Two-Particle Short Range Rapidity Correlations from the Bose-Einstein Effect and INTERMITTENCY A Quantitative Demonstration

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ABSTRACT

A measurement of the two-particle correlation of identified pions was performed in the E802/E859 magnetic spectrometer on the interval $1.5 \leq y \leq 2.0$, $\delta\phi = 0.4$ rad, for central ^{28}Si+Au collisions. It is demonstrated that the two-pion correlation in rapidity is entirely due to Bose-Einstein interference. The directly measured exponential correlation length is $\xi_y = 0.20 \pm 0.03$ for two π^-, with strength $R(0,0) \sim 1\%$, in agreement with previous E802 indirect measurements derived from an analysis of intermittency using Negative Binomial Distributions.

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INTERMITTENCY and CHAOS

The study of non-Poisson fluctuations of charged particle multiplicity distributions in small pseudorapidity intervals $\delta\eta \leq 1$ by many experiments has been heavily influenced by the utilization of normalized factorial moments (NFM)[1, 2] which are unity for a Poisson distribution:

$$F_q(\delta\eta) = \frac{< n(n-1)\ldots(n-q+1) >}{< n >^q} \quad , \tag{1}$$

where n is the multiplicity in a pseudorapidity interval (bin) of size $\delta\eta$ on a given event and the $< \ >$ brackets indicate averaging over all events. The normalized factorial moment with the clearest interpretation is

$$F_2 = \frac{< n(n-1) >}{< n >^2} = \frac{< n^2 > - < n >}{< n >^2} = \frac{\sigma^2 + < n >^2 - < n >}{< n >^2} = 1 + \frac{\sigma^2}{\mu^2} - \frac{1}{\mu} \tag{2}$$

where $\mu \equiv < n >$ is the mean and $\sigma \equiv \sqrt{< n^2 > - < n >^2}$ is the standard deviation. A mechanism, dubbed intermittency, was proposed[1], which would be indicated by a power-law increase of multiplicity distribution moments over pseudorapidity bins as the bin size is reduced:

$$F_q(\delta\eta) \propto (\delta\eta)^{-\phi_q} \quad . \tag{3}$$

The scale-invariant power-law dependence with singular behavior as $\delta\eta \longrightarrow 0$ was suggestive of the physics of phase transitions, chaos, and fractals.

Many experiments applied the formalism to their data, leading to the observation[2] of the predicted power law behavior in the region $1 \geq \delta\eta \geq 0.1$. However, the observation of tantalizing power-laws tended to obscure the fact that multiplicity distributions were well known to be non-Poisson because of short-range rapidity correlations in multi-particle production[3, 4]. On the other hand, the variation in NFM as a function of $\delta\eta$ means that the fluctuations, i.e. the shapes of the multiplicity distributions, change as a function of $\delta\eta$.

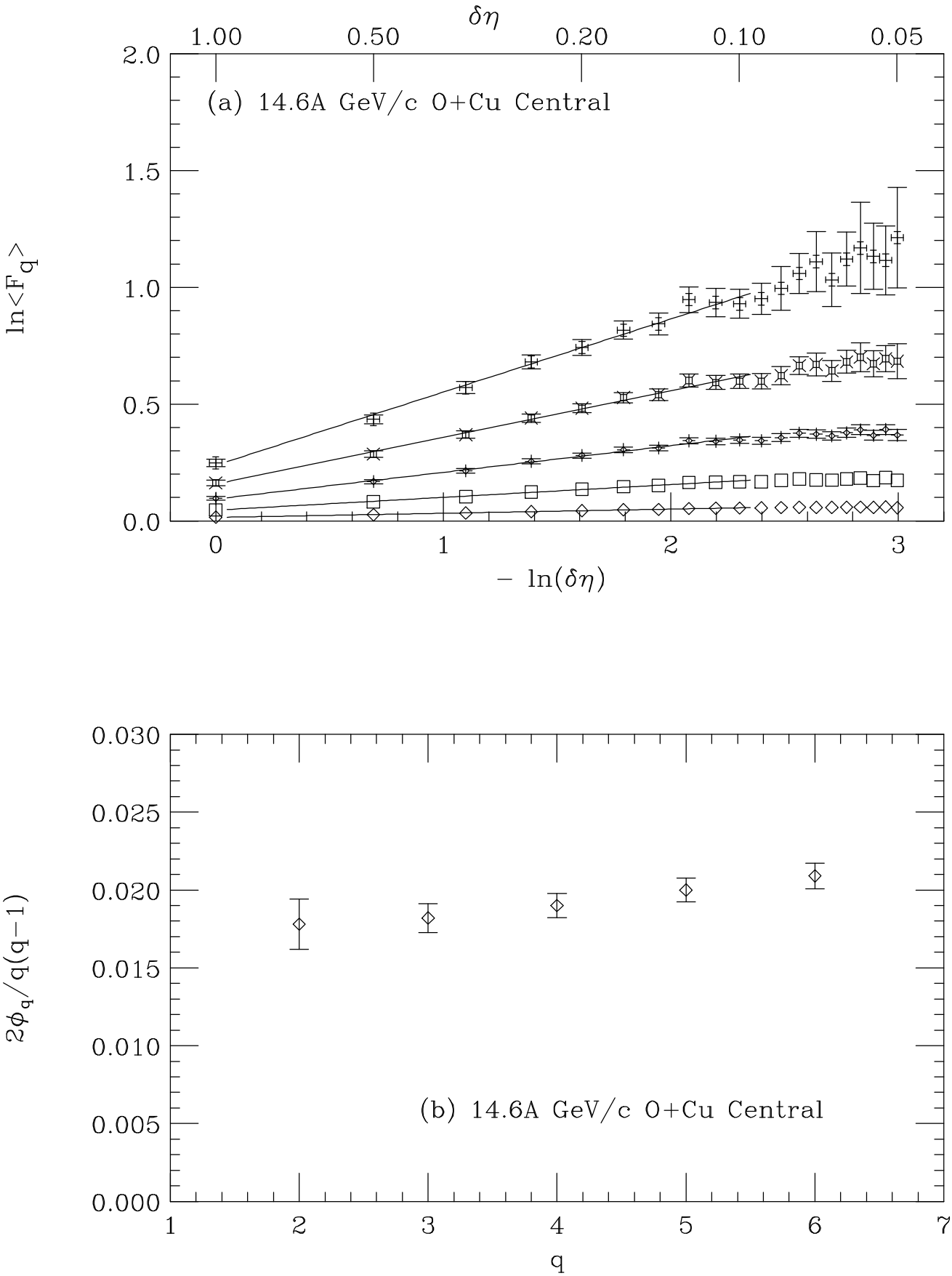
δη
1.00
0.50
0.20
0.10
0.05
2.0
1.5
1.0
0.5
0.0
(a) 14.6A GeV/c O+Cu Central
ln<F_q>
0
1
2
3
− ln(δη)
0.030
0.025
0.020
0.015
0.010
0.005
0.000
2φ_q/q(q−1)
(b) 14.6A GeV/c O+Cu Central
1
2
3
4
5
6
7
q

Figure 1

Problems with the Intermittency Formalism

- The plots are "upside down and backwards"—$\delta\eta$ decreases in the "$+x$" direction, and smaller $\delta\eta$ intervals result in larger "y" values. *It has all the earmarks of dividing by zero.*
- An intermittency plot with many orders can be made from experiments with few events, **even ONE EVENT!**
  See my 1993 talk.
- The points in $\delta\eta$ are not independent
- The different orders are not independent. F_q includes all random combinations of lower order correlations $\rightarrow$ use Normalized Factorial cumulants K_q.
- The normalized factorial cumulants, K_q, which are zero if there is no direct q particle correlation (Poisson distribution), are just the normalized factorial moments, F_q, with all q-fold combinations of lower order correlations subtracted[3]: $K_2 = F_2 - 1$, $K_3 = F_3 - (1 + 3K_2)$, $K_4 = F_4 - (1 + 6K_2 + 3K_2^2 + 4K_3) \ldots$
- The change in moments as a function of $\delta\eta$ means that the fluctuations, i.e. the shape of the multiplicity distribution, change as a function of $\delta\eta$.
- If the multiplicity distribution happens to be a well known function, e.g. NBD, which depends on only 2 parameters, e.g. $k(\delta\eta)$ and $\mu(\delta\eta)$, it is obvious that all the q-orders of moments only depend on the 2 parameters.
- The Normalized Factorial Moments and Cumulants of the NBD are particularly simple:

$$F_q = F_{(q-1)}(1 + \frac{q-1}{k}) \qquad K_q = \frac{(q-1)!}{k^{q-1}} \quad .$$

Beauty of the Negative Binomial Dist.

- The Negative Binomial Distribution of an integer m, $\mu \equiv < m >$, is:

$$P(m) = \frac{(m+k-1)!}{m!(k-1)!} \frac{(\frac{\mu}{k})^m}{(1+\frac{\mu}{k})^{m+k}}$$

- The NBD, with the additional parameter k compared to a Poisson, becomes Poisson in the limit $1/k = 0$. Thus statisticians use the NBD parameter $1/k$ as a test for whether a distribution is Poisson, i.e. as "a test for independence in rare events."

$$\frac{\sigma^2}{\mu^2} = \frac{1}{\mu} + \frac{1}{k(\delta\eta)} \qquad (4)$$

where $\mu \equiv < n(\delta\eta) >$ is the mean multiplicity on the interval, $\sigma \equiv \sqrt{< n^2 > - < n >^2}$.

- The NBD fluctuates more than a POISSON by an amount $1/k$.

- The normalized factorial moment F_2 is simply related to $1/k$

$$\frac{1}{k(\delta\eta)} = \frac{\sigma^2}{\mu^2} - \frac{1}{\mu} = F_2(\delta\eta) - 1 = K_2(\delta\eta) \qquad (5)$$

where $K_2(\delta\eta)$ is a normalized factorial cumulant[3].

Factorial Moments are simply Integrals

- The factorial moments and cumulants of the multiplicity on an interval $\delta\eta$ are simply related to the integrals of the q-particle inclusive rapidity densities $\rho_q(y_1, \ldots y_q)$, where $\rho_1(y) = dn/dy$ and $\rho_2(y_1, y_2)$ are the inclusive densities for a single particle (at rapidity y) or 2 particles (at rapidities y_1 and y_2), etc :

$$\int^{\delta\eta} dy_1 \; \rho_1(y_1) \;=\; < n > \tag{6}$$

$$\int^{\delta\eta} dy_1 dy_2 \; \rho_2(y_1, y_2) \;=\; < n(n-1) > = < n >^2 \; F_2 \tag{7}$$

$$\int^{\delta\eta} dy_1 \cdots dy_q \; \rho_q(y_1, \ldots y_q) \;=\; < n(n-1)\ldots(n-q+1) > = < n >^q \; F_q \tag{8}$$

- These integrals (or moments) are sensitive to any short-range rapidity correlation in particle production, since if there would be no correlation then

$$\rho_q(y_1, \ldots y_q) = \rho_1(y_1)\rho_1(y_2)\cdots\rho_1(y_q) \qquad , \tag{9}$$

  in which case all the F_q reduce to unity, a Poisson distribution.

- Mueller[3] introduced a series of functions to describe correlations in multiparticle emission. For instance, the normalized two-particle short-range rapididy correlation function $R_2(y_1, y_2)$ is defined as:

$$R_2(y_1, y_2) \equiv \frac{C_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} \equiv \frac{\rho_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1 = R(0,0)\, e^{-|y_1 - y_2|/\xi} \tag{10}$$

- $C_2(y_1, y_2) = \rho_2(y_1, y_2) - \rho_1(y_1)\rho_1(y_2)$ is the Mueller correlation function for 2 particles (which is zero for the case of no correlation) and ξ is the two-particle short-range rapidity correlation length[3, 4] for an exponential parameterization.

Measurements by AGS-E802/E859

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ANL - BNL - UCBerkeley - UCRiverside - Columbia - INS(Tokyo) - Kyoto - Kyushu - LLNL - MIT - NYU - Tokyo - Tsukuba - Yonsei

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- The E802 Collaboration[5] measured charged particle multiplicity distributions from central (ZCAL) collisions of O+Cu at 14.6A GeV/c. The fluctuations of these distributions as a function of the pseudrapidity interval was studied by the method of power-law evolution of normalized factorial moments and also by directly fitting the measurements to Negative Binomial Distributions (NBD).

- This led to a simple, elegant and quantitative understanding of multiplicity fluctuations, without resort to fractals or power-law divergences, in terms of the two-particle short-range rapidity correlation length which could be derived from the data and turned out to be much shorter and weaker than the values for hadron collisions—a small, but clearly finite scale, thus no intermittency.

- The surprisingly short two-particle rapidity correlation length, $\xi \sim 0.2$ also explains the large bin-by-bin multiplicity fluctuations in small $\delta\eta$ bins in heavy ion collisions, notably from Si+AgBr interactions in cosmic rays[6].

- A new, direct measurement of identified pion correlations in central Si+Au collisions is reported, which is in excellent agreement with the previous indirect measurement and confirms quantitatively the relationship of non-Poisson multiplicity fluctuations, the Negative Binomial Distribution, short-range rapidity correlations, intermittency and the Bose-Einstein effect, which has had much previous theoretical and experimental support.

- In nucleus-nucleus collisions, the conventional hadron short-range correlations are washed out by the random superposition of correlated sources, so that only the quantum-statistical Bose-Einstein (B-E) correlations of identical particles remain[7].

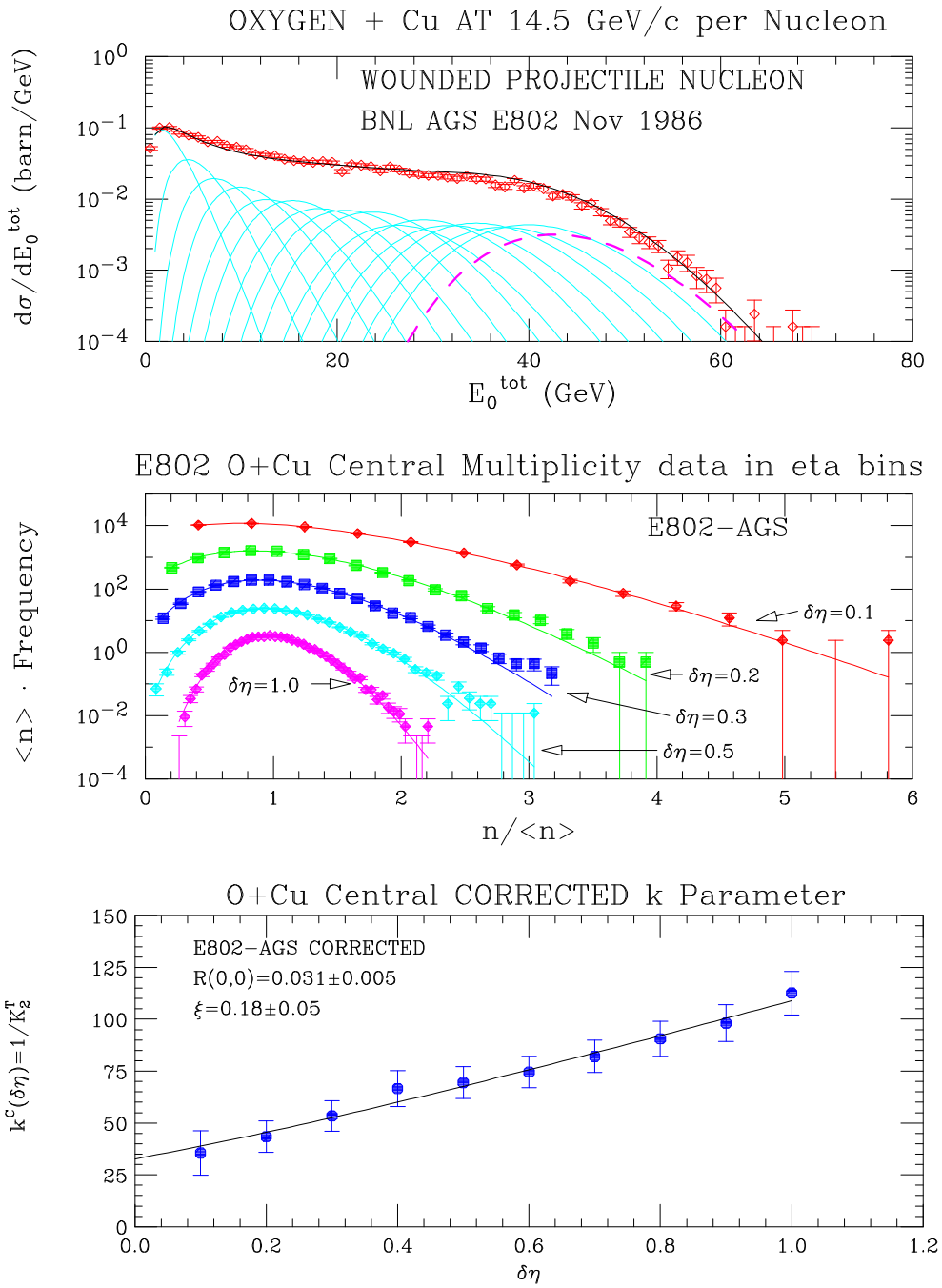
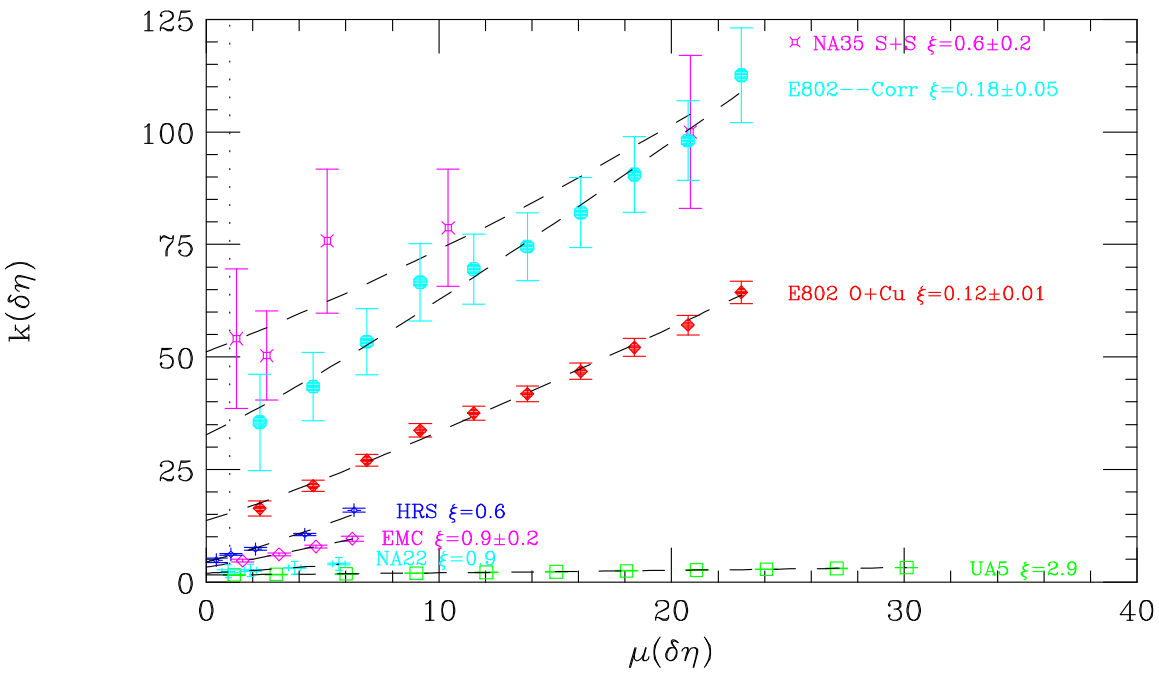
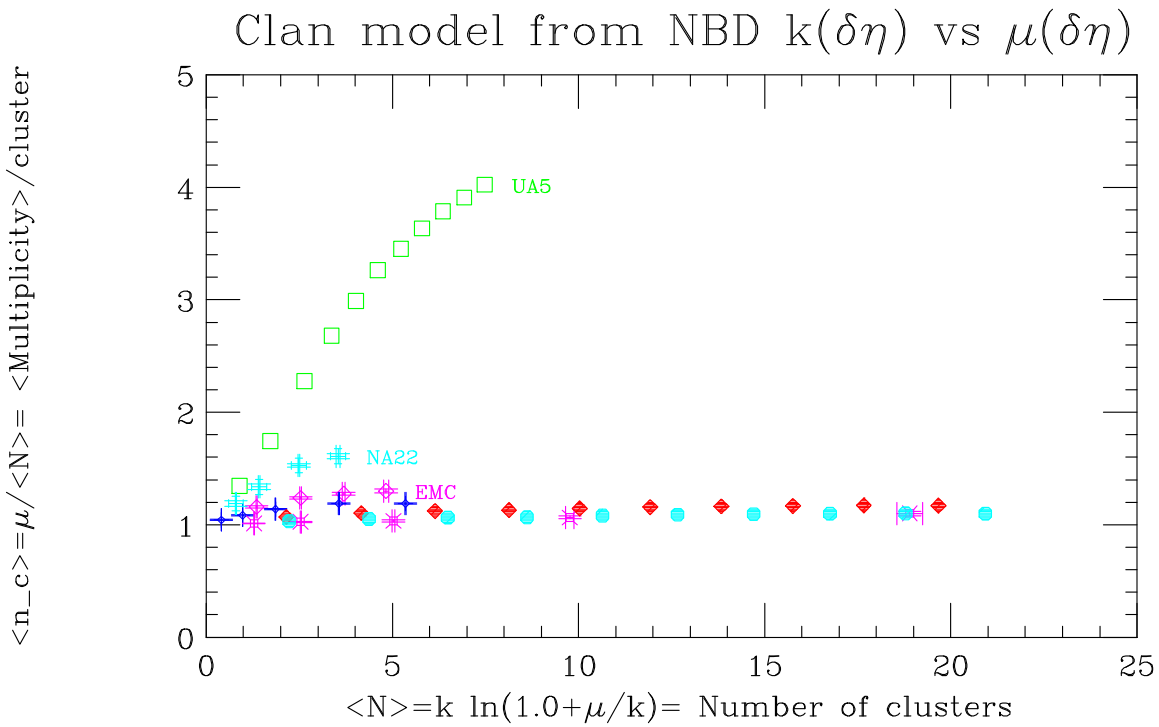


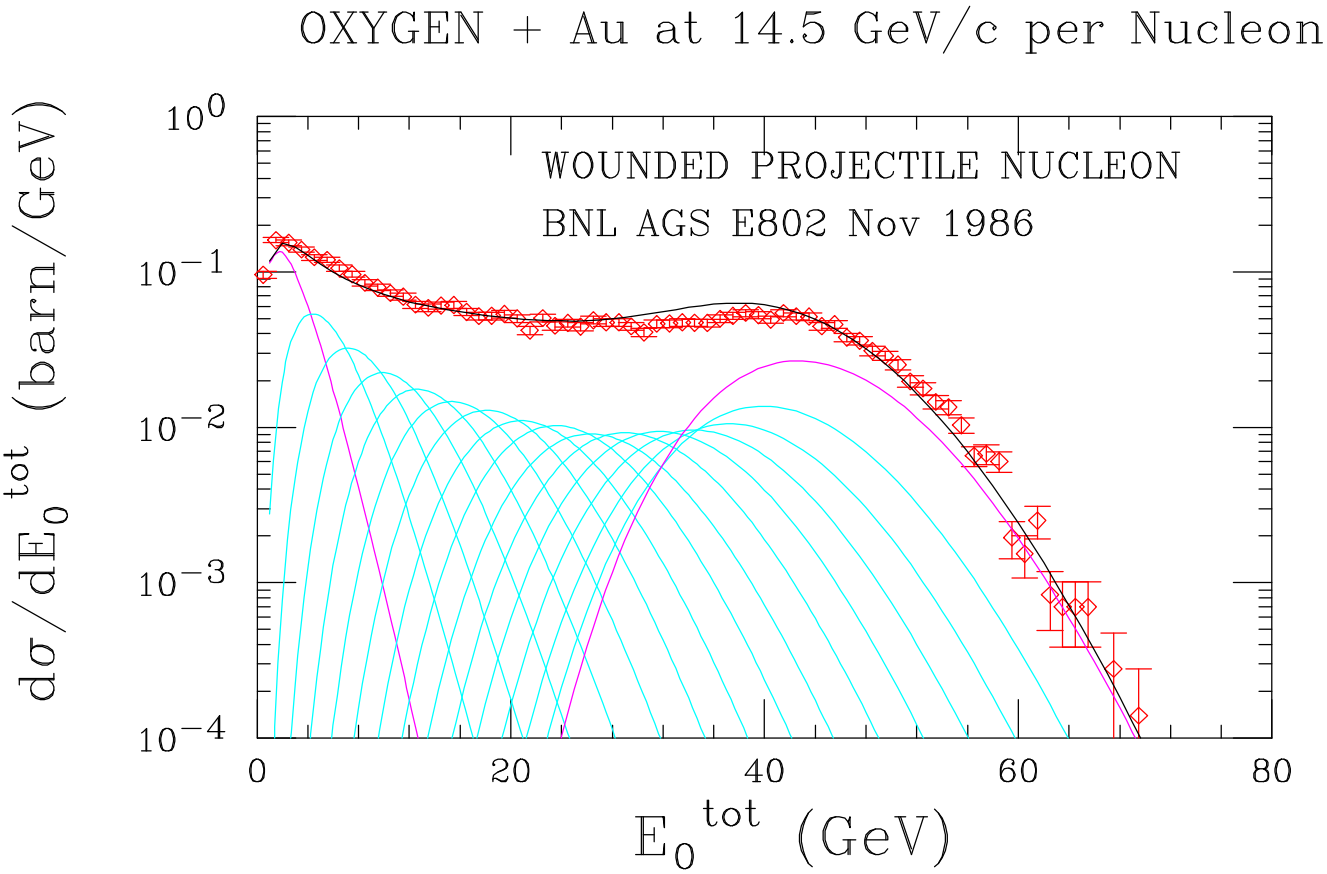
Figure 2: (a) Measured energy emission in $\delta\eta \sim 1$ for O+Cu collisions. The solid curve, which fits the data very well is the sum of the 16 lower curves which represent the energy emitted when from 1 to 16 projectile nucleons interact in the target nucleus with 'geometric' probabilities. (b) Multiplicity distributions measured in O+Cu central collisions as a function of the interval $\delta\eta$ (indicated) for the case when all 16 incident nucleons have interacted, as determined by a Zero degree calorimeter. The data labelled $\delta\eta = 1.0$ correspond to the highest energy (dashed) sub-curve in (a). The curves are NBD fits. (c) $k(\delta\eta)$ from the NBD fits corrected for instrumental effects. The solid line is a fit to Eq. 13 with the parameters indicated.

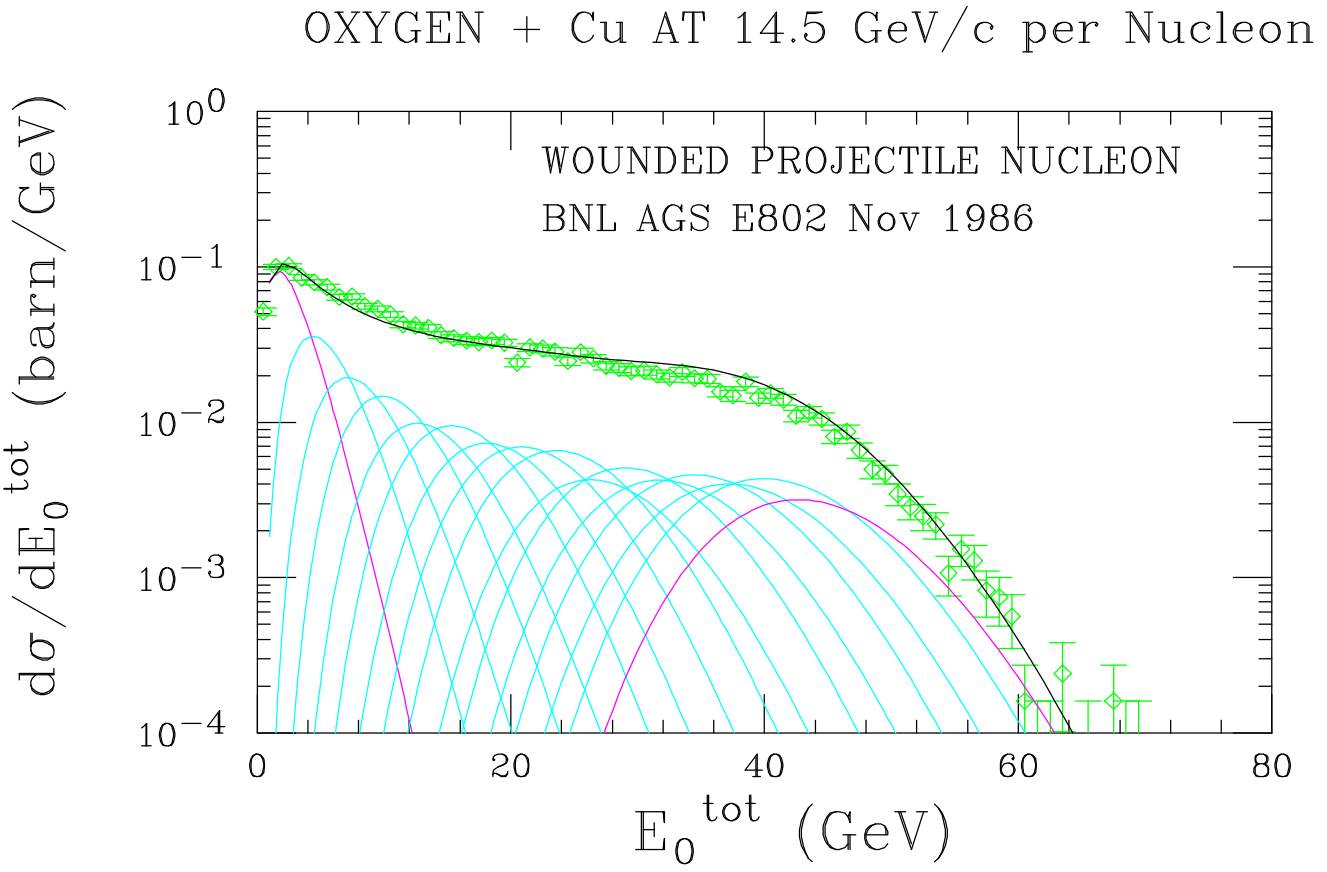
- By a stroke of good fortune (?), the E802 multiplicity distributions from central (ZCAL) collisions of ^{16}O+Cu at 14.6A GeV/c as a function of the pseudorapidity interval $\delta\eta \geq 0.1$ in the range $1.2 \leq \eta \leq 2.2$ exhibit excellent fits to NBD in all $\delta\eta$ bins, allowing, for the first time, a systematic formulation of the subject of intermittency in terms of distributions.
- The exact details of the centrality cut are important for the shapes of the distributions in and for the analysis by moments. The shapes of the distributions below the mean value are determined by the centrality cut.
- The E802 ZCAL centrality requirement—that the forward-going kinetic energy be less than that of a single projectile nucleon (13.6 GeV)—gave smooth distributions which vary uniformly with $\delta\eta$ and gave excellent NBD fits. **Is that just luck, or is it more meaningful?**
- In agreement with all previous measurements of NBD fits to multiplicity distributions in hadron and lepton reactions, the E802 k parameter shows an apparently linear increase with the $\delta\eta$ interval, albeit with a much steeper slope than for other reactions, and a non-zero intercept, $k(0) \neq 0$.

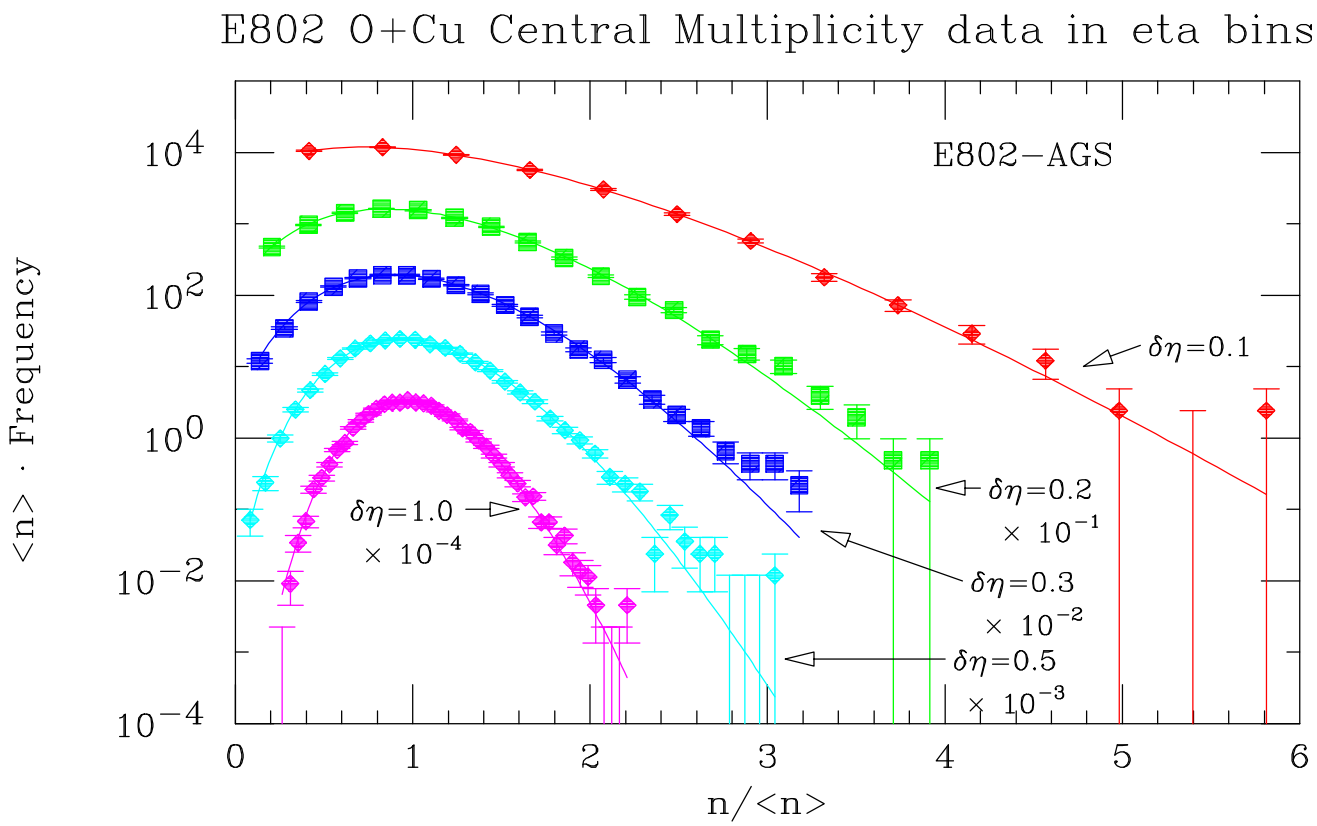
All Measurements of NBD fits Show 'Linear' $k(\delta\eta)$ with non-zero Intercept $k(0) \neq 0$ This Plot Leads Naturally to the Clan Model

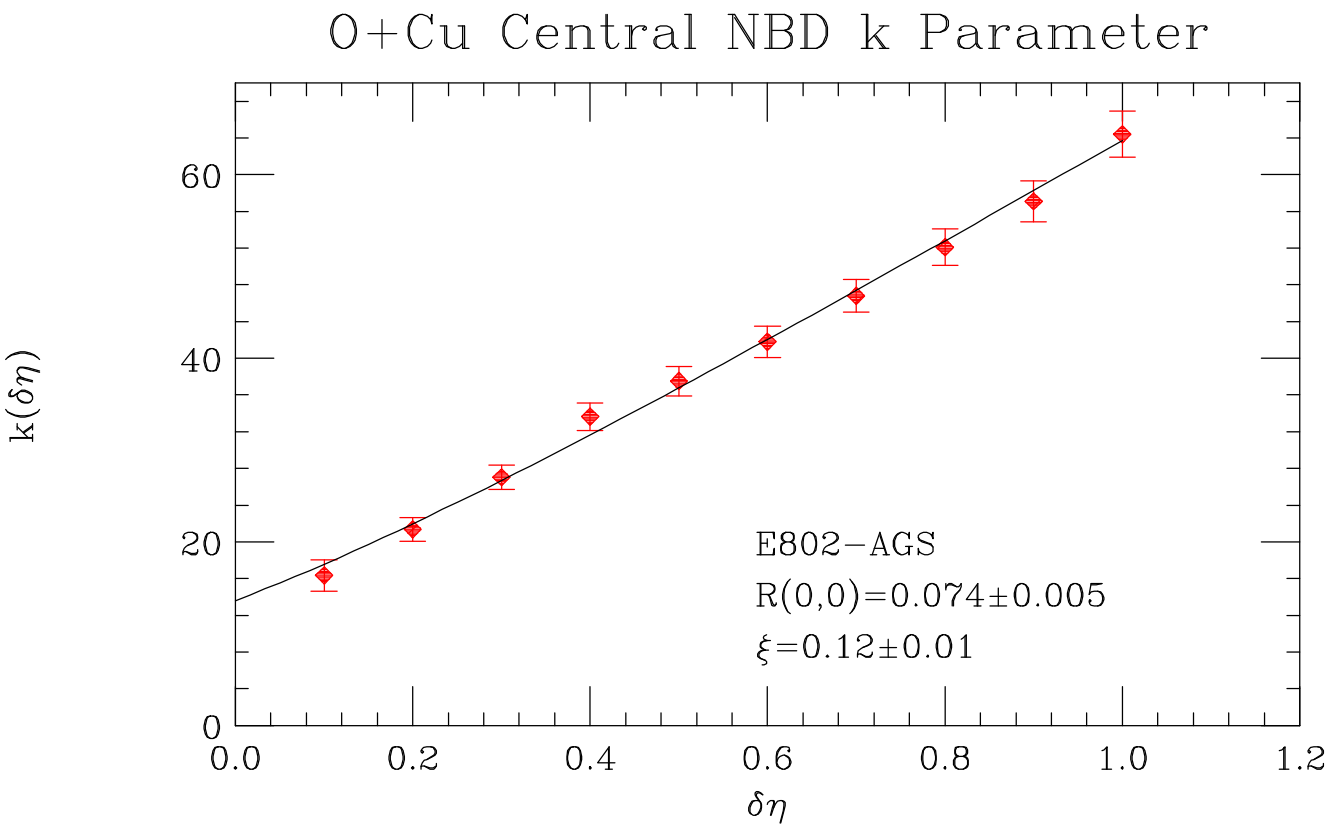





OXYGEN + Au at 14.5 GeV/c per Nucleon
WOUNDED PROJECTILE NUCLEON
BNL AGS E802 Nov 1986
dσ/dE0tot (barn/GeV)
E0tot (GeV)
10^0
10^-1
10^-2
10^-3
10^-4
0
20
40
60
80


OXYGEN + Cu AT 14.5 GeV/c per Nucleon
WOUNDED PROJECTILE NUCLEON
BNL AGS E802 Nov 1986
dσ/dE0tot (barn/GeV)
E0tot (GeV)
10^0
10^-1
10^-2
10^-3
10^-4
0
20
40
60
80


E802 O+Cu Central Multiplicity data in eta bins
E802−AGS
$\delta\eta$=0.1
$\delta\eta$=0.2
× 10^{-1}
$\delta\eta$=0.3
× 10^{-2}
$\delta\eta$=0.5
× 10^{-3}
$\delta\eta$=1.0
× 10^{-4}
<n> · Frequency
n/<n>


O+Cu Central NBD k Parameter
E802−AGS
R(0,0)=0.074±0.005
ξ=0.12±0.01
k($\delta\eta$)
$\delta\eta$

Distributions, Factorial Moments, Cumulants and the Two-Particle Rapidity Correlation

- In short-range rapidity (or pseudo-rapidity) correlation analyses, the normalized two-particle correlation function is usually taken as an exponential:

$$R_2(y_1, y_2) = \frac{\rho_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1 = R(0,0)\, e^{-|y_1 - y_2|/\xi} \quad , \tag{11}$$

  where $\rho_1(y)$ and $\rho_2(y_1, y_2)$ are the inclusive densities for a single particle (at rapidity y) or 2 particles (at rapidities y_1 and y_2), and ξ is the two-particle short-range rapidity correlation length[3, 4].

- The two-particle normalized factorial moment (cumulant), F_2 (K_2), is nothing other than the integral of the two-particle correlation function on the interval:

$$K_2(\delta\eta) = F_2(\delta\eta) - 1 = \frac{\int^{\delta\eta} dy_1 dy_2\, \rho_1(y_1)\rho_1(y_2) R_2(y_1, y_2)}{\int^{\delta\eta} dy_1 dy_2\, \rho_1(y_1)\rho_1(y_2)} \quad . \tag{12}$$

- The integrand in the numerator, is the Mueller correlation function, $C_2(y_1, y_2) = \rho_1(y_1)\rho_1(y_2)R_2(y_1, y_2) = \rho_2(y_1, y_2) - \rho_1(y_1)\rho_1(y_2)$, which is used in the discussion of conventional short-range correlations in multiparticle physics[3, 4]. The denominator of Eq. 12 is simply $< n(\delta\eta) >^2$, the square of the mean multiplicity on the interval.

- For the case of a NBD, $K_2(\delta\eta) = 1/k(\delta\eta)$.

- If the inclusive single particle density, $\rho_1(y) = dn/dy$, is assumed constant on the interval, then the integral can be performed analytically (specifically on the interval $0 \leq y_1 \leq \delta\eta$, $0 \leq y_2 \leq \delta\eta$) to obtain the normalized factorial moment $F_2(\delta\eta)$ or normalized factorial cumulant $K_2(\delta\eta)$ in terms of the parameters of Eq. 11 [8, 9, 10, 11, 12]:

$$K_2(\delta\eta) = R(0,0)\, G(\delta\eta/\xi) \quad , \tag{13}$$

  where the function $G(x)$ is defined as:

$$G(x) = 2\,\frac{(x - 1 + e^{-x})}{x^2} \quad . \tag{14}$$

- The correlation length ξ (actually ξ_η) and strength for central ^{16}O+Cu collisions were derived by E802[5] from a fit of the measured $K_2(\delta\eta) = 1/k(\delta\eta)$ to this integral function.

- It is clear in this formulation (Eq. 12) that the normalized factorial cumulant $K_2(\delta\eta)$ is simply the mean value of the normalized two-particle correlation function $R_2(y_1, y_2)$ on the interval $\delta\eta$, with weighting by the square of the inclusive single particle density:

$$K_2(\delta\eta) = < R_2(y_1, y_2) > |_{\delta\eta} \quad . \tag{15}$$

- For constant single particle density, the evolution of $K_2(\delta\eta)$ as a function of $\delta\eta$ depends on the scaled variable $x = \delta\eta/\xi$, and from the limits of $G(x)$ ($G(x) = 1$, for $x \ll 1$; $G(x) = 2/x$ for $x \gg 1$) is quite easy to understand:

$$K_2(\delta\eta) = < R_2(y_1, y_2) > |_{\delta\eta} = R(0,0) \qquad \text{for} \quad \delta\eta \ll \xi \tag{16}$$

$$K_2(\delta\eta) = < R_2(y_1, y_2) > |_{\delta\eta} = 2\,\frac{R(0,0)}{\delta\eta/\xi} \qquad \text{for} \quad \delta\eta \gg \xi \quad . \tag{17}$$

- For intervals much smaller than the correlation length, the average two-particle correlation strength is just the maximum value, while for intervals much larger than the correlation length, the average two-particle correlation strength becomes inversely proportional to the interval (for uniform density) since the correlation only exists for particles within approximately one correlation length of each other.

- In the limit of vanishing correlation length, $\xi \to 0$ (i.e. $\xi \ll \delta\eta_0$, where $\delta\eta_0$ is the smallest interval that can be resolved), the limit of Eq. 17 applies, and $K_2(\delta\eta)$ is in fact singular as $\delta\eta \to 0$. This would correspond to the case of true intermittency in multiparticle production, the apparent absence of scale, or more precisely a scale much smaller than the resolution of a particular measurement. Thus intermittency would require a zero intercept $k(0) = 0$ for the NBD analyses. Conversely, the non-zero intercept $k(0) = 1/K_2(0) \neq 0$ shown by all NBD data again implies that there is no intermittency.

If you are still Infatuated with Moments, Try These!
MJT PL B347 (1995) 431

- In my opinion, it is preferable to search for divergent $K_q(\delta\eta)$ as $\delta\eta \rightarrow 0$ by looking at convergent quantities like $k(\delta\eta) = 1/K_2(\delta\eta)$, since it is much easier to distinguish zero from non-zero, than finite from infinite.

- Thus, I have proposed a series of moments $T_q(\delta\eta)$ which are 'inverse' quantities. The moments $T_q(\delta\eta)$ should be examined as a function of $<n(\delta\eta)>$, where

$$T_q(\delta\eta) \equiv \left(\frac{(q-1)!}{K_q(\delta\eta)}\right)^{\frac{1}{q-1}}$$

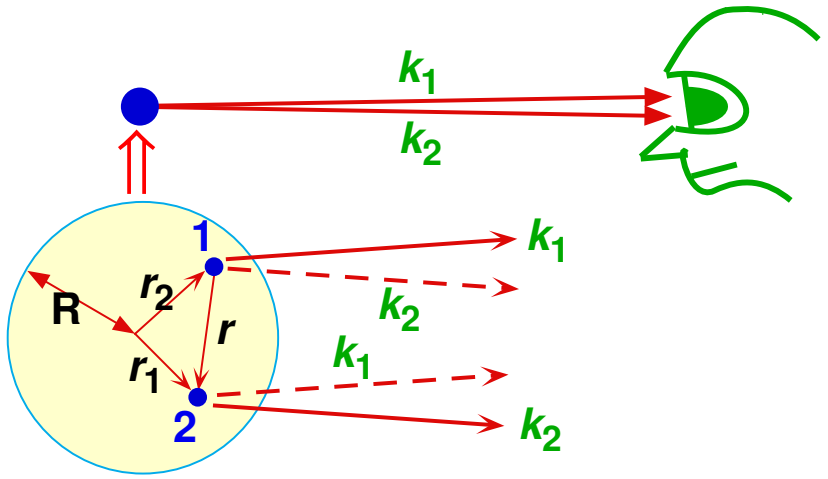
- In the case of the NBD, $T_q(\delta\eta) = k(\delta\eta)$, and this plot is identical to the $k(\delta\eta)$ vs $\mu(\delta\eta)$ plot for all orders. In the linked-pair approximation, where $K_q \equiv A_q K_2$, the plots for the different orders will have identical dependences on $< n(\delta\eta) >$, but the normalizations will be different, from which the factors A_q can be determined.

- Note that this is designed to be a linear plot on both axes, with a simple test for intermittency: intermittency exists only if the intercept $T_q(0) = 0$; if $T_q(0) \neq 0$ there is no intermittency.

- More generally, if $T_q(\delta\eta) \propto <n(\delta\eta)>^{\tau_q}$ as $<n(\delta\eta)> \rightarrow 0$: there is no intermittency if $\tau_q \leq 0$; and there is intermittency, with intermittency exponents $\phi_q^K = \tau_q\,(q-1)$, if $\tau_q > 0$. Obviously, if $T_q(0) \neq 0$, there is no intermittency, since $K_q(0)$ is finite.

Bose-Einstein Correlations
Drawn by Shoji Nagamiya

Hanbury-Brown/Twiss Effect



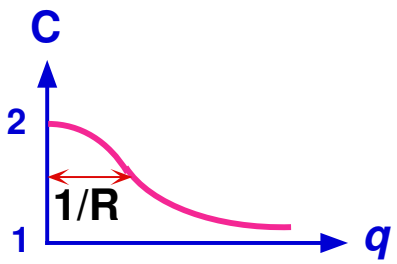
$$C = \left| \frac{e^{ik_1 r_1} e^{ik_2 r_2} + e^{ik_1 r_2} e^{ik_2 r_1}}{2} \right|^2$$

$$= 1 + \cos(q \cdot r)$$

$q = k_1 - k_2$, $r = r_1 - r_2$

$$= 1 + \exp(-q^2 R^2/2)$$

for Gaussian source



$$\rho = \frac{\text{Nucleon number}}{4\pi R^3/3}$$

Measurements in ^{28}Si+Au collisions

- The B-E correlation analysis is performed for pairs of negatively charged pions detected in the E802/E859 spectrometer from central (upper 10% TMA) collisions of 14.6A GeV/c ^{28}Si+Au$\rightarrow 2\pi^-$+X. Tracks with a measured $1/\beta$ (where β is the velocity) within 3σ of the value for a pion with the same reconstructed momentum, for momenta below 1.82 GeV/c (the π-K 3σ crossing), are identified as pions. For these data, the spectrometer spanned polar angles from 14°–28°, accepting pions with $150 \lesssim p_T \leq 700$ MeV/c and $1.5 < y < 2.0$, where the nucleon-nucleon center of mass rapidity is $y_{NN} = 1.7$. The azimuthal coverage ranged from 0.5 rad at the smallest polar angle to 0.25 rad at the largest, with an average value of $\delta\phi = 0.4$ rad.

- The short-range rapidity (or pseudo-rapidity) correlation notation, and the B-E notation are slightly different[13]:

$$\mathrm{C_2^{BE}}(p_1,p_2) = \frac{\rho_2(y_1,y_2)}{\rho_1(y_1)\rho_1(y_2)} = 1 + R_2(p_1,p_2) \quad , \tag{18}$$

  where $R_2(p_1,p_2)$ is the normalized two-particle correlation function which is usually taken as an exponential (see Eq. 11), although a gaussian form works just as well for the present data.

- In order to form a correlation function corresponding to Eq. 18, the measured correlation functions for B-E interferometry, $\mathcal{C}_2^{BE}(v)$, are defined to be the ratio of an *Actual* distribution of negative pion pairs to an event mixed *Background*,

$$\mathcal{C}_2^{BE}(v) = \frac{A(v)}{B(v)}, \tag{19}$$

  where $v = Q_{\mathrm{inv}}$, Δy, or $\Delta\eta$. We correct for the inefficient measurement of two tracks with small opening angles in the *Actual* distribution. The correlation functions (numerator and denominator) are restricted to the region where this correction is less than or equal to two [14]. The same set of pion pairs, and corrections, are used for all three of the correlation function projections. No corrections besides that for close-track inefficiency are applied to the data in this analysis—*i.e. no Coulomb corrections are applied.*

Forms of the Two-particle Correlation Function Used for HB-T Measurements

- The distribution in $Q_{\rm inv}$ for B-E correlations is traditionally fit to the gaussian form

$$\mathcal{C}_2^{BE}(Q_{\rm inv}) = \mathcal{N}_Q[1+\lambda_Q\, e^{-Q_{\rm inv}^2 R_{\rm inv}^2}] \quad (20)$$

or equivalently

$$\mathcal{C}_2^{BE}(Q_{\rm inv}) = \mathcal{N}_Q[1+\lambda_Q\, e^{-Q_{\rm inv}^2/(2\sigma_Q^2)}] \quad . \quad (21)$$

  The fit parameter $\mathcal{N}_Q$ is simply a normalization constant; its value depends on the number of events chosen for the $B(Q_{\rm inv})$ distribution.

  The factors in brackets are the Bose-Einstein 2-particle correlation functions $\mathrm{C_2^{BE}}(p_1,p_2)$ where the "chaoticity" parameter $\lambda_Q = 1.0$ for full Bose-Einstein interference.

- In Eq. 11, $R(0,0)$ represents the strength of the full-azimuth rapidity correlation. However, the present analysis occurs within the limited azimuthal aperture of the spectrometer, $\delta\phi = 0.40$ rad, so the following notation is used for the fits within the spectrometer aperture (cf. Eq. 21) :

$$\mathcal{C}_2^{BE}(y_1,y_2) = \mathcal{N}_y[1+\lambda_y\, e^{-|y_1-y_2|/\xi_y}] \quad (22)$$

$$\mathcal{C}_2^{BE}(\eta_1,\eta_2) = \mathcal{N}_\eta[1+\lambda_\eta\, e^{-|\eta_1-\eta_2|/\xi_\eta}] \quad . \quad (23)$$

- The key point for the present analysis is that the parameter $\mathcal{N}$ must be identical for the projections in the 3 different variables, $Q_{\rm inv}$, $\Delta\eta$, Δy, since the data are identical. Thus, this constraint is applied in a common fit of $\mathcal{C}_2^{BE}$ for all 3 projections (Eqs. 21, 22, 23), which greatly improves the statistical precision of the values of λ_η, ξ_η, λ_y and ξ_y.

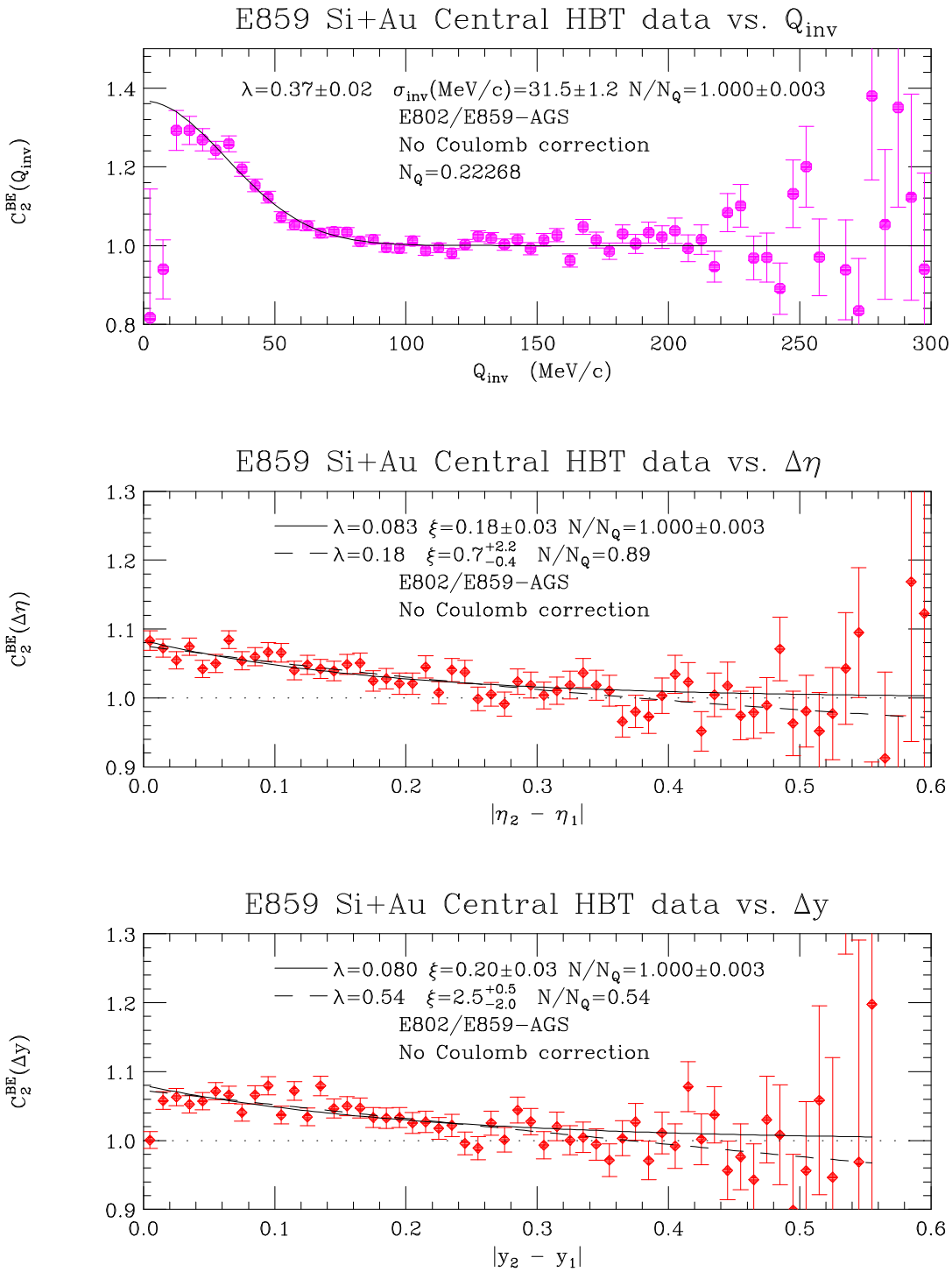


Figure 3: The Bose-Einstein correlation function C_2^{BE} as a function of the 3 variables Q_{inv}, $|\eta_2 - \eta_1|$, and $|y_2 - y_1|$ for pairs of identified π^- in central ^{28}Si+Au collisions. The lines are the fits discussed in the text. The plotted data have been divided by the fitted value of $\mathcal{N}_Q = 0.22268$.

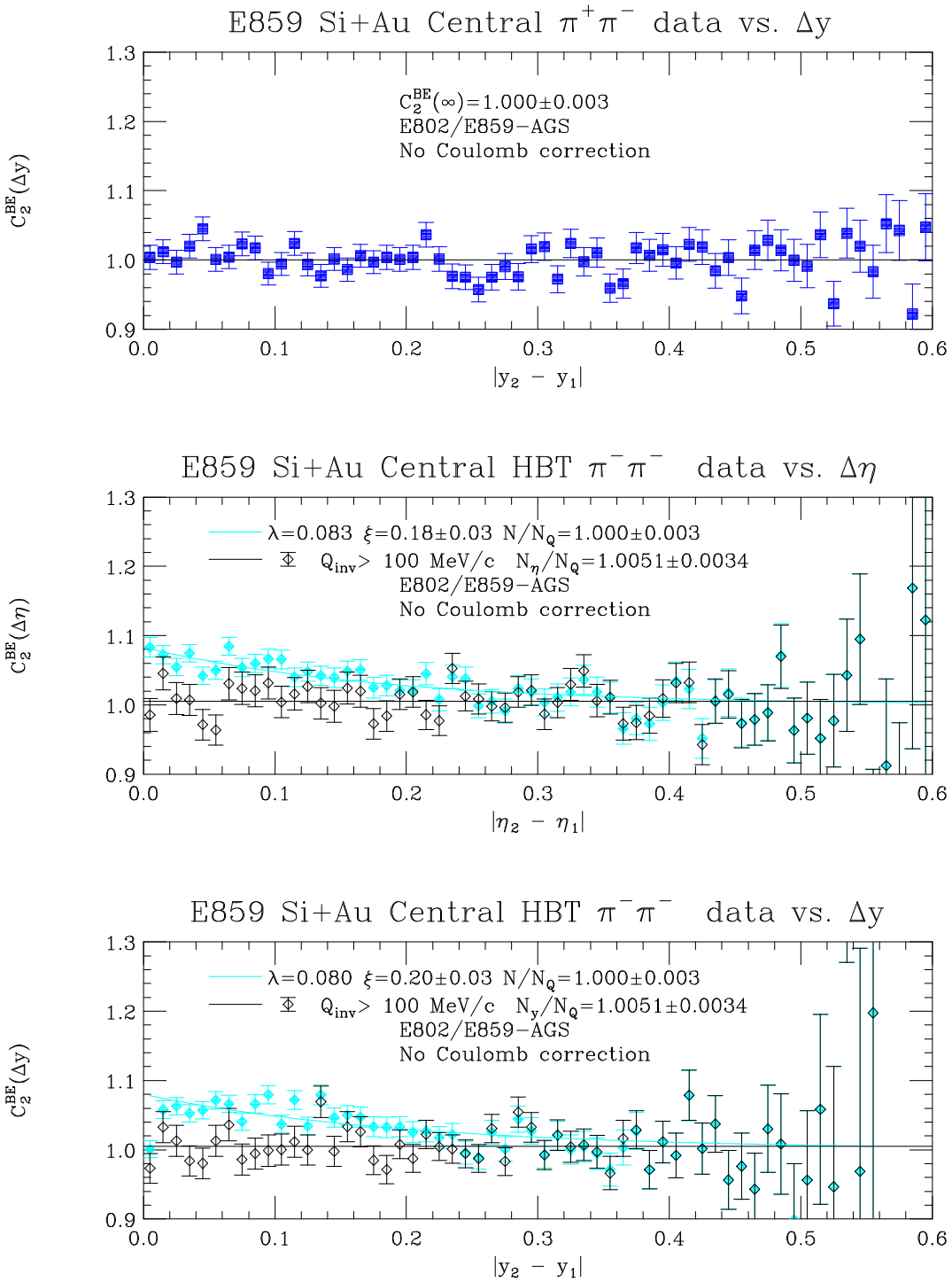


Figure 4: The Bose-Einstein correlation function C_2^{BE} as a function of $|y_2 - y_1|$ and $|\eta_2 - \eta_1|$ for $\pi^-\pi^-$ or $\pi^+\pi^-$ pairs in central ^{28}Si+Au collisions. (a) $\pi^+\pi^-$ data for $|y_2 - y_1|$. (b) and (c) $\pi^-\pi^-$ data with the requirement $Q_{\rm inv} \geq 100$ MeV/c in comparison to the uncut data. The horizontal lines shown are the best fits to a constant. The best fits for the strength of a correlation with $\xi = 0.2$ (fixed) are $\lambda = 0.013 \pm 0.007$ for (a), $\lambda = 0.010 \pm 0.007$ for (b),(c).

Some Details for the Experts

- The principal advantage of reconstructing the two-particle *Background* sample from the *Actual* sample of two-pions is that this procedure automatically solves the problem that the class of two-particle events may be different from the class of one-particle events[15, 16]. For heavy ion collisions, dynamical considerations and conservation of energy are not an issue since the central events have no structure (e.g. jets) and only 2 out of the more than 100 particle are used. One problem that event-mixing does not solve is that the integral of the *Actual* sample of correlated pairs (Eq. 7) is not equal to the integral of the background distribution of pairs of uncorrelated single particles (the square of Eq. 6)—even in the ideal case—since $F_2 > 1$ for most distributions.

- This may explain why some authors[15, 16] divide the correlation function (Eq. 18) by F_2. However, in such cases $\mathrm{C_2^{BE}} \rightarrow 1/F_2$ (rather than unity) in the region outside the B-E correlation.

- For the present analysis, the mixed event sample is purposely taken to be much larger than the *Actual* sample, so the correct normalization is obtained most simply by evaluating it as a parameter in the fit to the correlation function. The fit parameter $\mathcal{N}$ just depends on the number of mixed events chosen for the *Background* distribution compared to the *Actual* sample. The only assumption required is that there is no other correlation outside the region of the B-E correlation, so that $\mathrm{C_2^{BE}} \rightarrow 1$, $R_2 \rightarrow 0$. In practice, if the data exhibit a clear correlation peak and a significant region of constant $\mathcal{C}_2^{BE}$, then the normalization constant can be well determined by this procedure (e.g. see Fig. 3a). There remains the possiblity that the region of constant $\mathcal{C}_2^{BE}$ corresponds e.g. to a conventional two-particle correlation, $R_2 = \epsilon$, with much larger correlation length than the range of measurement. In this case, the above normalization procedure will lead to a systematic mismeasurement of λ_Q by a factor $1/(1+\epsilon)$.

Discussion and relation to other results

- In order to relate the strength parameter λ_η (Eq. 23) measured within the spectrometer ϕ aperture to the usual two-particle short-range strength $R(0,0)$ defined for the full azimuth (Eq. 11), the dependence of the two-particle normalized correlation function on $\Delta\phi$ must be known.

- The simplest model of the source to explain all the E802/E859 two-pion BE correlation data[14] is that of a spherically symmetric gaussian with finite lifetime:

$$R_2(|\vec{q}\,|, q_0) = \lambda_Q e^{-|\vec{q}|^2 R^2 - q_0^2 \tau^2} \quad . \tag{24}$$

  This expression can be evaluated to lowest order[17] in ΔE, $\Delta\eta$, $\Delta\phi$:

$$R_2(p_1+\Delta p, p_1) = \lambda_Q \exp -\{[(\Delta E/\beta)^2+(p_T\Delta\eta)^2+(p_T\Delta\phi)^2]R^2+[(\Delta E)^2]\tau^2\} \tag{25}$$

  which implies the relationship $\sigma_{\Delta\phi} = \sigma_{\Delta\eta} = 1/(\sqrt{2}p_T R)$ and $\sigma_{\Delta E} = 1/\sqrt{2(R^2/\beta^2+\tau^2)}$, where $\beta = P/E$ is the velocity of either pion[17].

- From the latest E802/E859 measurement[14] using the same data as the present analysis, $R = 2.80 \pm 0.11$ fm, with $< p_T >= 300$ MeV/c. This yields the prediction, $\sigma_{\Delta\phi} = \sigma_{\Delta\eta} = 0.166 \pm 0.007$, which is in excellent agreement with the gaussian fit in the present analysis, $\sigma_{\Delta\eta} = 0.149 \pm 0.015$. Consequently, we assume that $\sigma_{\Delta\phi} = \sigma_{\Delta\eta} = 0.149 \pm 0.015$ to extrapolate from the fit within the spectrometer aperture $\delta\phi = 0.40$ rad to the full azimuth.

- For simplicity in averaging, Eq. 25 is converted to an exponential form (cf. Eq. 11) by taking $\xi_\phi = 0.20 \simeq \xi_\eta$—the empirical ratio of ξ_η to $\sigma_{\Delta\eta}$ from the present fit (see Table 1) is $1.3(\sim\sqrt{2})$—and, similarly, by assuming $\xi_E \simeq \sqrt{2}\sigma_{\Delta E} = 1/\sqrt{R^2/\beta^2+\tau^2}$:

$$R_2(p_1+\Delta p, p_1) = \lambda_Q\, e^{-|E_1-E_2|/\xi_E}\, e^{-|\phi_1-\phi_2|/\xi_\phi}\, e^{-|\eta_1-\eta_2|/\xi_\eta} \quad . \tag{26}$$

  The average of this correlation function over the 3-dimensional interval $0 \le \eta_1, \eta_2 \le \delta\eta$, $0 \le \phi_1, \phi_2 \le \delta\phi$, $0 \le E_1, E_2 \le \delta E$ is then very similar in form to Eq. 13:

$$\begin{aligned} K_2(\delta\eta, \delta\phi, \delta E) &= < R_2(p_1+\Delta p, p_1) > |_{\delta\eta,\delta\phi,\delta E} \\ &= \lambda_Q\, H(\delta E/\xi_E)\, G(\delta\phi/\xi_\phi)\, G(\delta\eta/\xi_\eta) \end{aligned} \tag{27}$$

where $G(x)$ is the same function as Eq. 14 and $H(x)$ is another function[18, 19]. Comparison of Eq. 27 to Eqs. 11 – 13 makes it clear that the determination of λ_η involves an average over δE as well as $\delta\phi$:

$$R(0,0) = \lambda_Q \, H(\delta E/\xi_E) \, G(2\pi/\xi_\phi) \quad (28)$$

$$\lambda_\eta = \lambda_Q \, H(\delta E/\xi_E) \, G(\delta\phi/\xi_\phi) \quad . \quad (29)$$

- The ratio of Eqs. 28 and 29 can be used to obtain $R(0,0) = 0.9\%$ from the measured value of λ_η (Table 1), without knowledge of $H(\delta E/\xi_E)$. This result is for identical pions, and must be reduced by a factor of ~ 2 for comparison to a mixture of pions with equal numbers of π^+ and π^-. This yields an overall $R(0,0) \simeq 0.45\%$, which is comparable to, but a factor of ~ 2 lower than previous direct measurements[20, 21] of $\sim 1\%$ for non-identified charged particles.

- The same effect is seen in extrapolating the present $R(0,0)$ to correspond to the 200° aperture used in the E802 NBD analysis[5, 22, 23]: using $R(0,0) = 0.45\%$ yields $\bar{R}(0,0) \simeq 0.8\%$ in comparison with the measured $3.1\% \pm 0.5\%$ for non-identified charged particles. The agreement is quite good, considering the crude extrapolation approximations [24]. Furthermore, the value of $H(\delta E/\xi_E)$ is difficult to estimate in general. For the present analysis, $H(\delta E/\xi_E) \simeq 0.4$ can be estimated from Eq. 29 using the measured values (Table 1) of λ_η and λ_Q [25].

- On a final note, the E802 analysis was for ^{16}O+Cu, while the present analysis is for ^{28}Si+Au (and a different rapidity range), again leading to an uncertainty. However, to the extent that $\xi_\eta = \xi_\phi \simeq 1/(<p_T> R)$ is a reasonable approximation, the correlation lengths should be in the ratio of $A^{1/3}$. Thus, the present measurement for ^{28}Si, $\xi_\eta = 0.18 \pm 0.03$, should be shifted to 0.22 ± 0.03—an effect comparable to 1 standard deviation—for comparison with $\xi = 0.18 \pm 0.05$ from the ^{16}O+Cu NBD analysis. Hence, the effect of the extrapolation is minimal for the correlation length measurement, and the data agree very well.

Conclusions

- The two-pion correlation in rapidity has been determined using a sample of pion pairs in the E802/E859 spectrometer from central (upper 10% TMA) collisions of 14.6A GeV/c ^{28}Si+Au$\rightarrow 2\pi^- +$X, for which a Bose-Einstein correlation analysis is available.

- Within the spectrometer aperture, no rapidity correlation is observed for $\pi^+\pi^-$ pairs, while $\pi^-\pi^-$ pairs exhibit a rapidity correlation of maximum strength 8.0%±0.7%, with exponential correlation length $\xi_y = 0.20 \pm 0.03$.

- The $\pi^-\pi^-$ rapidity correlation vanishes (1.0%±0.7%) when the region of the Bose-Einstein correlation (in $Q_{\rm inv}$) is eliminated, thus demonstrating that the pion-pair short-range rapidity correlation in ^{28}Si+Au central collisions is entirely due to the Bose-Einstein effect.

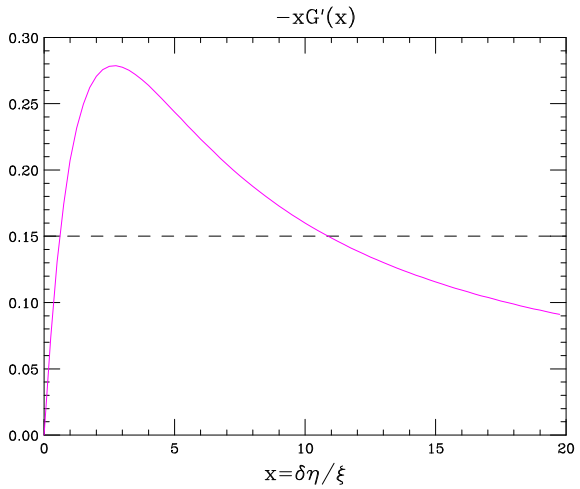
- The present direct measurements are in very good agreement with the previous indirect measurement[5] of the two-particle short-range rapidity correlation length and strength from the evolution with $\delta\eta$ of the NBD fit parameter $k(\delta\eta)$ in central ^{16}O+Cu collisions.

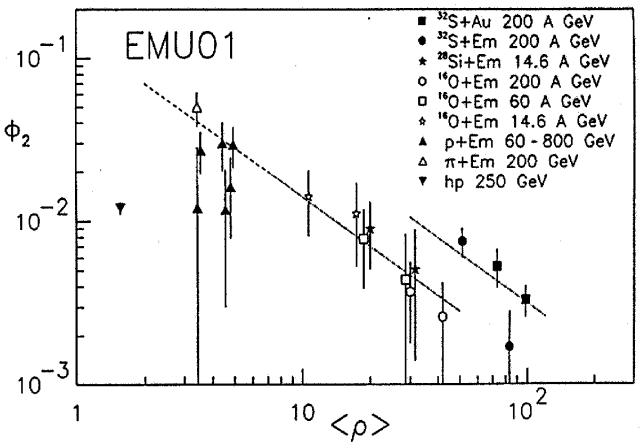
- Taken together, these measurements provide quantitative confirmation of the interrelationship of non-Poisson multiplicity fluctuations, the Negative Binomial Distribution, short-range rapidity correlations, intermittency and the Bose-Einstein effect, which has had much previous theoretical and experimental support.

- From Eq. 27 we can get a general formula for ϕ_2:

$$\begin{aligned} K_2(\delta\eta, 2\pi, \delta E) &= \lambda_Q \, H(\delta E/\xi_E) \, G(2\pi/\xi_\phi) \, G(\delta\eta/\xi_\eta) \\ &\simeq \sim 0.5 \qquad \xi_\phi/\pi \qquad G(x) \qquad (30) \\ \phi_2 = -d\ln F_2/d\ln\delta\eta &\simeq 0.5 \qquad \xi_\phi/\pi \qquad -xG'(x) \\ \phi_2 &\sim 1/(2 \qquad \pi <p_T> R) \qquad 0.15 \\ \phi_2 &\sim 0.075/ \qquad (\pi <p_T> R) \qquad (31) \end{aligned}$$

- The apparent power laws occur because $xG'(x) \sim 0.15$ is nearly constant over ranges where people measure $1.0 \geq \delta\eta \geq 0.1$, $5 \geq x \geq 0.5$.
- The well observed decrease of ϕ_2 with increasing dn/dy (i.e. with increasing R) for central collisions comes from the integral over ϕ, so is determined by ξ_ϕ.
- For the present data $< p_T >= 300$ MeV/c, and from the table $R^{-1} = \sqrt{2}\sigma = 44.5$ MeV/c, $\phi_2 \sim 0.01/\pi = 0.0035$ and should scale like $R_{\rm Si}/R$ for other reactions. This agrees with the spirit if not the fact of the measurements compiled by EMU01[26].





projection	form	λ	ξ or σ	$\mathcal{N}/\mathcal{N}_Q$	χ^2/NDF
$\lvert y_2 - y_1 \rvert$	exponential	0.080±0.007	0.20±0.03		
$\lvert \eta_2 - \eta_1 \rvert$	exponential	0.083±0.007	0.18±0.03		
$Q_{\rm inv}$ (MeV/c)	gaussian	0.367±0.020	31.5±1.2	1.000±0.003	198.8/169
$\lvert y_2 - y_1 \rvert$	gaussian	0.065±0.005	0.155±0.015		
$\lvert \eta_2 - \eta_1 \rvert$	gaussian	0.067±0.005	0.149±0.015		
$Q_{\rm inv}$ (MeV/c)	gaussian	0.367±0.021	31.3±1.1	1.0013±0.0026	181.7/169

Table 1: Fit parameters for the correlation functions with exponential form in Δy, $\Delta\eta$, and gaussian in $Q_{\rm inv}$ all constrained to the same normalization constant, $\mathcal{N}$ (upper half of table). The lower section of the table gives the parameters when gaussians forms are used for all 3 projections. The errors are statistical only. The systematic error in ξ is determined by the statistical error in $\mathcal{N}$, see text.

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[18] The weighting functions for the averages over η and ϕ are constant[19] as in the derivation of Eq. 13, but the energy distribution, roughly $\rho_1(E) \sim Ee^{-bE}$, is not constant and is further complicated by a cut-off at low energy imposed by the spectrometer.

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the more correct notation in which $R(0,0)$ implies an average over the full azimuth.

[23] A change in the value of $\xi_\phi = 0.2$ used in the extrapolation by ± 0.1 would change the extrapolated $R(0,0) = 0.45\%$ by $\pm 0.11\%$, and the $\bar{R}(0,0) = 0.8\%$ by $\pm 0.2\%$.

[24] Thus no errors are quoted for extrapolated quantities.

[25] The value of $\lambda_Q = 0.37 \pm 0.02$ for the present analysis with no Coulomb correction is reasonable in comparison to the fully corrected value[14] of 0.57 ± 0.02 which is, however, considerably smaller than the value of 1.0 expected for B-E correlations.

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