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SHORT RANGE RAPIDITY CORRELATIONS FROM THE BOSE-EINSTEIN EFFECT AND INTERMITTENCY A QUANTITATIVE DEMONSTRATION

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ABSTRACT

A measurement of the two-particle correlation of identified pions was performed in the E802/E859 magnetic spectrometer on the interval $1.5 \leq y \leq 2.0$, $\delta\phi = 0.4$ rad, for central ^{28}Si+Au collisions. It is demonstrated that the two-pion correlation in rapidity is entirely due to Bose-Einstein interference. The directly measured exponential correlation length is $\xi_y = 0.20 \pm 0.03$ for two π^-, with strength $R(0,0) \sim 1\%$, in agreement with previous E802 indirect measurements derived from an analysis of intermittency using Negative Binomial Distributions.

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LES CORRÉLATIONS D'ÉTENDUE COURTE EN RAPIDITÉ
DUES À L'EFFET BOSE-EINSTEIN
ET L'INTERMITTENCE
UNE DÉMONSTRATION QUANTITATIVE

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RÉSUMÉ

Une expérience sur les corrélations de deux-pions identifiés était fait par la Collaboration E802/E859 en utilisant le spéctromètre magnetique sur l'interval $1.5 \leq y \leq 2.0$, $\delta\phi = 0.4$ rad, dans les collisions centrales ^{28}Si+Au. On a démontré que la corrélation de deux-pions en rapidité est totalement due à l'interférence Bose-Einstein. La valeur de la longeur de corrélation mesurée est $\xi_y = 0.20 \pm 0.03$ pour deux π^-, qui s'accord bien avec les mesures indirectes déduites d'un analyse de l'intermittence en utilisant l'évolution de la forme de la Distribution Binôme Negative, déja publiées par E802.

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Introduction

The study of non-Poisson fluctuations of charged particle multiplicity distributions in small pseudorapidity intervals $\delta\eta \leq 1$ by many experiments has been heavily influenced by the utilization of normalized factorial moments (NFM)[1, 2] which are unity for a Poisson distribution. The normalized factorial moment with the clearest interpretation is

$$F_2 = \frac{< n(n-1) >}{< n >^2} = \frac{< n^2 > - < n >}{< n >^2} = \frac{\sigma^2 + < n >^2 - < n >}{< n >^2} = 1 + \frac{\sigma^2}{\mu^2} - \frac{1}{\mu} \quad (1)$$

where $\mu \equiv < n >$ is the mean and $\sigma \equiv \sqrt{< n^2 > - < n >^2}$ is the standard deviation. A mechanism, dubbed intermittency, was proposed[1], which would be indicated by a power-law increase of multiplicity distribution moments over pseudorapidity bins as the bin size is reduced:

$$F_q(\delta\eta) \propto (\delta\eta)^{-\phi_q} \quad . \quad (2)$$

The scale-invariant power-law dependence with singular behavior as $\delta\eta \to 0$ was suggestive of the physics of phase transitions, chaos, and fractals.

Many experiments applied the formalism to their data, leading to the observation[2] of the predicted power law behavior in the region $1 \geq \delta\eta \geq 0.1$. However, the observation of tantalizing power-laws tended to obscure the fact that multiplicity distributions were well known to be non-Poisson because of short-range rapidity correlations in multi-particle production[3, 4]. On the other hand, the variation in NFM as a function of $\delta\eta$ means that the fluctuations, i.e. the shapes of the multiplicity distributions, change as a function of $\delta\eta$.

In previous work, the E802 Collaboration[5] at the BNL-AGS analyzed the evolution of charged particle multiplicity distributions from central collisions of ^{16}O+Cu at $14.6A$ GeV/c as a function of the width of the pseudo-rapidity interval $\delta\eta$, in the range $1.2 \leq \eta \leq 2.2$, both by the method of normalized factorial moments and by direct measurements of the shape of the distributions. The charged multiplicity distributions (see Fig. 1) were well represented by Negative Binomial Distributions (NBD) and simply characterized by the NBD parameter $k(\delta\eta)$ which represents the first departure of a distribution from a Poisson:

$$\frac{1}{k(\delta\eta)} = \frac{\sigma^2}{\mu^2} - \frac{1}{\mu} = F_2(\delta\eta) - 1 = K_2(\delta\eta) \quad (3)$$

where $K_2(\delta\eta)$ is a normalized factorial cumulant[3, 6]. The factorial moments and cumulants of the multiplicity on an interval $\delta\eta$ are simply related to the integrals of the q-particle inclusive rapidity densities $\rho_q(y_1, \ldots y_q)$

$$\begin{aligned} \int^{\delta\eta} dy_1 \, \rho_1(y_1) &= <n> && (4) \\ \int^{\delta\eta} dy_1 dy_2 \, \rho_2(y_1, y_2) &= <n(n-1)>=<n>^2 F_2 && (5) \\ \int^{\delta\eta} dy_1 \cdots dy_q \, \rho_q(y_1, \ldots y_q) &= < n(n-1)\ldots(n-q+1) >=<n>^q F_q \quad . && (6) \end{aligned}$$

These integrals (or moments) are sensitive to any short-range rapidity correlation in particle production, since if there would be no correlation then

$$\rho_q(y_1, \ldots y_q) = \rho_1(y_1)\rho_1(y_2) \cdots \rho_1(y_q) \quad , \quad (7)$$

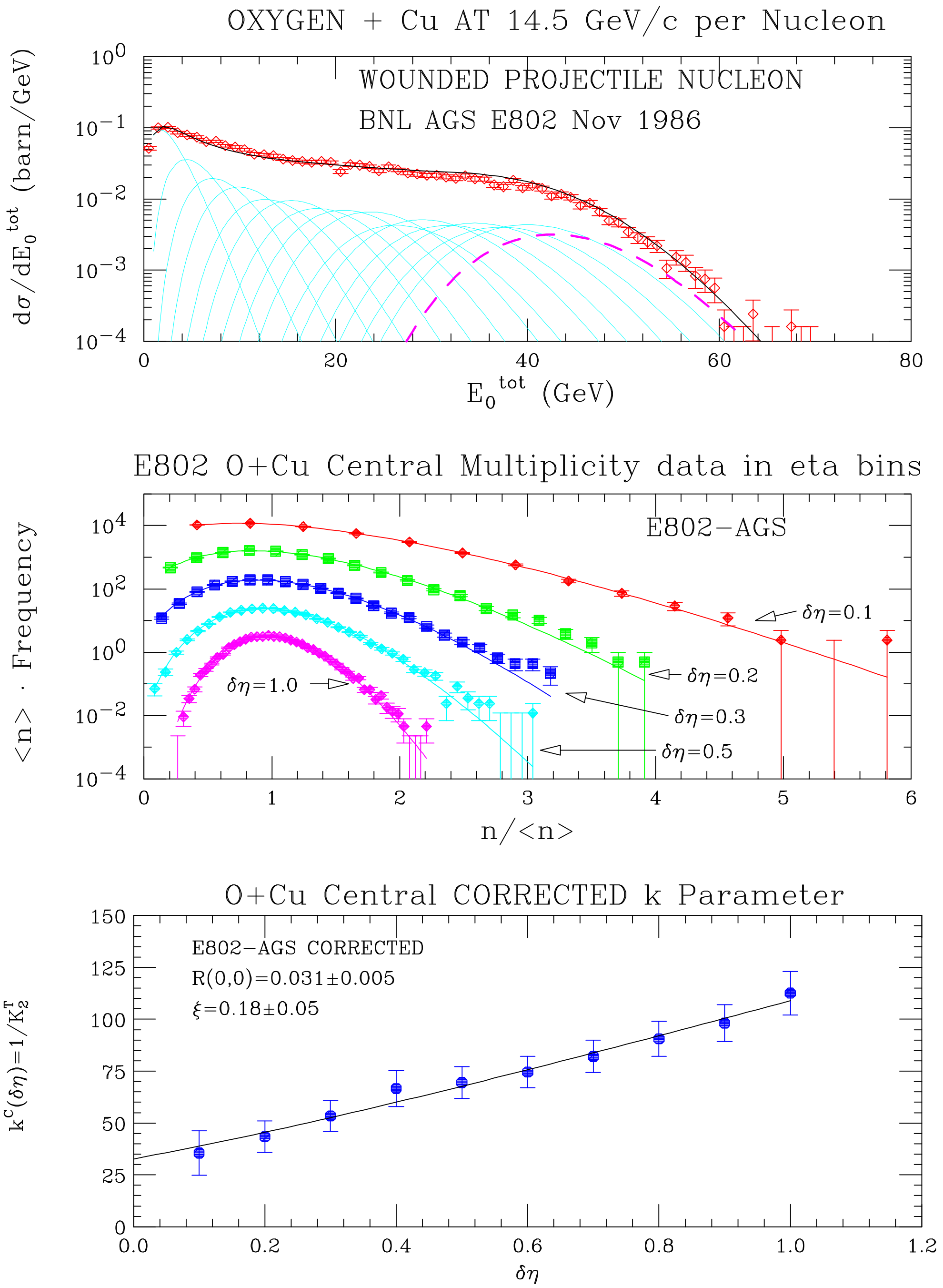

Figure 1: (a) Measured energy emission in $\delta\eta \sim 1$ for O+Cu collisions. The solid curve, which fits the data very well is the sum of the 16 lower curves which represent the energy emitted when from 1 to 16 projectile nucleons interact in the target nucleus with 'geometric' probabilities. (b) Multiplicity distributions measured in O+Cu central collisions as a function of the interval $\delta\eta$ (indicated) for the case when all 16 incident nucleons have interacted, as determined by a Zero degree calorimeter. The data labelled $\delta\eta = 1.0$ correspond to the highest energy (dashed) sub-curve in (a). The curves are NBD fits. (c) $k(\delta\eta)$ from the NBD fits corrected for instrumental effects. The solid line is a fit to Eq. 10 with the parameters indicated.

in which case all the F_q reduce to unity, a Poisson distribution. Mueller[3] introduced a series of functions to describe correlations in multiparticle emission. For instance, the normalized two-particle short-range rapididy correlation function $R_2(y_1,y_2)$ is defined as:

$$R_2(y_1,y_2) \equiv \frac{C_2(y_1,y_2)}{\rho_1(y_1)\rho_1(y_2)} \equiv \frac{\rho_2(y_1,y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1 = R(0,0)\, e^{-|y_1-y_2|/\xi} \quad , \tag{8}$$

where $\rho_1(y)$ and $\rho_2(y_1,y_2)$ are the inclusive densities for a single particle (at rapidity y) or 2 particles (at rapidities y_1 and y_2), $C_2(y_1,y_2) = \rho_2(y_1,y_2) - \rho_1(y_1)\rho_1(y_2)$ is the Mueller correlation function for 2 particles (which is zero for the case of no correlation), and ξ is the two-particle short-range rapidity correlation length[3] for an exponential parameterization. These equations may be combined to yield the relationship[4]

$$\frac{1}{k(\delta\eta)} = K_2(\delta\eta) = F_2(\delta\eta) - 1 = \frac{\int^{\delta\eta} dy_1 dy_2\, \rho_1(y_1)\rho_1(y_2)R_2(y_1,y_2)}{\int^{\delta\eta} dy_1 dy_2\, \rho_1(y_1)\rho_1(y_2)} \quad . \tag{9}$$

If the inclusive single particle density, $\rho_1(y) = dn/dy$, is assumed constant on the interval, then the integral can be performed analytically (specifically on the interval $0 \leq y_1 \leq \delta\eta$, $0 \leq y_2 \leq \delta\eta$) to obtain the normalized factorial moment $F_2(\delta\eta)$ or normalized factorial cumulant $K_2(\delta\eta)$ in terms of the parameters of Eq. 8 [7, 4]:

$$K_2(\delta\eta) = R(0,0)\, G(\delta\eta/\xi) \quad , \tag{10}$$

where the function $G(x)$ is defined as:

$$G(x) = 2\, \frac{(x - 1 + e^{-x})}{x^2} \quad . \tag{11}$$

The correlation length and strength for central ^{16}O+Cu collisions were derived by E802[5] from a fit of the measured $k(\delta\eta) = 1/K_2(\delta\eta)$ to this integral function[8] (see Fig. 1c).

The values of the two-particle correlation length and strength[5] determined for central ^{16}O+Cu collisions, $\xi = 0.18 \pm 0.05$ and $\bar{R}(0,0) = 0.031 \pm 0.005$, were much shorter and weaker than the values for hadron collisions. This result yielded a simple and elegant explanation of intermittency—the 'large' bin-by-bin fluctuations in individual event rapidity distributions from Si+AgBr interactions in cosmic rays are a consequence of the apparent statistical independence of the multiplicity in rapidity bins of size $\delta\eta \sim 0.2$ due to the surprisingly short two-particle rapidity correlation length.

In fact, the weakened, but finite, short-range rapidity correlations in the collisions of relativistic heavy ions had been predicted in the context of intermittency moment analyses[7, 4]. In nucleus-nucleus collisions, the conventional hadron short-range correlations should be washed out by the random superposition of many sources of correlated particles, so that eventually only the quantum-statistical Bose-Einstein (B-E) correlations of identical particles remain[7, 4]. In fact, the relationship between intermittency and B-E correlations has been convincingly demonstrated in other experiments[4] using non-identified charged particles. If B-E correlations were the entire effect, then direct measurements of B-E correlations in terms of the pseudorapidity or rapidity differences of the two particles, $\eta_2 - \eta_1 = \Delta\eta$, $y_2 - y_1 = \Delta y$, instead of the usual variables $Q_{\rm inv}$, $|\vec{q}|$, q_0—where $Q_{\rm inv} = \sqrt{|\vec{q}|^2 - q_0^2}$, $q = p_2 - p_1 = (\vec{q}, q_0)$ and $p = (\vec{P}, E)$—should reproduce the short-range rapidity correlation parameters derived by E802 from the evolution of $k(\delta\eta)$, when adjusted for the charged particle composition. This report presents such direct measurements of correlations in ^{28}Si+Au collisions.

Measurements of B-E correlations in ^{28}Si+Au

The B-E correlation analysis is performed for pairs of negatively charged identified pions detected in the E802/E859 spectrometer from 14.6A GeV/c ^{28}Si+Au$\rightarrow 2\pi^-$+X central (10% TMA) collisions[10]. For the present measurement, the 25 msr. spectrometer aperture spanned polar angles from 14°–28°, accepting pions with $150 \lesssim p_T \leq 700$ MeV/c and $1.5 < y < 2.0$, where the nucleon-nucleon center of mass rapidity is $y_{NN} = 1.7$. The azimuthal coverage ranged from 0.5 rad at the smallest polar angle to 0.25 rad at the largest, with an average value of $\delta\phi = 0.4$ rad.

Bose-Einstein intensity interferometry[11] exploits the fact that identical bosons emitted by a chaotic source of spatial extent R exhibit a correlation in relative momentum $q = p_2 - p_1$ which vanishes for $|q| \gtrsim \hbar/R$. The correlation function for B-E interferometry measurements is closely related to the normalized two-particle correlation function, $R_2(p_1, p_2)$:

$$\mathrm{C}_2^{\mathrm{BE}}(p_1, p_2) = \frac{\rho_2(p_1, p_2)}{\rho_1(p_1)\rho_1(p_2)} = 1 + R_2(p_1, p_2) \quad . \tag{12}$$

For $p_2 - p_1$ outside the region of B-E correlation, it is assumed that there is neglible other correlation so that $\rho_2(p_1, p_2) = \rho_1(p_1)\rho_1(p_2)$ and therefore $\mathrm{C}_2^{\mathrm{BE}} \rightarrow 1$ and $R_2 \rightarrow 0$. For small values of the argument $p_2 - p_1$, where there is full B-E correlation: $\mathrm{C}_2^{\mathrm{BE}} \rightarrow 2$, $R_2 \rightarrow 1$. The B-E correlation is traditionally represented by a gaussian in pair relative momenta quantities, such as Q_{inv}

$$\mathrm{C}_2^{\mathrm{BE}}(Q_{\mathrm{inv}}) = 1 + \lambda\, e^{-Q_{\mathrm{inv}}^2 R_{\mathrm{inv}}^2} \tag{13}$$

where the empirical parameter $\lambda \leq 1$ is introduced to account for the fact that not all detected pions come from a single chaotic source.

In order to form a correlation function corresponding to Eq. 12 from the measured sample of negative pion pairs (the *Actual* distribution of two-pion events), a *Background* sample must be found which exhibits all correlations induced by phase space, dynamics, experimental acceptance, etc., *except* those resulting from B-E correlations. The method chosen is that of event mixing. Two events from the *Actual* sample are selected at random, with a check that the same event isn't matched with itself. Then one pion from each event is chosen at random to form a background pair. In order to not be limited by the statistics of the mixed events, approximately 5 times as many background pairs are formed for the present analysis. The measured correlation functions $\mathcal{C}_2^{BE}(v)$, are defined to be the ratios of the *Actual* distribution of negative pion pairs to the event mixed *Background*, plotted as a function of a variable v,

$$\mathcal{C}_2^{BE}(v) = \frac{A(v)}{B(v)} \quad , \tag{14}$$

where $v = Q_{\mathrm{inv}}$, or Δy. We correct for the inefficient measurement of two tracks with small opening angles in the *Actual* distribution. The correlation functions (numerator and denominator) are restricted to opening angles where this correction is less than or equal to two. Precisely the same set of pion pairs, and corrections, are used for all of the correlation functions, which are in effect just projections in the different variables. No corrections besides that for close-track inefficiency are applied to the data in this analysis—*i.e. no Coulomb corrections are applied.*

The measured B-E correlation function in Q_{inv} is traditionally fit to the gaussian form

$$\mathcal{C}_2^{BE}(Q_{\mathrm{inv}}) = \mathcal{N}[1 + \lambda_Q\, e^{-Q_{\mathrm{inv}}^2/(2\sigma_Q^2)}] \quad . \tag{15}$$

The fit parameter $\mathcal{N}$ is the normalization constant, which just depends on the number of mixed events chosen for the *Background* distribution compared to the *Actual* sample. The correlation functions in rapidity is parameterized as exponential, as in Eq. 8 (although a gaussian works just as well for the present data):

$$\mathcal{C}_2^{BE}(y_1, y_2) = \mathcal{N}[1 + \lambda_y \, e^{-|y_1 - y_2|/\xi_y}] \quad . \tag{16}$$

The projection of $\mathcal{C}_2^{BE}$ onto the longitudinal direction implicitly integrates over the limited azimuthal aperture of the spectrometer, $\delta\phi = 0.40$ rad, and other variables such as $\Delta E = E_2 - E_1$, the energy difference of the two pions. Thus $\lambda_y \neq R(0,0)$, since $R(0,0)$ (Eq. 8) represents the strength of the two-particle rapidity correlation integrated over the full azimuth[9].

Results

The present measurement of the $\pi^-\pi^-$ correlation function $\mathrm{C}_2^{\mathrm{BE}}(p_1, p_2)$ is shown in Fig. 2 as a function of the variable Q_{inv}, together with the same data plotted as a function of the rapidity difference of the two pions, $|y_2 - y_1|$.

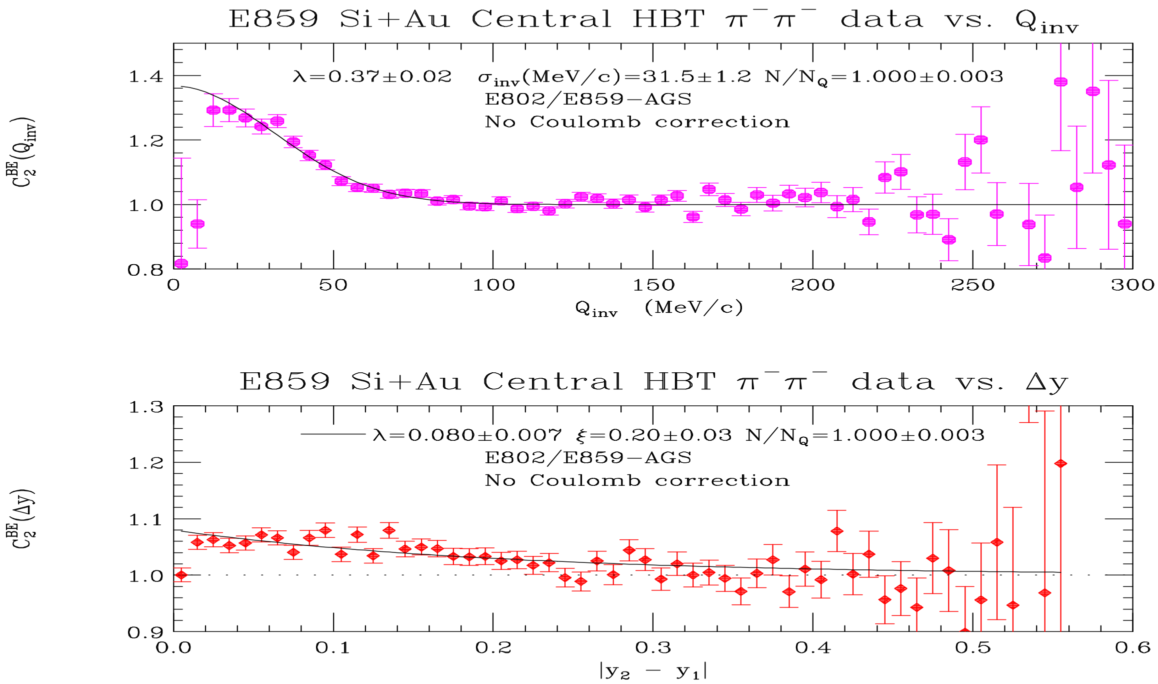

Figure 2: The Bose-Einstein correlation function $\mathrm{C}_2^{\mathrm{BE}}$ as a function of Q_{inv} and $|y_2 - y_1|$ for pairs of identified π^- in central ^{28}Si+Au collisions. The solid lines are the fit discussed in the text.

In both projections the plotted data have been divided by the best fit value of the normalization $\mathcal{N}_Q = 0.22268$ whose error then appears in the fitted value of $\mathcal{N}/\mathcal{N}_Q$. An evident correlation effect is visible for $Q_{\mathrm{inv}} \leq 80$ MeV$/c$, along with a clear region of constant $\mathrm{C}_2^{\mathrm{BE}}$ for $Q_{\mathrm{inv}} \geq 100$ MeV$/c$, where there is no correlation so that the normalization constant $\mathcal{N}_Q$ can be precisely determined. In the $|y_2 - y_1|$ projections, the data show an $\sim$8% drop over the range from 0 to 0.5; however it is not clear whether the correlation function has become constant or would continue to decrease for values greater than 0.5. The key point for the present analysis is that the parameter $\mathcal{N}$ just represents the relative number of events in the *Actual* and *Background* distributions which is identical for the projections in the different variables, Q_{inv}, Δy. Thus, this constraint is applied in a common fit of $\mathcal{C}_2^{BE}$ for both projections (Eqs. 15, 16, solid lines), which allows statistically significant values of the rapidity correlation parameters $\lambda_y = 0.080 \pm 0.007$ and $\xi_y = 0.20 \pm 0.03$ to be obtained. To be consistent with the custom in short-range rapidity correlation analyses, no Coulomb correction is applied to the data. This correction is significant for the two lowest points in $Q_{\rm inv}$ so they (and the lowest point in Δy) are not used in the fit—thus the absence of a Coulomb correction has no effect on the determination of the key normalization parameter. These values (for ^{28}Si+Au central collisions) agree impressively well with the previous indirect measurement[5, 9] (for ^{16}O+Cu central collisions).

In order to determine whether the measured rapidity correlation is entirely due to the Bose-Einstein effect, two tests were performed. The $\pi^+\pi^-$ correlation, which has no B-E interference[10], shows a constant value of $C_2{}^{\rm BE}$ in the $|y_2 - y_1|$ projection (see Fig. 3a)—this clearly establishes the correlation as being an identical particle effect. Secondly, for the

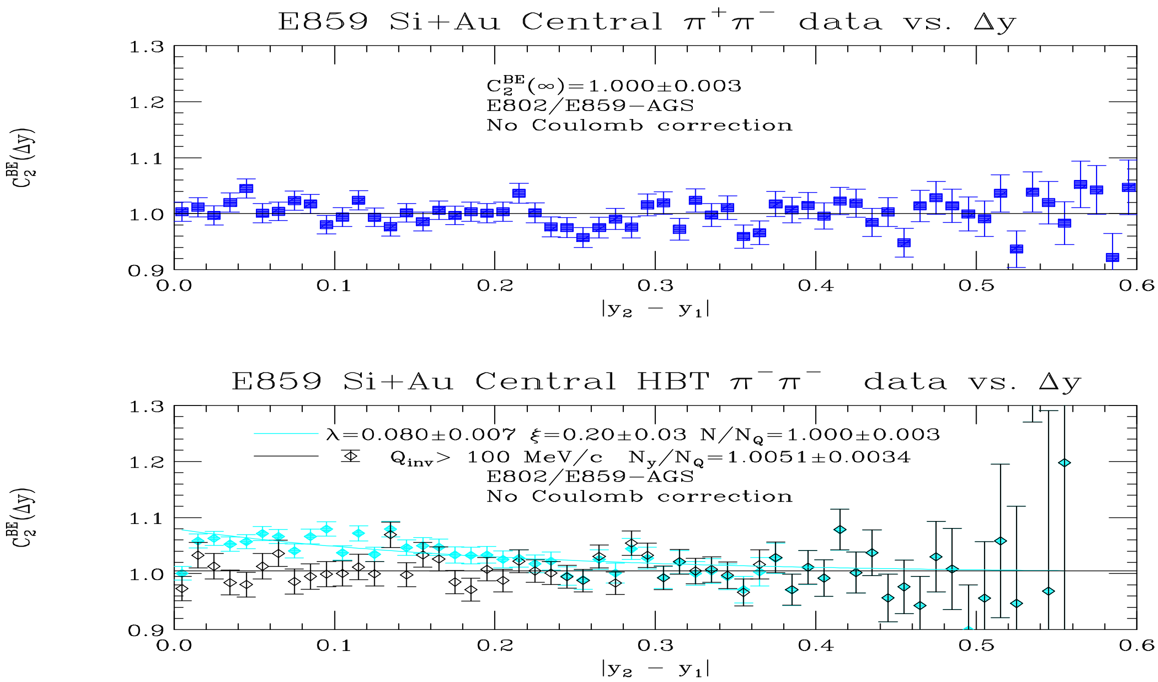

Figure 3: The Bose-Einstein correlation function $C_2^{\rm BE}$ as a function of $|y_2 - y_1|$ in central ^{28}Si+Au collisions. (a) $\pi^+\pi^-$ data for $|y_2 - y_1|$. (b) $\pi^-\pi^-$ data with the requirement $Q_{\rm inv} \geq$ 100 MeV/c in comparison to the uncut data (Fig. 2b). The horizontal lines shown are the best fits to a constant. The best fits for the strength of a correlation with $\xi = 0.2$ (fixed) are $\lambda = 0.013 \pm 0.007$ for (a), $\lambda = 0.010 \pm 0.007$ for (b).

identical $\pi^-\pi^-$, the region of $Q_{\rm inv} \leq 100$ MeV/c (where the correlation effect is exhibited) was eliminated and the data were again plotted in terms of $|y_2 - y_1|$ (Fig. 3b): the data at larger values of $|y_2 - y_1|$ are identical, but the correlation at low values has vanished as illustrated by direct comparison with the uncut data points. This demonstrates that the two-pion short-range rapidity correlation in ^{28}Si+Au collisions is entirely due to the Bose-Einstein interference.

Conclusions

The two-pion correlation in rapidity has been determined using a sample of pion pairs in the E802/E859 spectrometer from 14.6A GeV/c ^{28}Si+Au$\rightarrow 2\pi^-$+X, 10% central (TMA) collisions, for which a Bose-Einstein correlation analysis is available. Within the spectrometer aperture, no rapidity correlation is observed for $\pi^+\pi^-$ pairs, while $\pi^-\pi^-$ pairs exhibit a rapidity correlation of maximum strength 8.0%±0.7%, with exponential correlation length $\xi_y = 0.20 \pm 0.03$. The $\pi^-\pi^-$ rapidity correlation vanishes (1.0%±0.7%) when the region of the Bose-Einstein correlation (in $Q_{\rm inv}$) is eliminated, thus demonstrating that the pion-pair short-range rapidity correlation in ^{28}Si+Au central collisions is entirely due to the Bose-Einstein effect. The present direct measurements are in very good agreement with the previous indirect measurement[5] of the two-particle short-range rapidity correlation length and strength from the evolution with $\delta\eta$ of the NBD fit parameter $k(\delta\eta)$ in central ^{16}O+Cu collisions. Taken together, these measurements provide quantitative confirmation of the interrelationship of non-Poisson multiplicity fluctuations, the Negative Binomial Distribution, short-range rapidity correlations, intermittency and the Bose-Einstein effect, which has had much previous theoretical and experimental support.

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[4] Detailed citations are not reasonable in this report. See, for example [5] and M. J. Tannenbaum, *Phys. Lett.* **B347**, 431 (1995) for more complete references.

[5] E802 Collaboration, T. Abbott, *et al., Phys. Lett.* **B337**, 254 (1994); *Phys. Rev.* **C52**, 2663 (1995).

[6] The normalized factorial cumulants, K_q, which are zero if there is no direct q particle correlation (Poisson distribution), are just the normalized factorial moments, F_q, with all q-fold combinations of lower order correlations subtracted[3]: $K_2 = F_2 - 1$, $K_3 = F_3 - (1 + 3K_2)$, $K_4 = F_4 - (1 + 6K_2 + 3K_2^2 + 4K_3)$ ...

[7] See, for example, A. Capella, K. Fialkowski and A. Krzywicki, *Festschrift for Leon Van Hove and Proceedings Multiparticle Dynamics*, eds. A. Giovannini and W. Kittel (World Scientific, Singapore 1990); *Phys. Lett.* **B230**, 149 (1989); A. Capella, A. Krzywicki, and E. M. Levin, *Phys. Rev.* **D44**, 704 (1991).

[8] It is clear from Fig. 1c that $k(0) \neq 0$ and thus $1/K_2(0)$ is finite. In fact, the apparently divergent fits to Eq. 2 are just an artifact of fitting with the inverse form, $K_2(\delta\eta) = 1/k(\delta\eta)$, which causes the simple, almost linear relationship of Fig. 1c to appear divergent.

[9] The azimuthal correlation length λ_ϕ must be known in order to extrapolate the measurements in $\delta\phi = 0.40$ to the full azimuth.

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