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Twist-three fragmentation function contribution to the single spin asymmetry in pp collisions

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ABSTRACT

We study the twist-three fragmentation function contribution to the single transverse spin asymmetries in inclusive hadron production in pp collisions, $p^\uparrow p \to h + X$. In particular, we calculate the associated derivative terms which dominate the spin asymmetries in these processes. With certain parameterizations for the twist-three fragmentation function, we estimate its contribution to the single spin asymmetry of π^0 production at RHIC energy. We find that the contribution is sizable and might be responsible for the big difference between the asymmetries in η and π^0 productions observed by the STAR collaboration at RHIC.

Single transverse spin asymmetries (SSAs) in hadronic processes, such as the single inclusive hadron production nucleon–nucleon scattering, $p^\uparrow p \to hX$, have attracted much interests from both experimental and theoretical sides in the last few years, and great progress has been made in understanding the underlying physics [1]. Although the SSA is a simple observable, defined as the spin asymmetry when one flips the transverse spin of one of the hadrons involved in the scattering: $A_N = (d\sigma(S_\perp) - d\sigma(-S_\perp))/(d\sigma(S_\perp) + d\sigma(-S_\perp))$, it is far complicated to explain in the fundamental theory of strong interaction. The single transverse spin dependent differential cross section $d\sigma(S_\perp)$ is usually expressed as a correlation between the transverse polarization vector $S_\perp$ of one of the hadrons and the transverse momentum $P_{h\perp}$ of the final-state hadron. For example, in the process $p^\uparrow p \to hX$, it is the correlation between the polarization vector $S_\perp$ of the incoming nucleon and the transverse momentum $P_{h\perp}$ of the final-state hadron. In this Letter, we will focus on the SSAs in these processes, especially for the neutral meson π^0 and η productions, motivated by the recent experimental observations of large SSAs in forward direction of the polarized proton by the STAR collaboration at RHIC experiments [2,3]. In particular, it was found that the SSA for η meson is much larger than that for π^0 meson. This also confirms previous observations from the fixed target experiments [4]. In addition to these STAR results, both BRAHMS and PHENIX collaborations at RHIC have observed large single transverse spin asymmetries in charged and neutral meson production in the similar kinematic region [5,6].

In the QCD framework, there have been mainly two approaches to study the SSAs in high energy scattering processes: the transverse momentum dependent (TMD) parton distribution and fragmentation function approach [7–15] and the twist-three quark–gluon collinear correlation approach [16–21]. The TMD approach requires an additional hard momentum scale besides the transverse momentum $P_{h\perp}$ of the hadron, such as Q^2, the momentum transfer squared of the virtual photon in semi-inclusive deep inelastic scattering (SIDIS) process or Drell–Yan lepton pair production in pp collisions. On the other hand, the twist-three approach is more appropriate to describe the processes where all momentum scales are much larger than the nonperturbative scale $\Lambda_{\rm QCD}$ [17]. It has been shown, however, that these two approaches are consistent in the description of the SIDIS and Drell–Yan processes in the intermediate transverse momentum region where they both apply [22,23].

In this Letter, we follow the collinear factorization approach to study the single spin asymmetry coming from the twist-three fragmentation function contribution.[1] In this process, there is only

[1] The TMD approach has also been used to calculate the SSA in the single inclusive hadron production in $p^\uparrow p$ collisions in a model dependent way [10,24,25].

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one large momentum scale, the transverse momentum $P_{h\perp}$ of the final-state hadron, and the SSA is naturally suppressed by $1/P_{h\perp}$ at large transverse momentum $P_{h\perp}$ as a consequence of the twist-three effect. However, it is not clear how large the momentum should be for this behavior being manifest. At low transverse momentum, it certainly will fail, and a modification has to be made to account for the experimental observations [3,27].

In the twist-three collinear factorization approach, for the SSAs in hadron production in

$$p^{\uparrow}(A) + p(B) \to h + X, \tag{1}$$

the twist-three effects could come from the distribution functions of the incoming polarized nucleon (A) with momentum P_A or the unpolarized nucleon (B) with momentum P_B, or the fragmentation function for the final state hadron (h) with momentum P_h. Therefore, schematically, we can write down the single transverse spin dependent differential cross section in the following way [17,18],

$$\begin{aligned} d\sigma(S_\perp) = \epsilon_\perp^{\alpha\beta} S_{\perp\alpha} P_{h\beta} \int [dx][dy][dz] \\ \times \{\phi^{(3)}_{i/A}(x,x') \otimes \phi_{j/B}(y) \otimes D_{h/c}(z) \otimes \mathcal{H}^{(A)}_{ij\to c}(x,x',y,z) \\ + \phi_{i/A}(x) \otimes \phi^{(3)}_{j/B}(y,y') \otimes D_{h/c}(z) \otimes \mathcal{H}^{(B)}_{ij\to c}(x,y,y',z) \\ + \phi_{i/A}(x) \otimes \phi_{j/B}(y) \otimes D^{(3)}_{h/c}(z,z') \otimes \mathcal{H}^{(c)}_{ij\to c}(x,y,z,z')\}, \end{aligned} \tag{2}$$

where $\epsilon_\perp^{\alpha\beta}$ is defined as $\epsilon_\perp^{\alpha\beta} = \epsilon^{\mu\nu\alpha\beta} P_{A\mu} P_{B\nu}/P_A \cdot P_B$ with the convention of $\epsilon^{0123} = 1$, x (x') and y (y') are the momentum fractions of the incoming hadrons carried by the partons i and j, and z (z') the momentum fraction of the fragmenting quark carried by the final state hadron, respectively. Here we use $\perp$ to label the transverse direction in the center of mass frame of P_A and P_B. In the above equation, $\otimes$ stands for the convolution in the longitudinal momentum fractions x, y, and z. The leading twist parton distributions are labeled by $\phi_{i/A}(x)$ and $\phi_{j/B}(y)$. Because hadron A is transversely polarized, we immediately see that $\phi_{i/A}$ represents the leading-twist quark transversity distributions. However, for unpolarized hadron B, $\phi_{j/B}$ represents both leading twist quark and gluon distributions. Leading twist parton fragmentation function is represented by $D_{h/c}(z)$ where parton c can be a quark or gluon. In the above equation, the superscript (3) represents the twist-three correlations for the distribution functions or the fragmentation functions. These twist-three functions normally involve two variables, for which we have been shown explicitly in the above formula. For a complete analysis, all of the above three terms have to be taken into account. The first term in Eq. (2), representing the contributions from the twist-three parton distributions $\phi^{(3)}_{i/A}(x,x')$ of the polarized nucleon, have been calculated in Refs. [17–19]. The second term, for the contributions from the twist-three effect in the unpolarized nucleon $\phi^{(3)}_{j/B}(x,x')$, has been found very small in the forward region of the polarized nucleon [28]. The third term is least known in the literature. There have been earlier attempts to formulate this part of contribution in the twist-three collinear factorization approach [29]. However, the universality argument for the Collins fragmentation function [30,31] would indicate that the contribution calculated there vanishes. This universality has also been extended to the relevant processes in pp collisions [32]. Large azimuthal asymmetries from the Collins effect have been observed in semi-inclusive hadron production in deep inelastic scattering by the HERMES and COMPASS collaborations [33,34], and in di-hadron production in e^+e^- annihilation processes by the BELLE collaborations [35], respectively. In a recent publication, two of us have re-examined the twist-three fragmentation contribution to the single spin asymmetry [23]. In particular, we have identified the twist-three fragmentation function corresponding to the TMD Collins fragmentation function. In this Letter, we will extend this formalism to the twist-three fragmentation function contribution to the above process of (1), the single spin asymmetries in inclusive hadron production in $p^{\uparrow}p$ collisions.

The twist-three fragmentation function which corresponds to the universal Collins TMD fragmentation function, is defined as[2]

$$\begin{aligned} \hat{H}(z) = \frac{z^2}{2} \int \frac{d\xi^-}{2\pi} e^{ik^+\xi^-} \\ \times \frac{1}{2}\left\{ \mathrm{Tr}\,\sigma^{\alpha+}\langle 0| \left[iD_T^\alpha + \int_{\xi^-}^{+\infty} d\zeta^- g\, F^{\alpha+}(\zeta^-) \right] \psi(\xi)|P_h X\rangle \right. \\ \left. \times \langle P_h X|\bar{\psi}(0)|0\rangle + \text{h.c.} \right\}, \end{aligned} \tag{3}$$

where $k^+ = P_h^+/z$, $D_T^\alpha = \partial_T^\alpha - igA_T^\alpha$ is the transverse component of the covariant derivative, and $F^{\alpha+}$ is the gluon field strength tensor. $\hat{H}(z)$ can be written as the transverse momentum moment of the Collins fragmentation function $H_1^\perp(z, p_T^2)$ [15],

$$\hat{H}(z) = \int d^2 p_T \frac{|\vec{p}_T|^2}{2M_h} H_1^\perp(z, p_T^2). \tag{4}$$

In the above definition (Eq. (3)), we have set the hadron momentum in the $+\hat{z}$ direction. The index α is in the transverse direction perpendicular to the hadron momentum. This index is not considered to be summed up in Eq. (3). In order to uniquely define this transverse component (index), we introduce a light-like vector $n_h^\mu \propto (P_h^0, -\vec{P}_h)$ with normalization that $n_h \cdot n_h = 0$ and $n_h \cdot P_h = 1$.[3] Therefore, the transverse component for any vector p^μ can be defined as $p_T^\mu = p^\mu - p\cdot n_h P_h^\mu - p\cdot P_h n_h^\mu$. Here and in the following, we use subscript "T" to represent the transverse direction perpendicular to P_h which is different from "$\perp$" in Eq. (2) for transverse direction perpendicular to P_A and P_B. From this definition, we can immediately see that p_T is space-like, and perpendicular to $\vec{P}_h$: $\vec{p}_T \cdot \vec{P}_h = 0$.

As discussed in Ref. [23], the above twist-three fragmentation function belongs to more general two-variable dependent twist-three fragmentation function,

$$\begin{aligned} \hat{H}_D(z_1,z_2) = \frac{z_1 z_2}{2} \int \frac{d\xi^- d\zeta^-}{(2\pi)^2} e^{ik_2^+\xi^-} e^{ik_g^+\zeta^-} \\ \times \frac{1}{2}\{ \mathrm{Tr}\,\sigma^{\alpha+}\langle 0|iD_T^\alpha(\zeta^-)\psi(\xi^-)|P_h X\rangle \\ \times \langle P_h X|\bar{\psi}(0)|0\rangle + \text{h.c.}\}, \end{aligned} \tag{5}$$

where $k_i^+ = P_h^+/z_i$ and $k_g^+ = k_1^+ - k_2^+$. Similarly, we can define a F-type fragmentation function $\hat{H}_F(z_1,z_2)$ by replacing D_T^α with $F^{+\alpha}$

However, there has been no factorization argument based on the TMD parton distribution and fragmentation functions for this process [26].

[2] In the definition of $\hat{H}(z)$ in Ref. [23], a factor of 1/2 was missing.

[3] In the low transverse momentum semi-inclusive hadron production in DIS process, n_h will be proportional to the incoming nucleon momentum as we used in Ref. [23]. In current study, there is no natural available momentum for this choice. We can also perform the calculations without choosing the vector n_h, but keeping all transverse component in Eq. (3) perpendicular to the hadron momentum. This will lead to the same results.

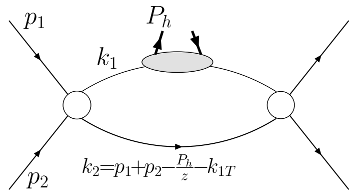

Fig. 1. Generic Feynman diagrams to calculate the derivative terms of the twist-three fragmentation function contributions to the single inclusive hadron production in pp scattering, $p^\uparrow p \to hX$: p_1 and p_2 are the two incident partons' momenta, k_1 and k_2 are the outgoing partons' momenta, where the quark k_1 fragments into final-state hadron P_h. The expansion of the scattering amplitude in terms of transverse momentum component of $k_1 = P_h/z + k_{1T}$ leads to the twist-three contributions from the fragmentation function $\hat{H}(z)$ defined in Eq. (3).

in Eq. (5). By using the equation of motion, D-type and F-type functions are related to each other [15,21],

$$\hat{H}_D(z_1, z_2) = PV\left(\frac{1}{\frac{1}{z_1}-\frac{1}{z_2}}\right)\hat{H}_F(z_1, z_2) + \delta\left(\frac{1}{z_1}-\frac{1}{z_2}\right)\frac{z_2}{z_1}\hat{H}(z_1). \tag{6}$$

Eq. (6) shows that $\hat{H}_D$ is more singular than $\hat{H}_F$ at $z_1 = z_2$. Moreover, recent study has shown that $\hat{H}_F(z_1, z_2)$ vanishes when $z_1 = z_2$ [31]. Therefore, it is more convenient to express the cross section in terms of $\hat{H}(z)$ and $\hat{H}_F(z_1, z_2)$, as we will follow in our calculations below.

Similar to the twist-three distribution function contributions [18], the twist-three fragmentation function contributions contain both the derivative terms, which are proportional to $z\frac{\partial}{\partial z}(\hat{H}(z)/z^2)$ as we will show below, and the non-derivative terms, which are proportional to either $\hat{H}(z)/z^2$ or $\hat{H}_F(z_1, z_2)$. It has been shown that the cross section of inclusive hadron production in pp collisions is dominated by the large z region of the fragmentation function for the RHIC kinematics [36]. If this were true, the derivative terms will have an enhancement factor $1/(1-z)$ compared to the non-derivative terms, thus are more important to the SSAs of inclusive hadron productions. As a first step, we keep only the derivative terms in the following calculations. The complete result including both the derivative and non-derivative terms will be left for a future publication.

We follow the techniques developed in the last few years [17–23] to derive the contributions from the twist-three fragmentation functions. First, we calculate the associated scattering amplitudes in terms of various twist-three matrix elements of hadrons in the fragmentation final state. From diagrammatic point of view, these individual contributions are not in a gauge invariant way. However, the final results will be gauge invariant when we sum up all the contributions. In the calculations, there are contributions from the matrix elements associated with $\partial_T\psi$, A_T, and $\partial_T A^+$ ($\partial^+ A_T$) operators. These contributions will form the gauge invariant results in terms of $\hat{H}(z)$, $\hat{H}_F(z, z_1)$ as defined above. In particular, $\partial_T\psi$ term corresponds to $\hat{H}(z)$ whereas A_T term corresponds to $\hat{H}_F$. Since the derivative terms are associated with the function $\hat{H}(z)$, we will need to calculate the contributions from the $\partial_T\psi$ part. In order to carry out this calculation, we have to perform the collinear expansion for the hard partonic part associated with this matrix element. In Fig. 1, we plot the generic Feynman diagram for the calculation of the derivative contributions. In this diagram, p_1 and p_2 are the two incoming partons' momenta from the polarized and unpolarized nucleons; k_1 and k_2 are the two outgoing partons' momenta where k_1 fragments into the final-state hadron P_h. The factorized expression for the spin-dependent cross section $\Delta\sigma(S_\perp)$ takes the following form

$$d\Delta\sigma(S_\perp) \propto \frac{1}{2S}\sum_{a,b,c}\int\frac{dz}{z^2}\frac{\hat{H}(z)}{z^2}\int\frac{dx}{x}h_a(x)\int\frac{dx'}{x'}f_b(x') \times i\gamma^\rho \not{P}_h \lim_{k_{1T}\to 0}\frac{\partial}{\partial k_{1T}^\rho}\mathcal{H}_{ab\to c}(x', x, z, k_{1T}), \tag{7}$$

where $h_a(x)$ is the quark transversity distribution of flavor a inside the transversely polarized nucleon A, $f_b(x')$ is the spin-averaged parton distribution function of flavor b inside the unpolarized nucleon B. The partonic scattering amplitude square $\mathcal{H}_{ab\to c}(x', x, z, k_{1T})$ is calculated from the Feynman diagram similar to Fig. 1 for the partonic process $ab \to c$, where k_{1T} is the transverse momentum component (perpendicular to the final state hadron momentum P_h) of the fragmenting quark c. In order to obtain the twist-three fragmentation contribution, we need to keep the transverse momentum dependence in $\mathcal{H}_{ab\to c}$. In particular, the linear expansion of $\mathcal{H}_{ab\to c}$ in terms of k_{1T} will lead to the twist-three fragmentation contribution.

The derivative terms arise from the on-shell condition for the unobserved particle k_2 in Fig. 1, i.e., from the k_{1T} expansion of the delta-function $\delta(k_2^2)$ in $\mathcal{H}_{ab\to c}(x', x, z, k_{1T})$. Since we can parameterize k_1 as $k_1 = P_h/z + k_{1T}$, the momentum of the unobserved particle is $k_2 = p_1 + p_2 - P_h/z - k_{1T}$. Thus, the on-shell condition will lead to the following expansion,

$$\delta\big((p_1+p_2-k_1)^2\big) = \delta\big((p_1+p_2-P_h/z)^2\big) - 2(p_1+p_2)\cdot k_{1T}\,\delta'\big((p_1+p_2-P_h/z)^2\big), \tag{8}$$

where the second term will result into a derivative term contribution to the transverse spin dependent differential cross section. As discussed above, for convenience, we use a light-like vector n_h to constrain the transverse momentum expansion. Therefore, k_{1T} can be expressed as $k_{1T}^\mu = k_1 - k_1\cdot P_h n_h^\mu - k_1\cdot n_h P_h^\mu$. Substituting this expression into the above equation, we will obtain the contribution from the matrix element associated with the operator $\partial_T\psi$ and that from the twist-three fragmentation function $\hat{H}(z)$. The derivative on the delta-function will be translated into the derivative on the twist-three fragmentation function $\hat{H}(z)$ by partial integral due to the fact

$$\delta'\big((p_1+p_2-P_h/z)^2\big) = \left[\frac{z^2}{2(p_1+p_2)\cdot P_h}\right]\frac{\partial}{\partial z}\delta\big(k_2^2\big). \tag{9}$$

Once the k_{1T} expansion is done, the rest of the calculation is straightforward. We consider all contributing partonic reactions and calculate the contributions to Eq. (7). The partonic channels we need to consider are $qq' \to qq'$, $q\bar{q}' \to q\bar{q}'$, $qq \to qq$, $q\bar{q} \to q\bar{q}$, $\bar{q}q \to \bar{q}q$, $\bar{q}q \to q\bar{q}$, $q\bar{q} \to \bar{q}q$, and $qg \to qg$. Upon calculating all associated hard-scattering functions $H_{ab\to c}$, we have found they are the same as the transverse spin transfer coefficients calculated in Ref. [37]. The situation here is similar to what has been found for the twist-three distribution functions contributions [17], where the hard-scattering functions are proportional to the unpolarized twist-two Born diagrams. Even though similarity exists, we want to emphasize that there is a major difference between these two contributions. The twist-three distribution contributions are calculated by taking the poles from the initial/final state interactions, and the derivative terms come from the expansion of the on-shell condition for the unobserved particle and the double pole contribution from the final state interaction diagrams [17]. For the twist-three fragmentation function contributions, on contrast, we do not take pole contributions, and the derivative terms only come from the expansion of the on-shell condition of the unobserved particle in the above $2\to 2$ processes. The final result for the derivative contributions of $\hat{H}(z)$ takes the following form,

$$
\begin{aligned}
E_h \frac{d^3\Delta\sigma(S_\perp)}{d^3P_h} &= \epsilon_{\perp\alpha\beta} S_\perp^\alpha \frac{2\alpha_s^2}{S} \sum_{a,b,c} \int_{x'_{\min}}^{1} \frac{dx'}{x'} f_b(x') \frac{1}{x} h_a(x) \\
&\times \int_{z_{\min}}^{1} \frac{dz}{z} \left[-z \frac{\partial}{\partial z} \left(\frac{\hat{H}(z)}{z^2} \right) \right] \\
&\times \frac{1}{x'S + T/z} \left\{ \frac{P_h^\beta + z(p_2 \cdot P_h n_h^\beta - p_2 \cdot n_h P_h^\beta)}{-z\hat{u}} \right\} \\
&\times H_{ab\to c}(\hat{s}, \hat{t}, \hat{u}),
\end{aligned} \tag{10}
$$

where $\sum_{a,b,c}$ represents the sum over all parton flavors, and

$$
x'_{\min} = \frac{-T/z}{S + U/z}, \qquad z_{\min} = -\frac{T+U}{S},
$$
$$
x = -\frac{x'U/z}{x'S + T/z}, \tag{11}
$$

with S, T, U defined as hadronic Madelstam variables: $S = (P_A + P_B)^2$, $T = (P_h - P_A)^2$, and $U = (P_h - P_B)^2$. The hard-scattering functions $H_{ab\to c}$ are given by

$$
\begin{aligned}
H_{qq'\to qq'} &= H_{q\bar{q}'\to q\bar{q}'} = \frac{N_c^2-1}{4N_c^2} \frac{4\hat{s}\hat{u}}{-\hat{t}^2}, \\
H_{qq\to qq} &= \frac{N_c^2-1}{4N_c^2} \left[\frac{4\hat{s}\hat{u}}{-\hat{t}^2} - \frac{1}{N_c} \frac{4\hat{s}}{-\hat{t}} \right], \\
H_{q\bar{q}\to q\bar{q}} &= H_{\bar{q}q\to\bar{q}q} = \frac{N_c^2-1}{4N_c^2} \left[\frac{4\hat{s}\hat{u}}{-\hat{t}^2} + \frac{1}{N_c} \frac{4\hat{u}}{\hat{t}} \right], \\
H_{\bar{q}q\to q\bar{q}} &= H_{q\bar{q}\to\bar{q}q} = -\frac{N_c^2-1}{N_c^3}, \\
H_{qg\to qg} &= \frac{N_c^2-1}{N_c^2} + \frac{1}{2} \frac{4\hat{s}\hat{u}}{-\hat{t}^2},
\end{aligned} \tag{12}
$$

with $\hat{s}$, $\hat{t}$, and $\hat{u}$ the usual partonic Mandelstam variables. Substituting the expression of n_h into Eq. (8), the factor in the bracket can also be written as,

$$
\begin{aligned}
&\left\{ \frac{P_h^\beta + z(p_2 \cdot P_h n_h^\beta - p_2 \cdot n_h P_h^\beta)}{-z\hat{u}} \right\} \\
&\quad \to \left(\frac{P_h^\beta}{z} \right) \frac{x - x'}{x(-\hat{u}) + x'(-\hat{t})}.
\end{aligned} \tag{13}
$$

In the forward rapidity region of the polarized nucleon, we have $x \gg x'$ and $-\hat{u} \gg -\hat{t}$, and we can further simplify the transverse spin dependent differential cross section as

$$
\begin{aligned}
&E_h \frac{d^3\Delta\sigma(S_\perp)}{d^3P_h}\bigg|_{\text{forward}} \\
&\quad = \epsilon_{\perp\alpha\beta} S_\perp^\alpha P_{h\perp}^\beta \frac{2\alpha_s^2}{S} \sum_{a,b,c} \int_{x'_{\min}}^{1} \frac{dx'}{x'} f_b(x') \frac{1}{x} h_a(x) \\
&\quad \times \int_{z_{\min}}^{1} \frac{dz}{z} \left[-z \frac{\partial}{\partial z} \left(\frac{\hat{H}(z)}{z^2} \right) \right] \\
&\quad \times \frac{1}{x'S + T/z} \frac{1}{-z\hat{u}} H_{ab\to c}(\hat{s}, \hat{t}, \hat{u}).
\end{aligned} \tag{14}
$$

This term is the most phenomenological relevant contribution to the single spin asymmetries of hadron production in the forward direction of the polarized nucleon.

To demonstrate the twist-three fragmentation function contribution to the SSA in inclusive hadron production in $p^\uparrow p$ collisions, we need the unknown, but universal, twist-three fragmentation function $\hat{H}(z)$. We notice that $\hat{H}(z)$ can be related to the Collins fragmentation function $H_1^\perp(z, p_\perp^2)$ as in Eq. (4), which has been studied from the available experimental data [38,39]. However, we emphasize that the Collins function $H_1^\perp(z, p_\perp^2)$ are fitted from small transverse momentum region where TMD factorization applies. To obtain the functional form for $\hat{H}(z)$, one has to assume a transverse momentum dependence in the Collins fragmentation functions. In principle, the twist-three fragmentation functions $\hat{H}(z)$ should be extracted from the experimental data of the SSAs at large transverse momentum region where collinear factorization applies, similar to what has been done for the twist-three distribution functions $T_{q,F}(x,x)$ in [18], or from the transverse momentum weighted azimuthal asymmetry measurements where $\hat{H}(z)$ enters directly.

In the following, we follow the ansatz in Ref. [18] to parameterize the twist-three fragmentation function $\hat{H}(z)$ for π^0 meson as

$$
\hat{H}(z) = C_f z^a (1-z)^b D(z), \tag{15}
$$

where $D(z)$ is the leading-twist unpolarized fragmentation function, C_f, a and b are unknown parameters. The factor z^a comes from the consideration that this novel fragmentation is mostly a valence-type fragmentation function. The other suppression factor $(1-z)^b$ usually appears in the twist-three functions from the power counting arguments at $z \to 1$ [40]. However, the twist-three fragmentation function for a scalar meson is not power suppressed in terms of $(1-z)$, similar to the power counting of the Boer–Mulders function of π meson at large-x [40]. Therefore, we set $b = 0$ in Eq. (15). For the purpose of estimating the SSAs and motivating future experimental measurements, we choose $C_f = -0.4$, and three different values for a: $a = 1, 2, 4$. We emphasize that our intention here is not to provide a precise parameterization for $\hat{H}(z)$, but to show that sizable asymmetries could be generated by the twist-three fragmentation function if a suitable parameterization is adopted. The more comprehensive parameterization including those for the charged mesons should be extracted from the measured SSAs through a global fit [18], which is beyond the scope of the current study.

To calculate the SSAs in Eq. (10), we have also adopted the quark transversity distributions from the parameterizations in Ref. [41] and the unpolarized fragmentation function in [42]. In Fig. 2, we show the predictions of the SSAs with the above parameterizations for the π^0 production. The three curves from up to bottom correspond to $a = 1$ (solid), $a = 2$ (dashed), and $a = 4$ (dotted), respectively. With our parametrization of $\hat{H}(z)$, the twist-three fragmentation function can generate a sizable SSA in inclusive π^0 production at RHIC energy $\sqrt{s} = 200$ GeV. These contributions are comparable to that of the twist-three distribution functions from the polarized nucleon [17,18].

We would like to emphasize that the predictions in Fig. 2 are just rough estimates and suffer certain theoretical uncertainties. The twist-three fragmentation parameterization in Eq. (15) is arbitrary, and the quark transversity distribution from Ref. [41] are upper bounds. To finally pin down these functions, we need to carry out a global fit and take into account all the contributions in Eq. (2) from the twist-three distribution and fragmentation functions.

The single transverse spin asymmetry of η meson has also been studied by the STAR collaboration at RHIC recently [2]. A significantly larger asymmetry A_N has been observed for η meson compared to π^0. As we discussed, in the twist-three collinear fac-

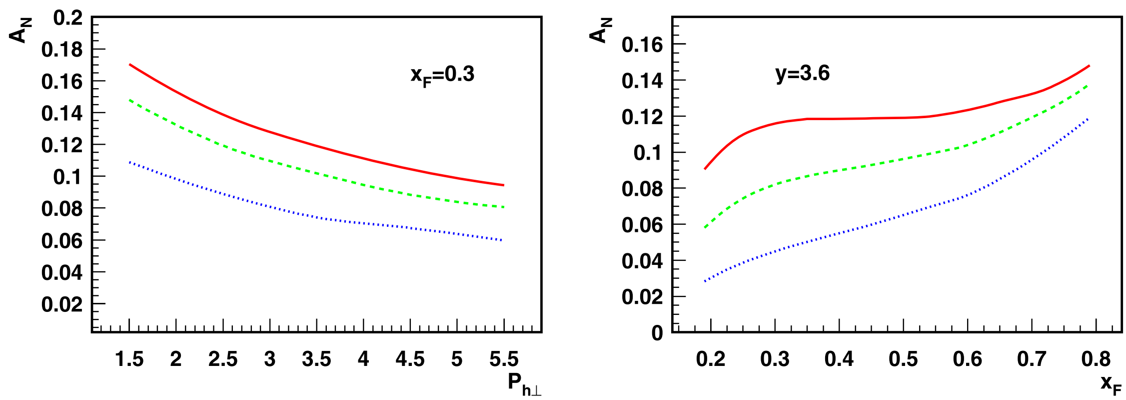

Fig. 2. Twist-three fragmentation contributions to the single spin asymmetries in hadron production in the forward direction of the polarized nucleon at RHIC at $\sqrt{s} = 200$ GeV, as functions of x_F, with the parameterization for the twist-three fragmentation function $\hat{H}(z)$ in Eq. (15) with parameters $C_f = -0.4$ GeV and $a = 1, 2, 4$ (from up to bottom), respectively.

torization, the dominate contribution comes from the twist-three distribution of the polarized nucleon, as shown in the first term of Eq. (2), and the twist-three fragmentation function, as shown in the third term of Eq. (2). The difference between η and π^0 from the first term will be mainly due to the difference in the unpolarized meson fragmentation function. However, from the measurements of spin-averaged cross section of η and π^0 by PHENIX collaboration at RHIC [43], one finds that for a large range of transverse momentum, η/π^0 ratio is a constant.[4] This indicates that the unpolarized fragmentation function for η and π^0 are very similar, and will lead to the similar SSAs for them if the first term in Eq. (2) dominates. In other words, it will be difficult to explain the large difference between the SSAs of η and π^0 mesons from the twist-three distribution contribution from the polarized nucleon.[5] Since the second term in Eq. (2) generates a very small asymmetry, one would expect that the large difference between η and π^0 would come from the twist-three fragmentation function contribution. The twist-three fragmentation function $\hat{H}(z)$ for η and π^0 in general need not to be the same. This might generate the needed difference observed by the experiments if η meson has a much larger twist-three fragmentation function $\hat{H}(z)$ compared to π^0. To test this scenario, it will be very important to study the associated Collins fragmentation for η meson in e^+e^- annihilation and/or semi-inclusive DIS processes and compare to that for π^0 meson. We hope that, in particular, the BELLE collaboration can carry out this measurement and cross check with the STAR observation.

Again, we would like to emphasize the twist-three fragmentation function contribution is part of the twist-three formalism for single spin asymmetry in inclusive hadron production in pp collisions as formulated in Eq. (2). To finally understand the underlying mechanism for these asymmetries, we need to carry out a global analysis in this formalism for all experimental data, including the SSAs for the neutral mesons (π^0 and η), and charged mesons ($\pi^\pm$, $K^\pm$), and so on. In particular, the BRAHMS collaboration at RHIC have also published results on SSAs in charged hadron productions, including proton (antiproton) and charged π and K [5]. These results have not yet been completely understood, especially for those of antiproton and K^-. We also notice that the BRHMS data [5] are at different energy than those at STAR [3]. In our calculations, we have not yet taken into account the energy dependence for the single spin asymmetries, except that coming from the different values of the parton distributions and fragmentation functions at these two energies. Preliminary comparison between the experimental data and the theory predictions on the SSAs of π indicates the consistency for this dependence [5].

In conclusion, in this Letter, we have studied the twist-three fragmentation function contribution to the inclusive hadron's SSA in pp scattering $p^\uparrow p \to hX$. With a simple parametrization for the twist-three fragmentation function, we estimated its contribution to the SSAs of π^0 production at RHIC energy. We find that the contribution of the twist-three fragmentation function is comparable to that of the twist-three distribution function from the polarized nucleon. We comment on the possibility to use our approach to describe the large difference of the SSAs between the η and π^0 mesons. We emphasize that one needs to include both contributions from the twist-three distribution and fragmentation functions into a global analysis, in order to better understand the single spin asymmetries in the inclusive hadron production.

[4] We notice that the PHENIX measurements are performed at midrapidity. We hope that this ratio at forward rapidity will be analyzed soon as well.

[5] It might still be possible that the strange quark contribution from the polarized nucleon may dominate and lead to a larger SSA in η meson production, which is, however, unlikely in the forward rapidity region (the valence region) of the polarized nucleon.

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