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How to Measure Specific Heat Using Event-by-Event Average p_T Fluctuations.

M. J. Tannenbaum*

Brookhaven National Laboratory

Upton, NY 11973-5000 USA

for the PHENIX Collaboration

Email: mjt@bnl.gov

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Abstract

A simple way to visualize event-by-event average p_T fluctuations is by assuming that each collision has a different temperature parameter (inverse p_T slope) and that the ensemble of events has a temperature distribution about the mean, $\langle T \rangle$, with standard deviation σ_T. PHENIX characterizes the non-random fluctuation of M_{p_T}, the event-by-event average p_T, by F_{p_T}, the fractional difference of the standard deviation of the data from that of a random sample obtained with mixed events. This can be related to the temperature fluctuation:

$$F_{p_T} = \sigma^{\rm data}_{M_{p_T}} / \sigma^{\rm random}_{M_{p_T}} - 1 \simeq (\langle n \rangle - 1)\sigma_T^2 / \langle T \rangle^2 \qquad .$$

Combining this with the Gavai, *et al.*,[1] and Korus, *et al.*,[2] definitions of the specific heat per particle, a simple relationship is obtained:

$$c_v/T^3 = \frac{\langle n \rangle}{\langle N_{tot} \rangle} \frac{1}{F_{p_T}} \qquad .$$

F_{p_T} is measured with a fraction $\langle n \rangle / \langle N_{tot} \rangle$ of the total particles produced, a purely geometrical factor representing the fractional acceptance, $\sim 1/33$ in PHENIX. Gavai, *et al.* predict that $c_v/T^3 = 15$, which corresponds to $F_{p_T} \sim 0.20\%$ in PHENIX, which may be accessible by measurements of M_{p_T} in the range $0.2 \leq p_T \leq 0.6$ GeV/c. In order to test the Gavai, *et al.* prediction that c_v/T^3 is reduced in a QGP compared to the ideal gas value (15 compared to 21), precision measurements of F_{p_T} in the range 0.20% for $0.2 \leq p_T \leq 0.6$ GeV/c may be practical.

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I. INTRODUCTION

R. Gavai, S. Gupta and S. Mukherjee [1] predict in "quenched QCD" at $2T_c$ and $3T_c$ that the specific heat, c_V/T^3, differs significantly from the value for an ideal gas—15 compared to 21 (see Fig. 1). Can this be measured?

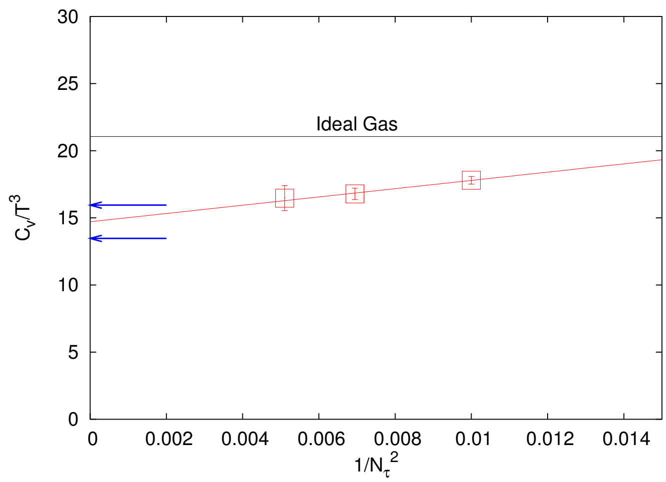

FIG. 1: Gavai, *et al.*, prediction for c_v/T^3 [1].

II. EVENT-BY-EVENT AVERAGE p_T FLUCTUATIONS AND SPECIFIC HEAT

A. Single particle distributions

The single particle transverse momentum (p_T) distribution averaged over all particles in all events for a p-p experiment (inclusive) or in all events of a given centrality class for an A+A experiment (semi-inclusive) is usually written in the form:

$$\frac{d\sigma}{dp_T} = \frac{b}{\Gamma(p)}(bp_T)^{p-1}e^{-bp_T} \qquad \text{or} \qquad \frac{d\sigma}{p_T dp_T} = \frac{b^2}{\Gamma(p)}(bp_T)^{p-2}e^{-bp_T} \qquad . \tag{1}$$

Equation 1 represents a Gamma distribution, where $\langle p_T \rangle = p/b$, $\sigma_{p_T}/\langle p_T \rangle = 1/\sqrt{p}$. Typically $b = 6$ (GeV/c)$^{-1}$ and $p = 2$ for p-p collisions. As shown in Fig. 2, the p parameter depends on the particle type in central Au+Au collisions, with $p < 2$ for $\pi^\pm$, $p \sim 2$ for $K^\pm$ and $p > 2$ for (anti-) protons, but the asymptotic slope tends to be the same for all particles. The 'inverse slope parameter', $T = 1/b$, is usually referred to as the 'Temperature parameter'.

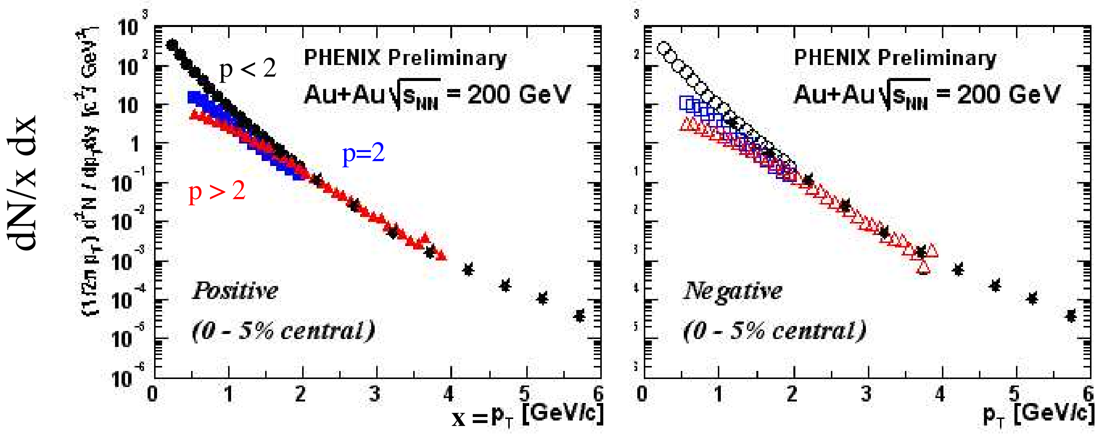

FIG. 2: Identified particle semi-inclusive invariant p_T spectra for Au+Au central collisions [3]. At the lowest p_T, the π^+ are the highest, followed by K^+ and p (left) and the same for the negatives (right).

B. Event-by-Event Average

For events with n detected charged particles with magnitudes of transverse momenta, p_{T_i}, the event-by-event average p_T, denoted M_{p_T}, is defined as:

$$M_{p_T} = \overline{p_T} = \frac{1}{n}\sum_{i=1}^{n} p_{T_i} \qquad . \tag{2}$$

By definition $\langle M_{p_T}\rangle \equiv \langle p_T\rangle = \mu$; however, it takes hard work to make one's data follow this identity to high precision ($\ll 1\%$). The standard deviation of M_{p_T} is defined the usual way:

$$\sigma^2_{M_{p_T}} \equiv \left\langle M^2_{p_T}\right\rangle - \langle M_{p_T}\rangle^2 = \frac{1}{n^2}\left(n\sigma^2_{p_T} + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n}\Big\langle (p_{T_i} - \langle p_T\rangle)(p_{T_j} - \langle p_T\rangle)\Big\rangle\right) \qquad . \tag{3}$$

If all the p_{T_i} on all events are random samples of the same p_T distribution, then:

$$\sigma^2_{M_{p_T}} = \frac{\sigma^2_{p_T}}{n} \qquad , \tag{4}$$

where $\sigma_{p_T} = \sqrt{\langle p_T^2\rangle - \langle p_T\rangle^2}$ is the standard deviation of Eq. 1, the inclusive p_T spectrum (averaged over all events).

A nice illustration of what can be revealed by the event-by-event average that is not shown by the inclusive average over all events was given by Korus, *et al.* [2]. Suppose that each collision has a different temperature parameter such that the ensemble of events has a mean, $\langle T\rangle$, with standard deviation, $\sigma_T = \sqrt{\langle T^2\rangle - \langle T\rangle^2}$, about the mean. It is easy to show that for this case:

$$\frac{\sigma^2_{M_{p_T}}}{\mu^2} - \frac{1}{n}\frac{\sigma^2_{p_T}}{\mu^2} = (1 - \frac{1}{n})\frac{\sigma^2_T}{\langle T\rangle^2} \quad . \tag{5}$$

C. Specific Heat

As pointed out by Korus, *et al.*,[2] if the parameter T would correspond to the actual temperature of the system, not just the inverse slope of the p_T distribution, then a basic equation of thermodynamics would relate the temperature fluctuations of a system to its total heat capacity [4–6]:

$$\frac{1}{C_V} = \frac{\sigma^2_T}{\langle T\rangle^2} \quad , \tag{6}$$

where C_V is an extensive quantity corresponding to the total number of particles in the system, $\langle N_{tot}\rangle$. Thus the specific heat per particle is $c_V = C_V/\langle N_{tot}\rangle$. Gavai, *et al.*,[1] refer to this same (dimensionless) quantity as c_v/T^3, resulting in the final equation:

$$\frac{c_v}{T^3} = \frac{1}{\langle N_{tot}\rangle}\frac{1}{\sigma^2_T/\langle T\rangle^2} \tag{7}$$

where n represents the number of particles used in the calculation of M_{p_T} (Eq. 5) from which $\sigma_T/\langle T\rangle$ is determined.

III. MEASUREMENTS OF M_{p_T}

The measured M_{p_T} distributions for two centrality classes in $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions in PHENIX [7] are shown in Figure 3 (data points) compared to a random baseline (histograms). Mixed-events are used to define the baseline for random fluctuations of M_{p_T}. This has the advantage of effectively removing any residual detector-dependent effects. The event-by-event average distributions are very sensitive to the number of tracks in the event (denoted n), so the mixed event sample is produced with the *identical* n distribution as the data. Additionally, no two tracks from the same data event are placed in the same mixed event in order to remove any intra-event correlations in p_T. Finally, $\langle M_{p_T}\rangle$ must exactly match the semi-inclusive $\langle p_T\rangle$.

The non-Gaussian, Gamma distribution shape of the M_{p_T} distributions is evident. The difference between the data and the mixed-event random baseline distributions is barely

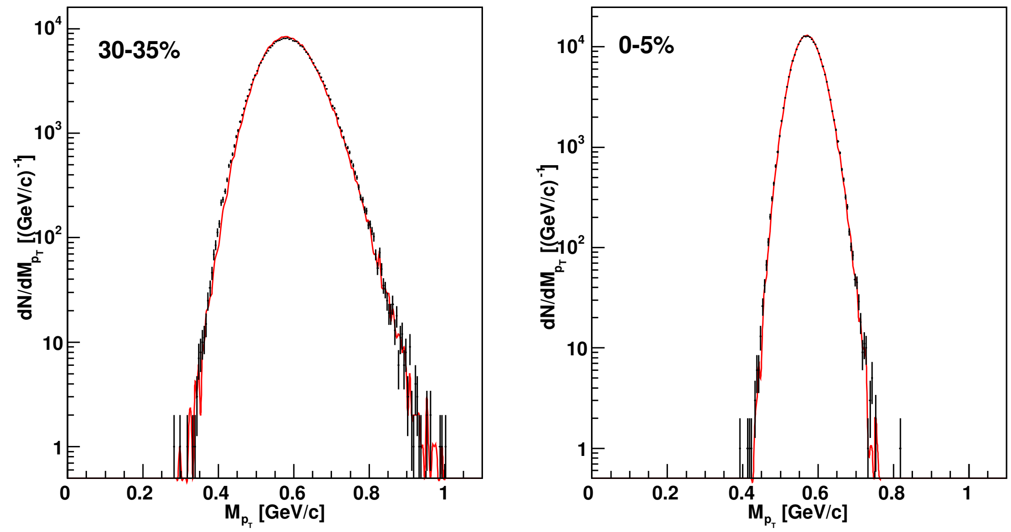

FIG. 3: M_{p_T} for 30-35% and 0-5% centrality classes[7]: data (points) mixed-events (histogram).

visible to the naked eye. PHENIX quantifies the non-random fluctuation by the fractional difference of the standard deviations of M_{p_T} for the data and the mixed-event (random) sample:

$$F_{p_T} \equiv \frac{\sigma_{M_{p_T},\mathrm{data}} - \sigma_{M_{p_T},\mathrm{mixed}}}{\sigma_{M_{p_T},\mathrm{mixed}}} \quad , \tag{8}$$

which is on the order of a few percent. The results are shown (Fig. 4-left) as a function of centrality (represented by N_{part}) for charged particle tracks in the range 0.2 GeV/c $\leq p_T \leq 2.0$ GeV/c; and, for the 20-25% centrality class ($N_{part} = 181.6$), over a varying p_T range, 0.2 GeV/c $\leq p_T \leq p_T^{\max}$ (Figure 4-right). The steep increase in F_{p_T} for the small increase in the number of tracks with increasing $p_T^{\max} > 1$ GeV/c is consistent with correlations due to jet production as shown by the dotted lines [7]. However, other explanations have been proposed [8]. Note that the errors are entirely systematic, due to time-dependent detector variations. Comparatively, statistical errors are negligible.

IV. HOW TO MEASURE c_v/T^3.

For the small values of F_{p_T} observed, one can make use of the identity

$$\frac{\Delta\sigma^2}{\sigma^2} = 2\frac{\Delta\sigma}{\sigma} = 2F \tag{9}$$

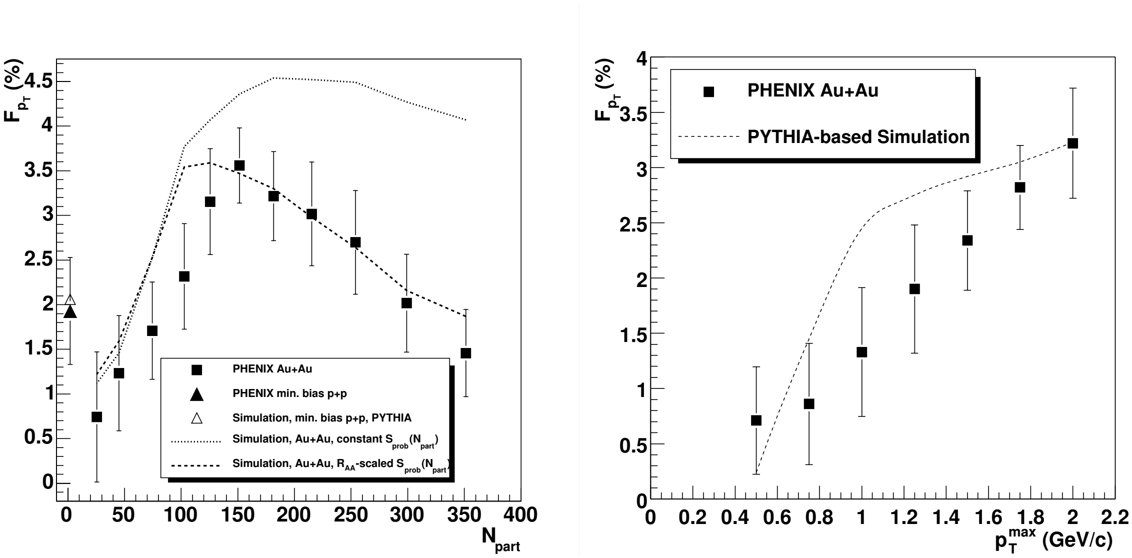

FIG. 4: F_{p_T} vs centrality and $p_T^{\rm max}$ compared to simulations [7].

to obtain the relation:

$$(1-\frac{1}{n})\frac{\sigma_T^2}{T^2} = \frac{\sigma_{M_{p_T}}^2}{\mu^2} - \frac{1}{n}\frac{\sigma_{p_T}^2}{\mu^2} = 2F_{p_T}\frac{1}{n}\frac{\sigma_{p_T}^2}{\mu^2} = \frac{2F_{p_T}}{np} \simeq \frac{F_{p_T}}{n} \qquad . \tag{10}$$

Combining Eq. 10 with Eq. 7, we obtain the simple and elegant expression:

$$\frac{c_v}{T^3} = \frac{\langle n\rangle}{\langle N_{tot}\rangle}\frac{1}{F_{p_T}} \qquad . \tag{11}$$

Note that F_{p_T} is measured with a fraction $\langle n\rangle / \langle N_{tot}\rangle$ of the total particles produced, which is a purely geometrical factor representing the fractional acceptance of the measurement. For example, if all particles are produced in a range $\delta\eta_{\rm FWHM}$ (assuming a flat or trapezoidal $dn/d\eta$ over this interval) and if including the neutrals gives a factor of 1.5 more total particles than charged particles; and if F_{p_T} is measured with charged particles in an acceptance $\delta\eta_c$, $\delta\phi_c/2\pi$, which due to the p_T cut only represents a fraction f_c of the charged particles on that solid angle, then:

$$\frac{\langle N_{tot}\rangle}{\langle n\rangle} = \frac{1.5\times 2\pi\times\delta\eta_{\rm FWHM}}{f_c\times\delta\phi_c\times\delta\eta_c} \qquad . \tag{12}$$

For RHIC at $\sqrt{s_{NN}} = 200$ GeV, $\delta\eta_{\rm FWHM} = \pm 3.5$ [9], and the PHENIX acceptance was $\delta\phi = \pi$, $\delta\eta = \pm 0.35$, f_c=0.9 for $p_T \geq 0.2$ GeV/c, resulting in $\langle N_{tot}\rangle / \langle n\rangle = 33$. From Fig. 4, F_{p_T} is of order 2% but most of that is due to jets, so the effect due to temperature fluctuations is $< F_{p_T}$, say 1%, so we obtain $c_v/T^3 > 1/(33 * 0.01) = 3$. This is to be compared to the Korus, *et al.* [2], result of $c_v/T^3 = 60 \pm 100$ [10] from the NA49 data. Recall that Gavai, *et al.*, [1] predict a value of 15 for c_v/T^3, which would correspond to a value of $F_{p_T} = 1\% \times 3/15 = 0.20\%$ for the data in Fig. 4 (see Fig. 5). Perhaps this precision can

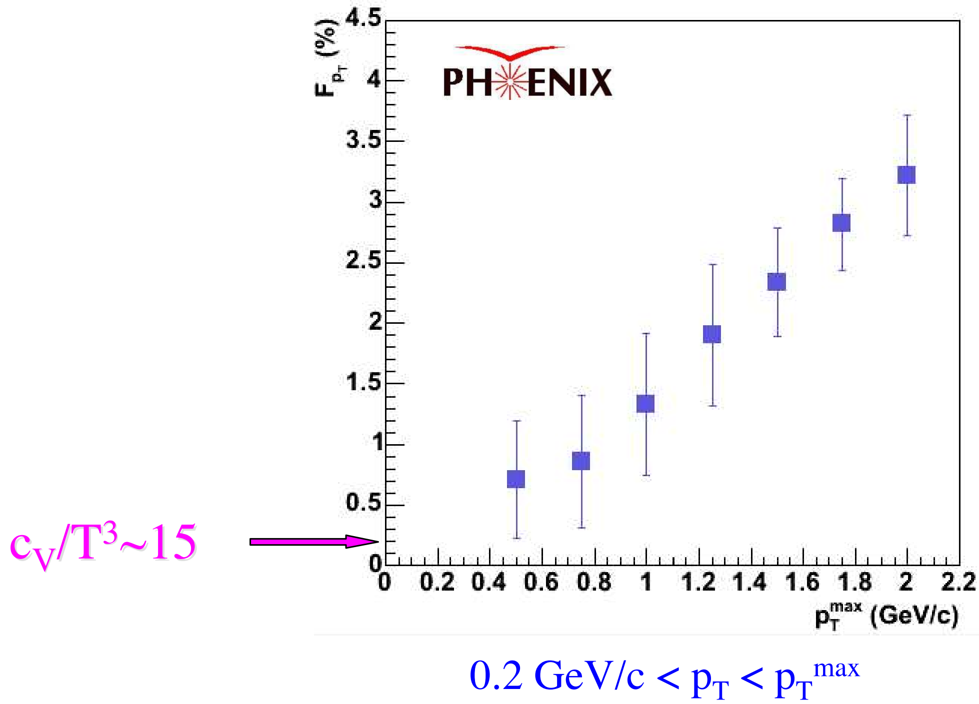

FIG. 5: Gavai *et al.*, prediction [1] compared to PHENIX measurement.

be achieved by concentrating on the region $p_T^{\max} \leq 0.6$ GeV/c, where jets have least effect. Also, as the present error is totally systematic due to run-by-run variation, there is hope that a substantial reduction should be possible.

V. CONCLUSIONS

In central heavy ion collisions, the huge correlations in p-p collisions are washed out [5]. The remaining correlations are: Jets; Bose-Einstein correlations; Hydrodynamic Flow. These correlations seem to saturate the present fluctuation measurements. No other sources of non-random fluctuations have been observed. This puts a severe constraint on the critical fluctuations that were expected for a sharp phase transition but is consistent with the present expectation from lattice QCD that the transition is a smooth crossover. In order to see the temperature fluctuations predicted by $c_v/T^3 \simeq 15$ in lattice gauge calculations, present sensitivity needs to be improved by an order of magnitude by removing the known sources of correlation and improving the measurement errors. An interesting check of whether temperature fluctuations, rather than the correlations noted above, produce the observed non-random fluctuations is provided by Eq. 10: for a pure $\sigma_T^2/\langle T\rangle^2$ fluctuation, F_{p_T} for a given centrality should increase linearly with the number of tracks measured (e.g. by increasing the solid angle—PHENIX *cf.* STAR).

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