Polarized p+A, single transverse spin asymmtries

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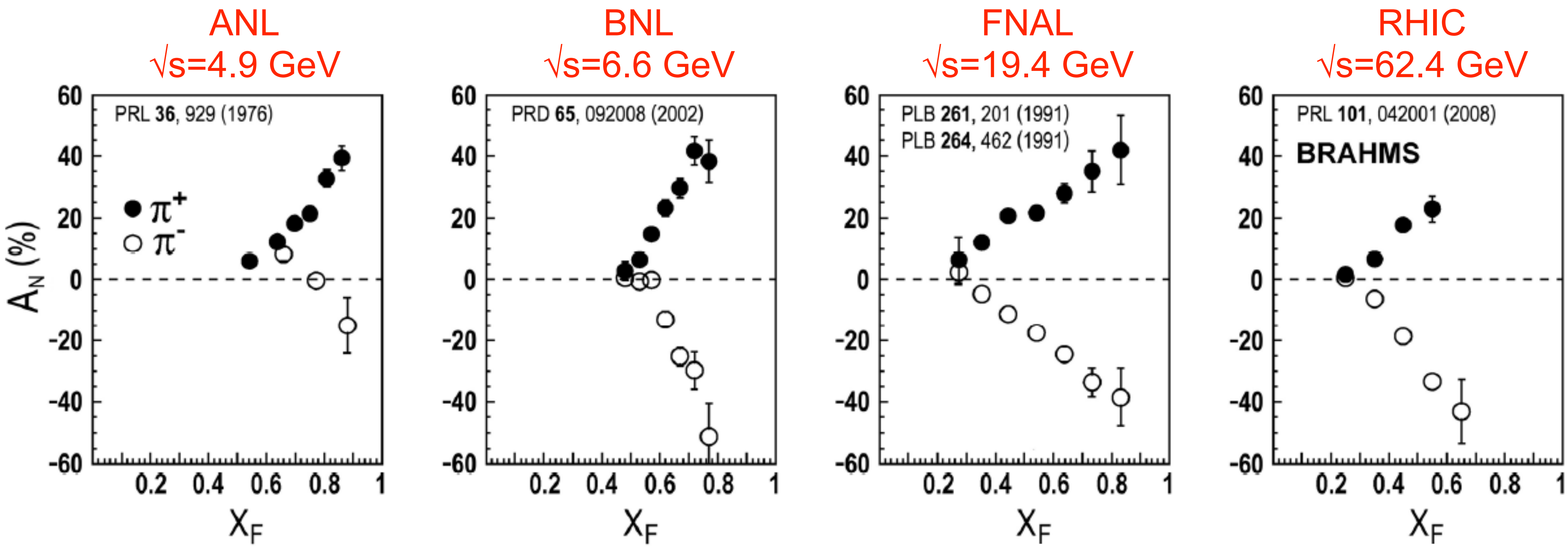
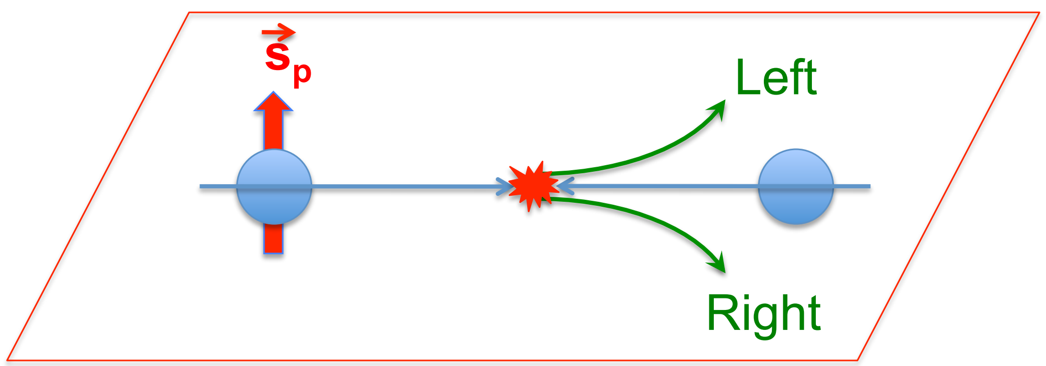
Outline

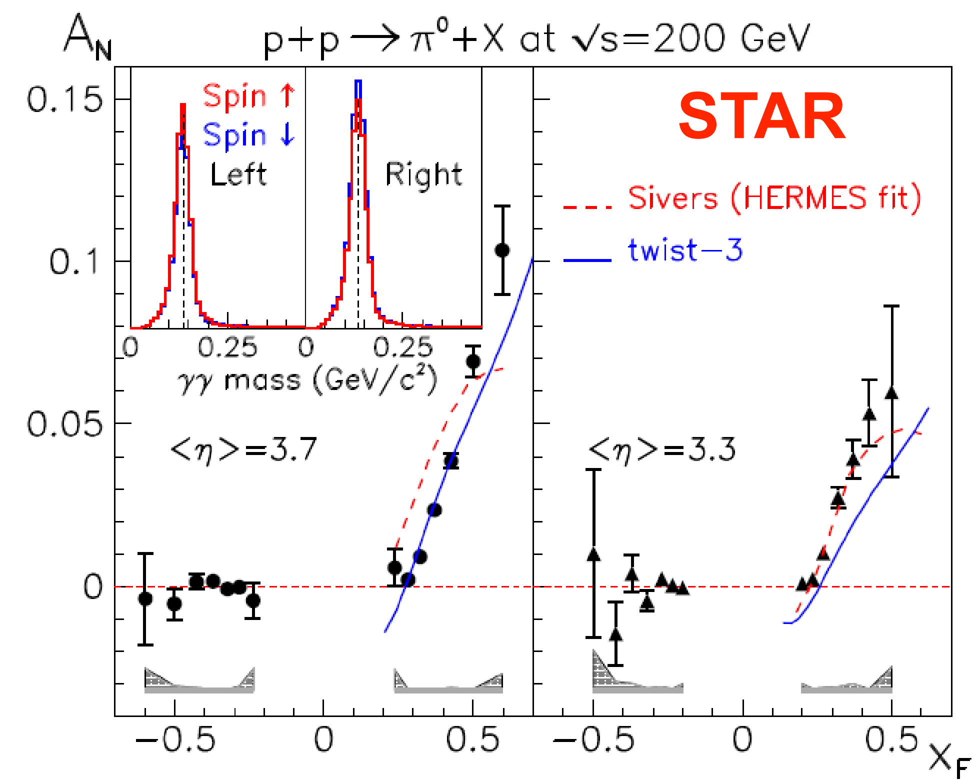
- Motivation
  - Why transverse spin and small-x could be probed at the same time?
  - How spin asymmetries could be more sensitive to saturation physics
- Single transverse spin asymmetry
  - Inclusive hadron production
  - Drell-Yan production (analyzing factorization properties)
- Summary

Single transverse-spin asymmetry (SSA)

- Consider a transversely polarized proton scatter with an unpolarized proton

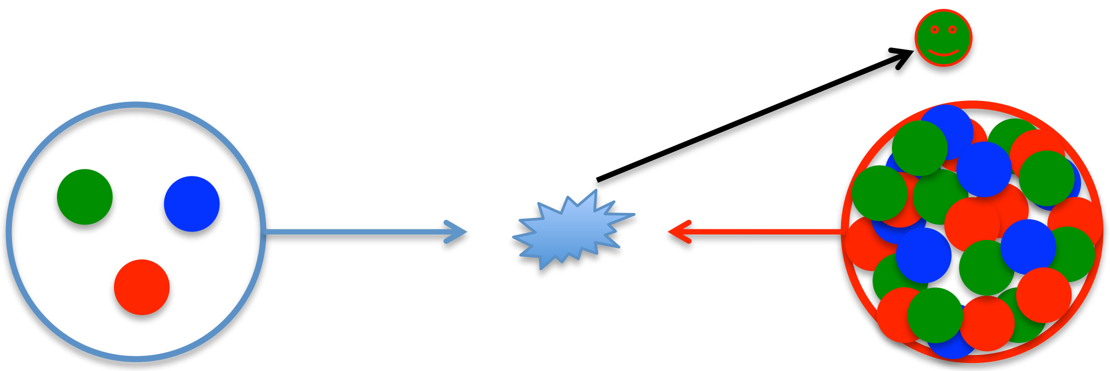






$$A_N \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$

Observation at high energy $\sqrt{s}$



projectile: $x_1 \sim \frac{p_\perp}{\sqrt{s}} e^{+y} \sim 1$ valence

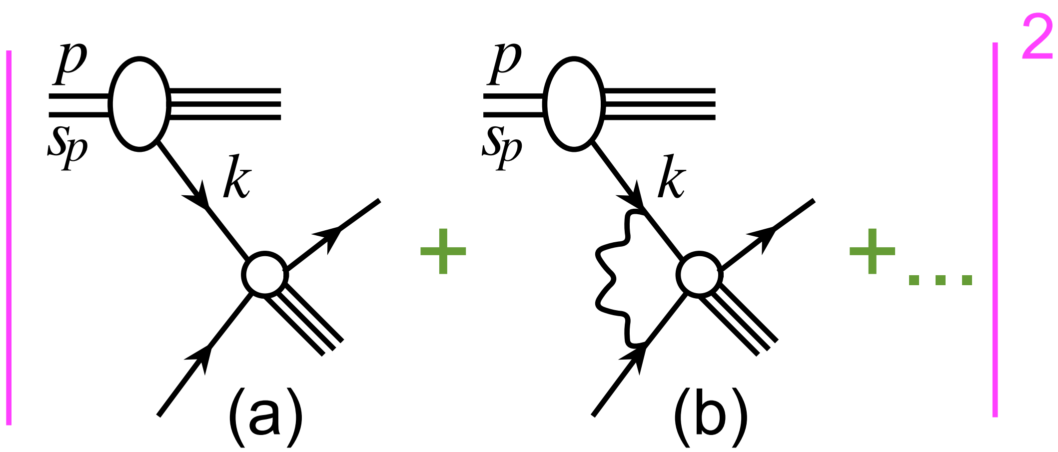
target: $x_2 \sim \frac{p_\perp}{\sqrt{s}} e^{-y} \ll 1$ gluon

- The spin asymmetry becomes the largest at forward rapidity region, corresponding to
  - The partons in the projectile (the polarized proton) have very large momentum fraction x: dominated by the valence quarks (spin effects are valence effects)
  - The partons in the target (the unpolarized proton or nucleus) have very small momentum fraction x: dominated by the small-x gluons
- Thus spin asymmetry in the forward region could probe both
  - The transverse spin effect from the valence quarks in the projectile: Sivers effect, Collins effect, and etc
  - The small-x gluon saturation physics in the target

Theoretical understanding in conventional factorization

- At leading twist formalism, assuming partons are collinear, the asymmetry is vanishing small

Kane-Pumplin-Repko, 1978

$\sigma(s_T) \sim$ 

$\Rightarrow$ $\Delta\sigma(s_T) \sim \mathrm{Re}[(a)] \cdot \mathrm{Im}[(b)]$

  - generate phase from loop diagrams, proportional to α_s
  - helicity is conserved for massless partons, helicity-flip is proportional to current quark mass m_q

Therefore we have

$$A_N \sim \alpha_s \frac{m_q}{\sqrt{s}} \frac{\delta q(x)}{\phi(x)}$$

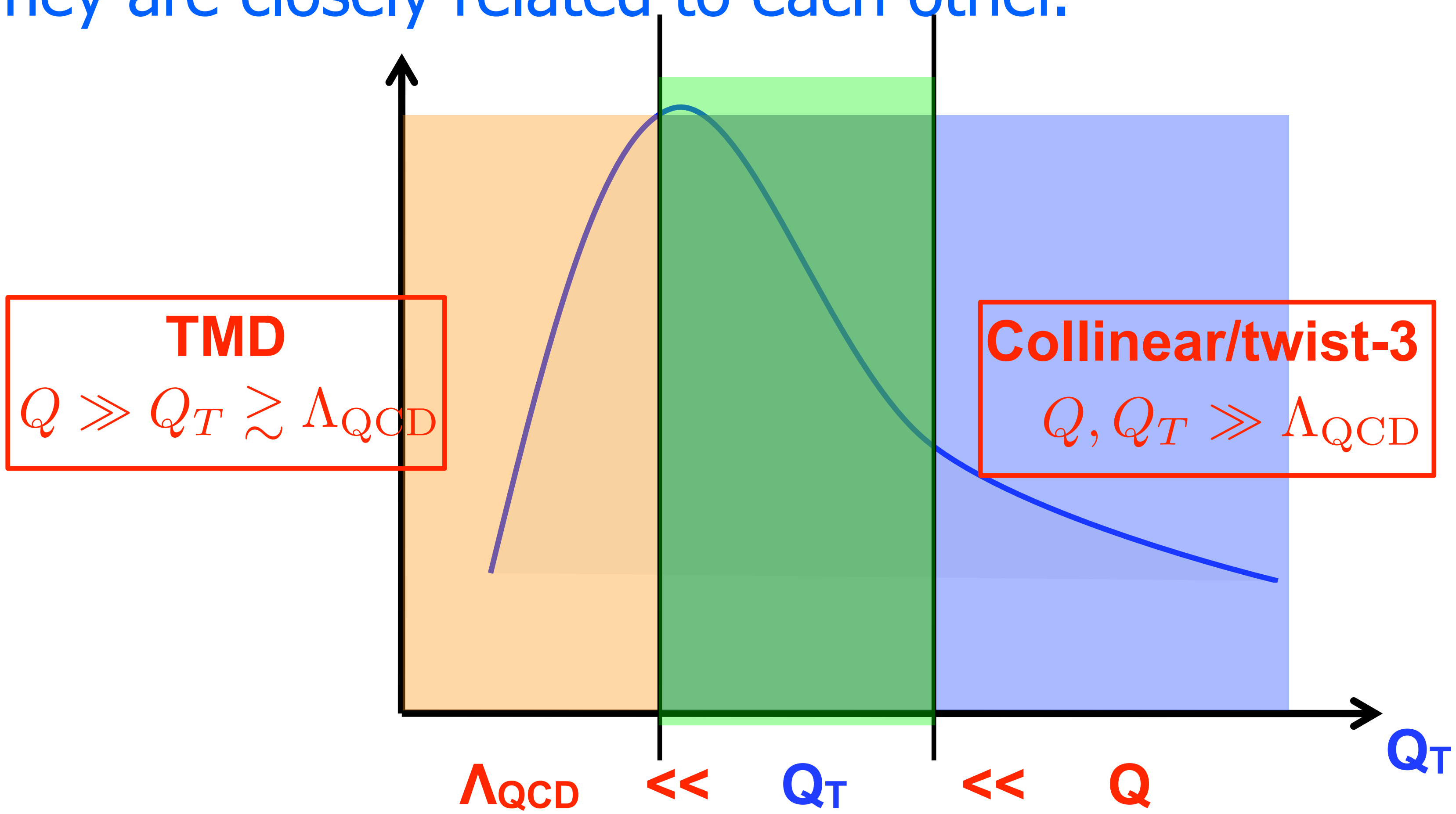
$\delta q(x)$: tranversity

$\phi(x)$: unpolarized parton distribution

- $A_N \neq 0$: result of parton's transverse motion or correlations!

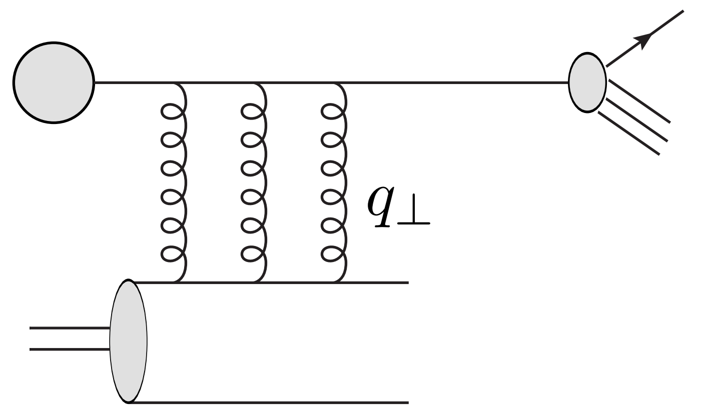
Understand SSA: related to parton transverse motion

- One could immediately think of two ways to include parton's transverse momentum into the formalism
  - Generalize the collinear distribution $f(x)$ to $f(x, k_\perp)$
  - Taylor expansion: $H(Q, k_\perp) = H(Q) + k_\perp H'(Q) + \cdots$, where $H'(Q) = dH(Q, k_\perp)/dk_\perp$ at $k_\perp = 0$, then ∫d²kt kt f(x,kt) = a higher-twist correlation
- The first one is called TMD approach (SIDIS, DY at low pt), the second one is called collinear twist-3 approach (pp→hX at high pt). They are closely related to each other.



Inclusive hadron production in small-x formalism

- At forward rapidity, the hadron is produced as follows (at LO)



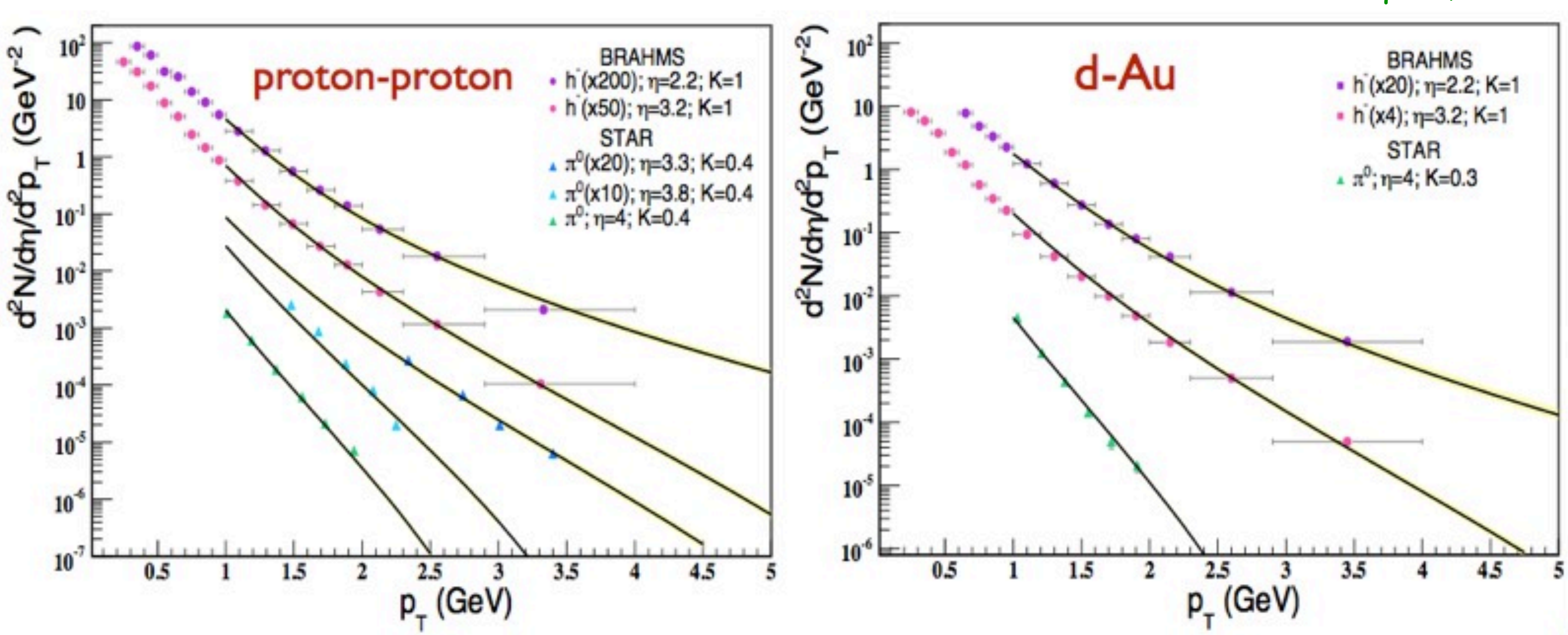
$$\frac{d\sigma}{dy d^2p_\perp} = \frac{K}{(2\pi)^2}\int d^2b \int_{x_F}^{1}\frac{dz}{z^2}x f_{q/p}(x) F(x_A, q_\perp) D_{h/q}(z)$$

$$F(x_A, q_\perp) = \int \frac{d^2r_\perp}{(2\pi)^2} e^{iq_\perp\cdot r_\perp}\frac{1}{N_c}\left\langle \mathrm{Tr}\left(U(0)U^\dagger(r_\perp)\right)\right\rangle_{x_A}$$

  - Dipole gluon distribution follows B-K evolution equation, which can be solved numerically

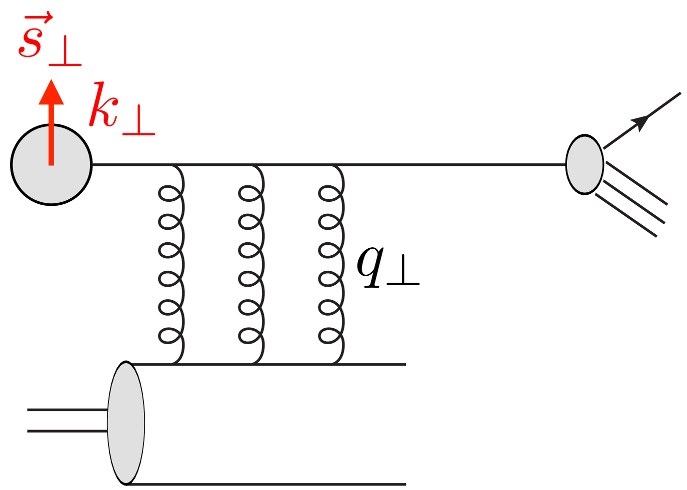
- Comparison with RHIC data

Albaete-Marquet, 2010



Naively incorporate Sivers effect

- Thinking about the incoming quark has a small kt-component, which generates a Sivers type correlation in the proton wave function (Sivers function)



$$p_\perp = z\,(k_\perp + q_\perp)$$

$$f_{q/p^\uparrow}(x, k_\perp) = f_{q/p}(x, k_\perp^2) + \frac{\epsilon_{\alpha\beta} s_\perp^\alpha k_\perp^\beta}{M_p} f_{1T}^{\perp,q}(x, k_\perp^2)$$

- Now spin-dependent cross section becomes

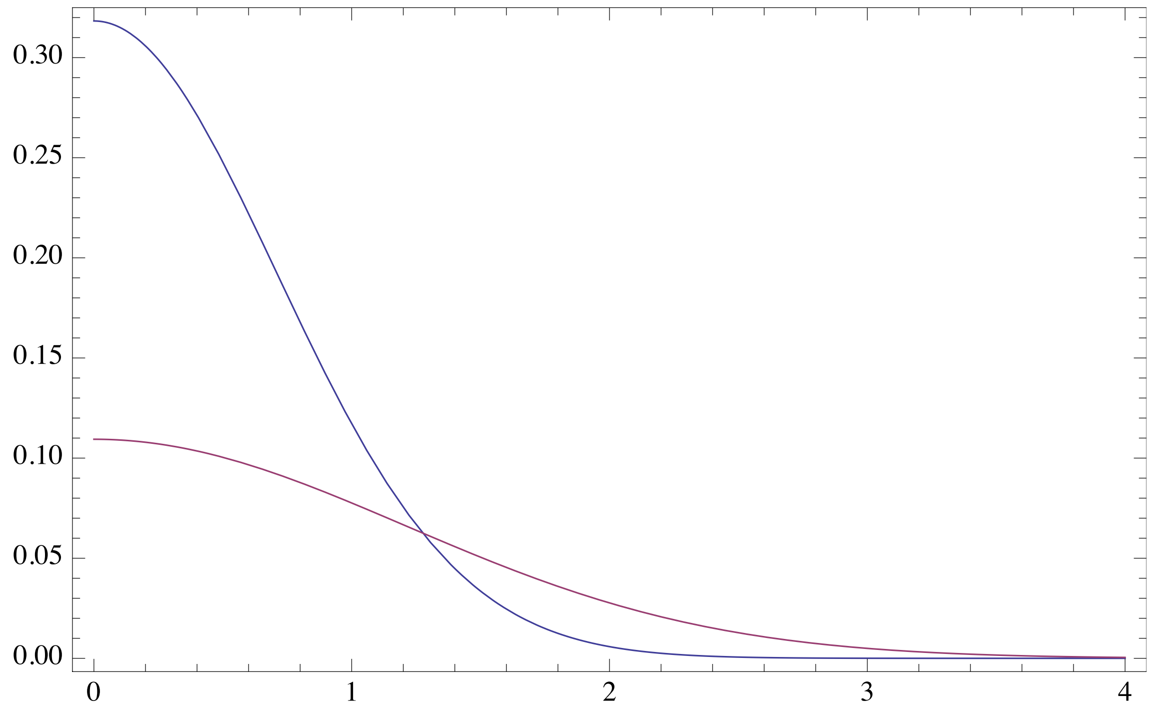
$$\frac{d\sigma}{dyd^2p_\perp} = \frac{K}{(2\pi)^2}\int d^2b \int_{x_F}^1 \frac{dz}{z^2}\int d^2k_\perp x\epsilon^{\alpha\beta} s_\perp^\alpha k_\perp^\beta f_{1T}^{\perp,q}(x, k_\perp^2) F(x_A, q_\perp = p_\perp/z - k_\perp) D_{h/q}(z)$$

  - Linear kt associated with Sivers function, need another kt to have kt-integral non-vanishing, which can only come from the gluon distribution
  - Spin asymmetry is sensitive to the slope of the dipole gluon distribution in kt-space

Take GBW (MV) model as an example

- Take GBW model as an example: Qs = 1GeV in proton

$$F(x, q_\perp) = \frac{1}{\pi Q_s^2(x)} e^{-q_\perp^2/Q_s^2(x)}$$



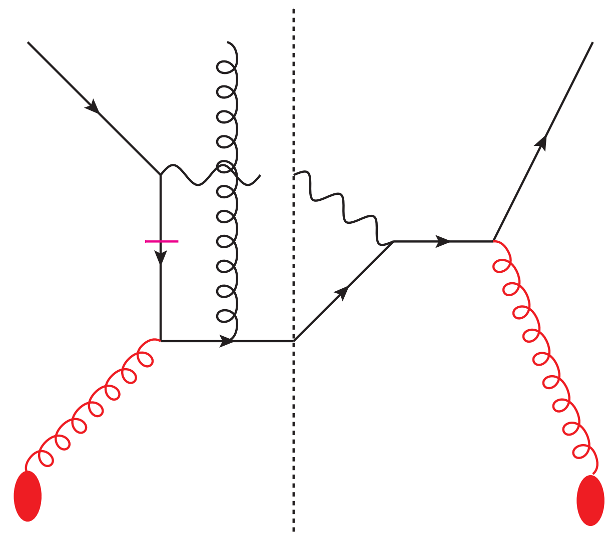
$$Q_{sA}^2 = cA^{1/3} Q_{sp}^2$$

DUsling-Gelis-Lappi-Venugolalan, arXiv:0911.2720

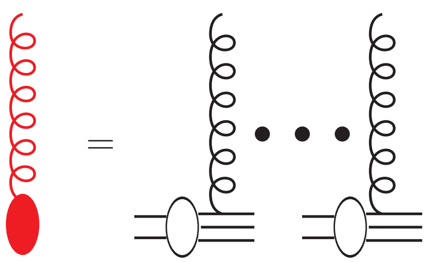
- Broadening might be difficult to see (as M. Chiu mentioned in his talk), but the slope could be easy to see
  - Comparing the A_N of pp and pA at small pt, which should give these information

Some cautions on this naive incorporation

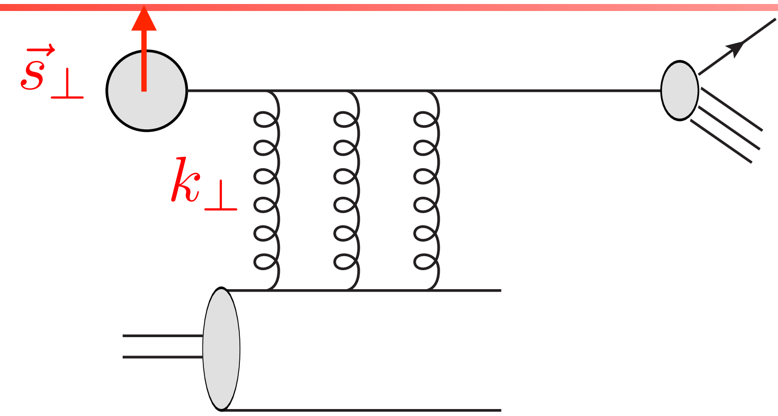
- How to study the process-dependence of the Sivers function (which Sivers function should one use?)
  - One might implement effectively through the method as in Gamberg-Kang, arXiv:1009.1936
  - Or study the Qiu-Sterman twist-3 effect (from the proton side) directly within the small-x formalism



Kang-Xiao-Yuan, in preparation



Collins effect: include spin in the fragmentation process



Kang-Yuan, arXiv: 1106.1375

$$P_{h\perp} = z\, k_\perp + P_{hT}$$

- Spin effect is always associated with the parton transverse momentum
  - generalized to include small transverse momentum in the fragmentation process

$$\frac{d\sigma}{dy_h d^2P_{h\perp}} = \frac{K}{(2\pi)^2}\int_{x_F}^{1}\frac{dz}{z^2}\int d^2P_{hT}\, x_1 q(x_1) N_F(x_2, k_\perp) D_{h/q}(z, P_{hT})$$

  - Now we could introduce Collins function in the game

Collins function

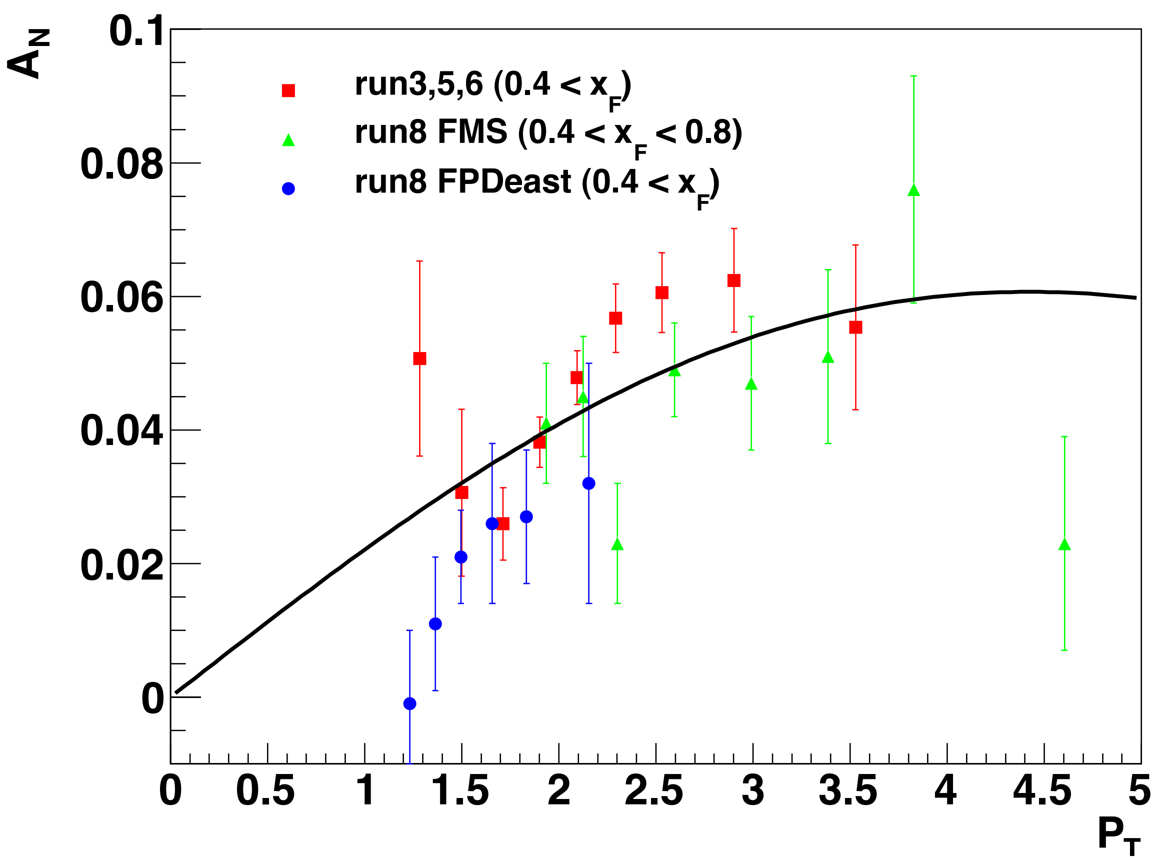
$$\frac{d\Delta\sigma}{dy_h d^2P_{h\perp}} = \frac{K}{(2\pi)^2}\int_{x_F}^{1}\frac{dz}{z^2}\int d^2P_{hT}\, I(S_\perp, P_{hT}) x_1 h(x_1) N_F(x_2, k_\perp)\delta\hat{q}(z, P_{hT})$$

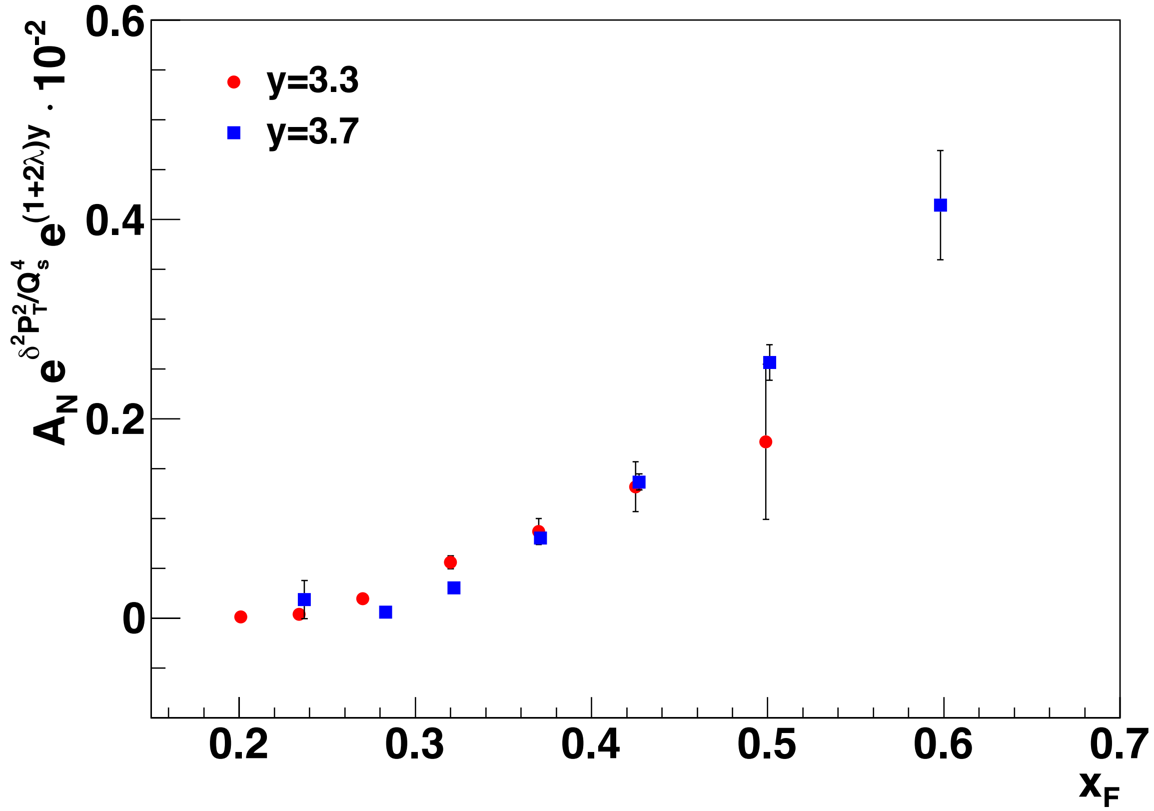
$$I(S_\perp, P_{hT}) = \epsilon^{\alpha\beta} S_\perp^\alpha \left[P_{hT}^\beta - \frac{n\cdot P_{hT}}{n\cdot P_J} P_J^\beta\right] = |S_\perp||P_{hT}|\sin(\phi_h - \phi_s)$$

  - Nice thing: Collins function is universal, independent of gauge link

Data seems to support scaling analysis

- Scaling analysis for pt and xf dependence





$$A_N \sim \frac{P_{h\perp}\Delta}{Q_s^2} e^{-\frac{\delta^2 P_{h\perp}^2}{(Q_s^2)^2}}$$

$$A_N e^{\delta^2 P_{h\perp}^2/Q_s^4} e^{(1+2\lambda)y_h} \sim x_F^{(1+\lambda)} \mathcal{F}(x_F)$$

- Compare pp and pA collisions

$$\frac{A_N^{pA\to h}}{A_N^{pp\to h}}\Big|_{P_{h\perp}\ll Q_s^2} \approx \frac{Q_{sp}^2}{Q_{sA}^2} e^{\frac{P_{h\perp}^2 \delta^2}{Q_{sp}^4}} \qquad \frac{A_N^{pA\to h}}{A_N^{pp\to h}}\Big|_{P_{h\perp}\gg Q_s^2} \approx 1$$

SSA of inclusive hadron production

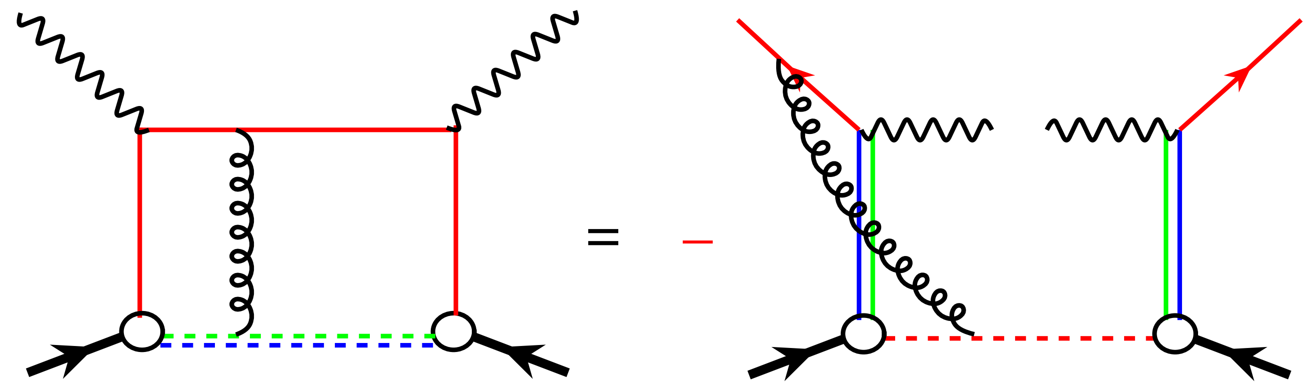
- The source of single spin correlation for $A^{\uparrow} + B \to h(p_\perp) + X$

$$\begin{aligned}
\Delta\sigma &= T_{a,F}(x,x) \otimes \phi_{b/B}(x') \otimes H_{ab\to c}(p_\perp, \vec{s}_T) \otimes D_{c\to h}(z) && \text{(I)} \\
&+ \delta q_{a/A}(x) \otimes T^{(\sigma)}_{b,F}(x',x') \otimes H'_{ab\to c}(p_\perp, \vec{s}_T) \otimes D_{c\to h}(z) && \text{(II)} \\
&+ \delta q_{a/A}(x) \otimes \phi_{b/B}(x') \otimes H''_{ab\to c}(p_\perp, \vec{s}_T) \otimes D^{(3)}_{c\to h}(z,z) && \text{(III)} \\
&+ m_q \delta q_{a/A}(x) \otimes \phi_{b/B}(x') \otimes H'''_{ab\to c}(p_\perp, \vec{s}_T) \otimes D_{c\to h}(z) && \text{(IV)}
\end{aligned}$$

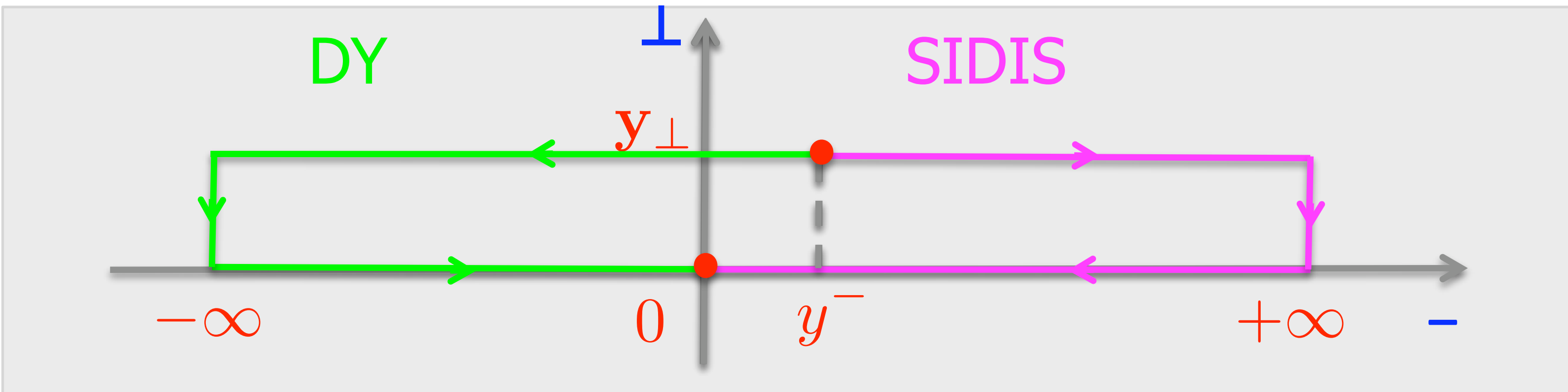
Term	meaning	collinear	small-x	Remarks
(I)	Sivers $T_{q,F}(x,x)$	Qiu-Sterman 91, 98 hep-ph/9806356	Boer-Dumitru-Hayashigaki, 2006 Kang-Xiao, 1212.4809	process dependence of Sivers function
(II)	Boer-Mulders $T^{(\sigma)}_{q,F}(x',x')$	Kanazawa-Koike, 2000 hep-ph/000727		small in the collinear formalism
(III)	Collins $D^{(3)}_{c\to h}(z,z)$	Kang-Yuan-Zhou, 2010 1002.0399	Kang-Yuan, 2011 1106.1375	Collins function is universal
(IV)	Kane-Pumplin-Repko $m_q\delta q(x)$	Kane-Pumplin-Repko, 1978	(different from KPR) Kovchegov-Sievert 1201.5890	small?? (because of quark mass?)

Why Drell-Yan is so interesting: physics of gauge link

- Rescattering (gauge link) determined by hard process and its color flow



SIDIS = – DY



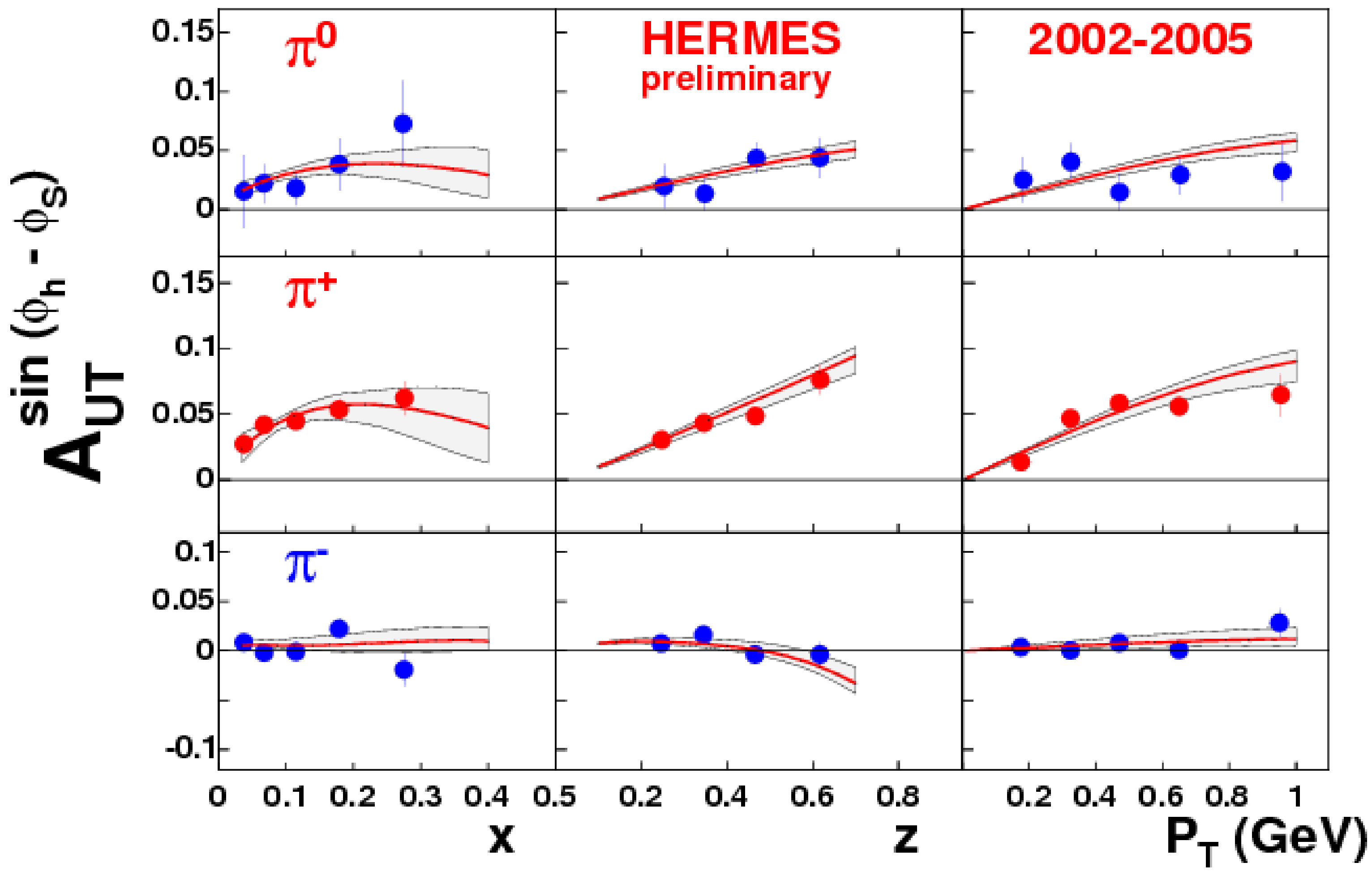
$$\Delta^N f^{\text{SIDIS}}_{q/h^\uparrow}(x, k_\perp) = -\Delta^N f^{\text{DY}}_{q/h^\uparrow}(x, k_\perp)$$

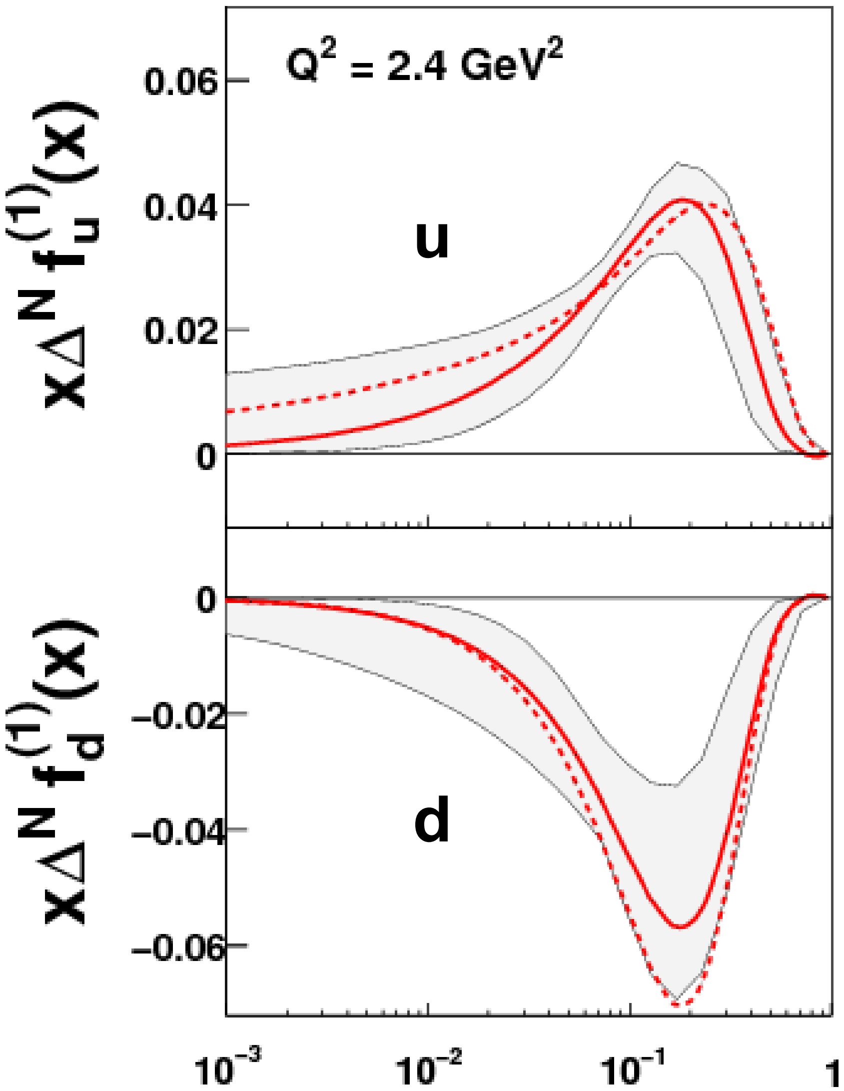
Central quest for the field at the moment

- Because gauge link is generated in the factorization procedure, for spin effect in small-x formalism, it is important to analyze the factorization properties

Sivers function from SIDIS $\ell + p^\uparrow \to \ell' + \pi(p_T) + X: \quad p_T \ll Q$

- Extract Sivers function from SIDIS (HERMES&COMPASS): a fit



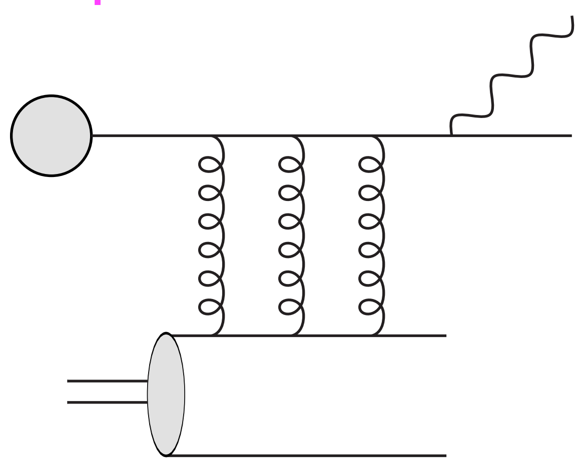


Anselmino, et.al., 2009

  - u and d almost equal size, different sign
  - d-Sivers is slightly larger
- Still needs DY results to verify the sign change, thus fully understand the mechanism of the SSAs

Drell-Yan production in small-x regime

- At leading order, Drell-Yan production is simple $p^\uparrow + A \to [\gamma^* \to]\ell^+\ell^- + X$
  - quark (from polarized proton) scatters off the classical gluon field to produce a virtual photon



Kopeliovich-Raufeisen-Tarasov 01, Baier-Mueller-Schiff 04, Gelis-Jalilian-Marian 02, 03, Stasto-Xiao-Zaslavsky, 2012

- When high-energy partons scatter off the classic gluon field, the interaction is eikonal in that the projectile propagate through the target without changing their transverse position but picking up an eikonal phase

Bjorken-Kogut-Soper, 1971

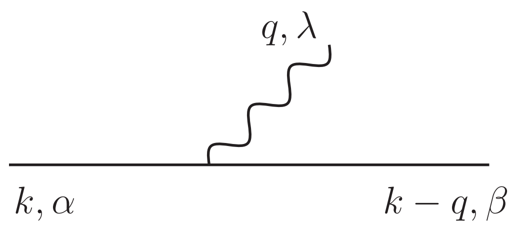
$$\mathcal{S}|k^+, b, i\rangle \otimes |\mathcal{A}\rangle = U^{ij}[\mathcal{A}]|k^+, b, j\rangle \otimes |\mathcal{A}\rangle$$

$$U(x) = \mathcal{P}\exp\left\{ig_s\int_{-\infty}^{+\infty} dx^+ T^c A_c^-(x^+, x_\perp)\right\}$$

Quark splitting wave function: keep quark kt from proton

- Quark to photon splitting wave function in light-front perturbation theory $q \to q + \gamma^*$

Kang-Xiao, arXiv:1212.4809



$z = q^+/k^+$ $\quad\quad$ $\epsilon_M^2 = (1-z)M^2$

- In momentum space

$$\phi_{\alpha\beta}^{\lambda}(k,q) = \frac{1}{\sqrt{8(k-q)^+k^+q^+}} \frac{\bar{u}_\beta(k-q)\gamma_\mu \epsilon^\mu(q,\lambda) u_\alpha(k)}{(k-q)^- + q^- - k^-}$$

- In transverse coordinate space: $\psi_{\alpha\beta}^{\lambda}(k,q^+,r) = \int d^2q_\perp e^{iq_\perp\cdot r}\phi_{\alpha\beta}^{\lambda}(k,q)$

$$\psi_{\alpha\beta}^{T\,\lambda}(k,q^+,r) = 2\pi\sqrt{\frac{2}{q^+}}e^{izk_\perp\cdot r} i\epsilon_M K_1(\epsilon_M|r|) \begin{cases} \frac{r\cdot\epsilon_\perp^1}{|r|}\left[\delta_{\alpha-}\delta_{\beta-} + (1-z)\delta_{\alpha+}\delta_{\beta+}\right], & \lambda = 1, \\ \frac{r\cdot\epsilon_\perp^2}{|r|}\left[\delta_{\alpha+}\delta_{\beta+} + (1-z)\delta_{\alpha-}\delta_{\beta-}\right], & \lambda = 2. \end{cases}$$

$$\psi_{\alpha\beta}^{L}(k,q^+,r) = 2\pi\sqrt{\frac{2}{q^+}}e^{izk_\perp\cdot r}(1-z)MK_0(\epsilon_M|r|)\delta_{\alpha\beta}$$

The multiple scattering could happen before or after

- The interaction with the target could happen before or after the splitting of the virtual photon

- The differential cross section for $q + A \to \gamma^* + X$

$$\frac{d\sigma(qA\to\gamma^* X)}{dq^+ d^2q_\perp} = \alpha_{\rm em} e_q^2 \int \frac{d^2b}{(2\pi)^2}\frac{d^2r}{(2\pi)^2}\frac{d^2r'}{(2\pi)^2} e^{-iq_\perp\cdot(r-r')}\sum_{\alpha\beta\lambda}\psi^{*\lambda}_{\alpha\beta}(k,q^+,r'-b)\psi^{\lambda}_{\alpha\beta}(k,q^+,r-b)$$
$$\times\left[1+S^{(2)}_{x_A}(v,v')-S^{(2)}_{x_A}(b,v')-S^{(2)}_{x_A}(v,b)\right]$$

  - multiple scattering is taken care of by

$$S^{(2)}_{x_A}(x,y)=\frac{1}{N_c}\left\langle {\rm Tr}\left(U(x)U^\dagger(y)\right)\right\rangle_{x_A}$$

Transform to momentum space

- To better compare with TMD factorization, let's transform to momentum space

$$\frac{d\sigma(qA\to\gamma^* X)}{dyd^2q_\perp} = \frac{\alpha_{\rm em}}{2\pi^2}e_q^2\int d^2bd^2p_\perp F(x_A,p_\perp)\left[H_T(q_\perp,k_\perp,p_\perp,z)+H_L(q_\perp,k_\perp,p_\perp,z)\right]$$

  - Unintegrated gluon distribution (dipole gluon distribution)

$$F(x_A,p_\perp) = \int\frac{d^2r_\perp}{(2\pi)^2}e^{ip_\perp\cdot r_\perp}\frac{1}{N_c}\left\langle {\rm Tr}\left(U(0)U^\dagger(r_\perp)\right)\right\rangle_{x_A}$$

  - Hard-part functions (transverse and longitudinal polarized photon)

$$\begin{aligned} H_T(q_\perp,k_\perp,p_\perp,z) &= \left[1+(1-z)^2\right]\left[\frac{q_\perp - zk_\perp}{(q_\perp-zk_\perp)^2+\epsilon_M^2} - \frac{q_\perp-zk_\perp-zp_\perp}{(q_\perp-zk_\perp-zp_\perp)^2+\epsilon_M^2}\right]^2 \\ H_L(q_\perp,k_\perp,p_\perp,z) &= 2(1-z)^2M^2\left[\frac{1}{(q_\perp-zk_\perp)^2+\epsilon_M^2} - \frac{1}{(q_\perp-zk_\perp-zp_\perp)^2+\epsilon_M^2}\right]^2 \end{aligned}$$

Unpolarized quark distribution

- The probability to find unpolarized quark in transversely polarized proton
  - Spin-averaged quark distribution
  - Sivers function: an asymmetric parton distribution in a polarized hadron (kt correlated with the spin of the hadron)

Spin-dependent

$$f_{q/p^\uparrow}(x, k_\perp) = f_{q/p}(x, k_\perp^2) + \frac{\epsilon_{\alpha\beta} s_\perp^\alpha k_\perp^\beta}{M_p} f_{1T}^{\perp,q}(x, k_\perp^2)$$

Spin-independent

Differential cross section in pp and pA collisions

$$f_{q/p^\uparrow}(x,k_\perp) = f_{q/p}(x,k_\perp^2) + \frac{\epsilon_{\alpha\beta} s_\perp^\alpha k_\perp^\beta}{M_p} f_{1T}^{\perp,q}(x,k_\perp^2)$$

- Spin-averaged virtual photon cross section

$$\frac{d\sigma(p^\uparrow A\to\gamma^* X)}{dy d^2q_\perp} = \frac{\alpha_{\rm em}}{2\pi^2}\sum_q e_q^2 \int_{x_p}^1 \frac{dz}{z} d^2k_\perp x f_{q/p}(x,k_\perp^2)\int d^2b d^2p_\perp F(x_A,p_\perp)$$
$$\times\left[H_T(q_\perp,k_\perp,p_\perp,z)+H_L(q_\perp,k_\perp,p_\perp,z)\right]$$

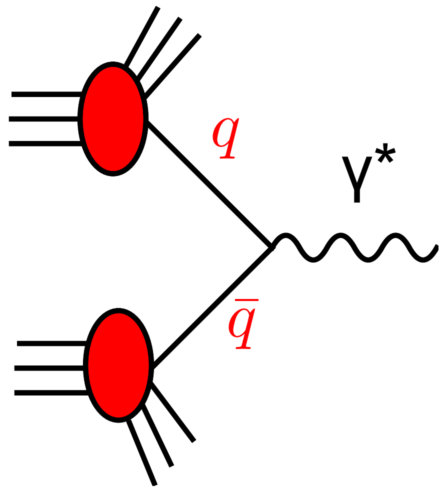
- Spin-dependent virtual photon cross section

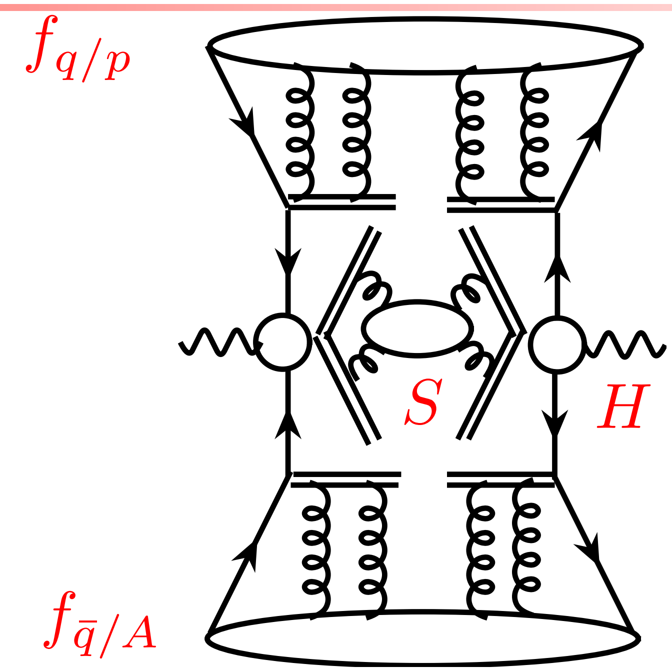
$$\frac{d\Delta\sigma(p^\uparrow A\to\gamma^* X)}{dy d^2q_\perp} = \frac{\alpha_{\rm em}}{2\pi^2}\sum_q e_q^2 \int_{x_p}^1 \frac{dz}{z} d^2k_\perp \frac{\epsilon_{\alpha\beta} s_\perp^\alpha k_\perp^\beta}{M_p} x f_{1T}^{\perp,q}(x,k_\perp^2)\int d^2b d^2p_\perp F(x_A,p_\perp)$$
$$\times\left[H_T(q_\perp,k_\perp,p_\perp,z)+H_L(q_\perp,k_\perp,p_\perp,z)\right]$$

- Single transverse spin asymmetry

$$A_N = \frac{d\Delta\sigma(p^\uparrow A\to\ell^+\ell^- X)}{dM^2 dy d^2q_\perp}\Big/\frac{d\sigma(p^\uparrow A\to\ell^+\ell^- X)}{dM^2 dy d^2q_\perp}$$

Compare to the usual TMD factorization formalism





- The spin-averaged cross section in TMD factorization formalism

$$\frac{d\sigma(p^\uparrow A\to \ell^+\ell^- X)}{dM^2 dy d^2q_\perp} = \frac{4\pi\alpha_{\rm em}^2}{3N_c M^4}\sum_q e_q^2 \int d^2k_\perp d^2\ell_\perp d^2\lambda_\perp \delta^2\left(k_\perp+\ell_\perp+\lambda_\perp-q_\perp\right)$$
$$\times x_p f_{q/p}(x_p,k_\perp^2) x_A f_{\bar{q}/A}(x_A,\ell_\perp^2) H(M^2,x_p,x_A) S(\lambda_\perp)$$

- For spin-dependent cross section

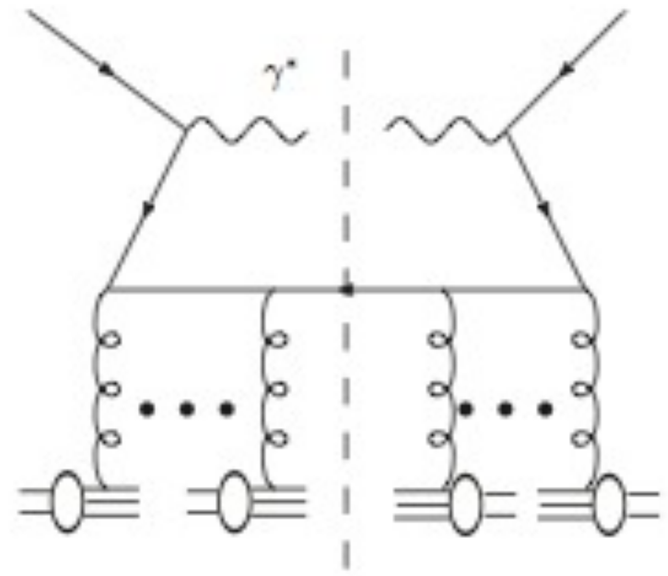
$$\frac{d\Delta\sigma(p^\uparrow A\to \ell^+\ell^- X)}{dM^2 dy d^2q_\perp} = \frac{4\pi\alpha_{\rm em}^2}{3N_c M^4}\sum_q e_q^2 \int d^2k_\perp d^2\ell_\perp d^2\lambda_\perp \delta^2\left(k_\perp+\ell_\perp+\lambda_\perp-q_\perp\right)$$
$$\times \frac{\epsilon_{\alpha\beta} s_\perp^\alpha k_\perp^\beta}{M_p} x_p f_{1T}^{\perp,q}(x_p,k_\perp^2) x_A f_{\bar{q}/A}(x_A,\ell_\perp^2) H(M^2,x_p,x_A) S(\lambda_\perp)$$

- How could the gluon distribution come in the game?

Quark distribution can be generated from the UGD

- Anti-quark distribution is generated from unintegrated gluon distribution

Marquet-Xiao-Yuan, 2009



  - This generation in the perturbative region can be easily computed

$$f_{\bar{q}/A}(x_A, \ell_\perp^2) = \frac{N_c}{8\pi^4} \int \frac{d\hat{z}}{x_A} \int d^2b d^2p_\perp F(x_A, p_\perp) A(p_\perp, \ell_\perp, \hat{z})$$

$$A(p_\perp, \ell_\perp, \hat{z}) = \left[\frac{\ell_\perp |\ell_\perp - p_\perp|}{(1-\hat{z})\ell_\perp^2 + \hat{z}(\ell_\perp - p_\perp)^2} - \frac{\ell_\perp - p_\perp}{|\ell_\perp - p_\perp|} \right]^2$$

TMD factorization in terms of dipole gluon distribution

- Since anti-quark distribution is expanded to NLO, we keep the LO for hard and soft factor

$$H(M^2, x_p, x_A) = 1 \qquad S(\lambda_\perp) = \delta^2(\lambda_\perp)$$

- Differential cross section in terms of dipole gluon distribution

$$\frac{d\sigma(p^\uparrow A \to \ell^+\ell^- X)}{dM^2 dy d^2q_\perp} = \frac{\alpha_{\rm em}^2}{6\pi^3 M^4}\sum_q e_q^2 \int d^2k_\perp x_p f_{q/p}(x_p, k_\perp^2) \int d\hat{z} \int d^2b d^2p_\perp F(x_A, p_\perp) A(p_\perp, q_\perp - k_\perp, \hat{z})$$

$$\frac{d\Delta\sigma(p^\uparrow A \to \ell^+\ell^- X)}{dM^2 dy d^2q_\perp} = \frac{\alpha_{\rm em}^2}{6\pi^3 M^4}\sum_q e_q^2 \int d^2k_\perp \frac{\epsilon_{\alpha\beta} s_\perp^\alpha k_\perp^\beta}{M_p} x_p f_{1T}^{\perp,q}(x_p, k_\perp^2)$$
$$\times \int d\hat{z} \int d^2b d^2p_\perp F(x_A, p_\perp) A(p_\perp, q_\perp - k_\perp, \hat{z})$$

Find the leading term in small-x formalism

- One could also find the leading term in the small-x formalism we have just derived (leading: $M \gg q_\perp \sim k_\perp \sim p_\perp(Q_s)$)

$$\frac{d\sigma(p^\uparrow A \to \gamma^* X)}{dy d^2 q_\perp} = \frac{\alpha_{\rm em}}{2\pi^2}\sum_q e_q^2 \int_{x_p}^1 \frac{dz}{z} d^2k_\perp x f_{q/p}(x, k_\perp^2) \int d^2b d^2p_\perp F(x_A, p_\perp) \times \left[H_T(q_\perp, k_\perp, p_\perp, z) + H_L(q_\perp, k_\perp, p_\perp, z)\right]$$

- Dominated by the large z-->1 region: introduce a delta-function, integrate out z first

$$\int d\hat{z}\delta(\hat{z} - 1/(1+\Lambda^2/\epsilon_M^2)) = 1 \qquad \Lambda^2 = (1-z)(q_\perp - k_\perp)^2 + z(q_\perp - k_\perp - p_\perp)^2$$

- At qt<<M, they are consistent with TMD factorization

$$\frac{d\sigma(p^\uparrow A \to \ell^+\ell^- X)}{dM^2 dy d^2q_\perp} = \frac{\alpha_{\rm em}^2}{6\pi^3 M^4}\sum_q e_q^2 \int d^2k_\perp x_p f_{q/p}(x_p, k_\perp^2) \int d\hat{z} \int d^2b d^2p_\perp F(x_A, p_\perp) A(p_\perp, q_\perp - k_\perp, \hat{z})$$

$$\frac{d\Delta\sigma(p^\uparrow A \to \ell^+\ell^- X)}{dM^2 dy d^2q_\perp} = \frac{\alpha_{\rm em}^2}{6\pi^3 M^4}\sum_q e_q^2 \int d^2k_\perp \frac{\epsilon_{\alpha\beta} s_\perp^\alpha k_\perp^\beta}{M_p} x_p f_{1T}^{\perp,q}(x_p, k_\perp^2) \times \int d\hat{z} \int d^2b d^2p_\perp F(x_A, p_\perp) A(p_\perp, q_\perp - k_\perp, \hat{z})$$

Connection to collinear factorization approach - I

- When $M \sim q_\perp \gg Q_s(p_\perp)$, the usual collinear factorization should work (dilute region)
  - We should treat kt and pt (parton intrinsic transverse momenta) as small compared with the Drell-Yan pair's momentum qt

$$H_T(q_\perp, k_\perp, p_\perp, z) = \left[1+(1-z)^2\right]\left[\frac{q_\perp - zk_\perp}{(q_\perp - zk_\perp)^2+\epsilon_M^2} - \frac{q_\perp - zk_\perp - zp_\perp}{(q_\perp - zk_\perp - zp_\perp)^2+\epsilon_M^2}\right]^2$$

$$H_L(q_\perp, k_\perp, p_\perp, z) = 2(1-z)^2 M^2\left[\frac{1}{(q_\perp - zk_\perp)^2+\epsilon_M^2} - \frac{1}{(q_\perp - zk_\perp - zp_\perp)^2+\epsilon_M^2}\right]^2$$

  - Drop kt, we could have

$$\int d^2k_\perp f_{q/p}(x, k_\perp^2) = f_{q/p}(x)$$

  - Drop pt, the hard-part function vanish, thus need to expand to higher order

Connection to collinear factorization approach - II

- In the dilute parton region, we have the relation between the UGD and collinear gluon distribution

$$\int d^2b\, d^2p_\perp p_\perp^2 F(x_A, p_\perp) = \frac{2\pi^2\alpha_s}{N_c} x_A f_{g/A}(x_A)$$

Baier-Mueller-Schiff 04

- Thus expand the hard-part function to the 2nd order

$$\int d^2b \int d^2p_\perp F(x_A, p_\perp) p_\perp^\rho p_\perp^\sigma \frac{1}{2} \frac{\partial}{\partial p_\perp^\rho \partial p_\perp^\sigma} \left[H_T(q_\perp, k_\perp = 0, p_\perp, z) + H_L(q_\perp, k_\perp = 0, p_\perp, z)\right]_{p_\perp \to 0}$$

- Eventually the spin-averaged cross section can be written as

$$\frac{d\sigma(p^\uparrow A \to \ell^+\ell^- X)}{dM^2 dy d^2q_\perp} = \frac{\alpha_{\rm em}^2 \alpha_s}{3\pi N_c M^2} \sum_q e_q^2 \int_{x_p}^1 \frac{dz}{z} x f_{q/p}(x) x_A f_{g/A}(x_A) H(q_\perp, z)$$

$$H(q_\perp, z) = \frac{1}{4} g_\perp^{\rho\sigma} \frac{\partial}{\partial p_\perp^\rho \partial p_\perp^\sigma} \left[H_T(q_\perp, k_\perp = 0, p_\perp, z) + H_L(q_\perp, k_\perp = 0, p_\perp, z)\right]_{p_\perp \to 0}$$

$$= \frac{z^2}{(q_\perp^2 + \epsilon_M^2)^2} \left\{ \left[1 + (1-z)^2\right] - \frac{2z^2 q_\perp^2 \epsilon_M^2}{(q_\perp^2 + \epsilon_M^2)^2} \right\}.$$

Connection to collinear factorization approach - III

- DY production (q+g channel) in collinear factorization approach

$$\frac{d\sigma(p^\uparrow A\to \ell^+\ell^- X)}{dM^2 dy d^2 q_\perp} = \sigma_0 \frac{\alpha_s}{4\pi^2}\sum_q e_q^2 \int \frac{dx}{x}\frac{dx_A}{x_A} f_{q/p}(x) f_{g/A}(x_A)\hat\sigma_{qg}(\hat s,\hat t,\hat u)\delta\left(\hat s+\hat t+\hat u - M^2\right)$$

- Partonic cross section $\hat\sigma_{qg}(\hat s,\hat t,\hat u) = 2\,T_R\left[-\frac{\hat s}{\hat t} - \frac{\hat t}{\hat s} - \frac{2M^2\hat u}{\hat s\hat t}\right]$

- Using the relations: $\hat s = \frac{q_\perp^2+\epsilon_M^2}{z(1-z)}, \qquad \hat t = -\frac{q_\perp^2+\epsilon_M^2}{z}, \qquad \hat u = -\frac{q_\perp^2}{1-z},$

- We have

$$\hat\sigma_{qg}(\hat s,\hat t,\hat u) = \frac{1}{1-z}\left\{\left[1+(1-z)^2\right] - \frac{2z^2 q_\perp^2\epsilon_M^2}{\left(q_\perp^2+\epsilon_M^2\right)^2}\right\}$$

- The small-x cross section is consistent with the above formalism

Connection to collinear factorization approach - IV

- The spin-dependent cross section

$$\frac{d\Delta\sigma(p^\uparrow A\to\gamma^* X)}{dyd^2q_\perp} = \frac{\alpha_{em}}{2\pi^2}\sum_q e_q^2\int_{x_p}^1 \frac{dz}{z}\, d^2k_\perp \frac{\epsilon_{\alpha\beta}s_\perp^\alpha k_\perp^\beta}{M_p} x f_{1T}^{\perp,q}(x,k_\perp^2)\int d^2b d^2p_\perp F(x_A,p_\perp)$$
$$\times\left[H_T(q_\perp,k_\perp,p_\perp,z)+H_L(q_\perp,k_\perp,p_\perp,z)\right]$$

  - Need a further expansion for the kt-part, since linear kt associated with Sivers function

$$\frac{d\Delta\sigma(p^\uparrow A\to\ell^+\ell^- X)}{dM^2dyd^2q_\perp} = \frac{\alpha_{em}^2}{6\pi^3M^2}\sum_q e_q^2\int_{x_p}^1 \frac{dz}{z}\, d^2k_\perp \frac{\epsilon^{\alpha\beta}s_\perp^\alpha k_\perp^\beta}{M_p} x f_{1T}^{\perp,q}(x,k_\perp^2)\int d^2b d^2p_\perp F(x_A,p_\perp)$$
$$\times k_\perp^\gamma p_\perp^\rho p_\perp^\sigma \frac{1}{2}\frac{\partial}{\partial k_\perp^\gamma\partial p_\perp^\rho\partial p_\perp^\sigma}\left[H_T(q_\perp,k_\perp,p_\perp,z)+H_L(q_\perp,k_\perp,p_\perp,z)\right]_{k_\perp\to0,p_\perp\to0}.$$

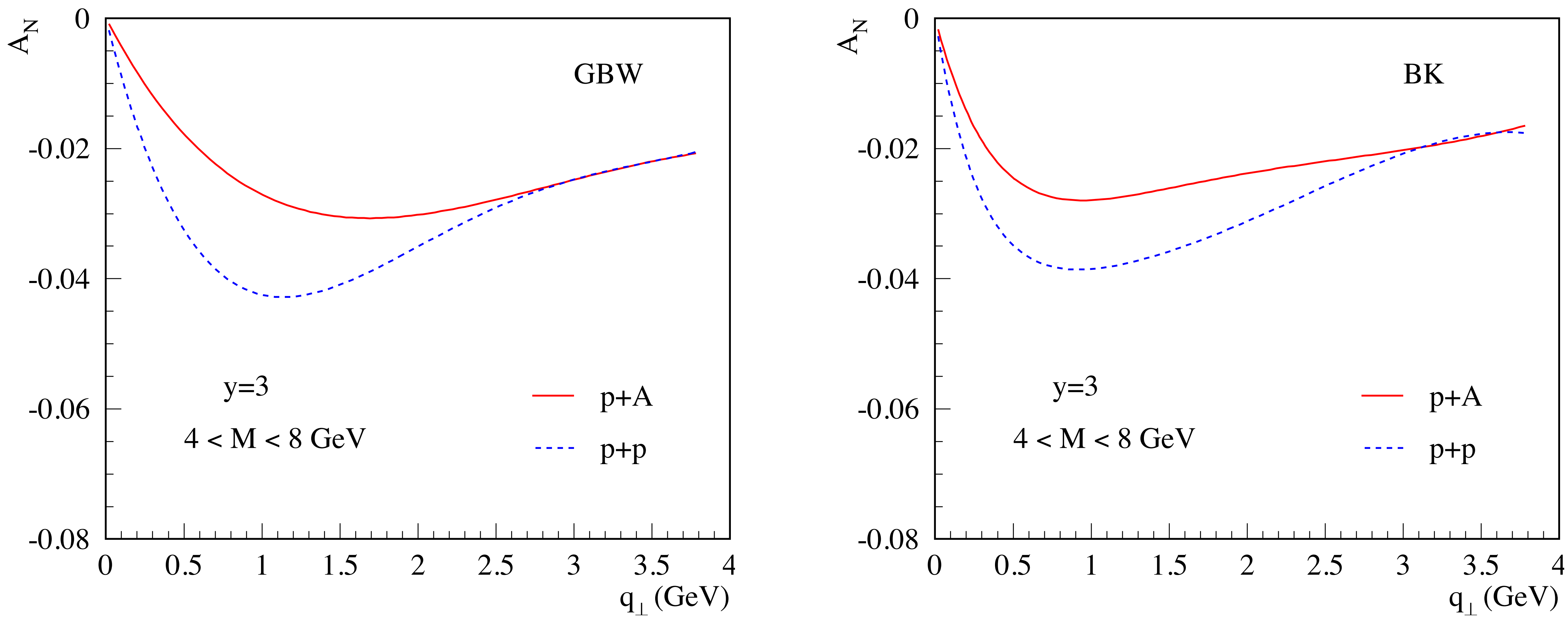
$$\frac{d\Delta\sigma(p^\uparrow A\to\ell^+\ell^- X)}{dM^2dyd^2q_\perp} = \frac{\alpha_{em}^2\alpha_s}{3\pi N_cM^2}\epsilon^{\alpha\beta}s_\perp^\alpha q_\perp^\beta\frac{1}{(2q_\perp^2)^3}\sum_q e_q^2\int_{x_p}^1 dz\, x\, T_{q,F}(x,x) x_A f_{g/A}(x_A)$$

  - In the forward limit, this is also consistent with collinear twist-3 formalism, even though they look very different at first

- (Polarized) Drell-Yan is still the cleanest process

Spin asymmetry at RHIC 510 GeV - I

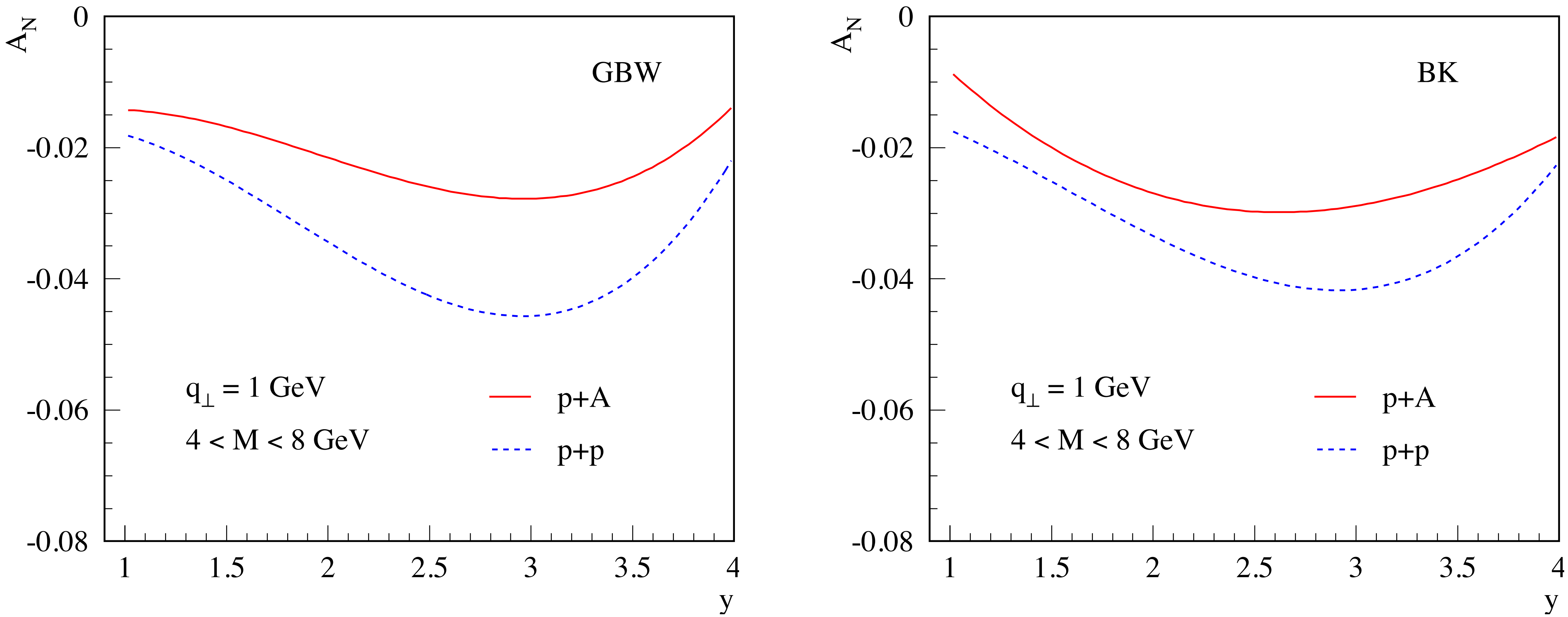
- Transverse momentum dependence



- Spin asymmetry is smaller in pA compared to pp, due to larger saturation scale

Spin asymmetry at RHIC 510 GeV - II

- Rapidity dependence



- The maximum happens at y~3, which corresponds to xp ~ 0.2 in the polarized proton (the Sivers function is largest at around this point)

Summary

- Polarized p+A (p+p) collisions is a good place to study both the transverse spin physics and small-x gluon saturation
- Polarized p+A collisions might add more to the saturation physics, as they could be sensitive to the slope of the unintegrate gluon distribution in the kt-space
- It will be interesting to study them at RHIC experiments
  - Inclusive hadron production
  - Drell-Yan production
  - Real photon/low mass dilepton production