E_T and other Event-by-Event Distributions

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- (Charged) Multiplicity n or dn/dy or dn/d□
- Transverse Energy E_T
- Average p_T per event , in contrast to
  - <pT> averaged over all events

Multiplicity in collision of nuclei

•The study of relativistic collisions involving nuclei has a long tradition, dating from studies in the 1930's of cosmic ray interactions in emulsions and in cloud chambers.

•One of the burning issues in the early 1950's was whether more than one meson could be produced in a single nucleon-nucleon collision (``multiple production'') or whether the multiple meson production observed in nucleon-nucleus interactions was the result of several successive n-n collisions, with each collision producing only a single meson (``plural production'').

•The issue was decided when multiple meson production was first observed in 1954 in collisions between neutrons of energy up to 2.2 GeV produced at the Brookhaven Cosmotron and protons in a hydrogen filled cloud chamber. Phys. Rev. **95**, 1026 (1954)

Single particle Inclusive Reactions

- A single particle "inclusive" reaction involves the measurement of just one particle coming out of a reaction,

$$a + b \rightarrow c + \text{anything} \quad .$$

The terminology comes from the fact that all final states with the particle c are summed over, or **included**.

- A "semi-inclusive" reaction refers to the measurement of all events of a given topology or class, e.g.

$$a + b \rightarrow n_1 \text{ particles of class } 1 + \text{anything} \quad .$$

Kinematics

A particle of mass m has energy E, longitudinal momentum p_L, transverse momentum p_T, rapidity y:

$$y = \ln \left(\frac{E + p_L}{m_T} \right) \tag{1}$$

$$\cosh y = E/m_T \qquad \sinh y = p_L/m_T \tag{2}$$

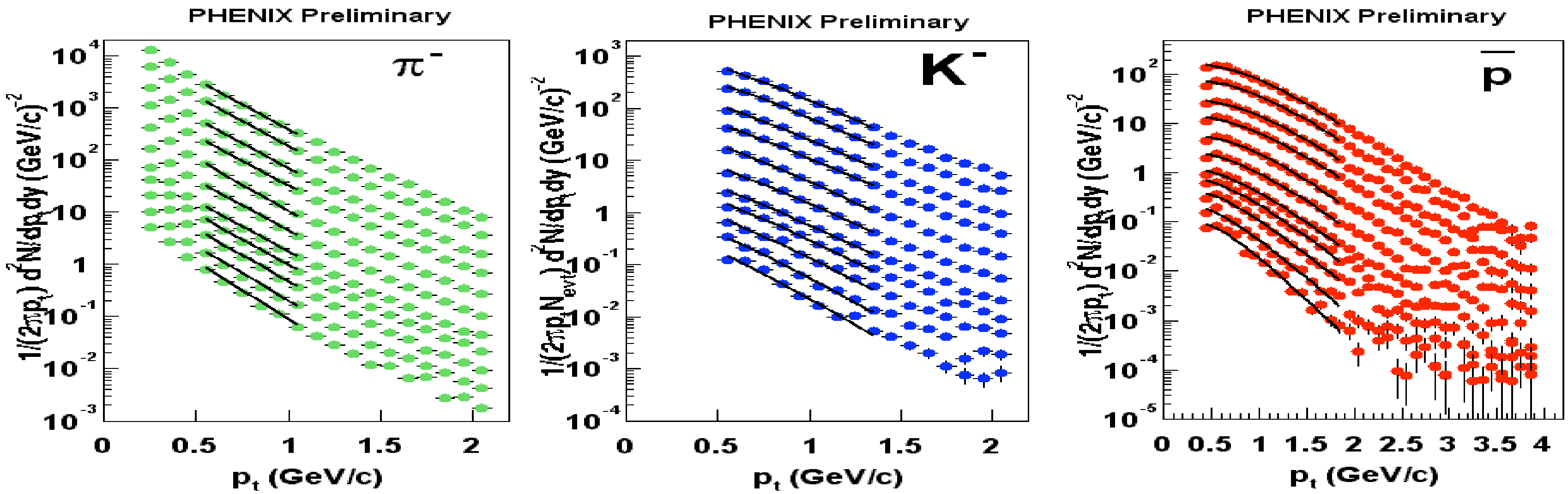
where

$$m_T = \sqrt{m^2 + p_T^2} \quad \text{and} \quad E = \sqrt{p_L^2 + m_T^2} \tag{3}$$

In the limit when $(m \ll E)$ the rapidity reduces to the pseudorapidity (η)

$$\eta = -\ln \tan \theta/2 \tag{4}$$

Semi-Inclusive p_T spectra Au+Au s_{NN}=200 GeV



- Measurements are presented in terms of the (Lorentz) Invariant single particle inclusive differential cross section (or Yield per event in the class if semi-inclusive):

$$\frac{Ed^3\sigma}{dp^3} = \frac{d^3\sigma}{p_T dp_T dy d\phi} = \frac{1}{2\pi}\,\mathbf{f}(p_T, y) \quad . \tag{5}$$

- A uniform azimuthal distribution is usually assumed, so that $\int d\phi = 2\pi$:

$$\frac{d^2\sigma}{p_T dp_T dy} = \mathbf{f}(p_T, y) \quad . \tag{6}$$

- $\langle p_T \rangle$ averaged over all events is obtained the usual way:

$$\langle p_T(y) \rangle = \frac{\int p_T\, dp_T\, p_T\, \mathbf{f}(p_T, y)}{\int \quad dp_T\, p_T\, \mathbf{f}(p_T, y)} \tag{7}$$

Integrals of the cross section=<n>

• Integrals of the single particle inclusive cross section (Eq. 5) are not equal to σ_I the interaction cross section, but rather equal to the mean multiplicity times the interaction cross section : $< n > \times \sigma_I$.

• Integration over p_T gives the average multiplicity density in rapidity, dn/dy:

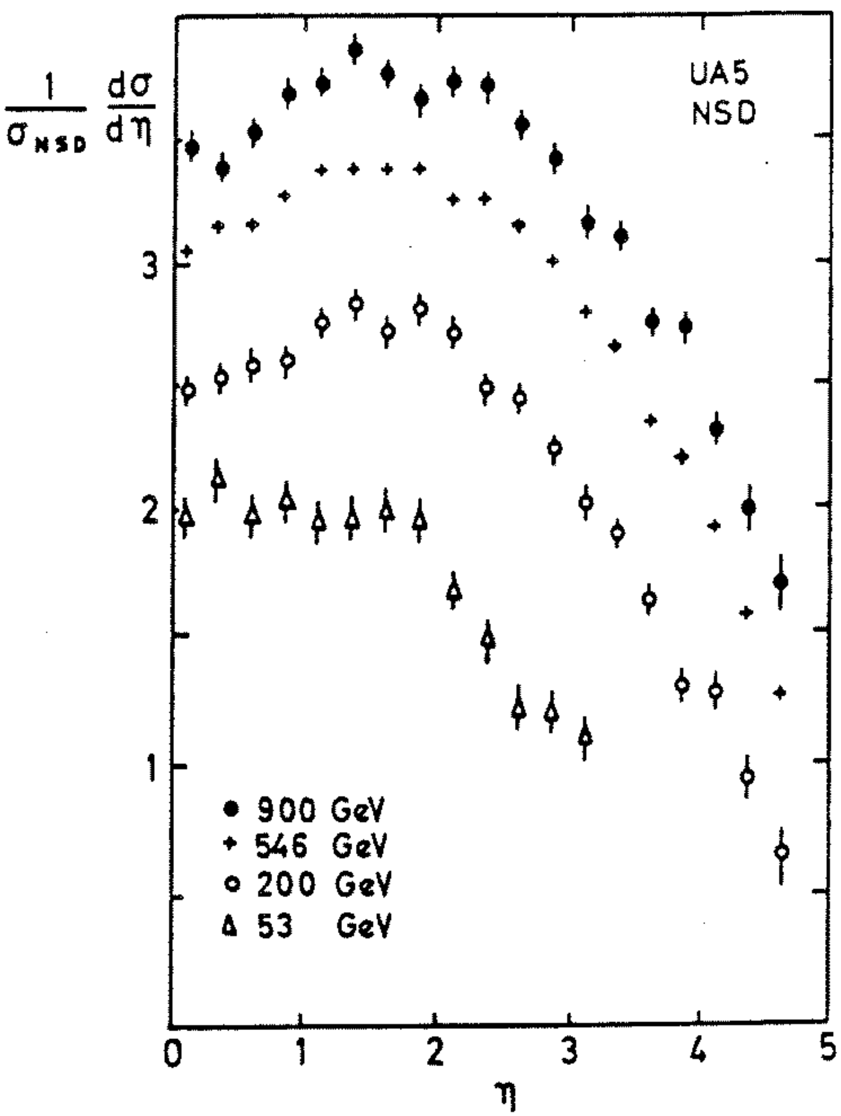
$$\frac{dn}{dy} = \frac{1}{\sigma_I}\frac{d\sigma}{dy} = \frac{1}{\sigma_I}\int dp_T\, p_T\, \mathbf{f}(p_T, y) = \rho(y) \quad . \tag{8}$$

• Integration over the kinematically possible rapidity gives the overall mean multiplicity per interaction, $< n >$:

$$< n > = \frac{1}{\sigma_I}\int dy\, dp_T\; p_T\; \mathbf{f}(p_T, y) = \int dy\, \frac{dn}{dy} = \int dy\, \rho(y) \tag{9}$$

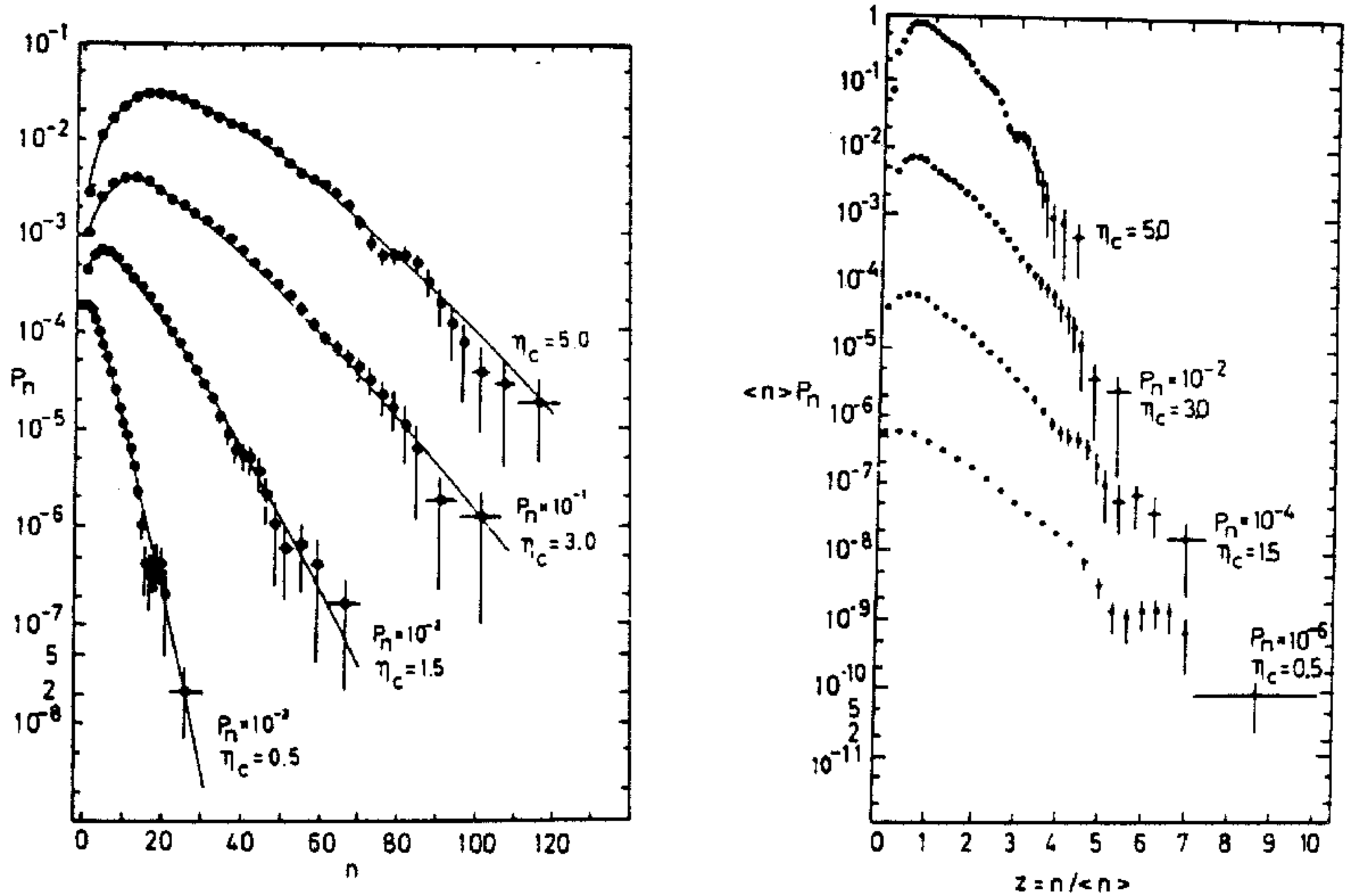
• Terminology is dn/dy for identified particles (known m), $dn/d\eta$ for non-identified particles (m unknown, assumed massless).

UA5 (1985) dn/dη in pbar-p



- Normalized to Non Single-Diffractive Cross Section
- This is an average quantity, averaged over all events.
- The event-by-event multiplicity distribution in regions of η is much more interesting: Instead of averaging over all events, plot a frequency distribution of the multiplicity n in an interval of pseudorapidity $|\eta| < \eta_c$
- Study of fluctuations in small intervals near mid-rapidity might allow observation of the ``real'' fluctuations freed of constraints like energy, momentum and charge conservation which need not be locally conserved.

UA5 Multiplicity Distribution $|\eta|<\eta_c$ shows huge variation event-by-event



UA5 PLB **160**, 193,199 (1985); **167**, 476 (1986)

Distributions are Negative Binomial, NOT POISSON: implies correlations

$\mathrm{E_T}$ distributions

- E_T is an event-by-event variable defined as:

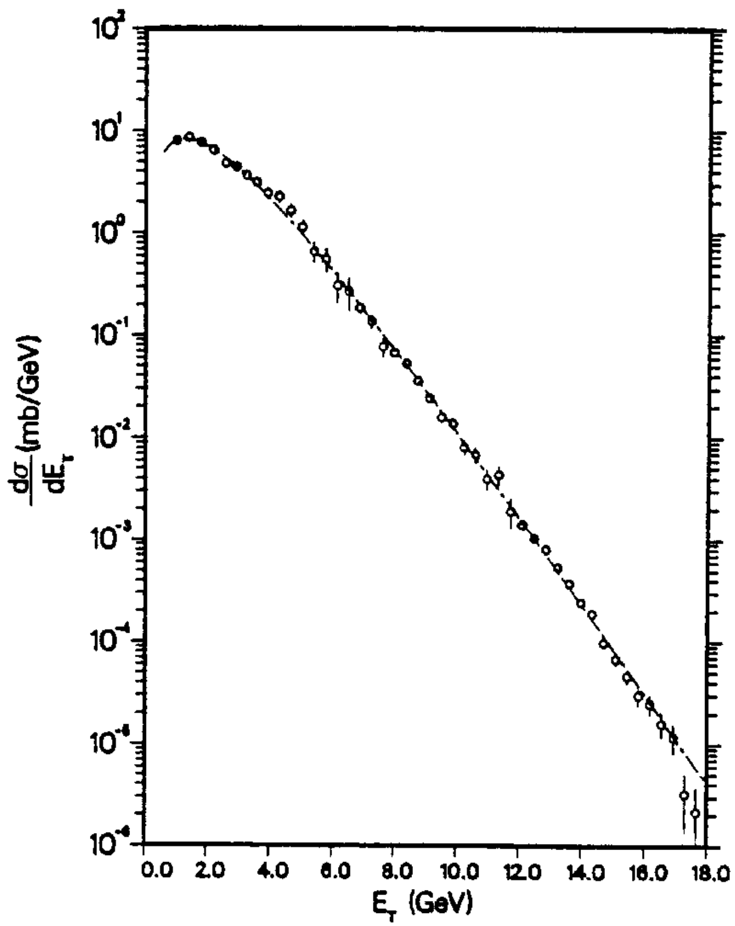
$$E_T = \sum_i E_i \sin\theta_i \qquad \text{and} \qquad dE_T(\eta)/d\eta = \sin\theta(\eta)\, dE(\eta)/d\eta \qquad , \qquad (10)$$

- The sum is taken over all particles emitted on an event into a fixed but large solid angle, (which is different in every experiment).
- Measured in Hadronic and Electromagnetic Calorimeters and even as sum of charged particles $\Sigma_i\, |p_{T_i}|$
- Introduced by High Energy Physicists as an "improved" method to detect and study the jets from "hard-scattering" compared to high p_T single particle spectra by which hard scattering was discovered in p-p collisions and used as a hard-probe in Au+Au collisions at RHIC. **Didn't work as expected: E_T distributions are dominated by soft particles near $\langle p_T\rangle$.**
- The importance of E_T distributions in relativistic heavy ion (RHI) collisions is that they are largely dominated by the *nuclear geometry* of the reaction and so provide a measure of the overall character or *centrality* of individual RHI interactions. **Bjorken Energy density**
- Normalization Problem: Measurement is not accurate but is very precise, good for A dependences in a single setup; not as good (larger systematic errors) for comparison of different experiments.

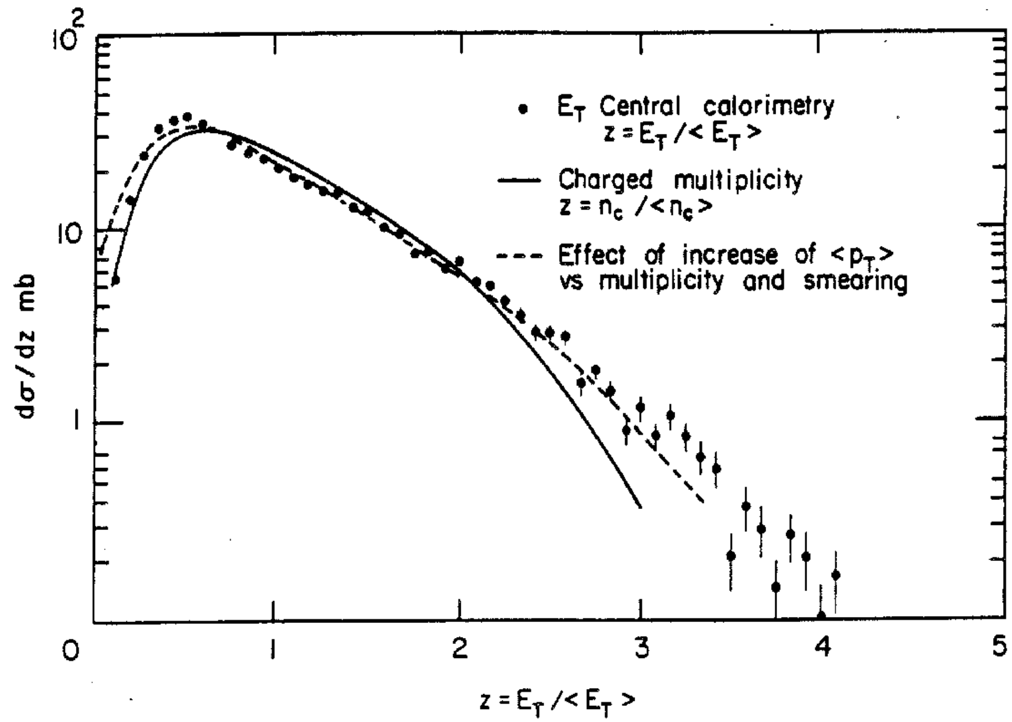
  ♡ correct from E_T in measured aperture to E_T in $\Delta\Phi = 2\pi$ $\Delta\eta = 1$ with or without HAD/EM calorimeter response correction.

  ♡ Measure $\langle E_T\rangle_{p-p}$ in same aperture and correct scale to $E_T/\langle E_T\rangle_{p-p}$, i.e. E_T in units of $\langle E_T\rangle$ per participant pair.

NA5 (CERN) (1980) First E_T dist. pp



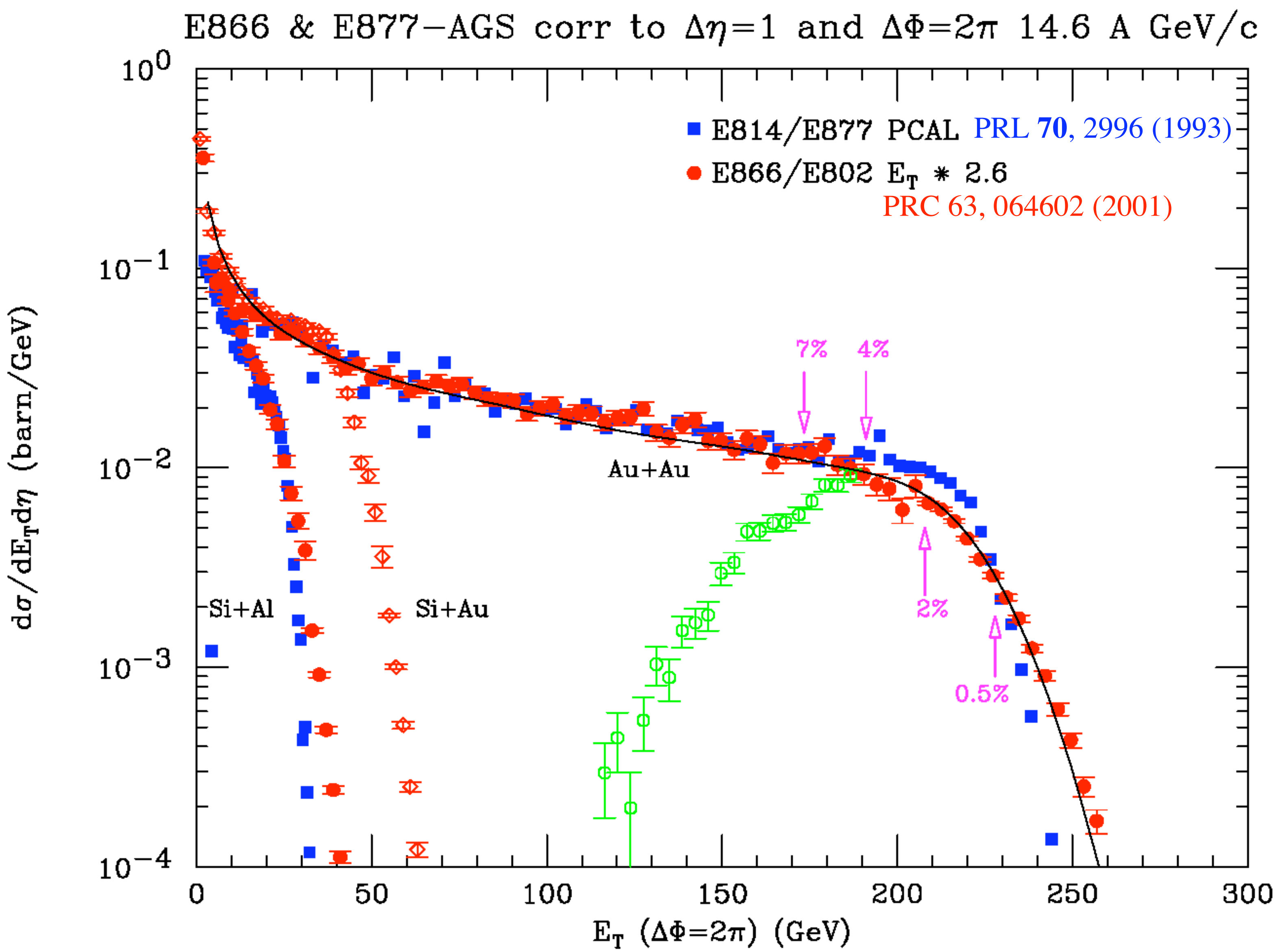
NA5 300 GeV PLB **112**, 173 (1980)
2□, -0.88<y<0.67 **NO JETS!**
s=23.7 GeV
Fit (by me) is □ dist p= 2.39 ± 0.06



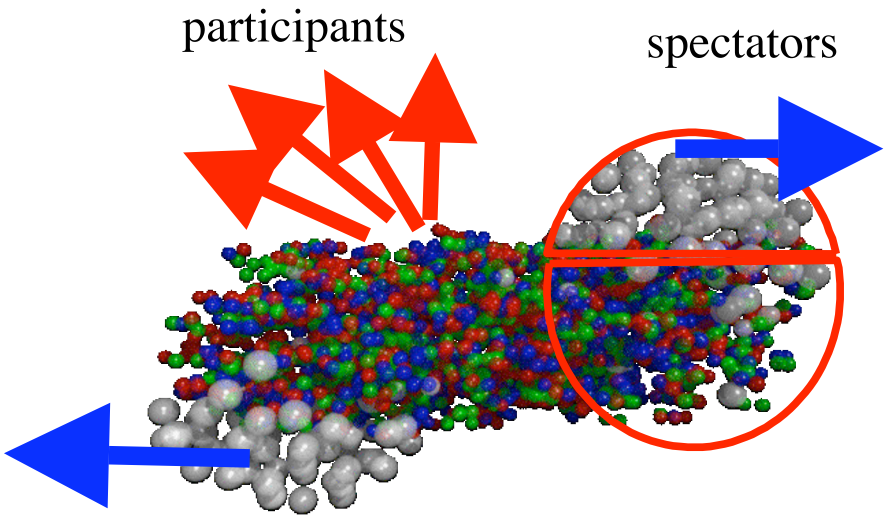
UA1 (1982) (C.Rubbia) s=540 GeV.
No Jets because E_T is like multiplicity (n), composed of many soft particles near $<p_T>$! CERN-EP-82/122.

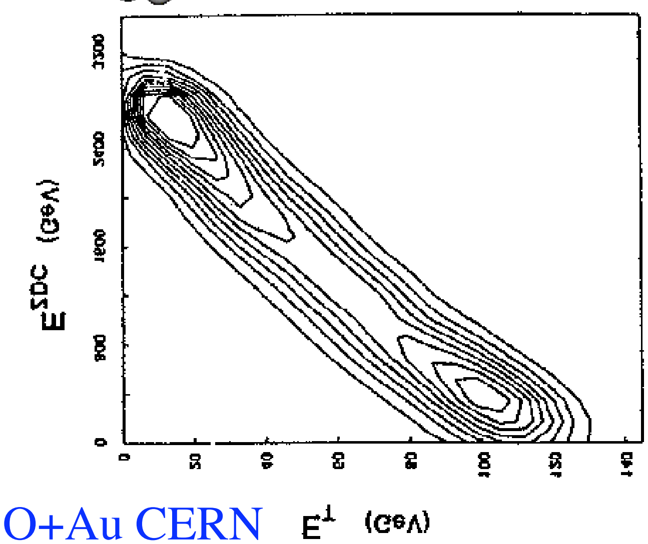
OOPS UA2 discovers jets 5 orders of magnitude down E_T distribution!

Typical E_T distributions in RHI collisions



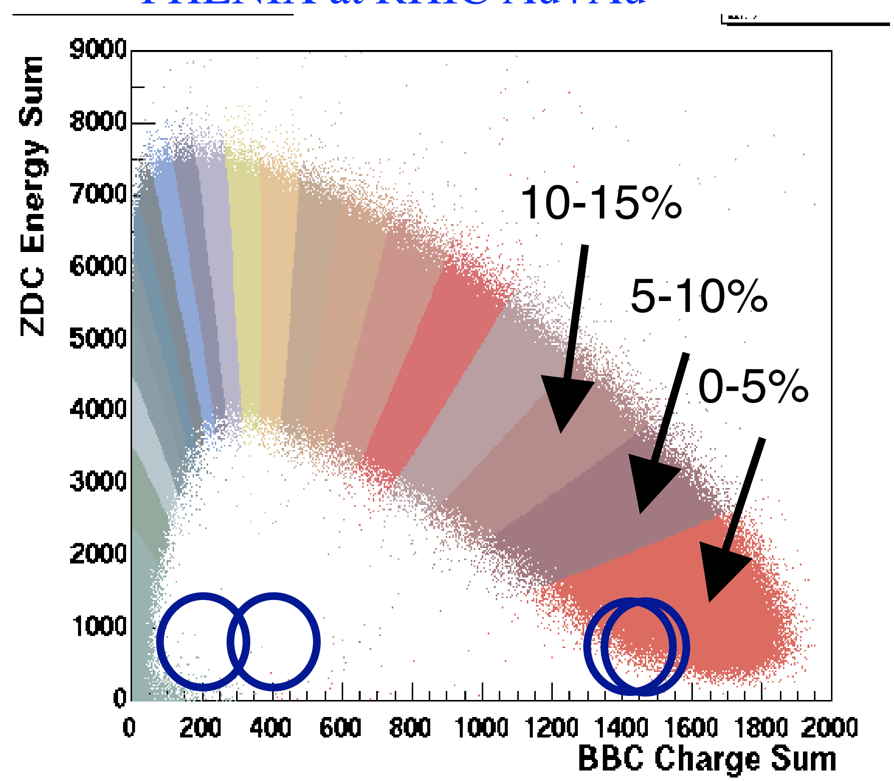
Collision Centrality Measurement
ZeroDegreeCalorimeter





WA80 O+Au CERN

PHENIX at RHIC Au+Au



Extreme-Independent or Wounded Nucleon Models

- Number of Spectators (i.e. non-participants) N_s can be measured directly in Zero Degree Calorimeters (more complicated in Colliders)

- Enables unambiguous measurement of (projectile) participants = A_p -N_s

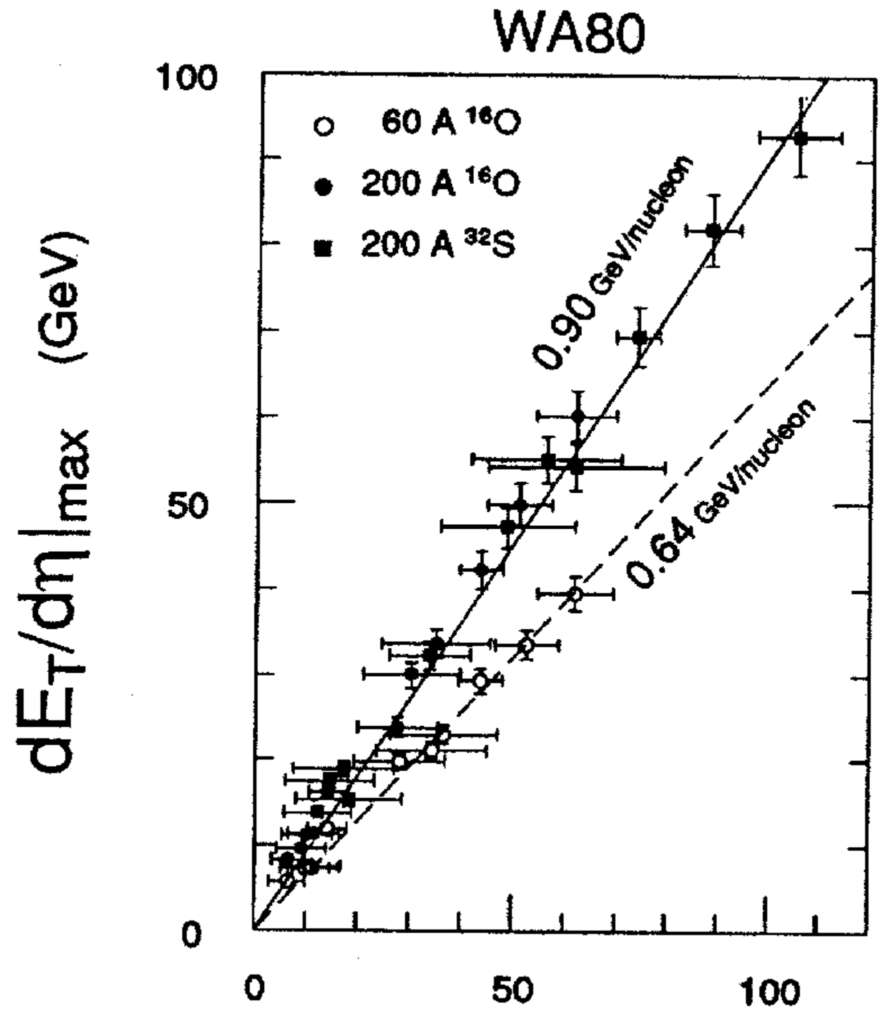
- For symmetric A+A collision N_{part}=2 $N_{projpart}$

- Uncertainty principle and time dilation prevent cascading of produced particles in relativistic collisions □h/m_{π}c > 10fm even at AGS energies: particle production takes place outside the Nucleus in a p+A reaction.

- Thus, Extreme-Independent models separate the nuclear geometry from the dynamics of particle production. The Nuclear Geometry is represented as the relative probability per B+A interaction w_n for a given number of total participants (WNM), projectile participants (WPNM), wounded projectile quarks (AQM), or other fundamental element of particle production.

- The dynamics of the elementary underlying process is taken from the data: e.g. the measured E_T distribution for a p-p collision represents, 2 participants, 1 n-n collision, 1 wounded projectile nucleon, a predictable convolution of quark-nucleon collisions.

WA80 proof of Wounded Nucleon Model at 60, 200 A GeV using ZDC

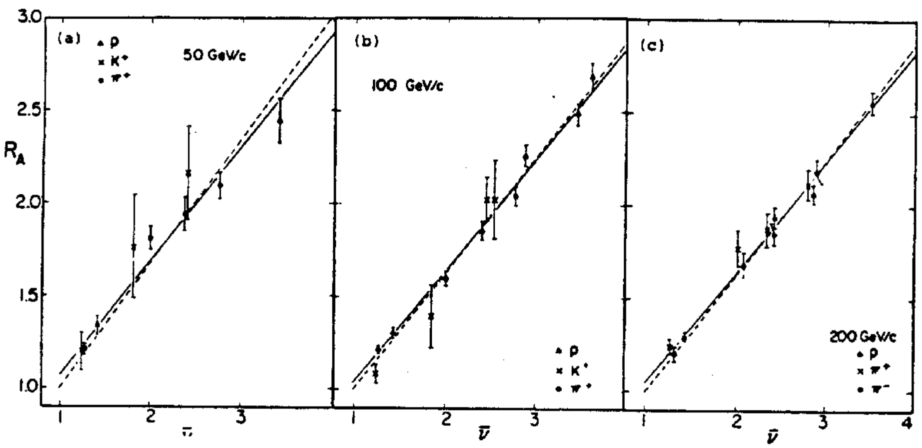


PRC **44**, 2736 (1991)

$\overline{W}$ = <Npart>

Original Discovery by W. Busza, et al at FNAL $\langle n\rangle_{pA}$ vs $\langle$□$\rangle$ (N_{coll})

PRD **22**, 13 (1980)



$$R_A = \langle n\rangle_{pA} / \langle n\rangle_{pp} = (1+\langle \nu\rangle)\ /\ 2$$

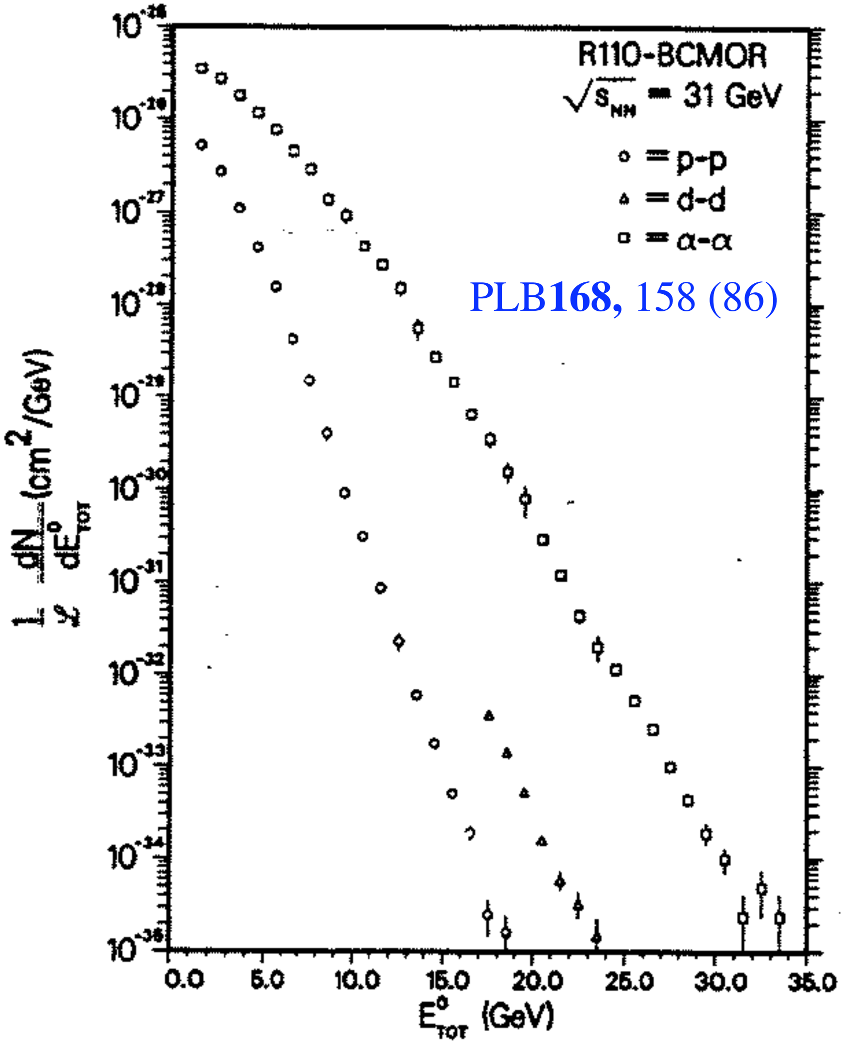
<Npart>$_{pA}$

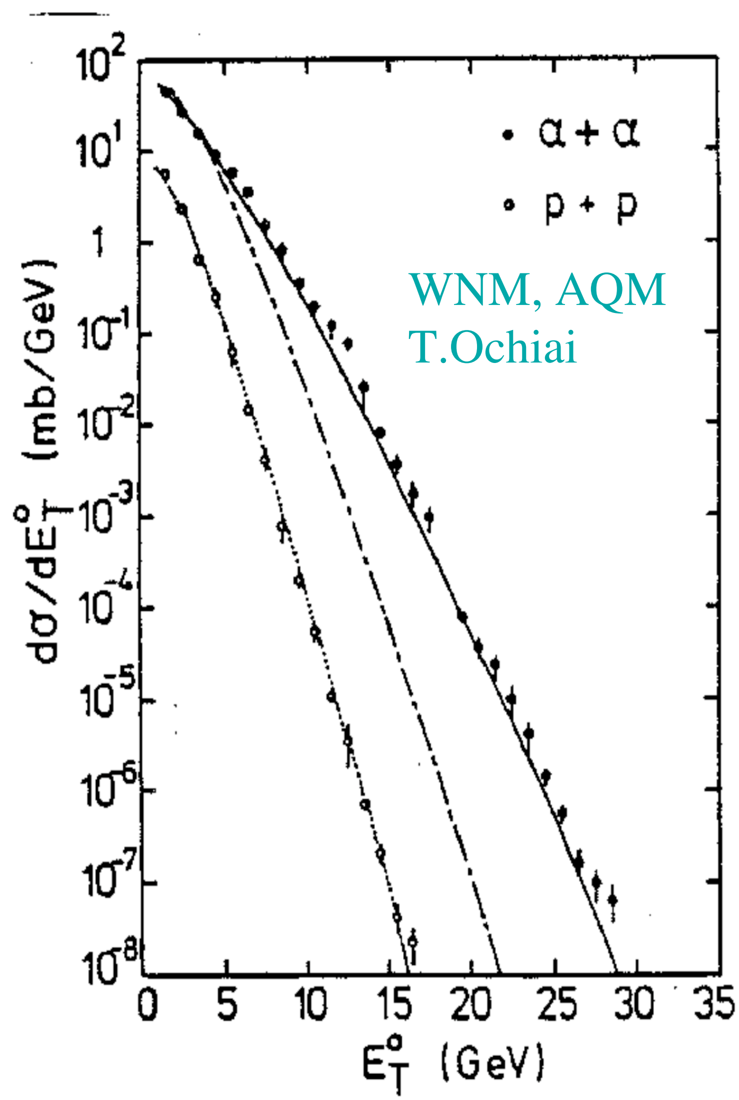
<Npart>$_{pp}$

Summary of Wounded Nucleon Models

- The classical Wounded Nucleon (N_{part}) Model (WNM) of Bialas, Bleszynski and Czyz (NPB **111**, 461 (1976)) works only at CERN fixed target energies, $\sqrt{s_{NN}}$~20 GeV.
- WNM overpredicts at AGS energies $\sqrt{s_{NN}}$~ 5 GeV (WPNM works at mid-rapidity)
- WNM underpredicts for $\sqrt{s_{NN}} \geq 31$ GeV

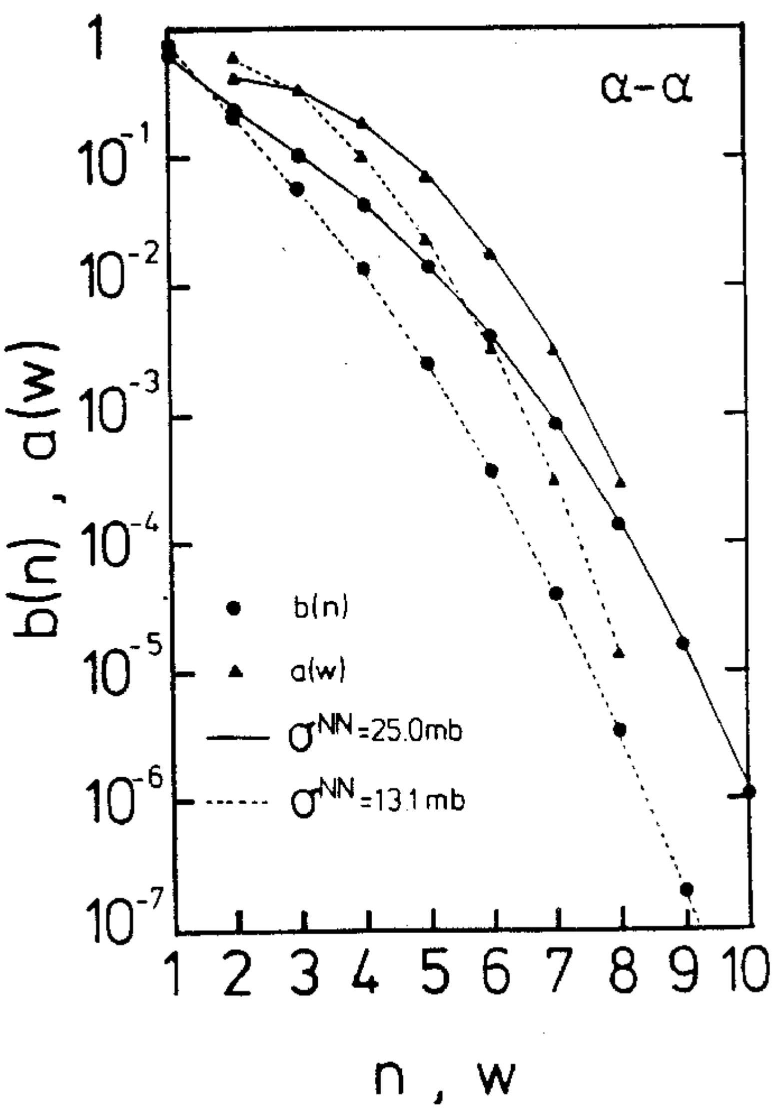
ISR-BCMOR-pp,dd,□□ $\sqrt{s_{NN}}$=31GeV WNM FAILS!





Note WNM edge is parallel to p-p data!

AQM works with σ_I (and p_0) or σ_{obs} for N_{part}



Observed σ_{obs}=13mb is ~ 0.5 σ_I=25 mb. Use either σ_{obs} for the nuclear geometry calculation or σ_I (properly taking account of the probability p_0~0.5 of detecting zero in the detector for a collision).

Calculation by T.Ochiai, ZPC **35**, 209 (87), PLB **206**, 535 (88).

This is the answer to Kopeliovich's complaint

Quick Course in Statistics

- Two of the most popular statistics are the sum and the average:

$$S_n \equiv \sum_{i=1}^{n} x_i \qquad \bar{x}_{(n)} \equiv \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n} S_n \tag{11}$$

where the x_i are n samples from a the same population or probability density function, $f(x)$.

- The probability distribution of a random variable S_n, which is itself the sum of n **independent** random variables with a common distribution $f(x)$:

$$S_n = x_1 + x_2 + \cdots + x_n \tag{12}$$

is given by $f_n(x)$, the n-fold convolution of the distribution $f(x)$:

$$f_n(x) = \int_0^x dy\, f(y)\, f_{n-1}(x-y) \quad . \tag{13}$$

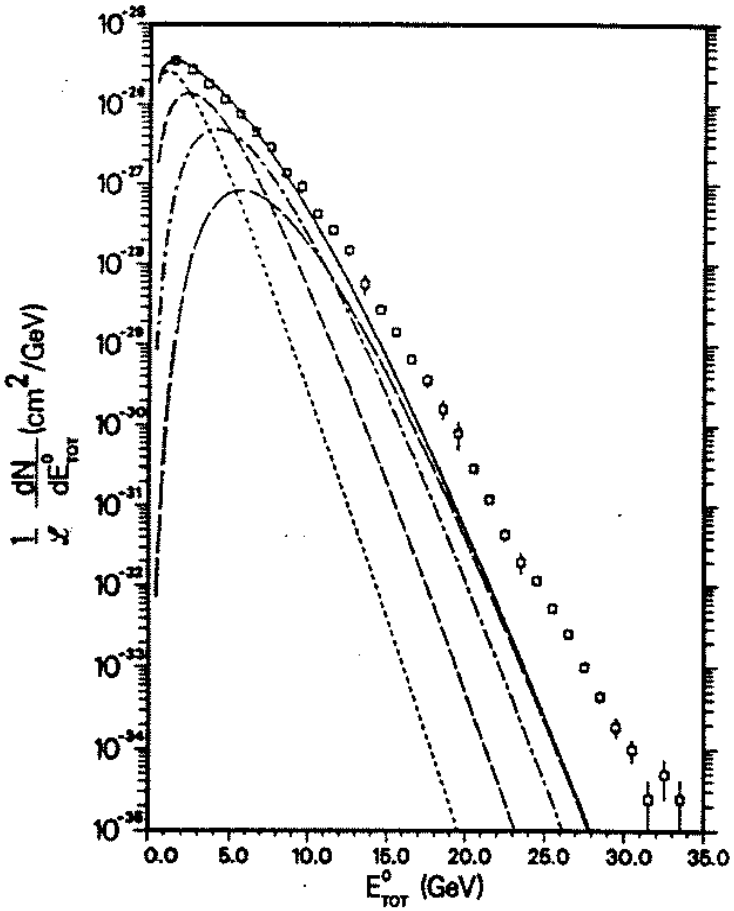
The mean, $\mu_n = \langle S_n \rangle$, and standard deviation, σ_n, of the n-fold convolution obey the familiar rule

$$\mu_n = n\mu \qquad \sigma_n = \sigma\sqrt{n} \quad , \tag{14}$$

where μ and σ are the mean and standard deviation of the distribution $f(x)$.

- A complementary case is that of a scale transformation $x \to nx$. The behavior of the mean and the standard deviation for a scale transformation is $\mu \to n\mu$, $\sigma \to n\sigma$, which is quite different than the behavior of the standard deviation under convolution (Eq. 14).

Details of WNM Calculation



The WPNM calculation for a B+A reaction

$$\left(\frac{d\sigma}{dE_T}\right)_{\rm WPNM} = \sigma_{BA} \sum_{n=1}^{B} w_n\, P_n(E_T) \qquad (15)$$

- σ_{BA} is the measured B+A cross section in the detector aperture,
- w_n is the relative probability for n projectile nucleons in the B+A reaction.
- $P_n(E_T)$ is the calculated E_T distribution on the detector aperture for n **independently interacting** projectile nucleons.
- If $f_1(E_T)$ is the measured E_T spectrum on the detector aperture for one projectile nucleon, and p_0 is the probability for the elementary collision to produce no signal on the detector aperture, then, the correctly normalized E_T distribution for one projectile nucleon collision is:

$$P_1(E_T) = (1-p_0) f_1(E_T) + p_0 \delta(E_T) \quad , \qquad (16)$$

where $\delta(E_T)$ is the Dirac delta function and $\int f_1(E_T)\, dE_T = 1$.

- $P_n(E_T)$ (including the p_0 effect) is obtained by convoluting $P_1(E_T)$ with itself $n-1$ times

$$P_n(E_T) = \sum_{i=0}^{n} \frac{n!}{(n-i)!\; i!}\, p_0^{n-i} (1-p_0)^i f_i(E_T) \qquad (17)$$

where $f_0(E_T) \equiv \delta(E_T)$ and $f_i(E_T)$ is the i-th convolution of $f_1(E_T)$:

$$f_i(x) = \int_0^x dy\, f(y)\, f_{i-1}(x-y) \quad . \qquad (18)$$

A Favorite Function

The Gamma distribution is an example of a probability density function (pdf) which has particularly simple properties under convolutions and scale transformations. The Gamma distribution is a function of a continuous variable x and has paramters p and b

$$f(x) = f_{\Gamma}(x, p, b) = \frac{b}{\Gamma(p)}(bx)^{p-1}e^{-bx} \tag{19}$$

where

$$p > 0, \quad b > 0, \quad 0 \leq x \leq \infty$$

$\Gamma(p) = (p-1)!$ if p is an integer, and $f(x)$ is normalized, $\int_0^\infty f(x)dx = 1$. The mean and standard deviation of the distribution are

$$\mu \equiv \langle x \rangle = \frac{p}{b} \qquad \sigma \equiv \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \frac{\sqrt{p}}{b} \qquad \frac{\sigma^2}{\mu^2} = \frac{1}{p} \qquad . \tag{20}$$

The n-fold convolution of the Gamma distribution (Eq. 19) is simply given by the function

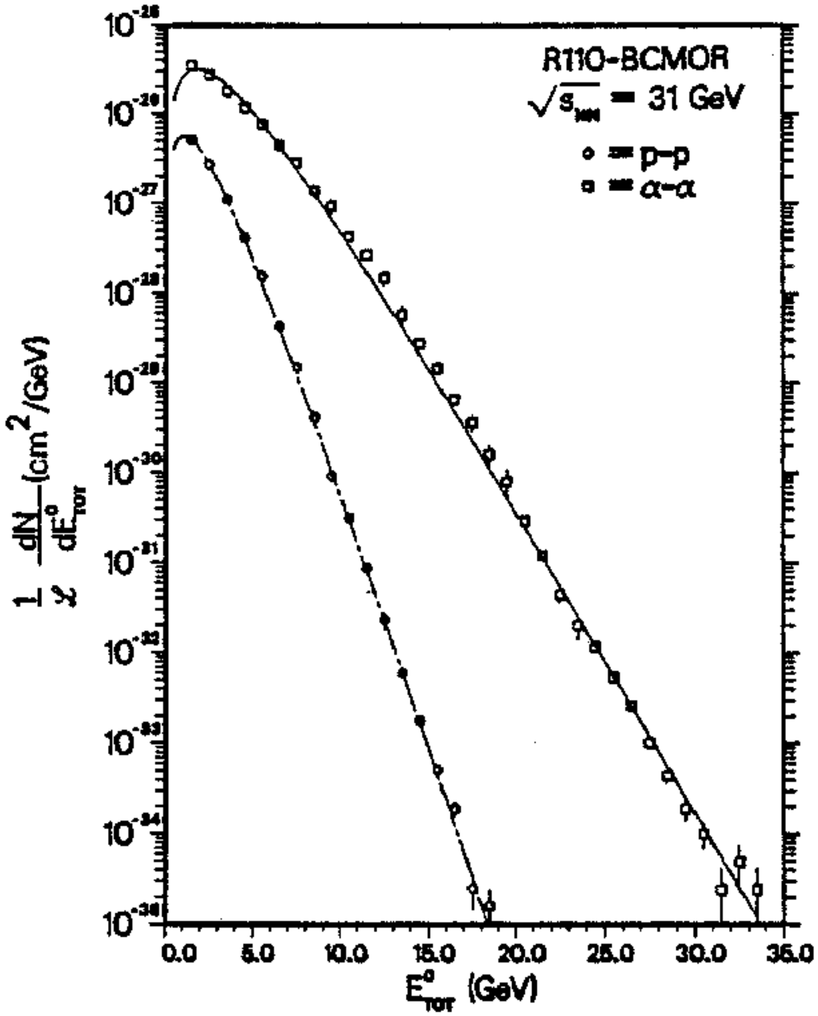
$$f_n(x) = \frac{b}{\Gamma(np)}(bx)^{np-1}e^{-bx} = f_{\Gamma}(x, np, b) \tag{21}$$

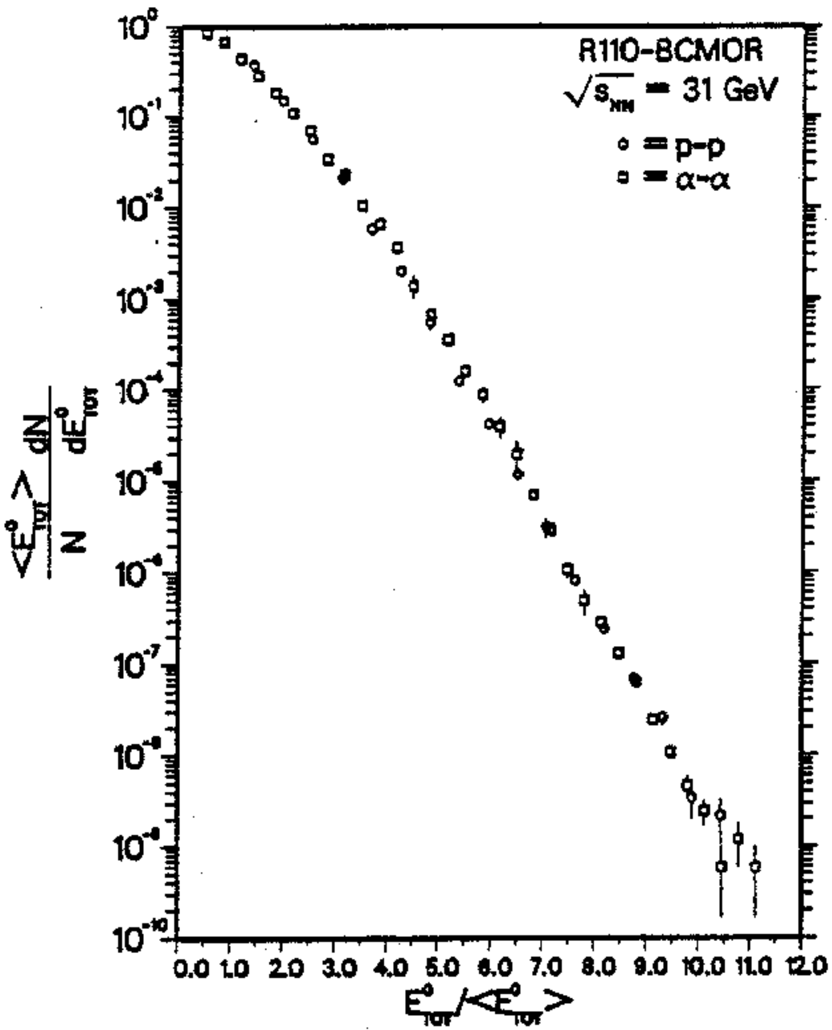
i.e. $p \rightarrow np$ and b remains unchanged. Note that the mean and standard deviation of Eq. 21

$$\mu_n = \frac{np}{b} \qquad \sigma_n = \frac{\sqrt{np}}{b} \qquad \frac{\sigma_n}{\mu_n} = \frac{1}{\sqrt{np}} \tag{22}$$

when compared to Eq. 20 explicitly obey Eq. 14. **To summarize, the n-th convolution of the Gamma distribution $f_{\Gamma}(x, p, b)$ is $f_{\Gamma}(x, np, b)$—b remains unchanged.**

But-Gamma Dist. fits uncover Scaling in the mean over10 decades??



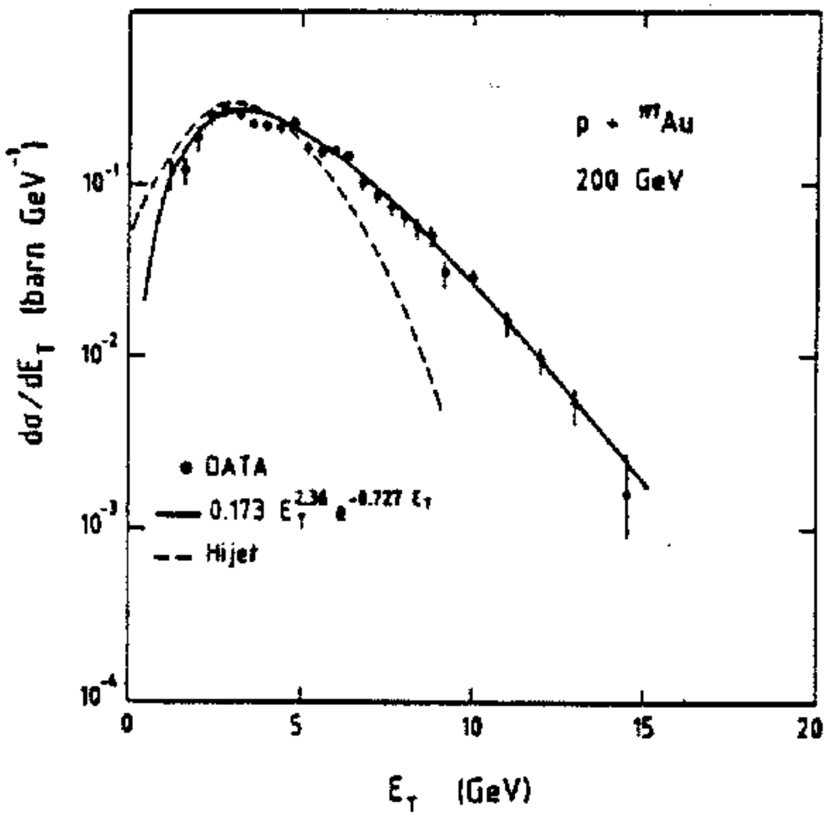


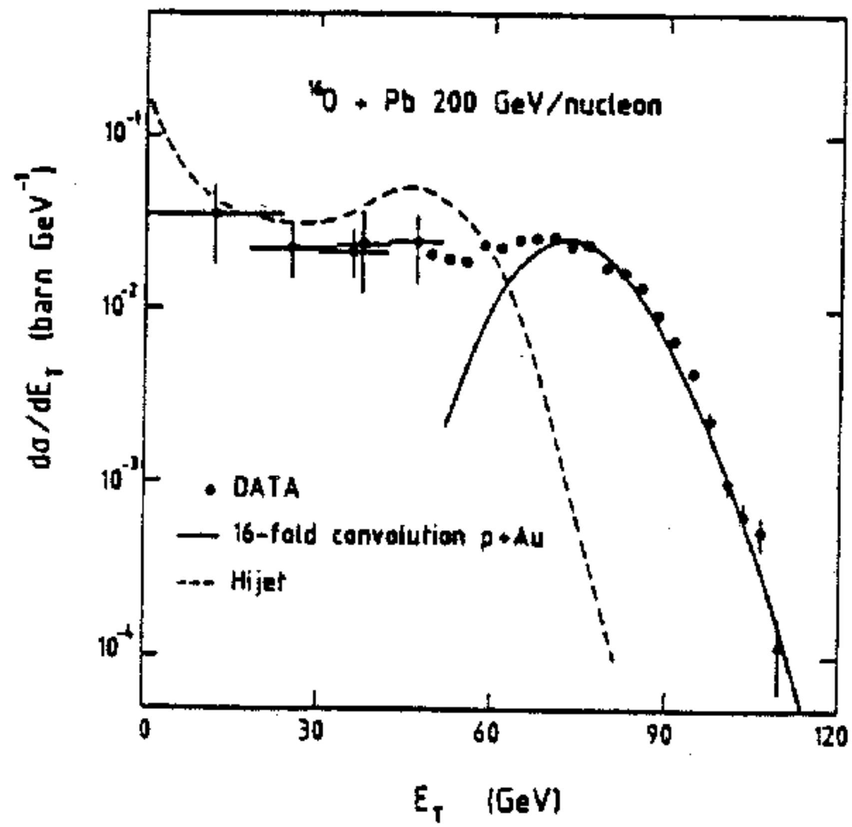
p-p p=2.50±0.06 - p=2.48±0.05

Is it Physics or a Fluke?

First RHI data NA35 (NA5 Calorimeter) CERN ^{16}O+Pb $\sqrt{s_{NN}}$=19.4 GeV midrapidity

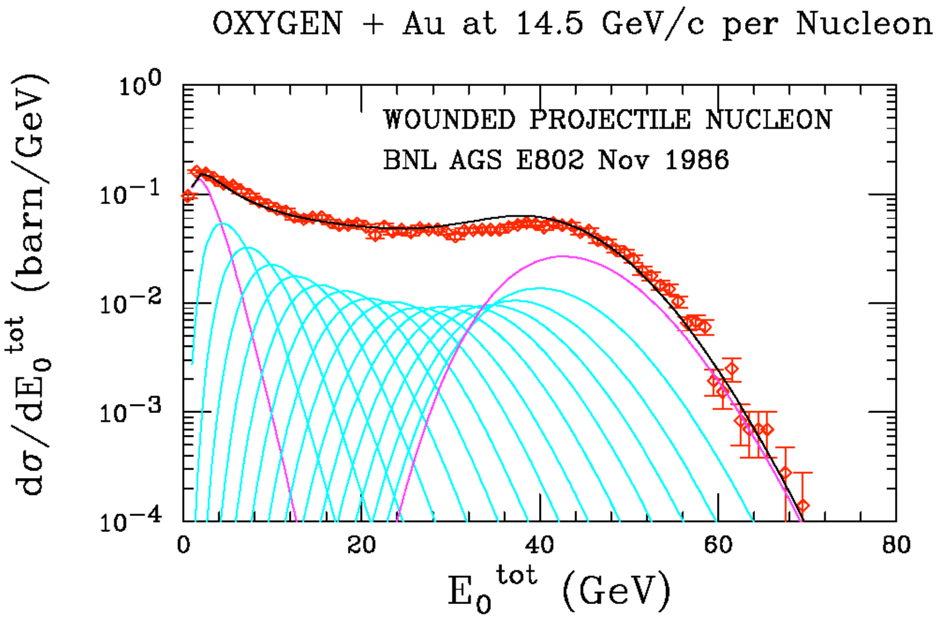
PLB **184,** 271 (1987)

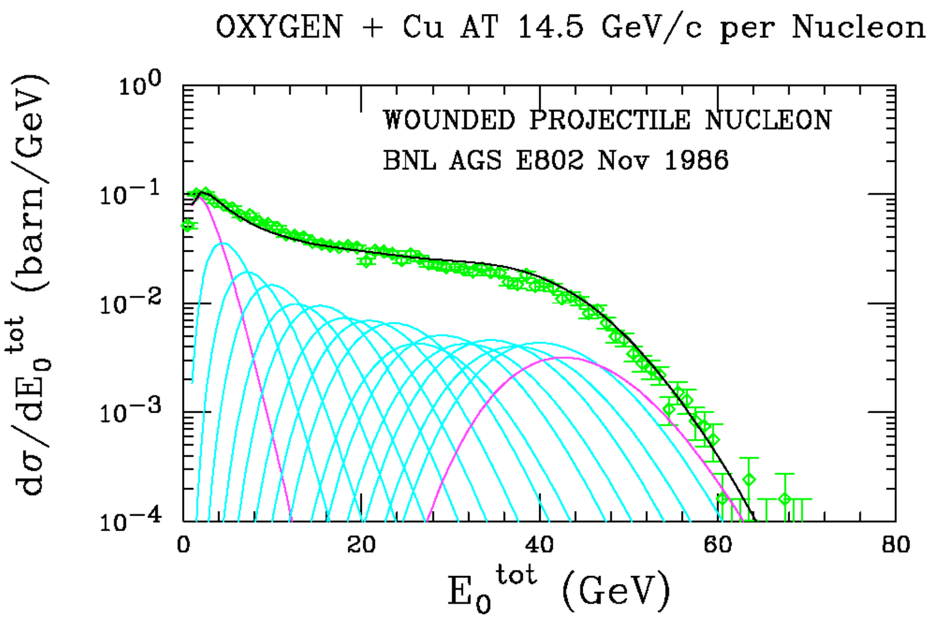




p+Au is a dist w. p=3.36

Upper Edge of O+Pb is 16 convolutions of p+Au. WPNM!!

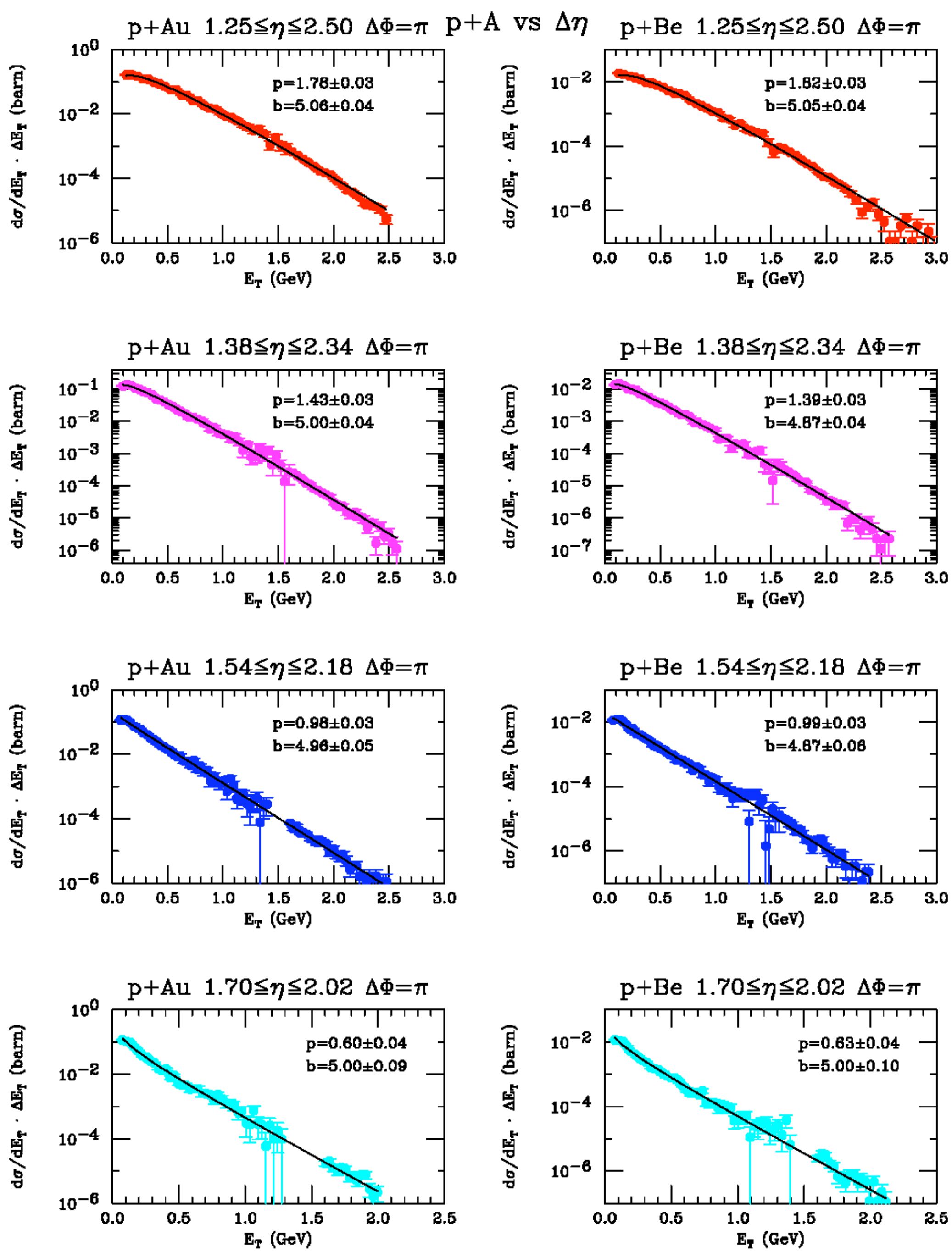




E802-O+Au, O+Cu midrapidity at $\sqrt{s_{NN}}$=5.4GeV WPNM works in detail

PLB **197**, 285 (1987)
ZPC **38**, 35 (1988)

- Maximum energy in O+Cu ~ same as O+Au--Upper edge of O+Au identical to O+Cu d[illegible]/dE * 6
- Indicates large stopping at AGS ^{16}O projectiles stopped in Cu so that energy emission (mid-rapidity) ceases
- Full O+Cu and O+Au spectra described in detail by WPNM based on measured p+Au

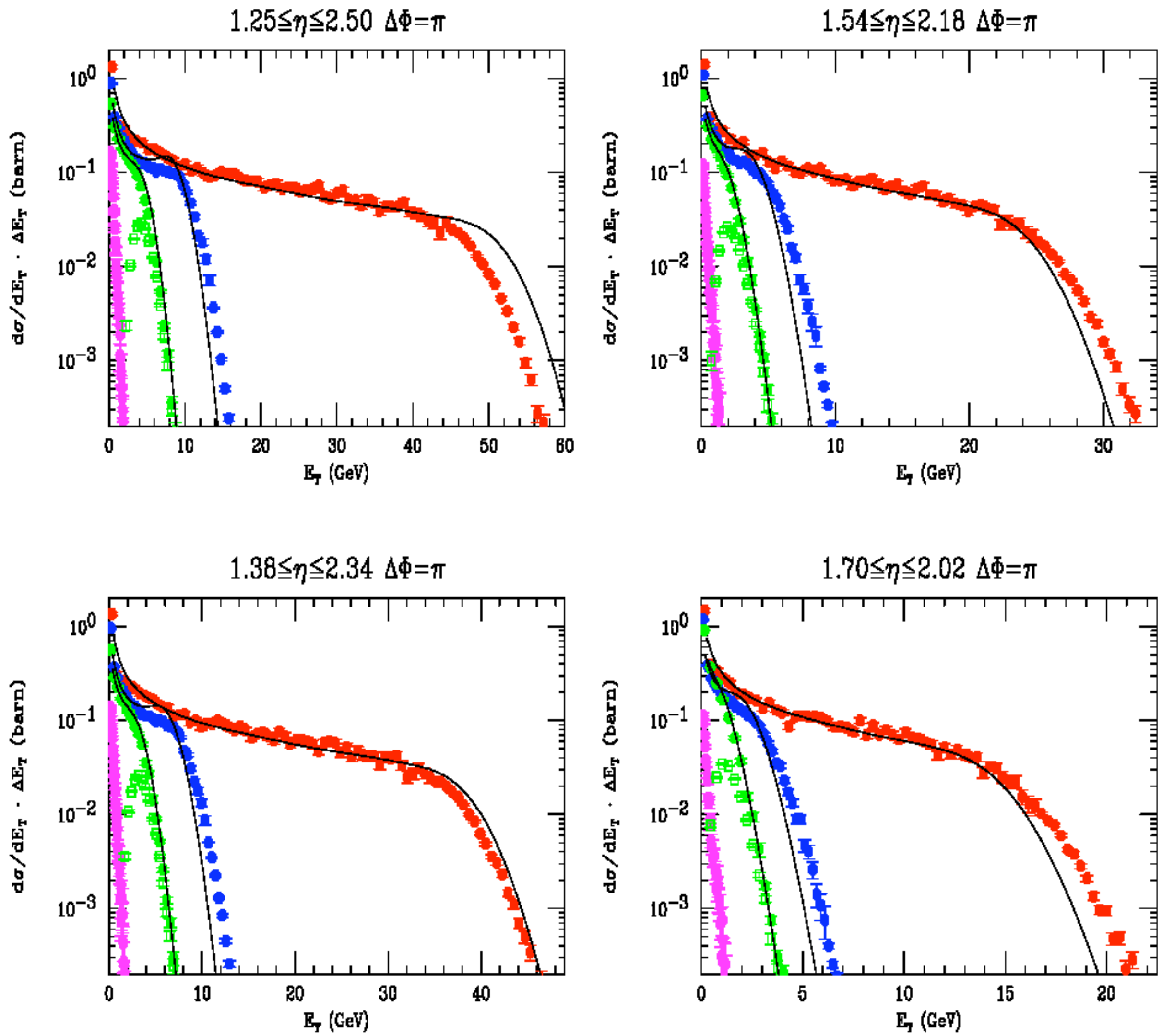


E802-AGS pBe & pAu have same shape at midrapidity over a wide range of □□

PRC **63**, 064602 (2001)

- confirms previous measurement PRC **45**, 2933 (1992) that pion distribution from second collision shifts by > 0.8 units in y, out of aperture. Explains WPNM.

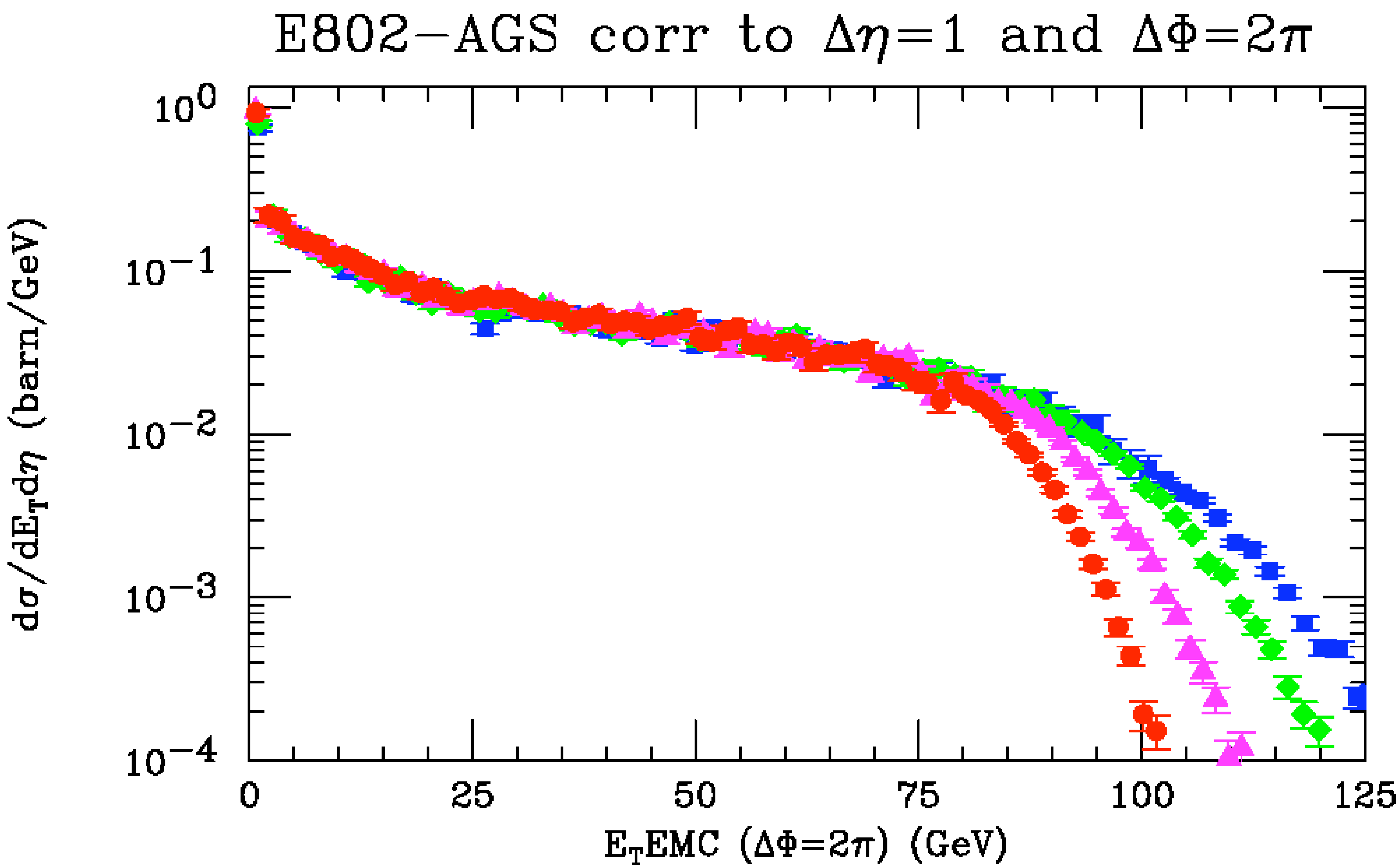
E802-AGS E_T ~follow WPNM vs □□



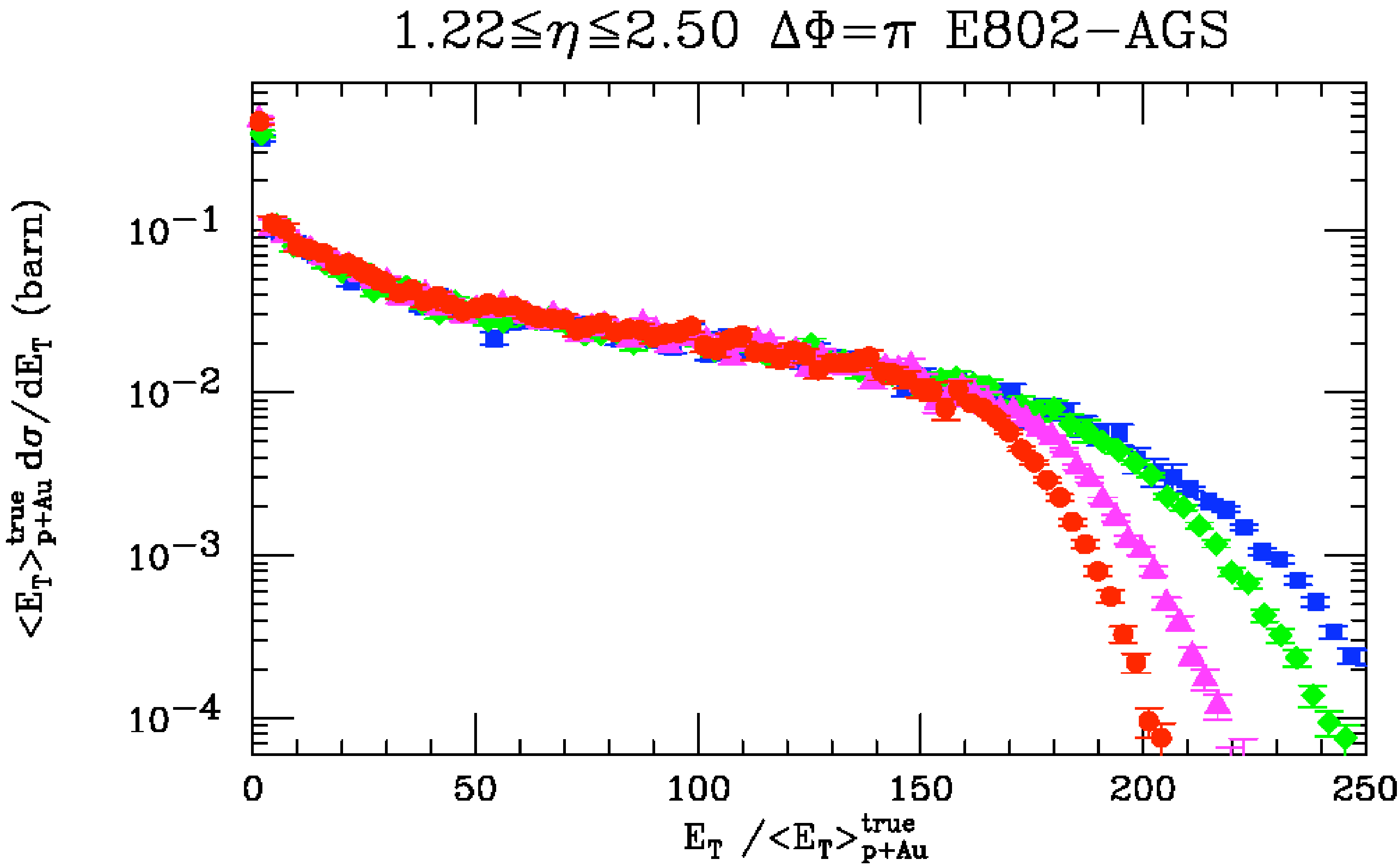
Spectra in different apertures hard to compare--Normalize 2 different ways

 PRC **63**, 064602 (2001)

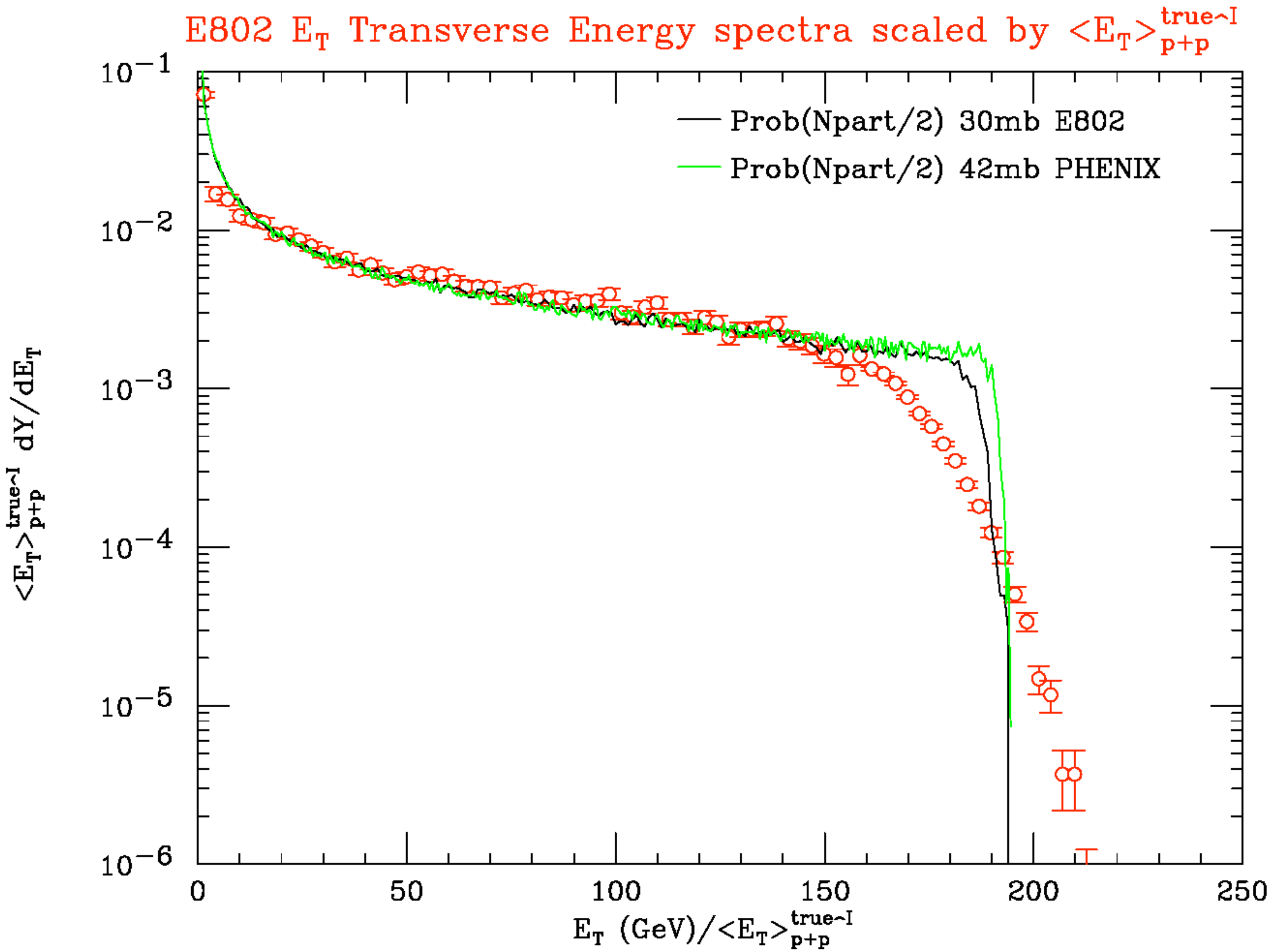
E802-AGS corrected to full azimuth



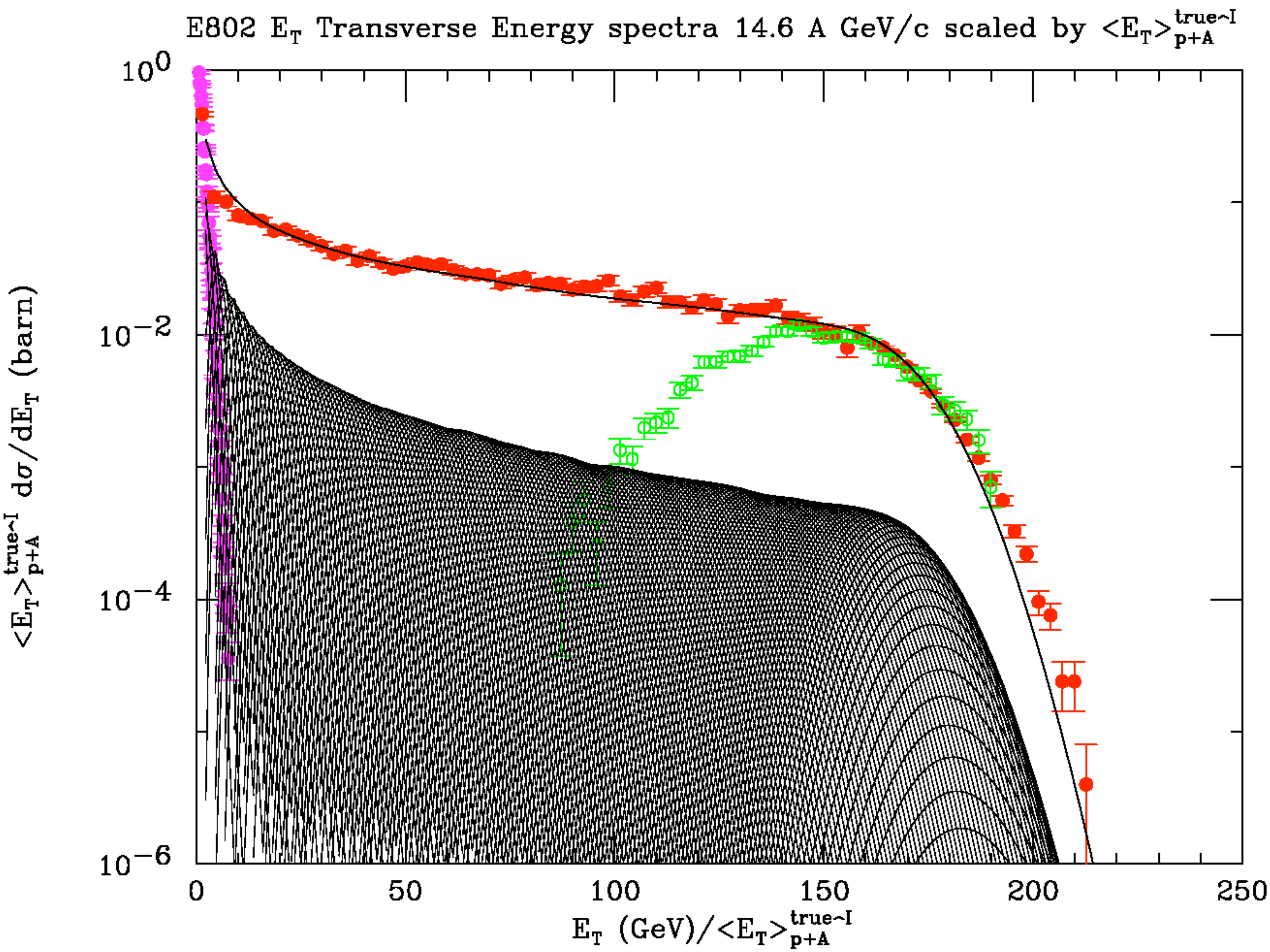
Normalized plots in units of $\langle E_T \rangle^{true} = (1-p_0)\langle E_T \rangle^{obs}$



Illustrate the Nuclear Geometry

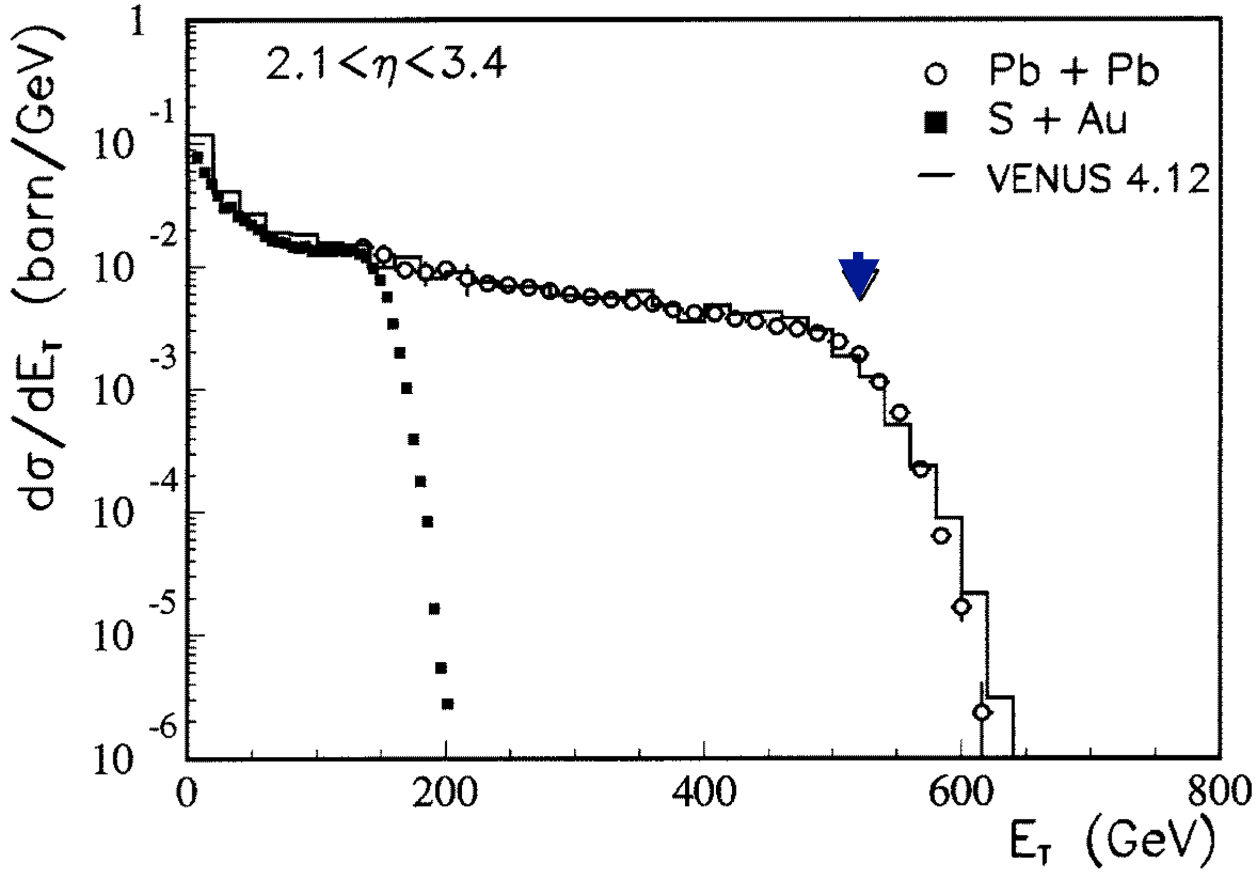


E802 W(p)NM calc Au+Au--details



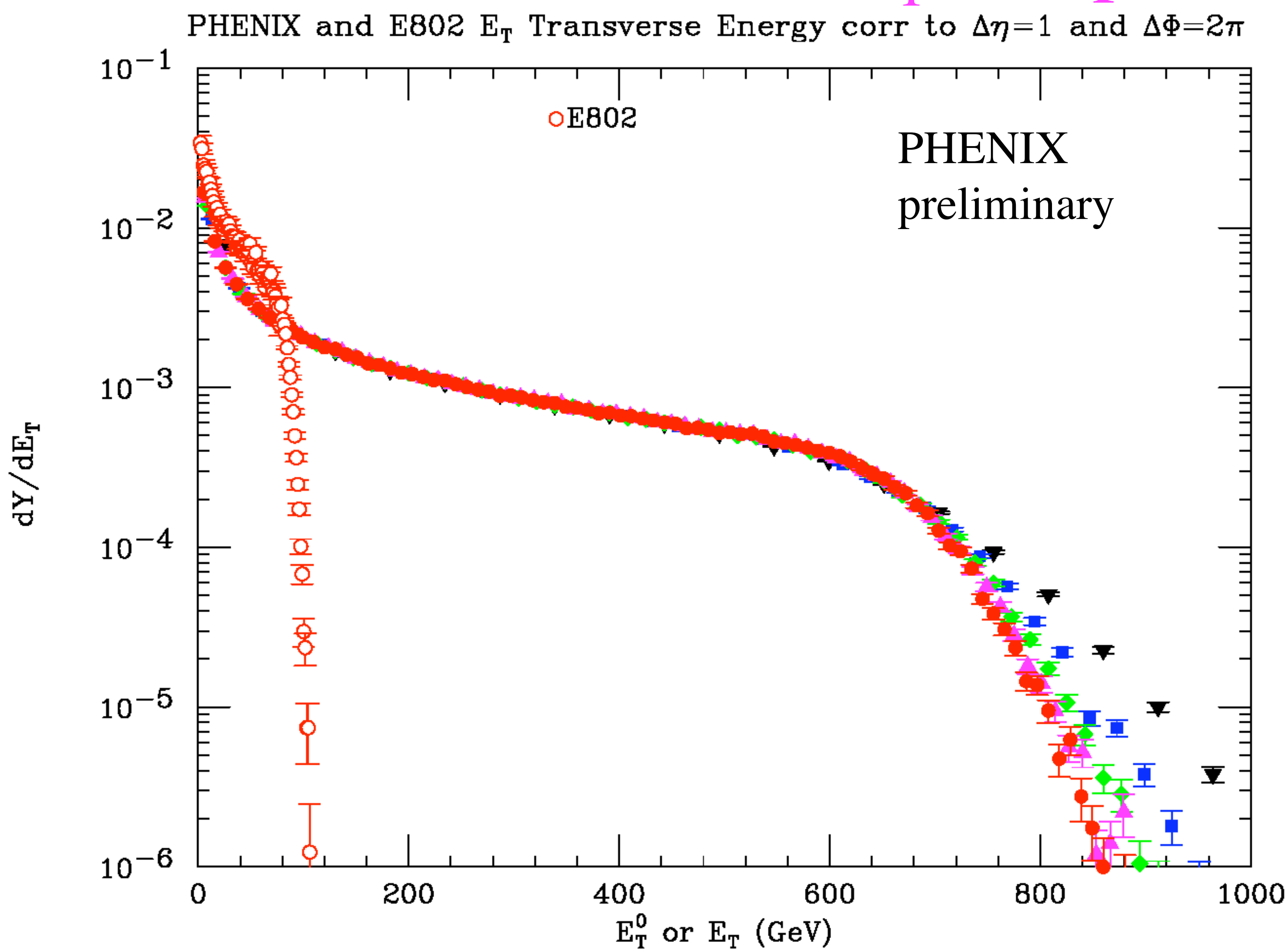
NA35-->NA49 Pb+Pb $\sqrt{s_{NN}}$=17 GeV

PRL **75**, 3814 (1995)



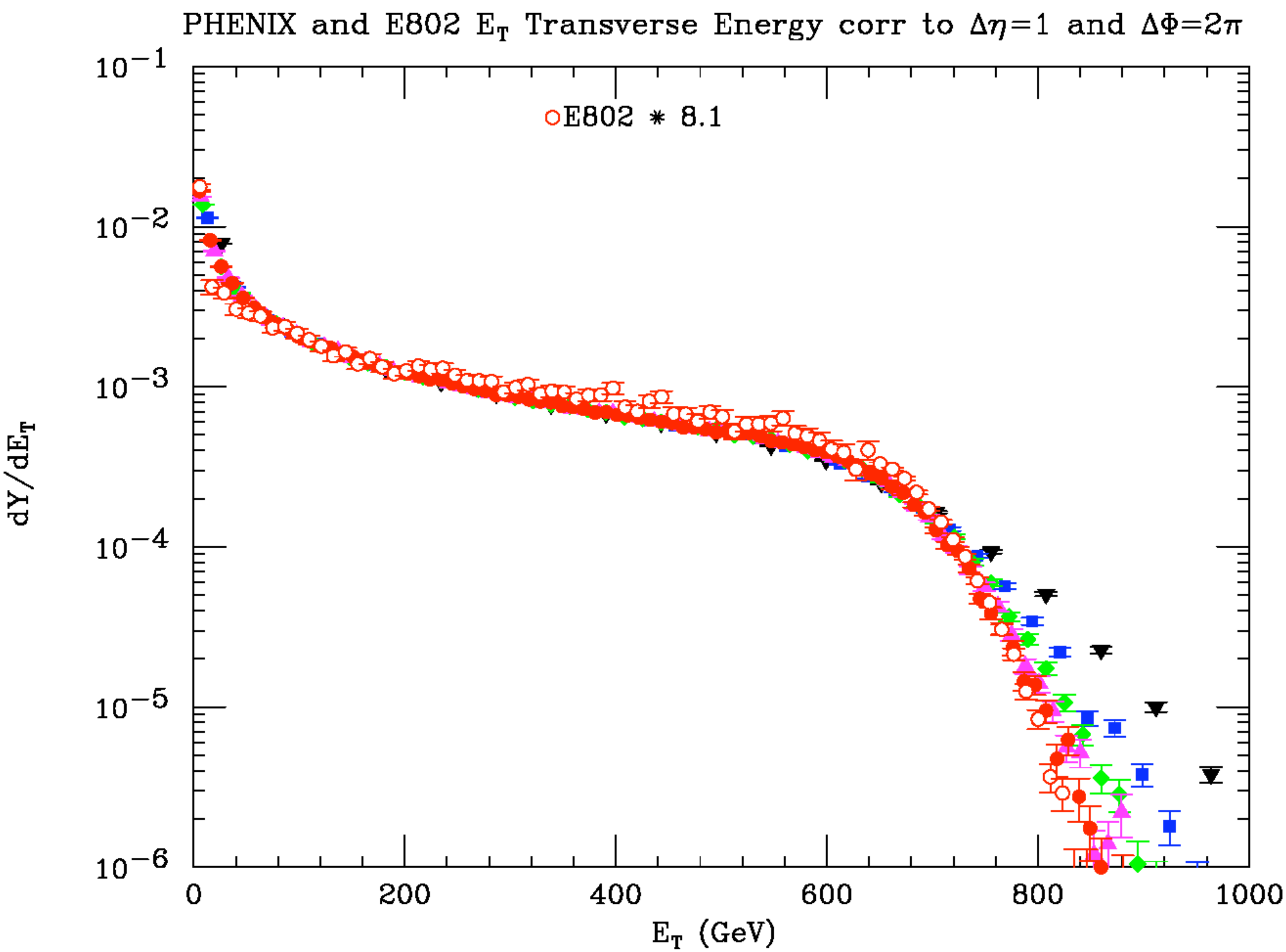
E_T(2.1-3.4)--> dE_T/d[illegible]=405 GeV@$\sqrt{s_{NN}}$=17 GeV

PHENIX and E802 E_T compared

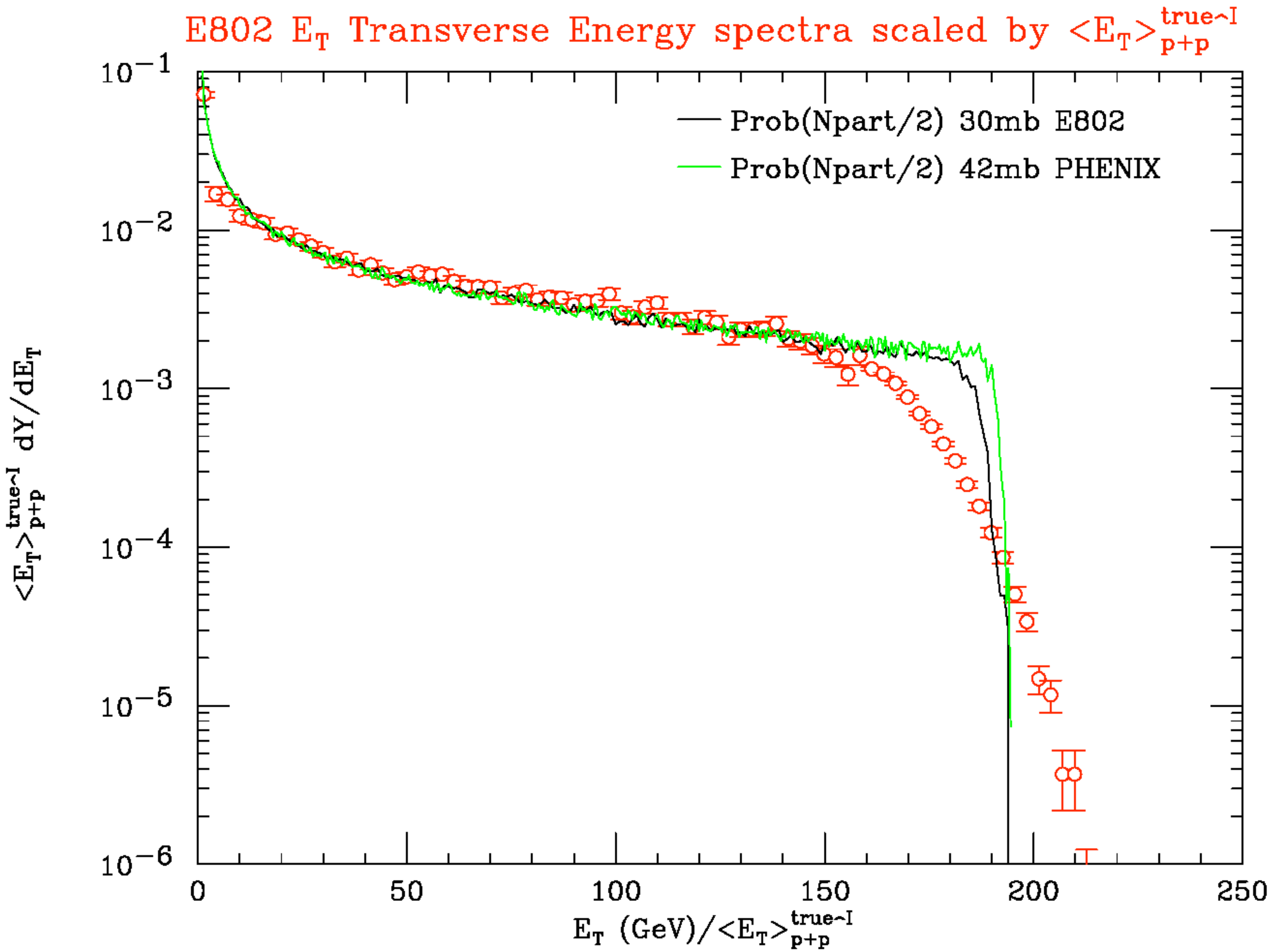


E877 $dE_T/d\square$=200 GeV@$\sqrt{s_{NN}}$=4.8 GeV PHENIX $dE_T/d\square$~680 GeV@$\sqrt{s_{NN}}$=200 GeV

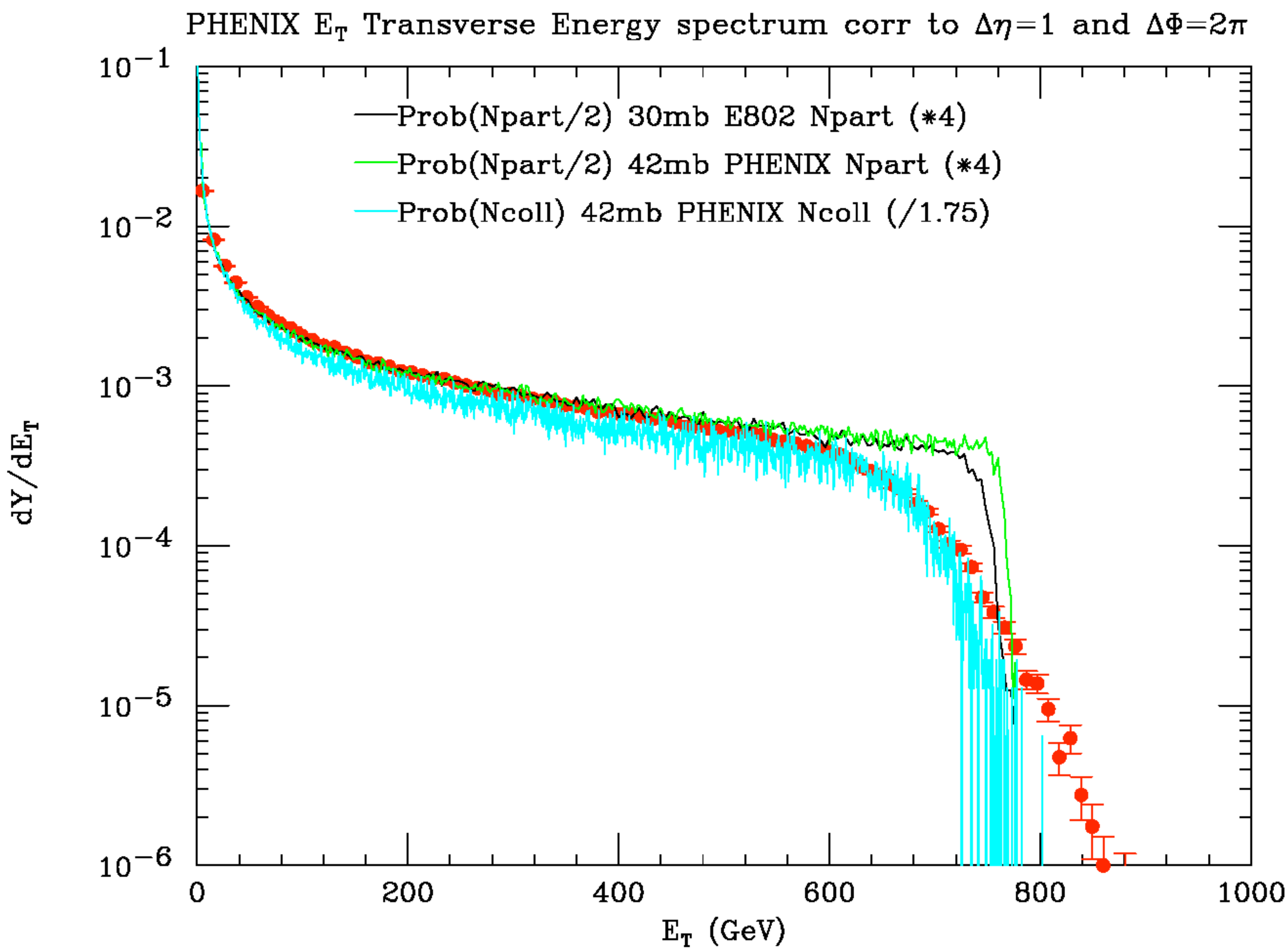
Au+Au E_T spectra at AGS and RHIC are the same shape!!!



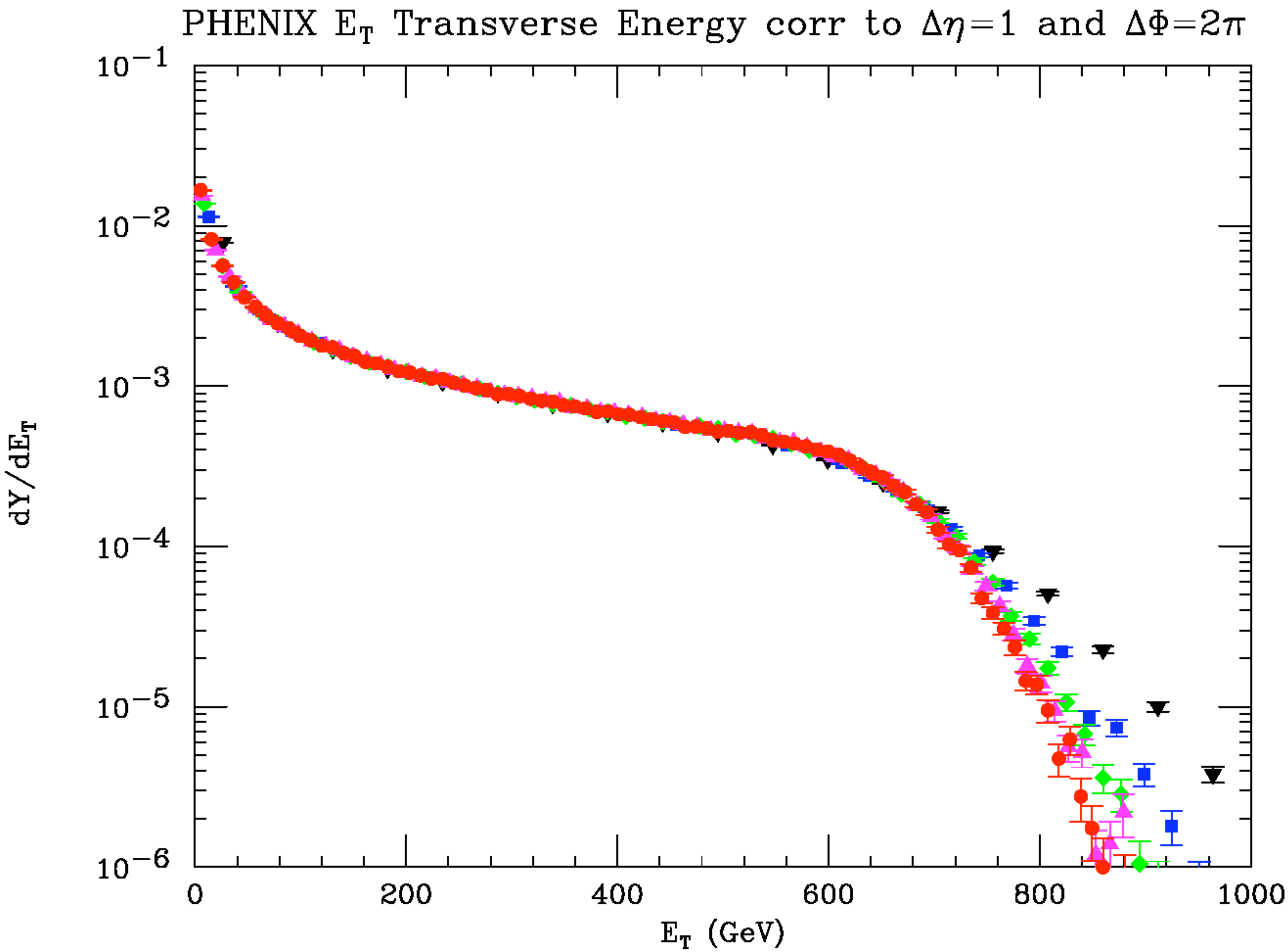
AuAu spectrum at AGS follows WN geometry



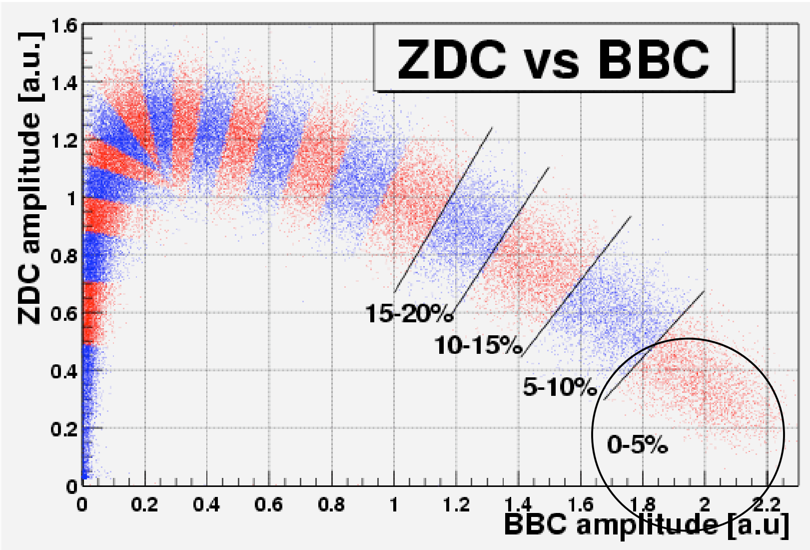
RHIC 2-3 times more E_T than WNM but:



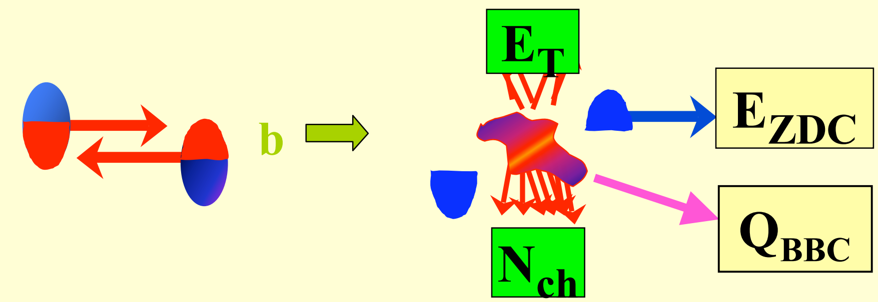
Are upper edge fluctuations random?



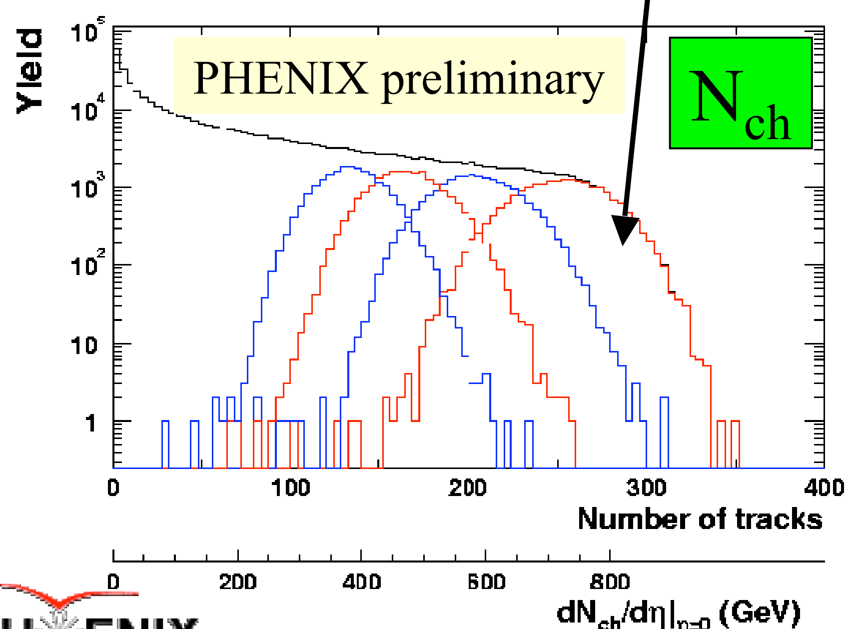
Fluctuations for central collisions

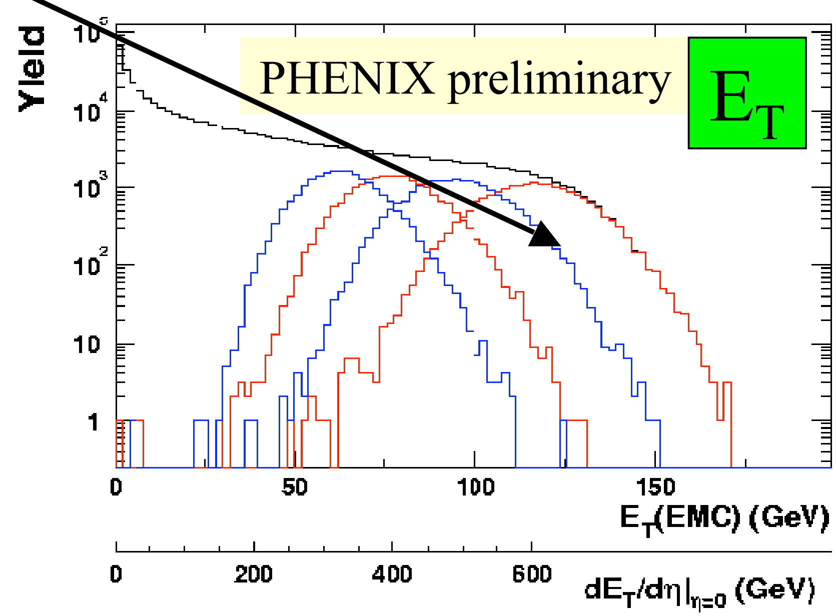


Define centrality classes: ZDC vs BBC

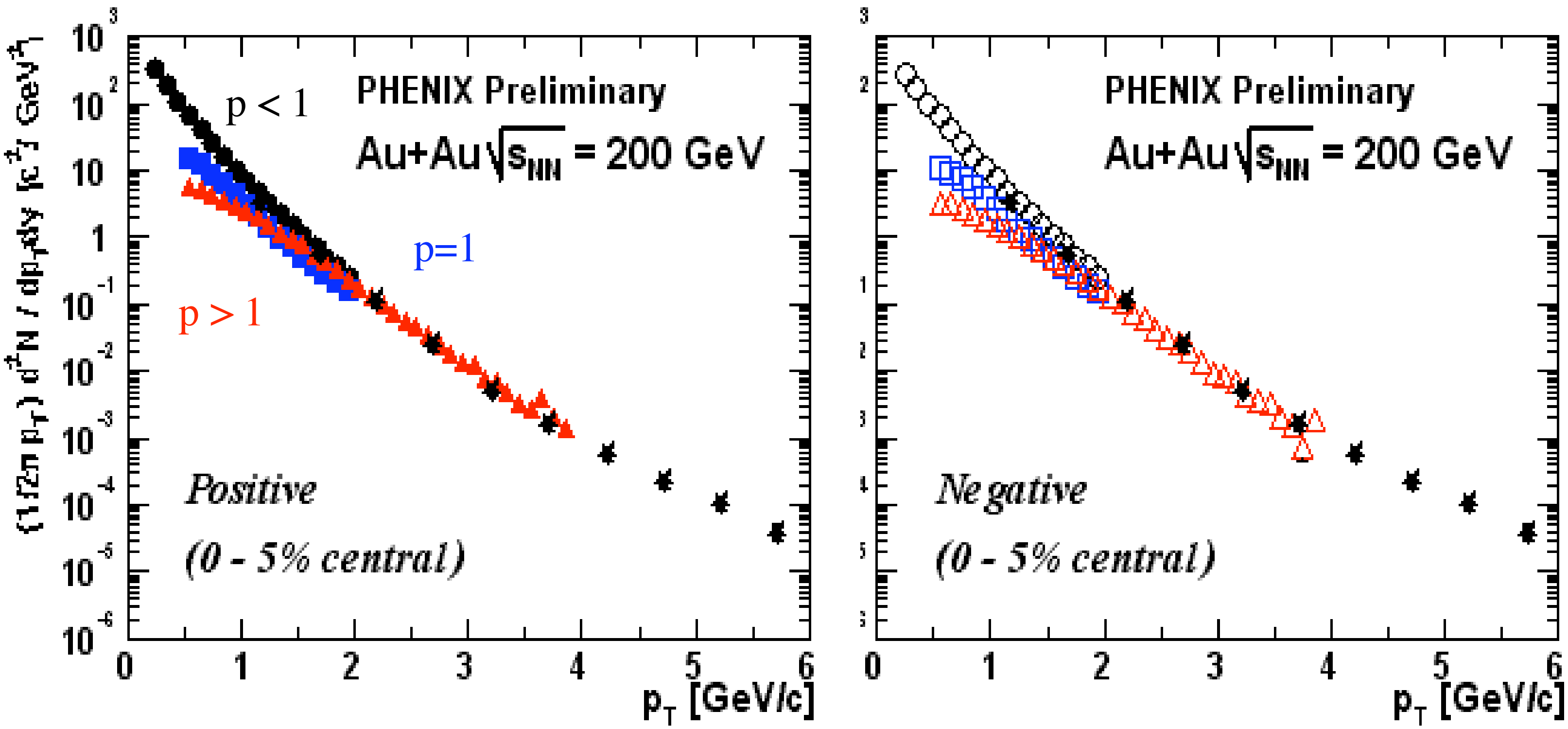


Extract N participants: Glauber model





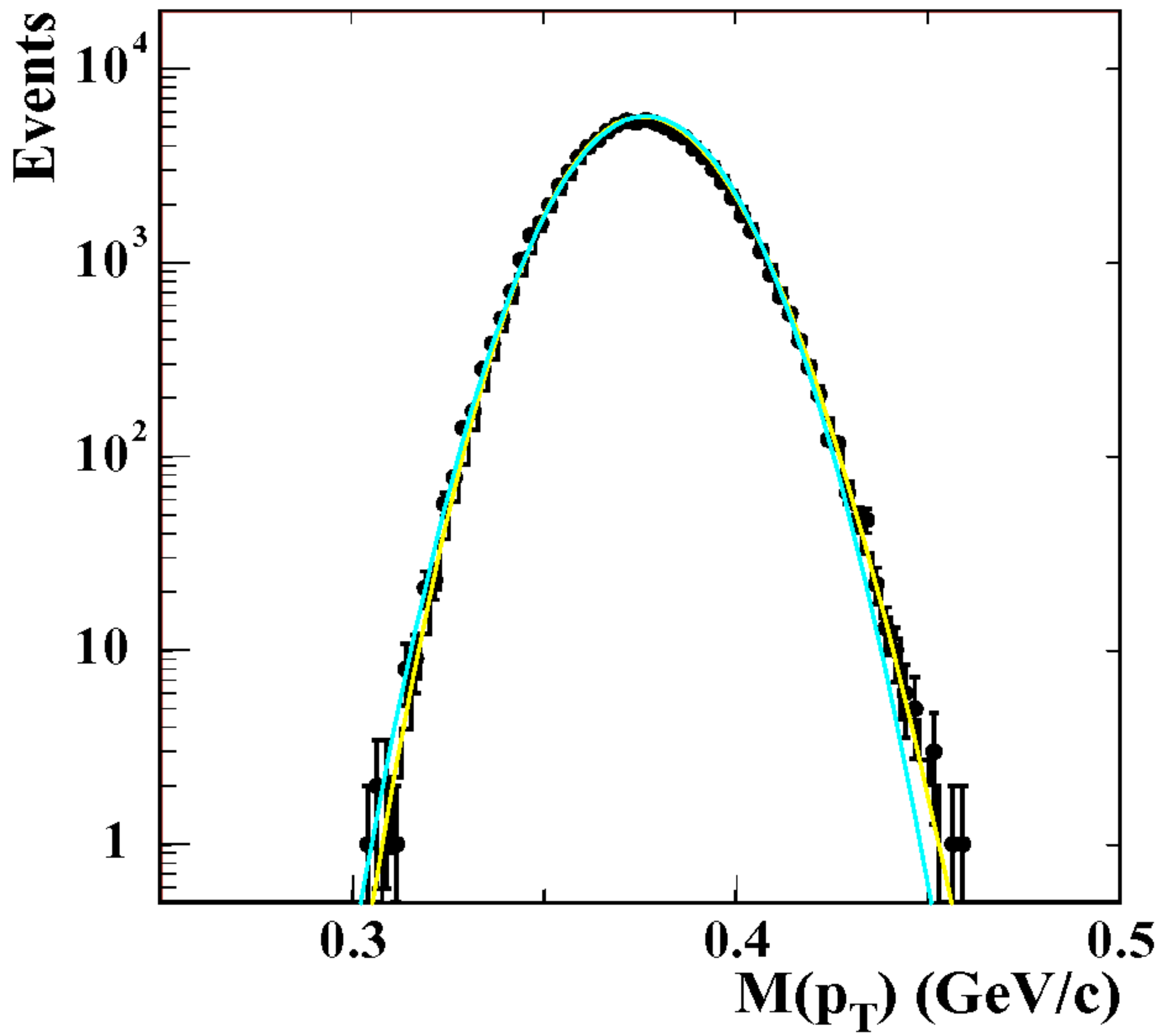
Inclusive p_T spectra are Gamma Distributions



Note: $d\square = p_T\, dp_T\, \square\, (p_T, p) = dp_T\, \square\, (p_T, p+1)$

Event by Event average p_T (Mp_T) Distributions It's not a gaussian, it's a gamma distribution

Analytical formula for statistically independent emission (the sum of independent x_i)



$$M_{p_T} = \overline{p_T}_{(n)} = \frac{1}{n}\sum_{i=1}^{n} p_{T_i} = \frac{1}{n} E_{Tc}$$

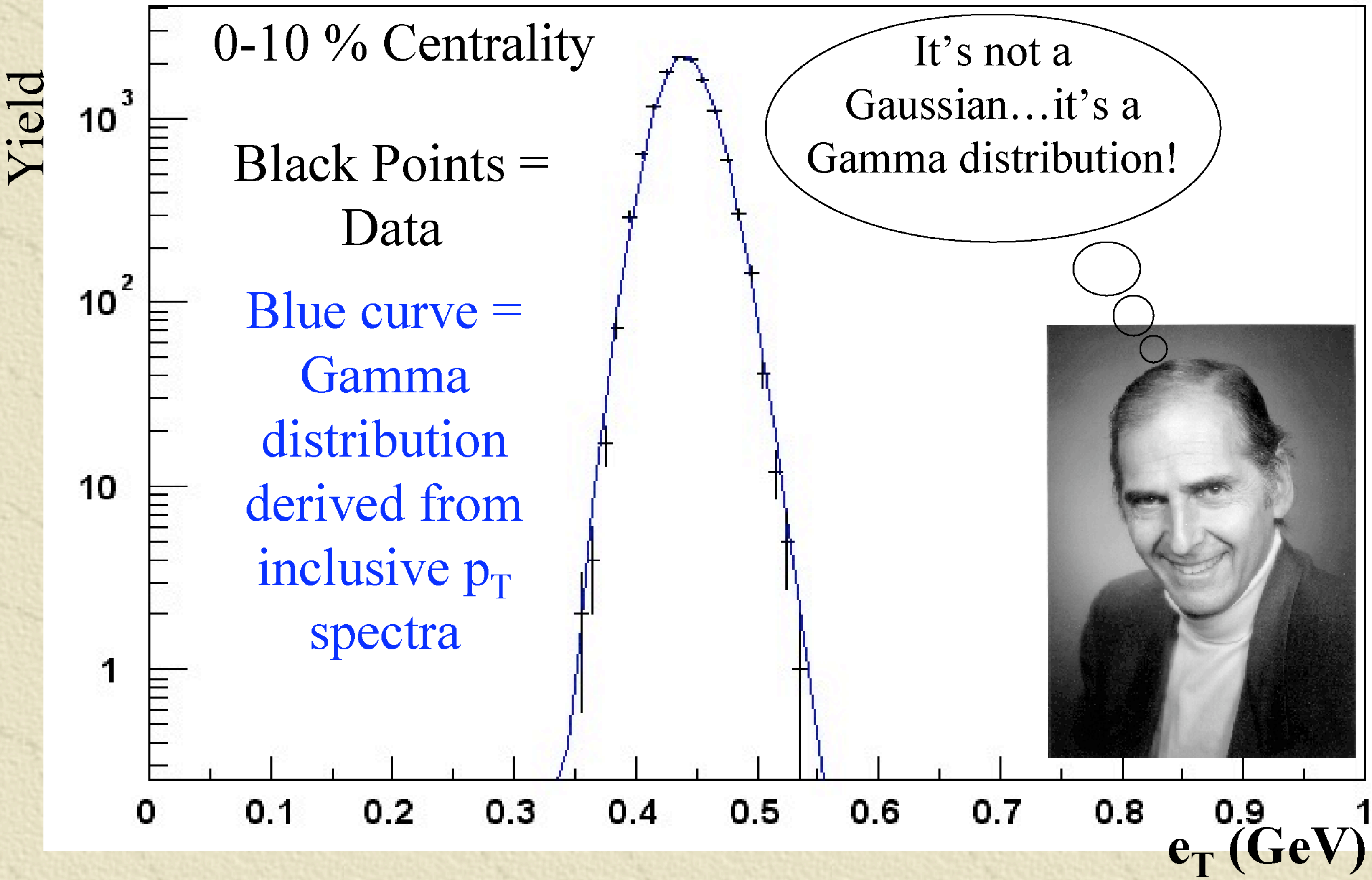
It depends on the 4 semi-inclusive parameters b, p, <n>, 1/k (NBD), which are derived from the quoted means and standard deviations of the semi-inclusive p_T and multiplicity distributions. The result is in excellent agreement with the NA49 Pb+Pb central measurement PLB **459**, 679 (1999)

See M.J.Tannenbaum PLB **498**, 29 (2001)

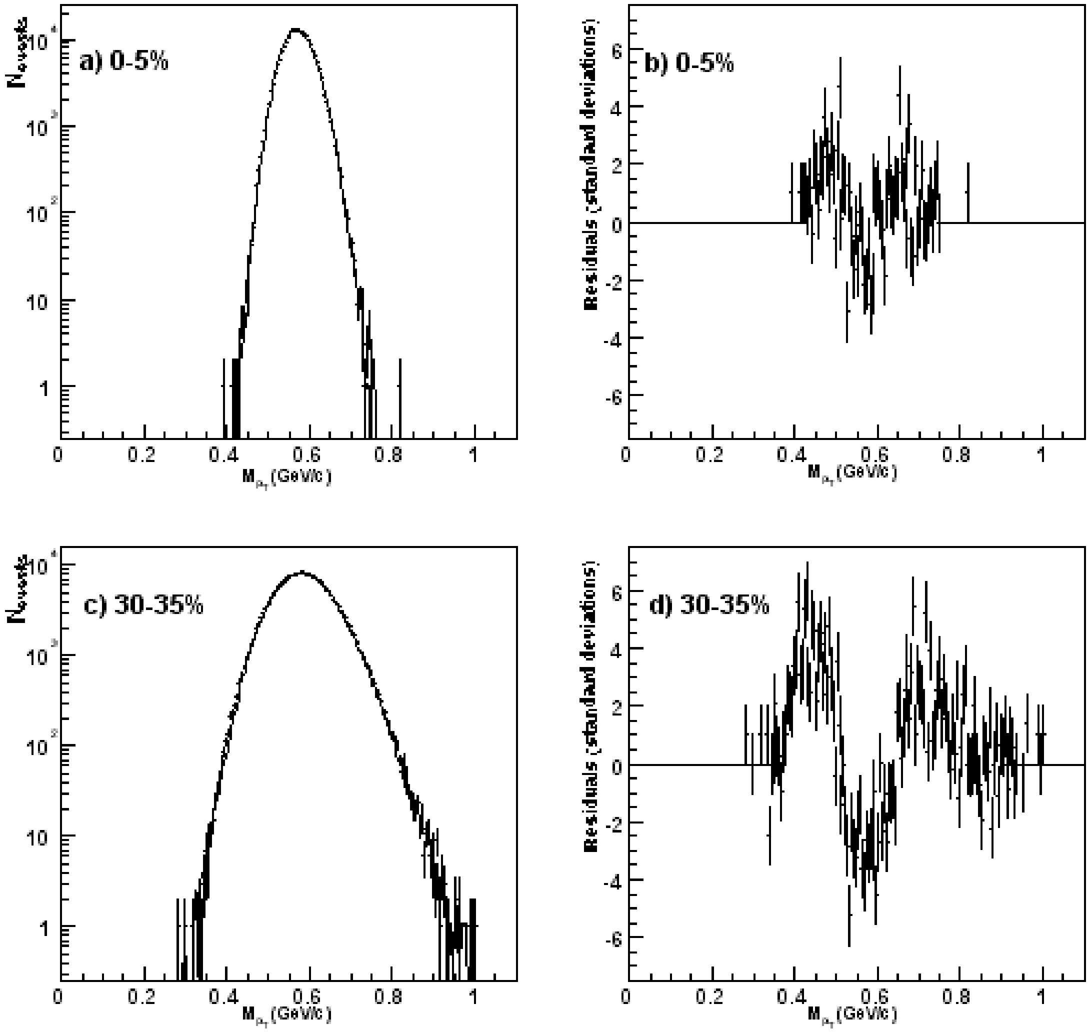
Average e_T Fluctuations

Run-2

Jeffery Mitchell



PHENIX Mp_T vs centrality

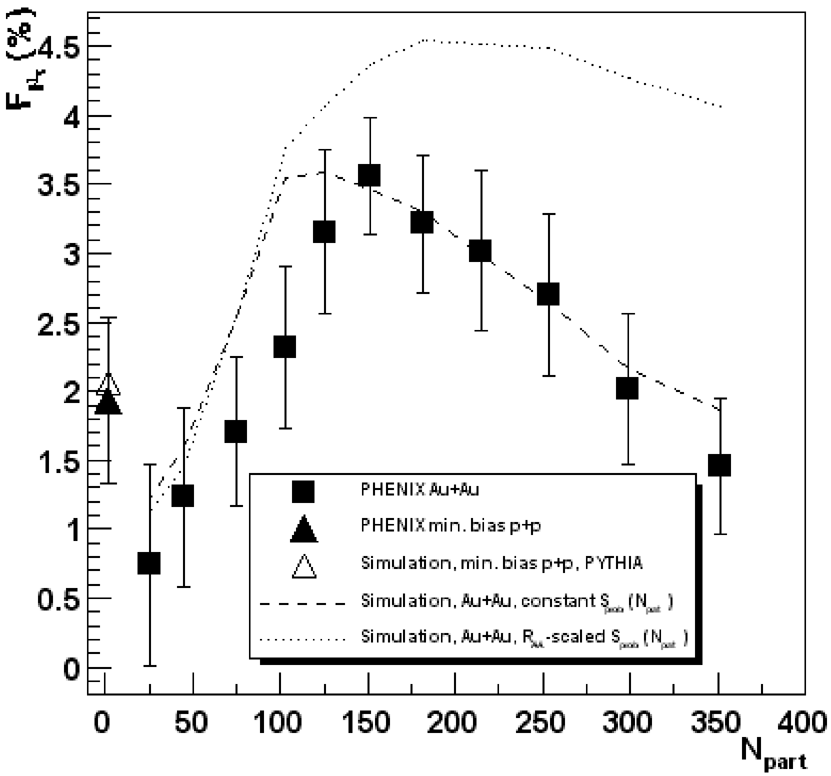


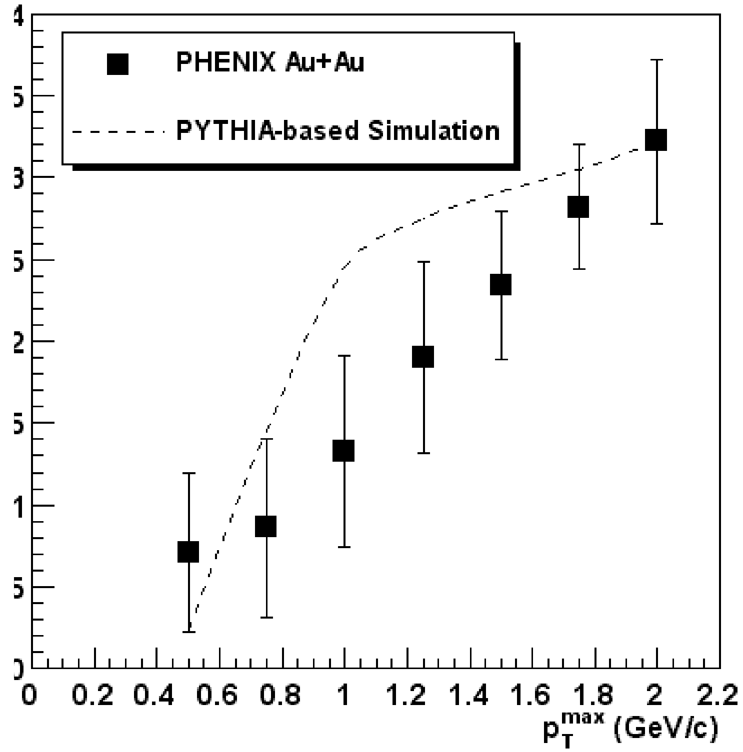
- compare Data to Mixed events for random.
- Must use exactly the same n distribution for data and mixed events and match inclusive <pT> to <Mp_T>
- best fit of real to mixed is statistically unacceptable
- deviation expressed as:

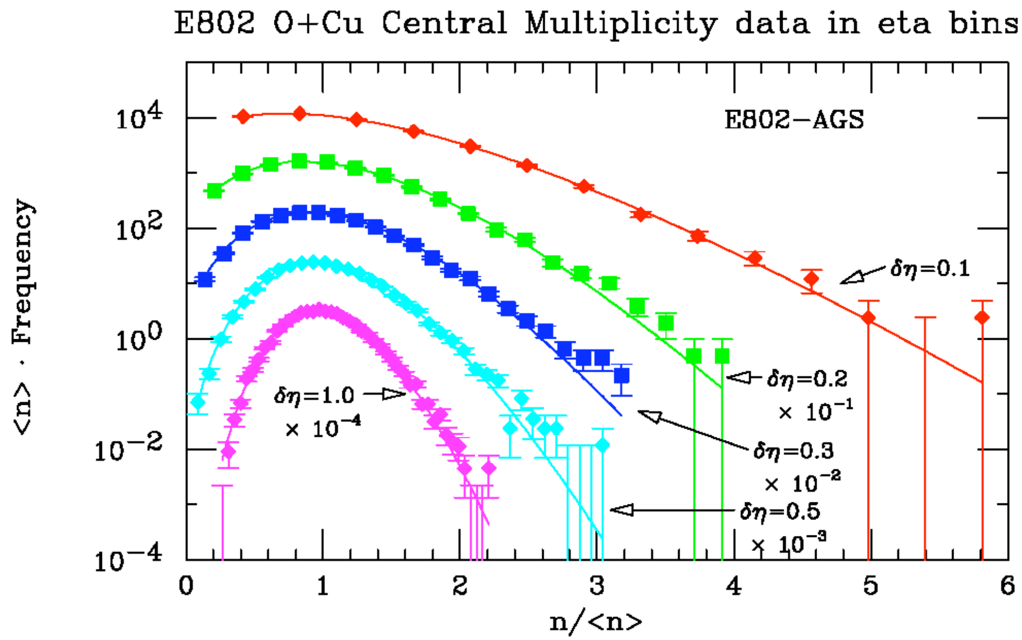
$Fp_T = \square_{Mp_T data} / \square_{Mp_T mixed} - 1$

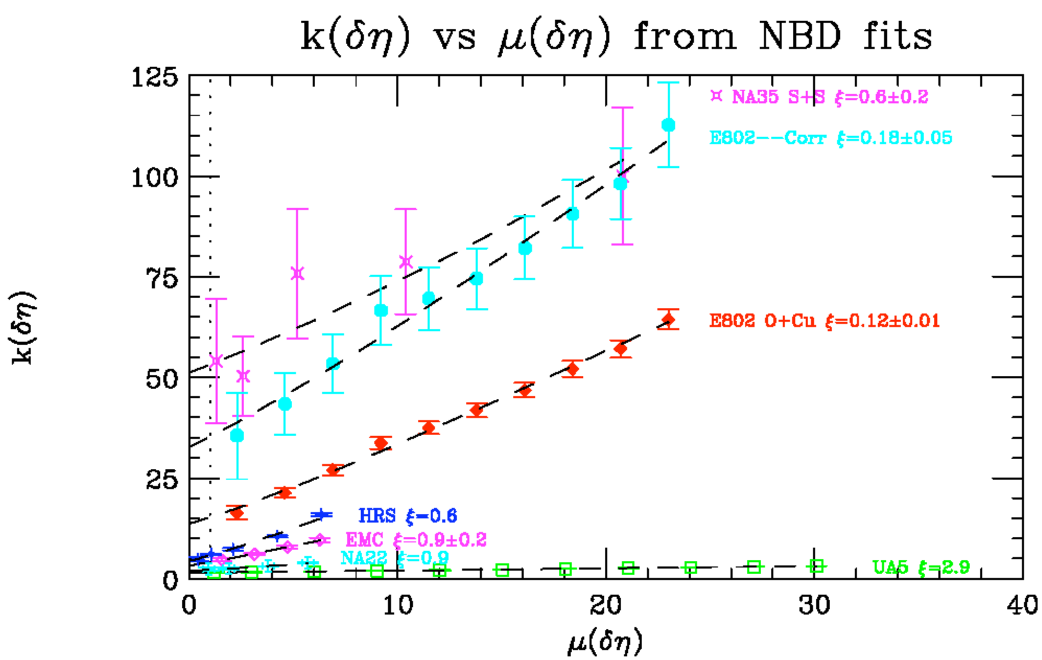
Corelation is a few percent of $\square_{MpT}$: Due to jets

PHENIX nucl-ex/0309xxx









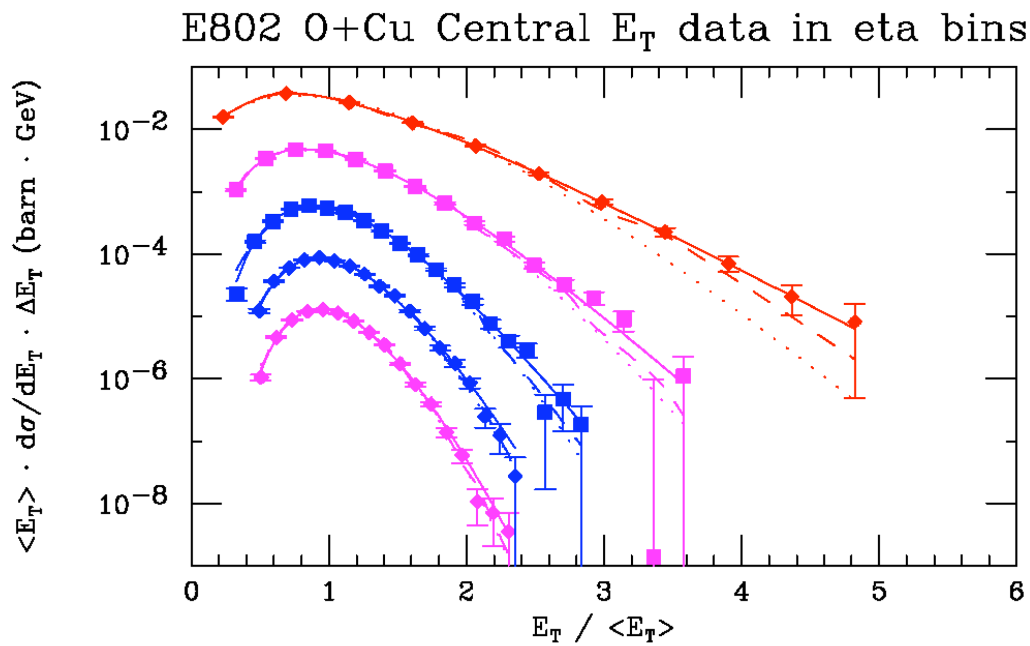
Large fluctuations in AGS O+Cu central *multiplicity*: distribution is NBD (correlations due to to B-E don’t vanish)

PRC **52**, 2663 (1995)

$\square^2/\square^2 = 1/\square + 1/k$

What happens at RHIC?

- Au+Au
- Lighter Ions



Is ET or multiplicity primary? Which fluctuates more?

PRC **63**, 064602 (2001)

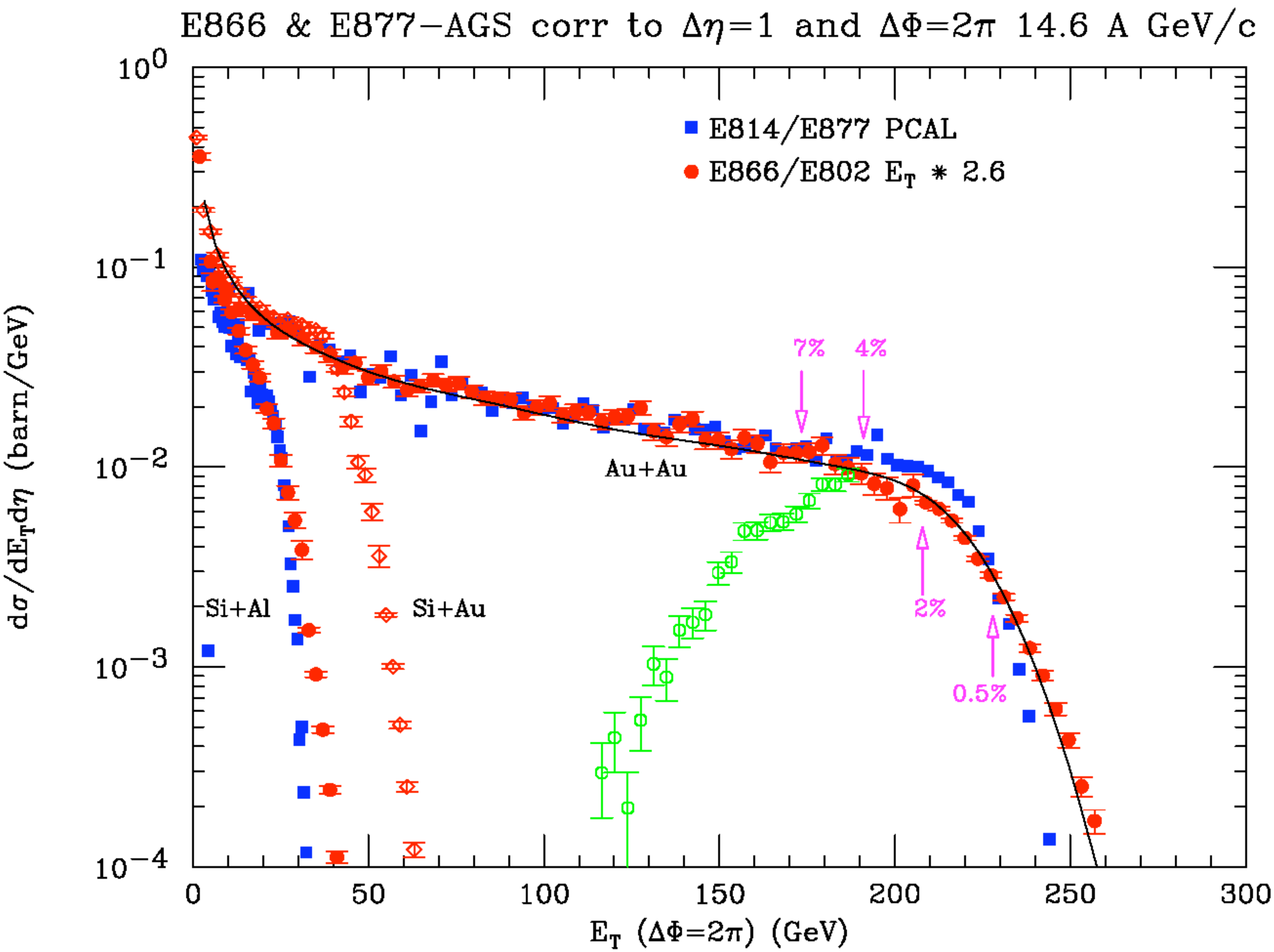
a) QCD-like: first create Energy which fragments to multiplicity
b) as in Mp_T, create multiplicity which then gets p_T according to inclusive distribution.

- solid lines fit to simple distribution

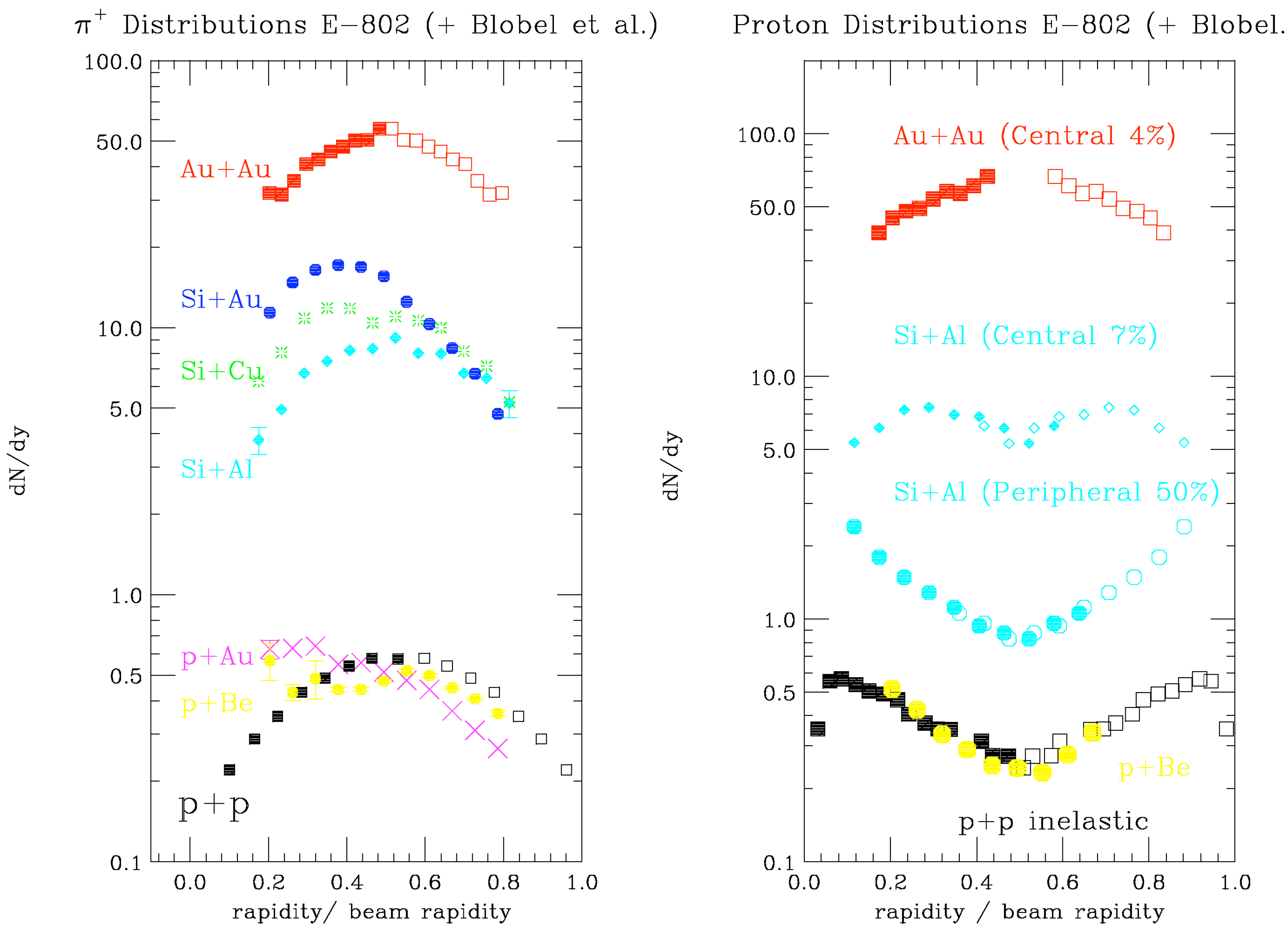
- dotted line mult NBD followed by for E_T per particle

$$\frac{d\sigma}{dE_T} = \sigma \sum_{n=1}^{n_{\max}} f_{\rm NBD}(n, 1/k, \mu)\, f_\Gamma(E_T, np, b)$$

??

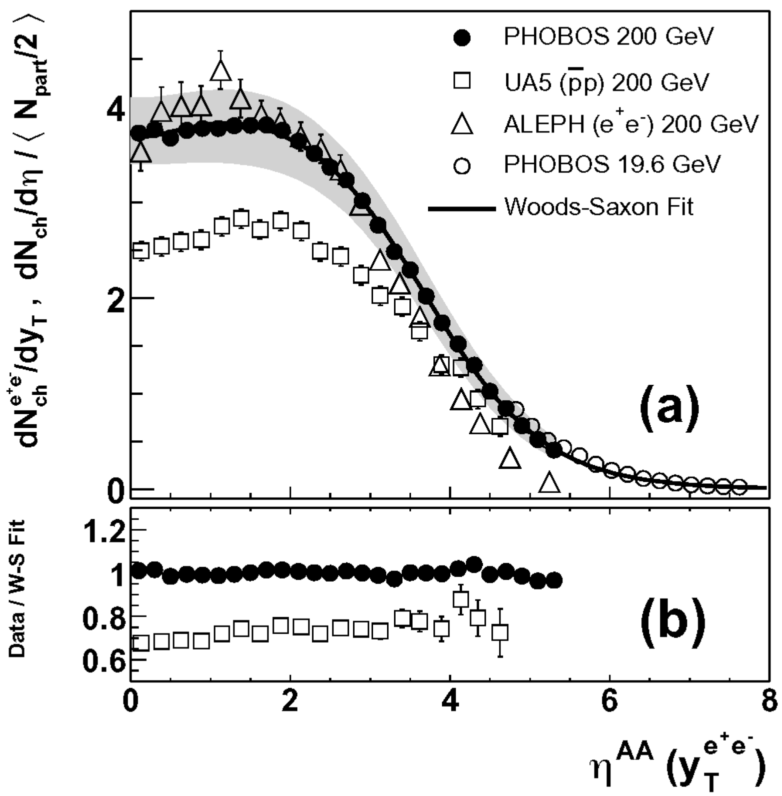

E866 & E877–AGS corr to Δη=1 and ΔΦ=2π 14.6 A GeV/c
E814/E877 PCAL
E866/E802 E_T * 2.6
7%
4%
2%
0.5%
Au+Au
Si+Al
Si+Au
dσ/dE_Tdη (barn/GeV)
E_T (ΔΦ=2π) (GeV)

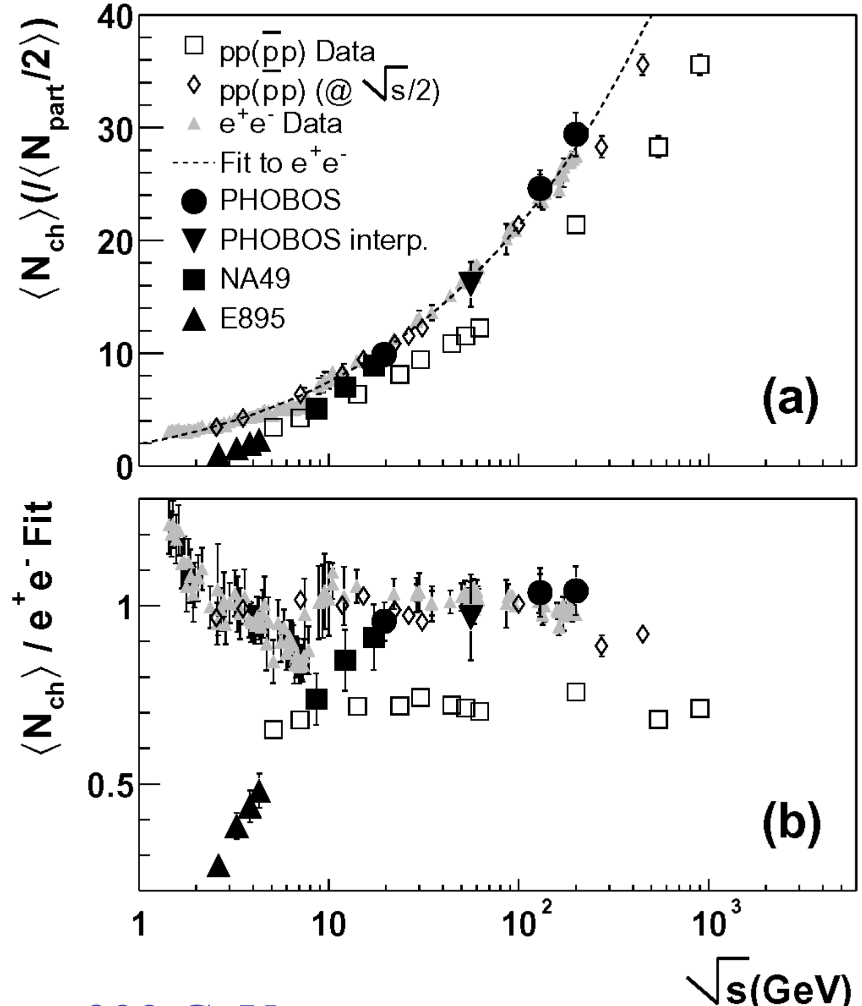
E802 measurements of dn/dy π^+ and p confirm stopping



For Nino
PHOBOS dn/d

cf. M.Basile, et al, PL**B92**, 367 (1980); **B95**, 311 (1980)





dn/d/<N_{part}/2> Au+Au ~ e^+e^- > pp @$\sqrt{s_{NN}}$=200 GeV