A short tutorial on the chemical potential

"A vague discomfort at the thought of the chemical potential is still characteristic of a physics education. This intellectual gap is is due to the obscurity of the writings o f J.Willard Gibbs who discovered and understood the matter 100 years ago."

C.Kittel;Preface to his book:Introduction to Solid State Physics

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- the chemical potential is $$\mu_i = \left(\frac{dF}{dN_i}\right)_{T,V,N} \quad \text{or} \quad \mu_i = \left(\frac{dG}{dN_i}\right)_{T,P,N}$$

- F=E-TS which is the Free energy ($_N$ means hold all constant except ith type)
- G=F+PV is the Gibbs free energy
  - so μ is the free energy it takes to make a new particle
- Take a reaction (e.g. or a phase transition)
  - we can write it as $$\sum b_i B_i = 0$$
  - Where b_i is the number of species B_i e.g.

$$-2\mathrm{H}_2 - O_2 + 2\mathrm{H}_2O \text{ or ice-water=0 or -u-}\bar{\mathrm{d}}+\pi = 0$$

(-2,-1,1 or 1, -1 or -1, -1, 1 are the b's)

- Now for equilibrium the condition is

$$\sum b_i \mu_i = 0$$

- In the case of ice to water you get that $\mu_{water} = \mu_{ice}$
  - This means that through the phase transition the two chemical potentials are equal, but that does not mean they stay the same. In fact they must change because a phase transition is
    - First order: first derivative of the free energy has a singularity
    - Second order: Second derivative of the free energy has a singularity
    - So μ can have a jump at the phase transition since it's a derivative of F

- in the case of our quarks we have

$$-u-\bar{d}+\pi=0 \qquad \text{so} \qquad \mu_u+\mu_{\bar{d}}=\mu_\pi$$

- Now if we assume that everything is a non-relativistic ideal gas we get for particle i that

$$\mu_i=-kT\ln\left[\left(\frac{2\pi}{h}\right)^{\frac{3}{2}}\left(\frac{1}{n_i}\right)\left(2\pi m_i kT\right)^{\frac{3}{2}}\right]$$

Where n_i is the number density

  – Now we should really figure this out for a relativistic gas but for illustration we just add the mass to the chemical potential so

$$\mu_i=-kT\ln\left[\left(\frac{2\pi}{h}\right)^{\frac{3}{2}}\left(\frac{1}{n_i}\right)\left(2\pi m_i kT\right)^{\frac{3}{2}}\right]+m_i$$

- Now here is the way I think about its connection to mass changes (chiral symmetry restoration)
  - For quarks

$$\mu_q = -kT \ln\left[\left(\frac{2\pi}{h}\right)^{\frac{3}{2}}\left(\frac{1}{n_q}\right)\left(2\pi m_q kT\right)^{\frac{3}{2}}\right] + m_q$$

where m_q =260 MeV below T_C and 5 MeV above

  - For pions

$$\mu_\pi = -kT \ln\left[\left(\frac{2\pi}{h}\right)^{\frac{3}{2}}\left(\frac{1}{n_\pi}\right)\left(2\pi m_\pi kT\right)^{\frac{3}{2}}\right] + m_\pi + \varepsilon_0$$

where ε_0 is the binding energy, m_π =135 MeV below T_C

ε_0=0 above T_C

ε_0 below T_C and m_π above T_C are not set

- This then allows the eqn :

$$\mu_u + \mu_{\bar{d}} = \mu_\pi$$

To be obeyed through the phase transition, but μ changes since the masses change