Chapter 13

Mechanical waves

Traveling waves

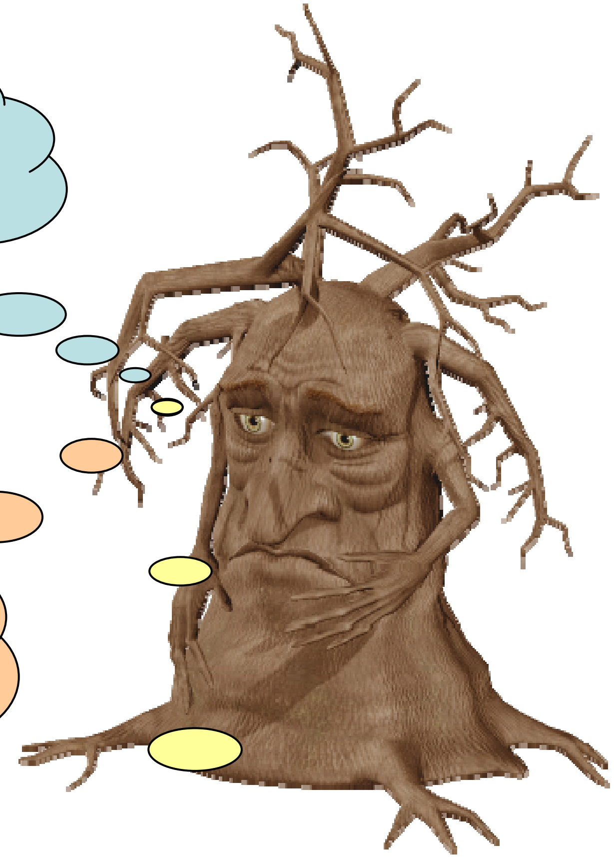
Serway/Jewett; Principles of Physics, 3/e
Figure 13.1

Transverse Waves

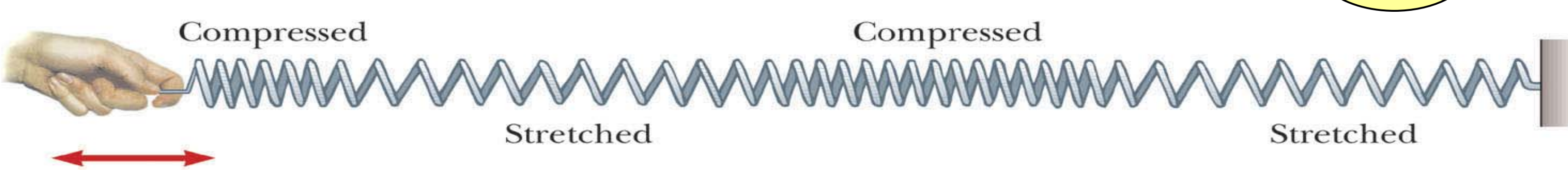
Serway/Jewett; Principles of Physics, 3/e
Figure 13.3

When you perturb a medium a wave often spreads out

If we perturb again and again we create a series of ***pulses***



One can do the same with a spring or a rope



Longitudinal Waves

• mechanica

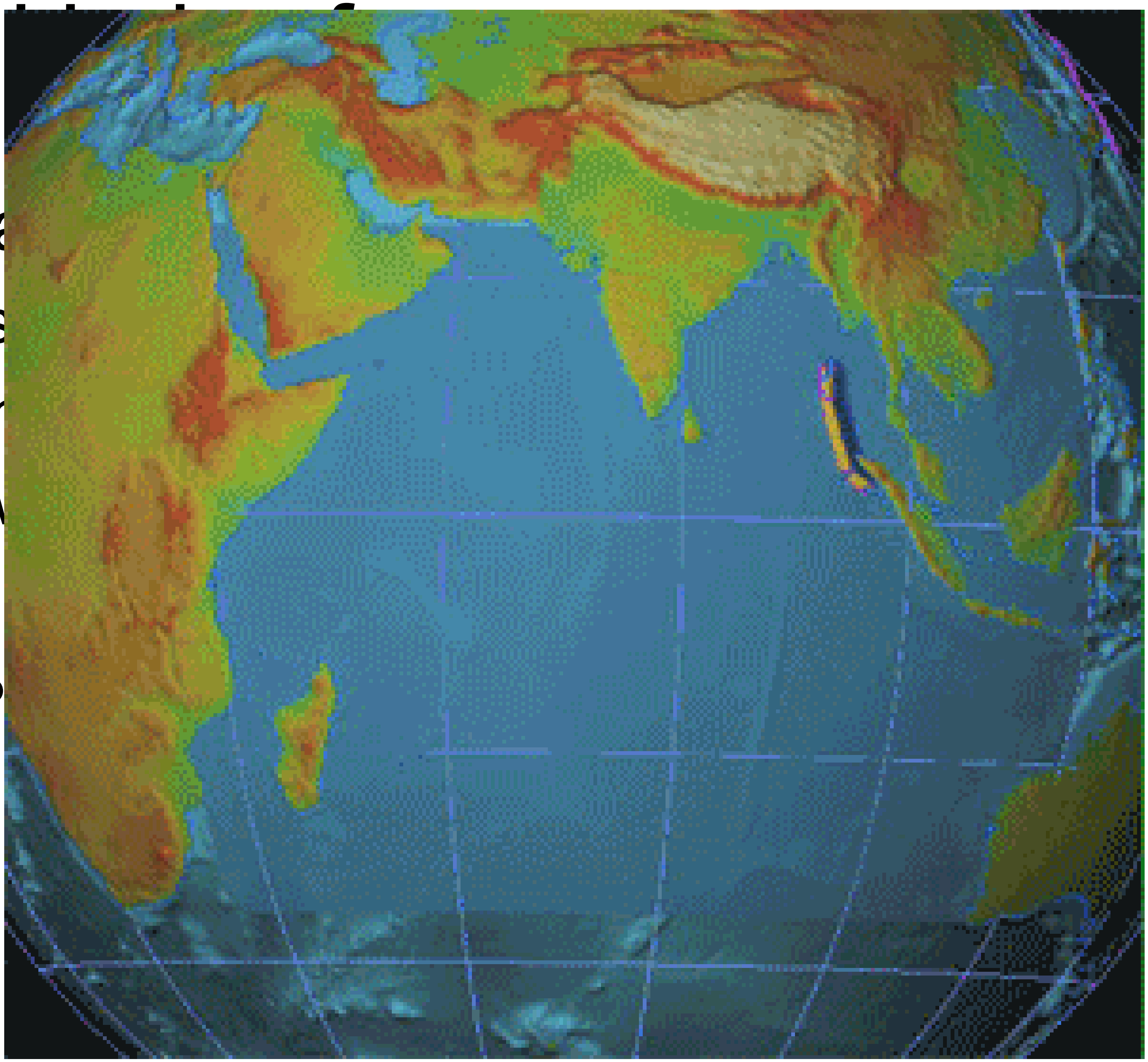
– transvers

– longitudir

– Seismic v

• Light???

• Matter???



stand up –sit down

L5 c1)

- 3. In a transverse wave on a spring, the coils of the spring vibrate in

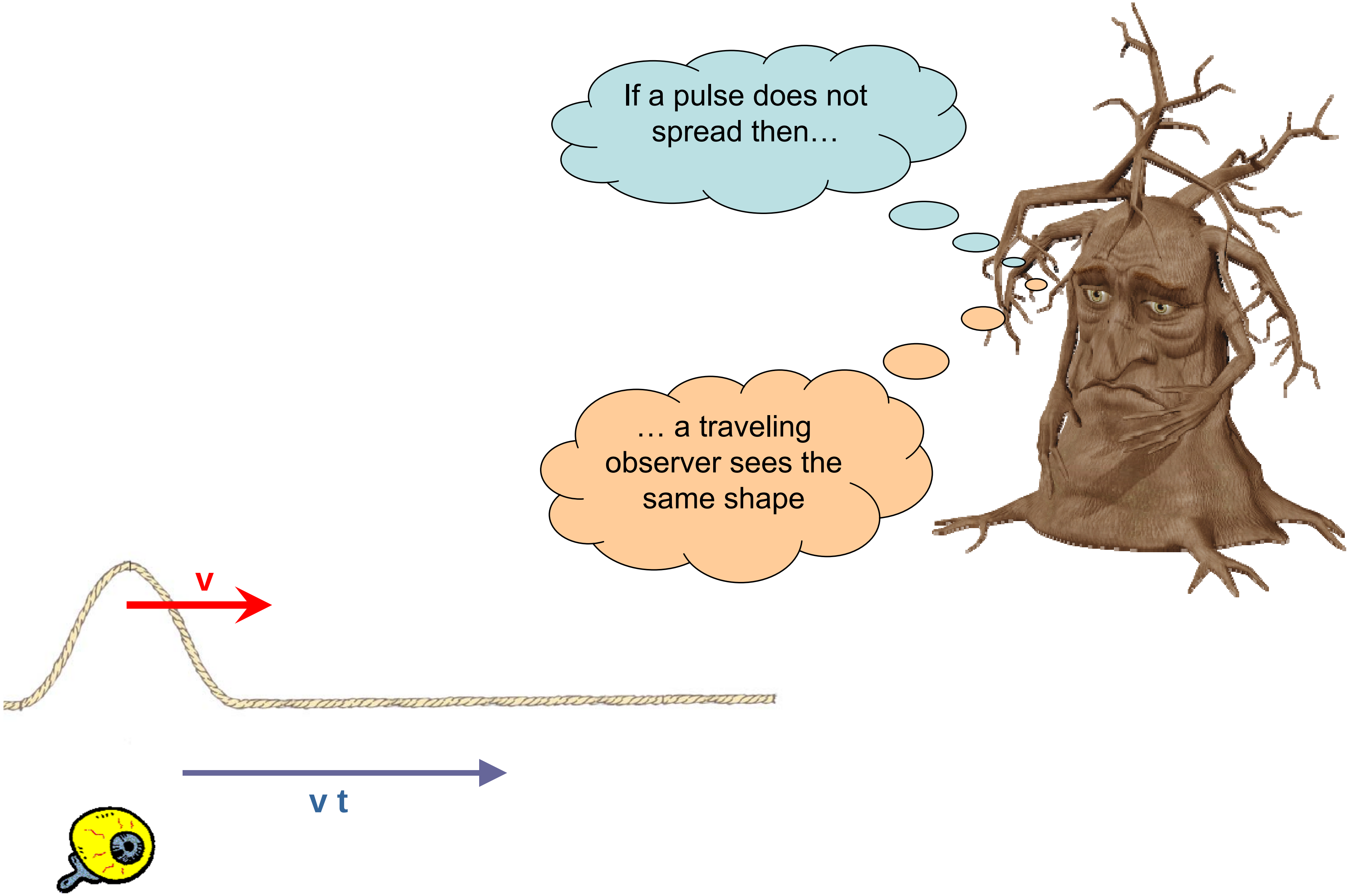
a. directions parallel to the length of the spring.

b. directions antiparallel to the length of the spring.

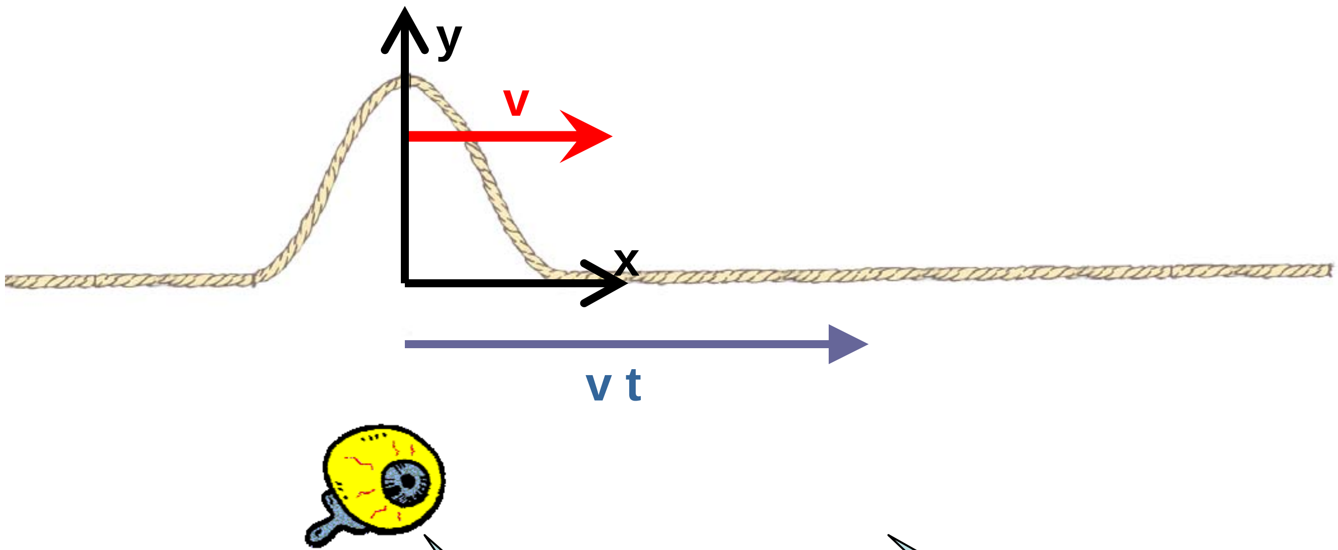
c. directions perpendicular to the length of the spring.

d. directions off axis by the helix angle of the spring.

e. directions parallel and antiparallel to the length of the spring.



If a pulse does not spread then…
… a traveling observer sees the same shape
v
v t



At *t=0* the shape is
D(x,t=0)
(D is displacement)

After the time *t* the shape is the same, but it is centered at position *v t*

$$D(x,t) = D(x - vt, 0)$$

The wave equation

A traveling wave looks like

$$D(x,t) = f(\underbrace{x - vt}_{z})$$

Suppose we change x

$$\left.\frac{dD}{dx}\right|_{t=\text{constant}} = \left.\frac{df}{dz}\frac{dz}{dx}\right|_{t=\text{constant}} = \frac{df}{dz}$$

$$\left.\frac{d^2D}{dx^2}\right|_{t=\text{constant}} = \frac{d^2 f}{dz^2}$$

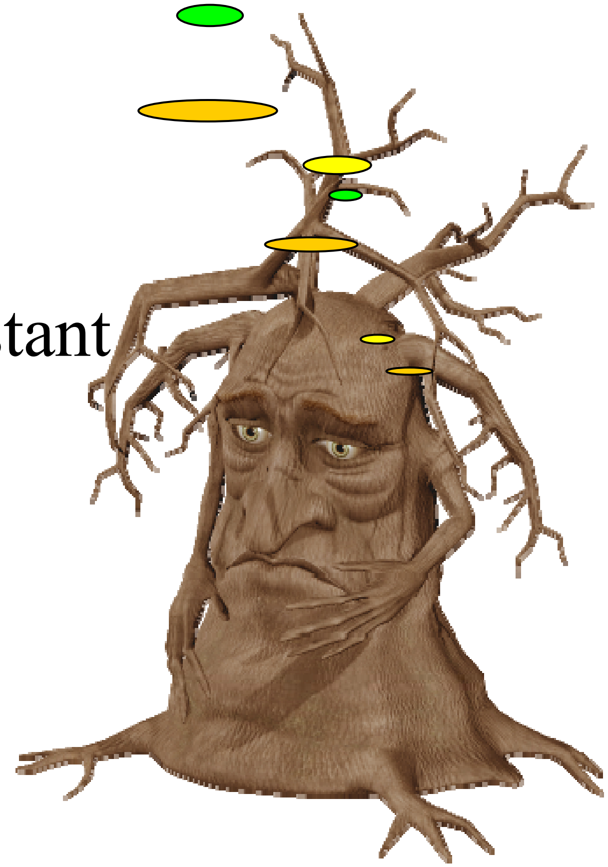
Now suppose we change t

$$\left.\frac{dD}{dt}\right|_{x=\text{constant}} = \left.\frac{df}{dz}\frac{dz}{dt}\right|_{x=\text{constant}} = -v\frac{df}{dz}$$

$$\left.\frac{d^2D}{dt^2}\right|_{x=\text{constant}} = v^2\frac{d^2 f}{dz^2}$$

$$\left.\frac{d^2D}{dt^2}\right|_{x=\text{constant}} = v^2 \left.\frac{d^2D}{dx^2}\right|_{t=\text{constant}}$$

The Wave Equation



A Solution

$$\left.\frac{d^2D}{dt^2}\right|_{x=\text{constant}} = v^2 \left.\frac{d^2D}{dx^2}\right|_{t=\text{constant}}$$

wave number

(-) moving to right
(+) moving to left

angular frequency =2πf

$$A\ solution\ is\ D(x,t) = A\sin(kx \mp \omega t)$$

Amplitude

check with (-)

$$\frac{d^2D}{dt^2} = -\omega^2 A\sin(kx-\omega t) \qquad \frac{d^2D}{dx^2} = -k^2 A\sin(kx-\omega t)$$

$$-\omega^2 A\sin(kx-\omega t) = -v^2k^2 A\sin(kx-\omega t)$$

$$\rightarrow v = \frac{\omega}{k}$$

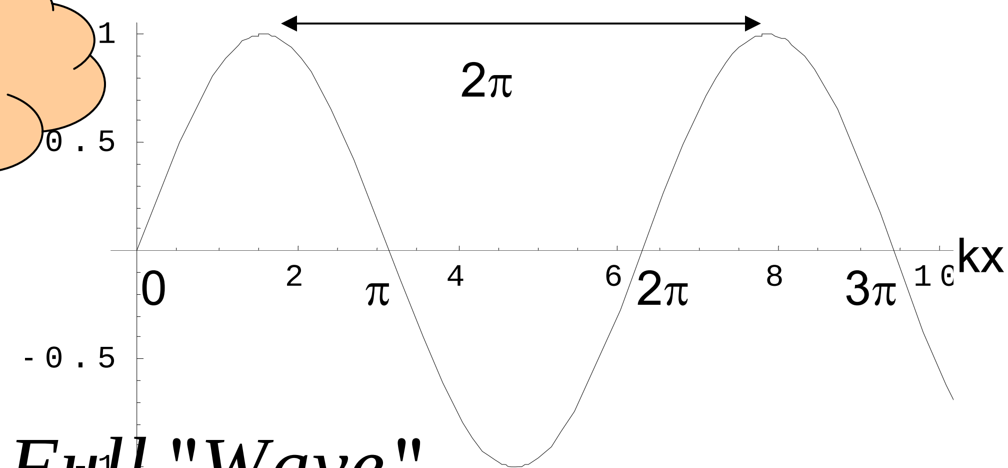
Sinusoidal traveling waves

A sinusoidal traveling wave is looks like a sine wave at *t=0*

The zeros are wherever *sin(kx)=0*

The distance between crests is *2π/k*

$$D(x,t=0) = A\sin(kx)$$



Full "Wave"

$$k\lambda = 2\pi \quad \rightarrow \quad k = \frac{2\pi}{\lambda}$$

This is nice because

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Lecture 6

$$\left.\frac{d^2 D}{dt^2}\right|_{x=\text{constant}} = v^2 \left.\frac{d^2 D}{dx^2}\right|_{t=\text{constant}}$$

wave number

(-) moving to right

(+) moving to left

angular frequency =2πf

$$A \text{ solution is } D(x,t) = A\sin(kx \mp \omega t)$$

Amplitude

$$\rightarrow v = \frac{\omega}{k} \qquad k = \frac{2\pi}{\lambda} \quad (\textit{wave number})$$

$$\omega = 2\pi f = \frac{2\pi}{T} \quad (\textit{angular frequency})$$

L6 C1)

- The wave number k of a sinusoidal wave is

a. $2\pi f$.

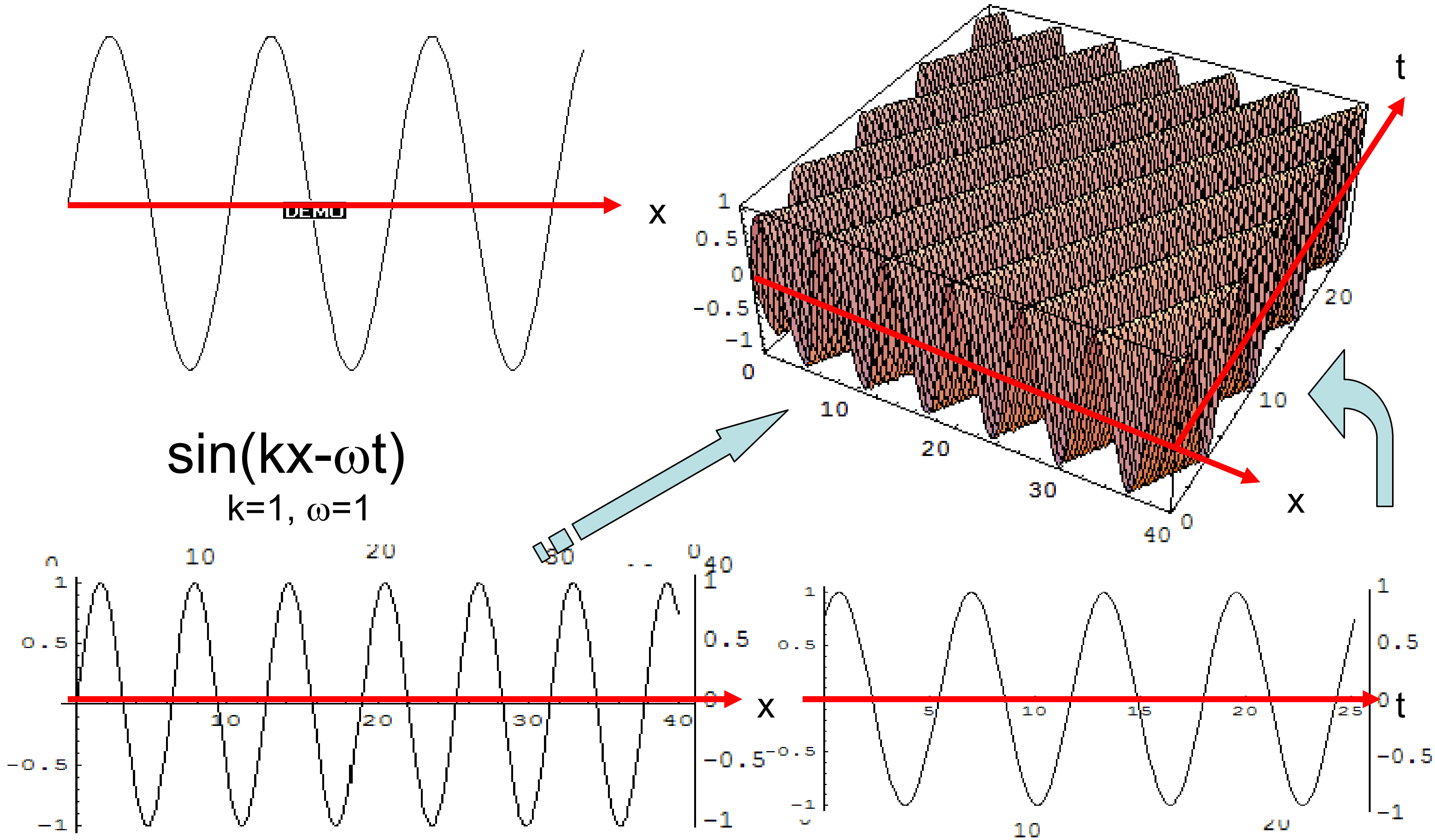
b. $\dfrac{\lambda}{T}$.

c. vT.

d. $\dfrac{2\pi}{\lambda}$.

e. $\dfrac{2\pi}{T}$.

snapshot and history graphs



snapshot graph, t=0

history graph, x=40

Since
y(x,t)=y(x-vt,0)

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}(x - vt)\right)$$
$$= A\sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi\, v}{\lambda}t\right)$$

The crests are at

$$\sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi\, v}{\lambda}t\right) = 1 \;\rightarrow$$
$$\frac{2\pi}{\lambda}x - \frac{2\pi\, v}{\lambda}t = \cdots \frac{\pi}{2} - 4\pi,\ \frac{\pi}{2} - 2\pi, \frac{\pi}{2},\ \frac{\pi}{2} + 2\pi,\ \frac{\pi}{2} + 4\pi \cdots$$

For ***fixed*** *x*, the **time** between crests is λ/v

$$T = \frac{\lambda}{v} \;\rightarrow\; v = \frac{\lambda}{T}$$

Many ways of writing the same thing…sorry

$$y(x,t) = A\sin(kx-\omega t) = A\sin\left(\frac{2\pi}{\lambda}x-\frac{2\pi}{T}t\right) = A\sin\left(\frac{2\pi}{\lambda}x-2\pi\ f\ t\right)$$

***A*: amplitude**

***k*: wave-number**

$k\ \lambda = 2\ \pi$

λ: wavelength

$\omega\ T = 2\ \pi$

$\omega / f = 2\pi$

$f\ T = 1$

ω: a
freq

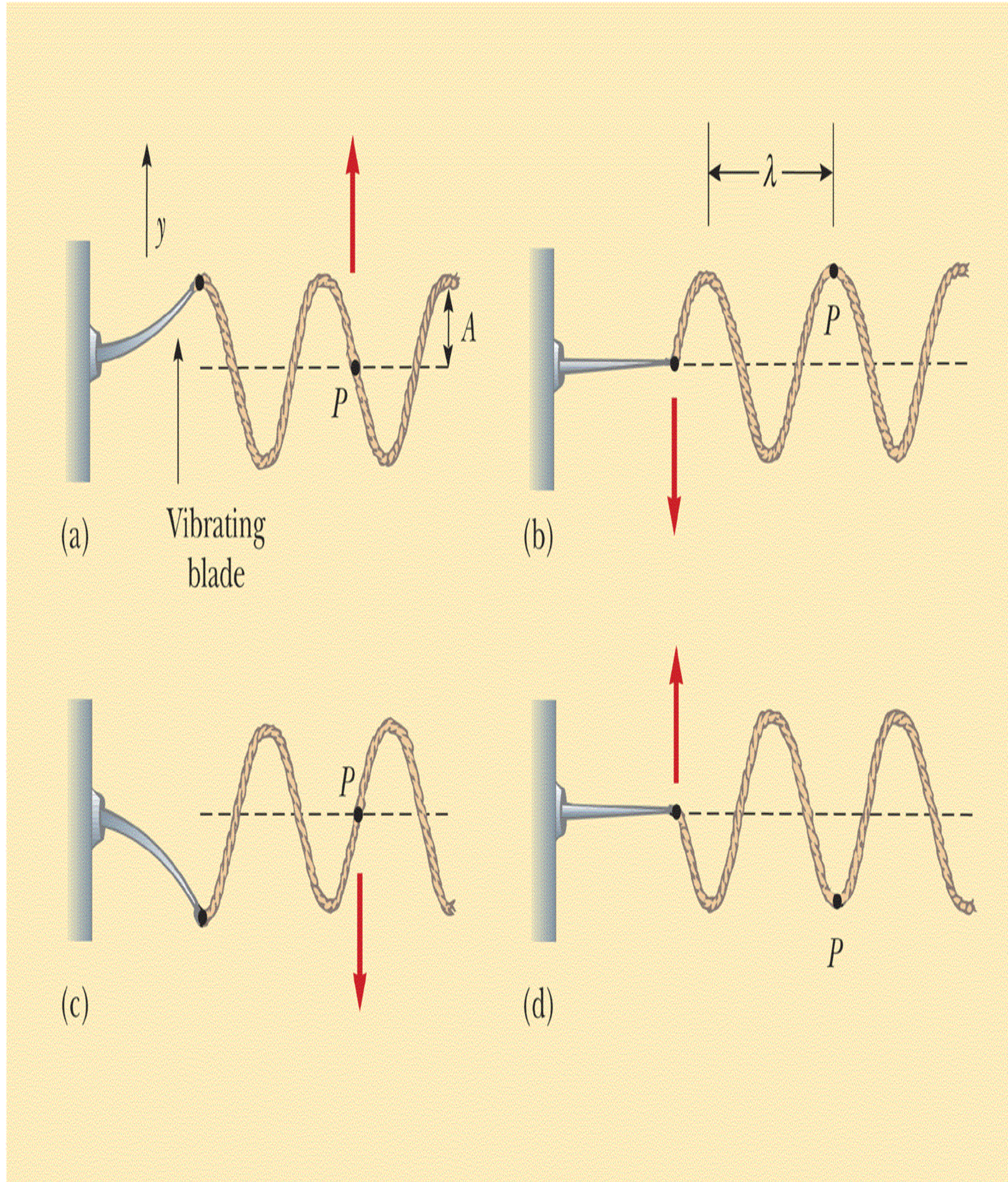
$v = \omega / k = \lambda f = \lambda / T$

***T*: period**

***f*: frequency**

Properties of traveling

A sinusc
traveling w
described
quantiti



And ***always*** any point on the rope just oscillates up and down

D(

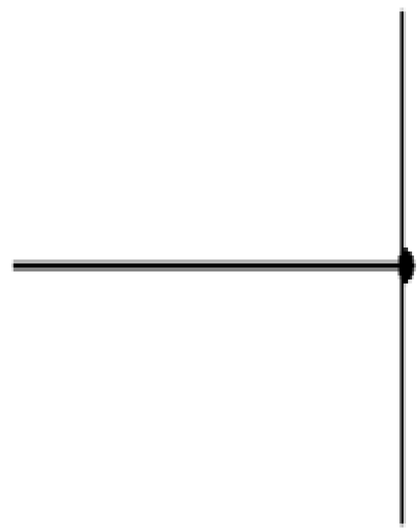
$k = \frac{\quad}{\lambda}$ ω $= \frac{2\pi}{T}$

When waves hit the wall

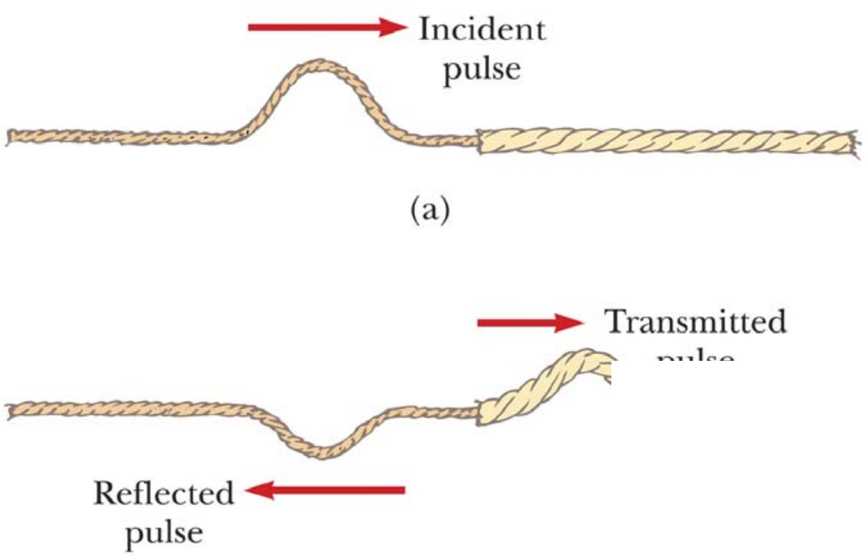
When a pulse finds an obstacle it will be reflected, at least partially

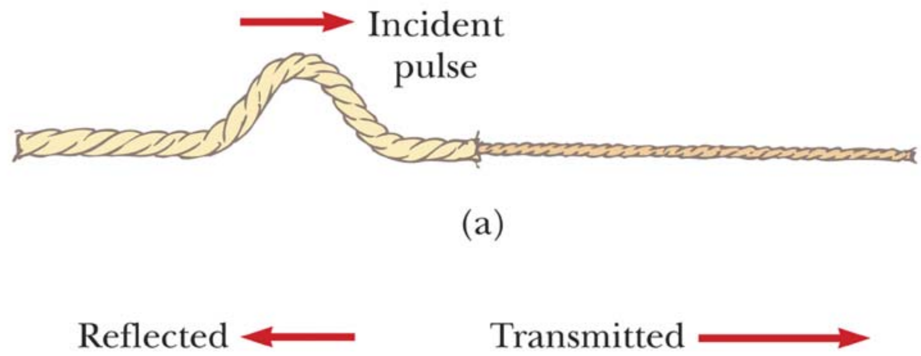
If the obstacle is very heavy a pulse on an ***attached*** string is complete reflected ***and*** inverted

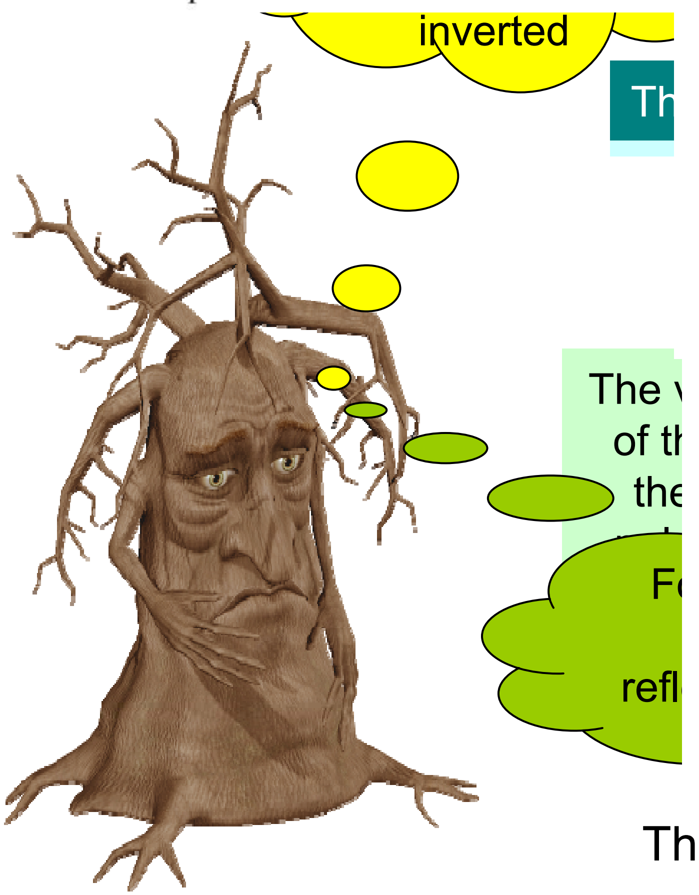
The puls

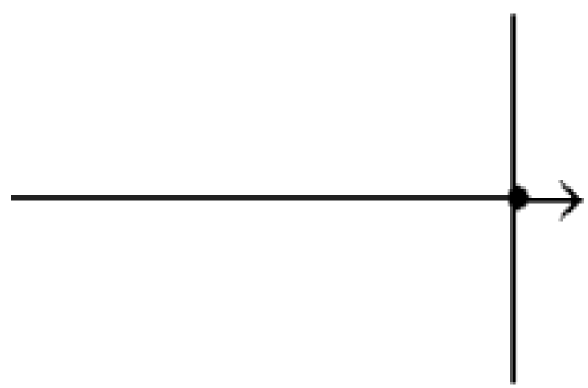


This is a phase change of $\Delta\phi=\pi$









This is a phase change of $\Delta\phi=0$

isv

- L6c2) When a traveling wave reaches a boundary with a ***medium of greater density***, the phase change in radians in the reflected wave is

a. 0.

b. $\pi/4$

c. $\pi/3$

d. $\pi/2$

e. π

Speed and tension

The **tauter** a string is the **faster** the speed of the waves

The **heavier** (per length) a string is the **slower** the speed of the waves

v* must grow with the tension *T

Do not confuse tension with period!

***v* must drop with μ=mass/length**

- T has units of force: mass •length/time^2
- μ has units of mass /length

The ***only*** combination that has units of speed is:

$$v = \sqrt{\frac{T}{\mu}}$$

These arguments determine *v* (up to a numerical constant which turns out to be 1)

L6c3) The speed of a wave on a stretched string can be calculated from

a. $\sqrt{\dfrac{m}{k}}$.

b. $\sqrt{\dfrac{T}{\mu}}$.

c. $\sqrt{\dfrac{\mu}{T}}$.

d. $\sqrt{\dfrac{T}{m}}$.

e. $\sqrt{\dfrac{g}{L}}$.

serway 13.8

- A transverse wave on a string is described by the wave function

  $D=(0.12\text{m})\sin(\frac{1}{8}\pi x+4\pi t)$

  a) determine the transverse speed and acceleration at t=0.2 s for the point on the string that is located at x=1.6m.

  b) What are the wavelength, period, and speed of propagation of this wave?

Exercise 13.8

- Transverse refers to the y direction
- Velocity is a t derivative
- Acceleration is two t derivatives
- λ and ω I can get form the formula

$$y = A\sin(kx + \omega t)\ \text{(left-moving)}$$

$$y = 0.12\sin(\pi x/8 + 4\pi t) \rightarrow k = \pi/8,\ \ \omega = 4\pi$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = 0.5\ \text{s}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/8} = 16\ \text{m}$$

$$v = \frac{\lambda}{T} = 32\ \text{m/s (moving left)}$$

$$y = 0.12\sin(\pi x/8 + 4\pi t)$$

$$v_y \equiv \frac{dy}{dt} = 0.48\pi\cos(\pi x/8 + 4\pi t)$$

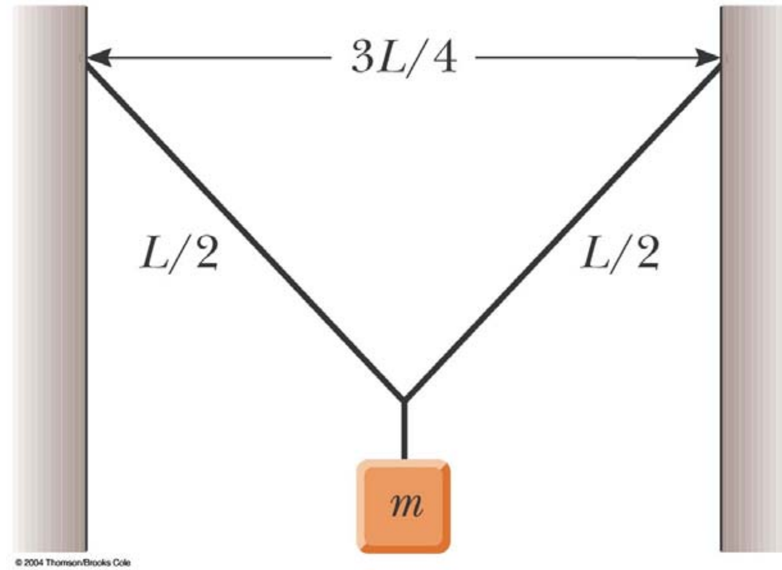
$$a_y \equiv \frac{d^2 y}{dt^2} = -1.92\pi^2 \sin(\pi x/8 + 4\pi t)$$

$$v_y(x = 1.6, t = 0.2) = 0.48\pi\cos(0.2\pi + 0.8\pi) = 1.5\ \text{m/s}$$

$$a_y(x = 1.6, t = 0.2) = -1.92\pi^2 \sin(0.2\pi + 0.8\pi) = 0$$

13.16

A light string with a mass per unit length of 8g/m has its ends tield to two walls separated by a distance equal to ¾ the length of the string.



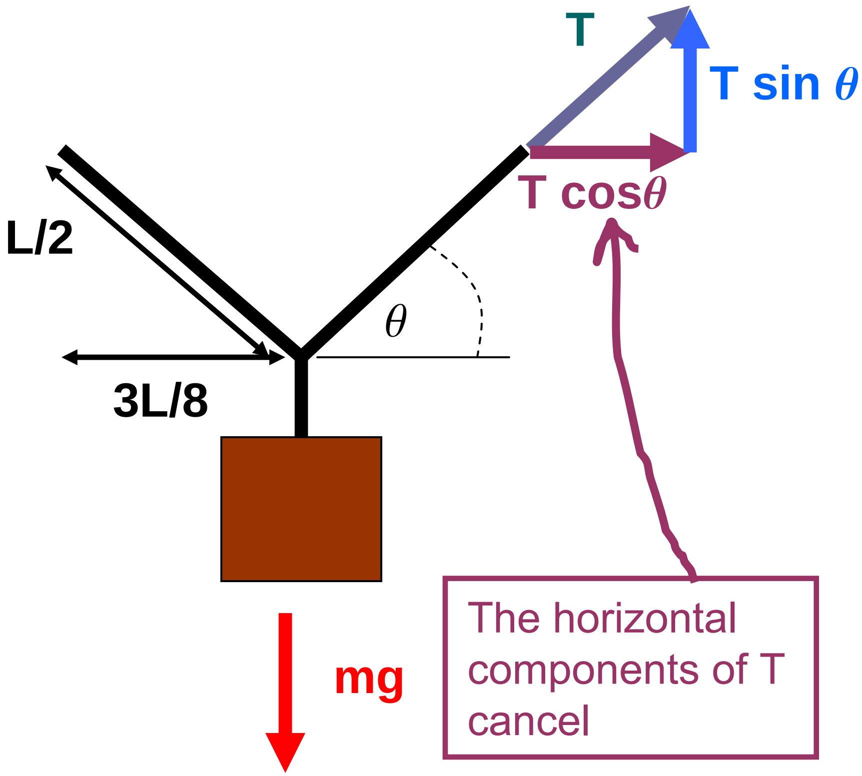
An object of mass m is suspended from the center of the string, creating tension in the string.

a) Find the expression for the transverse wave speed in the string as a function of the mass

b) How much mass should be suspended from the strong to produce a wave speed of 60 m/s?

Exercise 13.16

- The weight of m generates T
- Velocity determined by T and μ



$$\text{Equilibrium}: \quad mg = 2T\sin\theta$$

$$\Rightarrow T = \frac{mg}{2\sin\theta}$$

$$\Rightarrow v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{2\mu\sin\theta}}$$

$$\cos\theta = \frac{3L/8}{L/2} = 0.75 \Rightarrow \sin\theta = 0.66$$

$$v = \sqrt{\frac{9.8m}{2\cdot 0.008\cdot 0.66}} = 30.43\sqrt{m}$$

$$v = 60\ \text{m/s} \Rightarrow m = 3.89\text{kg}$$

L6 c4

In general the velocity of a wave is

a. f/T T=tension

b. λ/T T=period

c. $2\pi/\omega$

d. $2\pi/k$

e. ωt

L5 c5

- In a longitudinal wave on a spring, the coils of the spring vibrate in

a. directions away from the source of vibration.

b. directions toward the source of vibration.

c. directions perpendicular to the length of the spring.

d. directions off axis by the helix angle of the spring.

e. directions back and forth along the length of the spring.