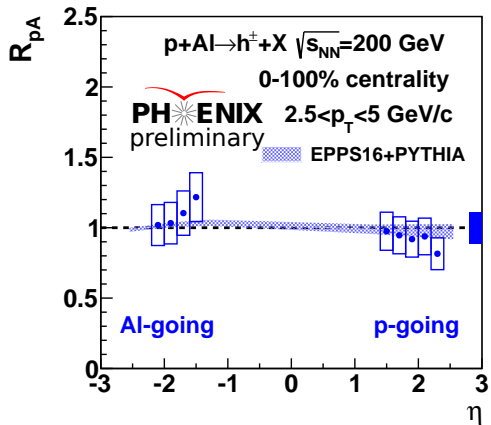
- Similar correlations for all energies
- Non-trivial fluctuations

Alternate viewpoints

- The CGC strikes back?

Particle production in small systems

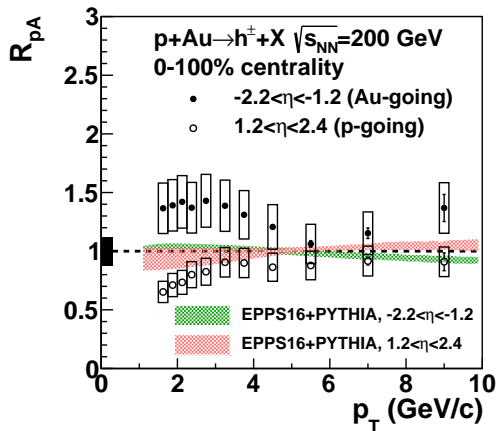
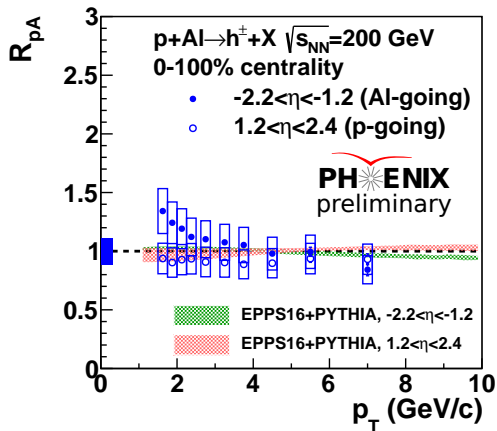
# Small systems nuclear modification



Forward modification consistent with nPDF effects (EPPS16)

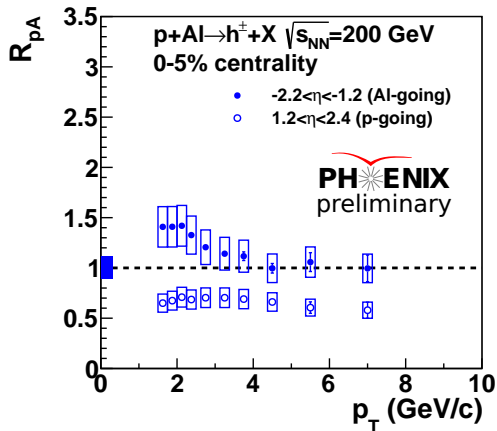


# Small systems nuclear modification

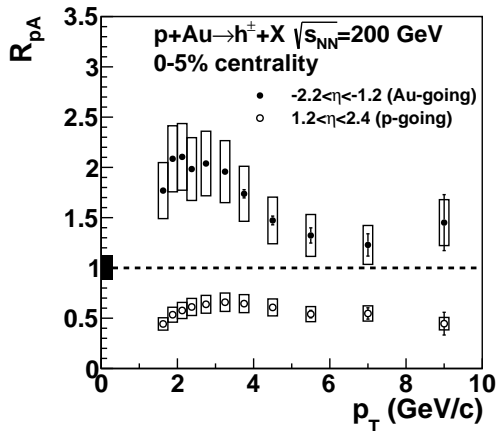


High- $p_T$  modification consistent with nPDF effects (EPPS16)

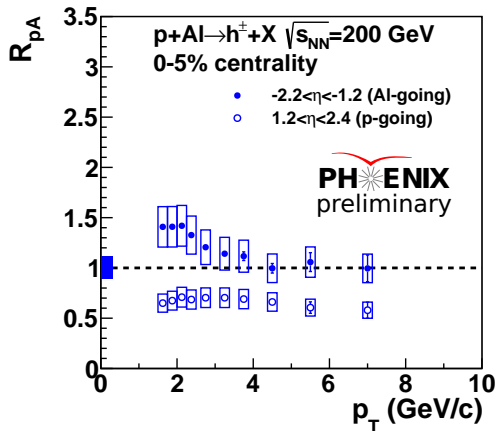
# Small systems nuclear modification



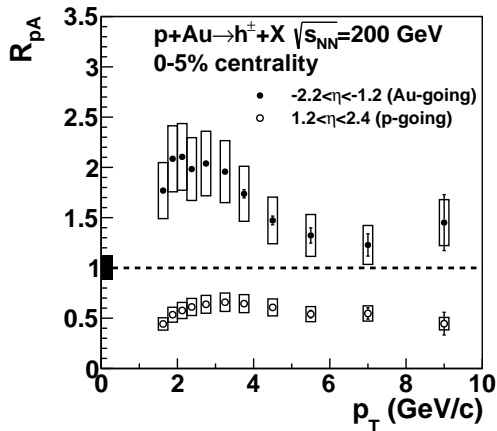
Stronger effects in central collisions



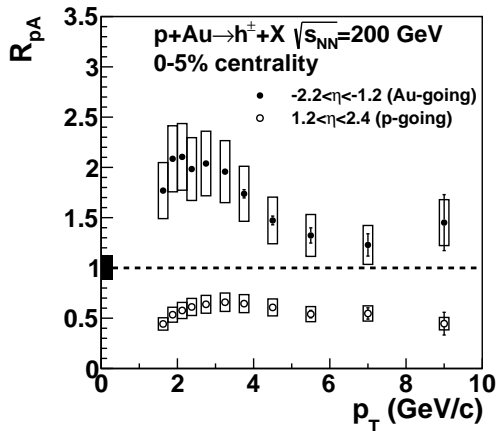
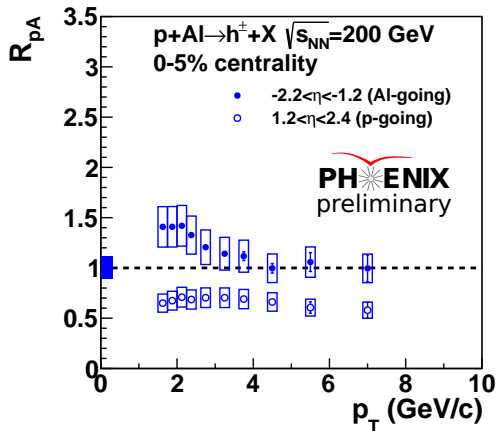
# Small systems nuclear modification



Strong enhancement for backward at intermediate  $p_T$ —why?



# Small systems nuclear modification

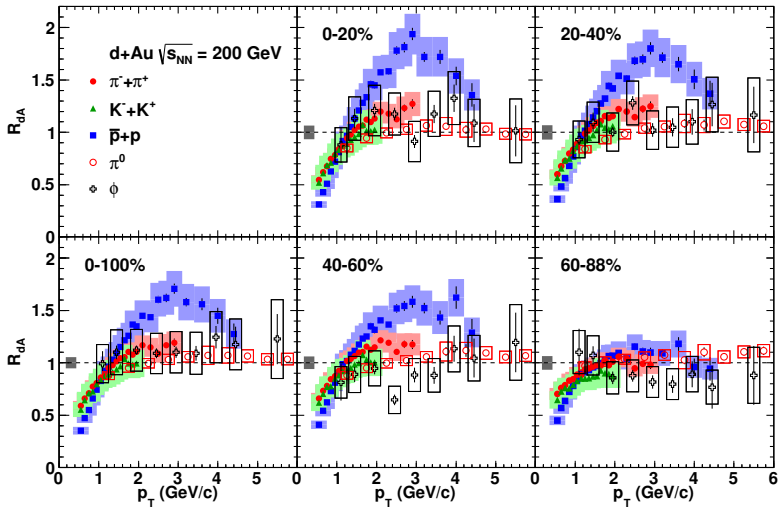


Strong enhancement for backward at intermediate  $p_T$ —why?

Don't forget: particle species dependence of Cronin! There must be final state effect(s)...

# Particle species dependence of “Cronin enhancement”

PHENIX, Phys. Rev. C 88, 024906 (2013)



$\pi^+, \pi^-, \pi^0$ ,

$K^+, K^-$ ,

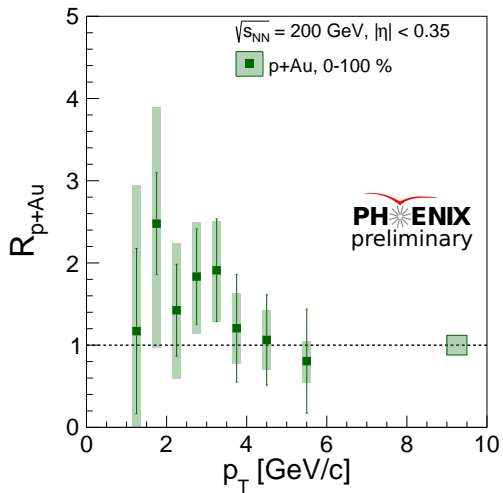
$p, \bar{p}$ ,

$\phi$

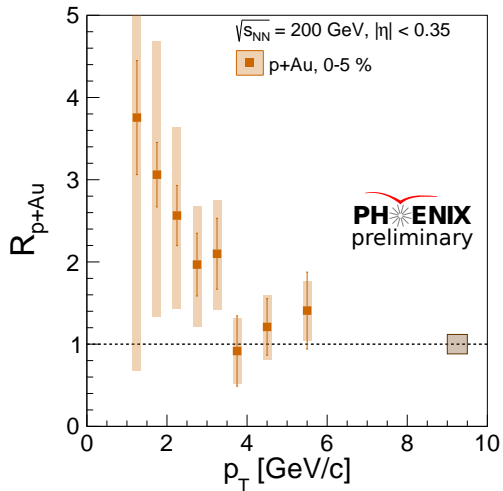
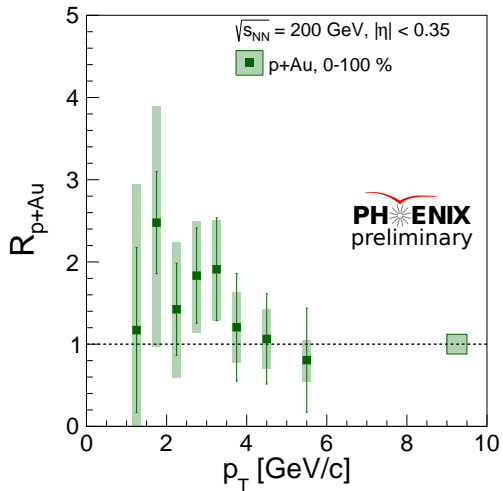
Protons much more strongly  
modified than pions

$\phi$  mesons confusing as always...

# Photons in small systems

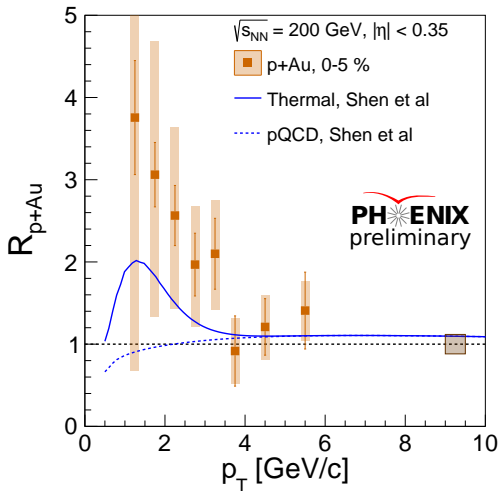
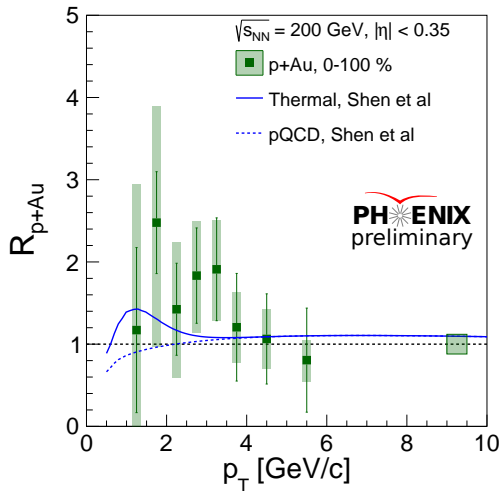


# Photons in small systems



Thermal photons in  $p+Au$ ?

# Photons in small systems



Thermal photons in  $p+Au$ ? Theory from Phys. Rev. C 95, 014906 (2017)



# Brief summary: particle production in small systems

Strong modifications at forward & backward rapidities

- Not nPDF effects alone
- Additional initial state effects possible (e.g. the usual multiple scattering)

Nuclear modification strongly dependent on particle species

- Must be final state effect(s)
- Hadronization, radial flow, etc...

# Intermission

Small systems geometry scan

# Testing hydro by controlling system geometry

arXiv:1805.02973, submitted to Nature Physics

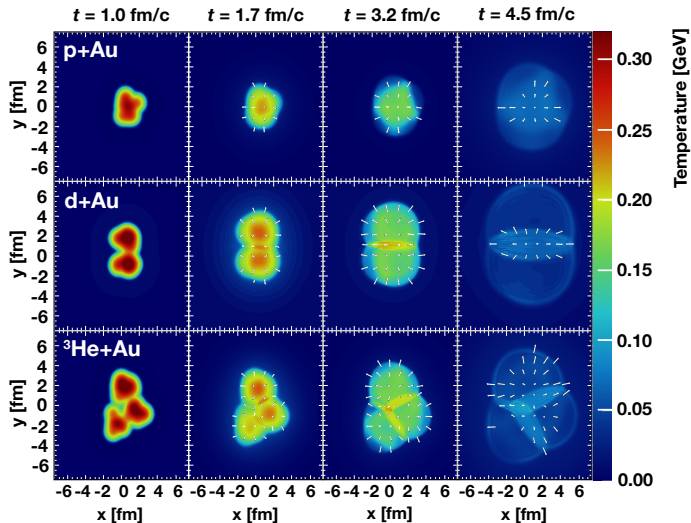
Hydrodynamics translates  
initial geometry into final  
state

Test hydro hypothesis by  
varying initial state

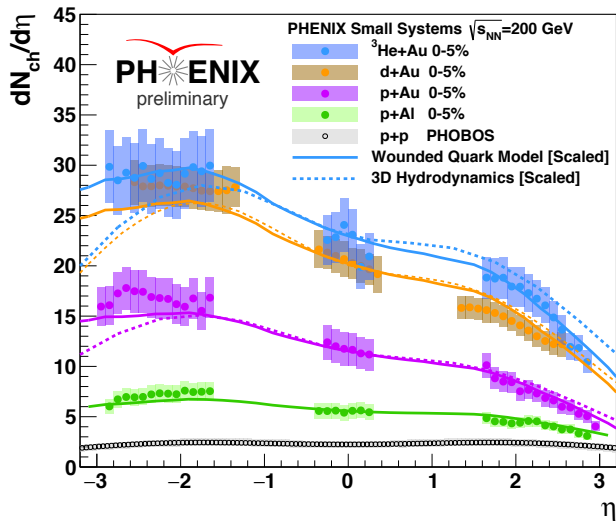
	$\varepsilon_2$	$\varepsilon_3$
$p+Au$	0.24	0.16
$d+Au$	0.57	0.17
$^3He+Au$	0.48	0.23

$$\varepsilon_2^{p+Au} < \varepsilon_2^{d+Au} \approx \varepsilon_2^{^3He+Au}$$

$$\varepsilon_3^{p+Au} \approx \varepsilon_3^{d+Au} < \varepsilon_3^{^3He+Au}$$



# Longitudinal dynamics in small systems

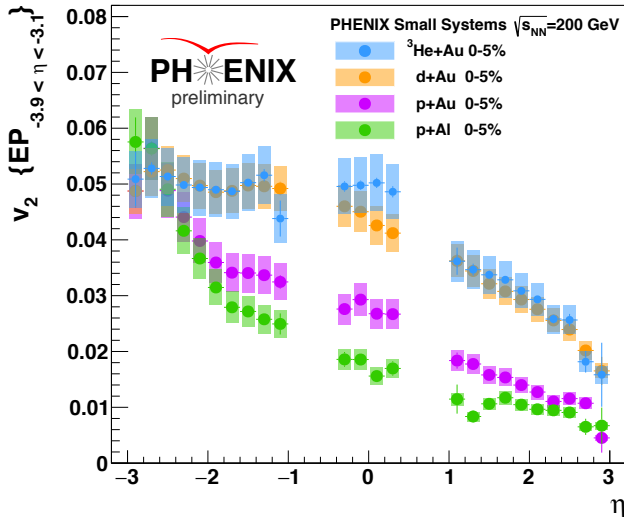


$p+\text{Al}$ ,  $p+\text{Au}$ ,  $d+\text{Au}$ ,  $^3\text{He}+\text{Au}$

Good agreement with wounded quark model

Good agreement with 3D hydro

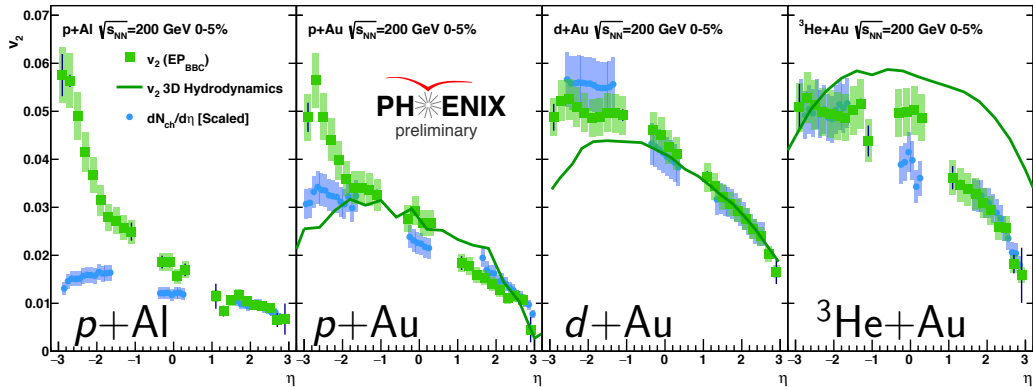
# Longitudinal dynamics in small systems



$p+\text{Al}$ ,  $p+\text{Au}$ ,  $d+\text{Au}$ ,  $^3\text{He}+\text{Au}$

Also have  $v_2$  vs  $\eta$

# Longitudinal dynamics in small systems

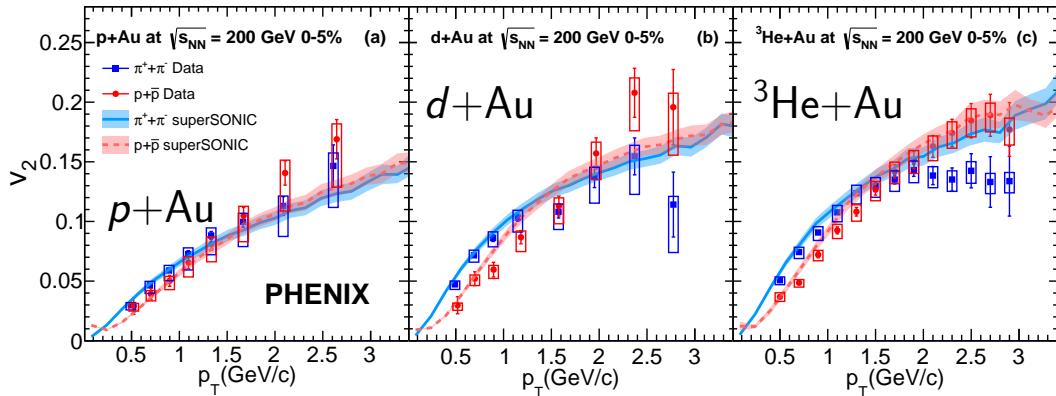


Good agreement with 3D hydro for  $p+Au$  and  $d+Au$

Apparent scaling between  $v_2$  and  $dN_{ch}/d\eta$ —coincidence?

# Small systems geometry scan

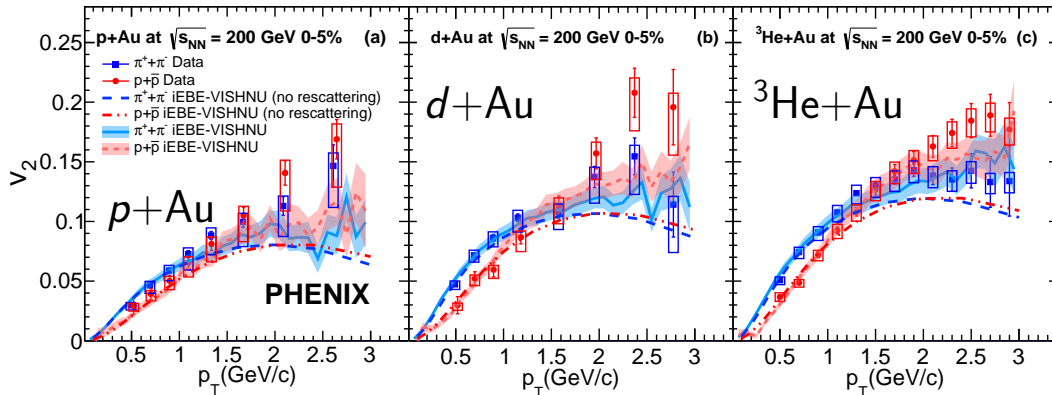
arXiv:1710.09736, accepted by Phys. Rev. C



Identified particle  $v_2$  vs  $p_T$  in  $p+Au$ ,  $d+Au$ , and  $^3He+Au$   
—Mass ordering well-described by hydro

# Small systems geometry scan

arXiv:1710.09736, accepted by Phys. Rev. C

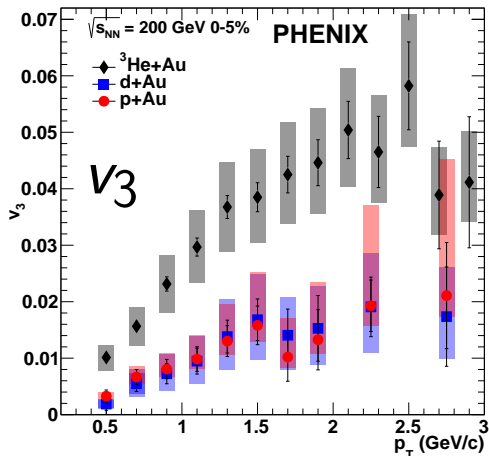
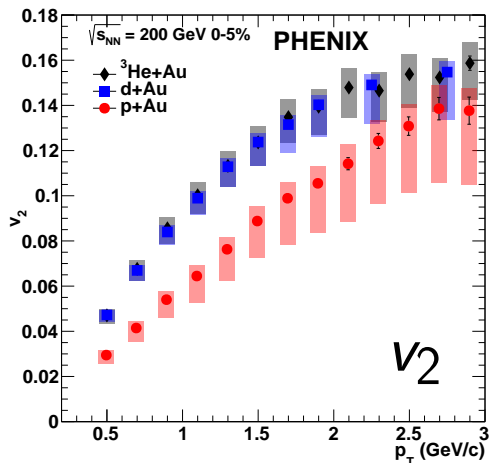


Identified particle  $v_2$  vs  $p_T$  in  $p+Au$ ,  $d+Au$ , and  $^3\text{He}+Au$   
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# Testing hydro by controlling system geometry

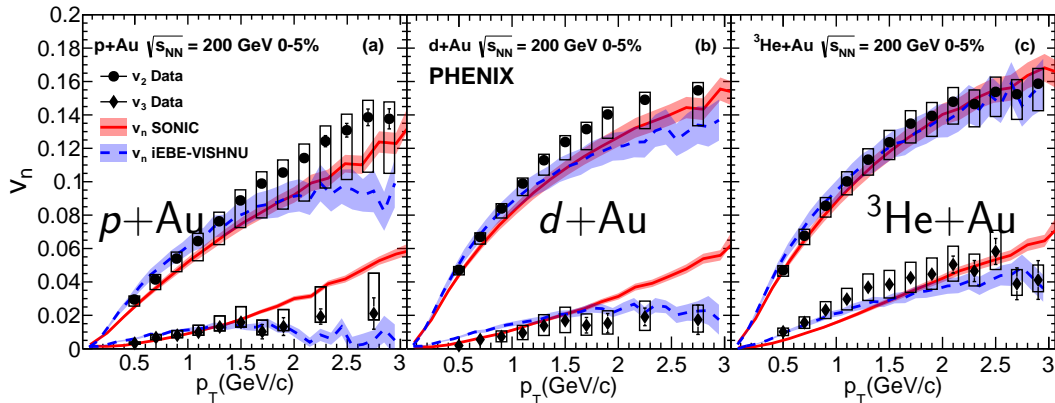
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$v_2$  and  $v_3$  ordering matches  $\varepsilon_2$  and  $\varepsilon_3$  ordering in all three systems  
—Regardless of mechanism, the correlation is geometrical

# Testing hydro by controlling system geometry

arXiv:1805.02973, submitted to Nature Physics



$v_2$  and  $v_3$  vs  $p_T$  described very well by hydro in all three systems  
—Strongly suggests QGP droplets in hydro evolution

# Brief summary: particle production in small systems

Comprehensive set of measurements for longitudinal dynamics

$v_2$  and  $v_3$  match  $\varepsilon_2$  and  $\varepsilon_3$  ordering in  $p+\text{Au}$ ,  $d+\text{Au}$ ,  $^3\text{He}+\text{Au}$

—Correlation is definitively geometrical in origin

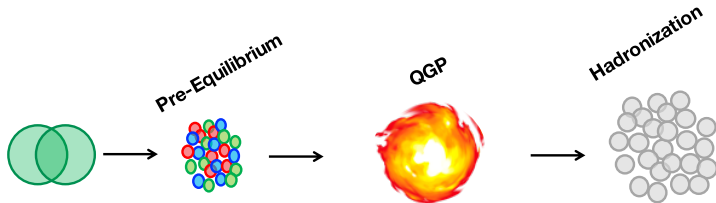
$v_2$  and  $v_3$  in  $p+\text{Au}$ ,  $d+\text{Au}$ ,  $^3\text{He}+\text{Au}$  are well-described by hydro theory

—Strongest evidence to date for QGP formation in small systems

# Intermission

Small systems beam energy scan

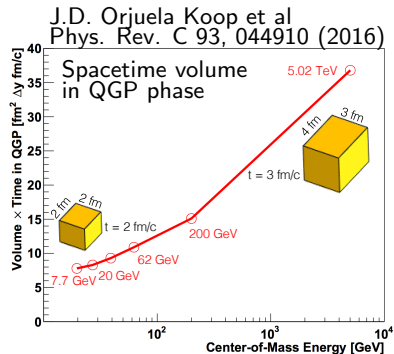
# Testing hydro by controlling system size and life time



Standard picture for A+A:  
QGP in hydro evolution

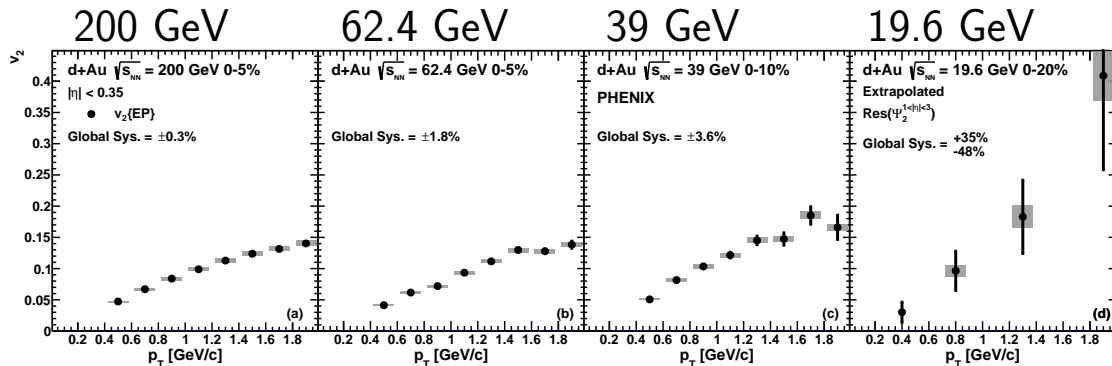
What about small systems?  
And lower energies?

Use collisions species and  
energy to control system  
size, test limits of hydro  
applicability



# $d+Au$ beam energy scan

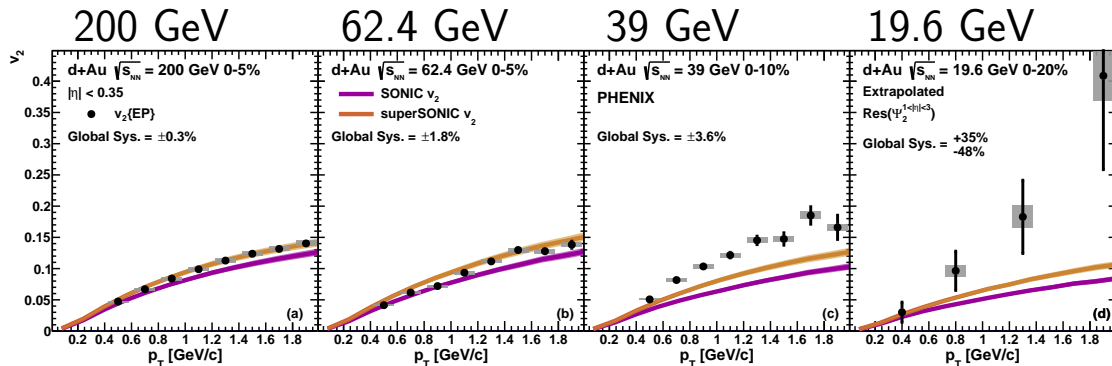
Phys. Rev. C 96, 064905 (2017)



Event plane  $v_2$  vs  $p_T$  measured for all energies

# $d+Au$ beam energy scan

Phys. Rev. C 96, 064905 (2017)

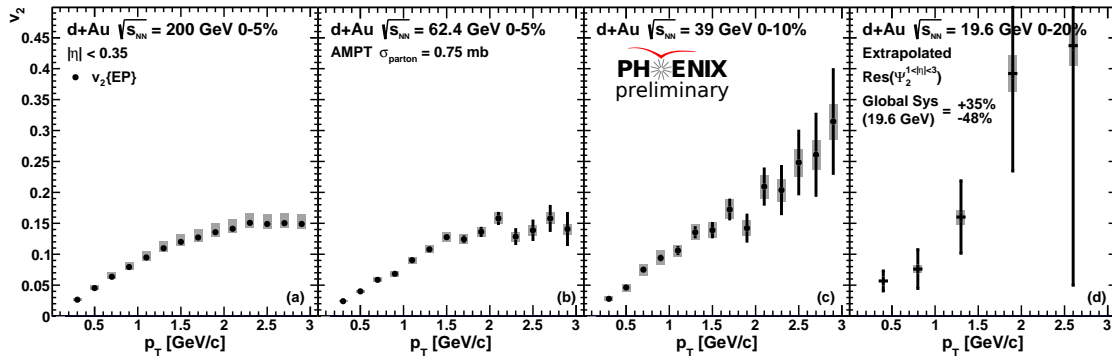


Event plane  $v_2$  vs  $p_T$  measured for all energies

Hydro theory agrees with higher energies very well,  
underpredicts lower energies—nonflow?

# $v_2$ vs $p_T$ , comparisons to AMPT

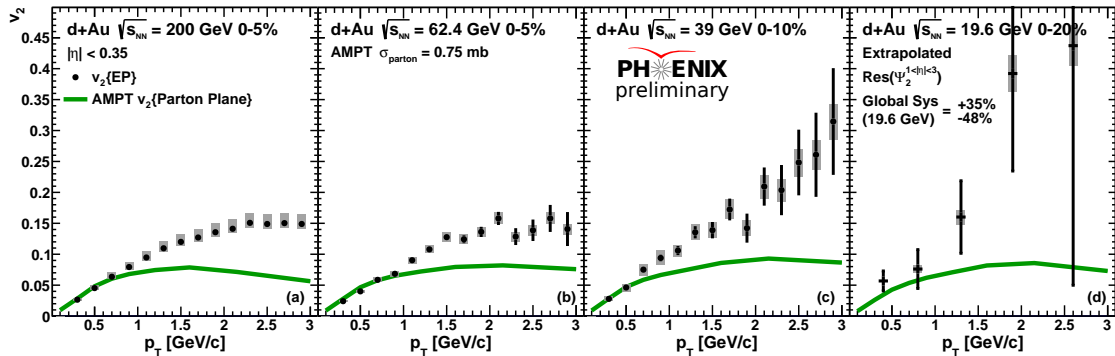
Phys. Rev. C 96, 064905 (2017)





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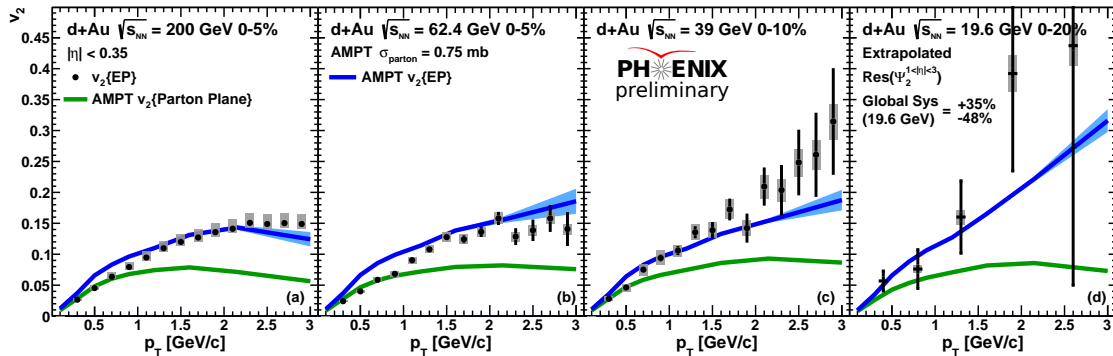
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AMPT flow only shows good agreement at low  $p_T$  and all energies

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Phys. Rev. C 96, 064905 (2017)

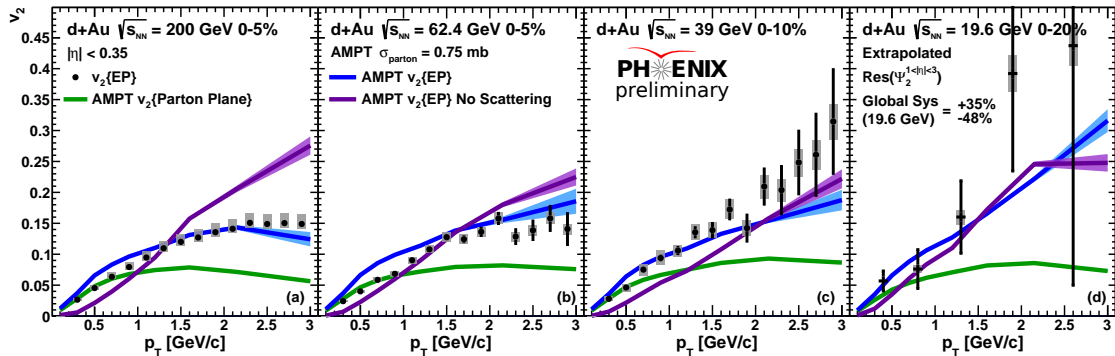


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AMPT flow+non-flow shows reasonable agreement for all  $p_T$  and all energies

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Phys. Rev. C 96, 064905 (2017)



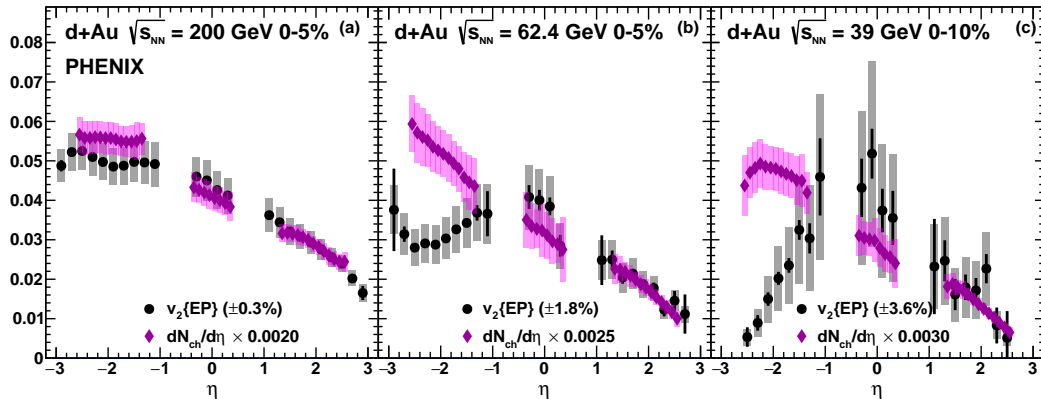
AMPT flow only shows good agreement at low  $p_T$  and all energies

AMPT flow+non-flow shows reasonable agreement for all  $p_T$  and all energies

AMPT non-flow only far under-predicts for low  $p_T$ , too high for high  $p_T$

# $v_2$ and $dN_{\text{ch}}/d\eta$ vs $\eta$

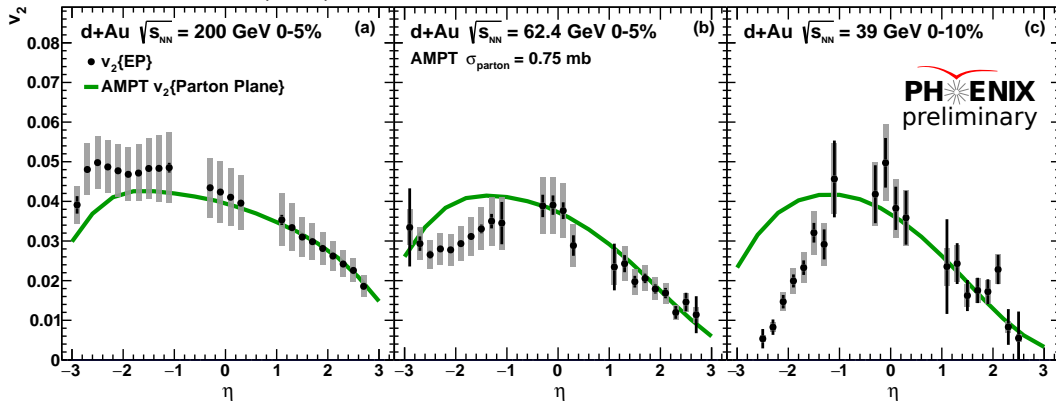
Phys. Rev. C 96, 064905 (2017)



BBC south ( $-3.9 < \eta < -3.1$ ) used to estimate the event plane  
200 GeV shows strong forward/backward asymmetry in  $v_2$  and  $dN_{\text{ch}}/d\eta$   
Asymmetry is large for  $dN_{\text{ch}}/d\eta$  at all energies, but not for  $v_2$

# $v_2$ vs $\eta$ , comparison with AMPT

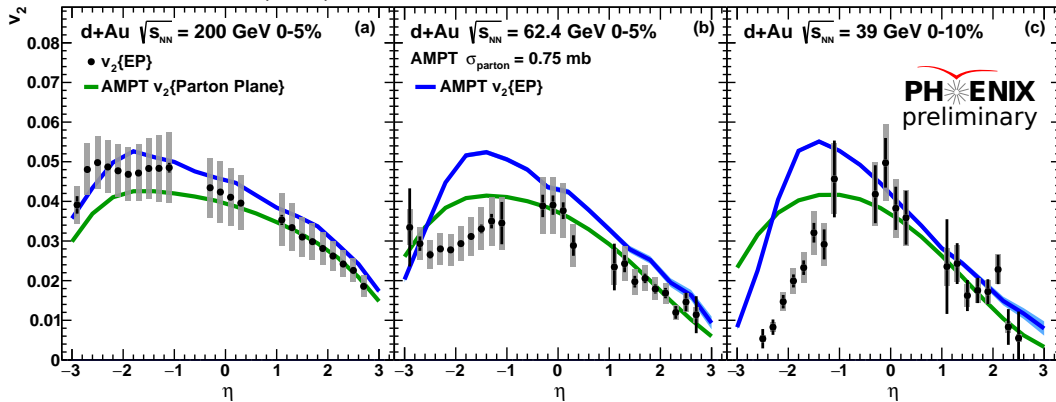
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AMPT flow only agrees with mid and forward rapidity very well, misses backward rapidity

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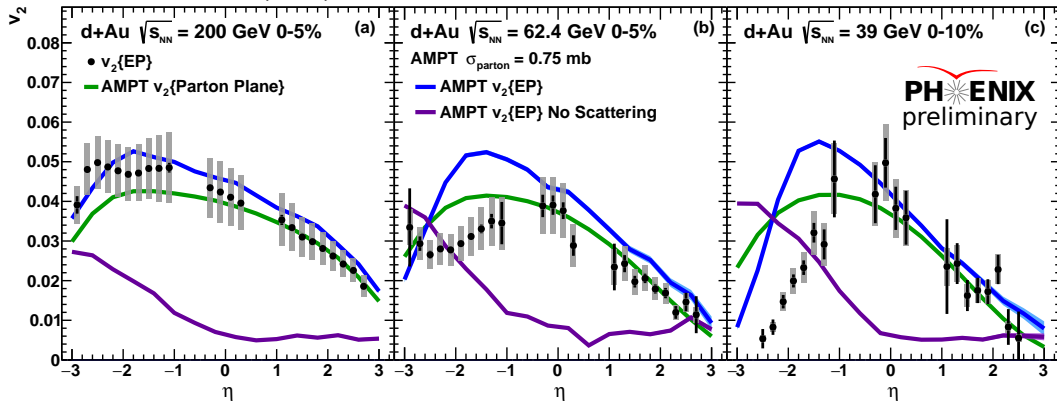
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- AMPT flow only agrees with mid and forward rapidity very well, misses backward rapidity
- AMPT flow+non-flow is very similar at mid and forward
- AMPT flow+non-flow shows striking anti-correlation at backward rapidity

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Phys. Rev. C 96, 064905 (2017)



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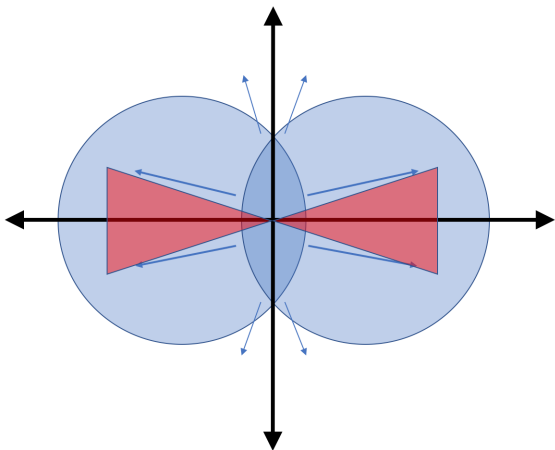
AMPT flow+non-flow is very similar at mid and forward

AMPT flow+non-flow shows striking anti-correlation at backward rapidity

AMPT non-flow only shows nothing at mid and forward, large  $v_2$  at backward rapidity near the

# Multi-particle correlations

flow and nonflow



$$v_n = \langle \cos(n(\phi_{\text{some particle}} - \psi_n)) \rangle$$

$$v_n^2 = \langle \cos(n(\phi_{\text{some particle}} - \phi_{\text{some other particle}})) \rangle$$

How to deal with “fake flow”?

—Kinematics

—Combinatorics

$$v_n^2 = \langle \cos(n(\phi_a - \phi_b)) \rangle$$

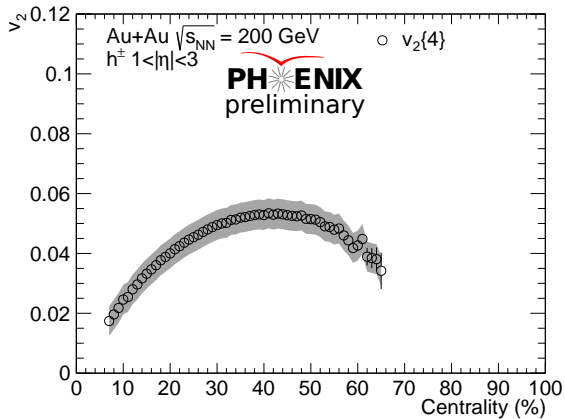
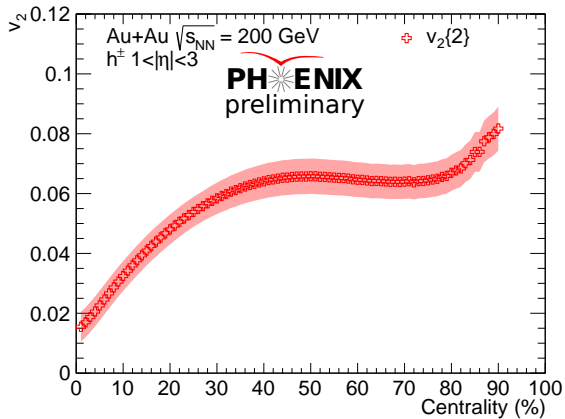
$$v_n^4 = \langle \cos(n(\phi_a + \phi_b - \phi_c - \phi_d)) \rangle$$

$$v_n^6 = \langle \cos(n(\phi_a + \phi_b + \phi_c - \phi_d - \phi_e - \phi_f)) \rangle$$

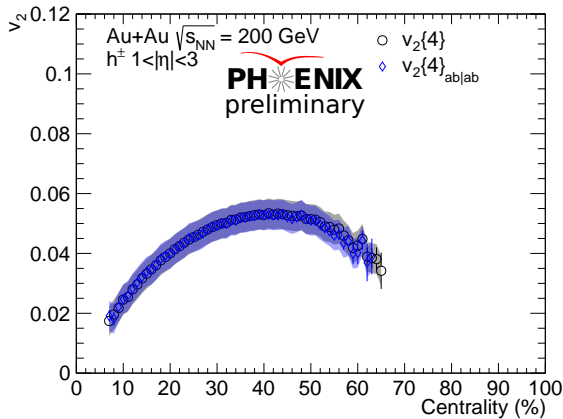
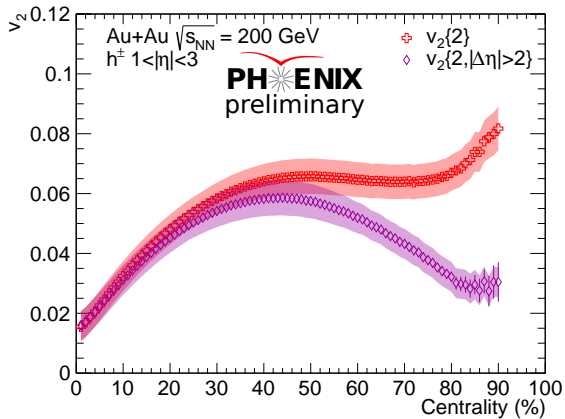
$$v_n^8 = \dots$$



# Nonflow approaches in AuAu

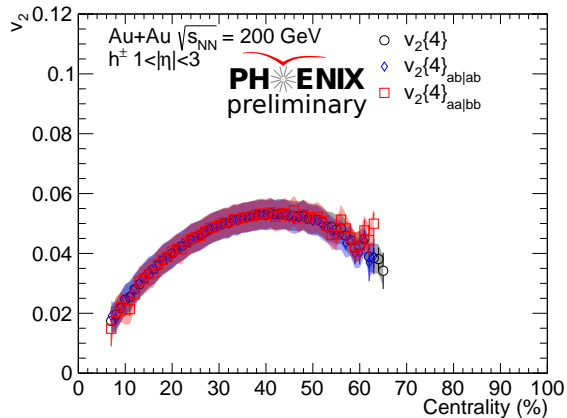
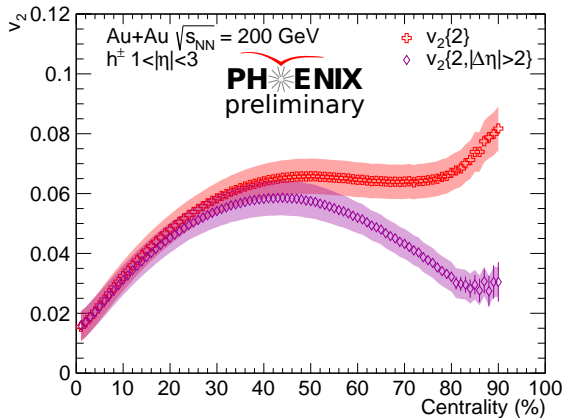


# Nonflow approaches in AuAu



- Large pseudorapidity separation
- Big difference for 2-particle (good)
- No difference for 4-particle (good)

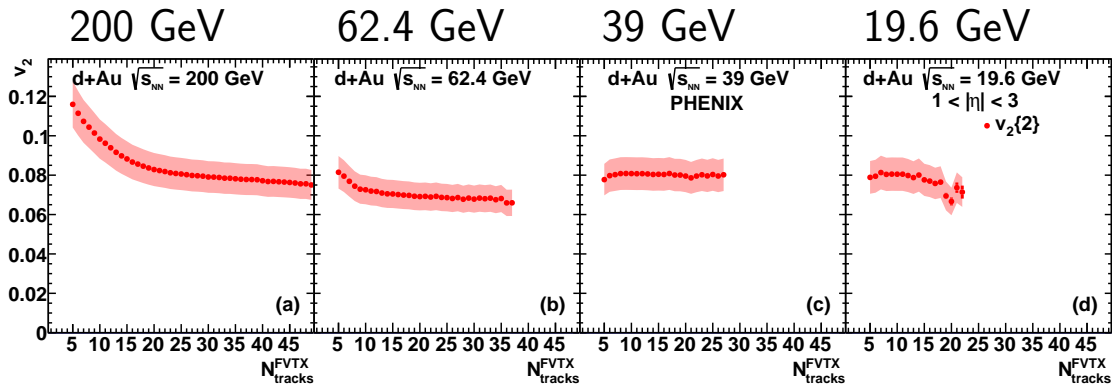
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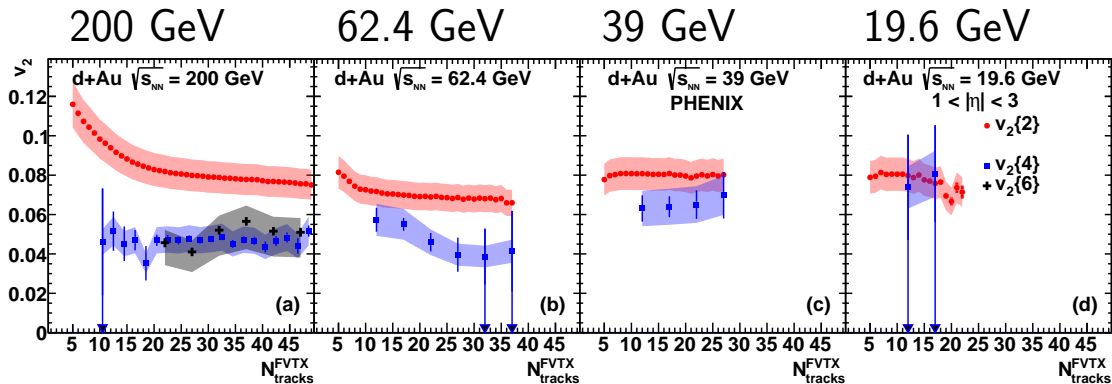
# $d+Au$ beam energy scan

Phys. Rev. Lett. 120, 062302 (2018)



# $d+Au$ beam energy scan

Phys. Rev. Lett. 120, 062302 (2018)



Measurement of  $v_2\{6\}$  in  $d+Au$  at 200 GeV and  $v_2\{4\}$  in  $d+Au$  at all energies

# $d$ +Au beam energy scan

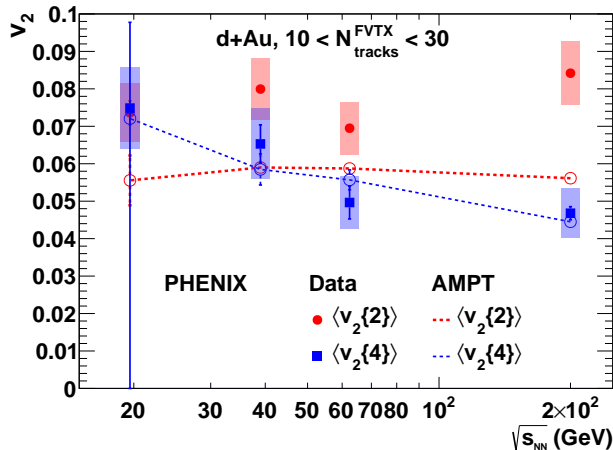
Phys. Rev. Lett. 120, 062302 (2018)

Select  $10 < N_{\text{tracks}}^{\text{FVTX}} < 30$ ,  
integrate

AMPT sees similar trend

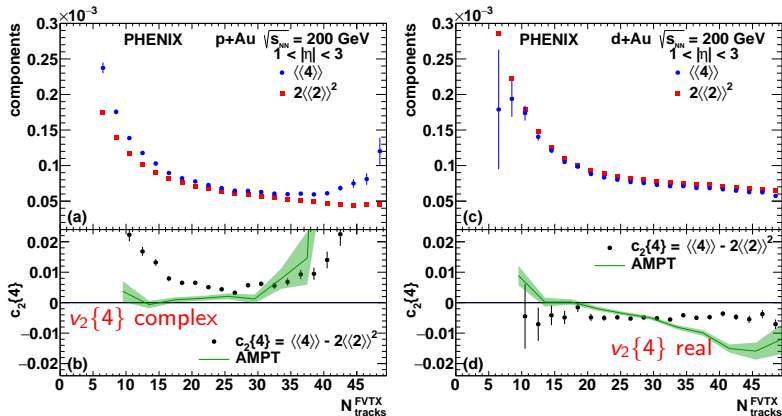
Fluctuations?

Not Bessel-Gaussian  
Not small-variance limit  
Need to understand  
fluctuations better



# Components and cumulants in p+Au and d+Au at 200 GeV

Phys. Rev. Lett. 120, 062302 (2018)



Is the sign of  $c_2\{4\}$  a good indicator of collectivity? No.  
Fluctuations could dominate in the p+Au...

## Back to basics (a brief excursion)

The (raw) moments of a probability distribution function  $f(x)$ :

$$\mu_n = \langle x^n \rangle \equiv \int_{-\infty}^{+\infty} x^n f(x) dx$$

The moment generating function:

$$M_x(t) \equiv \langle e^{tx} \rangle = \int_{-\infty}^{+\infty} e^{tx} f(x) dx = \int_{-\infty}^{+\infty} \sum_{n=0}^{\infty} \frac{t^n}{n!} x^n f(x) dx = \sum_{n=0}^{\infty} \mu_n \frac{t^n}{n!}$$

Moments from the generating function:

$$\mu_n = \left. \frac{d^n M_x(t)}{dt^n} \right|_{t=0}$$

Key point: the moment generating function uniquely describe  $f(x)$



## Back to basics (a brief excursion)

Can also uniquely describe  $f(x)$  with the cumulant generating function:

$$K_x(t) \equiv \ln M_x(t) = \sum_{n=0}^{\infty} \kappa_n \frac{t^n}{n!}$$

Cumulants from the generating function:

$$\kappa_n = \left. \frac{d^n K_x(t)}{dt^n} \right|_{t=0}$$

Since  $K_x(t) = \ln M_x(t)$ ,  $M_x(t) = \exp(K_x(t))$ , so

$$\mu_n = \left. \frac{d^n \exp(K_x(t))}{dt^n} \right|_{t=0}, \quad \kappa_n = \left. \frac{d^n \ln M_x(t)}{dt^n} \right|_{t=0}$$

End result: (details left as an exercise for the interested reader)

$$\begin{aligned} \mu_n &= \sum_{k=1}^n B_{n,k}(\kappa_1, \dots, \kappa_{n-k+1}) &&= B_n(\kappa_1, \dots, \kappa_{n-k+1}) \\ \kappa_n &= \sum_{k=1}^n (-1)^{k-1} (k-1)! B_{n,k}(\mu_1, \dots, \mu_{n-k+1}) &&= L_n(\kappa_1, \dots, \kappa_{n-k+1}) \end{aligned}$$

## Back to basics (a brief excursion)

Evaluating the Bell polynomials gives

$$\langle x \rangle = \kappa_1$$

$$\langle x^2 \rangle = \kappa_2 + \kappa_1^2$$

$$\langle x^3 \rangle = \kappa_3 + 3\kappa_1\kappa_2 + \kappa_1^3$$

$$\langle x^4 \rangle = \kappa_4 + 4\kappa_1\kappa_3 + 3\kappa_2^2 + 6\kappa_1^2\kappa_2 + \kappa_1^4$$

One can tell by inspection (or derive explicitly) that  $\kappa_1$  is the mean,  $\kappa_2$  is the variance, etc.

## Back to basics (a brief excursion)

Subbing in  $x = v_n$ ,  $\kappa_2 = \sigma^2$ , we find

$$\begin{aligned} \left( \langle v_n^4 \rangle &= v_n^4 + 6v_n^2\sigma^2 + 3\sigma^4 + 4v_n\kappa_3 + \kappa_4 \right) \\ - \left( 2\langle v_n^2 \rangle^2 &= 2v_n^4 + 4v_n^2\sigma^2 + 2\sigma^4 \right) \\ &\rightarrow \\ \langle v_n^4 \rangle - 2\langle v_n^2 \rangle^2 &= -v_n^4 + 2v_n^2\sigma^2 + \sigma^4 + 4v_n\kappa_3 + \kappa_4 \end{aligned}$$

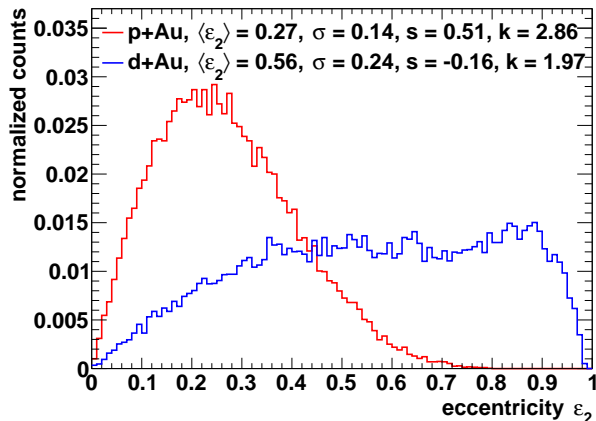
Skewness  $s$ :  $\kappa_3 = s\sigma^3$

Kurtosis  $k$ :  $\kappa_4 = (k - 3)\sigma^4$

$$\begin{aligned} v_n\{2\} &= (v_n^2 + \sigma^2)^{1/2} \\ v_n\{4\} &= (v_n^4 - 2v_n^2\sigma^2 - 4v_ns\sigma^3 - (k - 2)\sigma^4)^{1/4} \end{aligned}$$

So the fully general form is a bit more complicated than we tend to think...

# Eccentricity distributions and cumulants



$$\varepsilon_2\{4\} = (\varepsilon_2^4 - 2\varepsilon_2^2\sigma^2 - 4\varepsilon_2s\sigma^3 - (k-2)\sigma^4)^{1/4}$$

	p+Au	d+Au
$\varepsilon_2^4$	0.00531	0.0983
$2\varepsilon_2^2\sigma^2$	0.00277	0.0370
$4\varepsilon_2s\sigma^3$	0.00147	-0.0053
$(k-2)\sigma^4$	0.00031	-0.0001

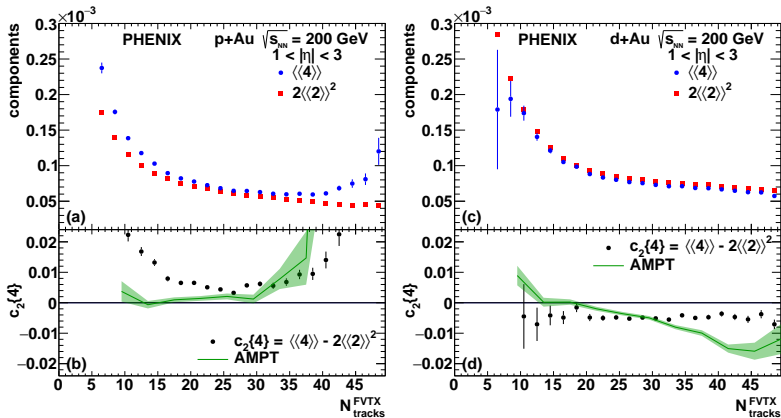
the variance brings  $\varepsilon_2\{4\}$  down (this term gives the usual  $\sqrt{v_2^2 - \sigma^2}$ )

positive skew brings  $\varepsilon_2\{4\}$  further down, negative skew brings it back up

kurtosis  $> 2$  brings  $\varepsilon_2\{4\}$  further down, kurtosis  $< 2$  brings it back up

—recall Gaussian has kurtosis = 3

# Eccentricity distributions and cumulants



$$v_2\{4\} = (v_2^4 - 2v_2^2\sigma^2 - 4v_2s\sigma^3 - (k-2)\sigma^4)^{1/4}$$

Eccentricity fluctuations alone go a long way towards explaining this  
 Additional fluctuations in the (imperfect) translation of  $\varepsilon_2$  to  $v_2$ ?

## Brief summary: small systems beam energy scan

Measurement of  $v_2$  vs  $p_T$  for  $d+Au$  at 200, 62.4, 39, and 19.6 GeV

—Hydro describes higher two energies well, misses lower two energies

—AMPT describes all data well with mix of flow and nonflow

Measurement of  $v_2$  vs  $\eta$  for  $d+Au$  at 200, 62.4, and 39 GeV

—Hydro theory at lower energies would be very useful

—Interesting anticorrelation between flow and nonflow at backward rapidity

Measurement of  $v_2\{6\}$  at 200 GeV and  $v_2\{4\}$  at all four energies

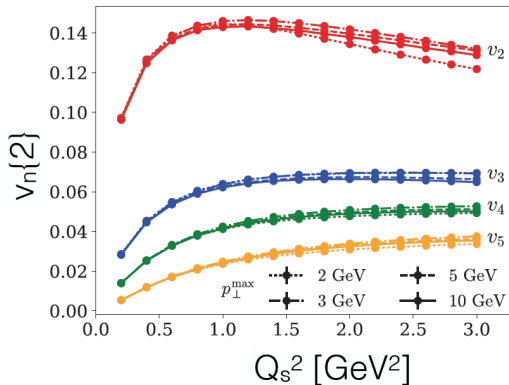
—Nonflow should be combinatorially suppressed

—Highly non-trivial fluctuations

The CGC strikes back?

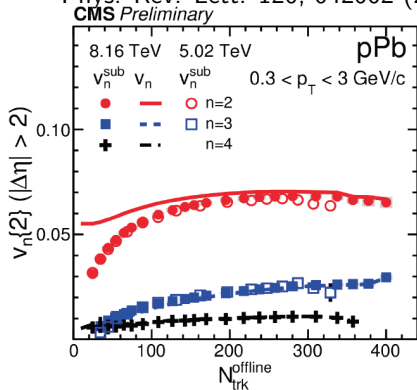
# CGC results on small systems

Mark Mace, QM18



Dusling, MM, Venugopalan PRL 120 (2018)

Phys. Rev. Lett. 120, 042002 (2018)



CMS-PAS-HIN-16-022

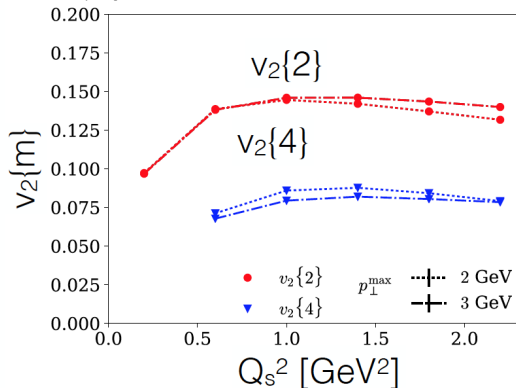
Can qualitatively reproduce harmonic ordering

Off from data by a factor of 2 to 3



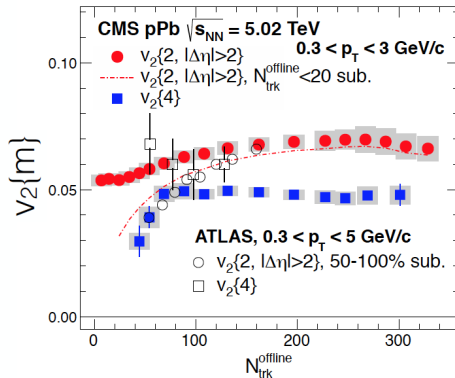
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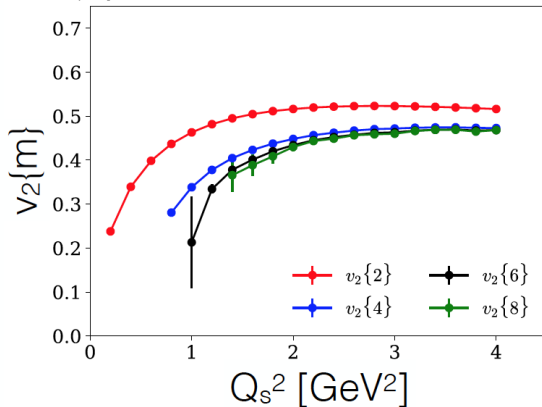
CMS PLB 724 (2013) 213

Can reproduce  $v_2\{2\}$  and  $v_2\{4\}$

Disagreement with data by a factor of 2, but qualitative features match

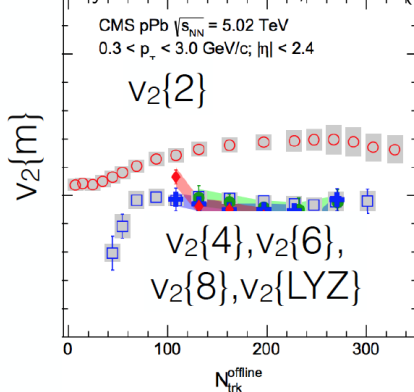
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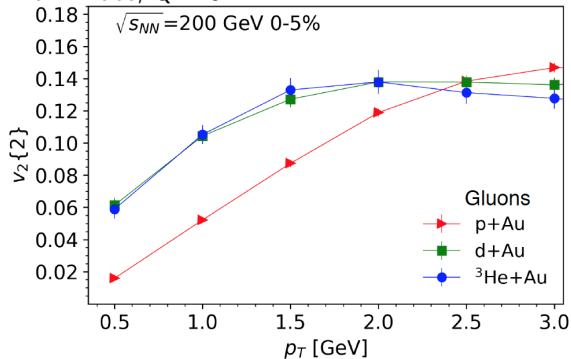
CMS PRL 115 (2015) 012301

Abelian calculations can produce  $v_2\{2\}$ ,  $v_2\{4\}$ ,  $v_2\{6\}$ ,  $v_2\{8\}$

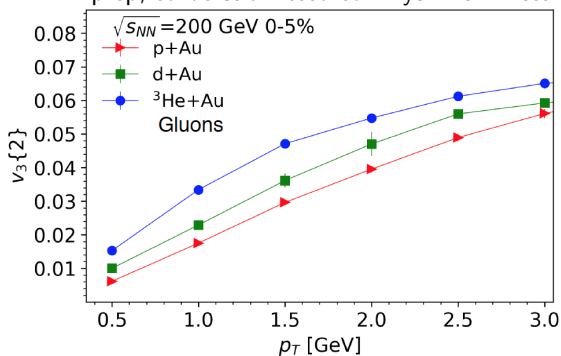
Disagreement with data by factor of 5, but qualitative features match

# CGC results on small systems

Mark Mace, QM18



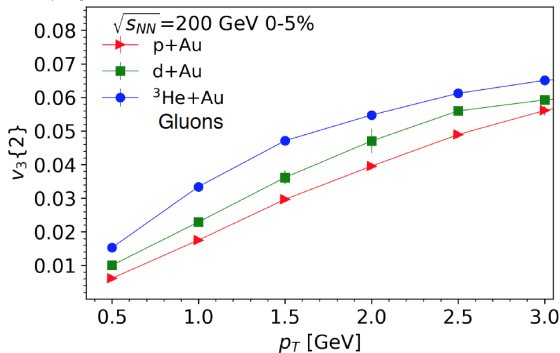
in prep, to be submitted to Phys. Rev. Lett.



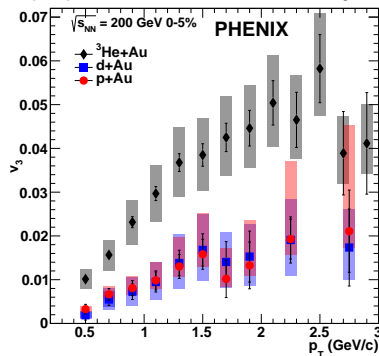
New for QM18:  $v_2$  and  $v_3$  for small systems

# CGC results on small systems

Mark Mace, QM18



in prep, to be submitted to Phys. Rev. Lett.



New for QM18:  $v_2$  and  $v_3$  for small systems

$v_3$  ordering is not quite right

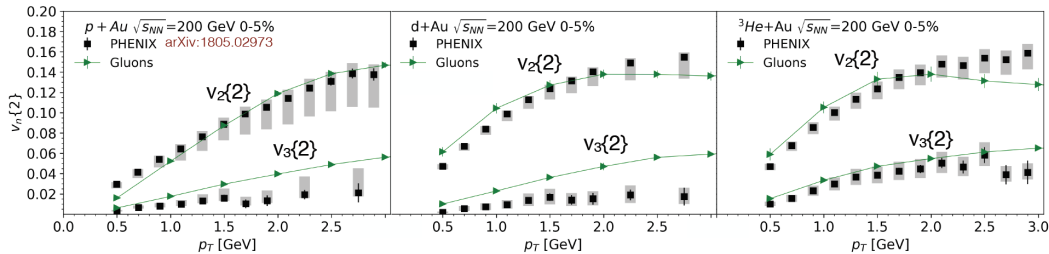
—CGC:  $p+\text{Au} < d+\text{Au} < ^3\text{He}+\text{Au}$

—Data:  $p+\text{Au} \approx d+\text{Au} < ^3\text{He}+\text{Au}$

# CGC results on small systems

Mark Mace, QM18

in prep, to be submitted to Phys. Rev. Lett.



Description of data now *quantitative* rather than *qualitative*

Hopefully coming soon: which set of refinements made this possible?

## Brief summary: the CGC strikes back?

CGC calculations now in some quantitative with RHIC data, but that means the LHC data needs to be revisited

Unclear which further theoretical refinements may be possible (not long ago it was assumed that odd harmonics were all zero)

$v_n$  and multiplicity distribution may provide additional discriminating power

# Final thoughts

Initial and final state effects are clear in the data  
—*Which* final state effects is perhaps not so clear

Low- $p_T$  photon enhancement observed in  $p+Au$   
—Consistent with EM radiation from QGP  
—Other explanations possible?

Wealth of data from small systems beam energy and geometry scans  
—Higher energies described by hydro, all energies described by AMPT  
—All geometries described by hydro, also somewhat described by CGC

# Final thoughts

Initial and final state effects are clear in the data  
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—All geometries described by hydro, also somewhat described by CGC

“The optimist regards the future as uncertain.”—Eugene Wigner

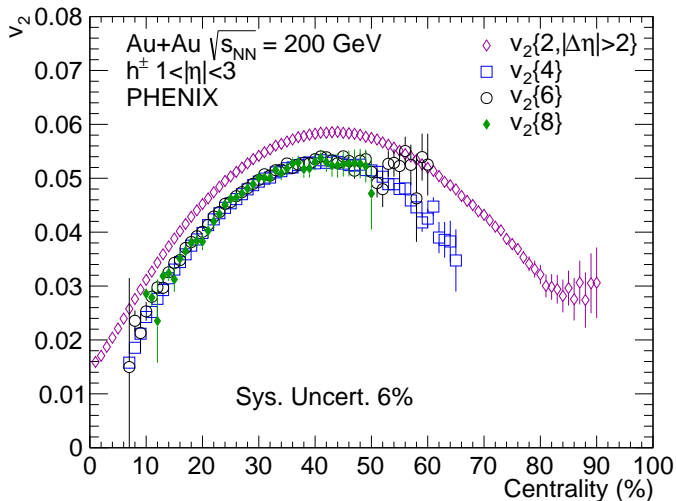


# Intermission

Additional material

# Collectivity in large systems

arXiv:1804.10024 (submitted to Phys Rev C)



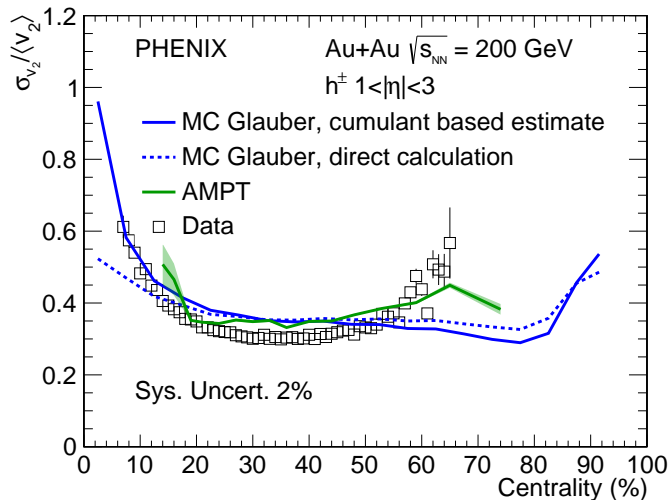
$$1 < |\eta| < 3$$

$$v_2\{2\}, v_2\{4\}, v_2\{6\},$$

$$v_2\{8\}$$

# Collectivity in large systems

arXiv:1804.10024 (submitted to Phys Rev C)



$$1 < |\eta| < 3$$

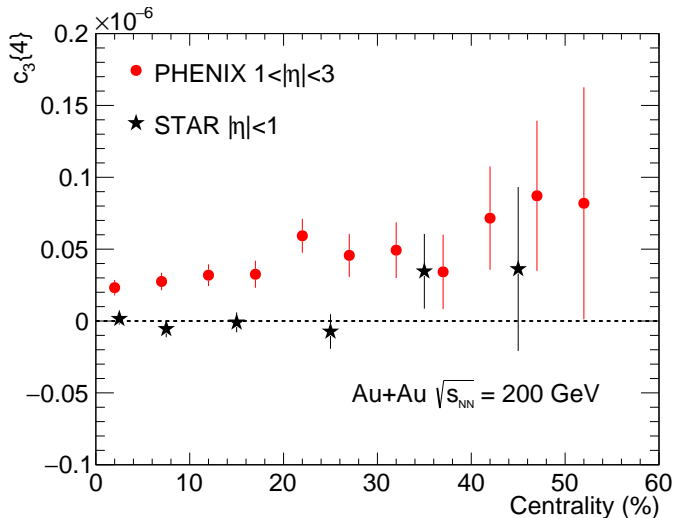
$$\sigma_{v_2} / \langle v_2 \rangle$$

Central: breakdown of small-variance limit

Peripheral: non-linearity in hydro response (e.g. J. Noronha-Hostler et al Phys. Rev. C 93, 014909 (2016))

# Collectivity in large systems

arXiv:1804.10024 (submitted to Phys Rev C)



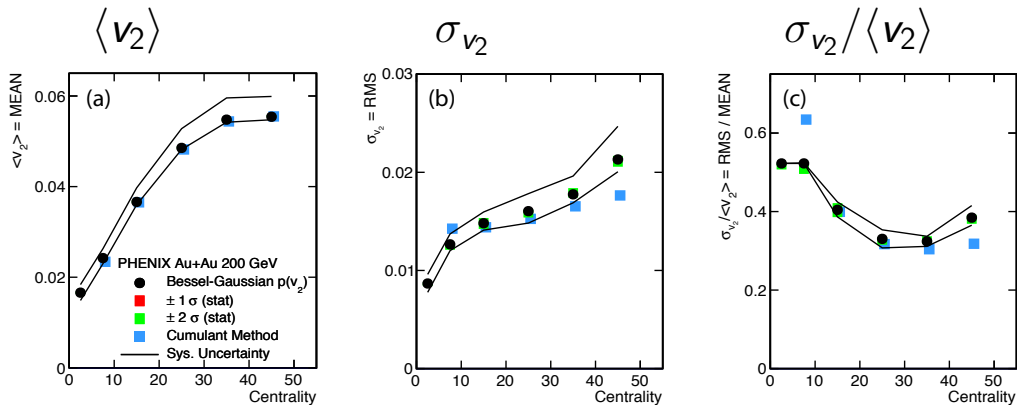
$$1 < |\eta| < 3$$

Cannot extract

$$\sigma_{v_3} / \langle v_3 \rangle$$

# Collectivity in large systems

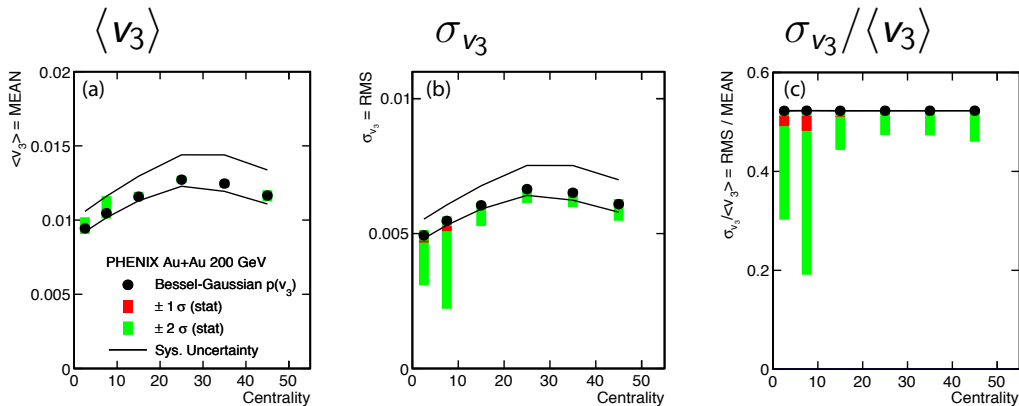
arXiv:1804.10024 (submitted to Phys Rev C)



Can extract  $\langle v_2 \rangle$  and  $\sigma_{v_2}$  separately using forward-fold

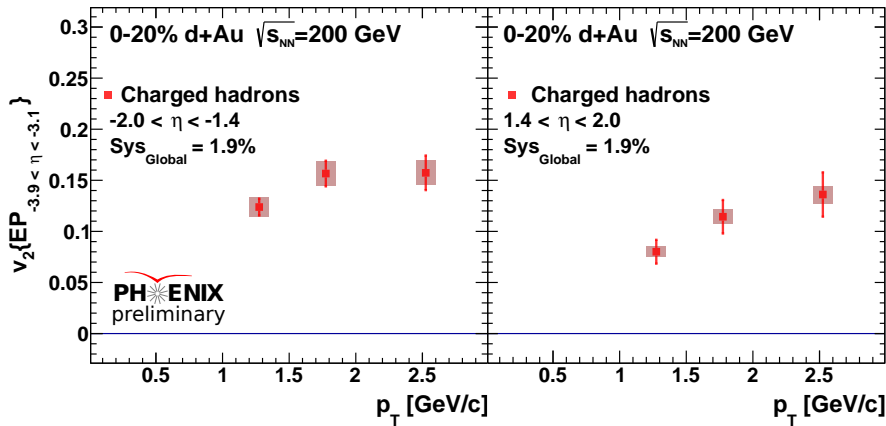
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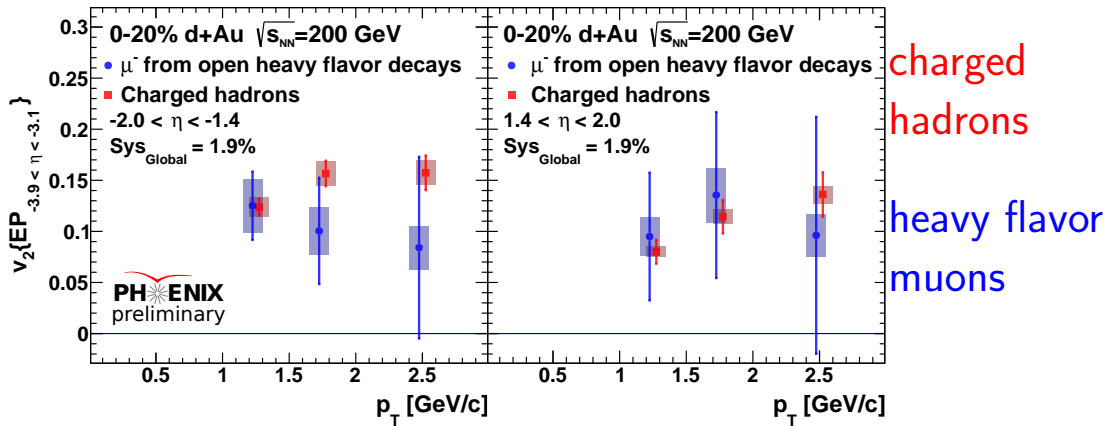
Can extract  $\langle v_3 \rangle$  and  $\sigma_{v_3}$  separately using forward-fold

# Small systems flow



charged  
hadrons

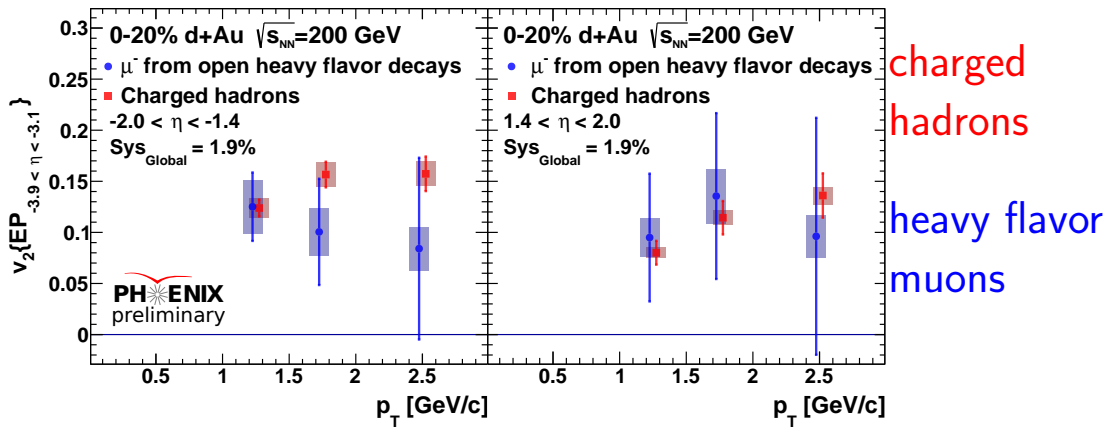
# Small systems flow—heavy flavor



Nonzero  $v_2$  for heavy flavor in  $d+Au$

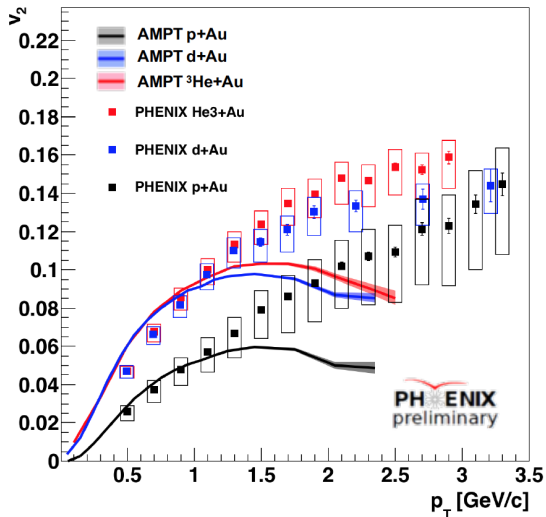


# Small systems flow—heavy flavor



Nonzero  $v_2$  for heavy flavor in  $d+Au$

$3.22\sigma$ ,  $2.16\sigma$  for  $v_2 > 0$  at backward, forward (99.9%, 98.5% one-sided)



## AMPT basic features

Initial conditions	HIJING
Particle production	String melting
Pre-equilibrium	None
Expansion	Parton scattering (tunable)
Hadronization	Spatial coalescence
Final stage	Hadron cascade (tunable)

