Short Range Rapidity Correlations from the Bose-Einstein Effect and Multiplicity Fluctuations in Heavy-Ion Collisions— An Experimental Demonstration

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A measurement of the two-particle correlation of identified pions was performed in the E802/E859 magnetic spectrometer on the interval $1.5 \leq y \leq 2.0$, $\delta\phi = 0.4$ rad, for central ^{28}Si+Au collisions. It is demonstrated that the two-pion correlation in rapidity is entirely due to Bose-Einstein interference. The directly measured exponential correlation length is $\xi_y = 0.20 \pm 0.03$ for two π^-, with strength $R(0,0) \sim 1\%$, in agreement with previous E802 indirect measurements derived from an analysis of intermittency using Negative Binomial Distributions.

1. INTRODUCTION

The study of non-Poisson fluctuations of charged particle multiplicity distributions in small pseudorapidity intervals $\delta\eta \leq 1$ by many experiments has been heavily influenced by the utilization of normalized factorial moments (NFM)[1,2] which are unity for a Poisson distribution:

$$F_q(\delta\eta) = \frac{< n(n-1)\ldots(n-q+1) >}{< n >^q} \quad , \qquad (1)$$

where n is the multiplicity in a pseudorapidity interval (bin) of size $\delta\eta$ on a given event and the $< >$ brackets indicate averaging over all events. The normalized factorial moment with the clearest interpretation is

$$F_2(\delta\eta) = \frac{< n(n-1) >}{< n >^2} = \frac{< n^2 > - < n >}{< n >^2} = 1 + \frac{\sigma^2}{\mu^2} - \frac{1}{\mu} \qquad (2)$$

where $\mu \equiv < n >$ is the mean and $\sigma \equiv \sqrt{< n^2 > - < n >^2}$ is the standard deviation. A mechanism, dubbed intermittency, was proposed[1], which would be indicated by a power-law increase of multiplicity distribution moments over pseudorapidity bins as the bin size is reduced:

$$F_q(\delta\eta) \propto (\delta\eta)^{-\phi_q} \quad . \qquad (3)$$

The scale-invariant power-law dependence with singular behavior as $\delta\eta \to 0$ was suggestive of the physics of phase transitions, chaos, and fractals.

Many experiments applied the formalism to their data, leading to the observation[2] of the predicted power law behavior in the region $1 \geq \delta\eta \geq 0.1$. However, the observation of tantalizing power-laws tended to obscure the fact that multiplicity distributions were well known to be non-Poisson because of short-range rapidity correlations in multi-particle production[3,4]. On the other hand, the observed variation in NFM as a function of $\delta\eta$ means that the fluctuations, i.e. the shapes of the multiplicity distributions, change as a function of $\delta\eta$.

In previous work, the E802 Collaboration[5] at the BNL-AGS analyzed the evolution of charged particle multiplicity distributions from central collisions of ^{16}O+Cu at 14.6A GeV/c as a function of the width of the pseudo-rapidity interval $\delta\eta$ in the range $1.2 \leq \eta \leq 2.2$, both by the method of normalized factorial moments and by direct measurements of the shape of the distributions. The charged multiplicity distributions (see Fig. 1) were well represented by Negative Binomial Distributions (NBD) and simply characterized by the

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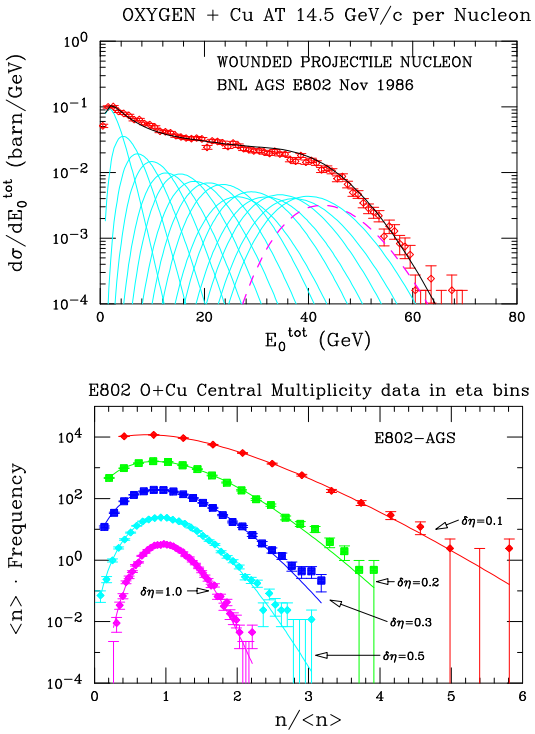

Figure 1. (a) Measured energy emission in $\delta\eta \sim 1$ for O+Cu collisions. The solid curve, which fits the data very well is the sum of the 16 lower curves which represent the energy emitted when from 1 to 16 projectile nucleons interact in the target nucleus with 'geometric' probabilities. (b) Multiplicity distributions measured in O+Cu central collisions as a function of the interval $\delta\eta$ (indicated) for the case when all 16 incident nucleons have interacted, as determined by a Zero degree calorimeter. The curves are NBD fits. The data labelled $\delta\eta = 1.0$ correspond to the highest energy (dashed) sub-curve in (a).

NBD parameter $k(\delta\eta)$ which represents the first departure of a distribution from a Poisson:

$$\frac{1}{k(\delta\eta)} = \frac{\sigma^2}{\mu^2} - \frac{1}{\mu} = F_2(\delta\eta) - 1 = K_2(\delta\eta) \qquad (4)$$

where $K_2(\delta\eta)$ is a normalized factorial cumulant[3,6].

The factorial moments and cumulants of the multiplicity on an interval $\delta\eta$ are simply related to the integrals of the q-particle inclusive rapidity densities $\rho_q(y_1, \ldots y_q)$

$$\int^{\delta\eta} dy_1\ \rho_1(y_1) = < n > \qquad (5)$$

$$\int^{\delta\eta} dy_1 dy_2\ \rho_2(y_1, y_2) = \\ < n(n-1) > = < n >^2\ F_2 \qquad (6)$$

$$\int^{\delta\eta} dy_1 \cdots dy_q\ \rho_q(y_1, \ldots y_q) = \\ < n(n-1)\ldots(n-q+1) > = < n >^q\ F_q \qquad (7)$$

These integrals (or moments) are sensitive to any short-range rapidity correlation in particle production, since if there would be no correlation then

$$\rho_q(y_1, \ldots y_q) = \rho_1(y_1)\rho_1(y_2)\cdots\rho_1(y_q) \quad , \qquad (8)$$

in which case all the F_q reduce to unity, a Poisson distribution. Mueller[3] introduced a series of functions to describe correlations in multiparticle emission. For instance, the normalized two-particle short-range rapidity correlation function $R_2(y_1, y_2)$ is defined as:

$$R_2(y_1, y_2) \equiv \frac{\rho_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1 \equiv \frac{C_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} \\ = R(0,0)\, e^{-|y_1 - y_2|/\xi} \quad , \qquad (9)$$

where $\rho_1(y)$ and $\rho_2(y_1, y_2)$ are the inclusive densities for a single particle (at rapidity y) or 2 particles (at rapidities y_1 and y_2), $C_2(y_1, y_2) = \rho_2(y_1, y_2) - \rho_1(y_1)\rho_1(y_2)$ is the Mueller correlation function for 2 particles (which is zero for the case of no correlation), and ξ is the two-particle short-range rapidity correlation length[3,4] for an exponential parameterization. These equations may be combined to yield the relationship[7,8]

$$\frac{1}{k(\delta\eta)} = K_2(\delta\eta) = F_2(\delta\eta) - 1 \qquad (10)$$

$$= \frac{\int^{\delta\eta} dy_1 dy_2\ \rho_1(y_1)\rho_1(y_2) R_2(y_1, y_2)}{\int^{\delta\eta} dy_1 dy_2\ \rho_1(y_1)\rho_1(y_2)} \quad ,$$

from which it is clear that the normalized factorial cumulant $K_2(\delta\eta)$ is simply the mean value of the normalized two-particle correlation function $R_2(y_1, y_2)$ on the interval $\delta\eta$, with weighting by the square of the inclusive single particle density:

$$K_2(\delta\eta) = < R_2(y_1, y_2) > |_{\delta\eta} \quad . \tag{11}$$

If the inclusive single particle density, $\rho_1(y) = dn/dy$, is assumed constant on the interval, then the integral in Eq. 10 can be performed analytically (specifically on the interval $0 \leq y_1 \leq \delta\eta$, $0 \leq y_2 \leq \delta\eta$) to obtain the normalized factorial moment $F_2(\delta\eta)$ or normalized factorial cumulant $K_2(\delta\eta)$ in terms of the parameters of Eq. 9 [7–11]:

$$K_2(\delta\eta) = R(0,0)\, G(\delta\eta/\xi) \quad , \tag{12}$$

where the function $G(x)$ is defined as:

$$G(x) = 2\,\frac{(x - 1 + e^{-x})}{x^2} \quad . \tag{13}$$

The correlation length and strength for central ^{16}O+Cu collisions were derived by E802[5] from a fit of the measured $k(\delta\eta) = 1/K_2(\delta\eta)$ to this integral function[12] (see Fig. 2).

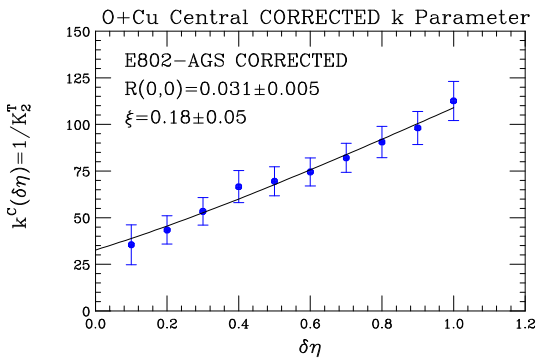

Figure 2. $k(\delta\eta)$ from the E802 NBD fits, corrected for instrumental effects. The solid line is a fit to Eq. 12 with the parameters indicated.

It is interesting to note that measurements of NBD fits to multiplicity distributions in limited $\delta\eta$ intervals show the same effect as Fig. 2 for all reactions studied[13]—linear dependences of the NBD parameter $k(\delta\eta)$ with the pseudorapidity interval $\delta\eta$, or equivalently with the mean multiplicity in the interval $\mu = < n(\delta\eta) >$, with non-zero intercept, $k(0) \neq 0$ (see Fig. 3a). Furthermore, when plotted this way, the data

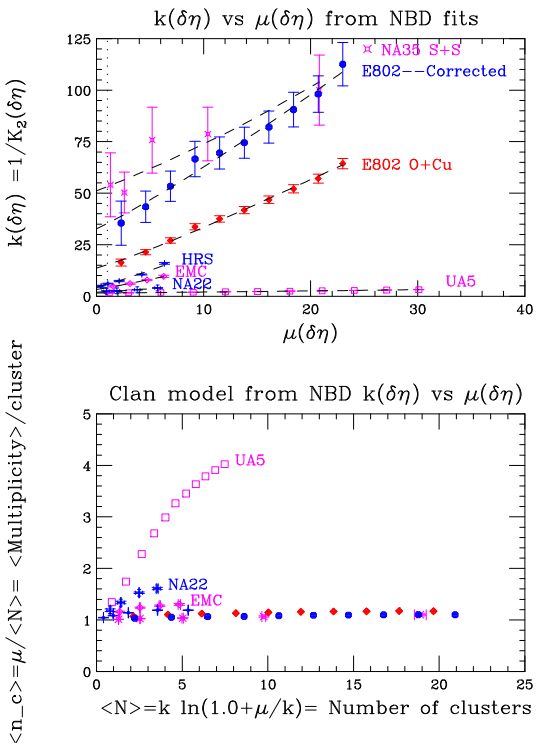

Figure 3. a) The NBD parameter $k(\delta\eta)$ as a function of the mean multiplicity in the pseudorapidity interval, $\mu(\delta\eta) = dn/d\eta \times \delta\eta$ for several experiments: UA5 $\bar{p}$+p, NA22 p+p, EMC μ-p, HRS $e^+ + e^-$, E802 O+Cu central (and E802 corrected), NA35 S+S central. See reference [13] for details. The lines are fits to Eq. 12. b) Parameters of the Clan Model from the same data.

directly give the two parameters of the clan model[8]: the mean number of clusters, $< N >= k \ln(1.0 + \mu/k)$, and the mean multiplicity in a cluster $< n_c >= \mu / < N >= (\mu/k)/\ln(1.0 + \mu/k)$,

as shown in Fig. 3b. For the heavy ion data, $k/\mu \gg 1$ for all cases, so that $< n_c > \simeq 1$ and $< N > \simeq \mu$ to an excellent approximation, with the result that the NA35 and both sets of E802 data are nearly indistinguishable on the line $< n_c > \simeq 1$. Only the UA5 data where $k/\mu < 1$, and to a certain extent NA22, show appreciable multiplicity/per cluster and it will be interesting at RHIC or LHC, which are in the UA5 domain, to see how the parameters evolve from p-p to heavy ion collisions.

From Fig. 3a, it is clear that the heavy ion data (E802 and NA35) are quantitatively, rather than qualitatively, different from the others in that $k(\delta\eta)$ is much larger and the dependence on $\delta\eta$ much steeper. This leads to values of the two-particle correlation length and strength, as determined by E802 (Fig. 2) for central ^{16}O+Cu collisions[14], $\xi = 0.18 \pm 0.05$ and $\bar{R}(0,0) = 0.031 \pm 0.005$, which are much shorter and weaker than the values for hadron collisions. This explains the small values of $K_2 = F_2 - 1$ and the continuous change of $F_2(\delta\eta)$ in the range $1 \geq \delta\eta \geq 0.1$ in heavy ion collisions. $F_2(\delta\eta)$ appears to be a power-law, but actually follows Eq. 12. The logarithmic derivative $\phi_2 = -R(0,0)\, xG'(x)$ appears to be a constant $\phi_2 \sim -0.2\, R(0,0)$, because

$$x\, G'(x) = x\, \frac{4(1-e^{-x}) - 2x(1+e^{-x})}{x^3} \qquad (14)$$

is slowly varying for $0.5 \leq x \leq 20$. The quantative result from E802 yielded a simple and elegant explanation of intermittency—the 'large' bin-by-bin fluctuations in individual event rapidity distributions from Si+AgBr interactions in cosmic rays[15,16] are a consequence of the apparent statistical independence of the multiplicity in rapidity bins of size $\delta\eta \sim 0.2$ due to the surprisingly short two-particle rapidity correlation length.

In fact, the weakened, but finite, short-range rapidity correlations in the collisions of relativistic heavy ions had been predicted in the context of intermittency moment analyses[9,10, 17,18]. In nucleus-nucleus collisions, the conventional hadron short-range correlations should be washed out by the random superposition of many sources of correlated particles[10,19,20], so that eventually only the quantum-statistical Bose-Einstein (BE) correlations of identical particles remain[9,10,21,17]. In fact, the relationship between intermittency and BE correlations has been convincingly demonstrated in other experiments[22–25] using non-identified charged particles. If BE correlations were the entire effect, then direct measurements of BE correlations in terms of the pseudorapidity or rapidity differences of the two particles, $\eta_2 - \eta_1 = \Delta\eta$, $y_2 - y_1 = \Delta y$, instead of the usual variables[26] $Q_{\rm inv}$, $|\vec{q}|$, q_0—where $Q_{\rm inv} = \sqrt{|\vec{q}|^2 - q_0^2}$, $q = p_2 - p_1 = (\vec{q}, q_0)$ and $p = (\vec{P}, E)$—should reproduce the short-range rapidity correlation parameters derived by E802 from the evolution of $k(\delta\eta)$, when adjusted for the charged particle composition. This report presents such direct measurements of correlations in ^{28}Si+Au collisions[27].

2. MEASUREMENTS OF BE CORRELATIONS IN ^{28}Si+Au

The BE correlation analysis is performed for pairs of negatively charged identified pions detected in the E802/E859 spectrometer from 14.6A GeV/c ^{28}Si+Au$\rightarrow 2\pi^-$+X central (10% TMA) collisions. For the present measurement[27], the 25 msr. spectrometer aperture spanned polar angles from 14°–28°, accepting pions with $150 \lesssim p_T \leq 700$ MeV/c and $1.5 \leq y \leq 2.0$, where the nucleon-nucleon center of mass rapidity is $y_{NN} = 1.7$. The azimuthal coverage ranged from 0.5 rad at the smallest polar angle to 0.25 rad at the largest, with an average value of $\delta\phi = 0.4$ rad.

Bose-Einstein intensity interferometry[28,26] exploits the fact that identical bosons emitted by a chaotic source of spatial extent R exhibit a correlation in relative momentum $q = p_2 - p_1$ which vanishes for $|q| \gtrsim \hbar/R$. The correlation function for BE interferometry measurements is closely related to the normalized two-particle correlation function, $R_2(p_1, p_2)$:

$$\mathrm{C}_2^{\rm BE}(p_1,p_2) = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)} = 1 + R_2(p_1,p_2) \quad .(15)$$

For $p_2 - p_1$ outside the region of BE correlation, it is assumed that there is neglible other correlation so that $\rho_2(p_1,p_2) = \rho_1(p_1)\rho_1(p_2)$ and therefore $\mathrm{C}_2^{\rm BE} \rightarrow 1$ and $R_2 \rightarrow 0$. For small values of the

argument $p_2 - p_1$, where there is full BE correlation: $C_2^{BE} \to 2$, $R_2 \to 1$. The BE correlation is traditionally represented by a gaussian in pair relative momenta quantities, such as $Q_{\rm inv}$

$$C_2^{BE}(Q_{\rm inv}) = 1 + \lambda\, e^{-Q_{\rm inv}^2 R_{\rm inv}^2} \qquad (16)$$

where the empirical parameter $\lambda \leq 1$ is introduced to account for the fact that not all detected pions come from a single chaotic source.

In order to form a correlation function corresponding to Eq. 15 from the measured sample of negative pion pairs (the *Actual* distribution of two-pion events), a *Background* sample must be found which exhibits all correlations induced by phase space, dynamics, experimental acceptance, etc., *except* those resulting from BE correlations. The method chosen is that of event mixing. Two events from the *Actual* sample are selected at random, with a check that the same event isn't matched with itself. Then one pion from each event is chosen at random to form a background pair. In order to not be limited by the statistics of the mixed events, approximately 5 times as many background pairs are formed for the present analysis. The measured correlation functions $\mathcal{C}_2^{BE}(v)$, are defined to be the ratios of the *Actual* distribution of negative pion pairs to the event mixed *Background*, plotted as a function of a variable v,

$$\mathcal{C}_2^{BE}(v) = \frac{A(v)}{B(v)} \qquad , \qquad (17)$$

where $v = Q_{\rm inv}$, Δy, or $\Delta\eta$. We correct for the inefficient measurement of two tracks with small opening angles in the *Actual* distribution. The correlation functions (numerator and denominator) are restricted to opening angles where this correction is less than or equal to two. Precisely the same set of pion pairs, and corrections, are used for all of the correlation functions, which are in effect just projections in the different variables. No corrections besides that for close-track inefficiency are applied to the data in this analysis—*i.e. no Coulomb corrections are applied.*

The measured BE correlation function[29] in $Q_{\rm inv}$ is traditionally fit to the gaussian form

$$\mathcal{C}_2^{BE}(Q_{\rm inv}) = \mathcal{N}[1 + \lambda_Q\, e^{-Q_{\rm inv}^2/(2\sigma_Q^2)}] \quad . \qquad (18)$$

The fit parameter $\mathcal{N}$ is the normalization constant, which just depends on the number of mixed events chosen for the *Background* distribution compared to the *Actual* sample. The only assumption required is that there is no other correlation outside the region of the B-E correlation, so that $C_2^{BE} \to 1$, $R_2 \to 0$. In practice, if the data exhibit a clear correlation peak and a significant region of constant $\mathcal{C}_2^{BE}$, then the normalization constant can be well determined by this procedure (e.g. see Fig. 4a). There remains the possiblity that the region of constant $\mathcal{C}_2^{BE}$ corresponds e.g. to a conventional two-particle correlation, $R_2 = \epsilon$, with much larger correlation length than the range of measurement. In this case, the above normalization procedure will lead to a systematic mismeasurement of λ_Q by a factor $1/(1+\epsilon)$.

The correlation functions in rapidity (or pseudo-rapidity) are parameterized as exponential, as in Eq. 9 (although a gaussian works just as well for the present data):

$$\mathcal{C}_2^{BE}(y_1, y_2) = \mathcal{N}[1 + \lambda_y\, e^{-|y_1 - y_2|/\xi_y}] \qquad (19)$$

$$\mathcal{C}_2^{BE}(\eta_1, \eta_2) = \mathcal{N}[1 + \lambda_\eta\, e^{-|\eta_1 - \eta_2|/\xi_\eta}] \quad . \qquad (20)$$

The projection of $\mathcal{C}_2^{BE}$ onto the longitudinal direction implicitly integrates over the limited azimuthal aperture of the spectrometer, $\delta\phi = 0.40$ rad, and other variables such as $\Delta E = E_2 - E_1$, the energy difference of the two pions. Thus λ_y, $\lambda_\eta \neq R(0,0)$, since $R(0,0)$ (Eq. 9) represents the strength of the two-particle rapidity correlation integrated over the full azimuth; and in general λ_y, $\lambda_\eta \leq \lambda_Q$—a full discussion is given below in section 4. It is evident, for the present analysis, that the same normalization constant $\mathcal{N}$ applies to any projection of the correlation function, since precisely the same *Actual* and *Background* events are used for all projections.

3. RESULTS

The present measurement of the $\pi^-\pi^-$ correlation function $C_2^{BE}(p_1, p_2)$ is shown in Fig. 4 as a function of the variable $Q_{\rm inv}$, together with the same data plotted as a function of the rapidity difference of the two pions, $|y_2 - y_1|$. The projection in $|\eta_2 - \eta_1|$ is very similar to $|y_2 - y_1|$ and is not shown here[27].

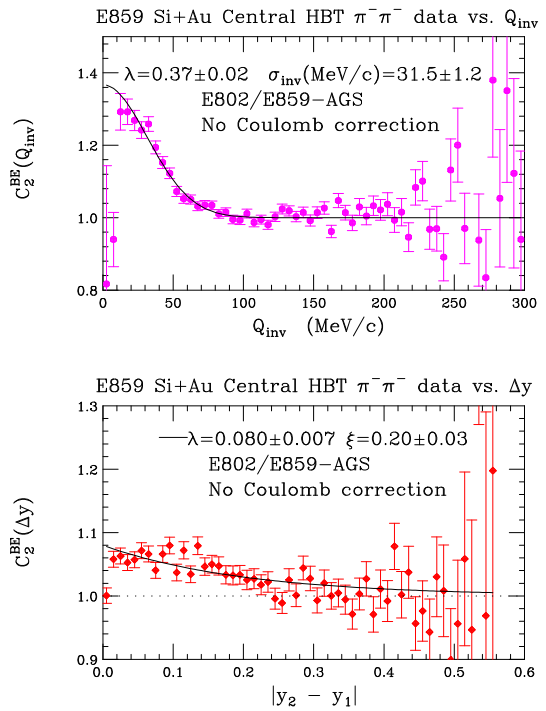

Figure 4. The Bose-Einstein correlation function C_2^{BE} as a function of Q_{inv} and $|y_2 - y_1|$ for pairs of identified π^- in central ^{28}Si+Au collisions. The solid lines are the fit discussed in the text.

In both projections the plotted data have been divided by the best fit value of the normalization $\mathcal{N}_Q = 0.22268$ whose error then appears in the fitted value of $\mathcal{N}/\mathcal{N}_Q = 1.000 \pm 0.003$. An evident correlation effect is visible for $Q_{inv} \leq 80$ MeV/c, along with a clear region of constant C_2^{BE} for $Q_{inv} \geq 100$ MeV/c, where there is no correlation so that the normalization constant $\mathcal{N}_Q$ can be precisely determined. In the $|y_2 - y_1|$ projection, the data show an ~8% drop over the range from 0 to 0.5; however it is not clear whether the correlation function has become constant or would continue to decrease for values greater than 0.5. As emphasized above, the key point for the present analysis is that the parameter $\mathcal{N}$ just represents the relative number of events in the *Actual* and *Background* distributions which is identical for the projections in the different variables, Q_{inv}, Δy, $\Delta\eta$. Thus, this constraint is applied in a common fit of C_2^{BE} for all 3 projections (Eqs. 18, 19, 20, solid lines on Fig. 4), which allows statistically significant values of the correlation parameters in rapidity $\lambda_y = 0.080 \pm 0.007$, $\xi_y = 0.20 \pm 0.03$, and pseudorapidity $\lambda_\eta = 0.083 \pm 0.007$, $\xi_\eta = 0.18 \pm 0.03$, to be obtained. To be consistent with the custom in short-range rapidity correlation analyses, no Coulomb correction is applied to the data. This correction is significant only for the two lowest points in Q_{inv} so they (and the lowest point in Δy) are not used in the fit—thus the absence of a Coulomb correction has no effect on the determination of the key normalization parameter. The fitted exponential correlation lengths $\xi_\eta = 0.18 \pm 0.03$ and $\xi_y = 0.20 \pm 0.03$ are equal within errors, as they should be since the pions are relativistic. Both values (for ^{28}Si+Au central collisions) agree impressively well with the previous indirect measurement[5] (for ^{16}O+Cu central collisions).

In order to determine whether the measured rapidity correlation is entirely due to the Bose-Einstein effect, two tests were performed. The $\pi^+\pi^-$ correlation, which has no BE interference[28], shows a constant value of C_2^{BE} in the $|y_2 - y_1|$ projection (see Fig. 5a)—this clearly establishes the correlation as being an identical particle effect. Secondly, for the identical $\pi^-\pi^-$, the region of $Q_{inv} \leq 100$ MeV/c (where the correlation effect is exhibited) was eliminated and the data were again plotted in terms of $|y_2 - y_1|$ (Fig. 5b): the data at larger values of $|y_2 - y_1|$ are identical, but the correlation at low values has vanished as illustrated by direct comparison with the uncut data points. This demonstrates that the two-pion short-range rapidity correlation in ^{28}Si+Au collisions is entirely due to the Bose-Einstein interference.

4. DISCUSSION AND RELATION TO OTHER RESULTS

In order to relate the strength parameter λ_η (Eq. 20) measured within the spectrometer ϕ aperture to the usual two-particle short-range strength $R(0,0)$ defined for the full azimuth

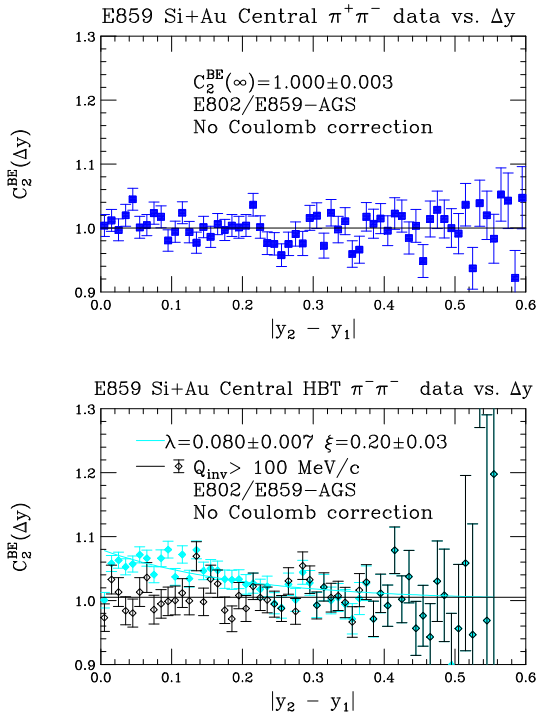

Figure 5. The Bose-Einstein correlation function C_2^{BE} as a function of $|y_2 - y_1|$ in central ^{28}Si+Au collisions. (a) $\pi^+\pi^-$ data for $|y_2 - y_1|$. (b) $\pi^-\pi^-$ data with the requirement $Q_{\rm inv} \geq 100$ MeV/c in comparison to the uncut data (Fig. 4b). The horizontal lines shown are the best fits to a constant. The best fits for the strength of a correlation with $\xi = 0.2$ (fixed) are $\lambda = 0.013 \pm 0.007$ for (a), $\lambda = 0.010 \pm 0.007$ for (b).

(Eq. 9), the dependence of the two-particle normalized correlation function on $\Delta\phi$ must be known. This can be reasonably well estimated for the case of BE correlations. The simplest model of the source to explain all the E802/E859 two-pion BE correlation data[30] is that of a spherically symmetric gaussian with finite lifetime:

$$R_2(|\vec{q}|, q_0) = \lambda_Q e^{-|\vec{q}|^2 R^2 - q_0^2 \tau^2} \quad . \tag{21}$$

This expression can be evaluated to lowest order[31] in ΔE, $\Delta\eta$, $\Delta\phi$:

$$R_2(p_1 + \Delta p, p_1) = \lambda_Q \exp -\{[(\Delta E/\beta)^2 + (p_T\Delta\eta)^2 + (p_T\Delta\phi)^2]R^2 + [(\Delta E)^2]\tau^2\} \tag{22}$$

which implies the relationship $\sigma_{\Delta\phi} = \sigma_{\Delta\eta} = 1/(\sqrt{2}p_T R)$ and $\sigma_{\Delta E} = 1/\sqrt{2(R^2/\beta^2 + \tau^2)}$, where $\beta = P/E$ is the velocity of either pion[31].

From the latest E802/E859 measurement[30] using the same data as the present analysis, $R = 2.80 \pm 0.11$ fm, with $< p_T >= 300$ MeV/c. This yields the prediction, $\sigma_{\Delta\phi} = \sigma_{\Delta\eta} = 0.166 \pm 0.007$, which is in excellent agreement with a direct gaussian fit of the present data[27], $\sigma_{\Delta\eta} = 0.149 \pm 0.015$. Consequently, we assume that $\sigma_{\Delta\phi} = \sigma_{\Delta\eta} = 0.149 \pm 0.015$ to extrapolate from the fit within the spectrometer aperture $\delta\phi = 0.40$ rad to the full azimuth. However, for simplicity in averaging, we convert Eq. 22 to an exponential form (cf. Eq. 9) by taking $\xi_\phi = \xi_\eta \simeq 1/(p_T R)$ —note that the empirical ratio of ξ_η to $\sigma_{\Delta\eta}$ from the present fit is $1.3(\sim \sqrt{2})$—and, similarly, by assuming $\xi_E \simeq \sqrt{2}\sigma_{\Delta E} = 1/\sqrt{R^2/\beta^2 + \tau^2}$:

$$R_2(p_1 + \Delta p, p_1) = \lambda_Q\, e^{-|E_1 - E_2|/\xi_E}\, e^{-|\phi_1 - \phi_2|/\xi_\phi}\, e^{-|\eta_1 - \eta_2|/\xi_\eta} \quad . \tag{23}$$

The average of this correlation function over the 3-dimensional interval $0 \leq \eta_1, \eta_2 \leq \delta\eta$, $0 \leq \phi_1, \phi_2 \leq \delta\phi$, $0 \leq E_1, E_2 \leq \delta E$ is then very similar in form to Eq. 12:

$$K_2(\delta\eta, \delta\phi, \delta E) = < R_2(p_1 + \Delta p, p_1) > |_{\delta\eta, \delta\phi, \delta E} = \lambda_Q\, H(\delta E/\xi_E)\, G(\delta\phi/\xi_\phi)\, G(\delta\eta/\xi_\eta) \tag{24}$$

where $G(x)$ is the same function as Eq. 13 and $H(x)$ is another function[32,33]. From the limits of $G(x)$ ($G(x) = 1$, for $x \ll 1$; $G(x) = 2/x$ for $x \gg 1$), it is easy to understand—for $\delta\phi$ and $\delta\eta$—why the effective strength of the correlation, $K_2 = < R_2 >$, decreases as the interval increases. For intervals much smaller than the correlation length, the average two-particle correlation strength is just the maximum value, while for intervals much larger than the correlation length, the average two-particle correlation strength becomes inversely proportional to the interval (for

uniform density) since the correlation only exists for particles within approximately one correlation length of each other.

Comparison of Eq. 24 to Eqs. 9 – 12 makes it clear that the determination of λ_η involves an average over δE as well as $\delta\phi$:

$$R(0,0) = \lambda_Q\, H(\delta E/\xi_E)\, G(2\pi/\xi_\phi) \quad (25)$$

$$\lambda_\eta = \lambda_Q\, H(\delta E/\xi_E)\, G(\delta\phi/\xi_\phi) \quad . \quad (26)$$

However, the ratio of Eqs. 25 and 26 can be used to obtain $R(0,0)$ from the measured value of λ_η, without knowledge of $H(\delta E/\xi_E)$ or λ_Q, if the azimuthal correlation length ξ_ϕ is known:

$$R(0,0) = \lambda_\eta \frac{G(2\pi/\xi_\phi)}{G(\delta\phi/\xi_\phi)} \quad . \quad (27)$$

The result, taking $\xi_\phi = 0.20 \simeq \xi_\eta$, is $R(0,0) = 0.9\%$ for identical pions, and must be reduced by a factor of ~ 2 for comparison to a mixture of pions with equal numbers of π^+ and π^-. This yields an overall $R(0,0) \simeq 0.45\%$, which is comparable to, but a factor of ~ 2 lower than previous direct measurements of $\sim 1\%$ for non-identified charged particles[34,35]. The same effect is seen in extrapolating the present $R(0,0)$ to correspond to the 200° aperture used in the E802 NBD analysis[5,14,36]: using $R(0,0) = 0.45\%$ yields $\bar{R}(0,0) \simeq 0.8\%$ (for Si+Au) in comparison with the measured 3.1%±0.5% for non-identified charged particles (O+Cu). The agreement is quite good, considering the crude extrapolation approximations[37] and the different reactions. Furthermore, the value of $H(\delta E/\xi_E)$ is difficult to estimate in general. For the present analysis, $H(\delta E/\xi_E) \simeq 0.4$ can be estimated from Eq. 26 using the measured values of λ_η and $\lambda_Q = 0.37 \pm 0.02$ [38].

To reiterate, the previous E802 analysis was for ^{16}O+Cu, while the present analysis is for ^{28}Si+Au (and a different rapidity range), again leading to an uncertainty. However, to the extent that $\xi_\eta = \xi_\phi \simeq 1/(< p_T > R)$ is a reasonable approximation, the correlation lengths should be in the ratio of $A^{1/3}$. Thus, the present measurement for ^{28}Si, $\xi_\eta = 0.18 \pm 0.03$, should be shifted to 0.22±0.03—an effect comparable to 1 standard deviation—for comparison with $\xi = 0.18 \pm 0.05$ from the ^{16}O+Cu NBD analysis. Hence, the effect of the extrapolation is minimal for the correlation length measurement, and the data agree very well. Interestingly, the present measurement of the two-particle rapidity correlation from BE interference in Si+Au central collisions at 14.6A GeV/c ($dn/dy \sim 35$), $R(0,0) \simeq 0.45\%$, $\xi_y = 0.20 \pm 0.03$, is in eerily good agreement with the only theoretical calculation of this effect in the literature[17,39].

5. CONCLUSIONS

The two-pion correlation in rapidity has been determined using a sample of pion pairs in the E802/E859 spectrometer from 14.6A GeV/c ^{28}Si+Au→$2\pi^-$+X, 10% central (TMA) collisions, for which a Bose-Einstein correlation analysis is available. Within the spectrometer aperture, no rapidity correlation is observed for $\pi^+\pi^-$ pairs, while $\pi^-\pi^-$ pairs exhibit a rapidity correlation of maximum strength 8.0%±0.7%, with exponential correlation length $\xi_y = 0.20 \pm 0.03$. The $\pi^-\pi^-$ rapidity correlation vanishes (1.0%±0.7%) when the region of the Bose-Einstein correlation (in $Q_{\rm inv}$) is eliminated, thus demonstrating that the pion-pair short-range rapidity correlation in ^{28}Si+Au central collisions is entirely due to the Bose-Einstein effect. The present direct measurements are in very good agreement with the previous indirect measurement[5] of the two-particle short-range rapidity correlation length and strength from the evolution with $\delta\eta$ of the NBD fit parameter $k(\delta\eta)$ in central ^{16}O+Cu collisions. Taken together, these measurements provide quantitative confirmation of the interrelationship of non-Poisson multiplicity fluctuations, the Negative Binomial Distribution, short-range rapidity correlations, intermittency and the Bose-Einstein effect, which has had much previous theoretical and experimental support.

5.1. Additional Discussion

From Eq. 24 we can get a general formula for the intermittency exponent ϕ_2:

$$K_2(\delta\eta, 2\pi, \delta E) = \lambda_Q\, H(\delta E/\xi_E)\, G(2\pi/\xi_\phi)\, G(\delta\eta/\xi_\eta)$$

$$\simeq \quad \sim 0.5 \qquad \xi_\phi/\pi \qquad G(x) \tag{28}$$

$$\phi_2 \;=\; -d\ln F_2/d\ln\delta\eta \;\simeq\; -dK_2/d\ln\delta\eta$$

$$\simeq \quad 0.5 \qquad \xi_\phi/\pi \qquad -xG'(x)$$

$$\phi_2 \;\sim\; 0.5 \qquad 1/(\pi < p_T > R) \quad 0.15$$

$$\phi_2 \;\sim\; 0.075/ \quad (\pi < p_T > R) \tag{29}$$

Thus, the apparent power law occurs because $xG'(x) \sim -0.15$ is nearly constant over the region where measurements are made ($1.0 \geq \delta\eta \geq 0.1$, $5 \geq x \geq 0.5$). The well observed decrease of ϕ_2 with increasing dn/dy (i.e. with increasing R) for central collisions—the so-called 'multiplicity' scaling rule[20,19,40,5]—is seen to be the result of the integral over ϕ, so is determined by ξ_ϕ. Using the parameters of the latest E802/E859 full BE analysis of the present data set[30], $<p_T>= 300$ MeV/c, and $R^{-1} = 70.4$ MeV/c, we obtain $\phi_2 \sim 0.018/\pi = 0.0055$ which should scale like $R_{\rm Si}/R$ for other reactions. Equation 29 for ϕ_2 clearly agrees with the E802 measurement[5] and the trend of the 'multiplicity' scaling rule[20,19,40,5], and thus bears further investigation.

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31. In the limit considered, the two pions are relativistic with nearly equal energies and are nearly parallel, with small difference in direction. Thus, the velocity of the pion pair is nearly the same as the velocity of either pion.
32. The weighting functions for the averages over η and ϕ are constant[33] as in the derivation of Eq. 12, but the energy distribution, roughly $\rho_1(E) \sim Ee^{-bE}$, is not constant and is further complicated by a cut-off at low energy imposed by the spectrometer.
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37. Thus no errors are quoted for extrapolated quantities.
38. The value of $\lambda_Q = 0.37 \pm 0.02$ for the present analysis with no Coulomb correction is reasonable in comparison to the fully corrected value[30] of 0.57 ± 0.02 which is, however, considerably smaller than the value of 1.0 expected for BE correlations.
39. The authors of reference[17] calculated the two-particle rapidity correlation from 'identical particle statistics' for $dn/dy = 25$ and $R = \tau = 3$fm, which happen to be very close to the conditions of the present measurement. Their results were: "the correlations from final-state interactions" (predominantly BE correlations) "extend only about two tenths of a unit of rapidity" with height $R(0,0)$ "about 0.5%" at mid-rapidity. "The height is determined mainly by the transverse radius of the source" (compare to Eq. 25).
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