Collectivity at RHIC

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THE UNIVERSITY of NORTH CAROLINA GREENSBORO

Collectivity at RHIC

What is collectivity?

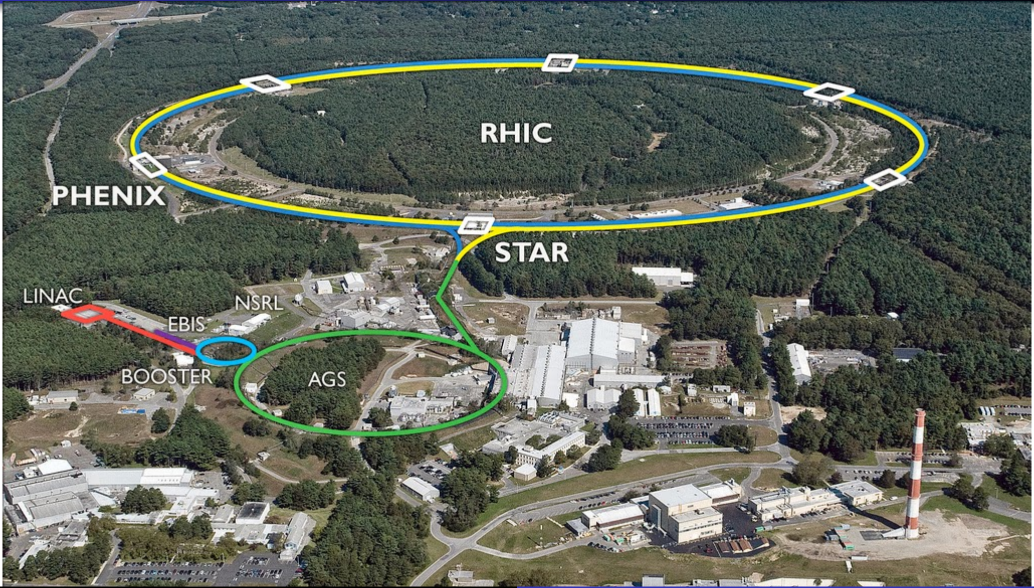
What is RHIC?

Why should we care about collectivity at RHIC?

Утро в сосновом лесу

Утро в сосновом лесу

The Relativistic Heavy Ion Collider



The Relativistic Heavy Ion Collider

RHIC is the only polarized proton collider in the world

RHIC is one of two heavy ion colliders, the other being the LHC

RHIC is a dedicated ion collider and is designed to collide many different species of ions at many different energies—vastly more flexible than the LHC

Collision Species	Collision Energies (GeV)
p↑+p↑	510, 500, 200, 62.4
p+Al	200
p+Au	200
d+Au	200, 62.4, 39, 19.6
^{3}He+Au	200
Cu+Cu	200, 62.4, 22.5
Cu+Au	200
Zr+Zr & Ru+Ru	200
Au+Au	200, 130, 62.4, 56, 39, 27, 19.6, 15, 11.5, 7.7, 5, ...
U+U	193

And lots more to come!

What is collectivity?

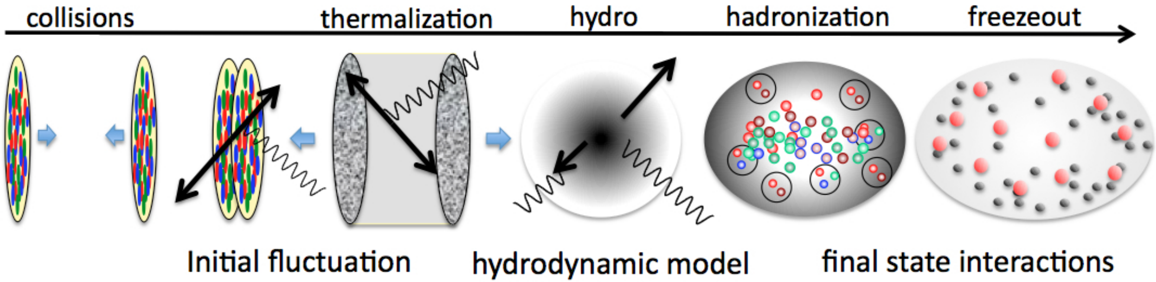
No fully agreed-upon definition...

For some, it means hydro evolution (too narrow)

For some, it means multiparticle correlations have a certain sign (too specific)

My proposal: existence of global correlations
—e.g. translation of initial geometry to final state observables

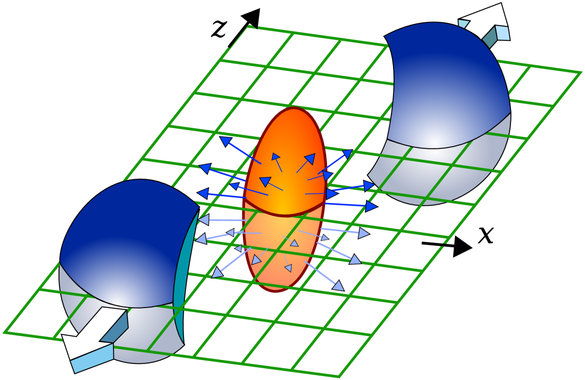
Standard model of heavy ion physics

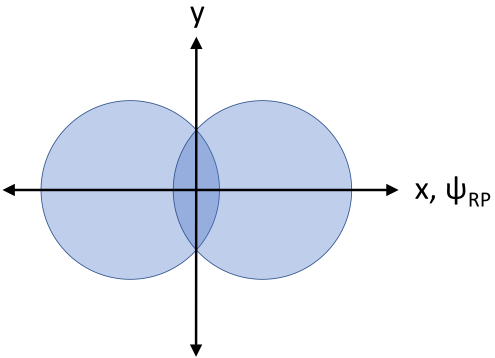


Initial state: very well-described by CGC, see talks by Adrian and Vladi in this session!

Thermalization may not be needed for hydro evolution to occur, see talk by Ryan in this session!

Azimuthal anisotropy measurements



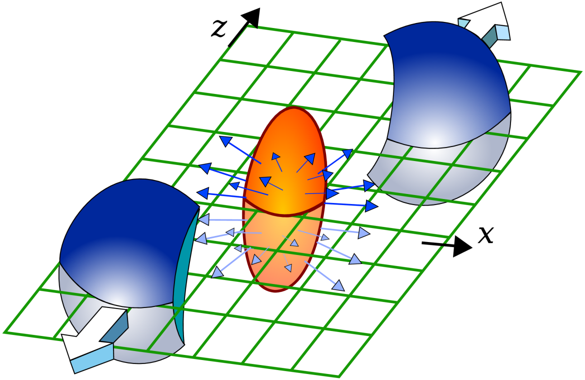


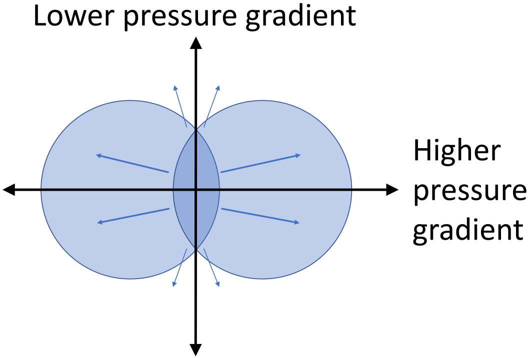
$$\frac{dN}{d\varphi} \propto 1 + \sum_{n=1}^{\infty} 2v_n \cos n\varphi \qquad v_n = \langle \cos n\varphi \rangle \qquad \varepsilon_n = \frac{\sqrt{\langle r^2 \cos n\varphi \rangle + \langle r^2 \sin n\varphi \rangle}}{\langle r^2 \rangle}$$

Hydrodynamics translates initial shape (ε_n) into final state distribution (v_n)

Overlap shape approximately elliptical, expect v_2 to be the largest

Azimuthal anisotropy measurements





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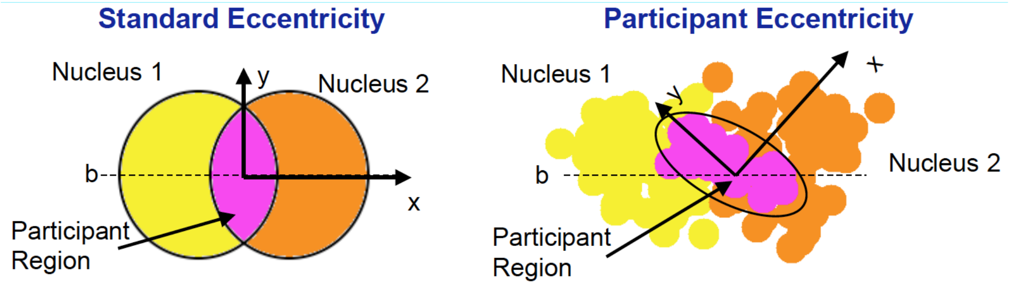
Important discovery in 2005

G. Roland, PHOBOS Plenary, Quark Matter 2005



A nucleus isn't just a sphere
Optical Glauber → Monte Carlo Glauber

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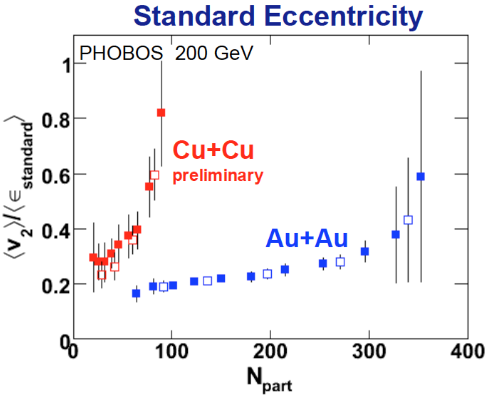

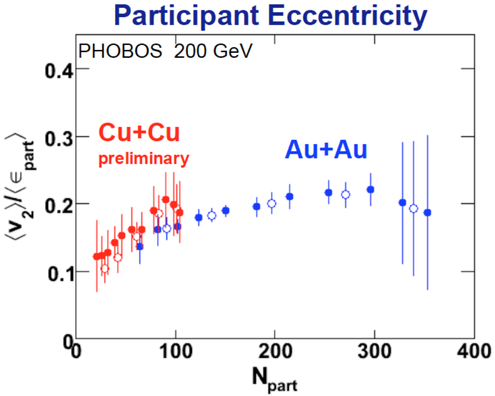

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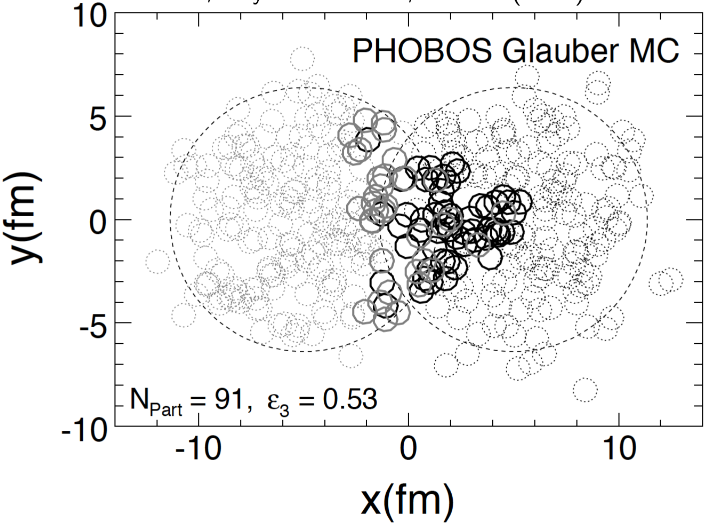
Alver and Roland, Phys. Rev. C 81, 054905 (2010)



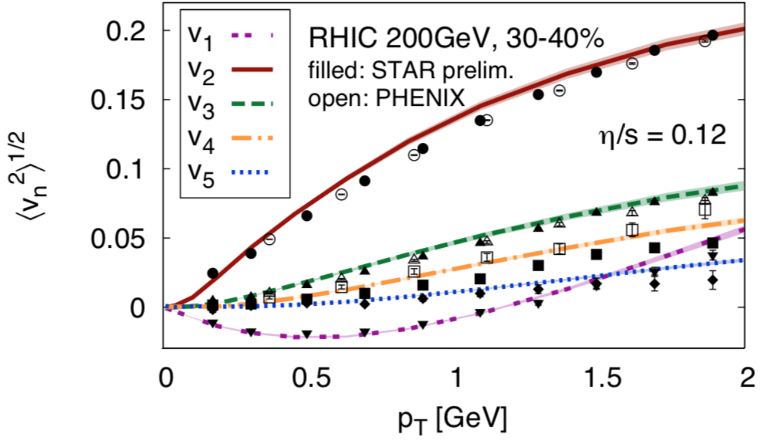
Nucleon fluctuations can produce non-zero ε_n for odd n

Symmetry planes ψ_n can be different for different harmonics

$$\varphi = \phi_{lab} - \psi_n$$

Data and theory for v_n

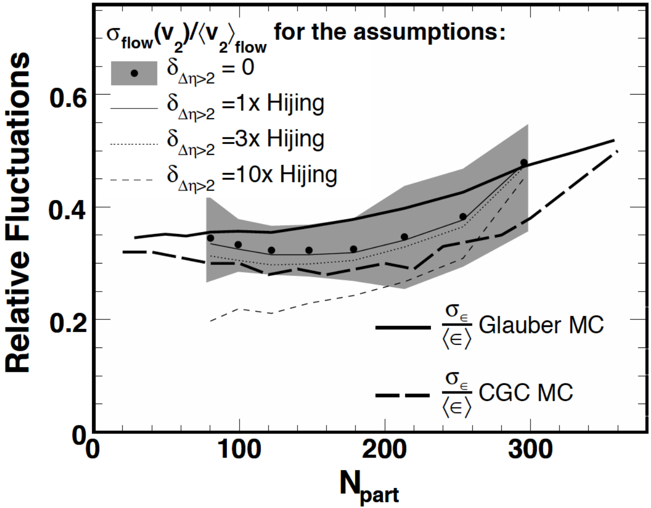
Gale et al, Phys. Rev. Lett. 110, 012302 (2013)



$$\frac{dN}{d\varphi} \propto 2v_1 \cos\varphi + 2v_2 \cos 2\varphi + 2v_3 \cos 3\varphi + 2v_4 \cos 4\varphi + 2v_5 \cos 5\varphi$$

Fluctuations in large systems

PHOBOS, Phys. Rev. C 81, 034915 (2010) [See also STAR, Phys. Rev. C 72, 014904 (2005)]



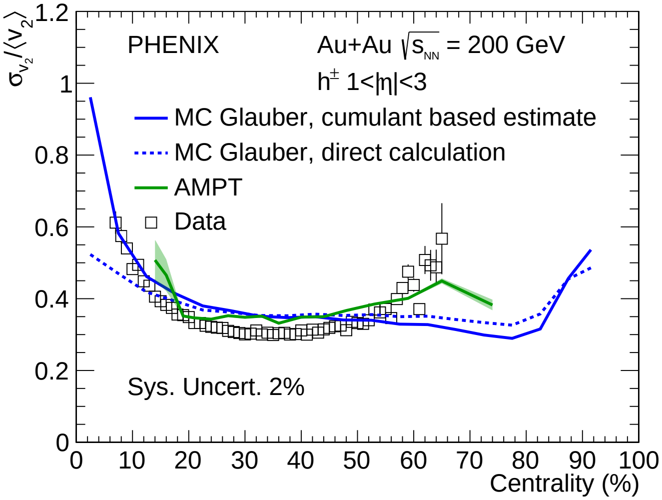
Fluctuations should also be translated, so measure $\sigma_{v_2}/\langle v_2 \rangle$

$|\eta| < 1$

Reasonable agreement with models of initial geomtry

Fluctuations in large systems

PHENIX, arXiv:1804.10024 (submitted to Phys. Rev. C)



Fluctuations should also be translated, so measure $\sigma_{v_2}/\langle v_2\rangle$

$1 < |\eta| < 3$

Generally good agreement with models of initial geometry

Fluctuations in large systems

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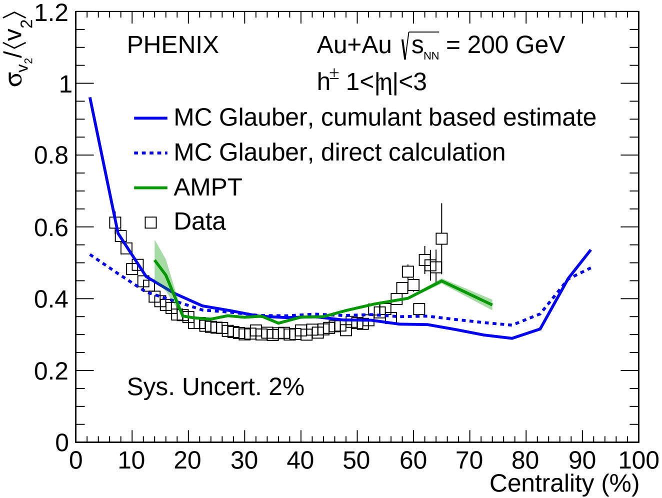

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$1 < |\eta| < 3$

Central: breakdown of small-variance limit

Peripheral: non-linearity in hydro response (e.g. J. Noronha-Hostler et al Phys. Rev. C 93, 014909 (2016))

Intermission

Small systems

A very brief history of recent heavy ion physics

1980s and 1990s—AGS and SPS... QGP at SPS!

Early 2000s—QGP at RHIC! No QGP at SPS. d+Au as control.

Mid-late 2000s—Detailed, quantitative studies of strongly coupled QGP. d+Au as control.

2010—Ridge in high multiplicity p+p (LHC)! Probably CGC!

Early 2010s—QGP in p+Pb!

Early 2010s—QGP in d+Au!

Mid 2010s and now-ish—QGP in high multiplicity p+p? QGP in mid-multiplicity p+p?? QGP in d+Au even at low energies???

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"Twenty years ago, the challenge in heavy ion physics was to find the QGP. Now, the challenge is to not find it." —Jürgen Schukraft, QM17

Intermission

Small systems geometry scan

Testing hydro by controlling system geometry

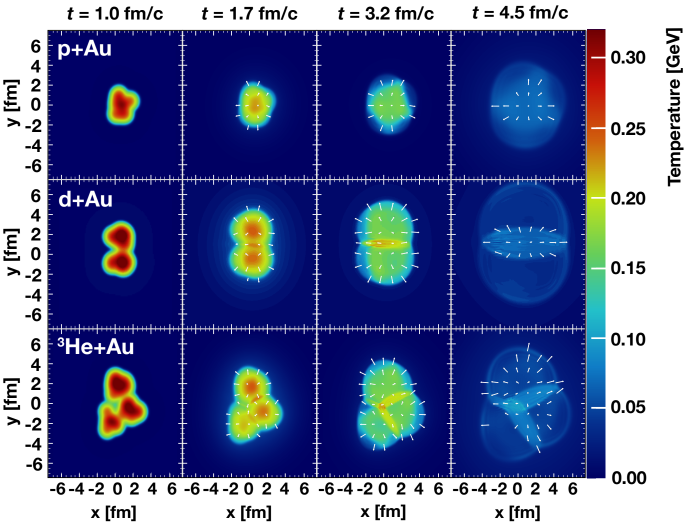
arXiv:1805.02973, submitted to Nature Physics

Hydrodynamics translates initial geometry into final state

Test hydro hypothesis by varying initial state

	ε_2	ε_3
p+Au	0.24	0.16
d+Au	0.57	0.17
^{3}He+Au	0.48	0.23

$$\varepsilon_2^{\text{p+Au}} < \varepsilon_2^{\text{d+Au}} \approx \varepsilon_2^{\text{3He+Au}}$$
$$\varepsilon_3^{\text{p+Au}} \approx \varepsilon_3^{\text{d+Au}} < \varepsilon_3^{\text{3He+Au}}$$



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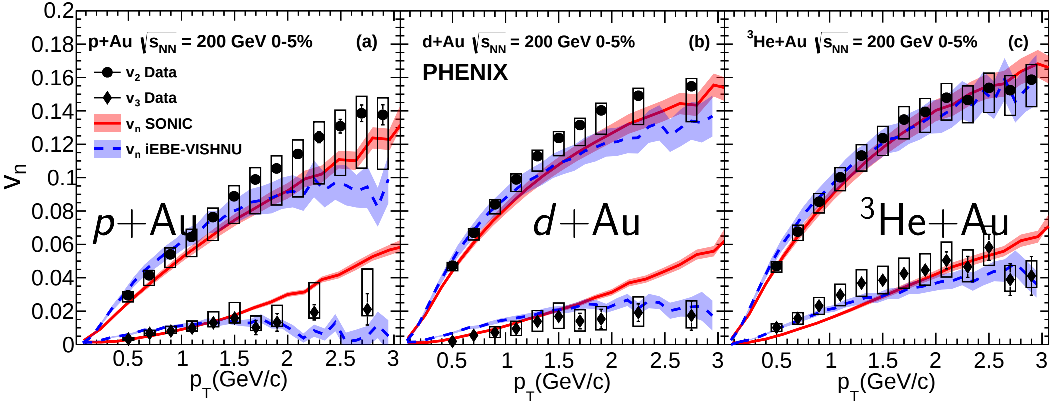


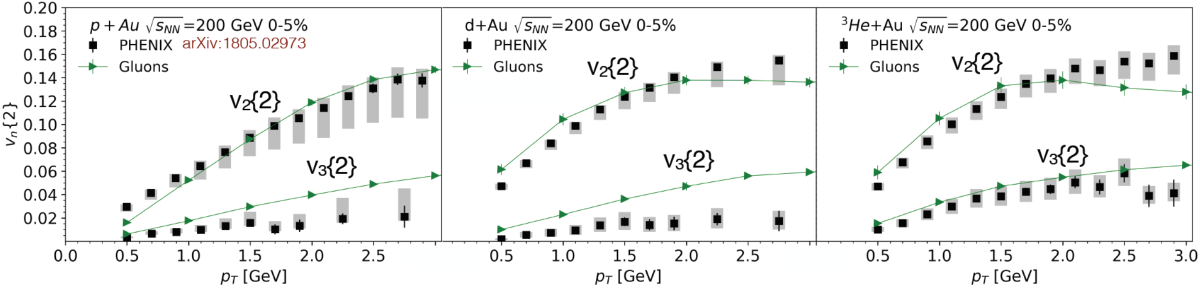
v_2 and v_3 vs p_T described very well by hydro in all three systems

—Suggests QGP droplets in hydro evolution

Hydro far from equilibrium—see Ryan's talk

CGC results on small systems

Mark Mace, Quark Matter 2018 arXiv:1805.09342, submitted to Phys. Rev. Lett.



v_2 is remarkably well-described

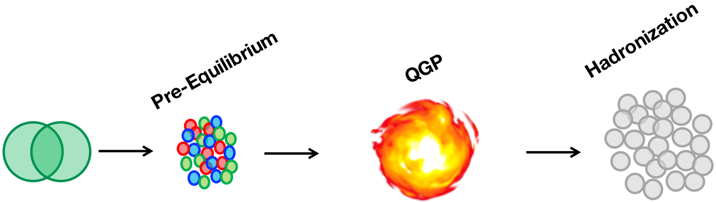
v_3 is also well-described, but hydro seems to do a bit better

More about CGC in small systems—Adrian and Vladi's talks

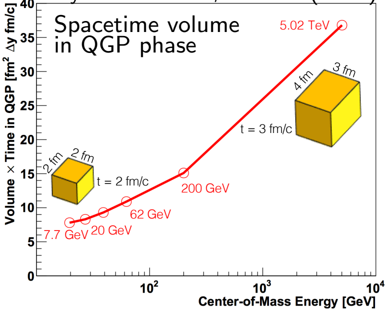
Intermission

Small systems beam energy scan

Testing hydro by controlling system size and life time



J.D. Orjuela Koop et al
Phys. Rev. C 93, 044910 (2016)

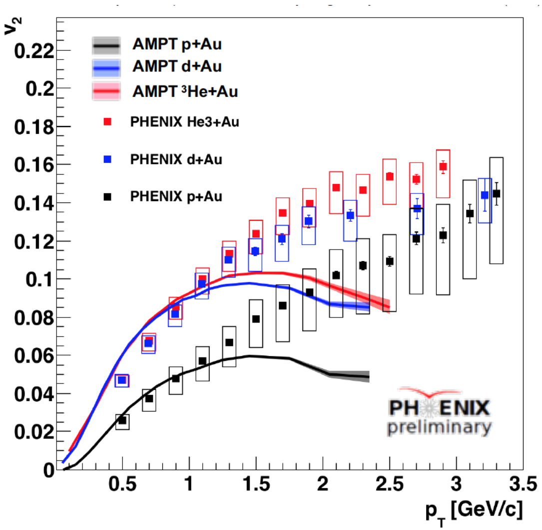


Standard picture for A+A: QGP in hydro evolution

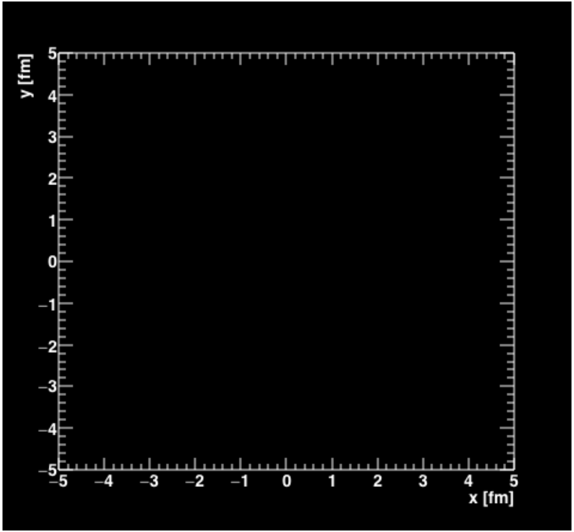
What about small systems? And lower energies?

Use collisions species and energy to control system size, test limits of hydro applicability

AMPT



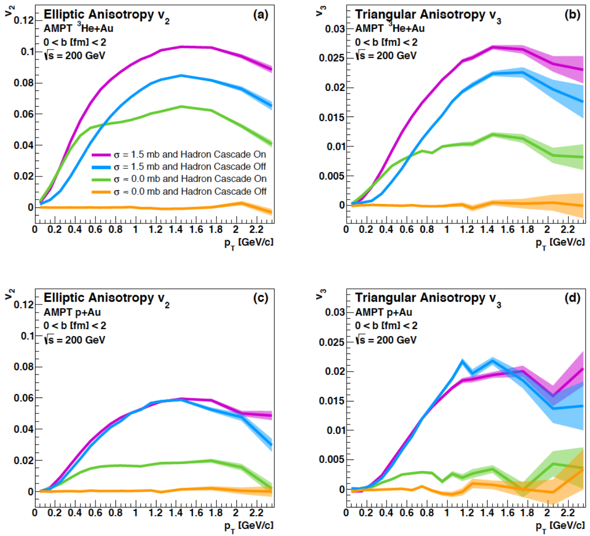
v_2
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.5
1
1.5
2
2.5
3
3.5
p_T [GeV/c]
AMPT p+Au
AMPT d+Au
AMPT ^{3}He+Au
PHENIX He3+Au
PHENIX d+Au
PHENIX p+Au
PHENIX
preliminary



y [fm]
x [fm]
5
4
3
2
1
0
−1
−2
−3
−4
−5

AMPT with no scattering

J.D. Orjuela Koop et al Phys. Rev. C 92, 054903 (2015)



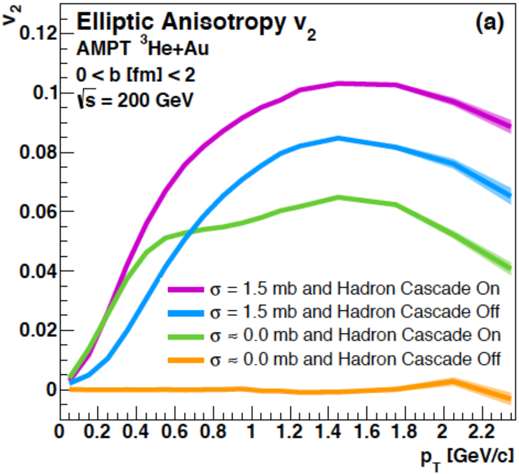
Turn off scattering in AMPT—remove all correlations with initial geometry $\sigma_{parton} = 0$ and $\sigma_{hadron} = 0$

Participant plane v_2 goes to zero

Other sources of correlation remain—non-flow

AMPT with no scattering

J.D. Orjuela Koop et al Phys. Rev. C 92, 054903 (2015)



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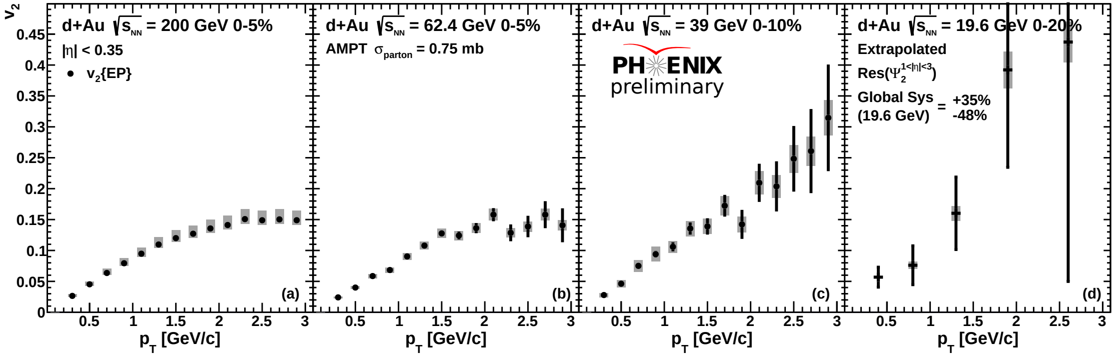
v_2 vs p_T, comparisons to AMPT

Phys. Rev. C 96, 064905 (2017)



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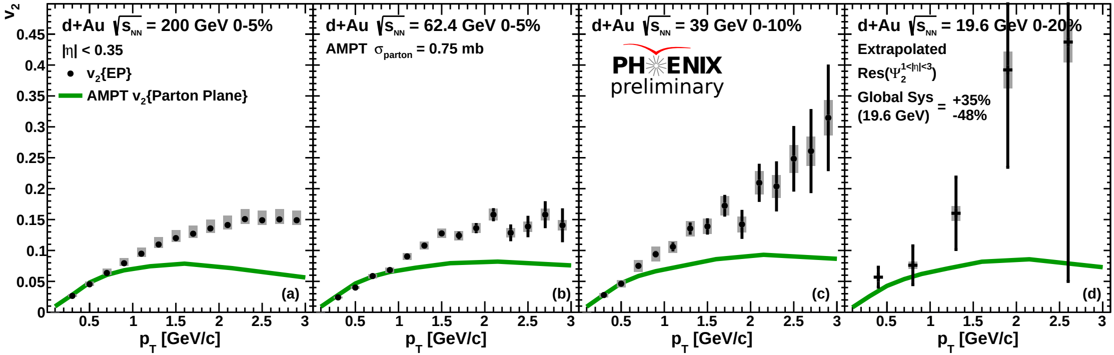

AMPT flow only shows good agreement at low p_T and all energies

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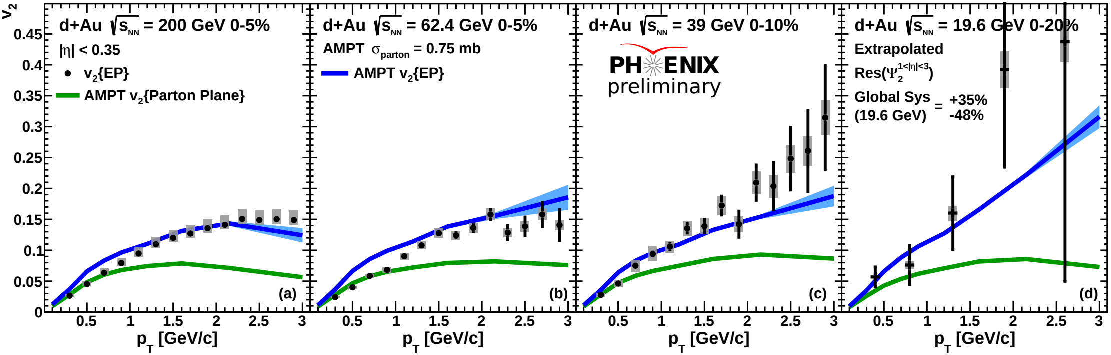

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AMPT flow+non-flow shows reasonable agreement for all p_T and all energies

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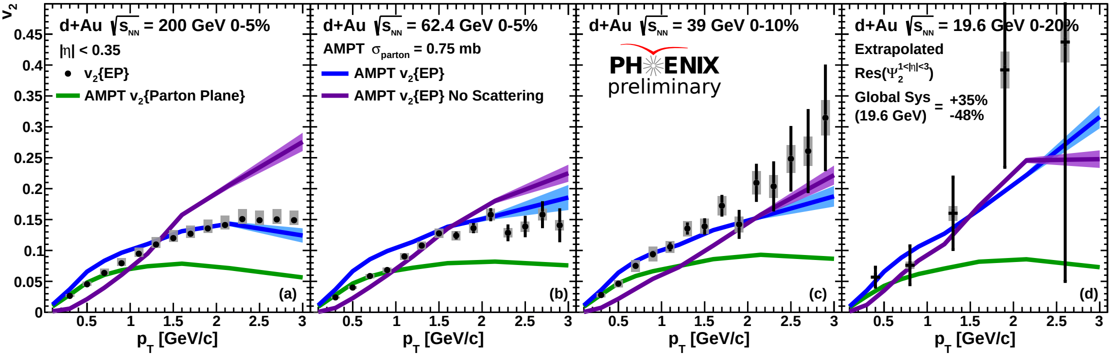


AMPT flow only shows good agreement at low p_T and all energies

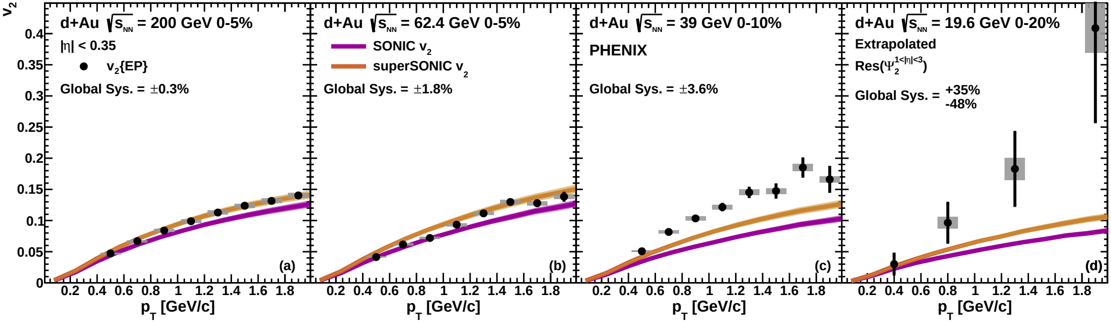
AMPT flow+non-flow shows reasonable agreement for all p_T and all energies

AMPT non-flow only far under-predicts for low p_T, too high for high p_T

d+Au beam energy scan

Phys. Rev. C 96, 064905 (2017)

200 GeV 62.4 GeV 39 GeV 19.6 GeV



Hydro theory agrees with higher energies very well, underpredicts lower energies—nonflow?

Intermission

Extremely small systems

Extremely small systems in hydro theory

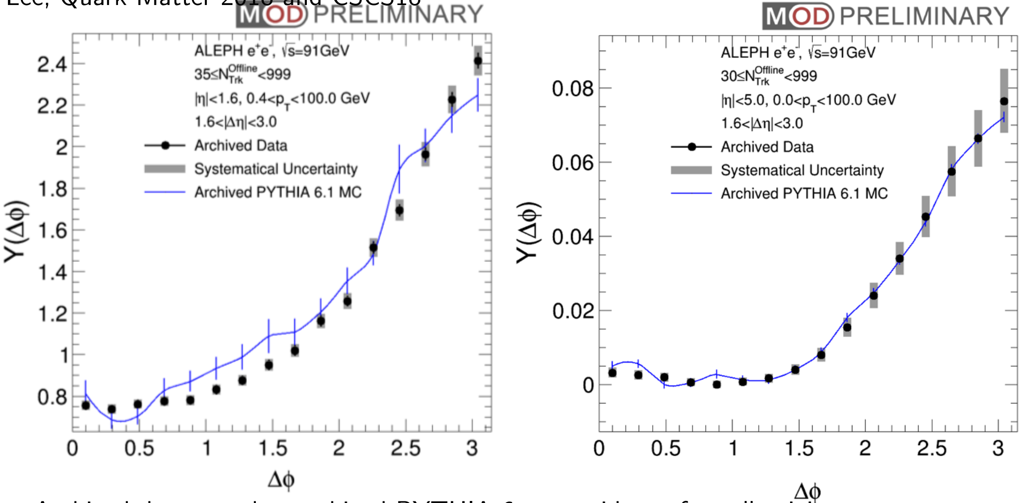
P. Romatschke, Eur. Phys. J. C 77, 21 (2017)

"I predict the breakdown of hydrodynamics at momenta of order seven times the temperature, corresponding to a smallest possible QCD liquid drop size of 0.15 fm."

"In view of the 'QGP drop size lower bound' of 0.15 fm, it is maybe not surprising that the matter created in $p+p$ collisions would behave hydrodynamically. At this scale, however, $p+p$ collisions may not be the ultimate drop size test. QCD-QED couplings allow fluctuations of electrons to e.g. quark pairs, thus opening up the possibility of local energy deposition reminiscent of $p+p$ collisions occurring in $e^{+}+e^{-}$ collisions (cf. Refs. [70–72]). Data on $e^{+}+e^{-}$ collisions taken at e.g. LEP should be re-analyzed with modern tools in order to find (or rule out) hydrodynamic behavior in these systems."

Extremely small systems at LEP

Y.-J. Lee, Quark Matter 2018 and CSCS18

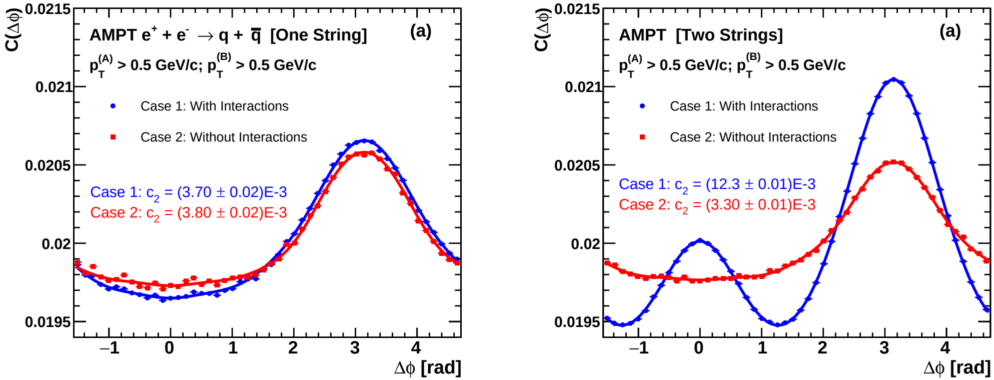


Archived data matches archived PYTHIA 6—no evidence for collectivity

Additional analysis with ALEPH (and OPAL) ongoing

Extremely small systems in AMPT

J.L. Nagle et al, Phys. Rev. C 97, 024909 (2018)



A single color string ($e^+ + e^-$ collisions) shows no sign of collectivity

Two color strings shows collectivity
—Small systems like $p/d/^3$He+Au have more

Final thoughts

Massive wealth of experimental data

Collectivity in A+A collisions well-established and widely accepted

Collectivity in small systems controversial and actively being researched

Three competing pictures:
—Hydro (collective)
—AMPT (collective)
—CGC (not collective)

Each of these has considerable success describing the data
—Clear need for additional observables to help discriminate

Extremely small systems, like $e^{+}+e^{-}$, $e+p$, $e+A$, may hold important information
—What will we find at the EIC?

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"The optimist regards the future as uncertain."—Eugene Wigner

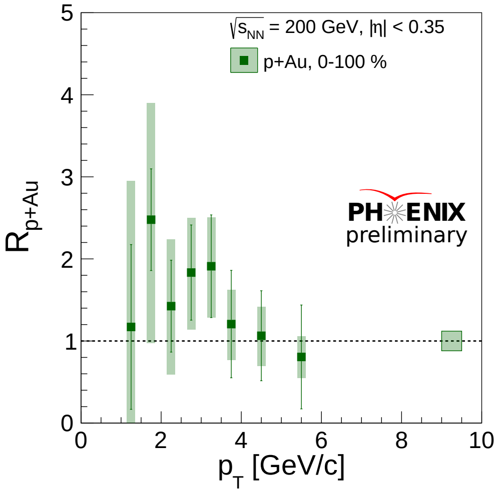
Intermission

Additional material

Intermission

Photons in small systems

Photons in small systems



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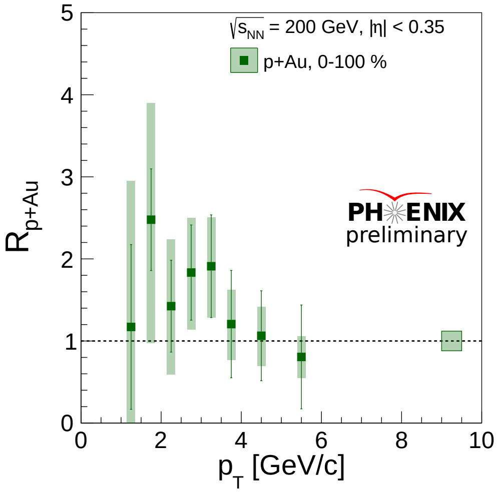
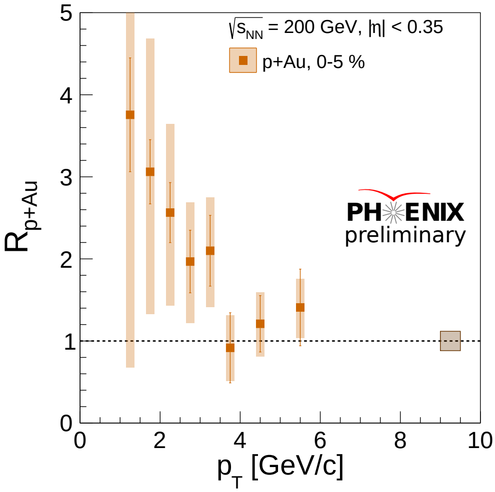
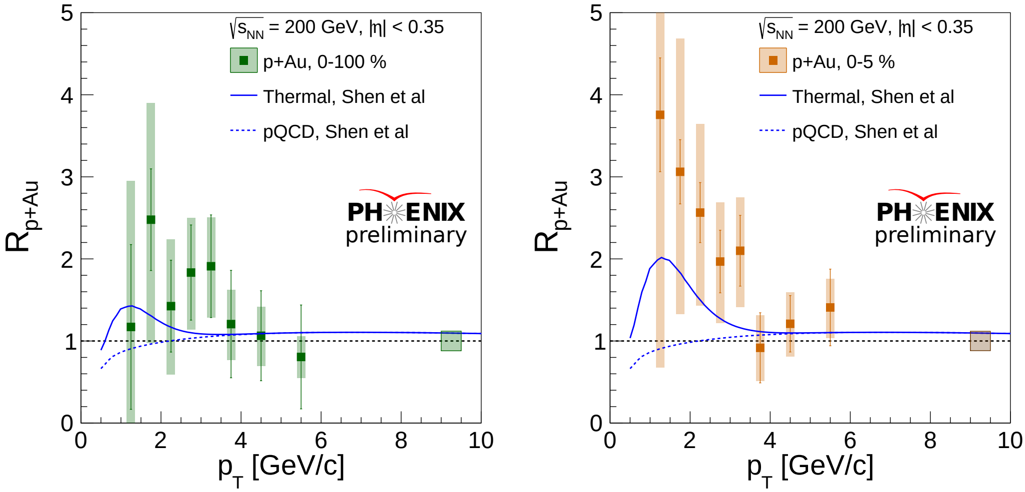




Thermal photons in p+Au?

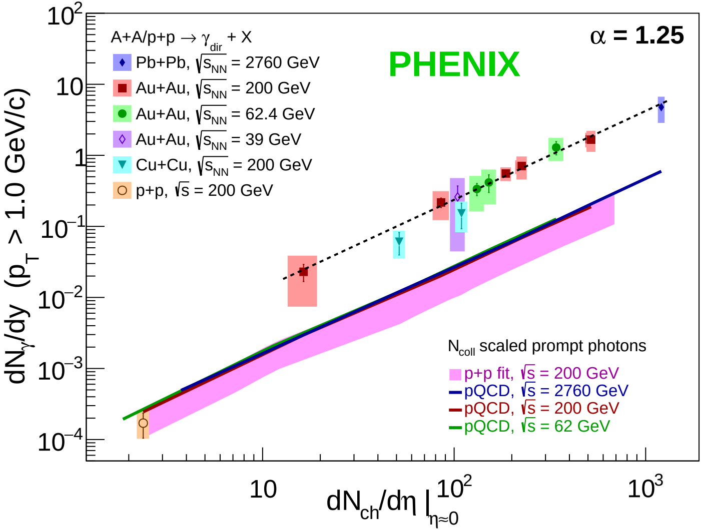
Photons in small systems



Thermal photons in p+Au? Theory from Phys. Rev. C 95, 014906 (2017)

Photon yields

arXiv:1805.04084, submitted to Phys. Rev. Lett.



Common scaling for Au+Au and Pb+Pb at different energies; very different from N_{coll}-scaled $p+p$

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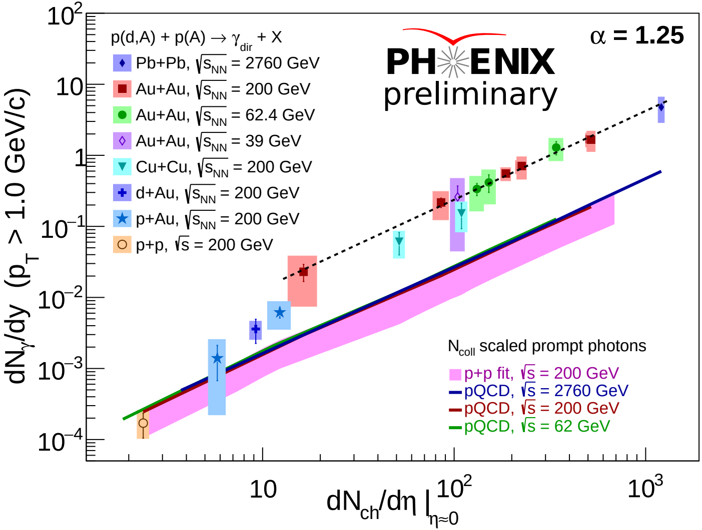

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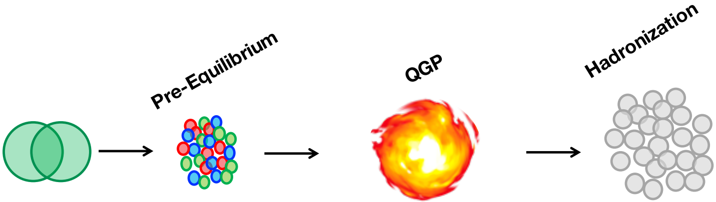
p/d+Au in between—onset of thermal radiation?

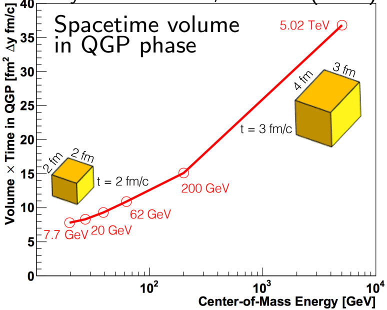
Intermission

Small systems beam energy scan

Testing hydro by controlling system size and life time



J.D. Orjuela Koop et al
Phys. Rev. C 93, 044910 (2016)



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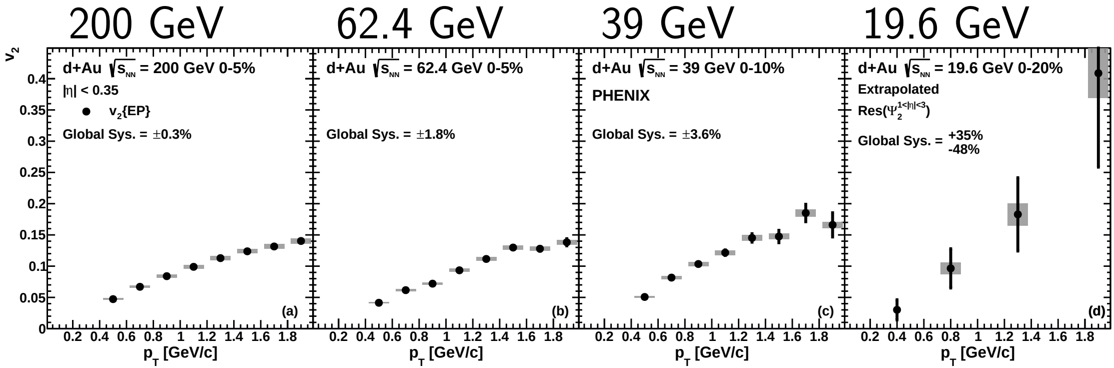
Phys. Rev. C 96, 064905 (2017)



Event plane v_2 vs p_T measured for all energies

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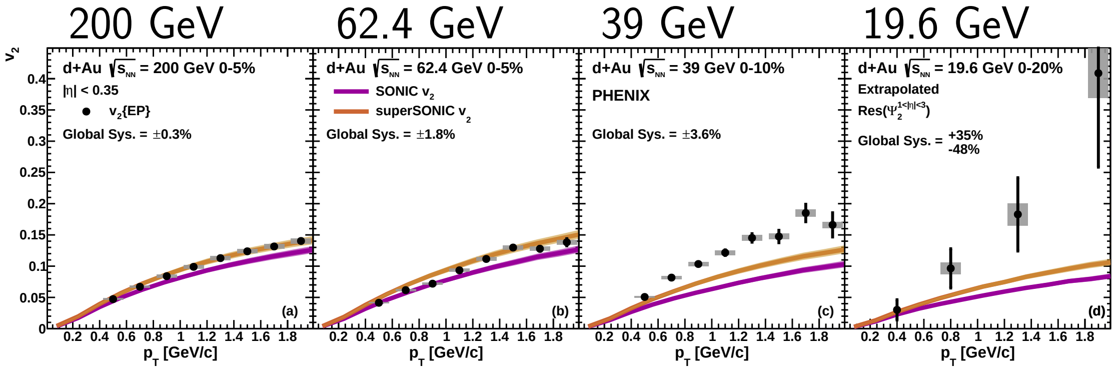

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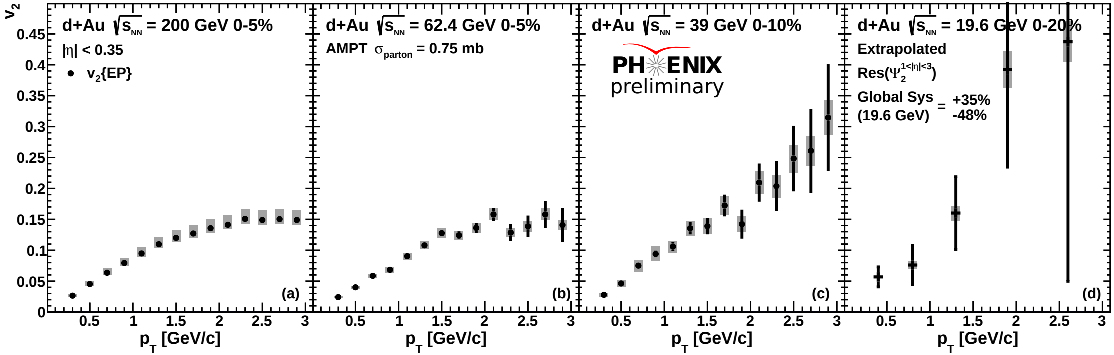
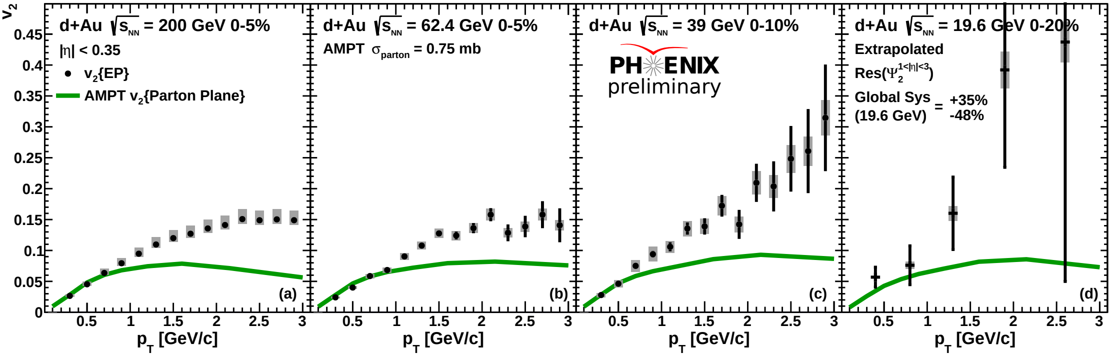


v_2 vs p_T, comparisons to AMPT

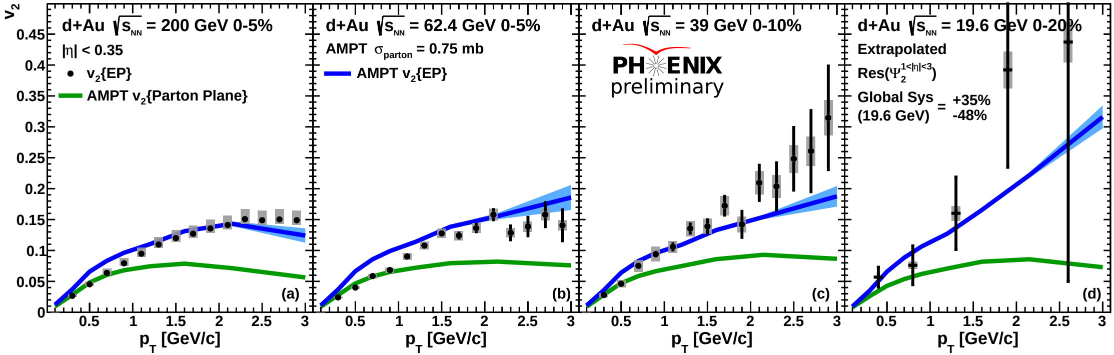
Phys. Rev. C 96, 064905 (2017)



AMPT flow only shows good agreement at low p_T and all energies

v_2 vs p_T, comparisons to AMPT

Phys. Rev. C 96, 064905 (2017)

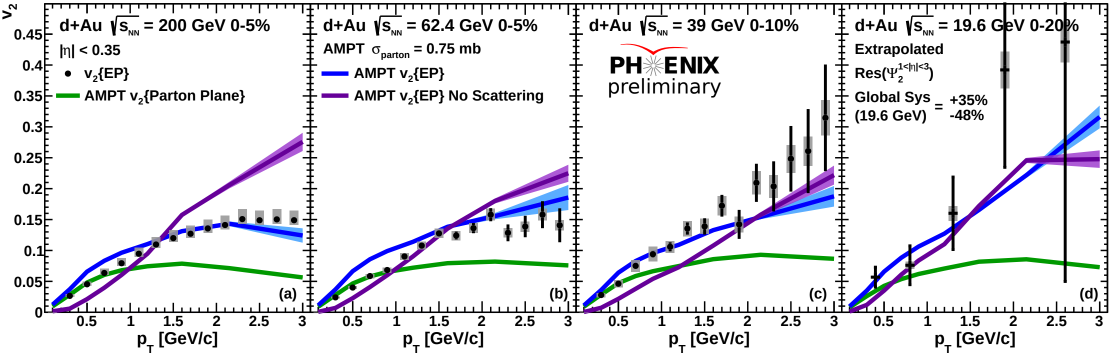


AMPT flow only shows good agreement at low p_T and all energies

AMPT flow+non-flow shows reasonable agreement for all p_T and all energies

v_2 vs p_T, comparisons to AMPT

Phys. Rev. C 96, 064905 (2017)



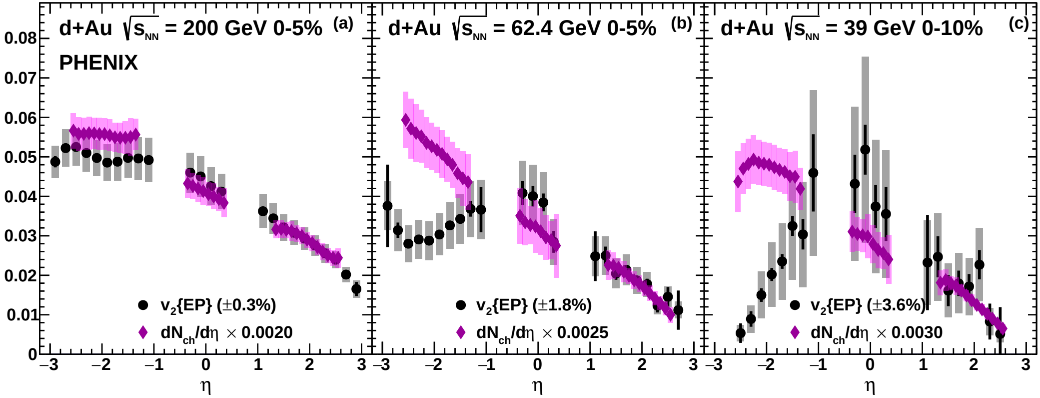
AMPT flow only shows good agreement at low p_T and all energies

AMPT flow+non-flow shows reasonable agreement for all p_T and all energies

AMPT non-flow only far under-predicts for low p_T, too high for high p_T

v_2 and $dN_{ch}/d\eta$ vs η

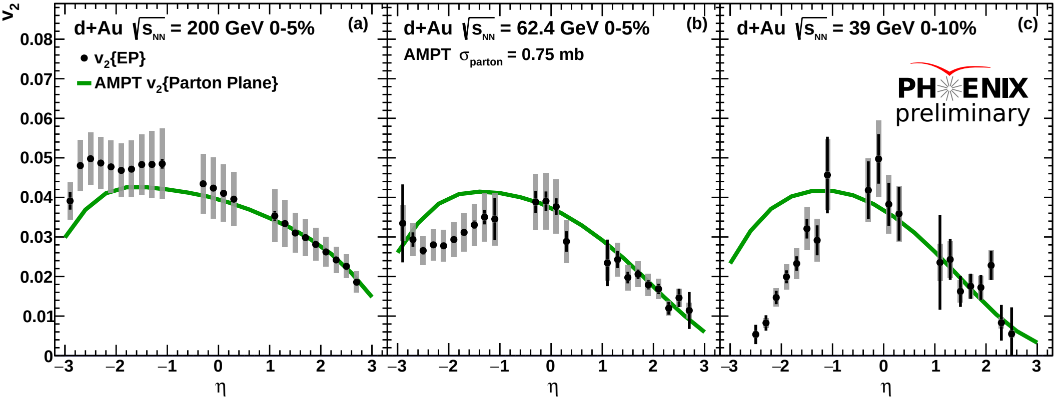
Phys. Rev. C 96, 064905 (2017)



- BBC south ($-3.9 < \eta < -3.1$) used to estimate the event plane
- 200 GeV shows strong forward/backward asymmetry in v_2 and $dN_{ch}/d\eta$
- Asymmetry is large for $dN_{ch}/d\eta$ at all energies, but not for v_2

v_2 vs η, comparison with AMPT

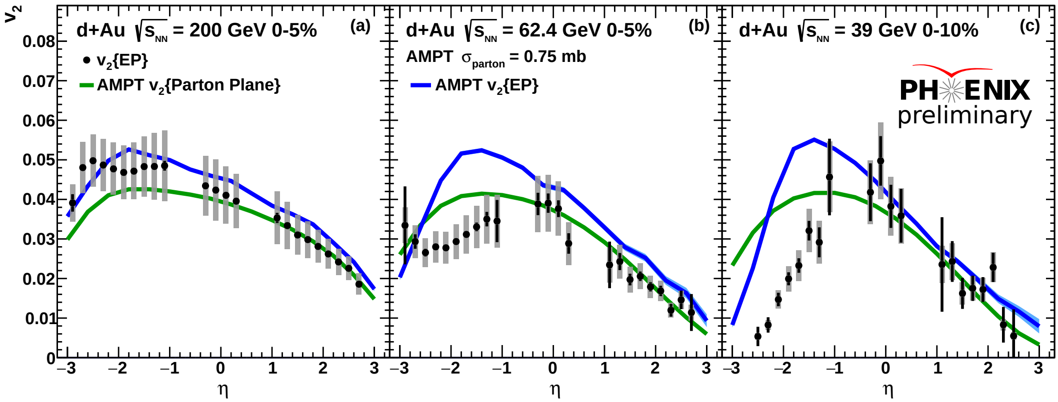
Phys. Rev. C 96, 064905 (2017)



AMPT flow only agrees with mid and forward rapidity very well, misses backward rapidity

v_2 vs η, comparison with AMPT

Phys. Rev. C 96, 064905 (2017)



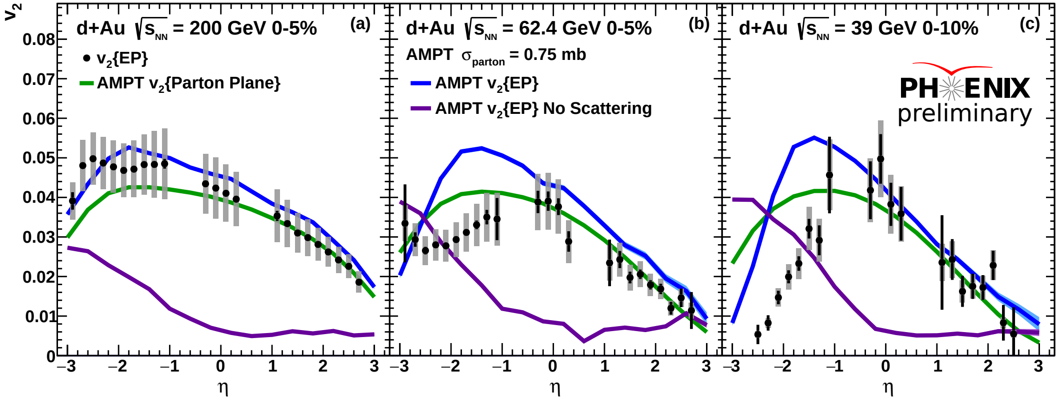
AMPT flow only agrees with mid and forward rapidity very well, misses backward rapidity

AMPT flow+non-flow is very similar at mid and forward

AMPT flow+non-flow shows striking anti-correlation at backward rapidity

v_2 vs η, comparison with AMPT

Phys. Rev. C 96, 064905 (2017)



AMPT flow only agrees with mid and forward rapidity very well, misses backward rapidity

AMPT flow+non-flow is very similar at mid and forward

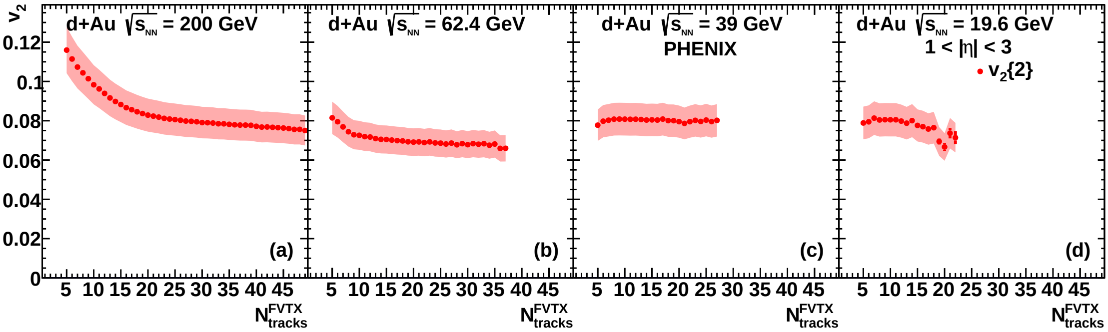
AMPT flow+non-flow shows striking anti-correlation at backward rapidity

AMPT non-flow only shows nothing at mid and forward, large v_2 at backward rapidity near the

d+Au beam energy scan

Phys. Rev. Lett. 120, 062302 (2018)

200 GeV 62.4 GeV 39 GeV 19.6 GeV



d+Au beam energy scan

Phys. Rev. Lett. 120, 062302 (2018)

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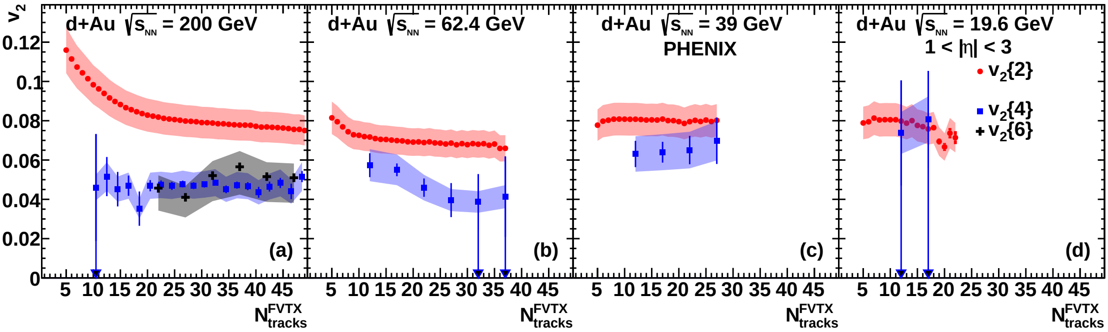


Measurement of $v_2\{6\}$ in d+Au at 200 GeV and $v_2\{4\}$ in d+Au at all energies

d+Au beam energy scan

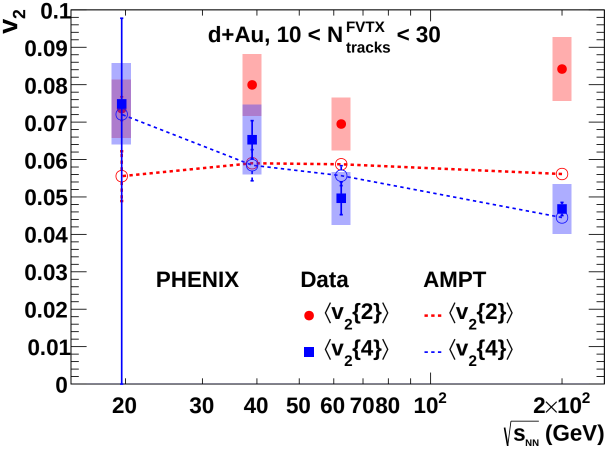
Phys. Rev. Lett. 120, 062302 (2018)

Select $10 < N^{\mathrm{FVTX}}_{\mathrm{tracks}} < 30$, integrate

AMPT sees similar trend

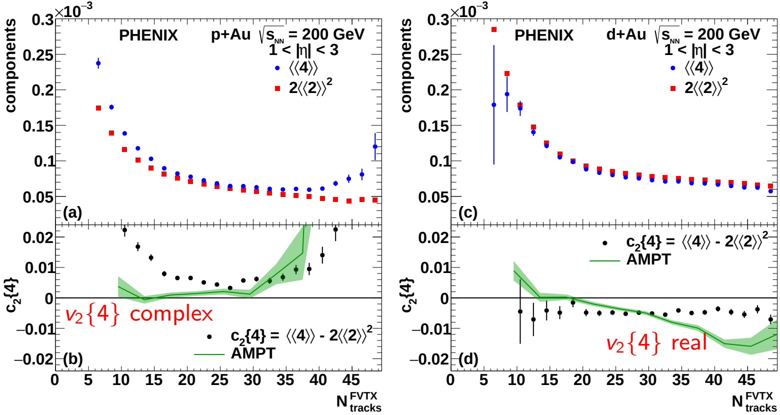
Fluctuations?

- Not Bessel-Gaussian
- Not small-variance limit
- Need to understand fluctuations better



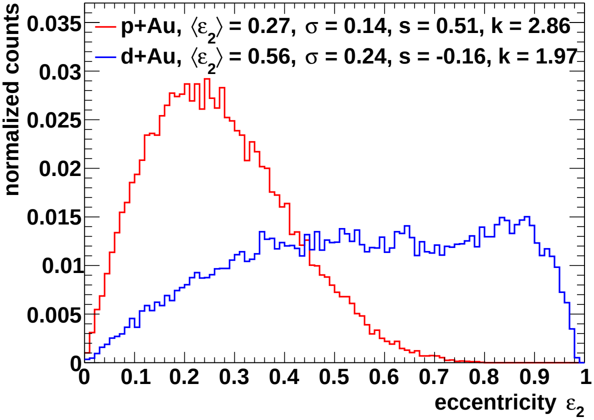
Components and cumulants in p+Au and d+Au at 200 GeV

Phys. Rev. Lett. 120, 062302 (2018)



Is the sign of $c_2\{4\}$ a good indicator of collectivity? No.
Fluctuations could dominate in the p+Au...

Eccentricity distributions and cumulants



$$\varepsilon_2\{4\} = (\varepsilon_2^4 - 2\varepsilon_2^2\sigma^2 - 4\varepsilon_2 s\sigma^3 - (k-2)\sigma^4)^{1/4}$$

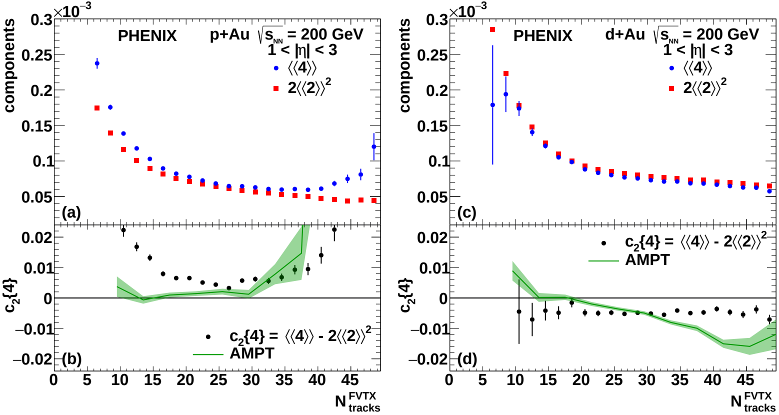
	p+Au	d+Au
ε_2^4	0.00531	0.0983
$2\varepsilon_2^2\sigma^2$	0.00277	0.0370
$4\varepsilon_2 s\sigma^3$	0.00147	-0.0053
$(k-2)\sigma^4$	0.00031	-0.0001

the variance brings $\varepsilon_2\{4\}$ down (this term gives the usual $\sqrt{v_2^2 - \sigma^2}$)

positive skew brings $\varepsilon_2\{4\}$ further down, negative skew brings it back up

kurtosis > 2 brings $\varepsilon_2\{4\}$ further down, kurtosis < 2 brings it back up
—recall Gaussian has kurtosis $= 3$

Eccentricity distributions and cumulants



$$v_2\{4\} = \left(v_2^4 - 2v_2^2\sigma^2 - 4v_2 s\sigma^3 - (k-2)\sigma^4\right)^{1/4}$$

Eccentricity fluctuations alone go a long way towards explaining this

Additional fluctuations in the (imperfect) translation of ε_2 to v_2?

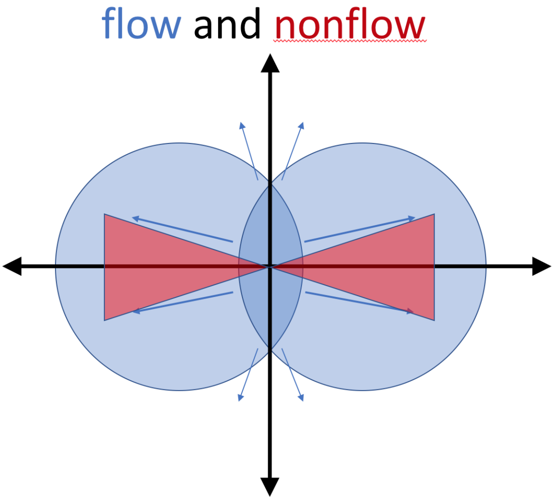
Brief summary: small systems beam energy scan

Measurement of v_2 vs p_T for d+Au at 200, 62.4, 39, and 19.6 GeV
—Hydro describes higher two energies well, misses lower two energies
—AMPT describes all data well with mix of flow and nonflow

Measurement of v_2 vs η for d+Au at 200, 62.4, and 39 GeV
—Hydro theory at lower energies would be very useful
—Interesting anticorrelation between flow and nonflow at backward rapidity

Measurement of $v_2\{6\}$ at 200 GeV and $v_2\{4\}$ at all four energies
—Nonflow should be combinatorially suppressed
—Highly non-trivial fluctuations

Multi-particle correlations



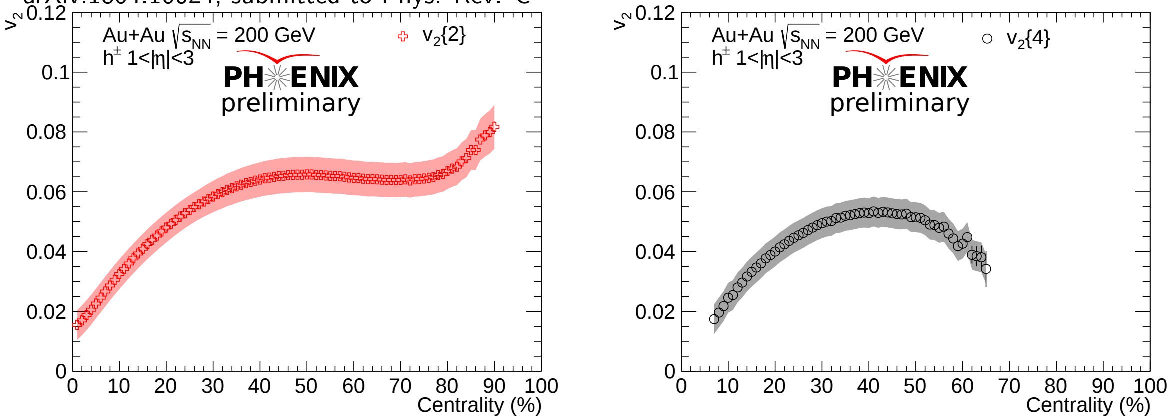
$$v_n = \langle \cos(n(\phi_{\text{some particle}} - \psi_n)) \rangle$$
$$v_n^2 = \langle \cos(n(\phi_{\text{some particle}} - \phi_{\text{some other particle}})) \rangle$$

How to deal with "fake flow"?
—Kinematics
—Combinatorics

$$v_n^2 = \langle \cos(n(\phi_a - \phi_b)) \rangle$$
$$v_n^4 = \langle \cos(n(\phi_a + \phi_b - \phi_c - \phi_d)) \rangle$$
$$v_n^6 = \langle \cos(n(\phi_a + \phi_b + \phi_c - \phi_d - \phi_e - \phi_f)) \rangle$$
$$v_n^8 = \ldots$$

Nonflow approaches in AuAu

arXiv:1804.10024, submitted to Phys. Rev. C



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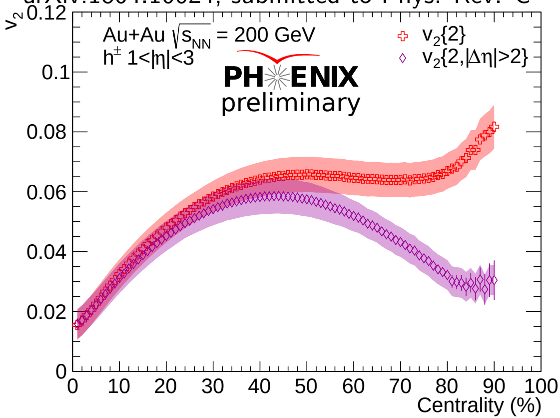
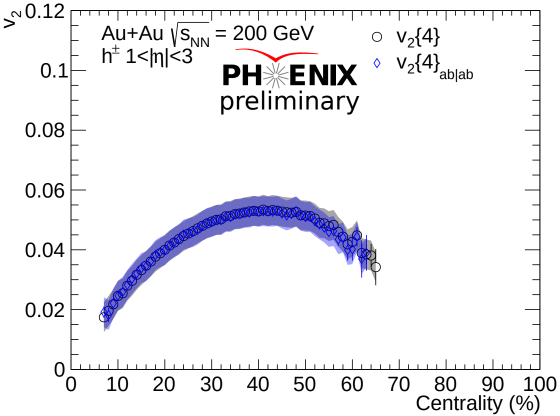




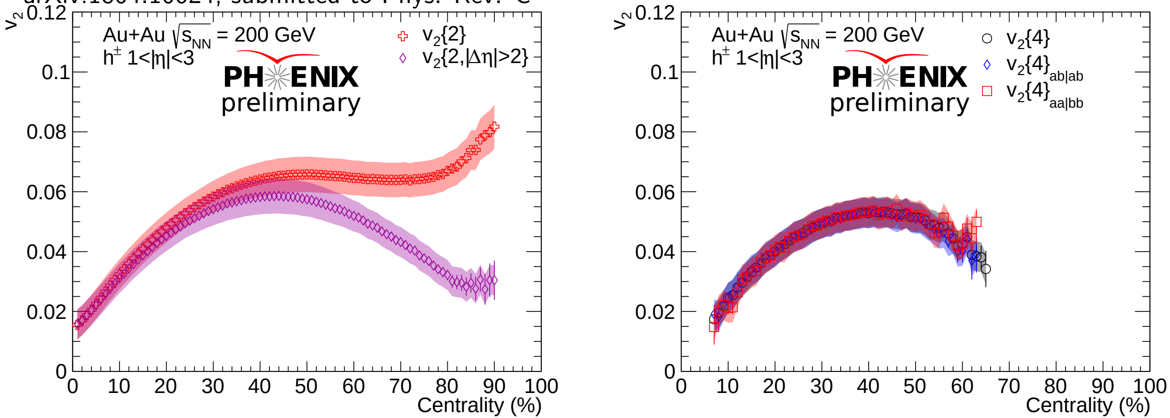
Large pseudorapidity separation

—Big difference for 2-particle (good)

—No difference for 4-particle (good)

Nonflow approaches in AuAu

arXiv:1804.10024, submitted to Phys. Rev. C



Large pseudorapidity separation

—Big difference for 2-particle (good)

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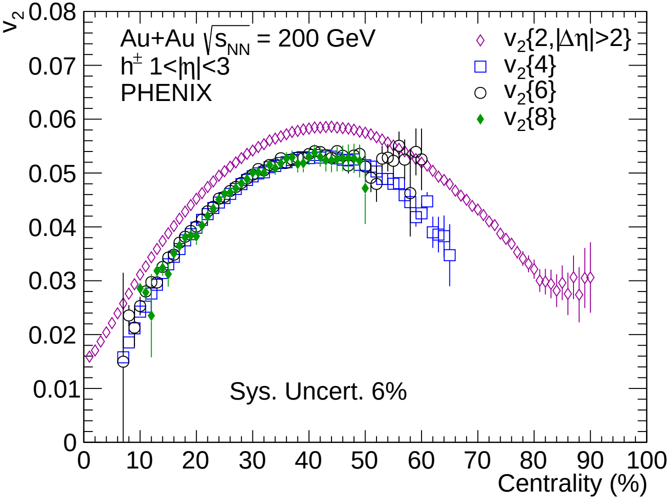
Collectivity in large systems

arXiv:1804.10024 (submitted to Phys. Rev. C)



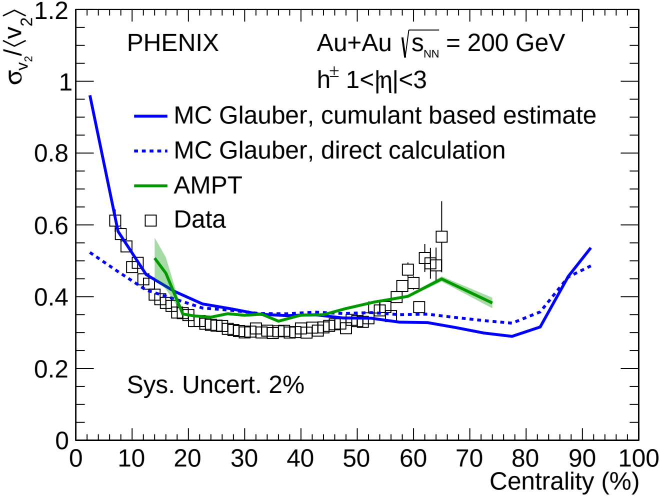
$$1 < |\eta| < 3$$

$v_2\{2\}$, $v_2\{4\}$, $v_2\{6\}$,

$v_2\{8\}$

Collectivity in large systems

arXiv:1804.10024 (submitted to Phys. Rev. C)



$1 < |\eta| < 3$

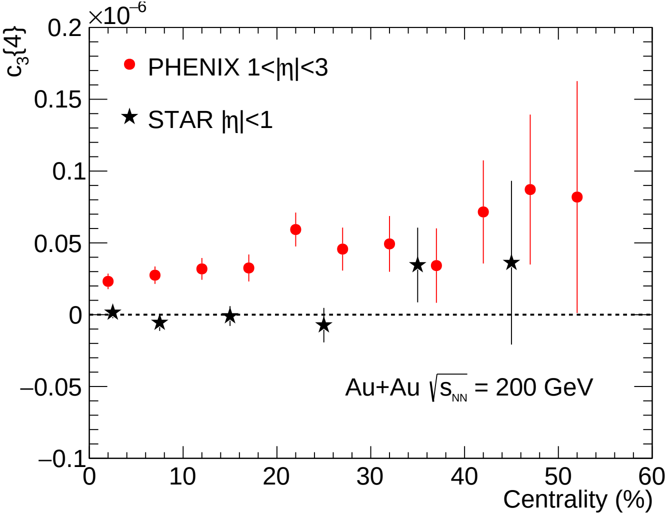
$\sigma_{v_2}/\langle v_2 \rangle$

Central: breakdown of small-variance limit

Peripheral: non-linearity in hydro response (e.g. J. Noronha-Hostler et al Phys. Rev. C 93, 014909 (2016))

Collectivity in large systems

arXiv:1804.10024 (submitted to Phys. Rev. C)



$1 < |\eta| < 3$

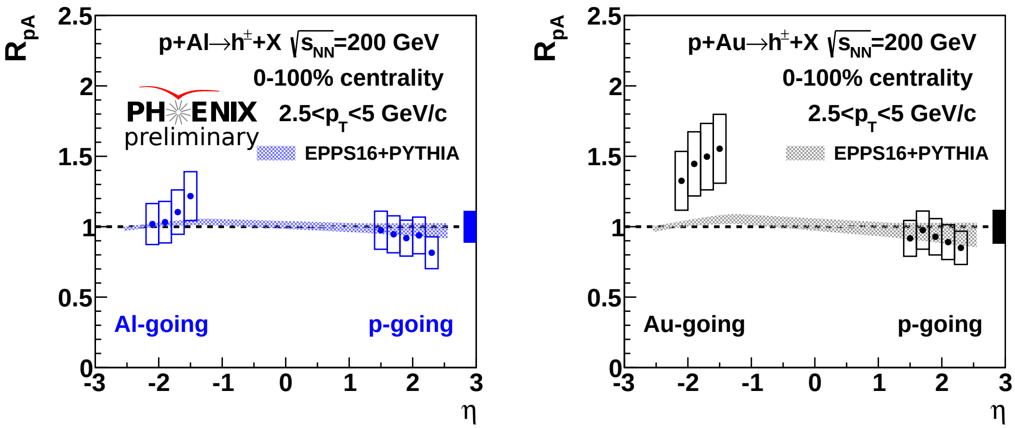
Cannot extract

$\sigma_{v_3} / \langle v_3 \rangle$

Intermission

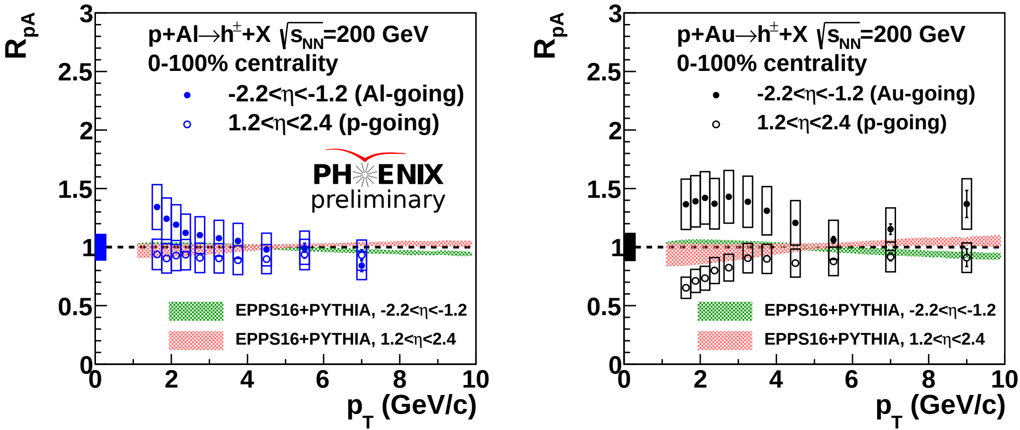
Particle production in small systems

Small systems nuclear modification



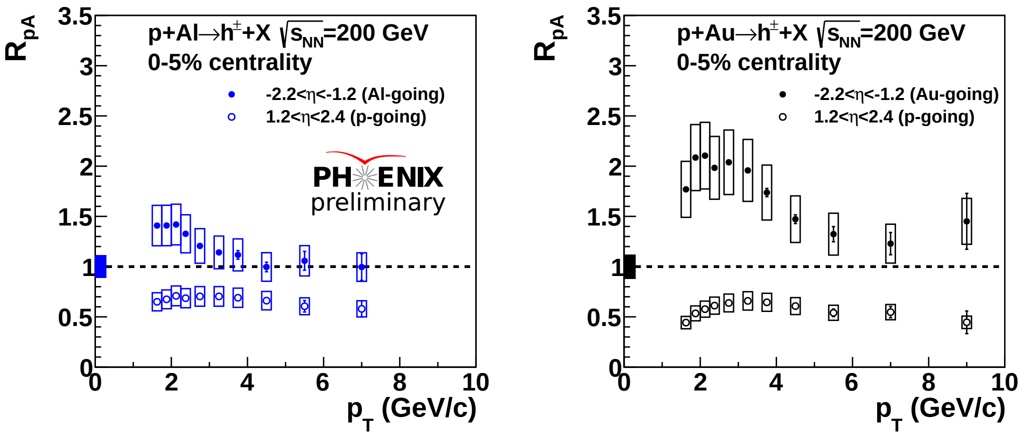
Forward modification consistent with nPDF effects (EPPS16)

Small systems nuclear modification



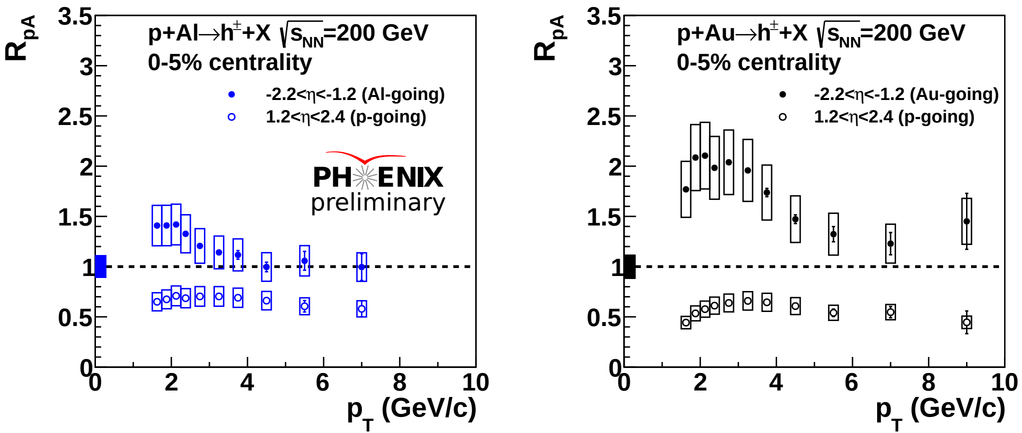
High-p_T modification consistent with nPDF effects (EPPS16)

Small systems nuclear modification



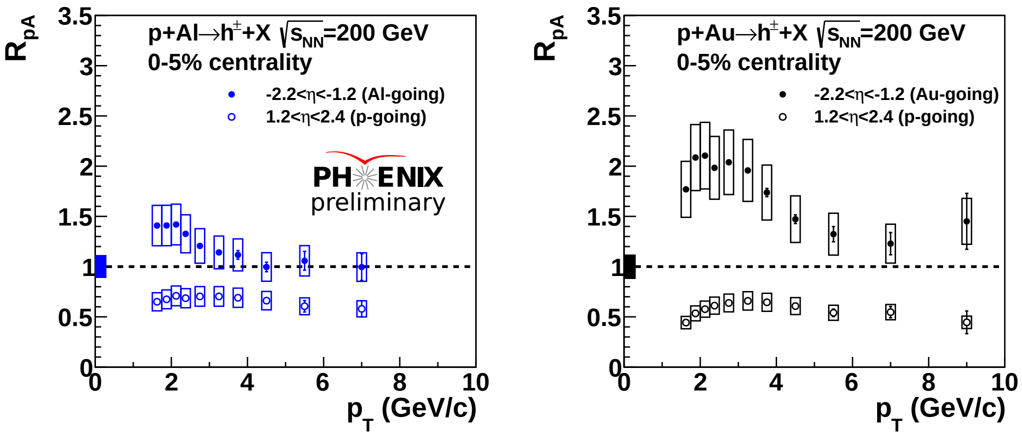
Stronger effects in central collisions

Small systems nuclear modification



Strong enhancement for backward at intermediate p_T—why?

Small systems nuclear modification

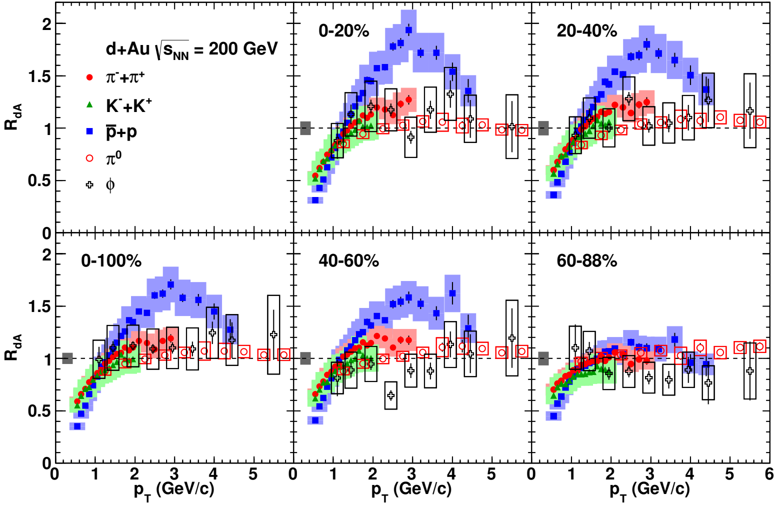


Strong enhancement for backward at intermediate p_T—why?

Don't forget: particle species dependence of Cronin! There must be final state effect(s)...

Particle species dependence of "Cronin enhancement"

PHENIX, Phys. Rev. C 88, 024906 (2013)



π^+, π^-, π^0,

K^+, K^-,

p, $\bar{p}$,

ϕ

Protons much more strongly modified than pions

ϕ consistent with other mesons

Brief summary: particle production in small systems

Strong modifications at forward & backward rapidities
—Not nPDF effects alone
—Additional initial state effects possible (e.g. the usual multiple scattering)

Nuclear modification strongly dependent on particle species
—Must be final state effect(s)
—Hadronization, radial flow, etc...

Back to basics (a brief excursion)

The (raw) moments of a probability distribution function $f(x)$:

$$\mu_n = \langle x^n \rangle \equiv \int_{-\infty}^{+\infty} x^n f(x) dx$$

The moment generating function:

$$M_x(t) \equiv \langle e^{tx} \rangle = \int_{-\infty}^{+\infty} e^{tx} f(x) dx = \int_{-\infty}^{+\infty} \sum_{n=0}^{\infty} \frac{t^n}{n!} x^n f(x) dx = \sum_{n=0}^{\infty} \mu_n \frac{t^n}{n!}$$

Moments from the generating function:

$$\mu_n = \left. \frac{d^n M_x(t)}{dt^n} \right|_{t=0}$$

Key point: the moment generating function uniquely describe $f(x)$

Back to basics (a brief excursion)

Can also uniquely describe $f(x)$ with the cumulant generating function:

$$K_x(t) \equiv \ln M_x(t) = \sum_{n=0}^{\infty} \kappa_n \frac{t^n}{n!}$$

Cumulants from the generating function:

$$\kappa_n = \left. \frac{d^n K_x(t)}{dt^n} \right|_{t=0}$$

Since $K_x(t) = \ln M_x(t)$, $M_x(t) = \exp(K_x(t))$, so

$$\mu_n = \left. \frac{d^n \exp(K_x(t))}{dt^n} \right|_{t=0}, \quad \kappa_n = \left. \frac{d^n \ln M_x(t)}{dt^n} \right|_{t=0}$$

End result: (details left as an exercise for the interested reader)

$$\mu_n = \sum_{k=1}^{n} B_{n,k}(\kappa_1, ..., \kappa_{n-k+1}) \qquad = B_n(\kappa_1, ..., \kappa_{n-k+1})$$

$$\kappa_n = \sum_{k=1}^{n} (-1)^{k-1}(k-1)! B_{n,k}(\mu_1, ..., \mu_{n-k+1}) \qquad = L_n(\kappa_1, ..., \kappa_{n-k+1})$$

Back to basics (a brief excursion)

Evaluating the Bell polynomials gives

$$\begin{aligned}
\langle x \rangle &= \kappa_1 \\
\langle x^2 \rangle &= \kappa_2 + \kappa_1^2 \\
\langle x^3 \rangle &= \kappa_3 + 3\kappa_1\kappa_2 + \kappa_1^3 \\
\langle x^4 \rangle &= \kappa_4 + 4\kappa_1\kappa_3 + 3\kappa_2^2 + 6\kappa_1^2\kappa_2 + \kappa_1^4
\end{aligned}$$

One can tell by inspection (or derive explicitly) that κ_1 is the mean, κ_2 is the variance, etc.

Back to basics (a brief excursion)

Subbing in $x = v_n$, $\kappa_2 = \sigma^2$, we find

$$\left(\langle v_n^4\rangle = v_n^4 + 6v_n^2\sigma^2 + 3\sigma^4 + 4v_n\kappa_3 + \kappa_4\right)$$
$$-\left(2\langle v_n^2\rangle^2 = 2v_n^4 + 4v_n^2\sigma^2 + 2\sigma^4\right)$$
$$\rightarrow$$
$$\langle v_n^4\rangle - 2\langle v_n^2\rangle^2 = -v_n^4 + 2v_n^2\sigma^2 + \sigma^4 + 4v_n\kappa_3 + \kappa_4$$

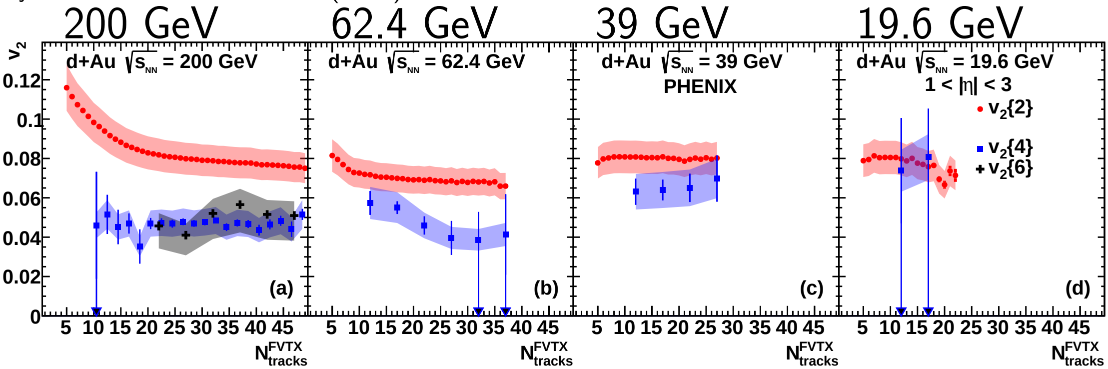
Skewness s: $\kappa_3 = s\sigma^3$
Kurtosis k: $\kappa_4 = (k-3)\sigma^4$

$$v_n\{2\} = (v_n^2 + \sigma^2)^{1/2}$$
$$v_n\{4\} = (v_n^4 - 2v_n^2\sigma^2 - 4v_n s\sigma^3 - (k-2)\sigma^4)^{1/4}$$

So the fully general form is a bit more complicated than we tend to think...

d+Au beam energy scan

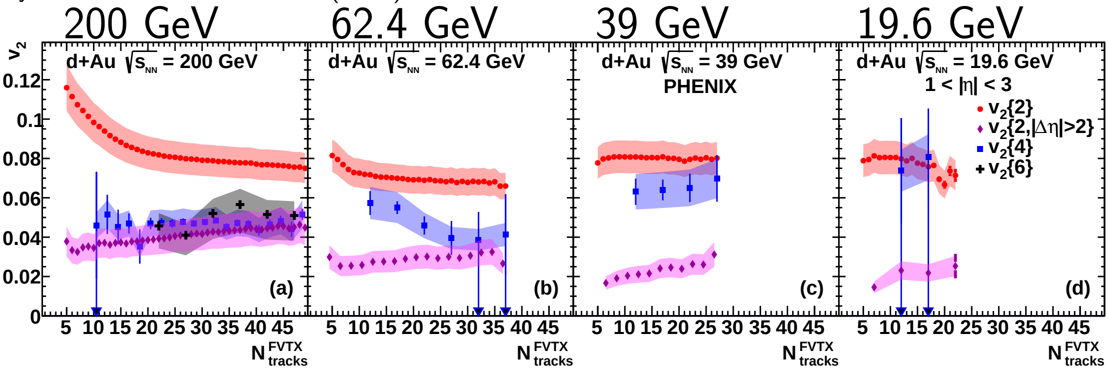
Phys. Rev. Lett. 120, 062302 (2018)



$v_2\{2\}$ and $v_2\{4\}$ vs $N_{\text{tracks}}^{\text{FVTX}}$, all tracks anywhere in FVTX

d+Au beam energy scan

Phys. Rev. Lett. 120, 062302 (2018)

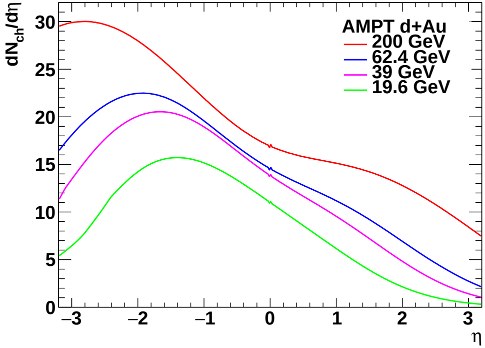


$v_2\{2\}$ and $v_2\{4\}$ vs $N_{\text{tracks}}^{\text{FVTX}}$, all tracks anywhere in FVTX

$v_2\{2, |\Delta\eta| > 2\}$ vs $N_{\text{tracks}}^{\text{FVTX}}$, one track backward, the other forward

How is $v_2\{4\} > v_2\{2, |\Delta\eta| > 2\}$ possible? Can blame fluctuations to a point, but...

d+Au beam energy scan

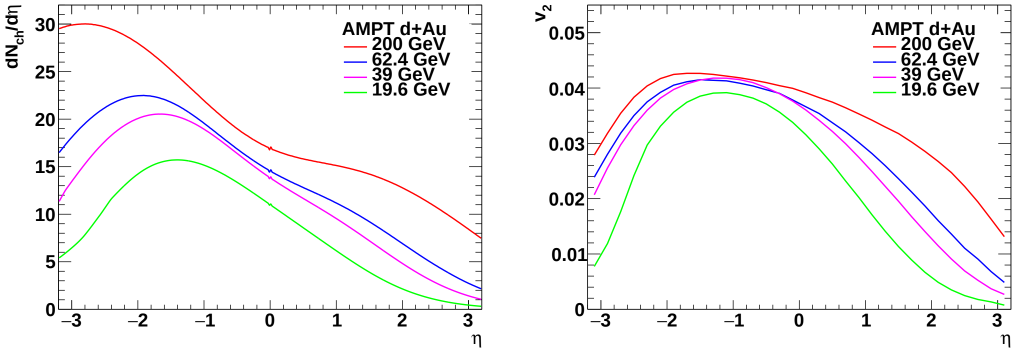


Asymmetric $dN_{\text{ch}}/d\eta$ and asymmetric v_2 vs η

The single subevent is weighted by $dN_{\text{ch}}/d\eta$ towards backward rapidity, where v_2 is also higher—the effect is more pronounced at lower energies

The two subevent is equally weighted between forward and back: $\sqrt{\langle v_2^B v_2^F \rangle}$

d+Au beam energy scan



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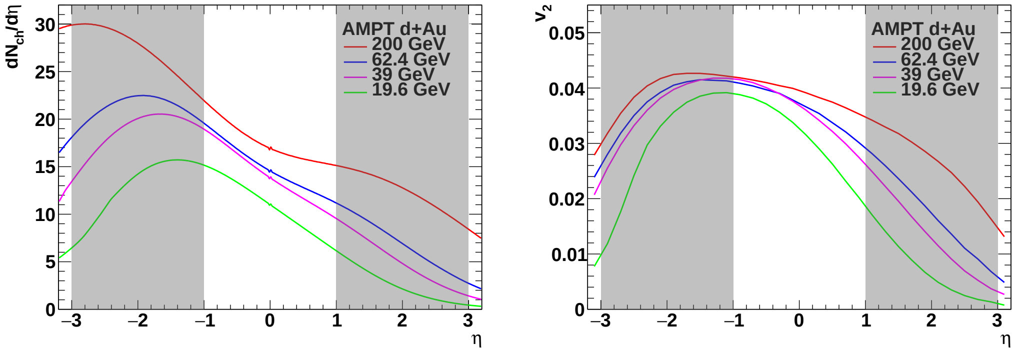

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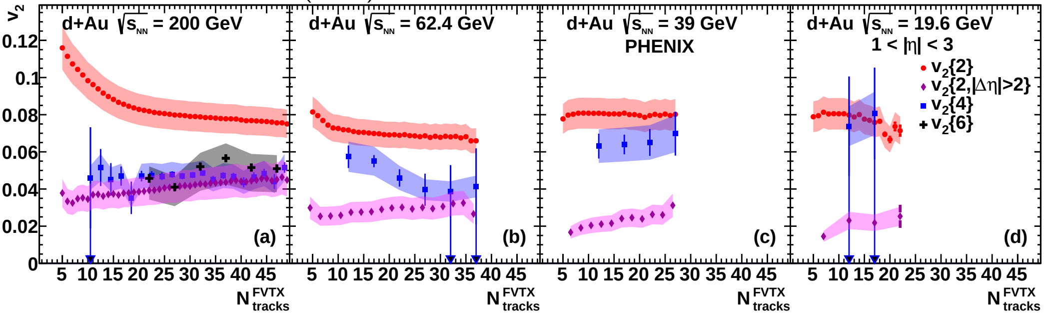
d+Au beam energy scan

Phys. Rev. Lett. 120, 062302 (2018)



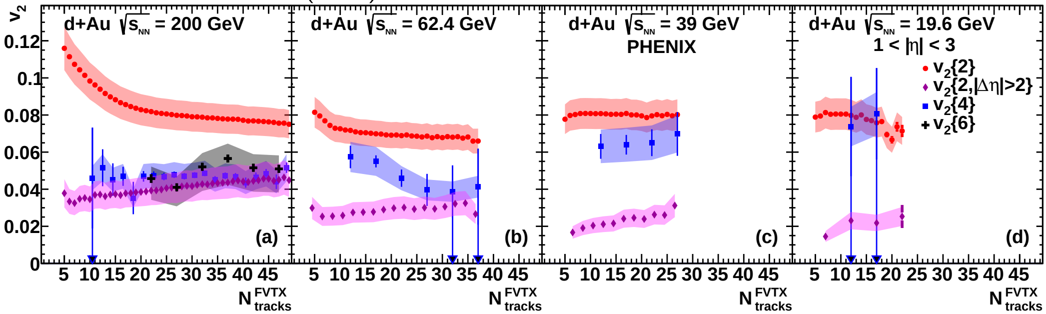
$v_2\{2\}$ and $v_2\{4\}$ vs $N_{\text{tracks}}^{\text{FVTX}}$—weighted average of v_2^B and v_2^F

$v_2\{2, |\Delta\eta| > 2\}$ vs $N_{\text{tracks}}^{\text{FVTX}}$—fixed, equal weighting $\sqrt{\langle v_2^B v_2^F \rangle}$

$dN_{\text{ch}}/d\eta$ and v_2 vs η alone may explain these results

d+Au beam energy scan

Phys. Rev. Lett. 120, 062302 (2018)



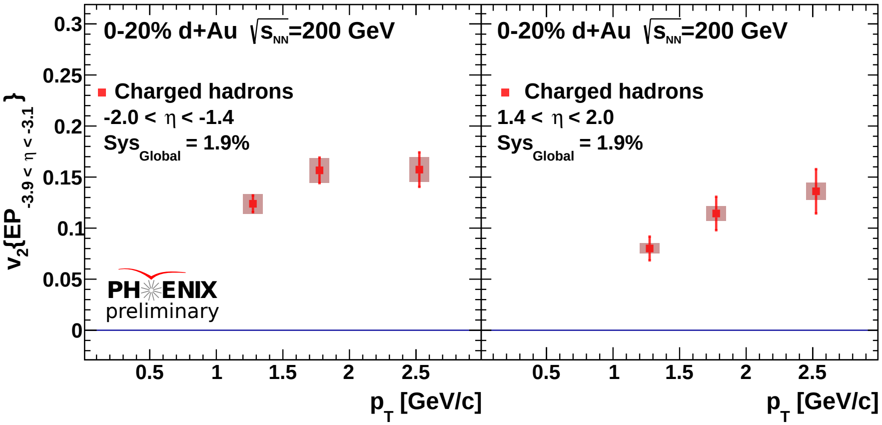
$v_2\{2\}$ and $v_2\{4\}$ vs $N_{\text{tracks}}^{\text{FVTX}}$—weighted average of v_2^B and v_2^F

$v_2\{2,|\Delta\eta|>2\}$ vs $N_{\text{tracks}}^{\text{FVTX}}$—fixed, equal weighting $\sqrt{\langle v_2^B v_2^F\rangle}$

$dN_{\text{ch}}/d\eta$ and v_2 vs η alone may explain these results

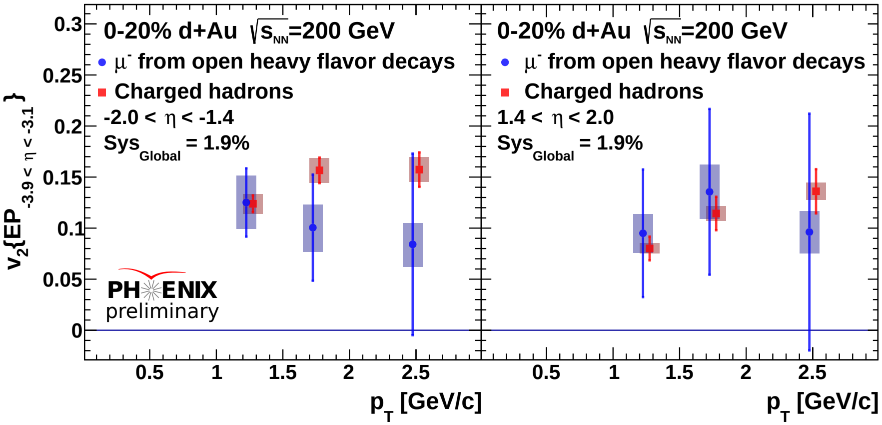
There can also be some event plane decorrelation, e.g.
$v_2\{2,|\Delta\eta|>2\} = \sqrt{\langle v_2^B v_2^F\rangle \cos(2(\psi_2^B - \psi_2^F))}$

Small systems flow



charged hadrons

Small systems flow—heavy flavor

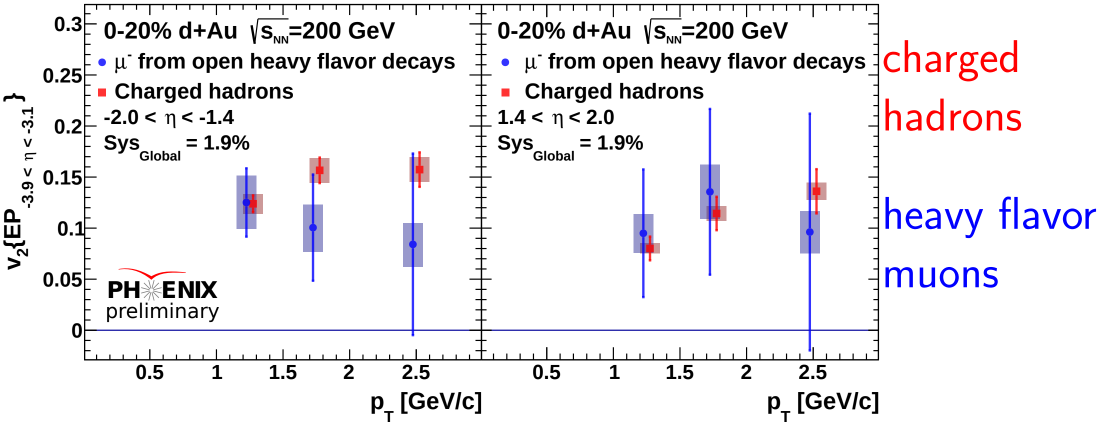


charged hadrons

heavy flavor muons

Nonzero v_2 for heavy flavor in d+Au

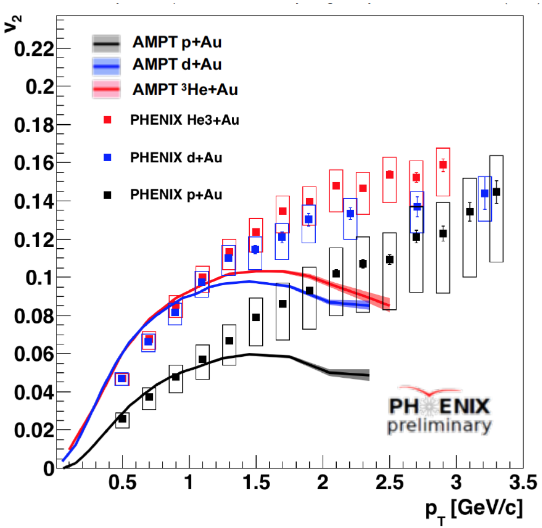
Small systems flow—heavy flavor



Nonzero v_2 for heavy flavor in d+Au

3.22σ, 2.16σ for $v_2 > 0$ at backward, forward (99.9%, 98.5% one-sided)

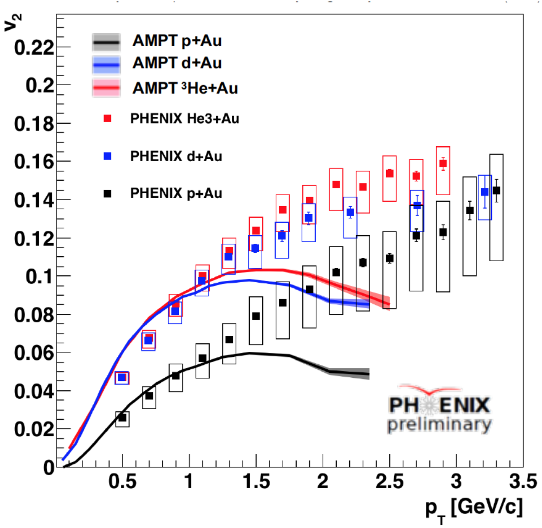
AMPT



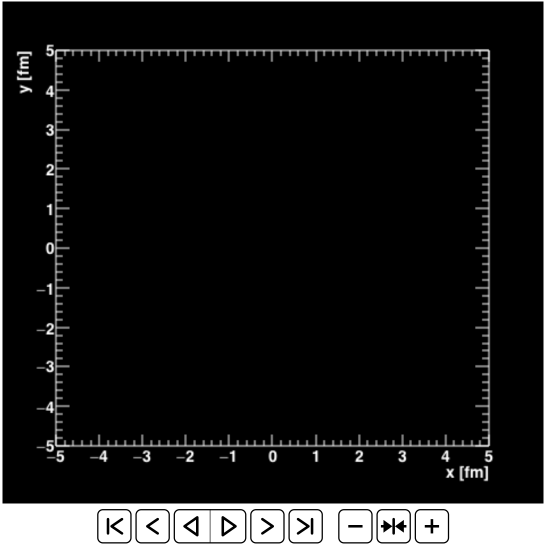
AMPT basic features

Initial conditions	HIJING
Particle production	String melting
Pre-equilibrium	None
Expansion	Parton scattering (tunable)
Hadronization	Spatial coalescence
Final stage	Hadron cascade (tunable)

AMPT



v_2
AMPT p+Au
AMPT d+Au
AMPT ³He+Au
PHENIX He3+Au
PHENIX d+Au
PHENIX p+Au
PHENIX preliminary
p_T [GeV/c]

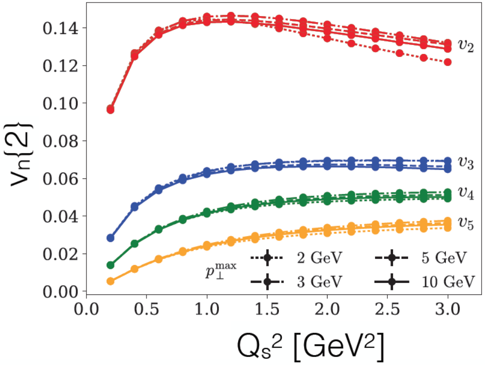


y [fm]
x [fm]

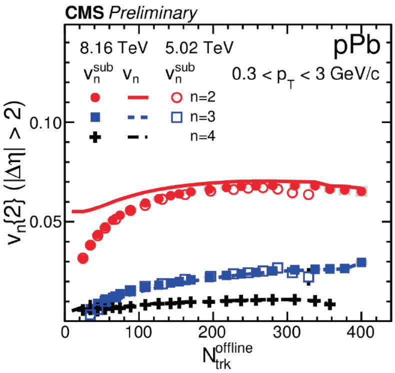
CGC results on small systems

Mark Mace, QM18

Phys. Rev. Lett. 120, 042002 (2018)



Dusling, MM, Venugopalan PRL 120 (2018)



CMS-PAS-HIN-16-022

"Simple parton model" with quarks scattering off dense gluon field

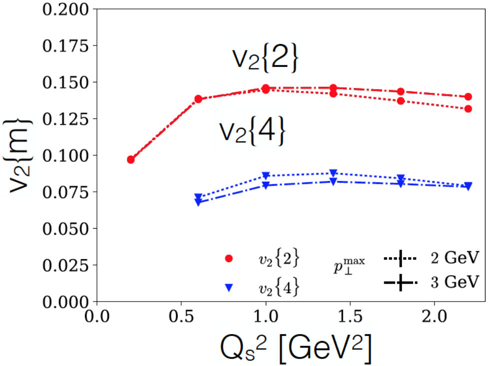
Can qualitatively reproduce harmonic ordering

Off from data by a factor of 2 to 3

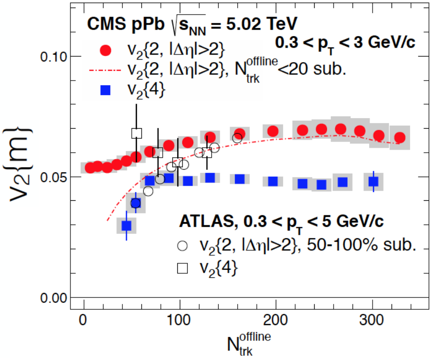
CGC results on small systems

Mark Mace, QM18

Phys. Rev. Lett. 120, 042002 (2018)



Dusling, MM, Venugopalan PRL 120 (2018)



CMS PLB 724 (2013) 213

"Simple parton model" with quarks scattering off dense gluon field

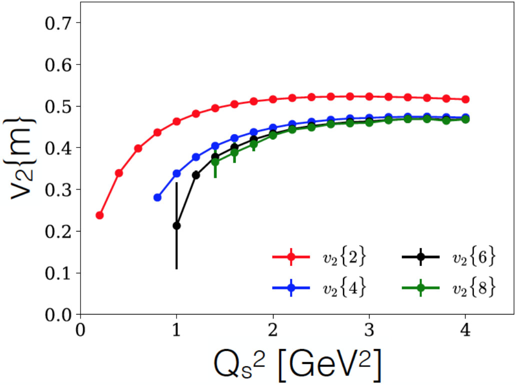
Can reproduce $v_2\{2\}$ and $v_2\{4\}$

Disagreement with data by a factor of 2, but qualitative features match

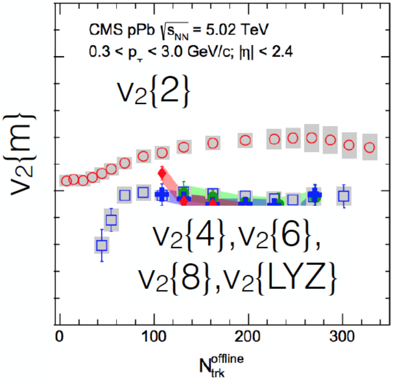
CGC results on small systems

Mark Mace, QM18

Phys. Rev. Lett. 120, 042002 (2018)



Dusling, MM, Venugopalan PRL 120 (2018)

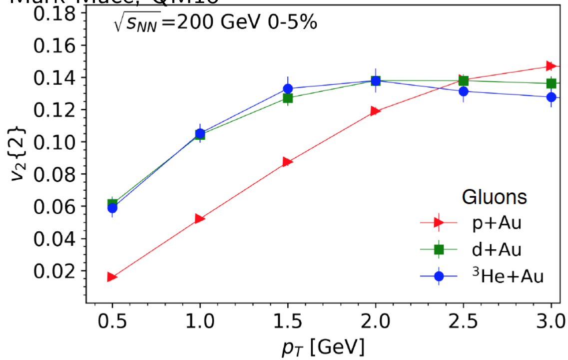


CMS PRL 115 (2015) 012301

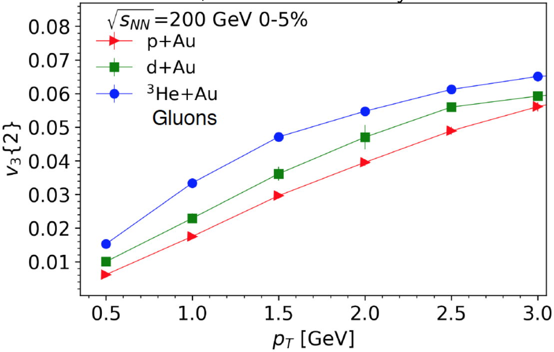
Abelian calculations can produce $v_2\{2\}$, $v_2\{4\}$, $v_2\{6\}$, $v_2\{8\}$
Disagreement with data by factor of 5, but qualitative features match

CGC results on small systems

Mark Mace, QM18



arXiv:1805.09342, submitted to Phys. Rev. Lett.

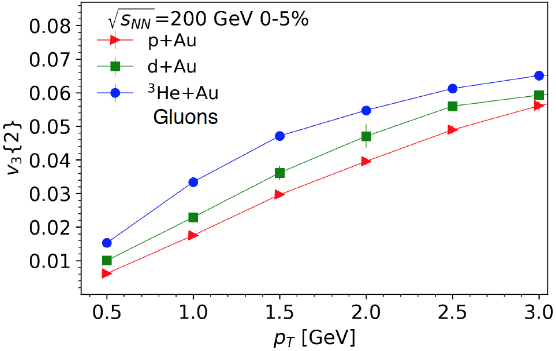


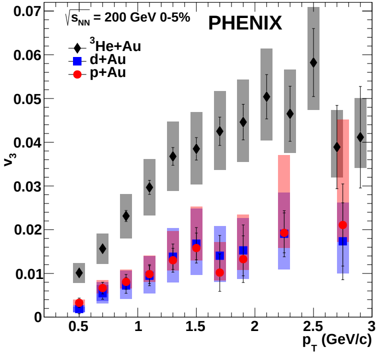
New for QM18: full calculation using dilute-dense framework, v_2 and v_3 for small systems geometry scan

CGC results on small systems

Mark Mace, QM18

arXiv:1805.09342, submitted to Phys. Rev. Lett.





New for QM18: full calculation using dilute-dense framework, v_2 and v_3 for small systems geometry scan

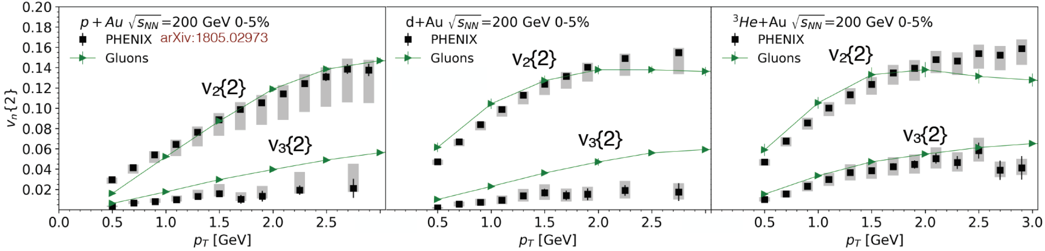
v_3 ordering is not quite right

—CGC: p+Au $<$ d+Au $<$ ^{3}He+Au

—Data: p+Au $\approx$ d+Au $<$ ^{3}He+Au

CGC results on small systems

Mark Mace, QM18 arXiv:1805.09342, submitted to Phys. Rev. Lett.



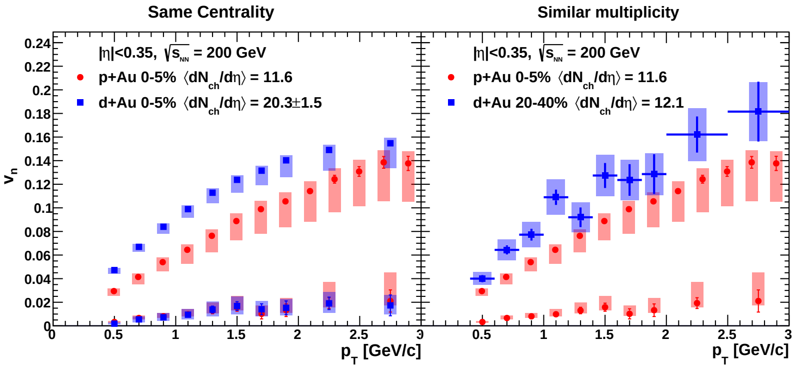
v_2 is quite close for the three systems

v_3 is rather far off

CGC results on small systems

Data: Phys. Rev. C 96, 064905 (2017) Theory: arXiv:1805.09342

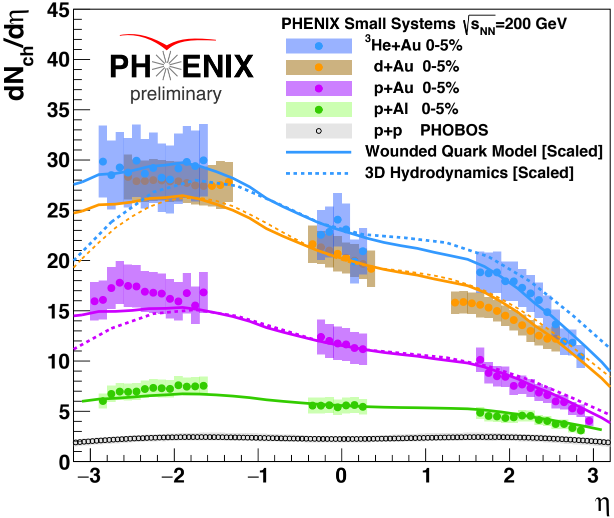
"Our prediction would therefore be that $v_{2,3}(p_\perp)$ for high multiplicity events across small systems should be identical for the same N_{ch}."



v_3 is same in p/d+Au for *different* N_{ch}

v_2 looks *different* for p/d+Au for similar N_{ch}, but need nonflow estimate...

Longitudinal dynamics in small systems

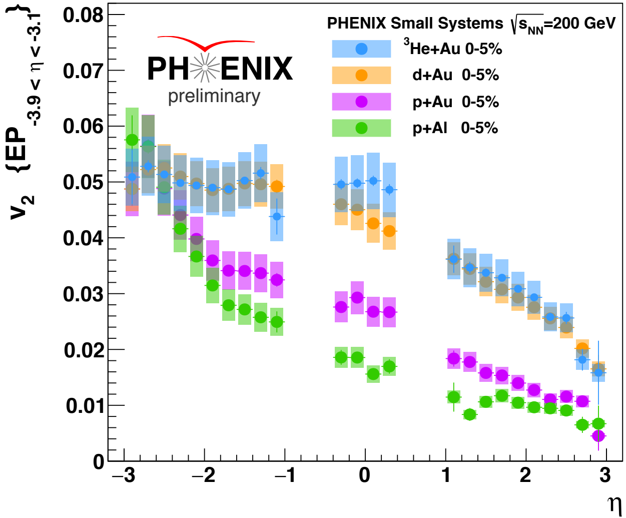


p+Al, *p*+Au, *d*+Au, ^{3}He+Au

Good agreement with wounded quark model

Good agreement with 3D hydro

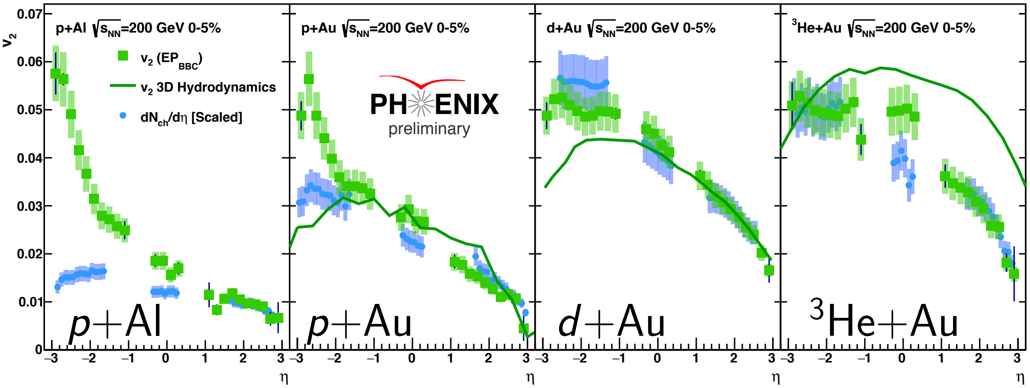
Longitudinal dynamics in small systems



p+Al, *p*+Au, *d*+Au, ^{3}He+Au

Also have v_2 vs η

Longitudinal dynamics in small systems

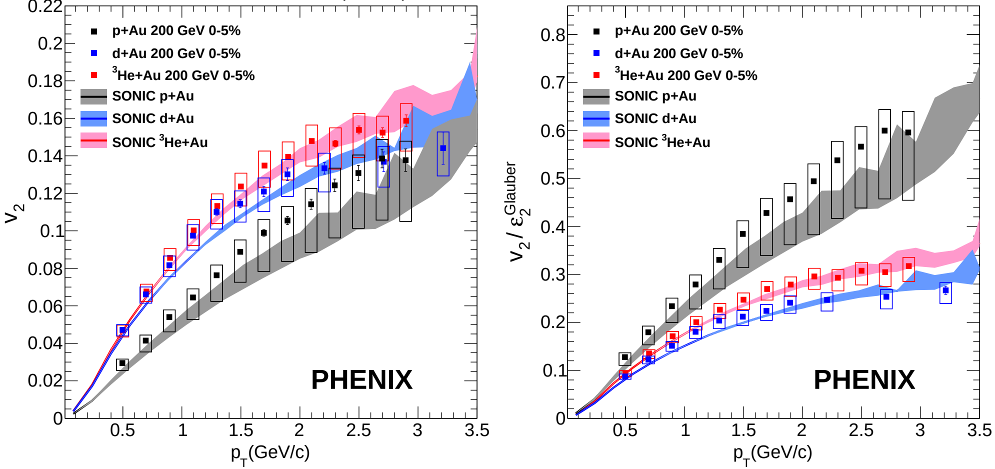


Good agreement with 3D hydro for p+Au and d+Au

Apparent scaling between v_2 and $dN_{ch}/d\eta$—coincidence?

Small systems geometry scan

PHENIX, Phys. Rev. C 95, 034910 (2017)

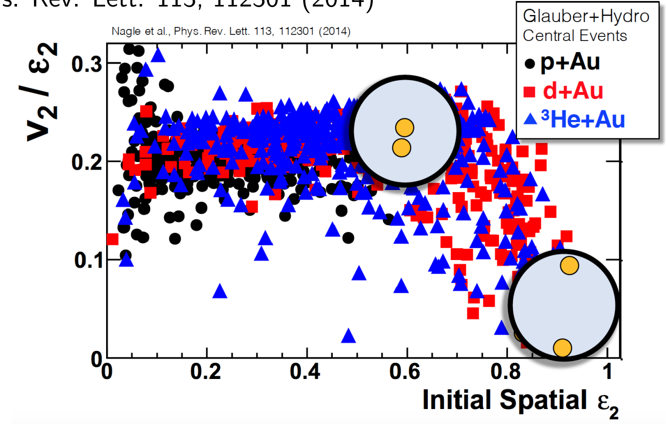


Hydro theory describes the data extremely well

Imperfect scaling with ε_2 captured by hydro—disconnected hot spots

Small systems geometry scan

J.L. Nagle et al, Phys. Rev. Lett. 113, 112301 (2014)



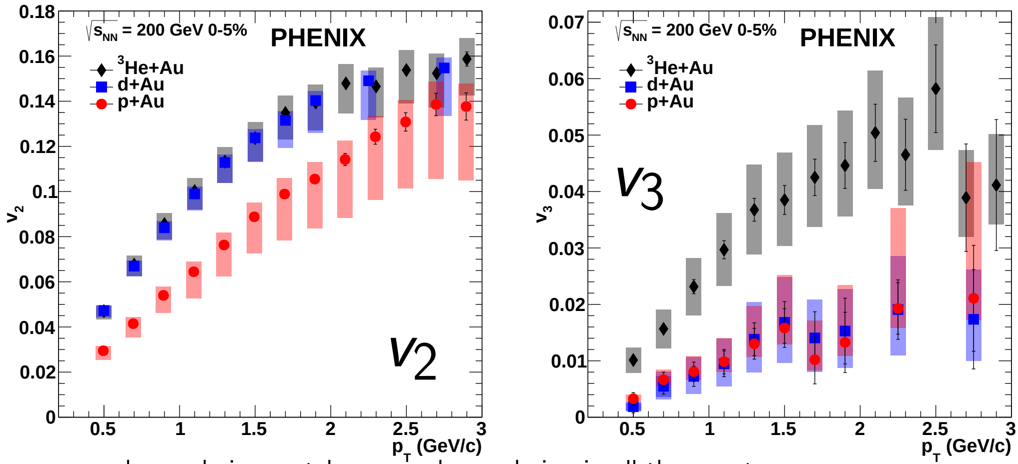
v_2/ε_2 relationship breaks for very large ε_2

The hydro hotspots are so far apart that they never connect

—Efficiency to translate ε_2 into v_2 goes down

Testing hydro by controlling system geometry

arXiv:1805.02973, submitted to Nature Physics



v_2 and v_3 ordering matches ε_2 and ε_3 ordering in all three systems

—Regardless of mechanism, the correlation is geometrical