Longitudinal dynamics and collective behavior in the d+Au beam energy scan

Ron Belmont
University of Colorado Boulder

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A very brief history of recent heavy ion physics

- 1980s and 1990s—AGS and SPS... QGP at SPS!
- Early 2000s—QGP at RHIC! No QGP at SPS? d+Au as control.
- Mid-late 2000s—Detailed, quantitative studies of strongly coupled QGP. d+Au as control.
- 2010—Ridge in high multiplicity p+p (LHC)! Probably CGC!
- Early 2010s—QGP in p+Pb!
- Early 2010s—QGP in d+Au!
- Mid 2010s and now-ish—QGP in high multiplicity p+p? QGP in mid-multiplicity p+p?? QGP in d+Au even at low energies???

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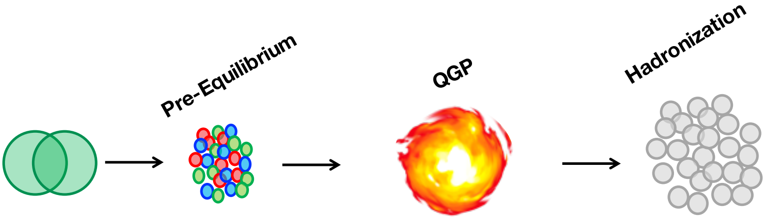
- Mid 2010s and now-ish—QGP in high multiplicity p+p? QGP in mid-multiplicity p+p?? QGP in d+Au even at low energies???

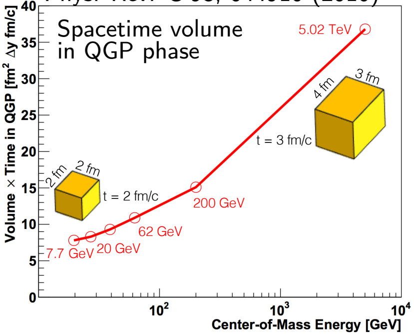
"Twenty years ago, the challenge in heavy ion physics was to find the QGP. Now, the challenge is to not find it." —Jürgen Schukraft, QM17

Testing hydro by controlling system size

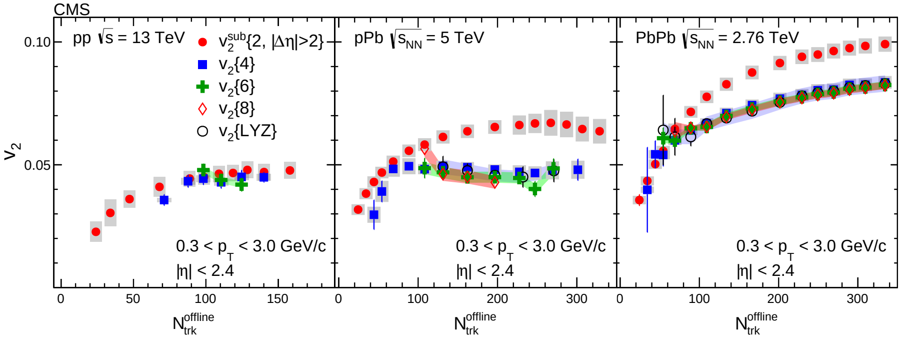


- Standard picture for A+A: QGP in hydro evolution
- What about small systems? And lower energies?
- Use collisions species and energy to control system size, test limits of hydro applicability

J.D. Orjuela Koop et al
Phys. Rev. C 93, 044910 (2016)



CMS, Phys. Lett. B 765 (2017) 193-220



- Multiparticle correlations: a strong case for collectivity
- Influence of fluctuations:

$$v_2\{2\} = \sqrt{v_2^2 + \sigma^2 + \delta} \qquad \delta \text{ non-flow, } \sigma^2 \text{ variance}$$

$$v_2\{2, |\Delta\eta| > 2\} = \sqrt{v_2^2 + \sigma^2} \qquad \text{eta gap removes some non-flow}$$

$$v_2\{4\} \approx v_2\{6\} \approx v_2\{8\} \approx \sqrt{v_2^2 - \sigma^2} \qquad \text{higher orders remove non-flow}$$

- Significant and well-known success in Au+Au and Pb+Pb

Multiparticle correlations in small systems

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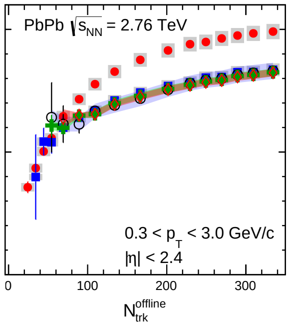

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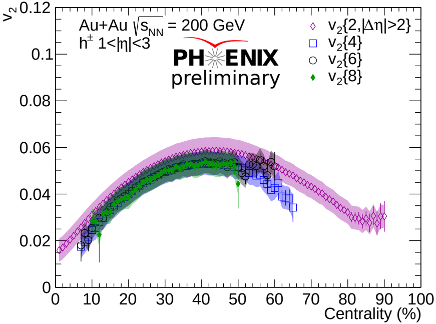

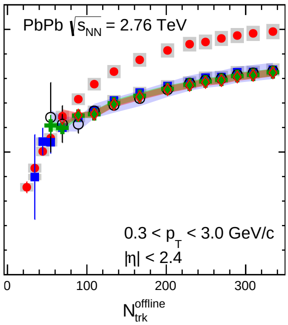

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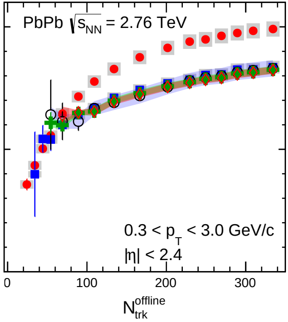

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Multiparticle correlations in small systems

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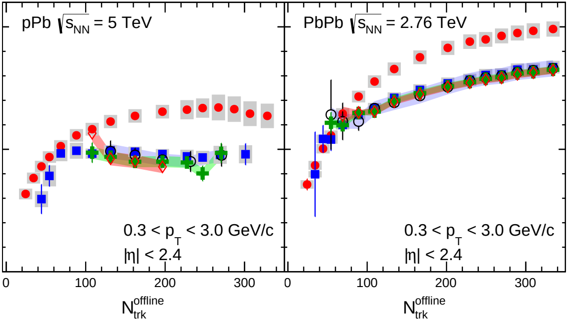

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- p+Pb remarkably similar to Pb+Pb

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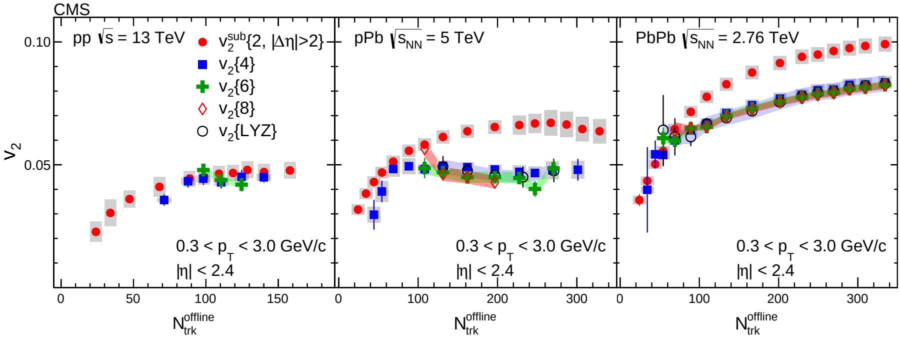

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- $\sigma = 0$ in p+p? Need to understand the fluctuations better

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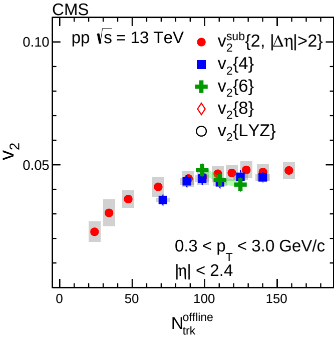

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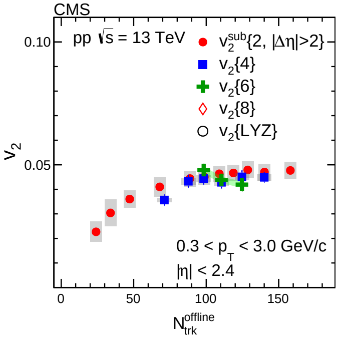

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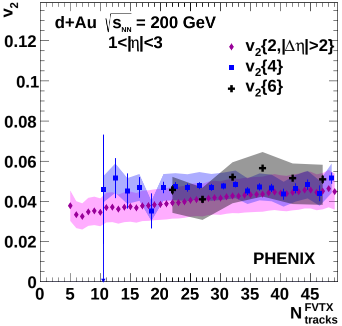

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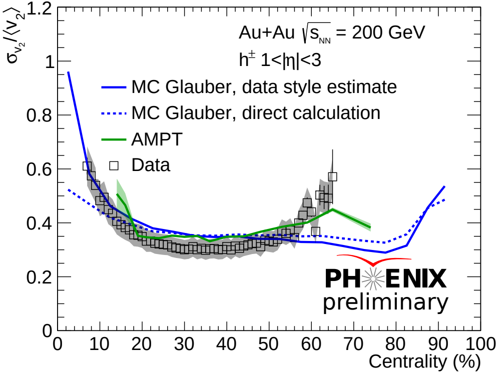
- $\sigma = 0$ in p+p and d+Au? Need to understand the fluctuations better

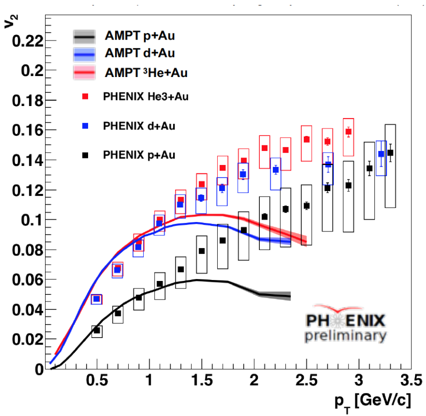
Fluctuations in Au+Au



Not a new idea, but:

$$\sqrt{\frac{v_2\{2\}^2 - v_2\{4\}^2}{v_2\{2\}^2 + v_2\{4\}^2}} = \frac{\sigma}{v_2}$$

- Standard picture of fluctuations works well for most centralities in Au+Au
- Up-tick in peripheral can be explained by non-linearity in hydro response (e.g. J. Noronha-Hostler et al Phys. Rev. C 93, 014909 (2016))
- Will revisit fluctuations again later...



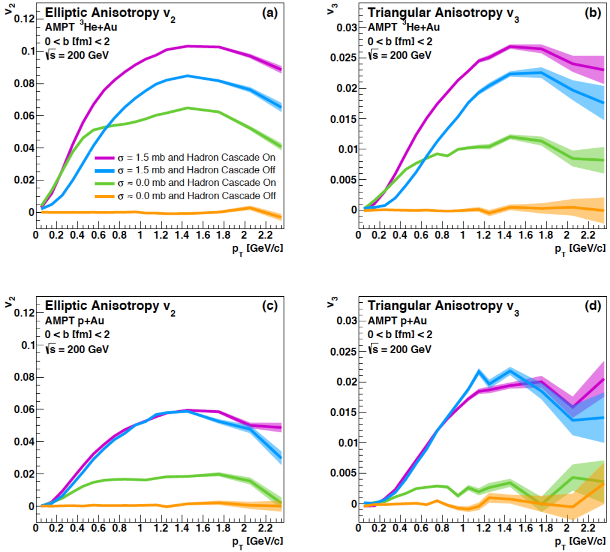
AMPT basic features

Initial conditions	MC Glauber
Particle production	String melting
Pre-equilibrium	None
Expansion	Parton scattering (tunable)
Hadronization	Spatial coalescence
Final stage	Hadron cascade (tunable)

- AMPT has significant success in describing flow-like signatures (for low p_T and p_T-integrated)
- AMPT produces final state particles over the full available phasespace —possible to perform exact same analysis on data and model

AMPT with no scattering

J.D. Orjuela Koop et al Phys. Rev. C 92, 054903 (2015)

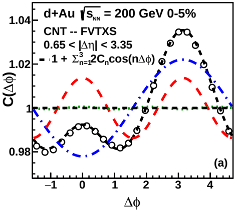


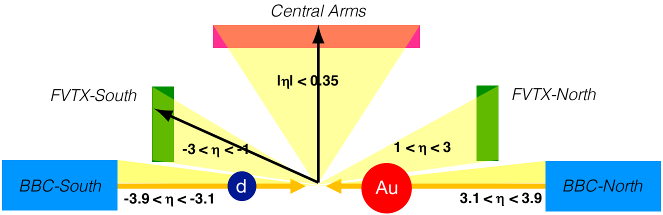
- Turn off scattering in AMPT—remove all correlations with initial geometry $\sigma_{parton} = 0$ and $\sigma_{hadron} = 0$
- Participant plane v_2 goes to zero
- Other sources of correlation remain—non-flow

Results

(Talk by D. McGlinchey)

arXiv:1708.06983





- $0.65 < |\Delta\eta| < 3.35$

Two particle correlations

(Talk by D. McGlinchey)

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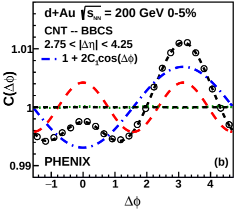

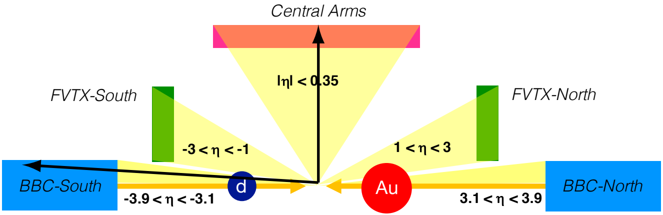

- $2.75 < |\Delta\eta| < 4.25$

(Talk by D. McGlinchey)

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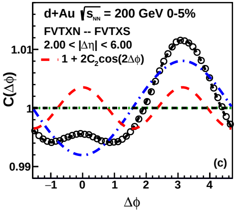
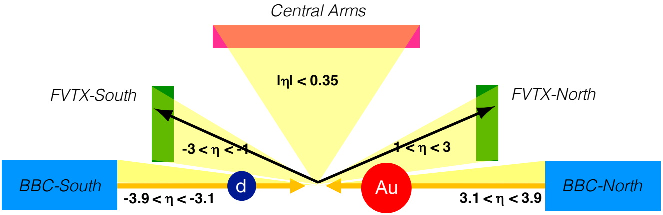
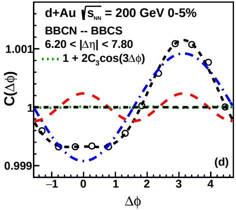


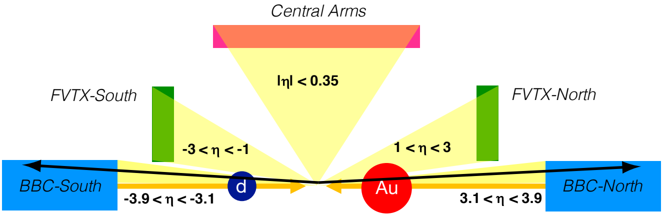


- $2.0 < |\Delta\eta| < 6.0$

(Talk by D. McGlinchey)

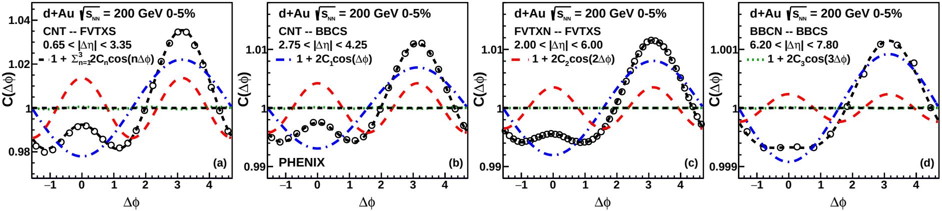
arXiv:1708.06983

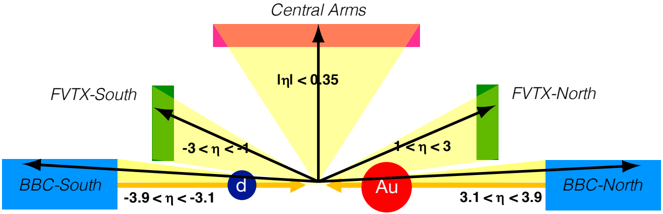




- $6.2 < |\Delta\eta| < 7.8$

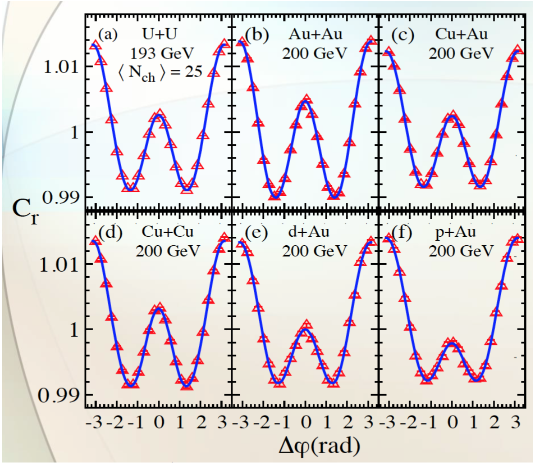
(Talk by D. McGlinchey) arXiv:1708.06983





- $0.65 < |\Delta\eta| < 7.8$
- Wide range of pseudorapidity separation has potential to provide significant leverage to disentangle various flow and non-flow effects
- Ridge observed for $|\Delta\eta| > 6.2$—long range indeed!

(Talk by N. Magdy)

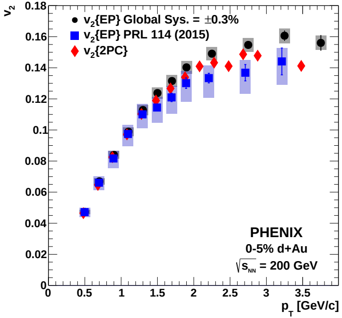


- $|\Delta\eta| > 0.7$ (STAR TPC $|\eta| < 1$)
- Observation of strong ridge structure in all systems at fixed multiplicity, including d+Au and p+Au

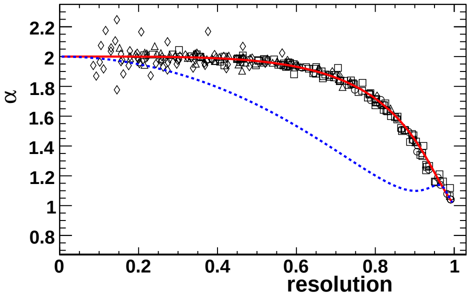
Event plane results

v_2 vs p_T 200 GeV method comparison

arXiv:1708.06983



J.Y. Ollitrault et al Phys Rev C 80, 014904 (2009)

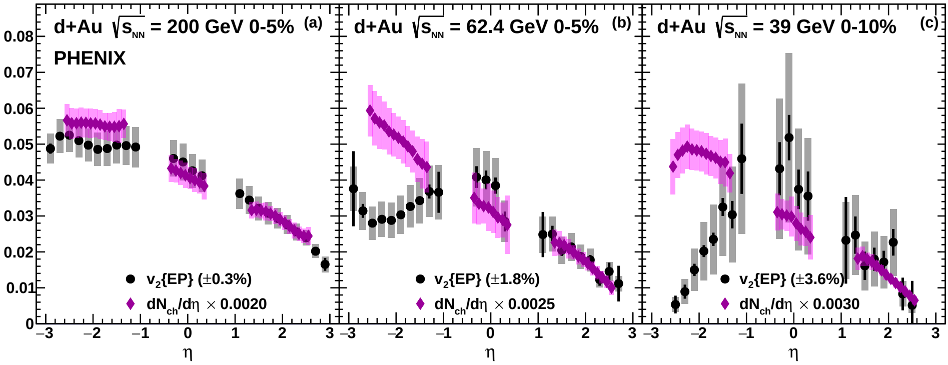


- Important to remember that $v_n\{\text{EP}\}$ is an estimator of $\langle v_n^\alpha \rangle^{1/\alpha}$
- High multiplicity → high resolution → $\alpha = 1$
- Low multiplicity → low resolution → $\alpha = 2$
- For all RHIC small systems results, $\alpha = 2$
  —same dependence on fluctuations as two-particle methods

v_2 and $dN_{ch}/d\eta$ vs η

(Talks by K. Hill, J. Nagle)

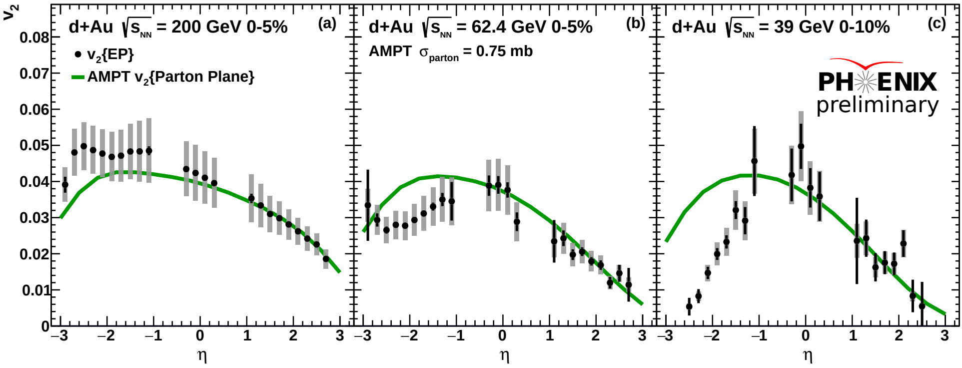
arXiv:1708.06983



- BBC south ($-3.9 < \eta < -3.1$) used to estimate the event plane
- 200 GeV shows strong forward/backward asymmetry in v_2 and $dN_{ch}/d\eta$ (both much larger at backward)
- Asymmetry is large for $dN_{ch}/d\eta$ at all energies, where as v_2 asymmetry seems to decrease with decreasing energy

v_2 vs η, comparison with AMPT

(Talks by D. McGlinchey, K. Hill, J. Nagle) arXiv:1708.06983



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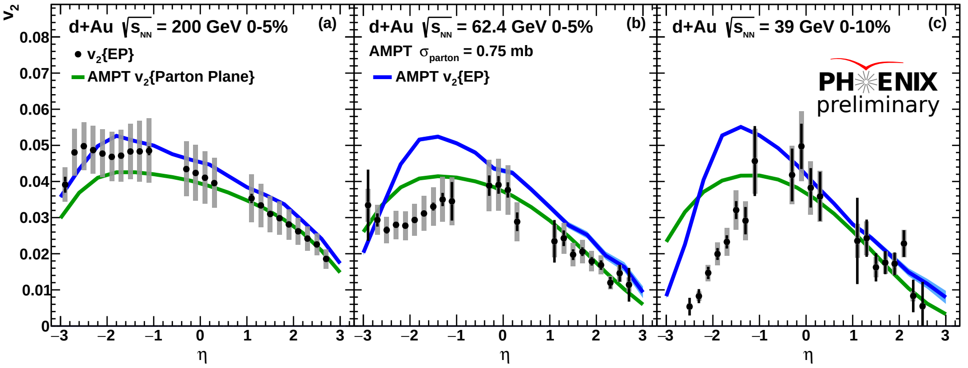

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- AMPT flow+non-flow shows striking anti-correlation at backward rapidity

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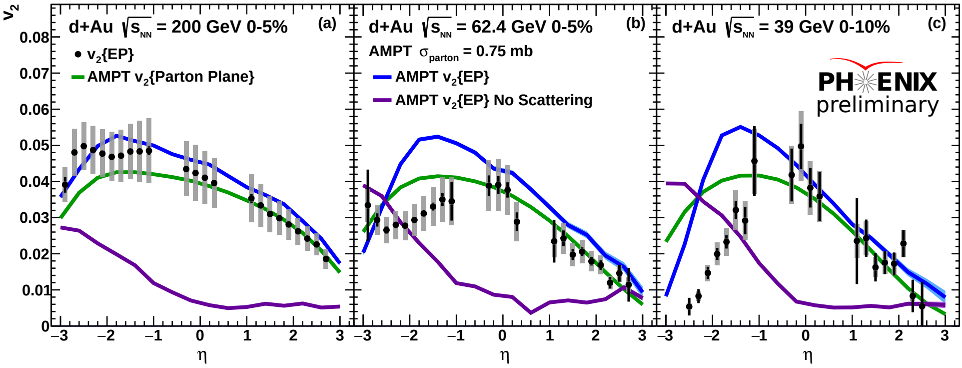

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- AMPT flow+non-flow is very similar at mid and forward
- AMPT flow+non-flow shows striking anti-correlation at backward rapidity
- AMPT non-flow only shows nothing at mid and forward, large v_2 at backward rapidity near the detector

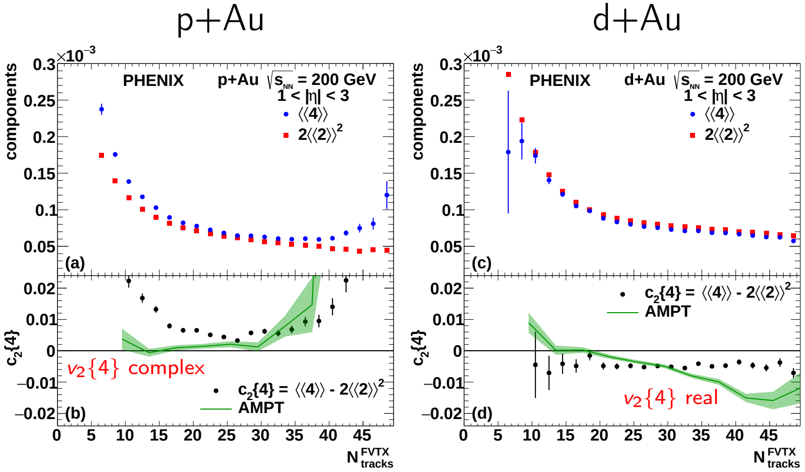
- More hydro theory calculations for η dependence would be very helpful
- The data shows large forward/backward asymmetry that decreases with energy, but is that what's really happening?
- AMPT flow only shows forward/backward asymmetry at all energies
- AMPT flow+non-flow shows strong anticorrelation between flow and non-flow at backward rapidity that brings v_2 backward down significantly
- **Flow and non-flow are *non-additive***

Intermission

Multiparticle correlations

Components and cumulants in p+Au and d+Au at 200 GeV

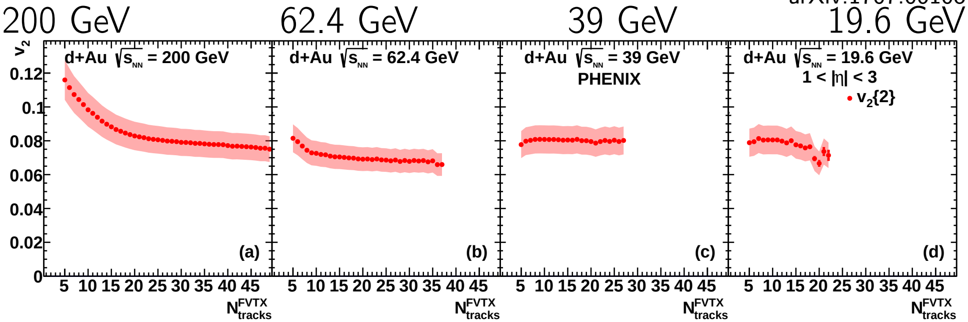
(Talk by J. Nagle) arXiv:1707.06108



- Is the sign of $c_2\{4\}$ a good indicator of collectivity? (Hint: no)
- Positive $c_2\{4\}$ doesn't mean absence of collectivity (many other p+Au results)
- Negative $c_2\{4\}$ doesn't mean collectivity, could be CGC (Talk by M. Mace)

$v_2\{2\}$ and $v_2\{4\}$ in the d+Au beam energy scan

arXiv:1707.06108



- $v_2\{2\}$ relatively constant with N_{tracks}^{FVTX} and collision energy

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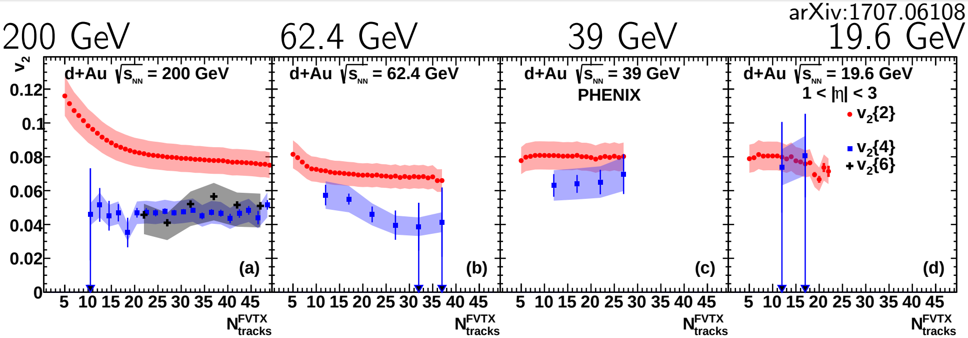

- $v_2\{2\}$ relatively constant with N_{tracks}^{FVTX} and collision energy
- Measurement of $v_2\{4\}$ in d+Au at all energies
- Measurement of $v_2\{6\}$ in d+Au at 200 GeV

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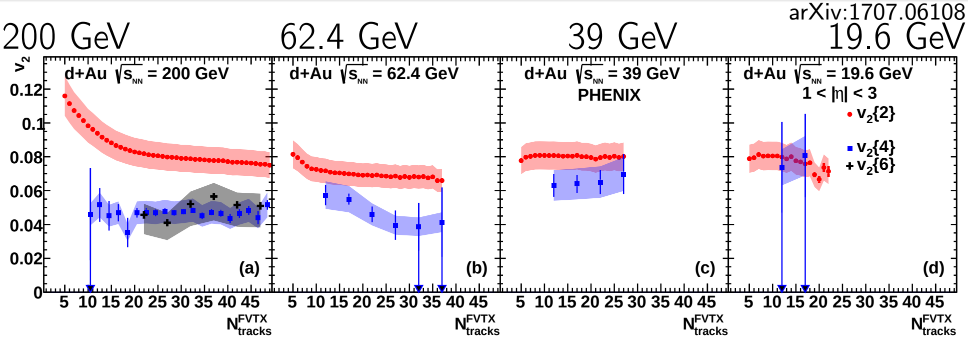
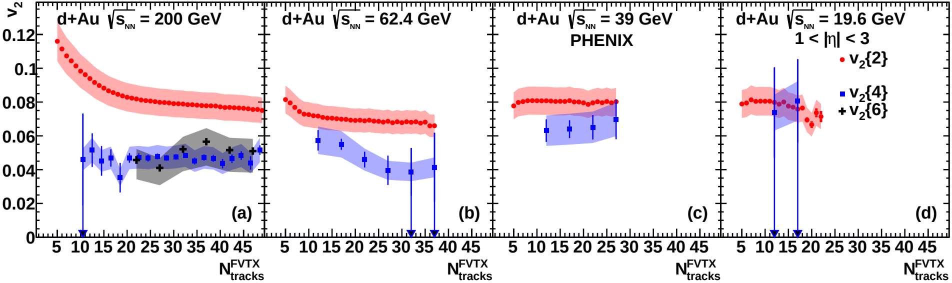


- $v_2\{2\}$ relatively constant with N^{FVTX}_{tracks} and collision energy
- Measurement of $v_2\{4\}$ in d+Au at all energies
- Measurement of $v_2\{6\}$ in d+Au at 200 GeV
- $v_2\{4\}$ increases and approaches $v_2\{2\}$
  (19.6 GeV looks a bit like CMS p+p...)

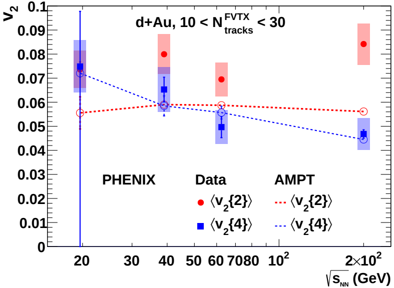
$v_2\{2\}$ and $v_2\{4\}$ in the d+Au beam energy scan

arXiv:1707.06108

200 GeV 62.4 GeV 39 GeV 19.6 GeV

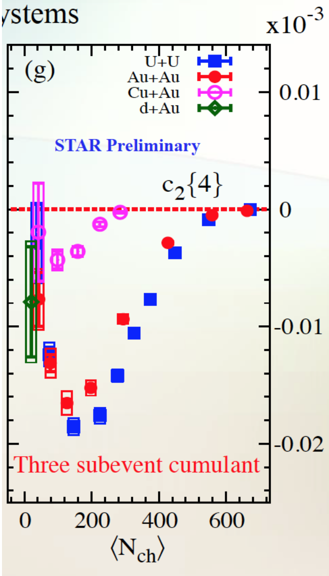


- Select $10 < N^{FVTX}_{tracks} < 30$, integrate
- AMPT sees similar trend
- Fluctuations?



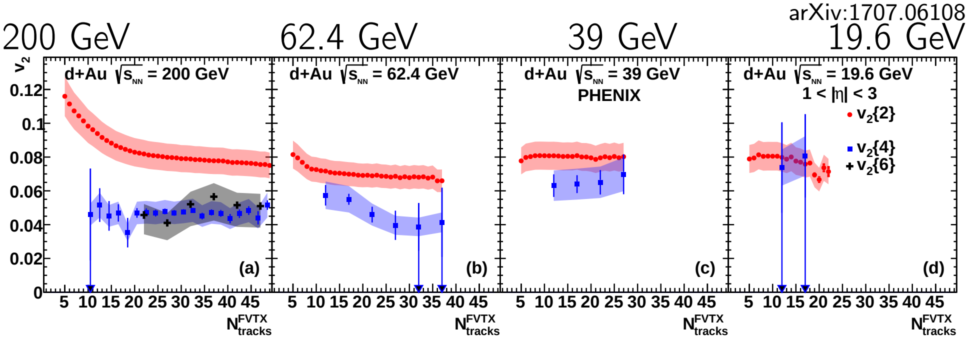
STAR cumulant results

(Talk by N. Magdy)



- Three subevent $c_2\{4\}$
- No p+Au point, but d+Au point is $\approx -8 \times 10^{-6}$
- $v_2\{4\} \approx 0.05$, in good agreement with PHENIX

Can we apply an eta gap to get a better handle on the non-flow?



- $v_2\{2\}$ and $v_2\{4\}$ vs N_{tracks}^{FVTX}, all tracks anywhere in FVTX

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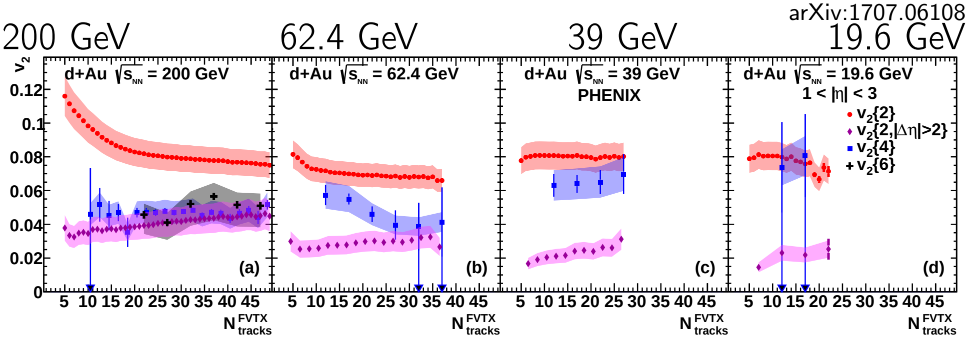

- $v_2\{2\}$ and $v_2\{4\}$ vs N^{FVTX}_{tracks}, all tracks anywhere in FVTX
- $v_2\{2,|\Delta\eta|>2\}$ vs N^{FVTX}_{tracks}, one track backward, the other forward

$$v_2\{2,|\Delta\eta|>2\} = \sqrt{v_2^2+\sigma^2} \qquad v_2\{2\} = \sqrt{v_2^2+\sigma^2+\delta}$$

$$v_2\{4\} \approx \sqrt{v_2^2-\sigma^2}$$

- How to understand this?

Understanding two-particle estimates of v_2 when using subevents

- $dN_{ch}/d\eta$ and v_2 are larger at backward rapidity, so $v_2\{2\}$ and $v_2\{4\}$ are weighted towards backward
- $v_2\{2, |\Delta\eta| > 2\}$ is weighted equally between forward and backward as $\sqrt{v_2^B v_2^F}$
- $v_2^B > v_2^F$, so $v_2^2 > v_2^B v_2^F$

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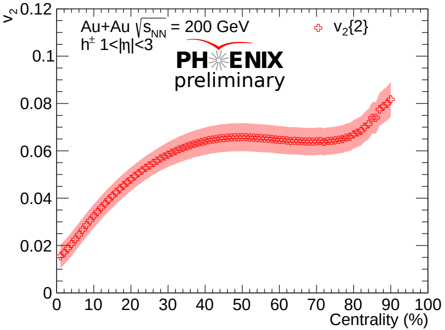
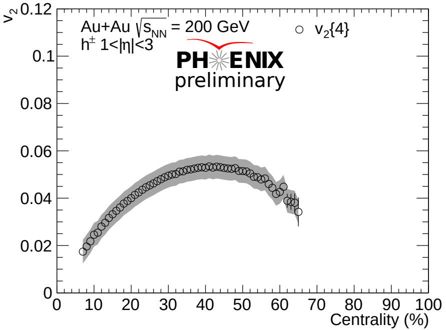
- Correlation strength between forward and backward $|\varsigma_{BF}| \leq \sigma_B \sigma_F$—fluctuations can contribute less than expected
- Event plane decorrelation small in Au+Au but could be larger in d+Au
- But that's already encoded in the v_2 vs η measurement discussed earlier

Understanding two-particle estimates of v_2 when using subevents





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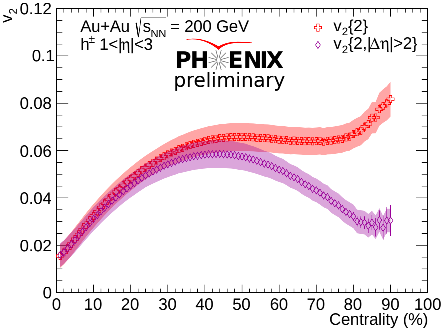
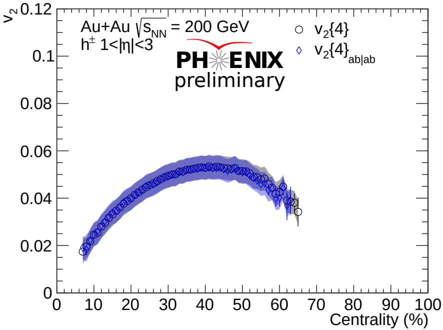
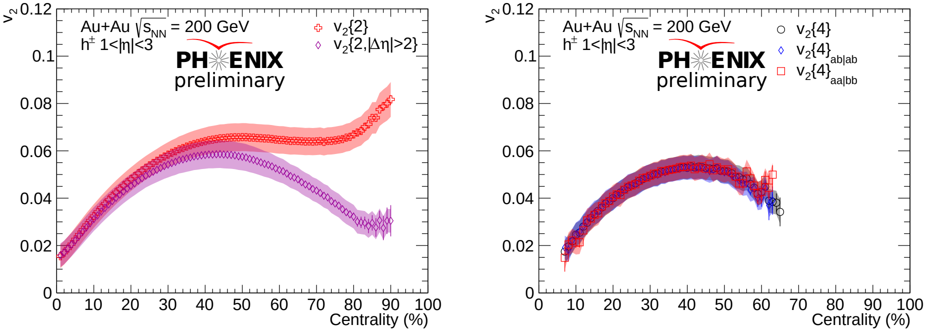




Understanding two-particle estimates of v_2 when using subevents

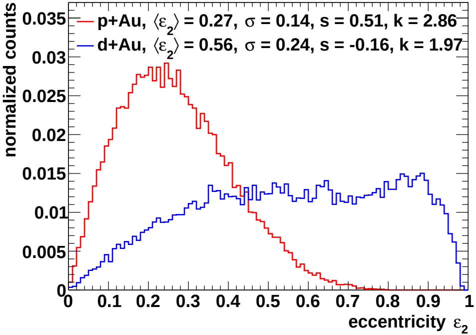


- Two-particle correlation without eta gap has significant non-flow
- Can't disentangle flow/non-flow/decorrelation effects (by looking at this plot)
- Four particle correlation with and without subevents is identical
- Non-flow and decorrelations don't affect four-particle results in Au+Au
- Decorrelation effects present but small (few %) in Au+Au (Talk by M. Nie)

Clearly, “fluctuations” are doing a lot of work for us. What do we mean, and how well do we understand them?

- We always say $v_2\{4\} \approx v_2\{6\} \approx v_2\{8\} \approx \sqrt{v_2^2 - \sigma^2}$
- Is that really true? Sometimes! But other times absolutely not...
- One of two assumptions (or both) are required to get there:
  - Small relative variance, $\sigma/v_n \ll 1$
    —higher moments suppressed by powers of σ, can do expansion about σ/v_n
  - Gaussian fluctuations
    —all standard cumulants are zero (odd standard moments are zero, even standard moments are $(m-1)!!$)
- Are these assumptions valid? Let's have a look...

Eccentricity distributions and cumulants



	p+Au	d+Au
$\langle\varepsilon_2\{2\}\rangle$	0.303	0.610
$\langle\varepsilon_2\rangle$	0.270	0.560
$\langle\varepsilon_2\{4\}\rangle$ Approx.	0.232	0.505
$\langle\varepsilon_2\{4\}\rangle$ Exact	0.166	0.508

- Eccentricity cumulants: $\varepsilon_2\{2\} = (\langle\varepsilon_2^2\rangle)^{1/2}$, $\varepsilon_2\{4\} = (-(\langle\varepsilon_2^4\rangle - 2\langle\varepsilon_2^2\rangle^2))^{1/4}$
- We don't have the v_n distribution but in the hydro limit $v_n \propto \varepsilon_n$
  (Well, not quite... could be nonlinear response)

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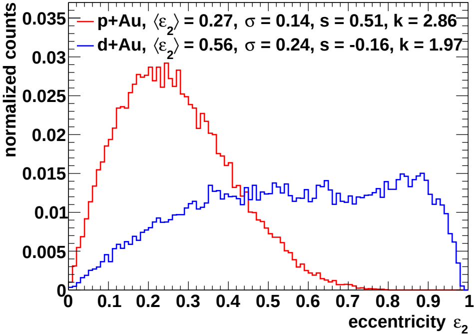

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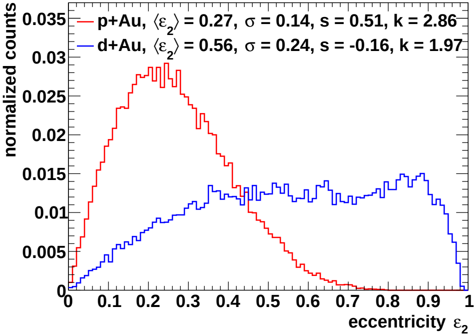
- We don't have the v_n distribution but in the hydro limit $v_n \propto \varepsilon_n$
  (Well, not quite... could be nonlinear response)
- Gaussian? No. Small relative variance? No.

Eccentricity distributions and cumulants



	p+Au	d+Au
ε_2^4	0.00531	0.0983
$2\varepsilon_2^2\sigma^2$	0.00277	0.0370
$4\varepsilon_2 s\sigma^3$	0.00147	-0.0053
$(k-2)\sigma^4$	0.00031	-0.0001

$$\varepsilon_2\{4\} = (\varepsilon_2^4 - 2\varepsilon_2^2\sigma^2 - 4\varepsilon_2 s\sigma^3 - (k-2)\sigma^4)^{1/4}$$

- the variance brings $\varepsilon_2\{4\}$ down (this term gives the usual $\sqrt{v_2^2 - \sigma^2}$)
- positive skew brings $\varepsilon_2\{4\}$ further down, negative skew brings it back up
- kurtosis > 2 brings $\varepsilon_2\{4\}$ further down, kurtosis < 2 brings it back up —recall Gaussian has kurtosis $= 3$

Final thoughts

- Ridge observed for $|\Delta\eta| > 6.2$—very long range
- Positive v_2 vs η observed from 200 GeV down to 39 GeV (and vs p_T from 200 GeV to 19.6 GeV)
  - (Nearly) All of these explained well by AMPT, many explained well by hydro
  - **Non-flow is not additive**
  - Possibly strong flow–non-flow anticorrelation at backward rapidity
- Understanding relationship between $v_2\{2, |\Delta\eta| > 2\}$ and $v_2\{2\}$, $v_2\{4\}$ requires careful consideration of longitudinal dynamics
- Subevents and eta gaps present additional hazards but also provide new opportunities
- Real-valued $v_2\{4\}$ observed from 200 GeV all the way down to 19.6 GeV But complex-valued $v_2\{4\}$ in p+Au at 200 GeV
  - Sign of $c_2\{4\}$ is not a good criterion for determining collectivity
  - But it is part of a suite of measurements to be used to build a case
  - How to get d/^{3}He+Au in CGC picture?

Additional Material

Back to basics (a brief excursions)

The (raw) moments of a probability distribution function $f(x)$:

$$\mu_n = \langle x^n \rangle \equiv \int_{-\infty}^{+\infty} x^n f(x) dx$$

The moment generating function:

$$M_x(t) \equiv \langle e^{tx} \rangle = \int_{-\infty}^{+\infty} e^{tx} f(x) dx = \int_{-\infty}^{+\infty} \sum_{n=0}^{\infty} \frac{t^n}{n!} x^n f(x) dx = \sum_{n=0}^{\infty} \mu_n \frac{t^n}{n!}$$

Moments from the generating function:

$$\mu_n = \left. \frac{d^n M_x(t)}{dt^n} \right|_{t=0}$$

Key point: the moment generating function uniquely describe $f(x)$

Back to basics (a brief excursions)

Can also uniquely describe $f(x)$ with the cumulant generating function:

$$K_x(t) \equiv \ln M_x(t) = \sum_{n=0}^{\infty} \kappa_n \frac{t^n}{n!}$$

Cumulants from the generating function:

$$\kappa_n = \left. \frac{d^n K_x(t)}{dt^n} \right|_{t=0}$$

Since $K_x(t) = \ln M_x(t)$, $M_x(t) = \exp(K_x(t))$, so

$$\mu_n = \left. \frac{d^n \exp(K_x(t))}{dt^n} \right|_{t=0}, \quad \kappa_n = \left. \frac{d^n \ln M_x(t)}{dt^n} \right|_{t=0}$$

End result: (details left as an exercise for the interested reader)

$$\begin{aligned} \mu_n &= \sum_{k=1}^{n} B_{n,k}(\kappa_1, \ldots, \kappa_{n-k+1}) \quad (= B_n(\kappa_1, \ldots, \kappa_{n-k+1})) \\ \kappa_n &= \sum_{k=1}^{n} (-1)^{k-1}(k-1)! B_{n,k}(\mu_1, \ldots, \mu_{n-k+1}) \quad (= L_n(\kappa_1, \ldots, \kappa_{n-k+1})) \end{aligned}$$

Evaluating the Bell polynomials gives

$$\begin{aligned}
\langle x \rangle &= \kappa_1 \\
\langle x^2 \rangle &= \kappa_2 + \kappa_1^2 \\
\langle x^3 \rangle &= \kappa_3 + 3\kappa_1\kappa_2 + \kappa_1^3 \\
\langle x^4 \rangle &= \kappa_4 + 4\kappa_1\kappa_3 + 3\kappa_2^2 + 6\kappa_1^2\kappa_2 + \kappa_1^4
\end{aligned}$$

One can tell by inspection (or derive explicitly) that κ_1 is the mean, κ_2 is the variance, etc.

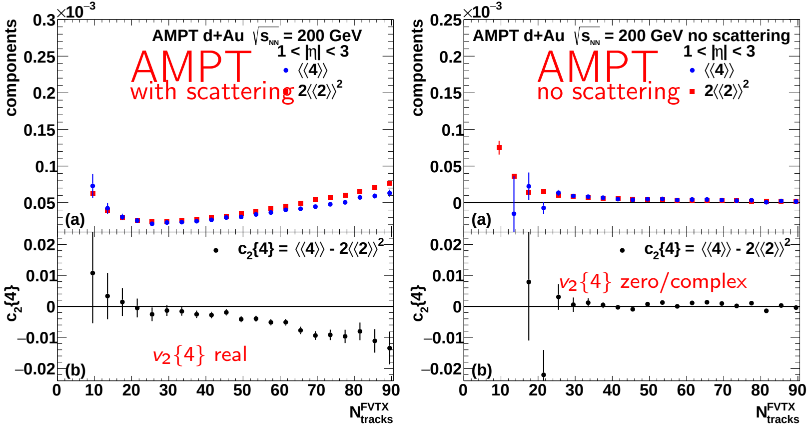
Subbing in $x = v_n$, $\kappa_2 = \sigma^2$, we find

$$\begin{aligned}
\Big(\langle v_n^4\rangle &= v_n^4 + 6v_n^2\sigma^2 + 3\sigma^4 + 4v_n\kappa_3 + \kappa_4\Big) \\
-\Big(2\langle v_n^2\rangle^2 &= 2v_n^4 + 4v_n^2\sigma^2 + 2\sigma^4\Big) \\
&\rightarrow \\
\langle v_n^4\rangle - 2\langle v_n^2\rangle^2 &= -v_n^4 + 2v_n^2\sigma^2 + \sigma^4 + 4v_n\kappa_3 + \kappa_4
\end{aligned}$$

Skewness s: $\kappa_3 = s\sigma^3$
Kurtosis k: $\kappa_4 = (k-3)\sigma^4$

$$\begin{aligned}
v_n\{2\} &= (v_n^2 + \sigma^2)^{1/2} \\
v_n\{4\} &= (v_n^4 - 2v_n^2\sigma^2 - 4v_n s\sigma^3 - (k-2)\sigma^4)^{1/4}
\end{aligned}$$

So the correct form is actually much more complicated than we tend to think...



- Turn off scattering in AMPT—remove all correlations with initial geometry
- Components show different trend but are still non-zero
- But $v_2\{4\}$ goes from real to ∼zero—connection between real $v_2\{4\}$ and geometry in d+Au

A word of caution about decorrelation measurements

- Flow is a vector quantity: $v_2^B v_2^F \to \vec{v}_2^B \cdot \vec{v}_2^F = v_2^B v_2^F \cos(2(\psi_2^B - \psi_2^F))$
- Decorrelation effect may be small ($\sim$few % in Au+Au), but in principle can influence covariance
- Usual observable:

$$\frac{\langle \vec{v}_2(\eta) \cdot \vec{v}_2(\eta_{ref}) \rangle}{\langle \vec{v}_2(-\eta) \cdot \vec{v}_2(\eta_{ref}) \rangle}$$

- Problem: v_2 and $dN_{ch}/d\eta$ in d+Au are highly asymmetric, so η and $-\eta$ are very different and physics effects could be hidden
- Could try instead:

$$\frac{\langle \vec{v}_2(\eta_a) \cdot \vec{v}_2(\eta_b) \rangle}{\sqrt{\langle v_2^2(\eta_a) \rangle \langle v_2^2(\eta_b) \rangle}}$$

- Problem: $v_2^2(\eta_a)$ and $v_2^2(\eta_b)$ could have large non-flow