k_T Issues

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April 5, 1999

- k_T is related to the net transverse momentum of a hard-scattering jet-pair, or a Drell-Yan pair, or a pair of high p_T photons, or the $\gamma+$ Jet pair for direct photon production.
- In leading order QCD or the Quark-Parton model, all the above pairs are coplanar with the incident beam axis: $k_T = 0$.
- However, early Drell-Yan and inclusive high p_T particle studies showed that k_T was measurable and non-zero. Systematic measurements were made at the ISR and Fermilab.
- Some experimentalists and theorists may view the issue of k_T differently—Experimentalists: multi-soft gluon, Gaussian;
Theorists: Hard-NLO gluons, power-law.
- The definitive work on k_T, actually on the p_T distribution of Drell-Yan pairs was made by G. Altarelli, R. K. Ellis and G. Martinelli in Phys. Lett. **151B**, 457 (1985), based on the ISR measurements. $\Rightarrow$ should be incorporated into event generators.
- The effect of k_T on the Gluon Spin structure function is mainly that it leads to an uncertainty in the value of Bjorken x of the inclusive direct photon measurements. This is illustrated and ways to measure k_T are discussed.

Yuji Goto's Pythia Results

On the following page I show Yuji Goto's results from Pythia relevant to knowing the structure function x value for a direct photon of transverse momentum p_T detected in PHENIX at $\sqrt{s} = 200$ GeV. The naive answer is

$$x_1 = x_2 = x_T = p_T/(\sqrt{s}/2) \tag{1}$$

The reason for the Jacobian peak at $\theta^* = 90°$ in the constiuent c.m. system for detection at $\theta = 90°$ in the $p - p$ c.m. system is that in the constituent c.m. system the momentum of the photon is $p^* = \sqrt{\hat{s}}/2$ and the transverse momentum of the photon (in both systems) is $p_T = p^* \sin\theta^*$, so that for a direct photon at p_T in PHENIX, the constituent c.m. energy $\hat{s}$ is

$$\sqrt{\hat{s}} = 2\, p_T/\sin\theta^* \qquad . \tag{2}$$

Since the cross section drops steeply as a function of $\hat{s}$ (think about the mass dependence in Drell-Yan), while the angular distribution varies only by a factor of 2-3 as $\cos\theta^*$ varies from 0.0 to 0.5 [see QCD subprocess angular distributions slide] and is actually much flatter for QCD-compton (varies a factor of 2 from 0.0 to 0.75, see below), the Jacobean peak strongly prefers the minimum $\hat{s}$ for a given p_T.

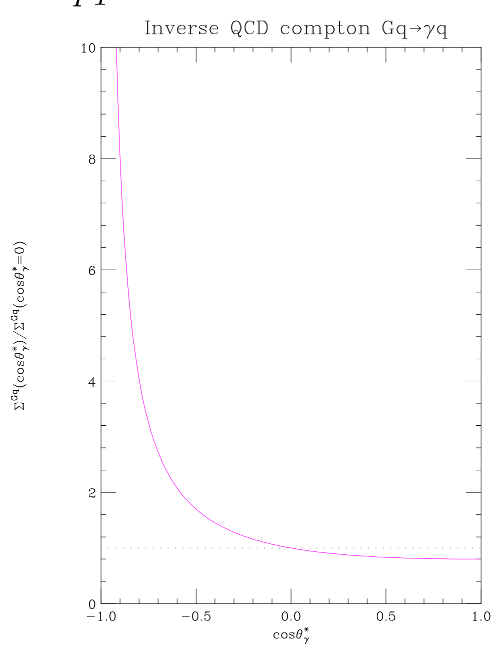


Figure 1: QCD compton subprocess angular distribution. This must be symmetrized unless you can tell a quark from a gluon

Yuji's Pythia Plots

♡ Yuji's results show 1) $x \ll x_T$, 2) there is no Jacobean peak: the condition $x_1 x_2 = \hat{s}/s = x_T^2$ is not satisfied in PYTHIA, unless $k_T = 0$, 3) about 1/2 the p_T of the photon seems to be due to motion towards the observer **!!!**

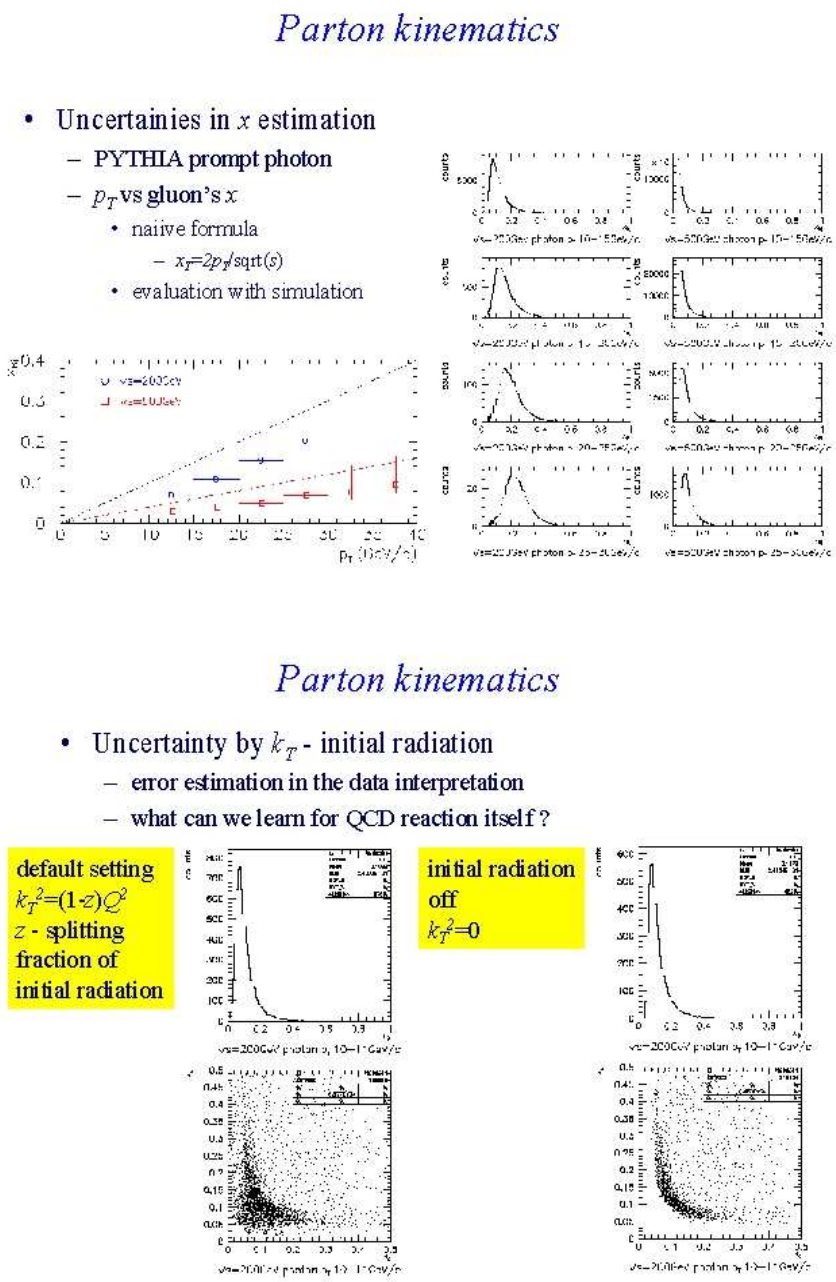


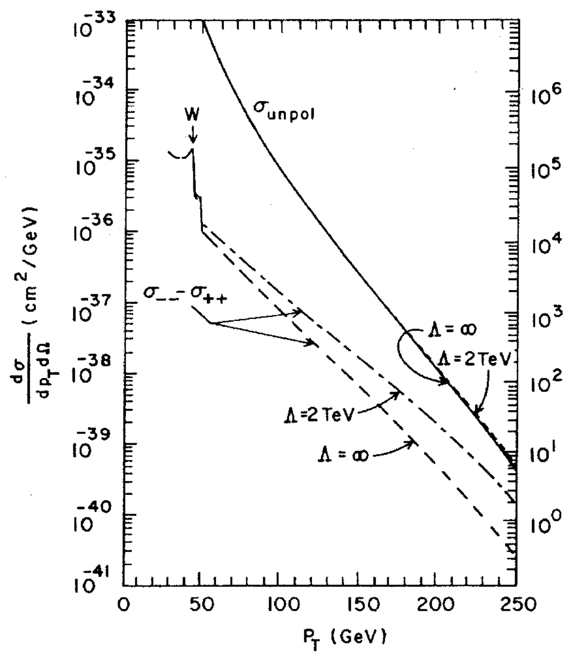
Figure 2: see `http://www.phenix.bnl.gov/WWW/publish/goto/HardWS/` for better quality

The Gluon Structure Function Appears Unexpectedly

1995-6—CDF reports possible "Substructure"
Jet Cross section deviates from 'QCD' at large p_T

Recall that the 1983—Substructure Model of Eichten, Lane, Peskin is Parity Violating

The effect looks exactly like 1983 figure of σ_{unpol} from calculations in BNL Memos, Newsletters done by Frank Paige (and me)!



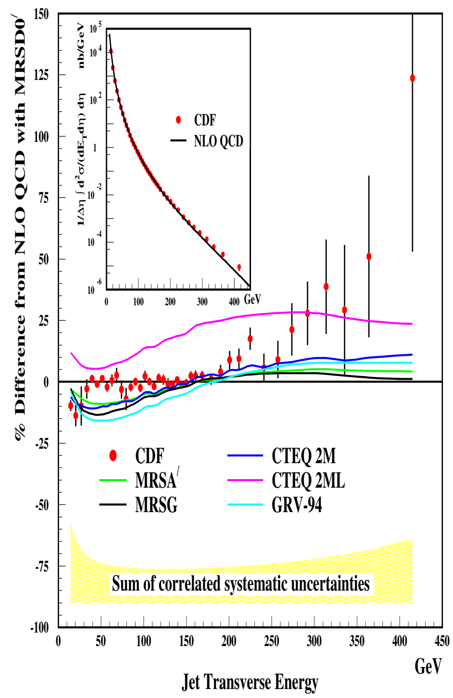


Figure 3: a) Prediction from 1983 for the effect of Quark Substructure on inclusive jet cross section at 800 GeV c.m. energy with and without Parity Violation capability. b) 1995-6 CDF Inclusive jet cross section and ratio to NLO QCD.

At PSU in 1990, I stated "Without the PVA handle, the CDF detector is limited to searching for substructure by deviations of jet production from QCD at large values of p_T... the advantage of the PVA signature is that **it is a clear indication of new physics**—qualitative not quantitative!

The CDF measurement above is a case in point: If the "% Difference from NLO QCD" were "% Parity Violation", the parity-violating signature would be unambiguous evidence of new physics. However, serious people say that CDF just used the **wrong gluon structure function!**

k_T and the Gluon Structure Function

• Is the gluon structure function (non-polarized) well known? I looked up the paper critical of CDF, and I present their 'even better than new' gluon structure function below, together with the previous 'standard' by Aurenche, et al. However, a recent paper [Ap1] [L. Apanasevich, et al, PR **D59**, 074007 (1999)] shows that including k_T vastly improves the agreement of the NLO QCD (CTEQ4M) predictions with the data. **But, clearly we must also measure the unpolarized Gluon Structure Function at RHIC!**

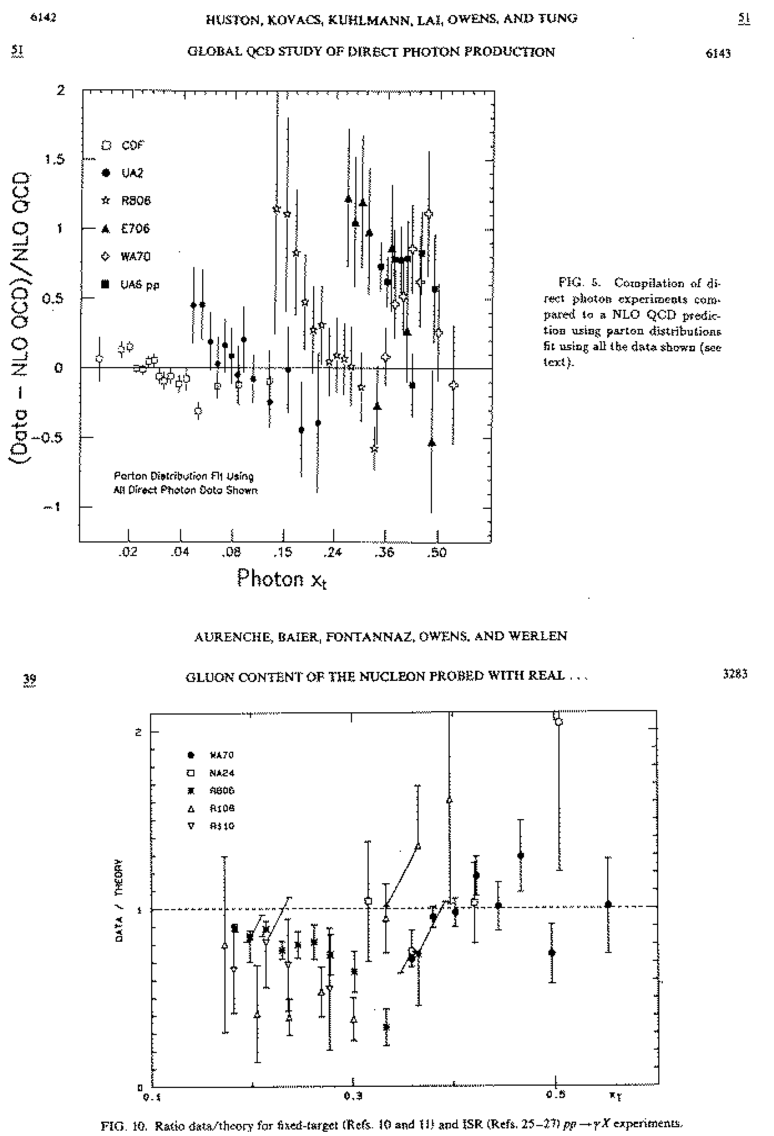


Figure 4: 2 favorite Gluon structure functions (?)

Phenomenology of k_T

• k_T denotes the magnitude of the effective transverse momentum vector $\vec{k}_T$ of each of the 2 colliding partons.

• The net p_T of the constituent c.m. system of the two colliding partons is

$$\vec{p}_T = \vec{k}_{T_1} + \vec{k}_{T_2} \tag{3}$$

Or, more accurately

$$p_T^2 = k_{T_1}^2 + k_{T_2}^2 = 2k_T^2 \tag{4}$$

• It is assumed that the two k_T act incoherently, or equivalently that one $\vec{k}_T$ acts in the scattering plane to make the energies of the two final state partons unequal, and one $\vec{k}_T$ acts perpendicular to the scattering plane to make the outgoing partons acoplanar with the direction of the colliding protons.

⋄ This simple concept leads to confusion since k_T is the magnitude of a vector, which is always positive like a radius, whereas the **components** of the vectors $\vec{p}_T$, $\vec{k}_T$, which act parallel or perpendicular to the scattering plane are more correctly written:

$$\begin{aligned} \langle p_{Tx}^2 \rangle &= \langle k_{T1x}^2 \rangle + \langle k_{T2x}^2 \rangle && (5)\\ \langle p_{Tx}^2 \rangle &= 2 \times \langle k_{Tx}^2 \rangle && (6)\\ \langle p_{Tx} \rangle &= \sqrt{2} \times \langle k_{Tx} \rangle = 0 && (7) \end{aligned}$$

where $k_T = k_{T_1} = k_{T_2}$.

⋄ The components k_{Tx} and k_{Ty} of the vector $\vec{k}_T$ are taken as gaussian, and following the notation of [Ap1]

$$\begin{aligned} k_{Tx} &= \sigma_{\text{1parton,1d}} && (8)\\ k_{Ty} &= \sigma_{\text{1parton,1d}} && (9)\\ k_T = \sqrt{k_{Tx}^2 + k_{Ty}^2} &= \sigma_{\text{1parton,2d}} && (10) \end{aligned}$$

Some Gaussian Integrals

• Recall how a the integral of a 1d Gaussian is obtained: take x to represent k_{Tx}, take y to represent k_{Ty}, and $r = \sqrt{x^2+y^2}$ to represent k_T; and in 2d, $dx\,dy = r\,dr\,d\phi$

$$\begin{aligned}
\mathrm{Prob}(x)\,dx &= \frac{1}{\sqrt{2\pi\sigma^2}}\exp -\frac{x^2}{2\sigma^2}\,dx && (11)\\
\mathrm{Prob}(y)\,dy &= \frac{1}{\sqrt{2\pi\sigma^2}}\exp -\frac{y^2}{2\sigma^2}\,dy && (12)\\
\mathrm{Prob}(x,y)\,dx\,dy &= \frac{1}{2\pi\sigma^2}\exp -\frac{x^2+y^2}{2\sigma^2}\,dx\,dy && (13)\\
&= \frac{1}{2\sigma^2}\exp -\frac{r^2}{2\sigma^2}\,dr^2 && (14)
\end{aligned}$$

◇ This is just Eq. 4 of [Ap1] and shows that the distribution of k_T^2 is exponential with mean value $\langle k_T^2\rangle = 2\sigma^2 \equiv 2\sigma_{1\mathrm{d}}^2 = \sigma_{2\mathrm{d}}^2$.

• A few more 1d and 2d Gaussian identities can be derived but are just stated here (to understand why $\sqrt{\langle k_T^2\rangle} = \langle k_T\rangle \times 2/\sqrt{\pi}$)

$$\begin{aligned}
\langle r^2\rangle &= 2\sigma_{1\mathrm{d}}^2 = \sigma_{2\mathrm{d}}^2 && (15)\\
\langle x^2\rangle = \langle y^2\rangle &= \sigma_{1\mathrm{d}}^2 = \frac{\langle r^2\rangle}{2} && (16)\\
\langle x\rangle = \langle y\rangle &= 0 && (17)\\
\langle |x|\rangle = \langle |y|\rangle &= \sqrt{\frac{2}{\pi}}\,\sigma_{1\mathrm{d}} = \sqrt{\frac{\langle r^2\rangle}{\pi}} && (18)\\
\langle r\rangle^2 &= \frac{\pi}{4}\sigma_{2\mathrm{d}}^2 = \frac{\pi}{4}\langle r^2\rangle && (19)
\end{aligned}$$

• Finally a point made very clearly by [Ap1]. k_T (really p_{Tx}) is the net momentum in the plane of the γ + Jet pair. Thus the γ is smeared in momentum **by half this value**, i.e.

$$\sigma_{\gamma,1\mathrm{d}} = k_T/2 = \sigma_{1\mathrm{parton},2\mathrm{d}}/2 \qquad (20)$$

Smearing of an Exponential by a Gaussian

• The k_T smearing of a p_T spectrum is very similar, if not identical, to the smearing effect of a gaussian momentum resolution.

◇ Suppose that x_o is a quantity to be measured, e.g. p_T, which is distributed with a steeply falling distribution, exponential for example (since the integral can be done analytically):

$$d\mathcal{P}(x_o) = f(x_o)\, dx_o = e^{-bx_o} dx_o \tag{21}$$

Further suppose that the true quantity x_o is measured with a Gaussian resolution function so that the result of the measurement is the quantity x, where

$$\mathcal{R}(x, x_o)\, dx = \mathrm{Prob}(x)|_{x_o}\, dx = \frac{1}{\sqrt{2\pi\sigma^2}} \exp - \frac{(x - x_o)^2}{2\sigma^2}\, dx \tag{22}$$

The result for the measured spectrum is simply

$$f(x)\, dx = \int\limits_{x_o=x-\infty}^{x_o=x+\infty} dx_o\; f(x_o)\; \mathrm{Prob}(x)|_{x_o}\, dx \tag{23}$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \int dx_o\; \exp -bx_o\; \exp - \frac{(x - x_o)^2}{2\sigma^2} \tag{24}$$

Complete the square:

$$f(x) = e^{-bx}\, e^{\frac{b^2\sigma^2}{2}} \times \frac{1}{\sqrt{2\pi\sigma^2}} \int\limits_{x_o=x-\infty}^{x_o=x+\infty} dx_o\; \exp - \frac{(x - b\sigma^2 - x_o)^2}{2\sigma^2} \tag{25}$$

The result, since the Gaussian is normalized over ($-\infty, +\infty$), is simply

$$f(x) = e^{\frac{b^2\sigma^2}{2}} \times e^{-bx} = e^{-b(x - b\sigma^2/2)} \tag{26}$$

• This deceptively simple formula (compare Eq. A5 in [Ap1], taking account of Eq. 20) has important implications. The measured spectrum is shifted higher than the true spectrum by $\Delta x = b\sigma^2/2$; or equivlalently, the measured spectrum at a true quantity x_o is higher than the true spectrum by a factor $\exp(b^2\sigma^2/2)$. Also, the steeper is the spectrum (larger b), the larger is the effect of the resolution smearing. This is a consequence of the fact that as the spectrum becomes steeper, it is relatively less probable to get larger values of the quantity of interest from the distribution itself, compared to the fluctuations due to resolution.

k_T is not a parameter, it can be measured

• In leading order QCD or the Quark-Parton model, the net transverse momentum $\langle p_T \rangle_{\rm pair} = \sqrt{2} \times \langle k_T \rangle$, of a hard-scattering jet-pair, or a Drell-Yan pair, or a pair of high p_T photons, or the $\gamma+$ Jet pair for direct photon production is zero. All the above pairs should be coplanar with the incident beam axis.

• However, early Drell-Yan and inclusive high p_T particle studies showed that k_T was measurable and non-zero.

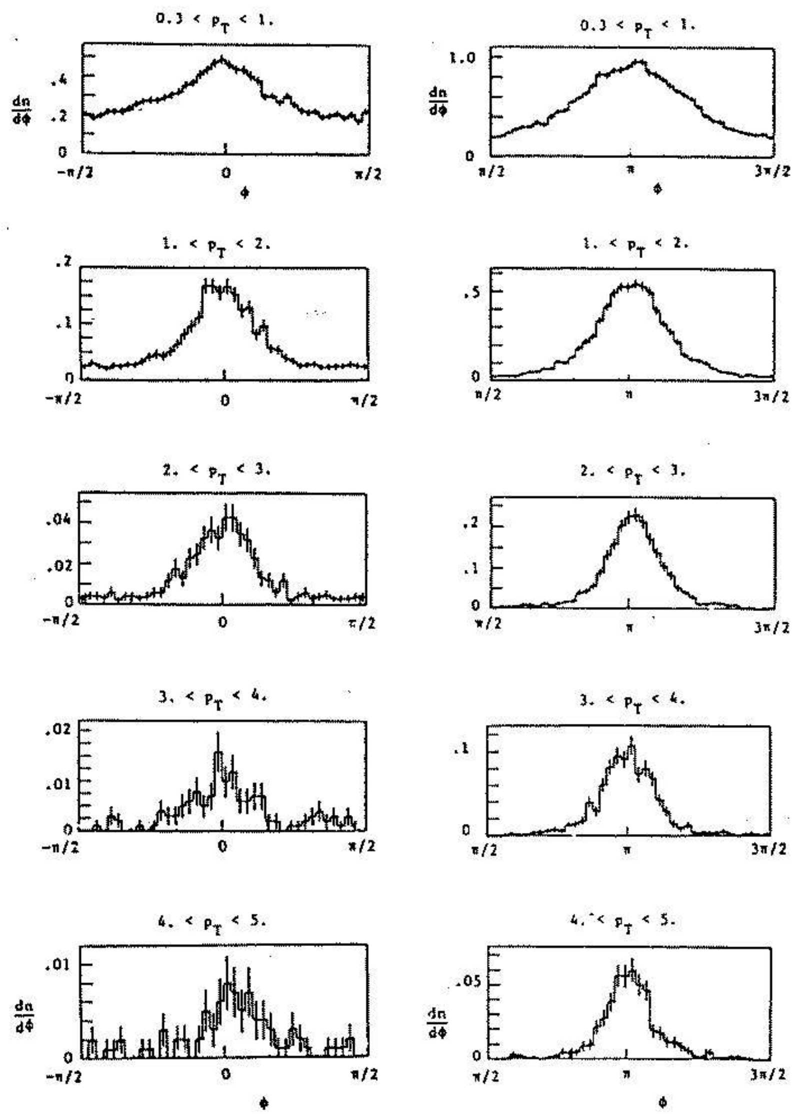
♡ The history of k_T is worth reviewing as k_T was predicted to be zero by theorists, but was discovered to be non-zero by experimentalists. The CCHK experiment [M. Della Negra, et al., Nucl. Phys. **B127**, 1 (1977)] discovered that back-to-back jets had considerable out of plane transverse momentum $p_{\rm out}$, and proposed that this was due to transverse momentum of partons inside a proton.

♡ This was elaborated by Feynman, Field and Fox, [Nucl. Phys. **B128**, 1, (1997), Phys Rev. **D18**, 3320 (1978)] who introduced the k_T phenomenology of a parton in a proton, which they discussed in terms of 'intrinsic transverse momentum' from confinement which would be constant as a function of x and Q^2, and NLO effects due to hard gluon emission which would vary with x and Q^2, but they used an constant 'effective' k_T to 'explain' the available measurements.

♡ A subsequent ISR experiment, CCOR, showed that k_T for jet-pairs was roughly the same as for Drell-Yan and increased similarly with $\sqrt{s}$ (and p_T) i.e. was not constant. See Fig. 1 in [Ap1].

• The definitive theoretical work on a calculation of k_T in QCD, actually on the p_T distribution of Drell-Yan pairs, was made by G. Altarelli, R. K. Ellis and G. Martinelli in [Phys. Lett. **151B**, 457 (1985)], inspired by the ISR measurements.

How Everything You Want To Know about JETS can be done in PHENIX with leading particles in each arm c.f. CCOR e.g. k_T from azimuthal correlations



Two particle correlation in azimuth of charged particles relative to a triggering neutral with transverse momentum $p_{Tt} \geq 7.0$ GeV/c which defines the zero of azimuth, $\phi = 0$. Charged particles with $|\eta| < 0.7$ in the same 'arm' as the trigger are on the left and opposite 'arm' to the trigger on the right. As the p_T of the observed charged particle increases, the width of the away side peak (plots on the right) narrows. This effect clearly shows that the jets **are not collinear in azimuth** (they have a net transverse momentum k_T. If there were only fragmentation transverse momentum, then $p_T \times \Delta\phi$ would remain constant which would equal to $< j_T >$, the mean transverse momentum of fragmentation. [See PL 97B (1980) 163 for details]

k_T Results from this Data

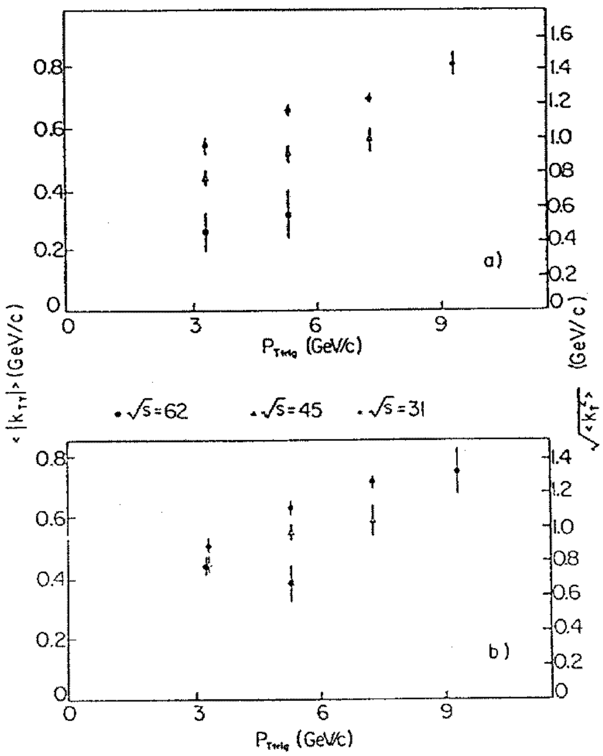
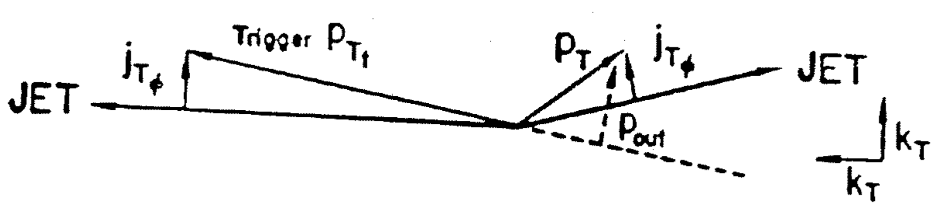
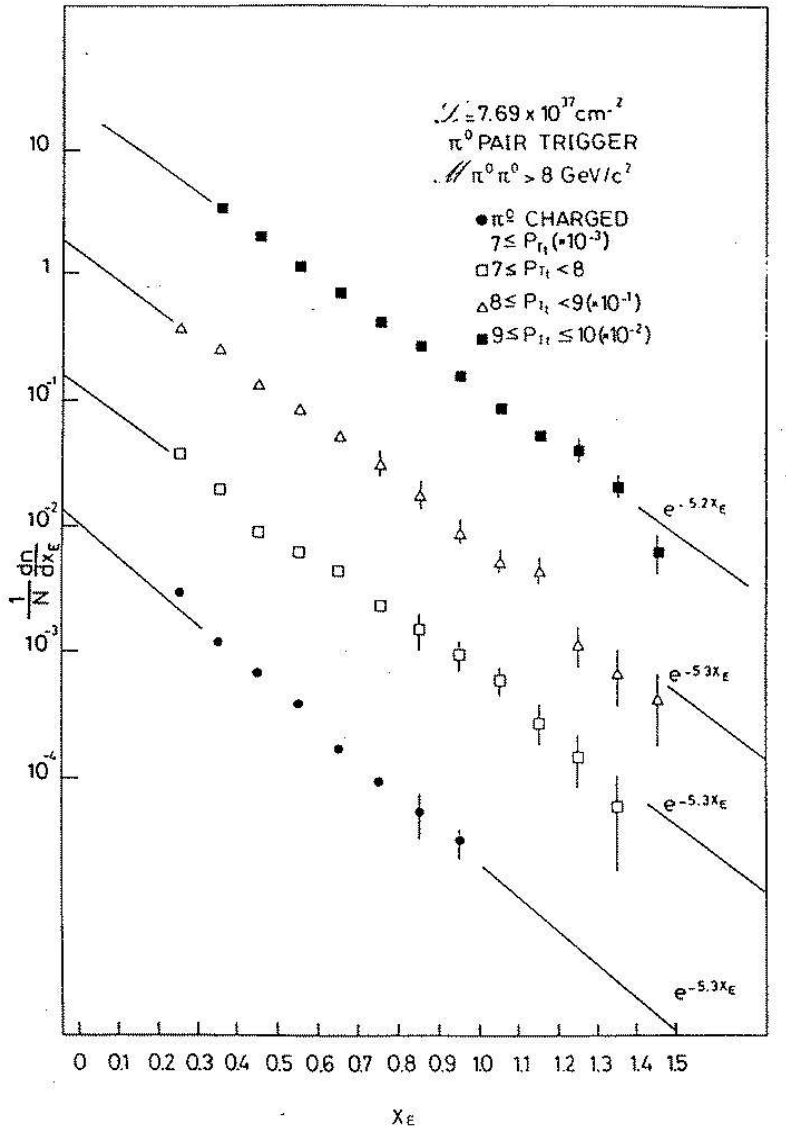


Figure 5: (a) $\langle|k_{Ty}|\rangle$ and $\sqrt{(\langle k_T^2\rangle)}$ as a function of p_{Ttrig} for three different $\sqrt{s}$ values, obtained from back-back correlations. (b) The same using events where the sum of charged paricle transverse momenta on the away side balances p_{Ttrig} [see Phys Lett **97B** (1980) 163].



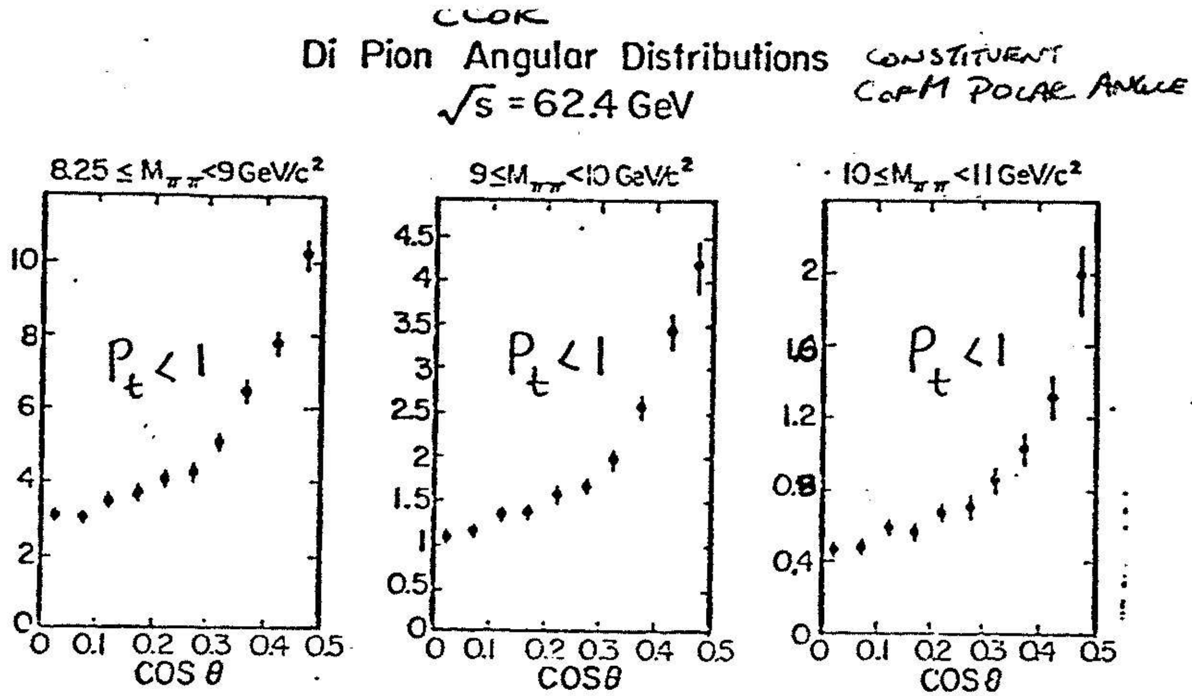
$$\langle|p_{\rm out}|\rangle^2 = \langle|j_{Ty}|\rangle^2 + x_E^2(\langle|j_{Ty}|\rangle^2 + 2\langle|k_{Ty}|\rangle^2) \tag{27}$$

Measurement of fragmentation function with the same data

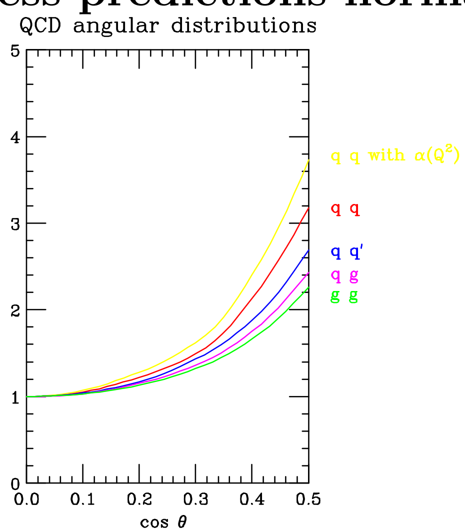


Distribution in x_E for a charged pion (or π^0) observed roughly back-to-back to a triggering π^0 of transverse momentum p_{Tt}, where both pions have $|\eta| < 0.5$ in the c.m. system. x_E is the ratio of the component of the p_T of the second pion, opposite in azimuth to the triggering pion, divided by p_{Tt}. **Exercise for students:** What do you have to know about the leading trigger particle to convert from $e^{-5.3x_E}$ to the jet fragmentation variable z [e^{-6z}].

Same Data Set— First measurement of QCD subprocess angular distributions



QCD Subprocess predictions normalized at 90°



Angular distributions of pairs of nearly back-to-back π^0 as a function of the invariant mass $M_{\pi\pi}$ of the pair. The net P_t of the pion pair is restricted as indicated on the figure and the net rapidity of the di-pion system is restricted to $|Y_{\pi\pi}| < 0.35$. The distribution plotted is the polar angular distribution of the dipion axis in the frame with zero net longitudinal momentum. The important feature of the analysis in these variables, which are more typically used for lepton pairs, is that the di-pion angular distribution at fixed mass corresponds closely to the distribution of scattered partons at fixed $\hat{s}$, thus the data and QCD prediction at the parton level can be directly compared without recourse to a Monte Carlo. [see Nucl Phys **B209** (1982) 284].

Correct k_T for $\sqrt{s} = 200$ GeV

♡ Altarelli, et al., predicted (in 1985) the value of $\langle p_T \rangle_{\rm pair}$ (which they called $\langle q_T \rangle$) for Drell-Yan pairs, which we have seen is the same as for di-hadrons. Interestingly, their predictions go to 200 GeV where the predicted $\langle q_T \rangle = \sigma_{\rm 2partons,2d} = 3.5 \pm 0.2$ GeV/c.

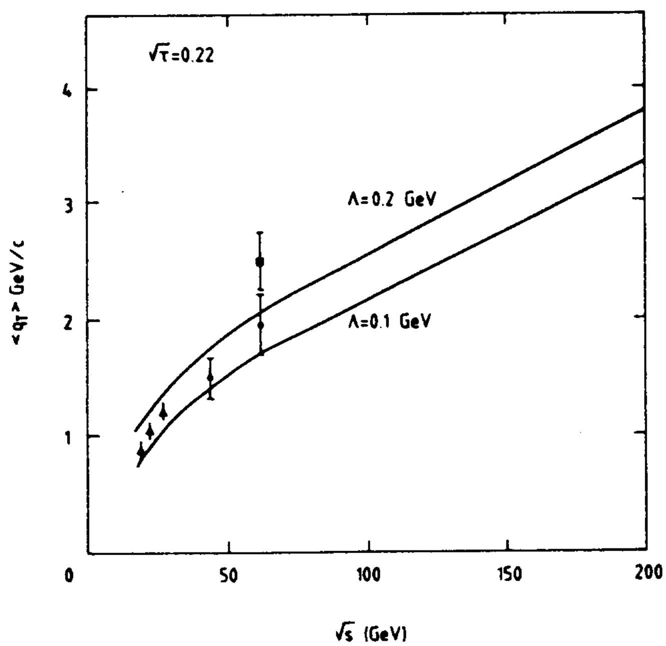


Figure 6: $\langle q_T \rangle$ vs $\sqrt{s}$ at fixed $\sqrt{\tau} = x_1 x_2 = 0.22$. Data shown are ISR and FNAL Drell-Yan. The curves are the theoretical predictions obtained using $\Lambda = 0.1 - 0.2$ GeV. No intrinsic q_T is included. At large values of $\sqrt{s}$, $\langle q_T \rangle$ increases linearly with $\sqrt{s}$. At smaller values, deviations from the linear law are visible, which are due to soft gluon and scaling violation pre-asymptotic effects

♡ Recall from above that

$$\langle k_T \rangle = \langle p_T \rangle_{\rm pair} / \sqrt{2} = 2.5 \text{ GeV/c} \tag{28}$$

$$\sqrt{\langle k_T^2 \rangle} = \langle k_T \rangle \times 2/\sqrt{\pi} = 2.82 \text{ GeV/c} \tag{29}$$

Finally, from Eq. 20 the Gaussian smearing is:

$$\sigma_{\gamma,\rm 1d} = k_T/2 = \sigma_{\rm 1parton,2d}/2 = 1.41 \text{ GeV/c.} \tag{30}$$

Conclusions

♡ There are two important things to note:

◇ $\sigma_{\gamma,1\mathrm{d}} = 1.41$ GeV/c is **much less** than exhibited by PYTHIA.

◇ The direct γ cross section from our proposal has an exponential value $b \simeq 0.40$ between 10 and 20 GeV/c, giving a shift in the p_T spectrum by

$$b\,\sigma^2_{\gamma,1\mathrm{d}}/2 = 0.4 \text{ GeV/c} \tag{31}$$

• This means that at $\sqrt{s} = 200$ GeV/c, x_T is an excellent estimator of Bjorken x to $\approx 3-4\%$ and therefore **PHTHIA's treatment of k_T is WRONG**

• **Of course, to get the Physics Correct, we should all try to measure k_T at RHIC.**

THIS IS MY ANALYSIS

IT SHOULD SERVE AS A CHALLENGE FOR SOMEONE ELSE

TO DO BETTER!