Lecture 8
The Stern-Gerlach Experiment

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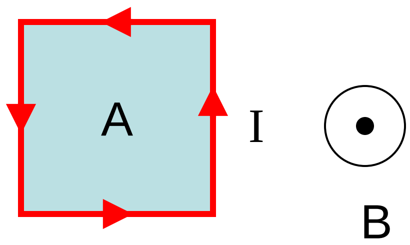
qm notes

updated for Fall 2005

Spin

- we are going to try to understand the spin of the electron. First we need to review the interaction of magnetic moments with a magnetic field
- The electron happens to have a magnetic moment which we will assume comes from the spin. We are going to try to understand what causes this. For the moment we will picture a magnetic moment as a loop of current I with a side L and area A like so:



Now $A=L^2$ and the magnetic moment $\mu=IA$. There is also a magnetic field B pointing out of the page. The potential energy of the loop is

$$U=-\vec{\mu}\cdot\vec{B}$$

so if we have an electron with a spin $\vec{s}$ which we will assume is proportional to $\vec{\mu}$, i.e. $\vec{\mu}\propto\vec{s}$ we will have for the electron that its potential energy in a magnetic field is also $U=-\vec{\mu}\cdot\vec{B}$

We want to figure out some way to figure out which way the spin is pointing – so if we could make a force which depends on the direction of spin – this would do it.

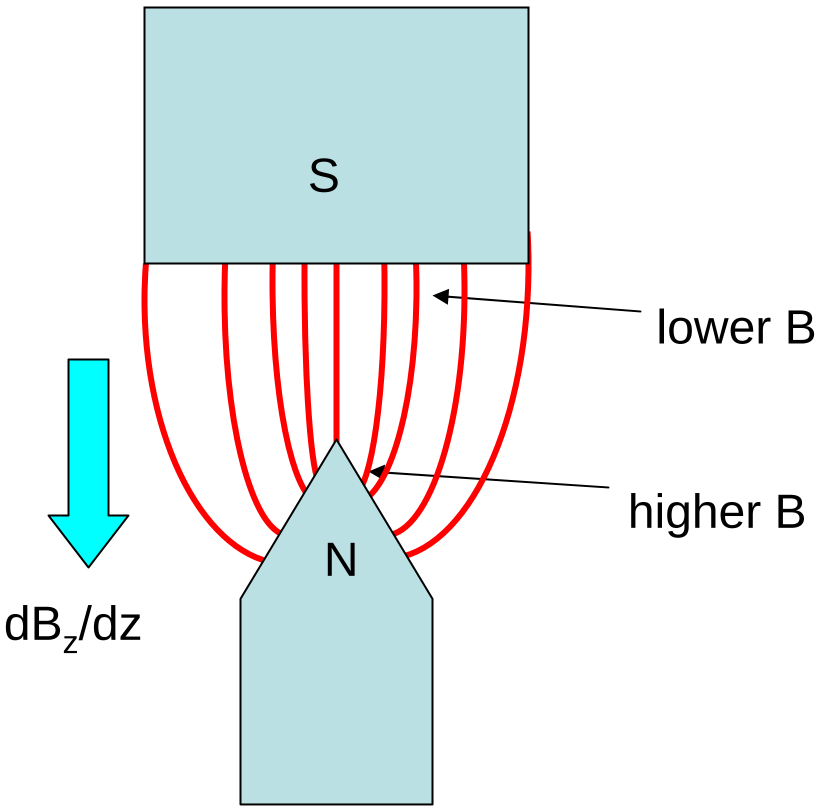
We know that force is a derivative of the potential energy.

So let's work in the z direction

$$\vec{\mathrm{F}}_z = -\frac{\partial U}{\partial z} = \frac{\partial}{\partial z}(\vec{\mu}\cdot\vec{B}) \approx \mu_z \frac{\partial \vec{B}_Z}{\partial z}$$ where I have assumed

μ is a constant and that B changes only in the z direction (I will set up the experiment that way) So this says that if I have a changing B field that there will be a force on the electron depending on the z component of μ or in other words, depending on the direction of the spin with respect to the magnetic field

- So can we make a magnetic field which has a changing z component in the z direction i.e. a dB_z/dz? We can. We shape our pole tips as follows where the field lines are in red. We remember that the strength of the B field is proportional to the density of lines. The density is highest near the bottom pointed tip so the field gets stronger toward the bottom and dB_z/dz points downward



Now since $$\vec{F}_z \approx \mu_z \frac{\partial \vec{B}_Z}{\partial z}$$

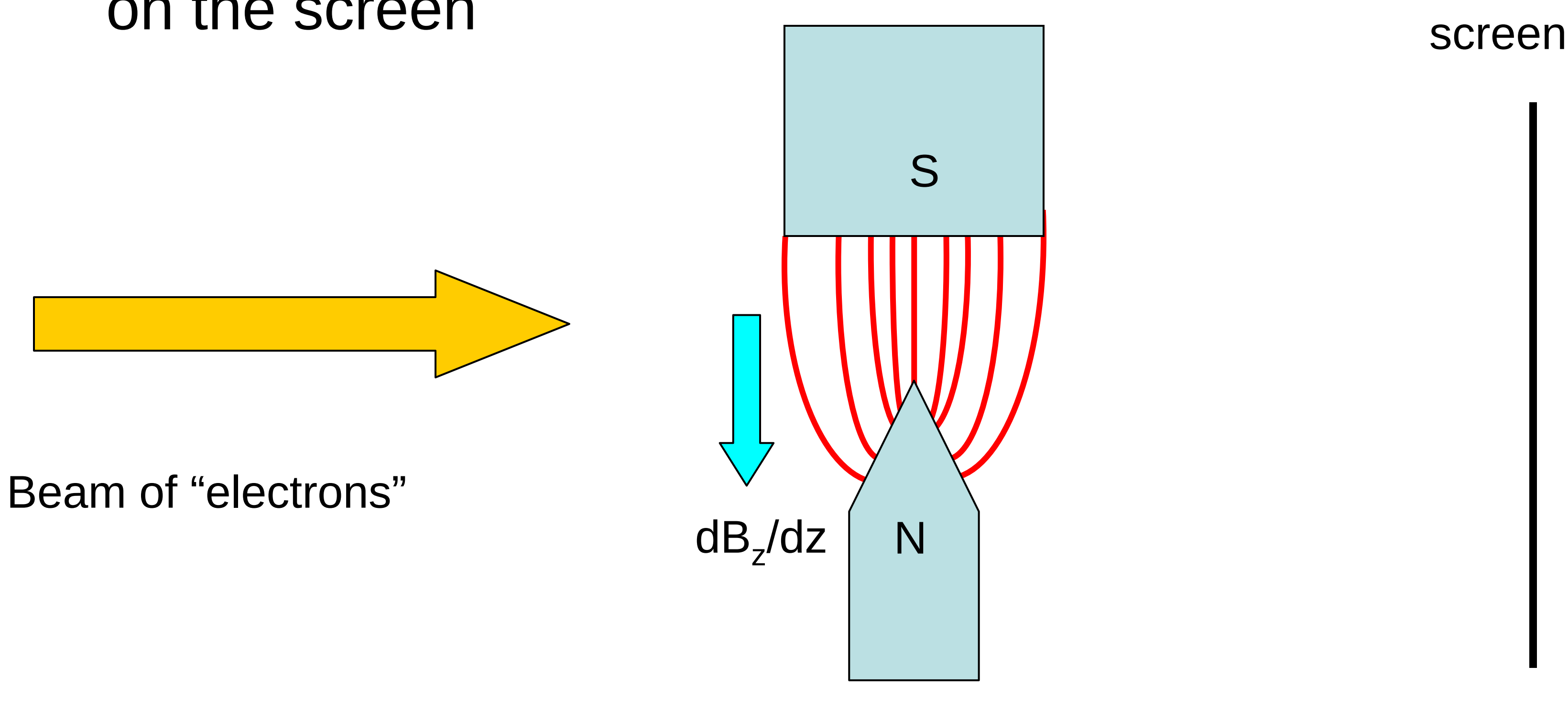
and μ and s point in opposite directions (the electron is negatively charged) then

if $\mu_z > 0$ $(s_z<0)$ so F_z is ↓

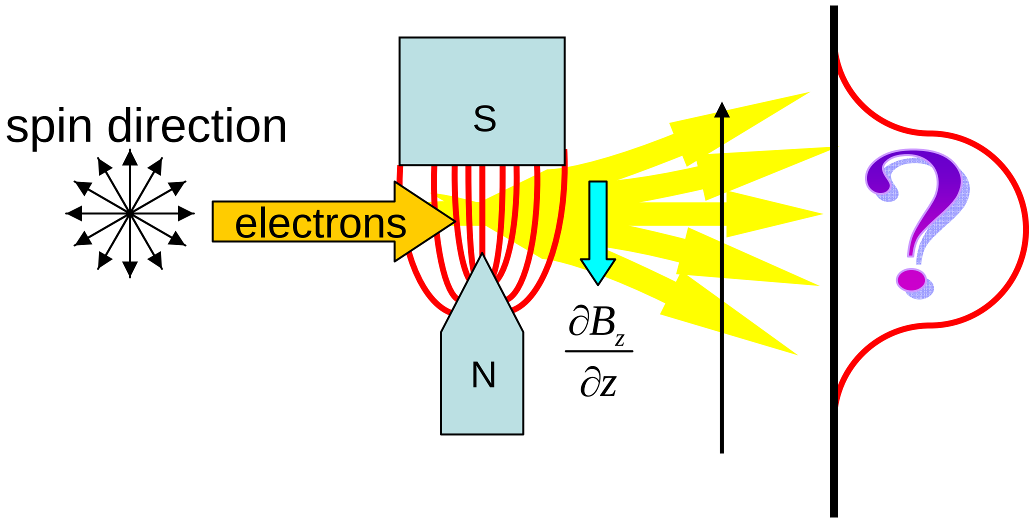
if $\mu_z < 0$ $(s_z>0)$ so F_z is ↑

Basically the force on the electron is proportional to the component of the spin in the z direction

- In the real experiment, an electron beam was not used. It was a Silver beam. Silver has one outer electron with an angular momentum of L=0. So the entire electron's magnetic moment is due to the spin of the outer electron. But for us, lets think of electrons. We set the experiment up as follows, and then look to see what pattern we see on the screen

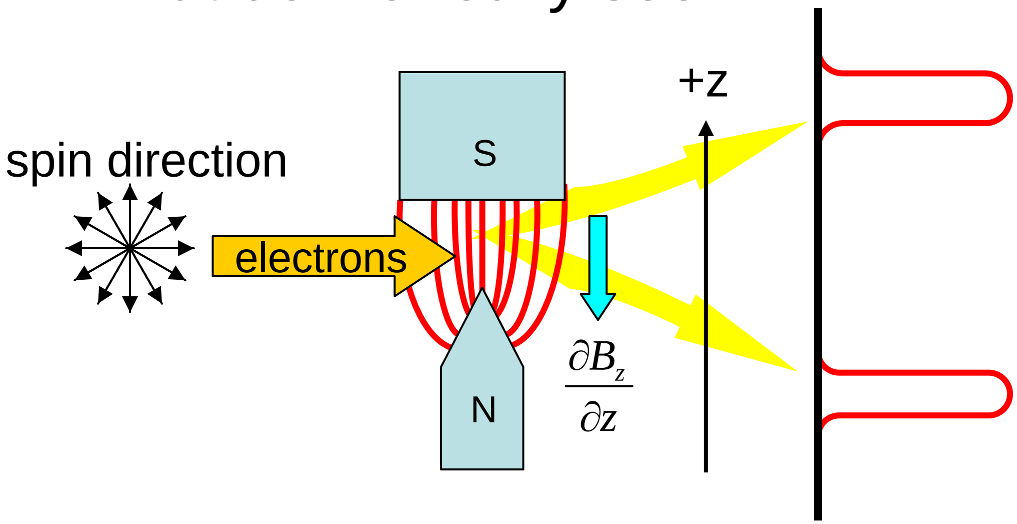


what might we expect?



Classically you might expect one wide spread reflecting the fact that the spin is randomly pointed and hence the z component of the magnetic momentum is spread around. Note that this initially random direction in like a ball in 4π

what do we really see?



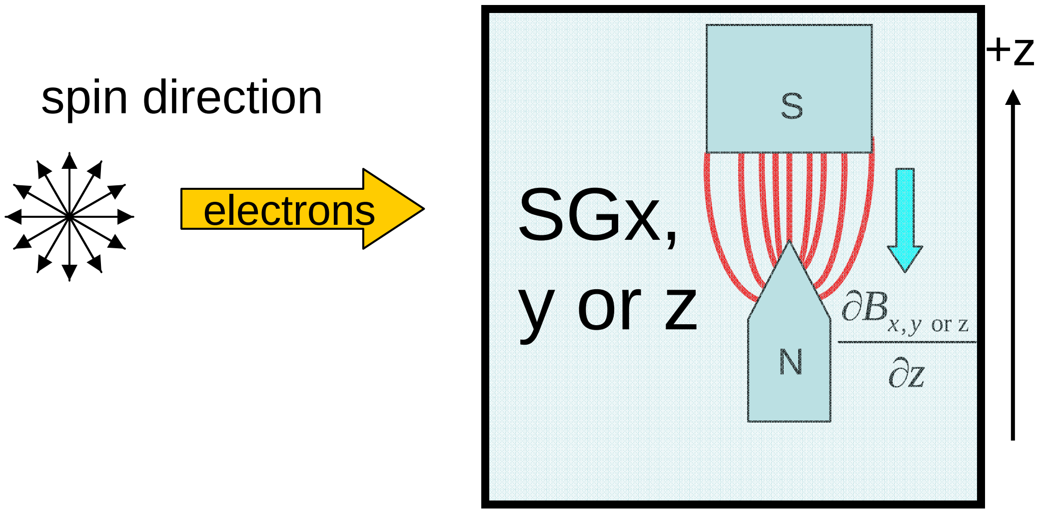
In reality, the beam is split neatly in half, as if the beam was initially ONLY pointed either in the +z or –z direction.

Now I can make a similar Stern-Gerlach aparatus which picks out the x component or y component of μ, by changing different components of the B field as follows.

$$F_z = -\frac{\partial U}{\partial z} = \frac{\partial}{\partial z}(\vec{\mu}\cdot\vec{B}) \approx \mu_x \frac{\partial \vec{B}_x}{\partial z} \text{ or } \approx \mu_y \frac{\partial \vec{B}_y}{\partial z}$$

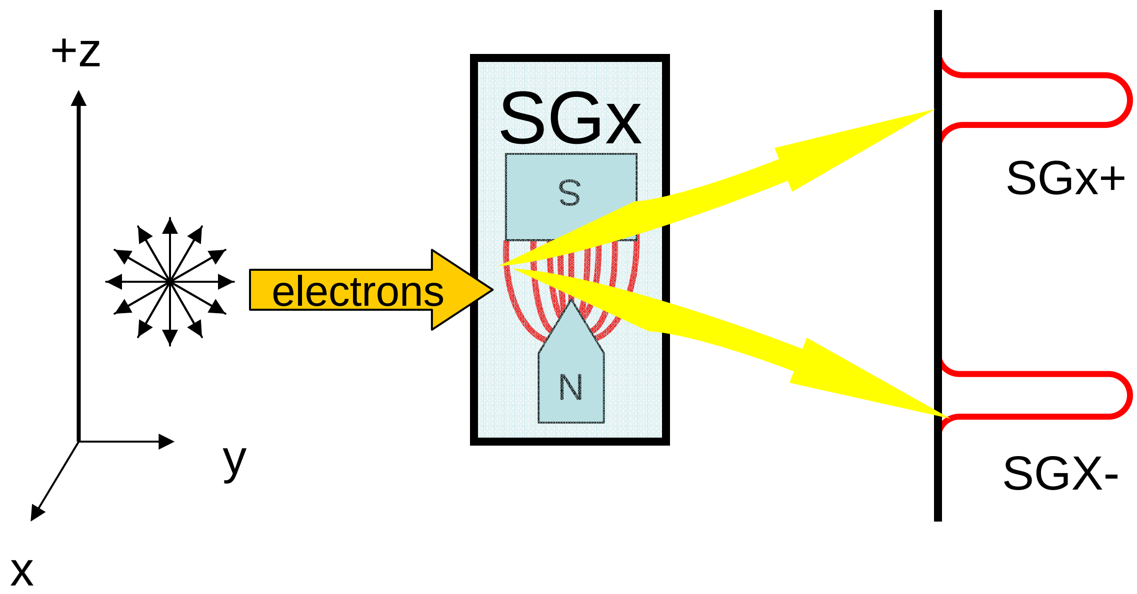
These have to be cleverly designed but I will not worry about that.

I will represent the apparatus as a box so we can have a box which is labeled SGx, SGy, and SGz which push on the various components of μ



So remember SGx will put a force in the z direction on the component of the spin in the x direction. Similarly for y and z

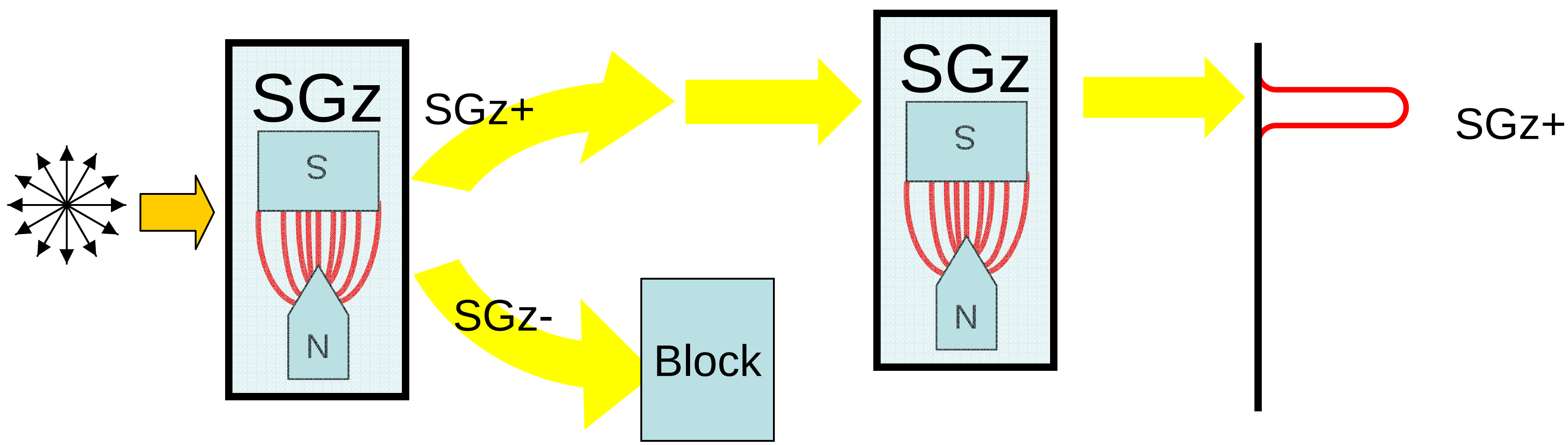
Now lets try putting it through an SGx apparatus

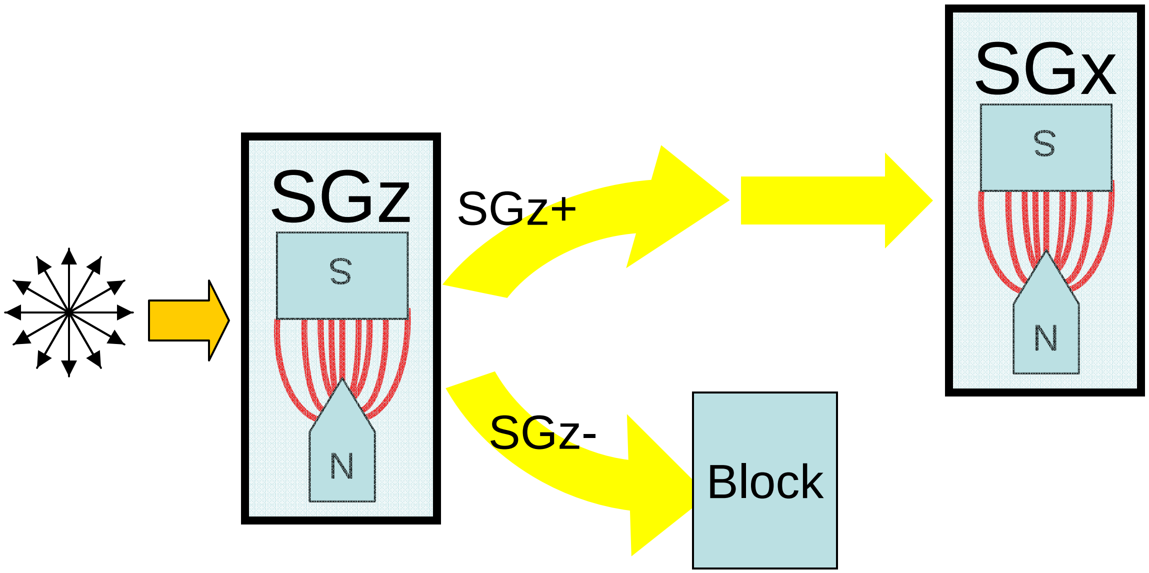


we see that it still spits in two as if the spin were either in the +x or –x direction !

Maybe somehow the electrons “guess” which direction the apparatus is set and align themselves accordingly

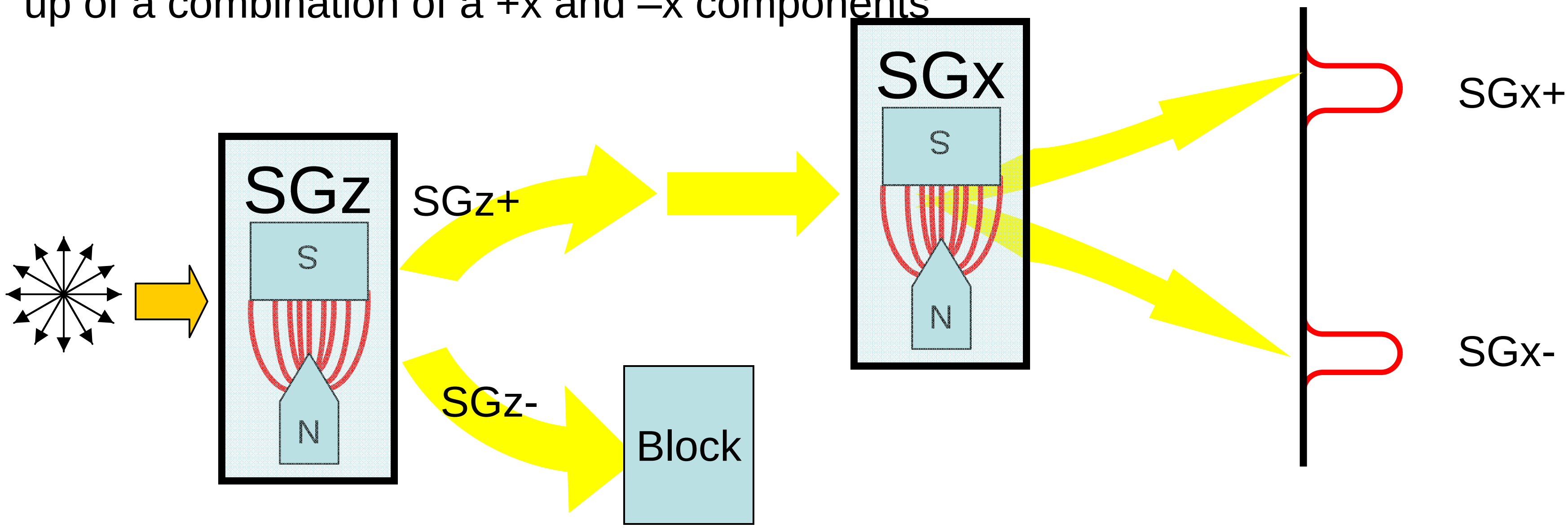
Now lets try using two SG apparatus. We use an SGz and then block off the bottom “-” component before we let it go through the second. Not surprisingly only the top bump is present. There is no SGz- compontent. It looks like once its “+z” it always stays “+z”

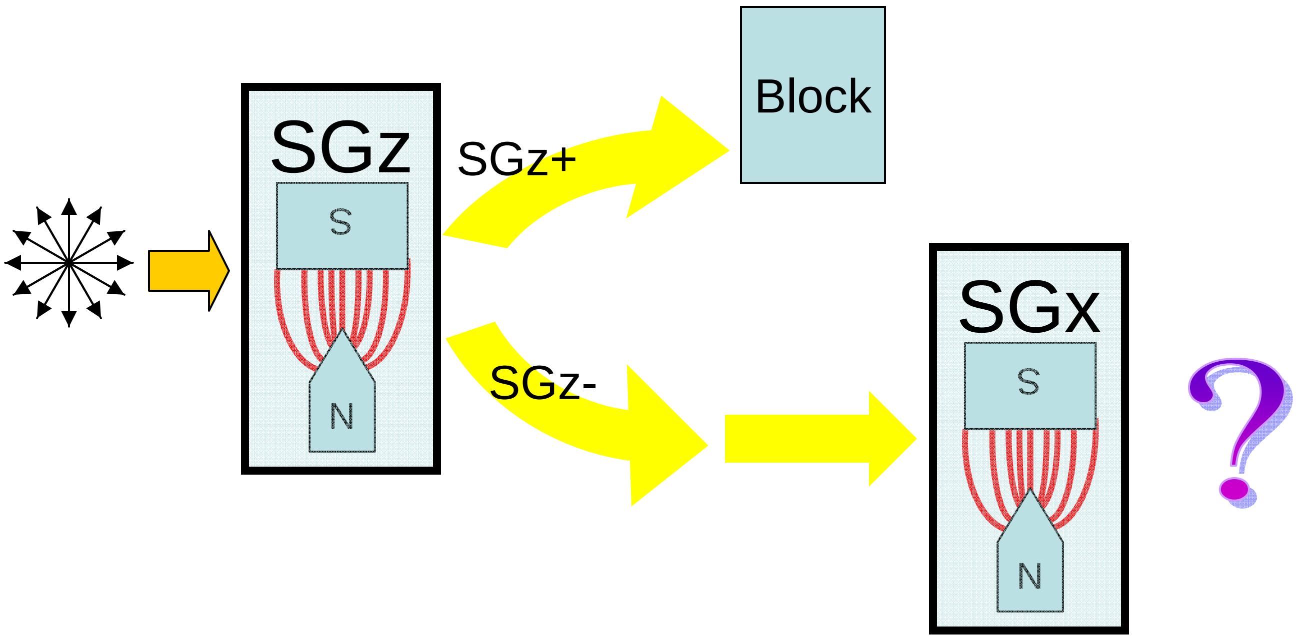




Now supposing we do the same thing, but this time, instead of "testing" the second beam with SGz lets test it with SGx.

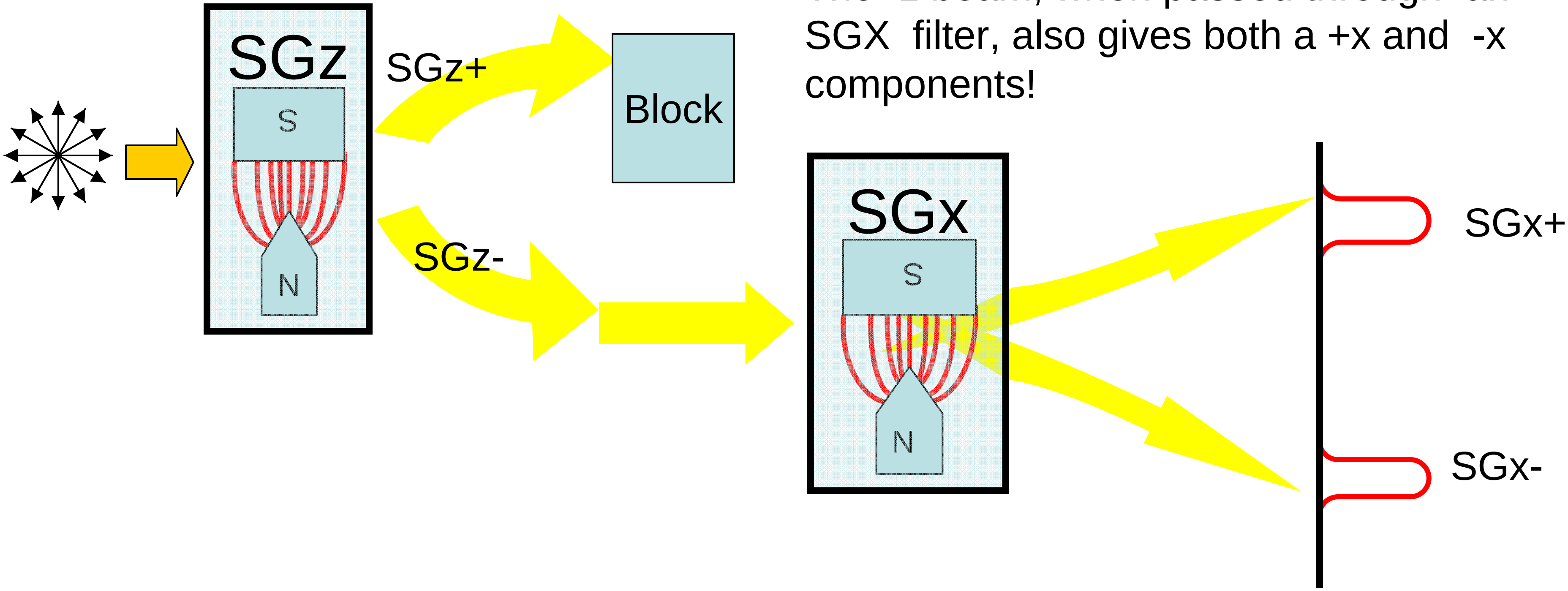
The +z beam, when passed through as SGX filter, gives both a +x and -x components, but note that the intensity is less! Its as if the SGz+ beam was made up of a combination of a +x and –x components

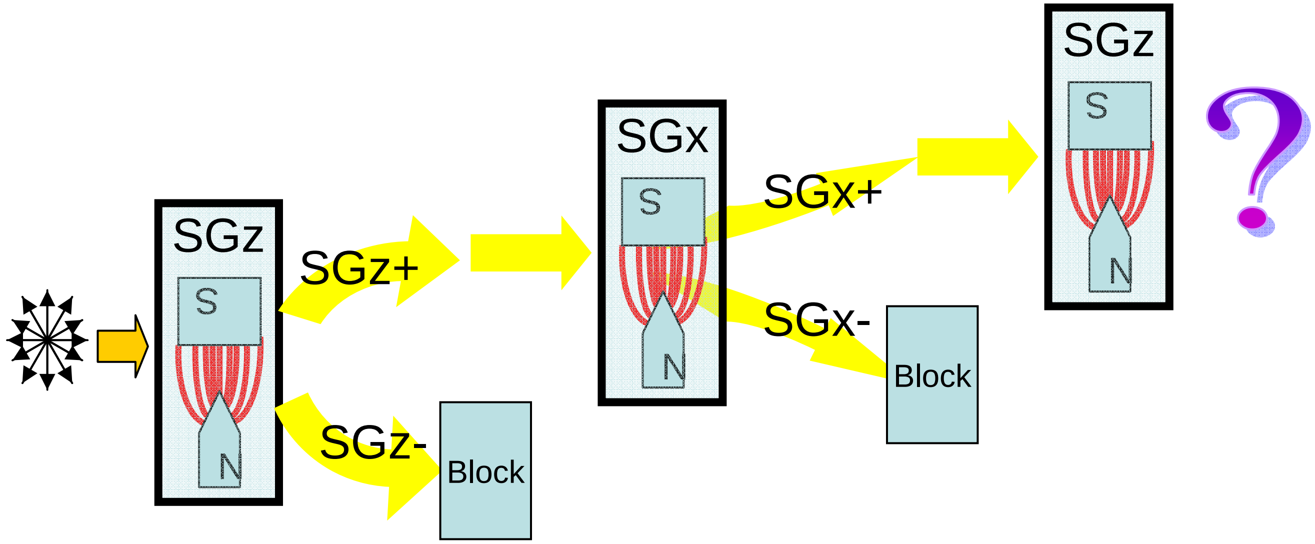




How about it we try the same with a –z?

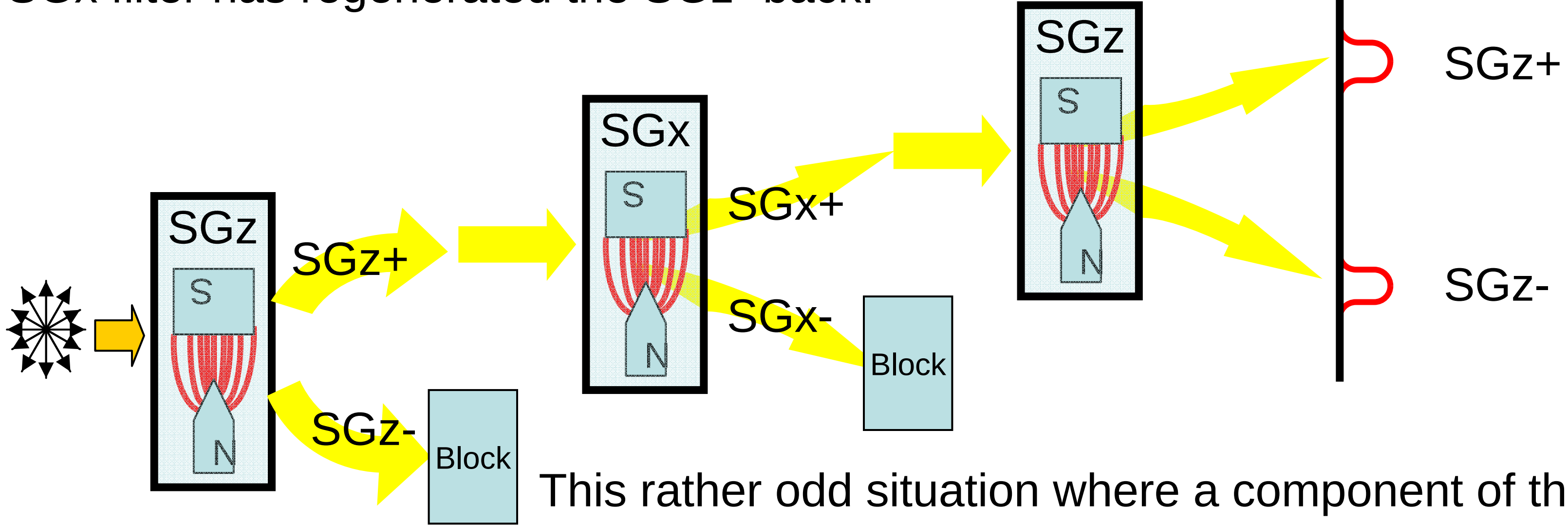
The -z beam, when passed through an SGX filter, also gives both a +x and -x components!





Now lets try using SGx to split up a SGz+ beam up into + and – x components and then test this to see if there is a –z component. Note this is exactly like the situation before where we used two SGz filters but we have added and extra SGx in between.

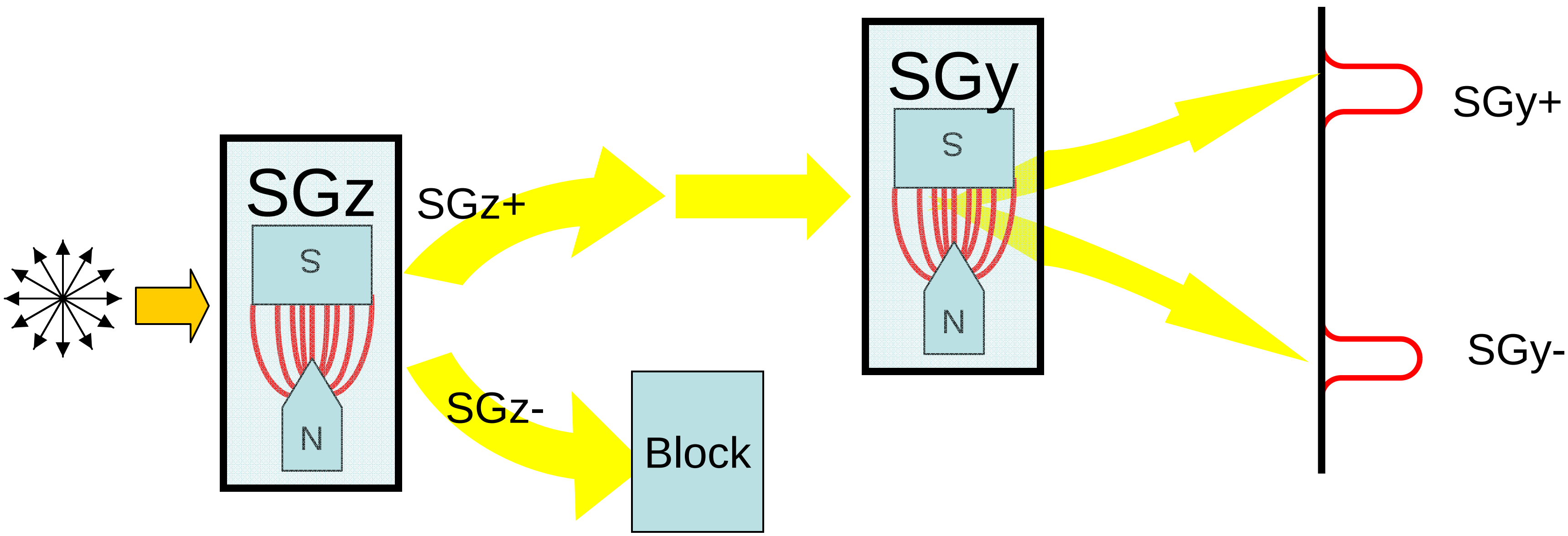
We seem to get the –z component back again. Its like the SGx filter has regenerated the SGz- back.



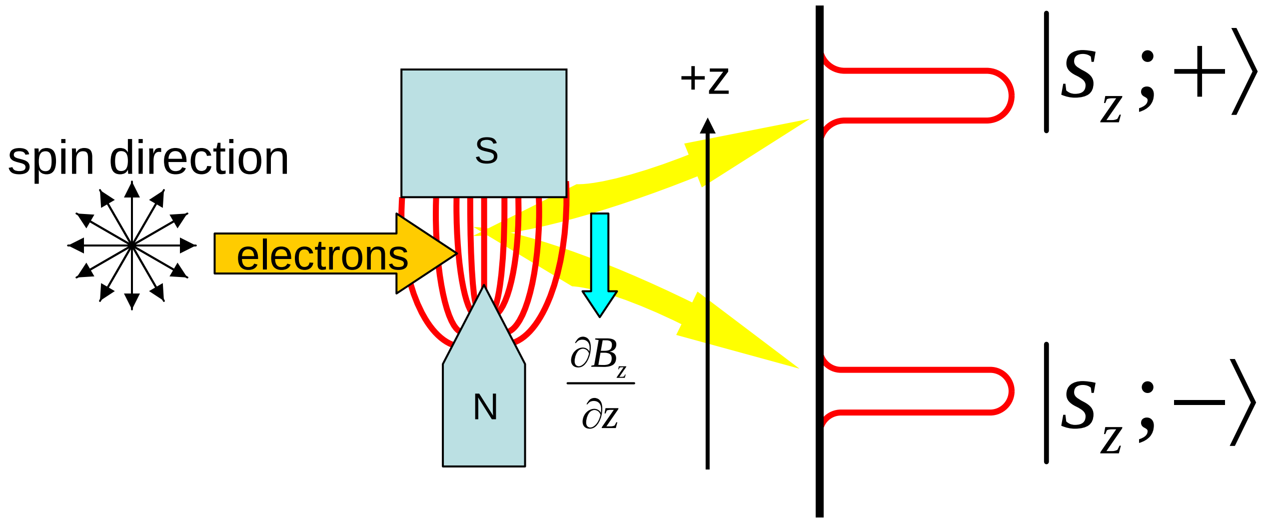
This rather odd situation where a component of the beam is regenerated, is reminiscent of polarized light where we are able to make a set of two perpendicular polarizers pass some light through by adding a third on at 45° in between

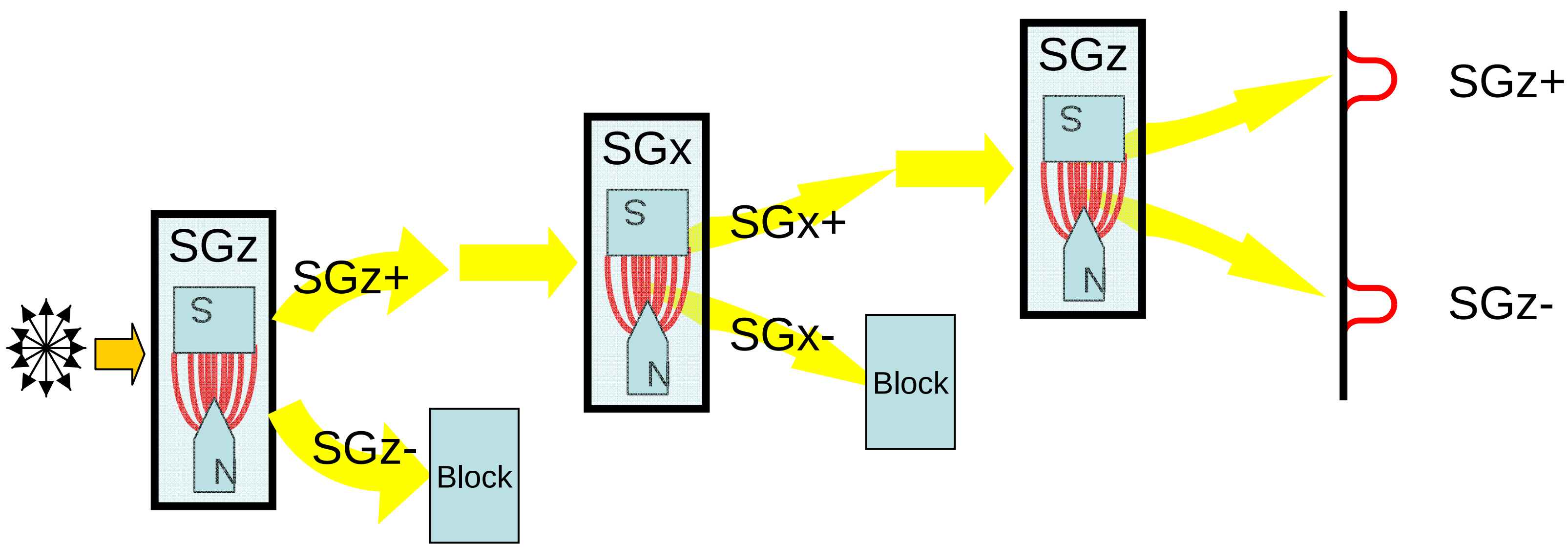
Now we remember there was also a SGy polarizer. Lets use it.

When we pass an SGz+ beam thorough an SGy filter we again see two components.

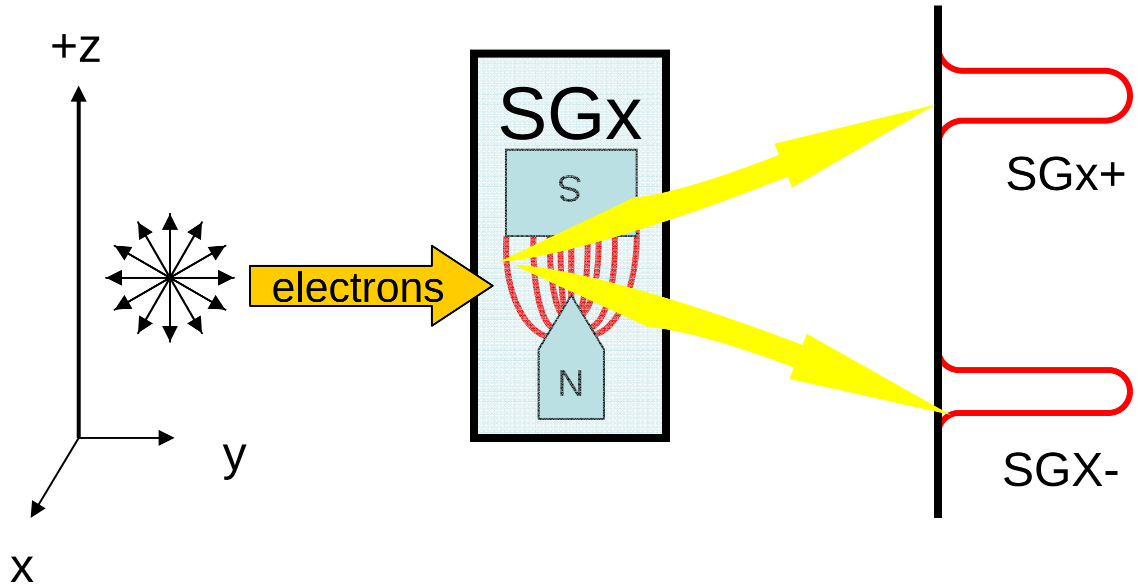


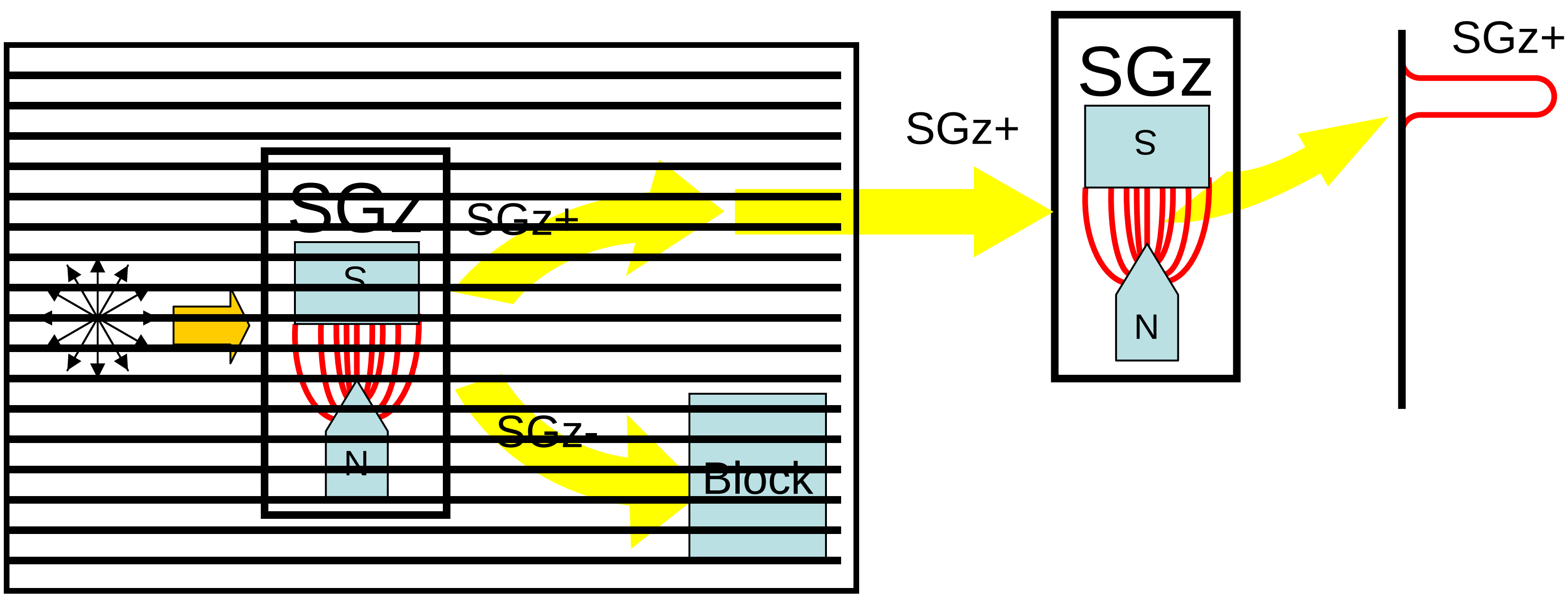
- rest are spares

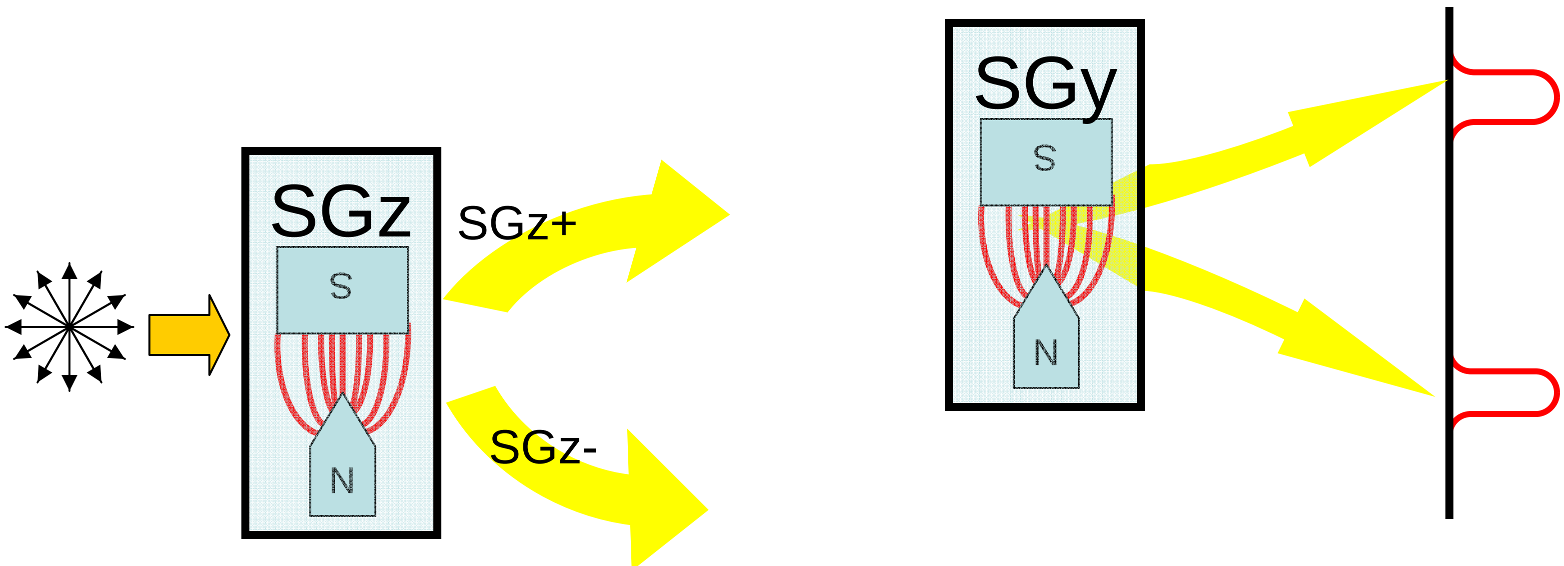

spin direction
S
electrons
N
$\frac{\partial B_z}{\partial z}$
+z
$|S_z;+\rangle$
$|S_z;-\rangle$

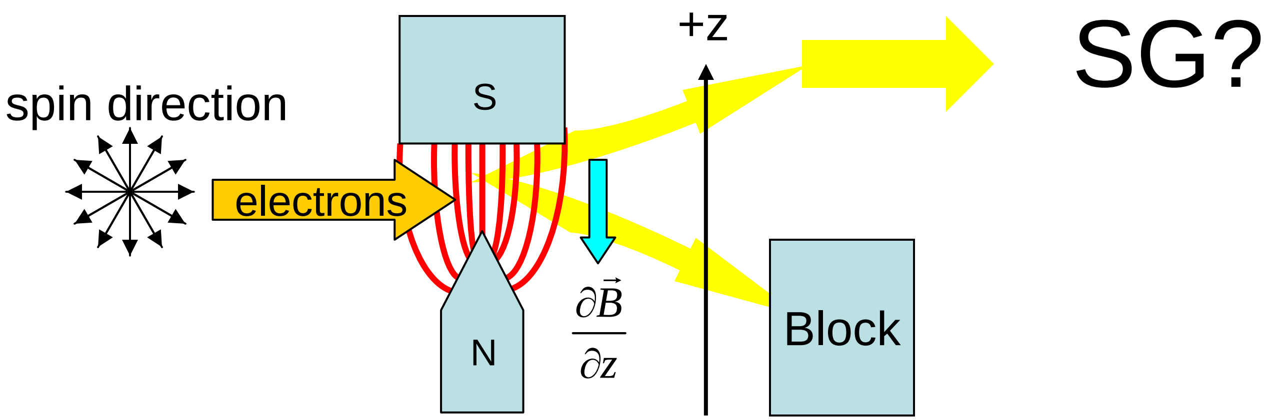

SGz
S
N
SGz+
SGz-
Block
SGx
S
N
SGx+
SGx-
Block
SGz
S
N
SGz+
SGz-

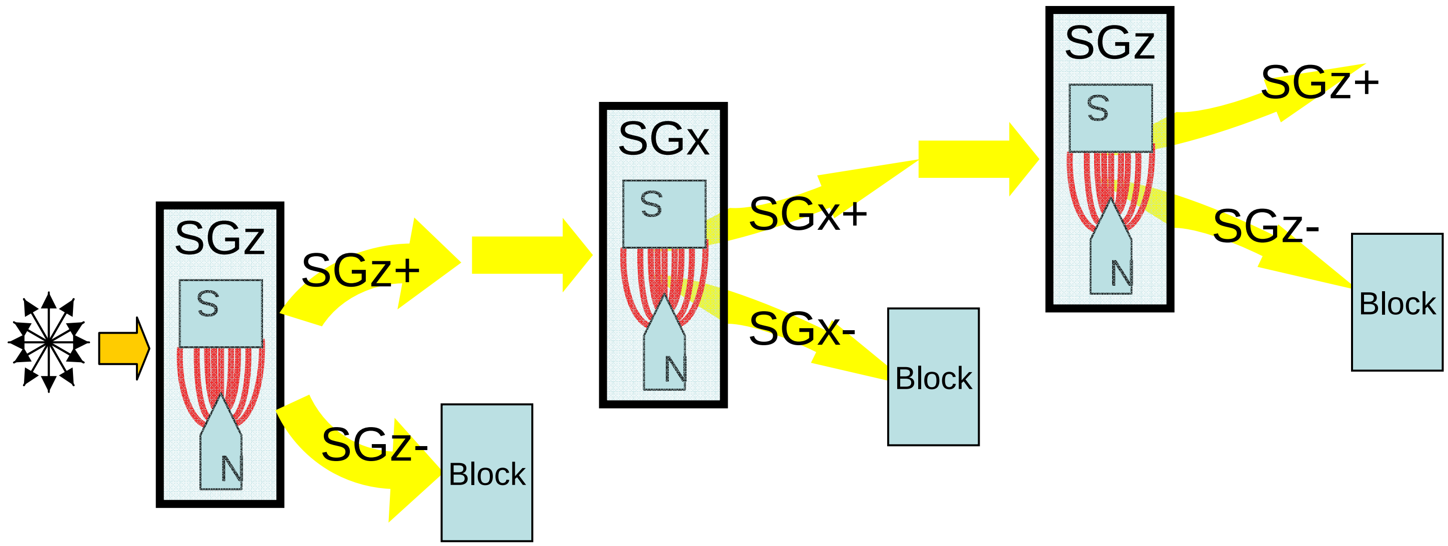
Now lets try putting it through an SGx apparatus

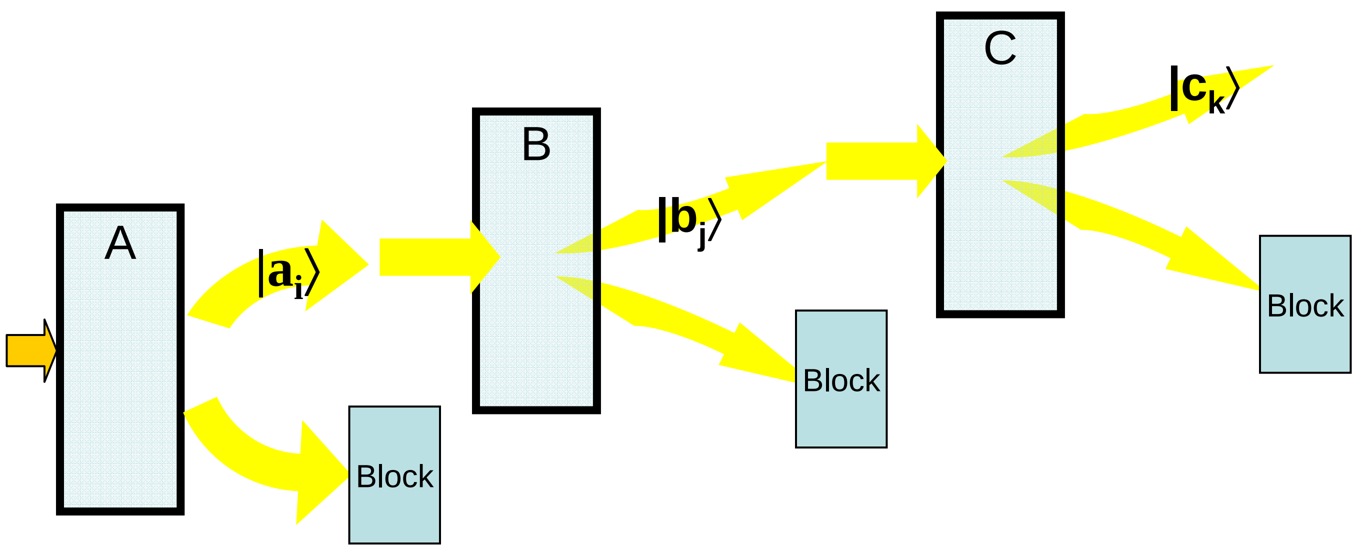


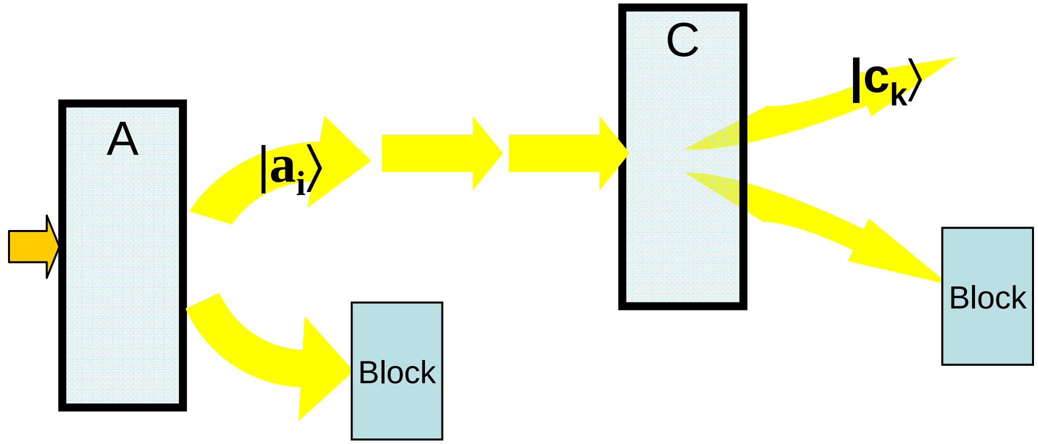



SGz
S
N
SGz+
SGz-
SGy
S
N


spin direction
S
electrons
N
$\frac{\partial \vec{B}}{\partial z}$
+z
Block
SG?


SGz
S
N
SGz+
SGz-
Block
SGx
S
N
SGx+
SGx-
Block
SGz
S
N
SGz+
SGz-
Block


A
$|a_i\rangle$
Block
B
$|b_j\rangle$
Block
C
$|c_k\rangle$
Block


A
$|a_i\rangle$
Block
C
$|c_k\rangle$
Block