using the CG-coeff

Physics 156B

r. seto

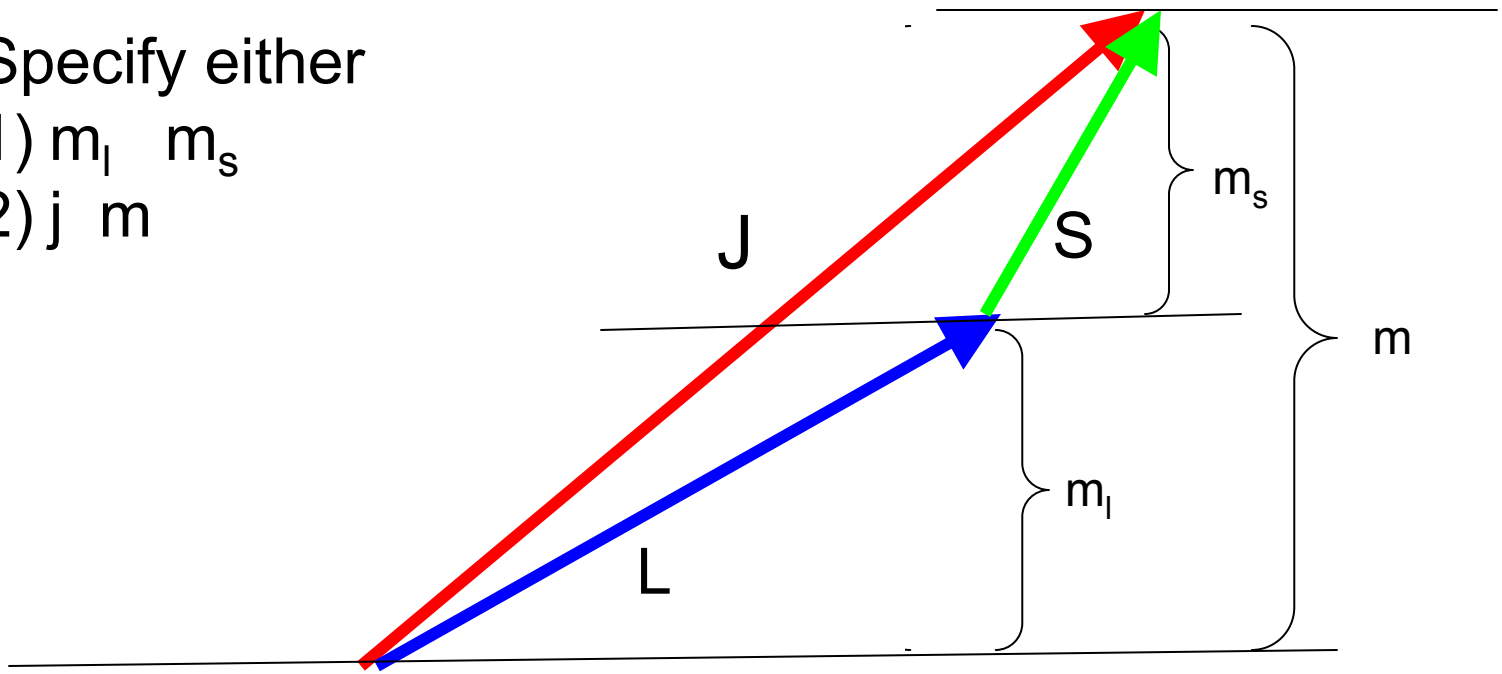
“Vector” angular momentum

L and S are given

Specify either

1) m_l m_s

2) j m

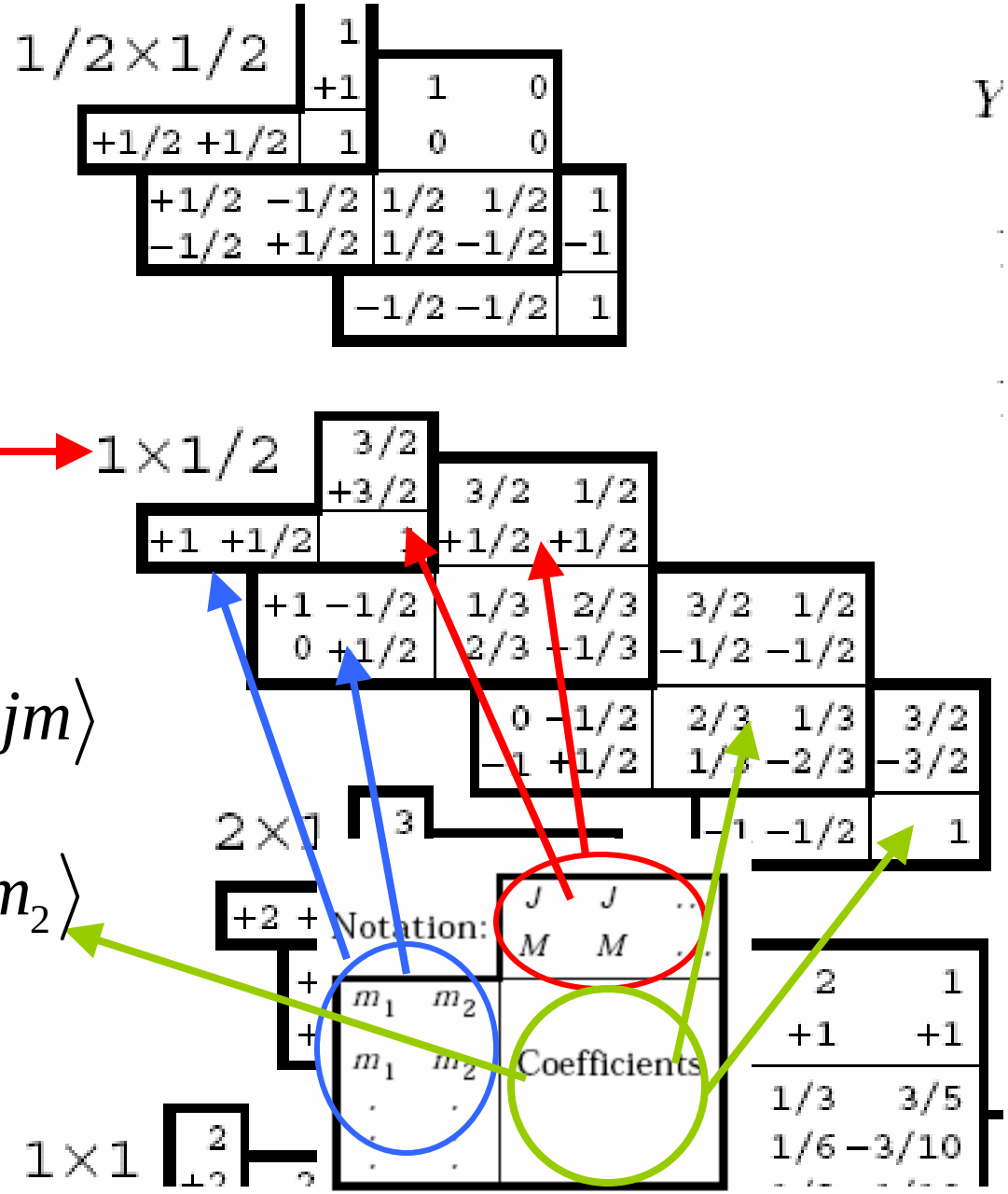


navigating the chart

$J = J_1 + J_2$

$J \rightarrow j\ m$
$J_1 \rightarrow j_1\ m_1$
$J_2 \rightarrow j_2\ m_2$

$j_1 = 1$
$j_2 = 1/2$



$$|jm\rangle = \sum_{m_1 m_2} |m_1 m_2\rangle\langle m_1 m_2 | jm\rangle$$

$$|m_1 m_2\rangle = \sum_{jm} |jm\rangle\langle jm | m_1 m_2\rangle$$

$$\langle m_1 m_2 | jm\rangle = \langle jm | m_1 m_2\rangle$$

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

1x1/2

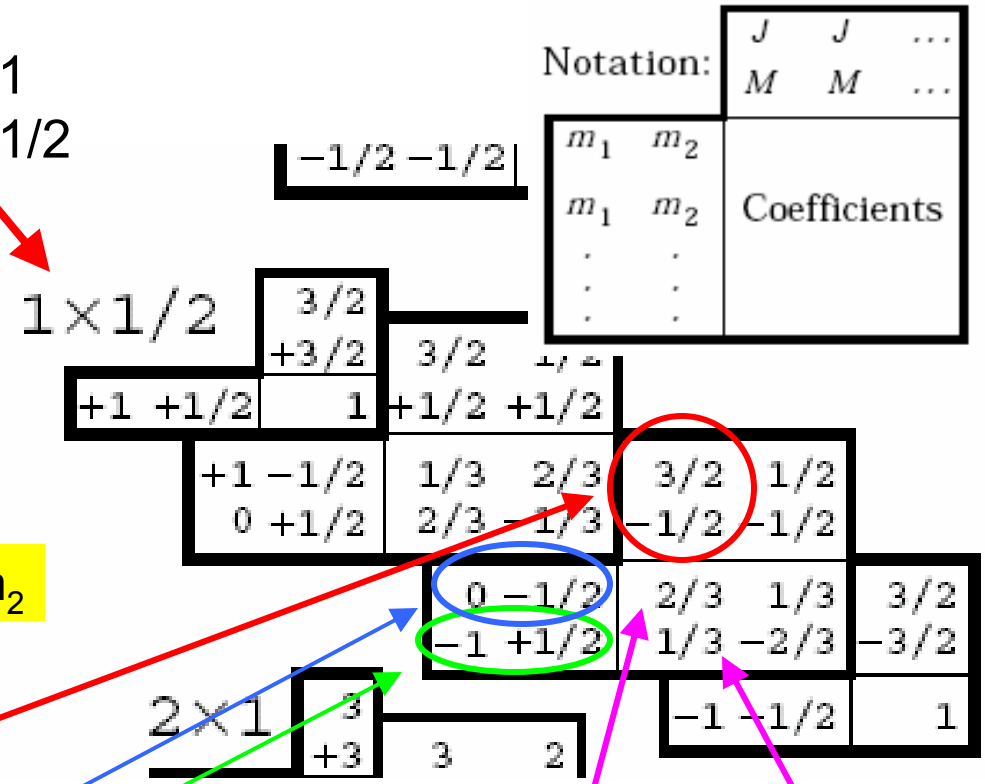
$m_1\ m_2$ states (6)
$m_1 = 1\ m_2 = 1/2,\ -1,2$
$m_1 = 0\ m_2 = 1/2,\ -1,2$
$m_1 = -1\ m_2 = 1/2,\ -1,2$

j, m states
j=1+1/2=3/2
1+1/2-1=1/2
6 states
j=3/2 m=3/2,1/2, -1/2 -3/2
j=1/2 m=1/2, -1/2

$m = m_1 + m_2$

lets make $|j=3/2\ m=-1/2\rangle$
if $m_1=0\ \ m_2=-1/2$
if $m_1=-1\ \ m_2=1/2$

$j_1 = 1$
$j_2 = 1/2$



$$\left|\frac{3}{2},-\frac{1}{2}\right\rangle = \left|0\ ,-\frac{1}{2}\right\rangle\sqrt{\frac{2}{3}} + \left|-1,\frac{1}{2}\right\rangle\sqrt{\frac{1}{3}}$$

$$\left|j=\frac{3}{2}m=-\frac{1}{2}\right\rangle =$$

$$\left|m_1=0\ \ m_2=-\frac{1}{2}\right\rangle\left\langle m_1=0\ \ m_2=-\frac{1}{2}\middle|j=\frac{3}{2}m=-\frac{1}{2}\right\rangle + \left|m_1=-1\ \ m_2=\frac{1}{2}\right\rangle\left\langle m_1=-1\ \ m_2=\frac{1}{2}\middle|j=\frac{3}{2}m=-\frac{1}{2}\right\rangle$$

$$= \left|m_1=0\ \ m_2=-\frac{1}{2}\right\rangle\sqrt{\frac{2}{3}} + \left|m_1=-1\ \ m_2=\frac{1}{2}\right\rangle\sqrt{\frac{1}{3}}$$

1x1/2 other way

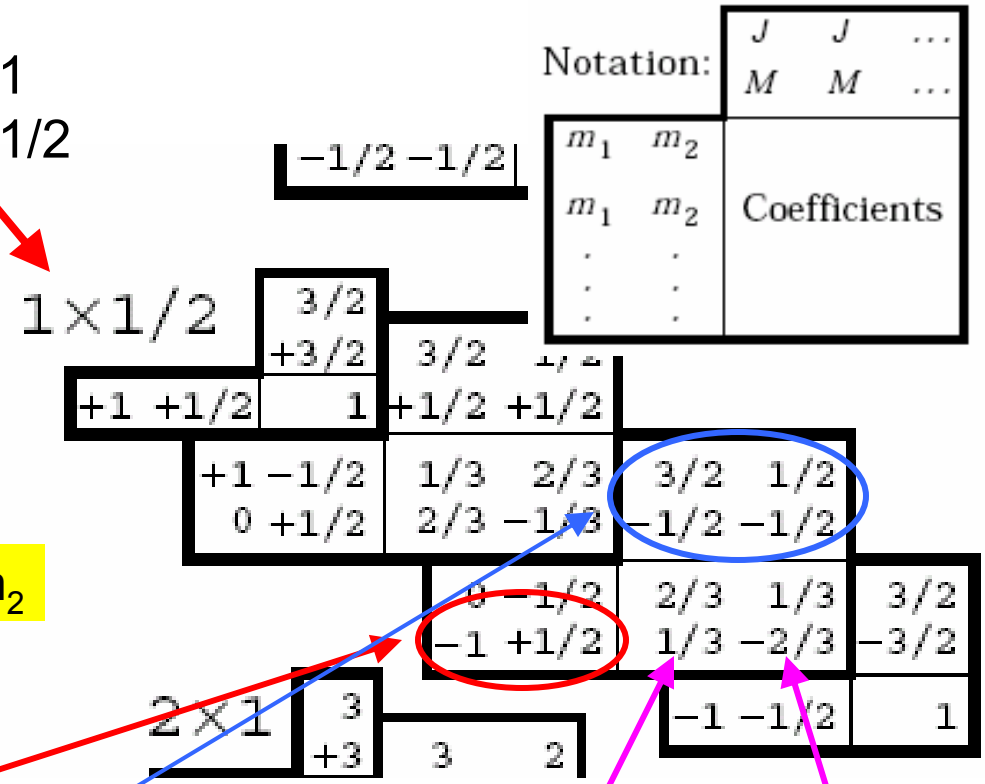
$j_1 = 1$
$j_2 = 1/2$

$m_1\ m_2$ states (6)
$m_1 = 1\ m_2 = 1/2,\ -1,2$
$m_1 = 0\ m_2 = 1/2,\ -1,2$
$m_1 = -1\ m_2 = 1/2,\ -1,2$

j, m states
j=1+1/2=3/2
1+1/2-1=1/2
6 states
j=3/2 m=3/2,1/2, -1/2 -3/2
j=1/2 m=1/2, -1/2

$m = m_1 + m_2$

lets make $|m_1=-1\ m_2=1/2\rangle$
so m=-1/2 and j=3/2 or 1/2



$$\left|-1,\frac{1}{2}\right\rangle = \left|\frac{3}{2},-\frac{1}{2}\right\rangle\sqrt{\frac{1}{3}} - \left|\frac{1}{2},-\frac{1}{2}\right\rangle\sqrt{\frac{2}{3}}$$

$$\left|m_1=-1\ \ m_1=\frac{1}{2}\right\rangle =$$

$$\left|j=\frac{3}{2}\ \ m_2=-\frac{1}{2}\right\rangle\left\langle j=\frac{3}{2}\ \ m_2=-\frac{1}{2}\middle|m_1=-1\ \ m_1=\frac{1}{2}\right\rangle + \left|j=\frac{1}{2}\ \ m_2=-\frac{1}{2}\right\rangle\left\langle j=\frac{1}{2}\ \ m_2=-\frac{1}{2}\middle|m_1=-1\ \ m_1=\frac{1}{2}\right\rangle$$

$$= \left|j=\frac{3}{2}\ \ m_2=-\frac{1}{2}\right\rangle\sqrt{\frac{1}{3}} - \left|j=\frac{1}{2}\ \ m_2=-\frac{1}{2}\right\rangle\sqrt{\frac{2}{3}}$$