A view from INT - Phase transitions as of 5/8/98

I start with a summary of the stuff I learned. Any corrections for my misconceptions is welcome. A tutorial is offered below which will help the curious. I also warn the reader that these conclusions are subject to phase transitions themselves depending on the mood of the theoretical community. :)

As I understand things there is a growing consensus of the following:

1) along the high baryon density axis the transition is first order.
   This comes from renormalization group arguments - I thought I understood this at the time but I cant reconstruct the argument- I am asking Steve Hsu now of Oregon State- to summarize if for me
2) Along the high temperature axis - in the relevant case of 2 light quarks and one semi-light quark (strange) it seems to be second order or a cross over. This from the lattice calculations of the Columbia and I think other groups. There is one NOTABLE EXCEPTION that comes from the Tsukuba group which gets a first order transition. This appears to have to do with the formulation of the problem. The two formulations are the (1)Wilson and (2)Kogut -Suskind and they have to do with the way fermions are put on the lattice. In the continuum limit these should give the same answer so there is definitely something funny going on here. The Tokyo group uses the Kogut-Suskind and the rest of the groups use the Wilson. See below for a tutorial.

A tutorial:

Phase transitions - general

As you know the usual phase transition diagram has two axis, baryon density and temperature. One of the basic questions is the nature of the phase transition as you raise the temperature or the baryon density. Usually these are categorized into several types

1) first order - this means that there is a latent heat and that the order parameter has a discontinuity giving us a infinity (or pole) in the first derivative (hence “first order”) See later for definition of order parameter. The transition of liquid water to steam is this sort of transition
2) Second order - this means that there is a kink in the order parameter, hence the second derivative has an infinity (or pole). There is no latent heat
3) Cross over - this means that there is really no infinity anywhere. BUT don’t despair, these cross overs are usually pretty fast so they look to an experimentalist like 1 or 2 above.

Note that in real life situations we will always have a cross over, since in the strict sense phase transitions happen in infinite volumes.

Some theoretical limits going to high temperature:

Everyone agrees that with no quarks (or in other words infinite mass quarks - or pure gauge fields - or gluons only) the phase transition is first order. The Wilson loop parameter servers as the order parameter in this case and corresponds to confinement - but its the confinement of gluons into glueballs. (note Wilson loop parameter = Polyakov loop)

Also with 3 zero mass quarks (so now gluons can split into q-qbar pairs) and zero chemical potential (i.e. net baryon number=0) the transition is first order. I note that this is an interesting argument which I think is pretty solid and has to do with the renomalization group. See the tutorial on RG arguments

Order parameters -

Definition - The order parameter is any parameter that signifies the order in the system. An example is the density of water as it goes from liquid to gas. Another is the magnetization of a magnet. Each order parameter can have its own properties and hence can have a phase transition as a function of temperature.

For us there are two of importance.

1) The confinement order parameter (the value of the wilson loop)
2) the chiral order parameter - the expectation value of our field ϕ or $<\psi\bar{\psi}>$ (that should be psi bar psi) or the quark condensate

In pure guage simulations the order parameter is the Wilson loop parameter which is really the energy it takes to pull a pair of gluons apart and hence is a good approximation to the CONFINEMENT phase transition (albeit for a glueball).

However once you put in quarks this thing is not well defined and you have to look at the chiral phase transition - the chiral order parameter which is the expectation value of the field <phi> or the value of the condensate.

A comment: Question – what is ϕ or $<\psi\bar{\psi}>$ (that should be psi bar psi) or the quark condensate? The picture is the following. The vacuum is a sea of q-qbar pairs which fills the universe. It is the expectation value of this sea that is our condensate. Since the q-qbar pair is a scalar the lorentz properties don't bother us. A bare quark exchanges gluons with this sea to give it a dressed mass, and presumably gets confined. (You don't like this?? You think I'm crazy? You think this is the aether coming back to haunt us? Good! You are starting to get the picture. See the April issue of Physics Today – "Back to Basics at Ultrahigh Temperatures" by Wilczeck.)

Lattice formulations:

These have to do with the way fermions are put on the lattice. It turns out that there is a doubling whenever you put fermions on a lattice. That is you cannot put a single fermion on a lattice. If you try you get 2. How do we deal with this? Wilson put in a special term into the action which breaks the chiral symmetry explicitly which kill the second of the

pair. The trouble is that you have a lot of constants in this term that have to be tuned as you go to the continuum limit so that the second of the pair really disappears and that the quarks stay massless. Kogut and Suskind said, let's make use of the pairs and make them the different flavors. It means though that you can add fermions only in pairs and that the pair must be degenerate in mass. That's OK for u and d quarks since mass(u)=mass(d) to a good approximation. But how do they add the third –strange quarks? Kogut and Suskind have a recipe. They take the fermionic determinant (exp(-I integral(Action)) which gives two fermions (up and down). To get the third they take the sqrt . Does this make sense? It apparently does in the continuum limit but for the lattice I don't know and no one has proved that it is OK. The fact that the Tokyo group gets a different answer than everyone else should give one pause.

Note on sigma models.

There are two basic types:

1) The linear sigma model -

This has in it a field (particle types - think of this as a higgs - we usually designate this as the field phi) which in SU(2) has two components the pion and sigma. In O(4) it is the pion field with 4 components - the 3 pions and the sigma. In SU(3) it is the pseudoscaltar octet (with pions(3), sigma, Kaons(3), eta) It doesn't work to well in SU3 since the theory is good to the extent that the bare quark masses are =0 which is not good for s quarks. This field ϕ is supposed to be some sort of glop of gluons and quarks that provides the symmetry breaking term. It provides the usual mexican hat picture of the potential with excitation in the radial direction being the sigma (remember particles are really excitations of a field) and the pions being the zero energy (massless) excitations in the theta direction.

This model has one SERIOUS drawback. It is not good near the phase transition. However theorists seem to push it close to the phase transition and then they pray...

2)The NON-Linear sigma model basically locks down the sigma so that it cannot be excited, i.e. it is stuck at the bottom of the potential. You can think of it as a mexican hat with the brim which in infinitely deep. This means that the sigma is infinitely heavy and you can expand in 1/(sigma_mass). The sigma is at something like 600 MeV (this could be 1 GeV etc- people think it is too broad to see) so to this works to energies of 100 MeV or so.

This model is used to do LOTS of calculations and has had tremendous success in many things like pion scattering. There is no phase transition here since there is no sigma.

Symmetry Breaking

I will try to put together a quick tutorial write up on symmetry breaking (and chiral symmetries) if people are interested.

Renomalization group -

I don't claim to be an expert on R-G. But they make powerful and very general arguments. I think that arguments based on RG ideas are pretty solid. R-G arguments are basically symmetry arguments. QCD has a symmetry called the Z(3) symmetry in the case of massless quarks R-G says that if two theories have the same symmetries (that is they fall into the same "universality class") then they have the same behavior at certain points (fixed points). So if we can find a model which has a Z(3) Symmetry then its a good model for QCD at the fixed point of interest to us - the phase transition point! There is such a model - the "pots model".

It turns out that the pots model does not have a 2nd order transition. Lattice calculations give a strong transition- so we have two facts

1) there is a phase transition – from lattice calculations
2) It is not 2nd order – from RG arguments

Hence it is first order.

Here is an argument for confinement – I also make an attempt to explain Z(3)

What is Z(3)? It turns out that you can make a transformation of the fields of QCD in which you essentially multiply the fields by some operator that commutes with all the generators times some constant z (which can be complex-hence it introduces a phase). The operator turns out to be a unitary matrix. In the case of SU(3) it turns out that there are three constants z=1,exp(-i pi/3), exp(-i 2pi/3).

Now lets look at the thermal expectation value of the integral of a quantity which we call the Action <L>=exp(-i integral(A_0)) where A_0 is the gauge field It turns out that this is = exp(beta F) where F is the free energy of a free quark in the thermal bath of gluons and beta is 1/kT. Now since Z(3)is a symmetry then under the transformation above <L>=<zL>. so if the symmetry is good, this can only be true if <L>=0 which must mean that F= infinite i.e. we have confinement. Now if Z(3)is broken then <L> does not =0 and F is finite and we can have a unconfined quarks. So for pure glue it is the Z(3) (3 for color by the way) is the symmetry that is broken. It is DIFFERENT than the chiral symmetry. It is the symmetry that corresponds to confinement. It goes backwards. 1) at T=0 the symmetry is OK and we have confinement. 2) as we go to high T, (from lattice calculations) the symmetry is broken and we have deconfinement.