From RSC Marseille 9/96
A discussion by Bunce, Makdisi, Tannenbaum (BMT)

One interesting thing that came up in the discussion of parity violation was the use of two-spin or single spin parity-violating asymmetries. It is clear (from Soffer's article) that any small parity violating effect is a factor of 2 larger for the two-spin asymmetry in a symmetric detector. However, Gerry Bunce pointed out that for our expected measurement condition of equal distribution among the 4 possible helicities $++$, $+-$, $-+$, $--$, the two-spin asymmetry error is a factor of $\sqrt{2}$ larger than $1/\sqrt{N}$, where N is the total number of interactions, since only the $++$ and $--$ helicities are useful for a symmetric apparatus, while the single-spin error is a factor of $\sqrt{2}$ smaller, because independent single spin asymmetries can be constructed for both beams with the above helicity arrangement: $(--)+(-+)-(+-)-(++)$ and $(--)+(+-)-(++)-(-+)$. This nullifies the factor of 2 advantage, but single-spin wins because the polarization ($\sim 70\%$) enters only once in the measurement. Furthermore an additional systematic check is provided in that both asymmetries should be equal. Thus, I guess that the 2-spin parity violating asymmetry, which Frank Paige and I introduced in 1983, can be now be ignored as we approach the real world of actual measurements.

Based on present Meeting, we can now correct the above summary

- According to Basim Kamal, A_{LL}^{PV} and its error only depend on the single power of the polarization, instead of $P_1 P_2$ which we had been using. His arguments are persuasive. Thus the single-spin doesn't obviously win since the polarization enters once in both cases.
- From the discussion, we have understood that while the two individual single-spin asymmetries do indeed have errors $1/\sqrt{N}$, they are not statistically independent since they are composed of different combinations of the same quantities. Thus, an additional improvement by a factor of $1/\sqrt{2}$ is not in general picked up in the average. The correct statistical analysis is included below.

The single-spin and two-spin asymmetries are both useful for Parity Violation Measurements

Introduction—Parity Violation Estimates from Substructure Model of Eichten, Lane, Peskin Calculations in BNL Memos, Newsletters done with/by Frank Paige, so they are correct.

From Snowmass '82—the Substructure is pure $(--)$

$$\sigma = \sigma_{QCD} + \sigma_\Lambda = \sigma_{QCD} + \frac{1}{4}(\sigma_\Lambda^{--} + \sigma_\Lambda^{++} + \sigma_\Lambda^{-+} + \sigma_\Lambda^{+-}) = \sigma_{QCD} + \frac{1}{4}\sigma_\Lambda^{--}$$

$$A_{LL}^{PV} = \frac{\sigma^{--} - \sigma^{++}}{\sigma^{--} + \sigma^{++}} \cong \frac{\sigma_\Lambda^{--}}{2\sigma_{QCD}} = \frac{2\sigma_\Lambda}{\sigma_{QCD}} \equiv \epsilon$$

A Brief History of A_{LL}^{PV}

- Page, Trueman, Tudron [PRD 19 (1979)], discussing Parity Violation in $W^\pm$ production at hadron colliders, first use $\sigma^{--} - \sigma^{++}$ and compare to σ_{unpol}.
- K. Hidaka [NPB192 (1981)] introduces A_{LL}^{PV} but he calls it $-A_{3LL}$.
- F. Paige (and MJT) in BNL33119 (1983) use $\sigma^{--} - \sigma^{++}$ for quark substructure model and compare to σ_{unpol}. This plot (see Fig. 1a) appears in CBA Newsletter 4(3/83).
- Then, MJT and Ron Longacre in BNL32888(1983) and MJT in OG769 (1983) mention A_{LL}^{PV} calling it PVA, but use $\epsilon = (\sigma^{--} - \sigma^{++})/2\sigma_{unpol}$ to calculate the plot which eventually appears in PSU p206 and in the original RSC letter of intent April 1991. In a footnote in BNL32888, MJT and RL noted *"Since the parity violation (ϵ) is actually only a single beam asymmetry, the effect is proportional to ths sum of the beam polarizations rather than the product"*.
- ISABELLE is definitively cancelled in August 1983.

- Larry Trueman presents the 1983 BNL work at the 1984 Spin Conference in Marseille and Fig. 1a) appears in the proceedings (e.g. see R. Ruckl, J. de Physique **46** (1985) C2-55, T. L. Trueman, *ibid.* C2-721).
- ...
- Polarized Collider Workshop at Penn State Univ. (PSU) 11/90.
- Bourrely, Guillet and Soffer NPB361 (1991) introduce the notation A_{LL}^{PV}, but calculate A_L. From Guillet at PSU and Bourrely and Soffer PLB314(1993) and NPB423(1994) it is clear that they have the correct A_{LL}^{PV} as in Eq. 14 (ahead).
- The Great Particle World paper (1992) uses the correct notation.
- The PHENIX part of the RSC proposal (R5) of September 1992 changes notation from σ^{--}... to $N(--)$... and explicitly defines A_{LL}^{PV} with $1/P^2$. The Guilty! But the draft of 7/21/92 didn't have this. Of course, the good news is that we calculated the errors CONSERVATIVELY!
- N.B. Gerry Bunce was not part of PHENIX at that time.

My plan in PHENIX
is to measure the Parity Violation Asymmetry
For every case in which the standard A_{LL} is measured

But first One page of P.R.

1995-6—CDF reports possible "Substructure"
Jet Cross section deviates from 'QCD' at large p_T
The effect looks exactly like 1983 figure of σ_{unpol}!

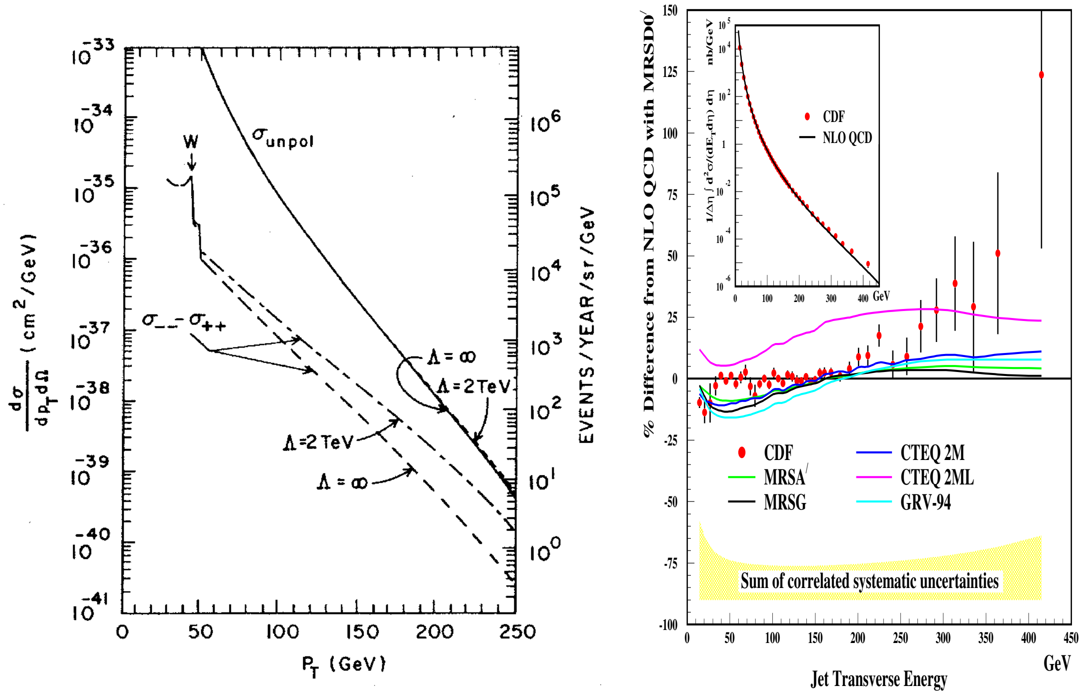

Figure 1: a) Prediction from 1983 for the effect of Quark Substructure on inclusive jet cross section with and without Parity Violation capability. b) 1995-6 CDF Inclusive jet cross section and ratio to NLO QCD.

At PSU in 1990, I stated "Without the PVA handle, the CDF detector is limited to searching for substructure by deviations of jet production from QCD at large values of p_T... the advantage of the PVA signature is that **it is a clear indication of new physics** and, in addition, Λ can be determined by the dependence of the PVA on p_T—thus, the handedness and any details of any new coupling can be measured."

The 1995-6 CDF measurement [NYTimes 2/8/96] is a case in point (see Fig. 1b). If the "% Difference from NLO QCD" were "% Parity Violation", the parity-violating signature would be unambiguous evidence of new physics.

Statistical Errors on Asymmetries

The error on an asymmetry is easy to understand in terms of the Binomial Distribution: N repeated independent trials with probability p for success and $q = 1 - p$ for failure. *i.e.* N events are divided into two classes, N_1 and N_2, where $N = N_1 + N_2$. Then, if $m = N_1$ is the number of events in class 1 after N trials

$$<m>=<N_1>= Np \qquad \text{and} \qquad \sigma_m^2 = Np(1-p) \tag{1}$$

The estimator of p is $\bar{p} = m/N$ and

$$<\bar{p}>= \frac{<m>}{N} = p \qquad , \qquad \sigma_{\bar{p}}^2 = \frac{\sigma_m^2}{N^2} = \frac{p(1-p)}{N} \tag{2}$$

The measured asymmetry in these events is A :

$$A = \frac{N_1 - N_2}{N_1 + N_2} = \bar{p} - (1 - \bar{p}) = 2\bar{p} - 1 \tag{3}$$

or

$$\bar{p} = \frac{1 + A}{2} \qquad , \qquad \bar{q} = 1 - \bar{p} = \frac{1 - A}{2} \tag{4}$$

Substituting Eqs. 3,4 into Eq. 2 yields the familiar result:

$$< A >= 2 <\bar{p}> -1 = 2p - 1 \tag{5}$$

$$\delta^2 A \equiv \sigma_A^2 = 4\sigma_{\bar{p}}^2 = 4\frac{(1+A)(1-A)}{4N} = \frac{1 - A^2}{N} \tag{6}$$

Note that POLARIZATIONS do not enter yet.
The statistical error comes from dividing events into two classes.

Two Spin Asymmetry A_{LL} in General

Assume that $\mathcal{L}$ is the same for all helicity combinations
$\bar{\sigma}^{++} = N(++)/P_{fac}\mathcal{L}(++) = N(++)/P_{fac}\mathcal{L}$
Where $\bar{\sigma}^{++}$ is the estimator of the theoretical quantity
$P_{fac} = 1, P_1, P_2, P_1P_2$ is the appropriate polarization factor for the quantity being measured
$N = N(++) + N(--) + N(+-) + N(-+)$ is the total number of events
Assume that all asymmetries are small so $1 - A^2 \sim 1$

$$A_{LL} = \frac{\sigma_{ll}}{\sigma_{uu}} = \frac{1}{P_1P_2}\frac{N(++) + N(--) - N(+-) - N(-+)}{N(++) + N(--) + N(+-) + N(-+)} \tag{7}$$

$$\delta A_{LL} = \frac{1}{P_1P_2}\frac{1}{\sqrt{N}} \tag{8}$$

If parity is conserved

$$\sigma^{++} = \sigma^{--} \qquad \sigma^{+-} = \sigma^{-+}$$

$$<N(++)>=<N(--)> \qquad <N(+-)>=<N(-+)>$$

$$A_{LL}^{1+} = \frac{1}{P_1P_2}\frac{N(++) - N(+-)}{N(++) + N(+-)} \qquad \delta A_{LL}^{1+} = \frac{1}{P_1P_2}\frac{1}{\sqrt{N/2}} \tag{9}$$

$$A_{LL}^{1-} = \frac{1}{P_1P_2}\frac{N(--) - N(-+)}{N(--) + N(-+)} \qquad \delta A_{LL}^{1-} = \frac{1}{P_1P_2}\frac{1}{\sqrt{N/2}} \tag{10}$$

If parity is conserved, we get a check C_{LL}

$$C_{LL} = A_{LL}^{1-} - A_{LL}^{1+} = 0 \qquad \delta C_{LL} = \frac{2}{P_1P_2}\frac{1}{\sqrt{N}} \tag{11}$$

Of course there are A_{LL}^{2+} and A_{LL}^{2-} ...

And we should redo with terms in the errors $\sqrt{(1 - P_{fac}^2 A^2)}$ ...

A_L Single Spin, Parity Violating Asymmetry

$$A_L = \frac{1}{P_1}\frac{N(-)-N(+)}{N(-)+N(+)} \qquad \textbf{where} \qquad \delta A_L = \frac{1}{P_1}\frac{1}{\sqrt{N'}} \tag{12}$$

BUT this Asymmetry Doesn't exist for both beams polarized

Two Spin Parity Violating Asymmetry A_{LL}^{PV}

First I introduce new Statistically Independent Numbers

$N_s \equiv N(--) + N(++)$, with Asymmetry $A_s = \frac{N(--)-N(++)}{N(--)+N(++)}$

$N_o \equiv N(-+) + N(+-)$, with Asymmetry $A_o = \frac{N(-+)-N(+-)}{N(-+)+N(+-)}$

and $N = N_s + N_o$, with Asymmetry $A_{so} = P_1 P_2 A_{LL}$

N_s are the number of events with same initial helicity states

N_o are the number of events with opposite initial helicity states

Original Notation. Items in Orange are not correct

$$A_{LLorig}^{PV} = \frac{\sigma^{--} - \sigma^{++}}{\sigma^{--} + \sigma^{++}} \neq \frac{1}{P_1 P_2}\frac{N(--)-N(++)}{N(--)+N(++)} = \frac{1}{P_1 P_2} A_s \tag{13}$$

Now in Kamal's Clear Notation

$$A_{LL}^{PV} = \frac{\sigma_{lu} + \sigma_{ul}}{\sigma_{uu} + \sigma_{ll}} = \frac{A_{Lu} + A_{uL}}{(1 + A_{LL})} \neq \frac{N(--)-N(++)}{N(--)+N(++)} \quad \text{unless } P_1 = P_2 = 1 \tag{14}$$

Bunce Makdisi Tannenbaum to rescue Kamal (or vice-versa)

Actually Guillet, Soffer, Taxil really had it (see PSU)

Two-spin parity violating single-Asymmetry

$$A_{Lu} = \frac{\sigma_{lu}}{\sigma_{uu}} = \frac{1}{P_1}\frac{N(--)+N(-+)-N(+-)-N(++)}{N(--)+N(-+)+N(+-)+N(++)} \tag{15}$$

$$\delta A_{Lu} = \frac{1}{P_1}\frac{\sqrt{1 - P_1^2 A_{Lu}^2}}{\sqrt{N}} \tag{16}$$

$$A_{uL} = \frac{\sigma_{ul}}{\sigma_{uu}} = \frac{1}{P_2}\frac{N(--)+N(+-)-N(-+)-N(++)}{N(--)+N(+-)+N(-+)+N(++)} \tag{17}$$

$$\delta A_{uL} = \frac{1}{P_2}\frac{\sqrt{1 - P_2^2 A_{uL}^2}}{\sqrt{N}} \tag{18}$$

Now Define

$$\bar{A}_L \equiv \frac{A_{Lu} + A_{uL}}{2} \tag{19}$$

Take $P_1 = P_2 = P \sim 0.7$. Then, directly from Eqs. 15,17

$$\bar{A}_L = \frac{A_{Lu} + A_{uL}}{2} = \frac{1}{2P}\frac{2N(--) - 2N(++)}{N} = \frac{1}{P}\frac{N(--) - N(++)}{N_s}\frac{N_s}{N} \tag{20}$$

$$\bar{A}_L = \frac{1}{P}A_s\bar{p}_s \qquad \text{where} \qquad \bar{p}_s = \frac{N_s}{N_s + N_o} \tag{21}$$

As A_s and $\bar{p}_s$ are statistically independent quantities

$$\frac{\delta^2\bar{A}_L}{\bar{A}_L^2} = \left(\frac{\delta A_s}{A_s}\right)^2 + \frac{\delta^2\bar{p}_s}{\bar{p}_s^2} \tag{22}$$

From Eq. 2 and Eq. 6 (to order $1 - A^2$)

$$\frac{\sigma^2_{\bar{p}_s}}{\bar{p}_s^2} = \frac{1 - \bar{p}_s}{\bar{p}_s N} = \frac{1 - \bar{p}_s}{N_s} \qquad \delta^2 A_s = \frac{1 - A_s^2}{N_s} \tag{23}$$

$$\frac{\delta^2\bar{A}_L}{\bar{A}_L^2} = \frac{1 - A_s^2}{N_s A_s^2} + \frac{1 - \bar{p}_s}{N_s} = \frac{1 - A_s^2\bar{p}_s}{N_s A_s^2} \tag{24}$$

Multiply both sides by $\bar{A}_L^2$ and from Eq. 21

$$\frac{\bar{A}_L}{A_s} = \frac{\bar{p}_s}{P} \tag{25}$$

$$\delta^2\bar{A}_L = \frac{1 - A_s^2}{P^2 N_s}\bar{p}_s^2 + \frac{A_s^2\bar{p}_s^2}{P^2}\frac{(1 - \bar{p}_s)}{N_s} \tag{26}$$

Remembering that $N_s = \bar{p}_s N$

$$\delta^2\bar{A}_L = \frac{\bar{p}_s}{P^2 N}\left(1 - A_s^2 + A_s^2(1 - \bar{p}_s)\right) = \frac{\bar{p}_s}{P^2 N}\left(1 - A_s^2\bar{p}_s\right) \tag{27}$$

Using Basim's New definition of A_{LL}^{PV} (Eq. 14)

$$A_{LL}^{PV} = \frac{2\bar{A}_L}{(1 + A_{LL})} = \frac{2A_s\bar{p}_s}{P(1 + \frac{A_{so}}{P^2})} = \frac{2A_s\bar{p}_s}{P(1 + \frac{2\bar{p}_s - 1}{P^2})} \tag{28}$$

Again A_s and $\bar{p}_s$ are independent:

$$\left(\frac{\delta A_{LL}^{PV}}{A_{LL}^{PV}}\right)^2 = \left(\frac{\delta A_s}{A_s}\right)^2 + \frac{\delta^2 Q}{Q^2} \tag{29}$$

Where $Q = \bar{p}_s/(1 + \frac{2\bar{p}_s - 1}{P^2})$, and by my algebra:

$$\frac{\delta Q}{Q} = \frac{\delta \bar{p}_s}{\bar{p}_s} \frac{(\frac{1}{P^2} - 1)}{1 + A_{LL}} \tag{30}$$

THEREFORE !!!! BMT are correct sometimes

$$\left(\frac{\delta A_{LL}^{PV}}{A_{LL}^{PV}}\right)^2 \geq \frac{\delta^2 \bar{A}_L}{\bar{A}_L^2} \qquad \textbf{for} \qquad P \leq \frac{1}{\sqrt{2 + A_{LL}}} \tag{31}$$

But if RHIC is Good and $P > 0.707$, then Basim is correct!

Note that even if $A_{LL} = 1$, $<\bar{p}_s>= \frac{1+P^2}{2} \leq 1$.

SYMMETRIES and ANTISYMMETRIES

Remember, A_{LL}^{PV} is symmetric in some variable(Soffer)

$$A_{LL}^{PV}(y) = +A_{LL}^{PV}(-y) \tag{32}$$

Define the New Antisymmeric Variable

$$\tilde{A}_L = \frac{A_{Lu} - A_{uL}}{2} = \frac{1}{2P} \frac{2N(-+) - 2N(+-)}{N} = \frac{1}{P} \frac{N(-+) - N(+-)}{N_o} \frac{N_o}{N} \tag{33}$$

$$\tilde{A}_L = \frac{1}{P} A_o \bar{p}_o \qquad \text{where} \qquad \bar{p}_o = \frac{N_o}{N_s + N_o} = 1 - \bar{p}_s \tag{34}$$

A_o and $\bar{p}_o$ are statistically independent so the error follows $\bar{A}_L$

$$\delta^2 \tilde{A}_L = \frac{\bar{p}_o}{P^2 N} \left(1 - A_o^2 \bar{p}_o\right) \tag{35}$$

Anti-symmetric two spin PVA(Soffer) following Kamal

$$\tilde{A}_{LL}^{PV} = \frac{\sigma_{lu} - \sigma_{ul}}{\sigma_{uu} - \sigma_{ll}} = \frac{2\tilde{A}_L}{(1 - A_{LL})} \tag{36}$$

$\tilde{A}_{LL}^{PV}$ is anti-symmetric

$$\tilde{A}_{LL}^{PV}(y) = -\tilde{A}_{LL}^{PV}(-y) \tag{37}$$

The error $\delta \tilde{A}_{LL}^{PV}$ is left as an exercise