Seminar class on RHIC Physics Lecture 2 – Kinematics and Cross sections

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UCR
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April 20, 2006 2PM BNL time 11AM UCR time
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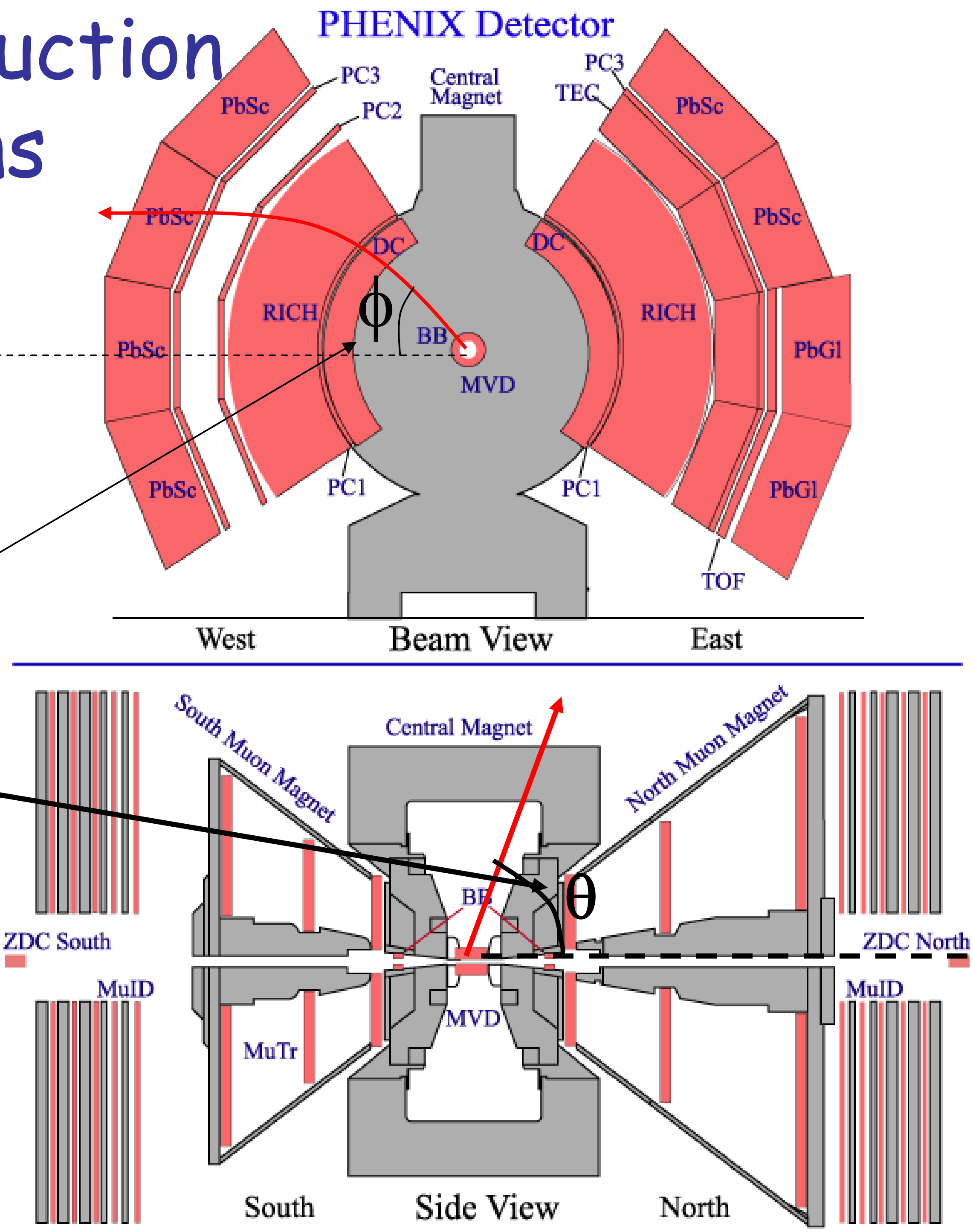
Website:https://www.phenix.bnl.gov/WWW/publish/seto/class/UCR%20RHIC%20seminar/

First: An introduction to Cross sections

- See in background abbotkinematics and yvesschutz
- What do we measure?
  - Number of particles per unit (solid) angle in our detector
  - two angles – ϕ and θ



rapidity and pseudorapidity (Abbot)

- Theta is something not nice to use since it is not boost invariant along the beam direction – define a new variable y
  - next week we will spend time on y and understand its properties

$$y = \tan^{-2}\left(\frac{p}{E}\right) = \frac{1}{2}\ln\left(\frac{p+E}{p-E}\right) = \frac{1}{2}\ln\left(\frac{1+\beta\cos\theta}{1-\beta\cos\theta}\right)$$

  - We usually cant measure y, but we can measure theta which we convert into something that is "Almost" y which we call the pseudorapidity

$$\eta = \frac{1}{2}\ln\left(\frac{1+\cos\theta}{1-\cos\theta}\right)$$

First some basic eqn for reference

$\beta = \frac{v}{c}$

$\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$E = \sqrt{p^2 + m^2 c^4} = p_\mu p^\mu$

$T = E - \mathrm{mc}^2$

cp=m$\beta\gamma$

E=mγ

we will set c=1 and measure energy in GeV, momentum in GeV/c, mass in GeV$/c^2$

distance in fm = fermi = femtometer = 10^{15} m, and time in fm$/c$

$\hbar$c=197 Mev-fm

we can derive the fact that

$\beta = \frac{p}{E}$ and $\gamma = \frac{E}{m}$

We will also need hyperbolic functions

$\sinh y = \frac{e^y - e^{-y}}{2}$ $\cosh y = \frac{e^y + e^{-y}}{2}$ $\tanh y = \frac{\sinh y}{\cosh y}$ $\cosh^2 y - \sinh^2 y = 1$ $\frac{d}{dy} \sinh y = \cosh y$

Rapidity

Now β_z (velocity) is not a nice thing to use in a cross section since it is dependent on the frame (we will assume that the beam is along the z-axis), and neither is p_z. However we will define y=rapidity which is Lorentz invariant.

$$y = \tanh^{-1}\beta = \frac{1}{2}\ln\frac{1+\beta}{1-\beta} = \tanh^{-1}\frac{p}{E} = \frac{1}{2}\ln\frac{E+p}{E-p}$$

We will almost always use the latter two forms. In fact sometimes calculators do not have inverse tanh so I use the last.

Example- the RHIC proton beam Kinetic Energy of 100 GeV/c. What is its rapidity?

$$E=T+m=100+0.938=100.938 \text{ GeV} \qquad p=\sqrt{E^2-m^2}=\sqrt{100.938^2-0.938^2}=100.934 \text{ GeV}/c = \frac{1}{2}\ln\frac{E+p}{E-p} = \frac{1}{2}\ln\frac{202}{0.004}=5.4$$

Let see the Lorentz properties of this thing. Remember how to add velocities? No? you are not alone

We will start with a man in a car moving in the z direction with a velocity β_0. He throws a ball in the z direction with velocity v'. What is the velocity in the lab?

$$v=\frac{v'+v_0}{1+\frac{v_0 v'}{c^2}} \quad \text{or} \quad \beta = \frac{\beta+\beta_0}{1+\beta_0\beta}$$ (which of couse reduces to $v = v' + v_0$ for v_0 and v' small) where all velocities are in the z direction.

BUT $\quad y = y' + y_0$

Proof

$$y = y' + y_0 \quad \rightarrow \quad \frac{1}{2}\ln\frac{1+\beta}{1-\beta} = \frac{1}{2}\ln\frac{1+\beta}{1-\beta} + \frac{1}{2}\ln\frac{1+\beta_0}{1-\beta_0} \rightarrow$$

$$\left(\frac{1+\beta}{1-\beta}\right)=\left(\frac{1+\beta}{1-\beta}\right)\left(\frac{1+\beta_0}{1-\beta_0}\right) \quad \rightarrow \text{ algebra } \rightarrow \beta = \frac{\beta+\beta_0}{1+\beta_0\beta} \quad \text{as above}$$

Hence differences in rapidity (or rapidity intervals) are Lorentz invariant - in particular dy.

Now we have been having all velocities in the z direction. Let us assume that in general velocities can be in any direction, but all boosts between frames will be in the z direction. Then we can rewrite the formulas as

$$\beta_z = \frac{p_z}{E} \qquad y = \tanh^{-1}\beta_z = \frac{1}{2}\ln\frac{1+\beta_z}{1-\beta_z} = \tanh^{-1}\frac{p_z}{E} = \frac{1}{2}\ln\frac{E+p_z}{E-p_z}$$

Note that unlike momentum, y is a scalar quantity associated with the z-axis

Now the nice thing about our agreement that all Lorentz boosts be along the z-axis is the p_x and p_y are also invariants. In particular the transverse momentum

$p_T = \sqrt{p_x^2 + p_y^2}$ is an invariant under boosts along the z-axis

Now lets define a "transverse mass" or "transverse energy" as

$$m_T^2 = m^2 + p_T^2$$

Now from above we know that $\tanh y = \frac{\sinh y}{\cosh y} = \frac{p_z}{E}$ This means that p_z must be some constant b with units of energy times sinh y and E must be the same constant b times cosh y.

$E^2 - p_z^2 = m_T^2 = b^2\cosh^2 y - b^2\sinh^2 y = b^2(\cosh^2 y - \sinh^2 y) = b^2$ so $b = m_T$ and we have

$E = m_T\cosh y$ $\qquad P_z = m_T\sinh y$ $\qquad$ another nice relation is $y = \ln\frac{E+p_z}{m_T}$

■ Pseudorapidity

Now $y=\frac{1}{2}\ln\frac{1+\beta_z}{1-\beta_z}=\frac{1}{2}\ln\frac{1+\beta\cos\theta}{1-\beta\cos\theta}$

To figure out y, we will need not only the momentum of the particle, but also the mass so we can figure out E or β. Sometime we "cheat" and assume β=1, i.e. the particle is very relativistic. Then we can use a new variable, the

pseudorapidity $\eta=\frac{1}{2}\ln\frac{1+\cos\theta}{1-\cos\theta}$

Note that for β~1 η~y

Also η can be a stand in for the angle θ. (e.g. when we speak of the angular coverage of the NCC and from η=1 to 3.

Measuring Yields

- What can the yield depend on?
  - 3-momentum
  - lets break this down into transverse momentum- P_T (or M_T,) y, and ϕ
- How do you measure how many particles were produced in a collision when your detector only measures some?
  - Make some assumptions
  - uniform in ϕ (makes sense)
  - pick a central area around y=0 (θ=0) and just say you measure there i.e. dN/dy (y=0 for a region of Δy=1)
    - There is an assumption that the yield is flat in y around y=0

How is it that we can characterize the result of a collision? The thing is for me to tell you how many particles come out at what momenta and in what direction *every time we have a collision* i.e. we might want to specify

$$\frac{dN}{d^3 p}$$

There is a sublety here in the comment "*every time we have a collision*". We wil talk about that later

The problem is that since $d^3 p$ is not an invariant since $d^3 p = dp_x\, dp_y dp_z$ and while wrt to boosts along the z-axis dp_x and dp_y are invariants, dp_z is not. So it is frame dependent. Suppose we want to compare a yield between two labs - e.g. RHIC and CERN. You measure different yields - is this

i) because there is some physics difference in what is happening at the two energies or

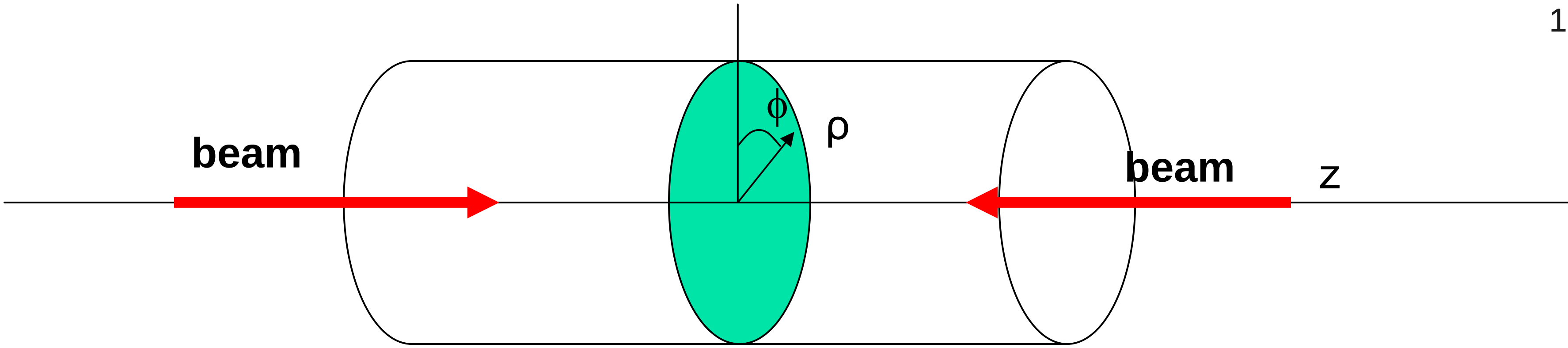
2) is it just because at RHIC we are in the center of mass frame, and at CERN we are in the target rest frame?

So what to do? First lets write $d^3 p$ in cylyndrical coordinates along the z-axis.

$d^3 p = p_T\, dp_T d\phi\, dp_z$ (remember $d^3 r = \rho d\rho d\phi dz$) where ρ is the radial direction

Now we can figure out $\frac{dp_z}{dy} = \frac{d(m_T \sinh y)}{dy} = m_T \cosh y = E$ so

$dy = \frac{dp_z}{E}$ hence $\frac{1}{E}\, d^3 p = p_T\, dp_T d\phi\, \frac{dp_z}{E} = p_T\, dp_T d\phi\, dy$ is an invariant since p_T, ϕ, and y are all invariants with respect to boosts along the z axis.



Now $m_T \frac{dm_T}{dp_T} = m_T \frac{d\sqrt{m^2+p_T{}^2}}{dp_T} = m_T \frac{1}{2\sqrt{m^2+p_T{}^2}} 2\, p_T = p_T$ so $m_T\, dm_T = p_T\, dp_T$

and $m_T\, dm_T\, d\phi\, dy = p_T\, dp_T\, d\phi\, dy = \frac{1}{E} d^3 p$ is also an invariant.

So lets look at the quantity

$E \frac{dN}{d^3 p} = \frac{dN}{m_T\, dm_T\, d\phi dy}$ - this in an invariant since dN is an inviant (its just a number) and $m_T\, dm_T\, d\phi\, dy$

So if we talk about this quantity - a person in any moving reference frame along the z-axis will measure the same thing.

In practice we integrate this thing over ϕ since we assume that the distribution is symmetric in ϕ and we plot

The invariant yield $\frac{1}{2\pi m_t} \frac{dN}{dm_T\, dy}$

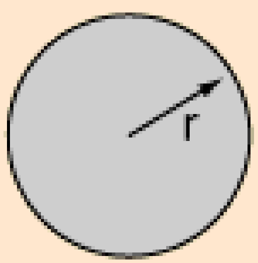
Everybody will agree to what this number is no matter what reference frame he is in.

Cross section

The concept of cross section, as its name suggests, is that of effective area for collision. The cross section of a spherical target is

$$\sigma = \pi r^2$$

The units of cross section are then area units, but for nuclear scattering the effective area is on the order of the cross sectional area of a nucleus. For a gold nucleus of mass number A=197, the radius determined from the nuclear radius relationship is about 7 fermis.



Gold nucleus
Z=79, A=197

$r = 7 \text{ fermi} = 7 \times 10^{-15} \text{ m}$

$A = \pi r^2 = 154 \text{ fermi} = 1.54 \times 10^{-28} \text{ m}^2$

$A = 1.54 \text{ barns}$

$1 \text{ barn} = 10^{-28} \text{ m}^2 = 100 \text{ fm}^2$

relating to yeild

- σ is proportional to the probability P that particles are produced. the probability
- P must be proportional to N
- Define
  - σ^b as the total cross section σ_{Tot} times P^b (the probability that given a collision the that a particle b is produced)
  - What is N?
    - It is the number of particles produced per collision. Lets assume that only 1 particle of type b is produced every 10 collisions. Then N^b=0.1
    - Ie N^b is the probability that a particle b is produced i.e. it is really P^b
      - Its funny because N^b can be > 1 if two particles can be produced per collision but it works out
  - $\sigma^b = \sigma_{Tot} N^b$ - now let relate this to the invariant yield and we often plot this in terms of the cross section rather than the yeild

$$\frac{1}{2\pi m_T}\frac{dN^b}{dm_T dy} = \frac{1}{\sigma_{Tot}}\frac{1}{2\pi m_T}\frac{d\sigma^b}{dm_T dy}$$

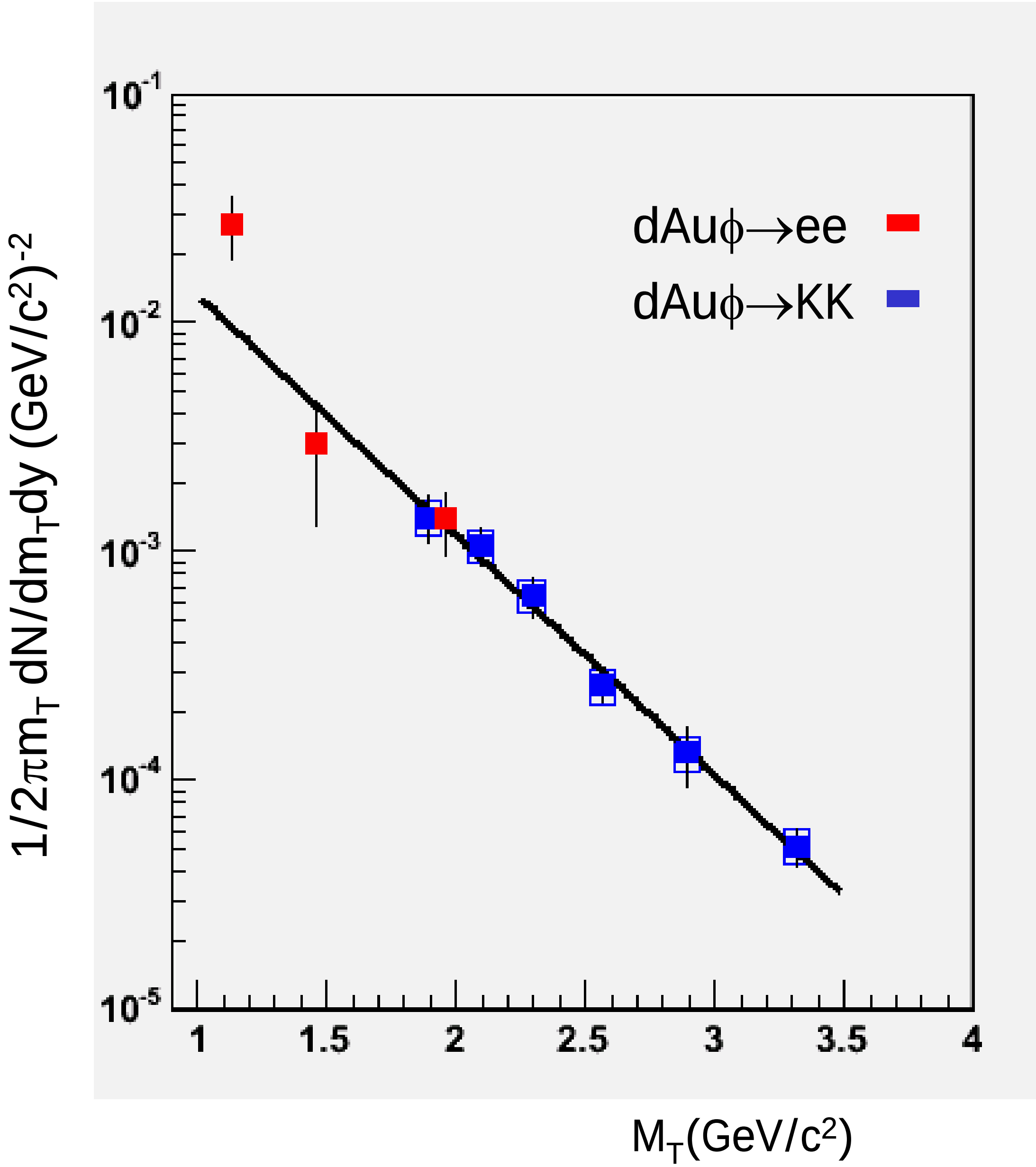
M_T=sqrt($m^2+p_T^2$)

Transverse momentum is tougher – use experience – a funny magic variable M_T
$M_T = \text{sqrt}(m^2+p_T^2)$
from experience particle distribution look like this

Note the log plot of

$$\frac{1}{2\pi m_T}\frac{dN}{dm_T dy}$$

Is a pretty good exponential
We will try to make some sense of this later but for now we just will assume its an exponential in M_T
And integrate to get dN/dy



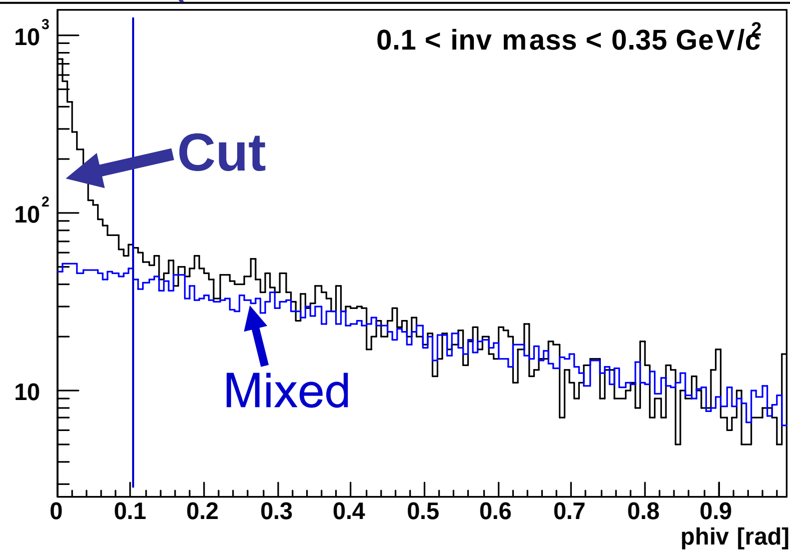
A talk from jan 2004 old analysis - here is where things were

Light vec… (φ) from … PHENIX

…Seto
…y of California, Riverside
for the PHENIX Collaboration
Quark Matter 2004
January 13, 2004

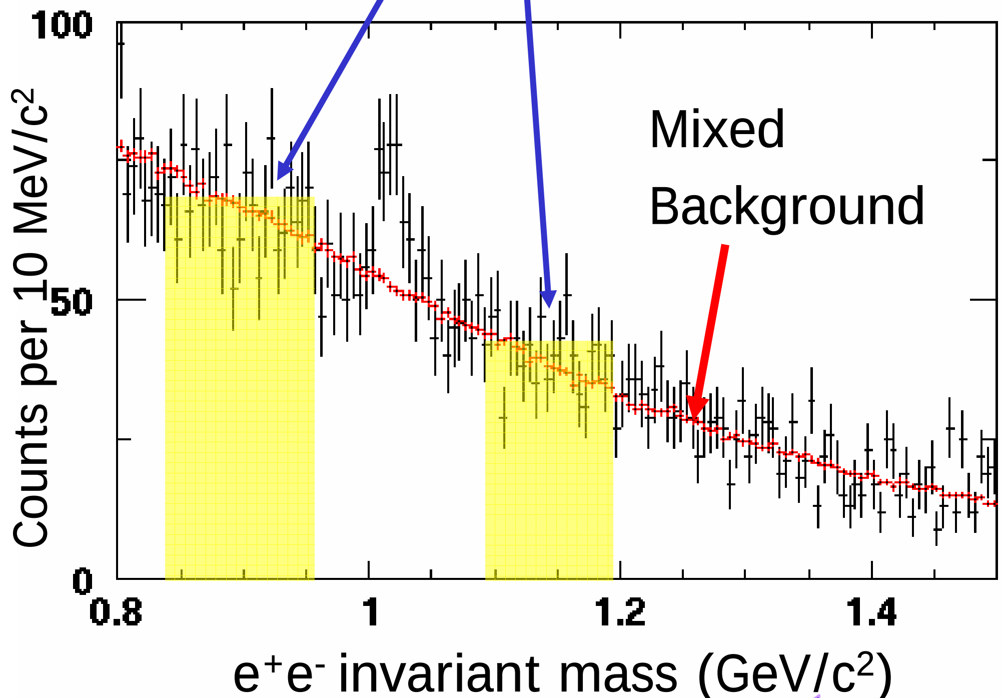
Conversion cuts, mixed background

PhiV (100<mass<400MeV)



- Rejecting conversions
  - PhiV=Angle plane of pair makes with plane normal to beam direction
    - Zero mass pairs➔PhiV~0
  - Reject conversion pairs if
    - If M_{ee}<100
    - If 100<M_{ee}<400 and phiV<100 mrad

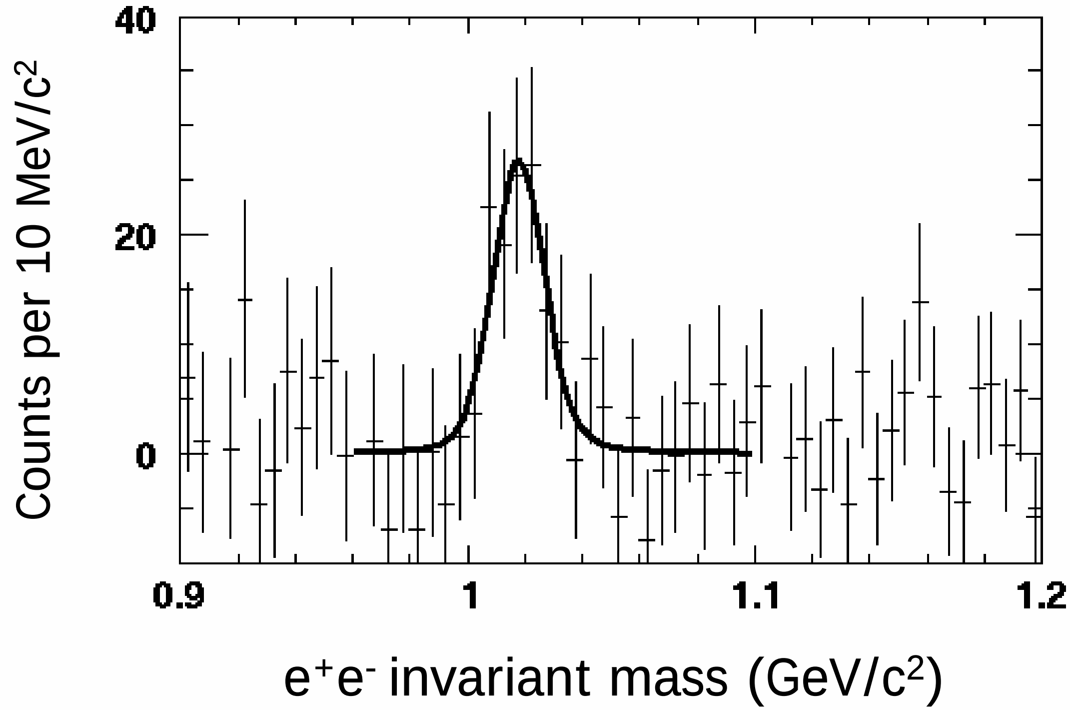
Sidebands for background normalization



old

ee Invariant Mass Spectra 200 GeV dAu- all m_T

- N_ϕ~120
- Fit is to relativistic B-W convoluted with Gaussian
  - $M=1.0177 \pm 0.0023$ GeV
  - $\Gamma=0.00446$ GeV(fixed)
  - $\sigma_{exp}=0.0081 \pm 0.0021$ GeV
  - χ^2/DOF=13.6/13
- Consistent with PDG

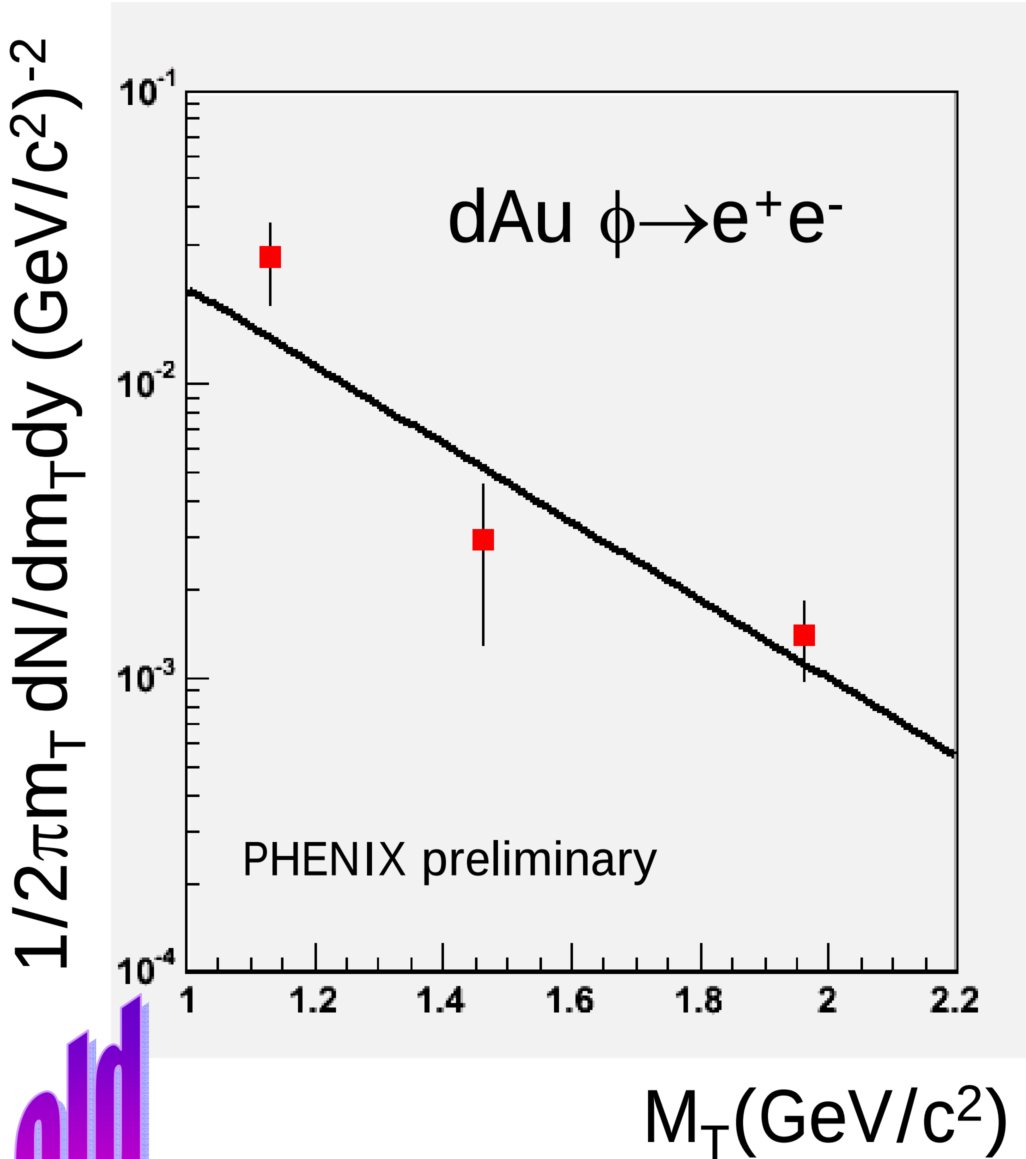


- Now
  - break into 3 m_T bins
  - count signal by summing mass bins $\pm 3\sigma$ around mass peak
  - Do corrections and

old

Poster: Electro 4
Yuji Tsuchimoto

dN/dm$_T$ and yield



old

dN/dy=.056±.015(stat) ±50%(syst)
T=326 ±94(stat) ± 53%(syst) MeV
(PHENIX preliminary)

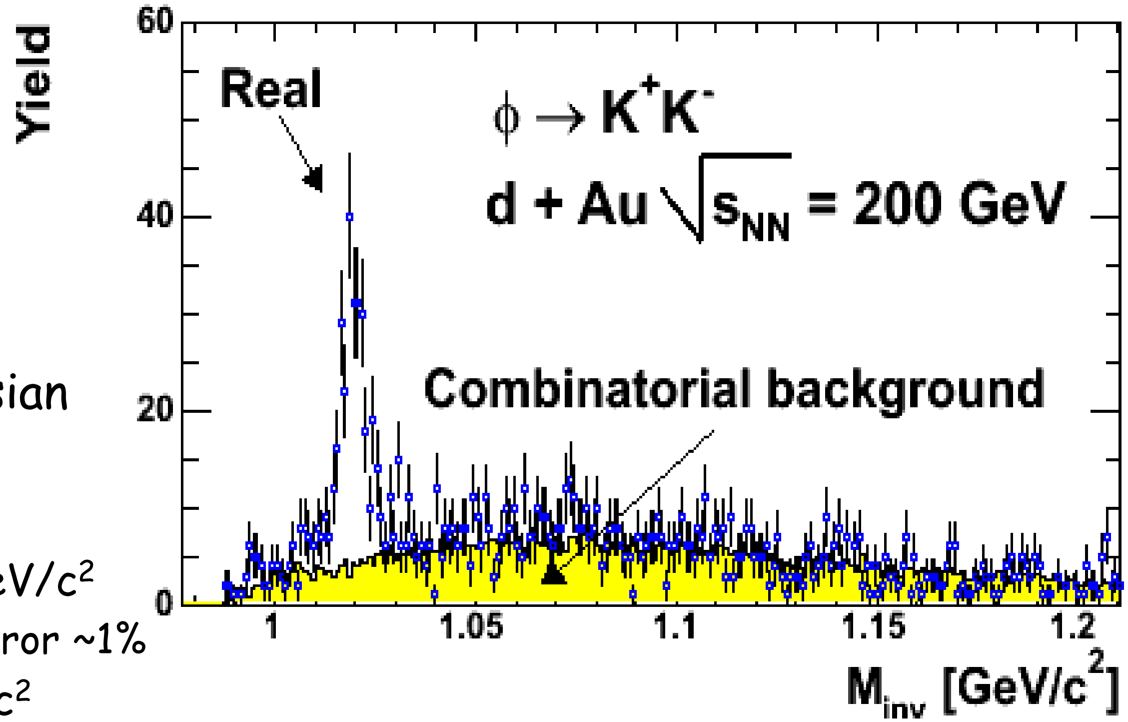
- major contributions to the systematic error
  - normalization of the background and signal extraction and the way the variations affect T and hence dN/dy
  - run-by run variation from the Electron-RICH-Trigger

200 GeV dAu – K^+K^- invariant mass

- PID in TOF only (smaller acceptance)
  - Higher pt
- Nevt = 62 M
- Min. bias

- Fit to Relativistic BW convoluted with a Gaussian
  - N = 207 ± 16
  - S/B ~ 5/1
  - m= 1.0193 ± 0.0003 GeV/c^2
    - Momentum scale error ~1%
  - Γ= 4.750 ± 0.67 MeV/c^2
  - σ=1.2 MeV (fixed)

  - PDG M=1.01946 GeV/c^2
    Γ= 4.26 MeV/c^2



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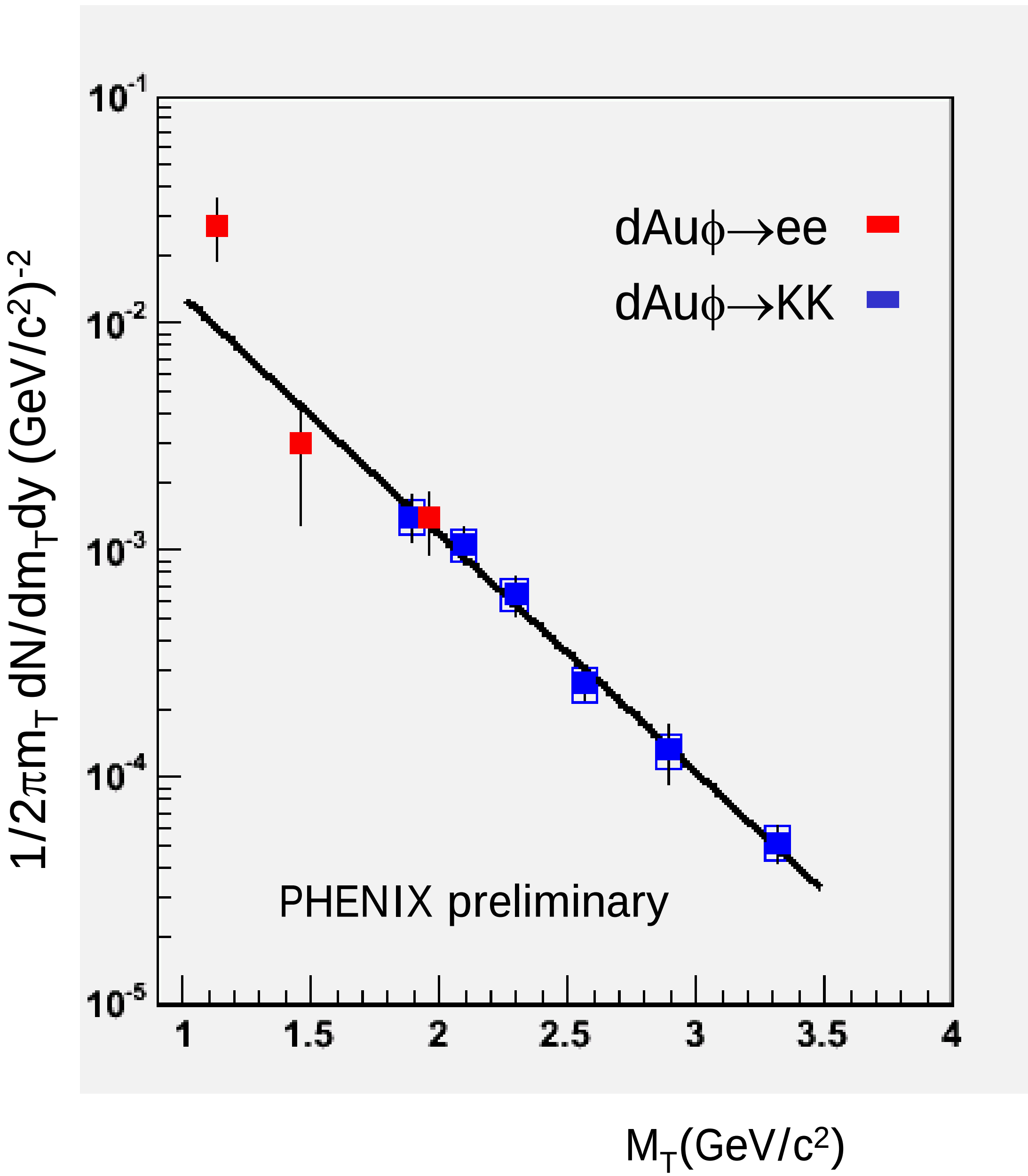
Poster: Spectra 9
Dipali Pal

Minimum-bias m_T distribution of ϕ



$\phi \rightarrow$KK min bias

dN/dy = 0.0468 +/- 0.0092(stat) (+0.0095,-0.0092) (syst.)

T (MeV) = 414 +/- 31 (stat) +/- 23 (syst)

(PHENIX preliminary)

Overall fit
dN/dy~ .0485
T~408
χ^2/DOF=6.7/7

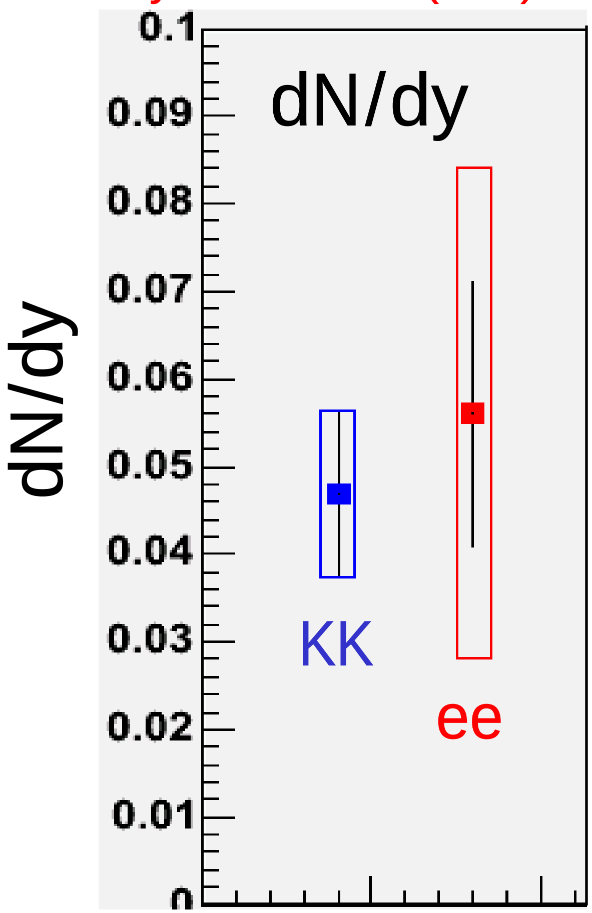
old

Compare ee with KK results

KK channel
dN/dy = 0.0468 +/- 0.0092(stat)
(+0.0095,-0.0092) (syst.)

ee channel

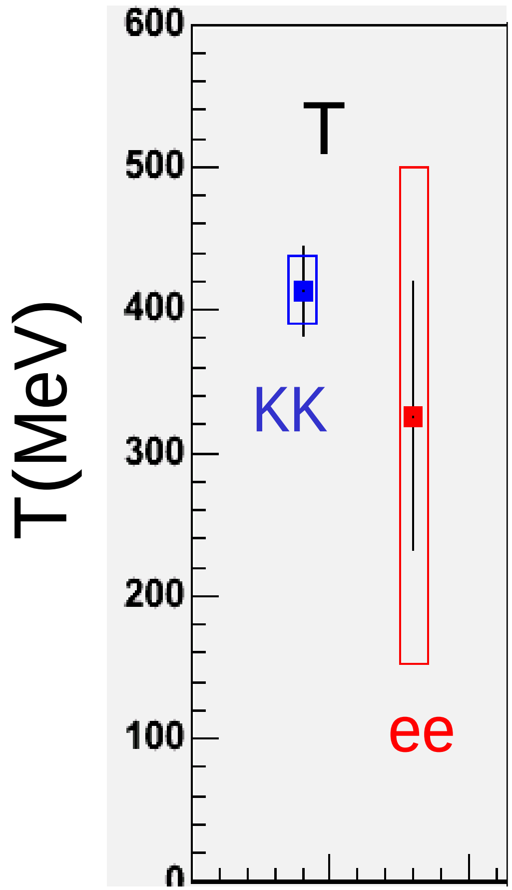
dN/dy=.056±.015(stat) ±50%(syst)



KK channel
T (MeV) = 414 +/- 31 (stat)
+/- 23 (syst)

ee channel

T=326 ±94(stat) ±53%(syst) MeV



- Yields consistent with each other
  - BR in normal ratio

PHENIX preliminary

old

- Some help from Astrid:
- I would recommend to at least take a peek at the tutorials I organized last year, where people(experts) gave excellent talks accompanied by working examples on most of the topics below with a lot of detail and insight.
- > 1) Fun4All, CabanaBoy, CVS, ... all these beginning steps/pieces of analysis?
- > ***Stefan***
- I am not sure about cabana boy. But for the Fun4all framework:
- https://www.phenix.bnl.gov/WWW/offline/wikioff/index.php?title=Framework_of_Fun4ALL
- > 2) How PHENIX data is handled. That is, what happens between the signals in the EMCal, TOF, MuId, etc. and the DST files (uDST, nDST, pDST) that we analyze?
- > *** Astrid ***
- This took many speakers but for starters:
- EMCal: https://www.phenix.bnl.gov/WWW/offline/wikioff/index.php?title=How_to_analyze_EMCal_Cluster_Data
- Muons(which all I know is what i read from this page):
- https://www.phenix.bnl.gov/WWW/offline/wikioff/index.php?title=How_to_Extract_Muons_from_nDst
- dst's and condor:
- https://www.phenix.bnl.gov/WWW/offline/wikioff/index.php?title=How_to_Run_Jobs_at_RCF
- > 3) The different triggers .... (what EXACTLY are they, what EXACTLY
- > do they do, how EXACTLY do you use them...)?
- ***Ken Barish***
- Triggers: http://www.phenix.bnl.gov/WWW/intro/dataflow/trigger.html

Are these good enough? – or would it be useful is someone went through these tutorials in class and explained them (i.e. the folks in ****)