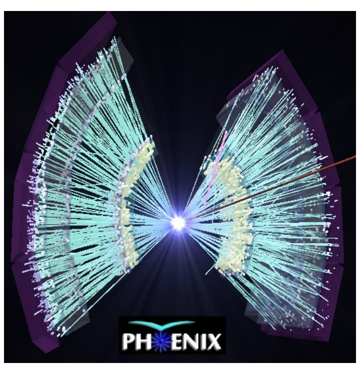
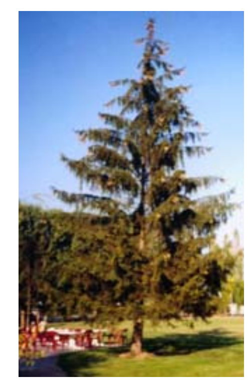
Where Feynman, Field and Fox Failed and How we Fixed it at RHIC

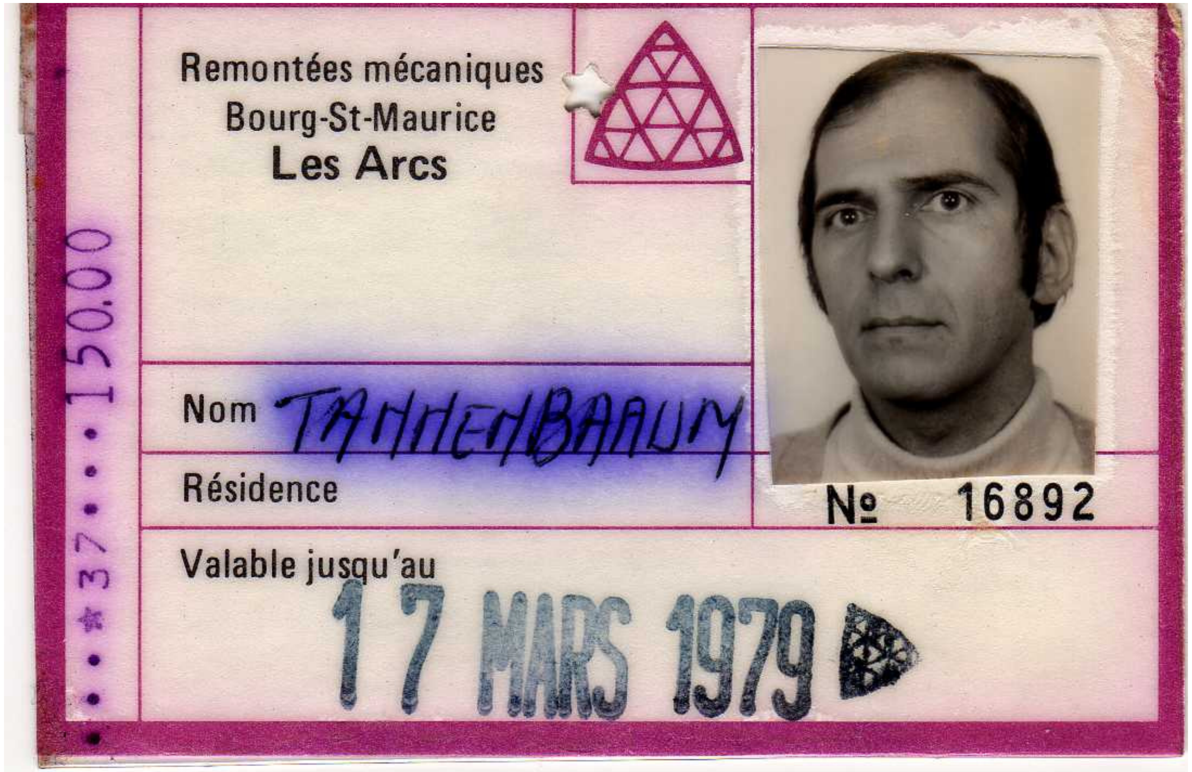
M. J. Tannenbaum
Brookhaven National Laboratory
Upton, NY 11973 USA



XLIII Rencontres de Moriond
(recent Super Bowl was XLII)
QCD and High Energy Interactions
La Thuile, Italy, Mar 8-15, 2008



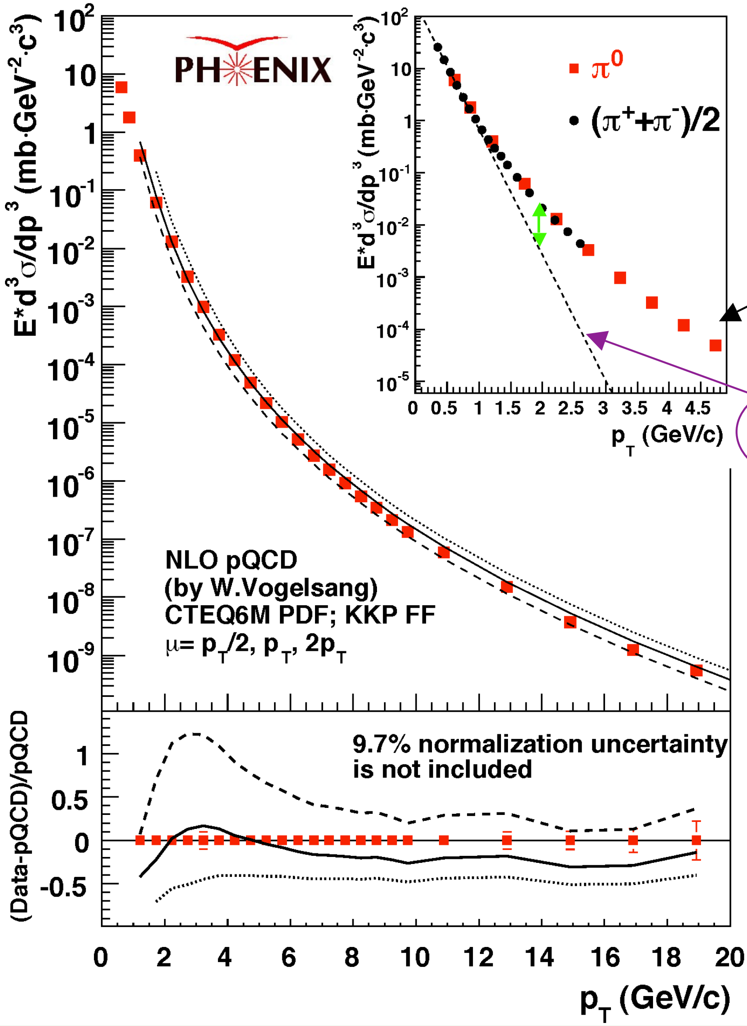
Il était une fois



Many results in p-p collisions that were new and exciting in 1979 are relevant for RHIC 2008

π^0 production in p-p collisions at RHIC

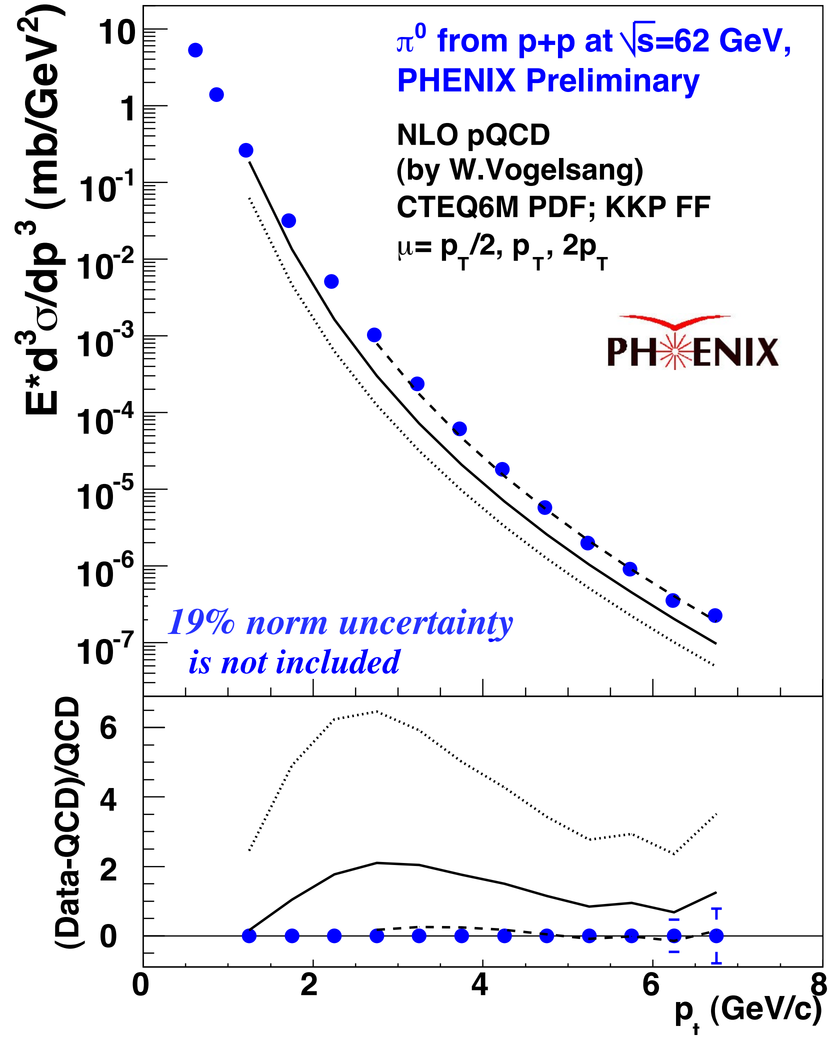
PHENIX, PRD**76**(2007)051006(R)



hard scattering dominates after ~3 orders of magnitude

power-law

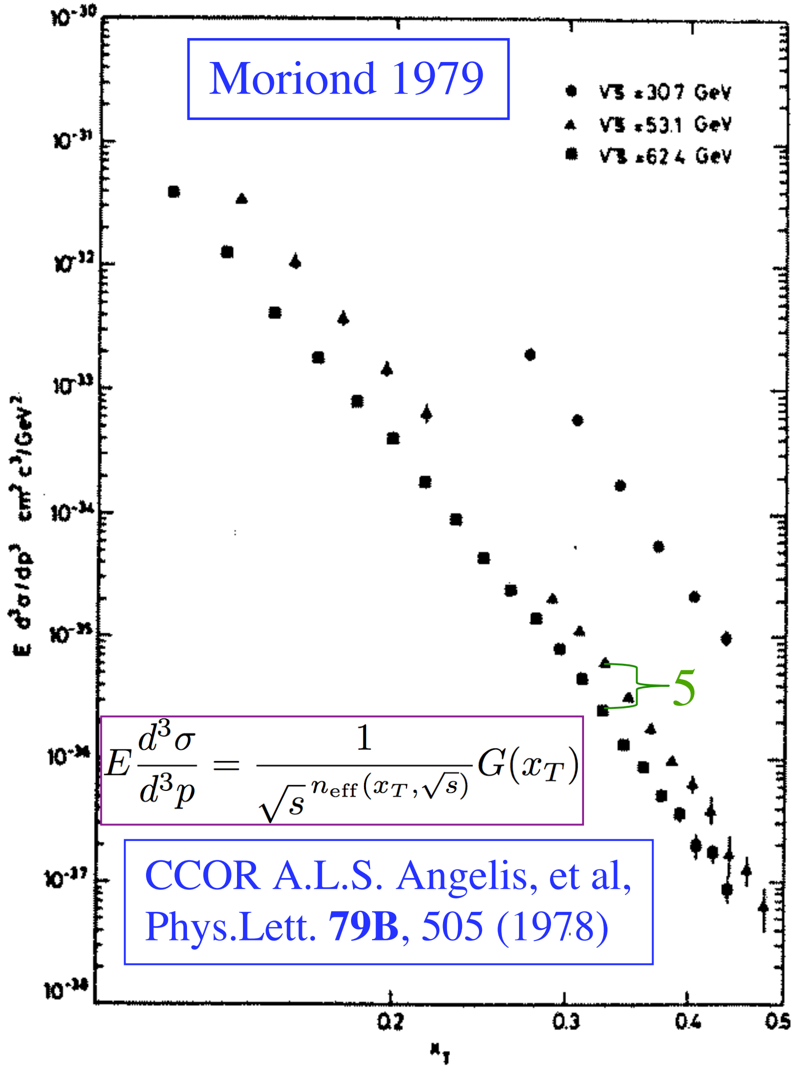
$e^{-5.6\,p_T}$

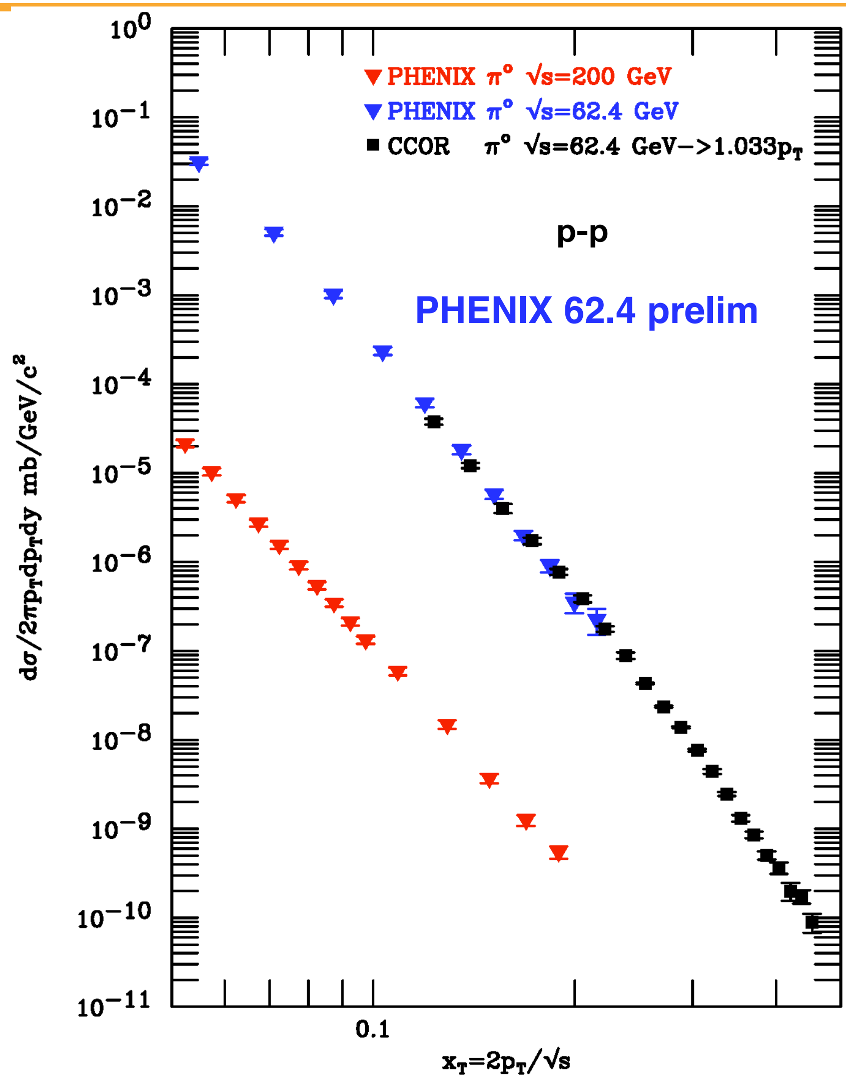


NLO-pQCD precison agreement
Strattmann Vogelsang hep-ph/0702083

No surprise (to me) that NLO pQCD agrees with data

ISR π^0 vs RHIC π^0 p-p

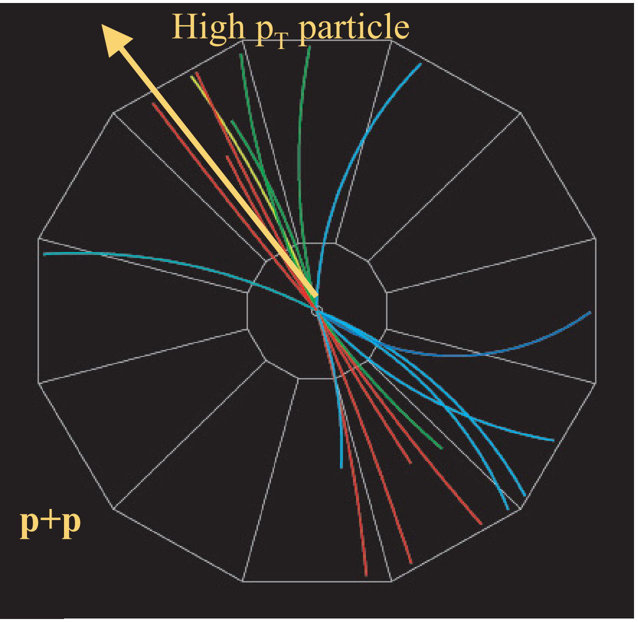




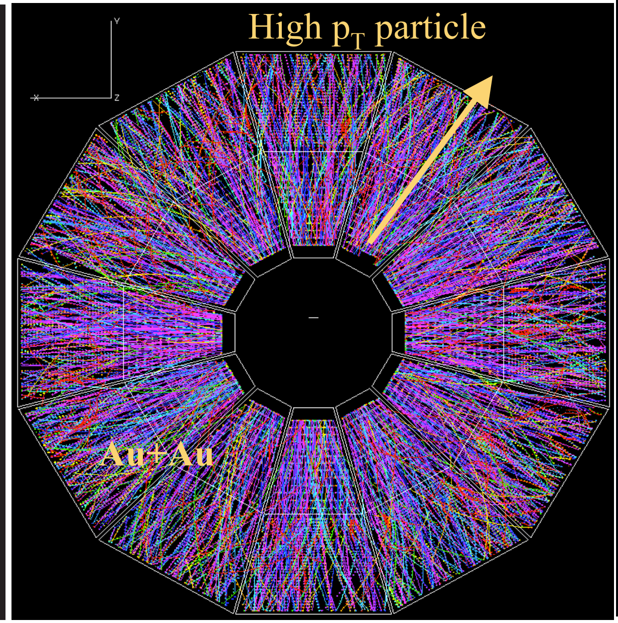
π^0 invariant cross section in p-p at $\sqrt{s}$=200 GeV is a pure power law for p_T > 3 GeV/c, n=8.10±0.05. Power at 62.4 ISR (x_T>0.27 is n=11.03±0.16)

Au+Au Central Collisions cf. p-p

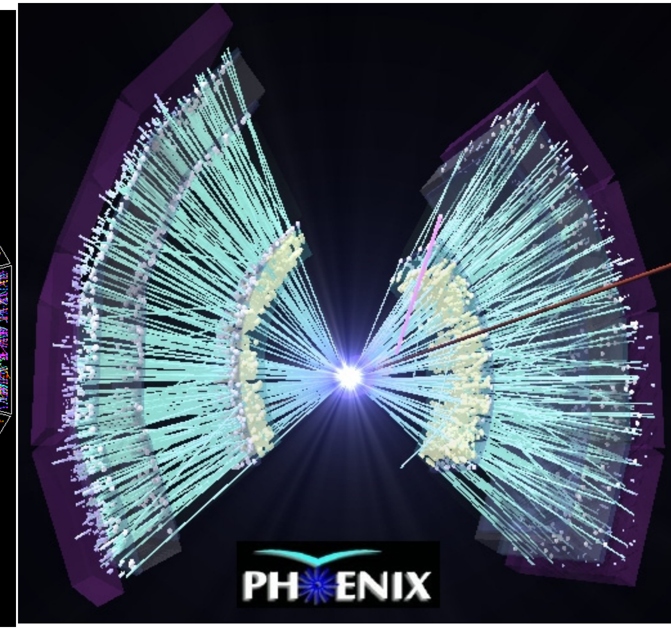
STAR-Jet event in pp

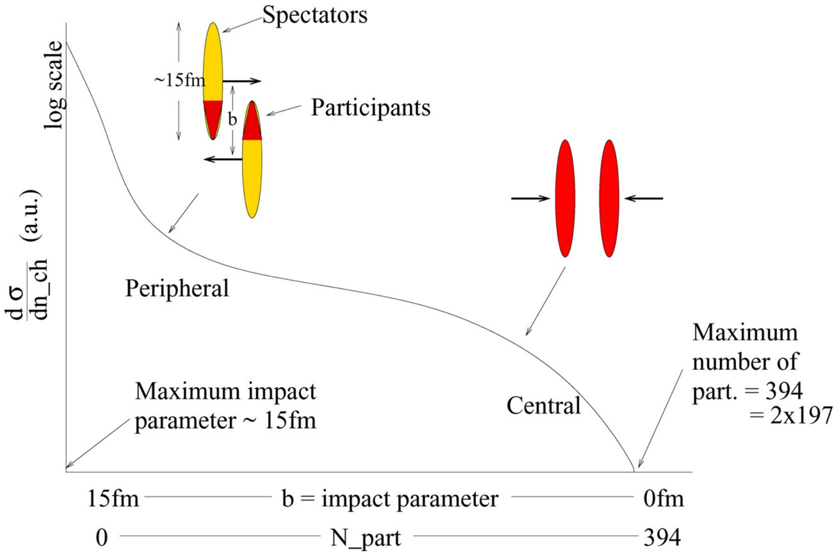


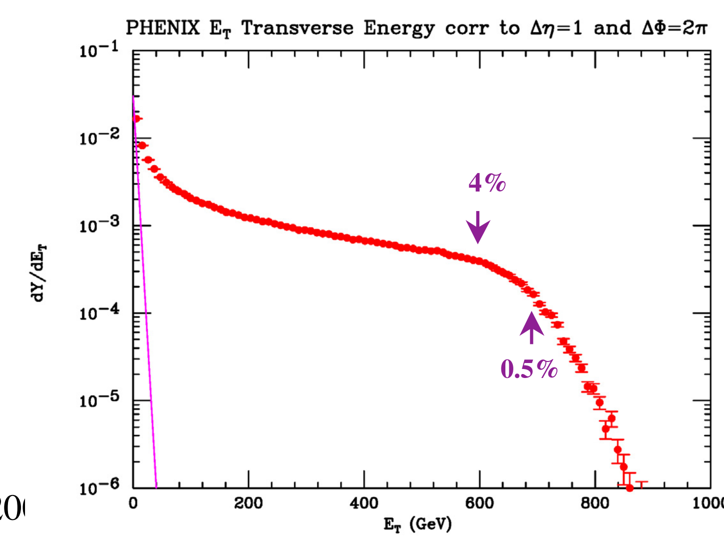
STAR Au+Au central



PHENIX Au+Au central







Latest π^0 Au+Au arXiv:0801.4020

Power Law p_T>3GeV/c all centralities n=8.10±0.05

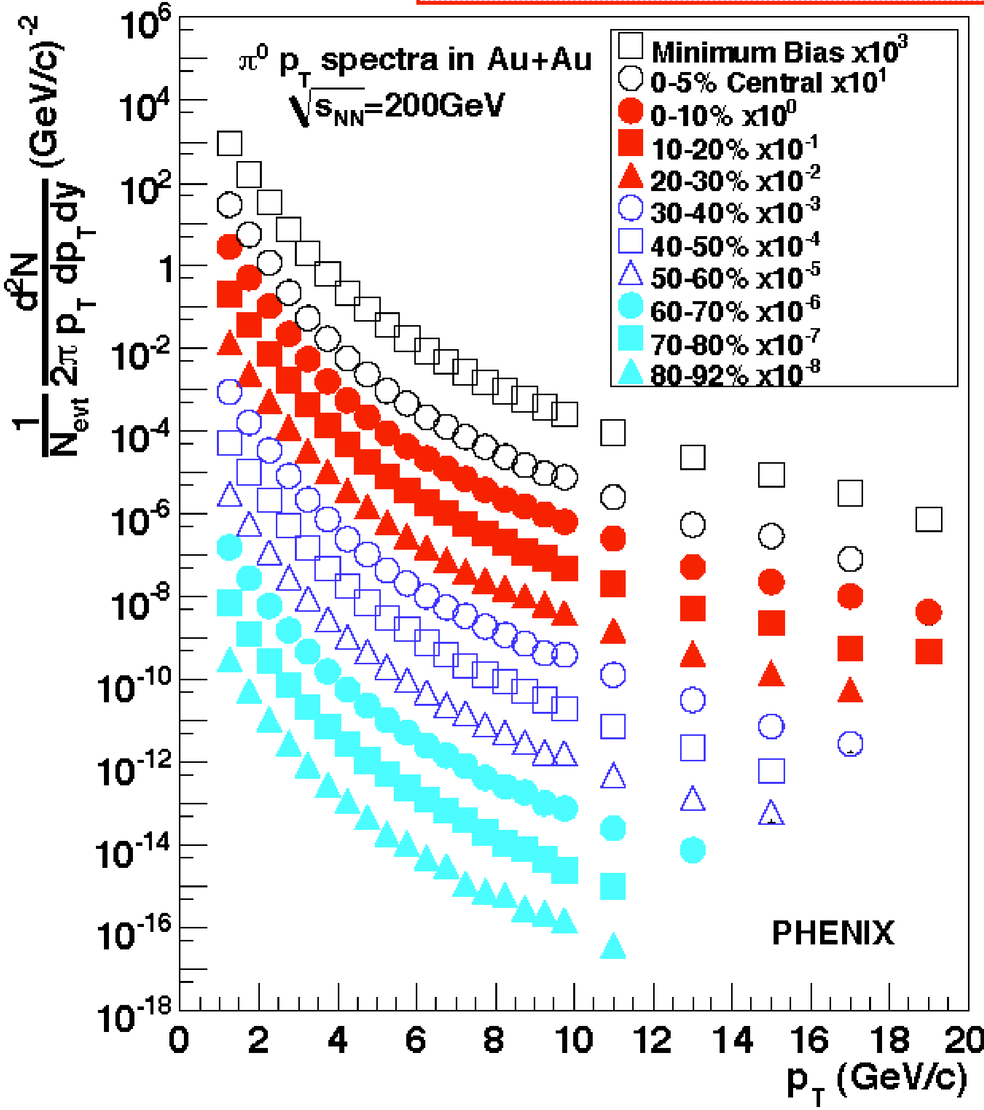
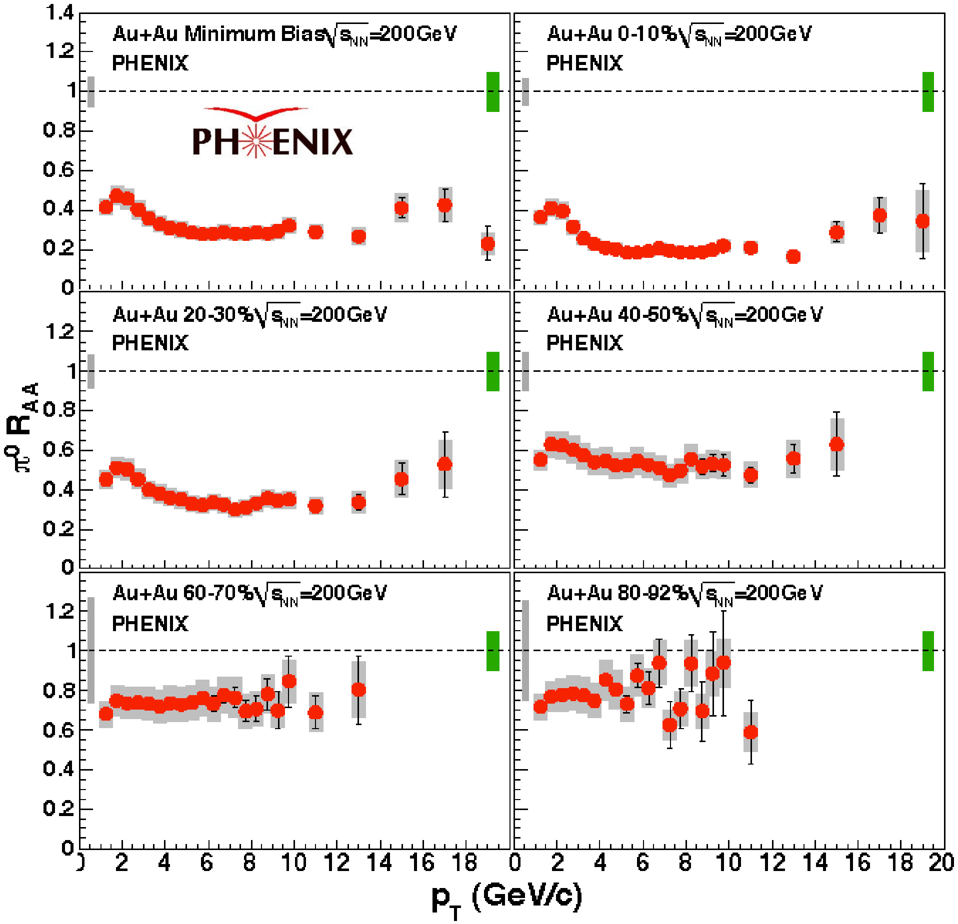


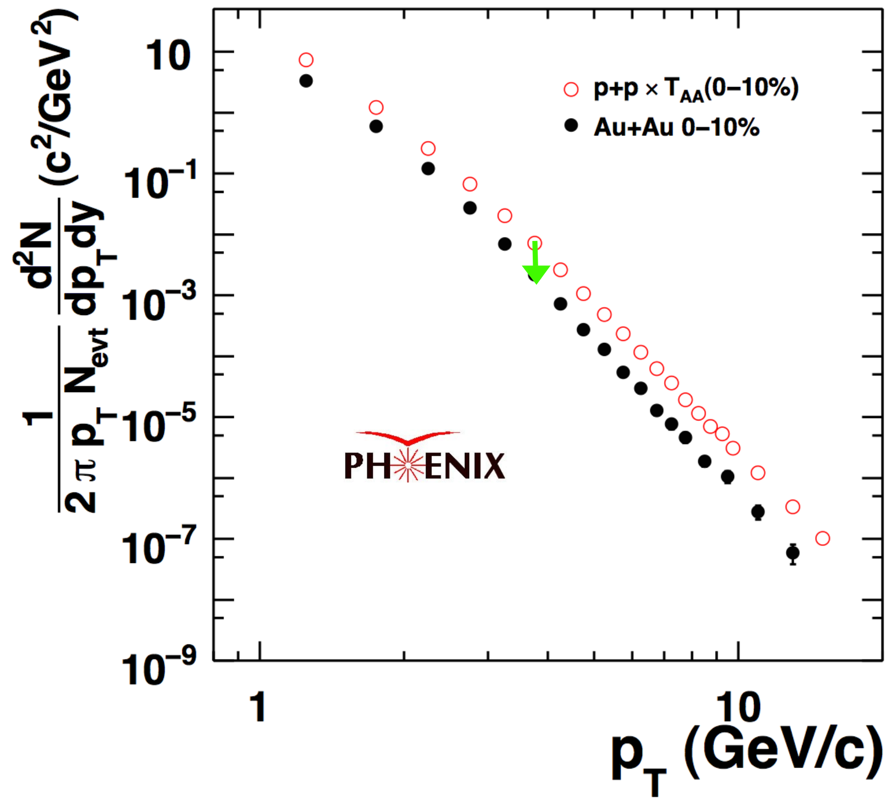
Table 5: Fit parameters for p_T>3 GeV/c

System	A	n	χ^2/NDF
p+p	14.61±1.45	8.12±0.05	5.68/17
Au+Au 0-5 %	81.18±10.30	8.20±0.07	9.66/16
Au+Au 0-10 %	75.28±8.89	8.18±0.06	10.62/17
Au+Au 10-20 %	64.62±7.64	8.19±0.06	10.04/17
Au+Au 20-30 %	49.33±5.78	8.18±0.06	6.63/16
Au+Au 30-40 %	30.85±3.53	8.10±0.06	10.63/16
Au+Au 40-50 %	22.58±2.61	8.13±0.06	3.50/15
Au+Au 50-60 %	12.40±1.48	8.06±0.07	8.09/15
Au+Au 60-70 %	6.25±0.78	8.03±0.07	2.89/14
Au+Au 70-80 %	3.38±0.45	8.12±0.08	8.42/13
Au+Au 80-92 %	1.19±0.18	8.03±0.09	9.84/13
Au+Au 0-92 %	29.31±3.07	8.17±0.05	6.83/17

Suppression of π^0 is arguably the major discovery at RHIC. Energy loss in medium?

Au Au $\sqrt{s_{NN}}$=200 GeV arXiv:0801.4020



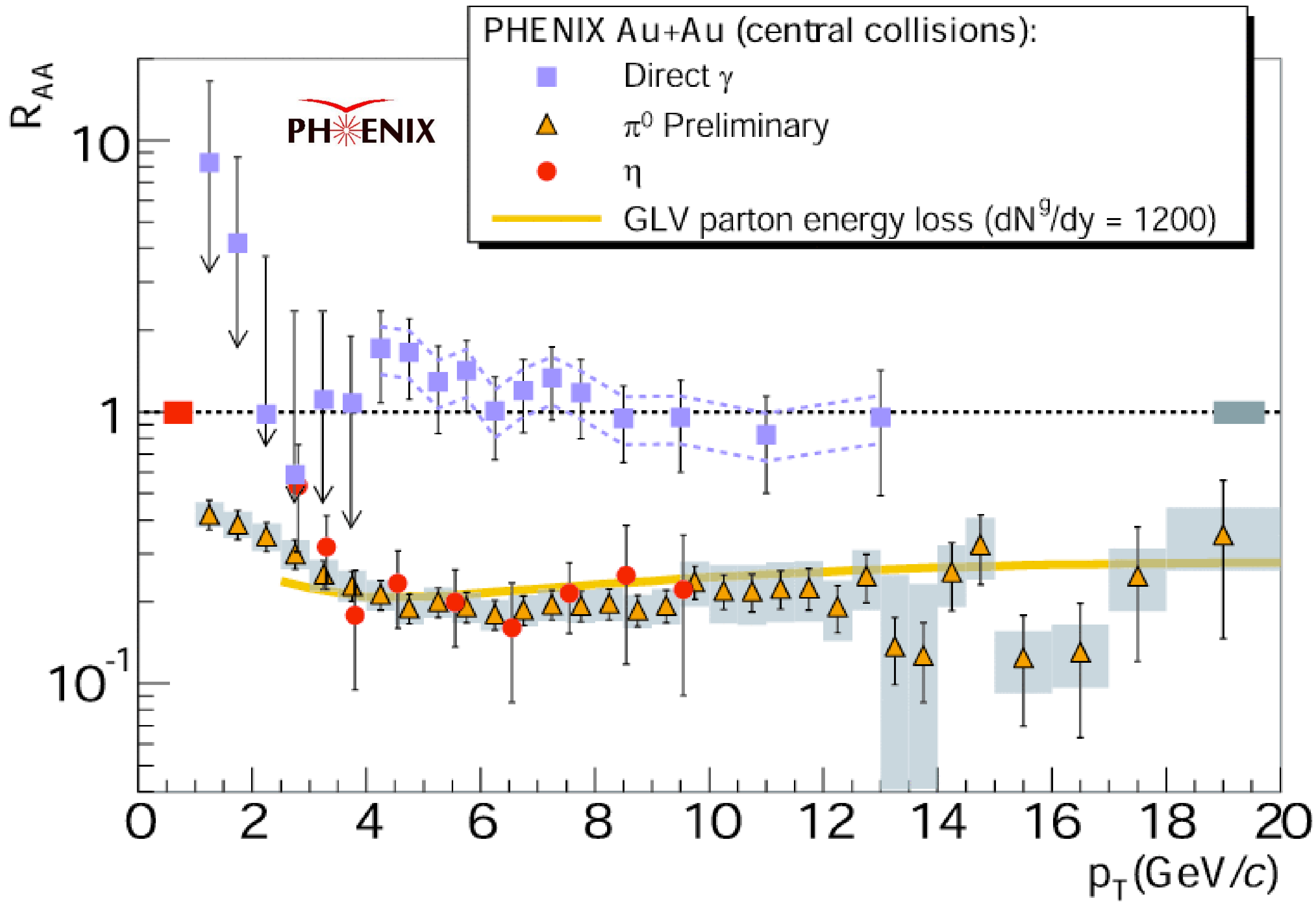


Nuclear Modification Factor

$$R_{AA}(p_T) = \frac{d^2N^{\pi}_{AA}/dp_T dy N^{inel}_{AA}}{\langle T_{AA}\rangle d^2\sigma^{\pi}_{pp}/dp_T dy}$$

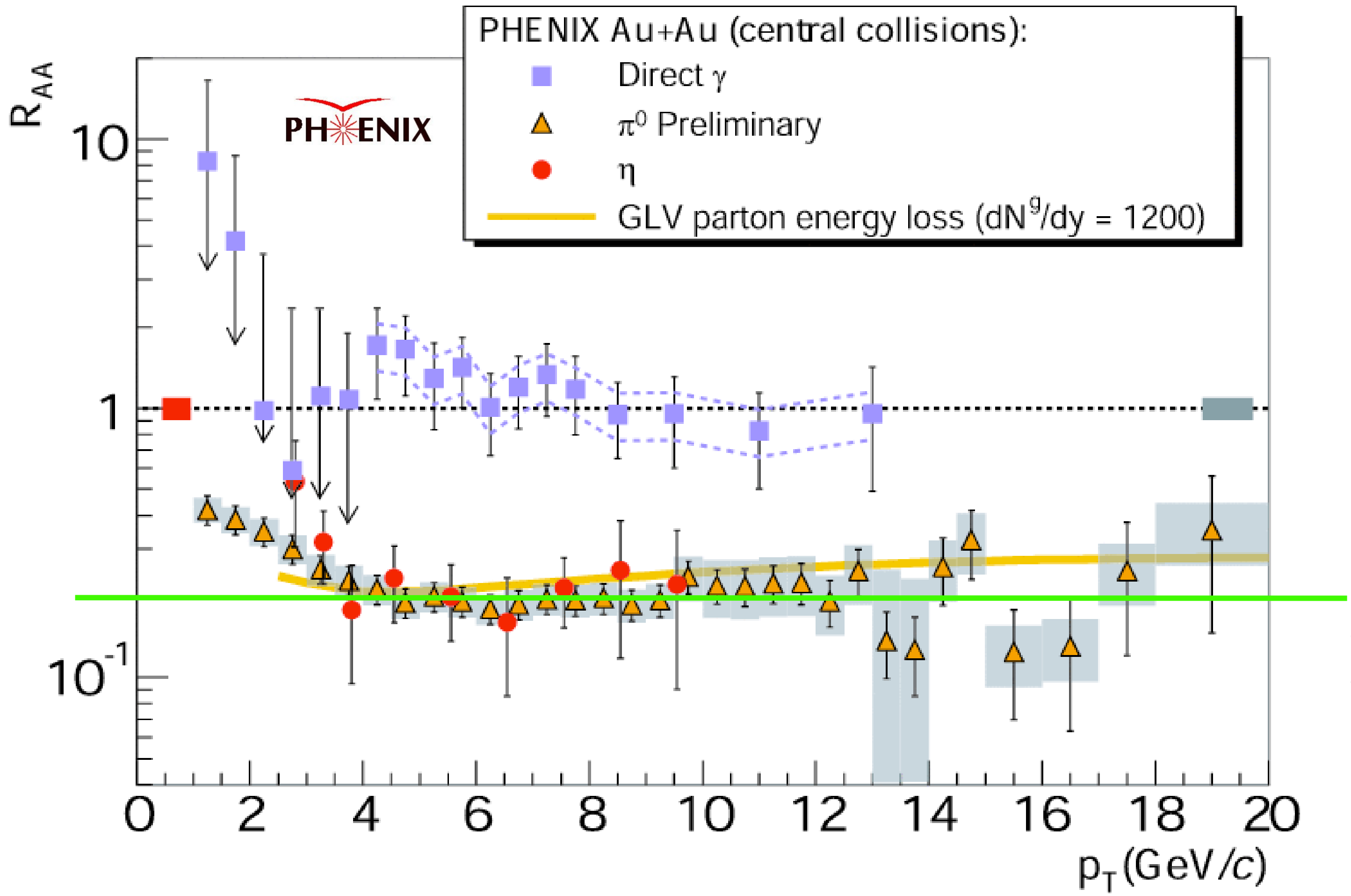
Original π^0 discovery, PHENIX PRL **88** (2002)022301

Status of R_{AA} in AuAu at $\sqrt{s_{NN}}$=200 GeV QM05



Direct γ are not suppressed. π^0 and η suppressed even at high p_T
Implies a strong medium effect (energy loss) since γ not affected.
Suppression is flat at high $p_{T.}$ Are data flatter than theory?

Status of R_{AA} in AuAu at $\sqrt{s_{NN}}$=200 GeV QM05



MJT
don't laugh
see arXiv:
0801.1665

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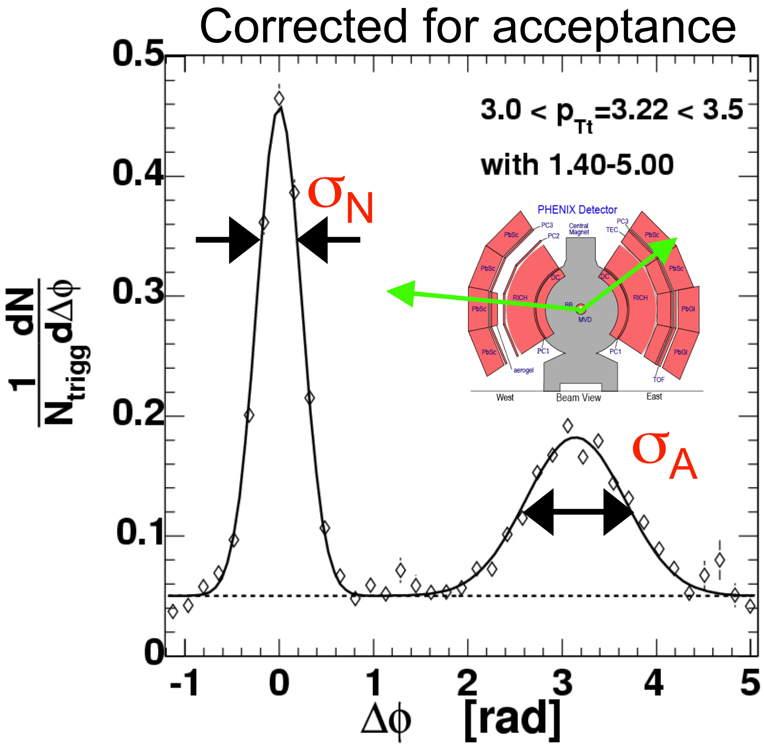
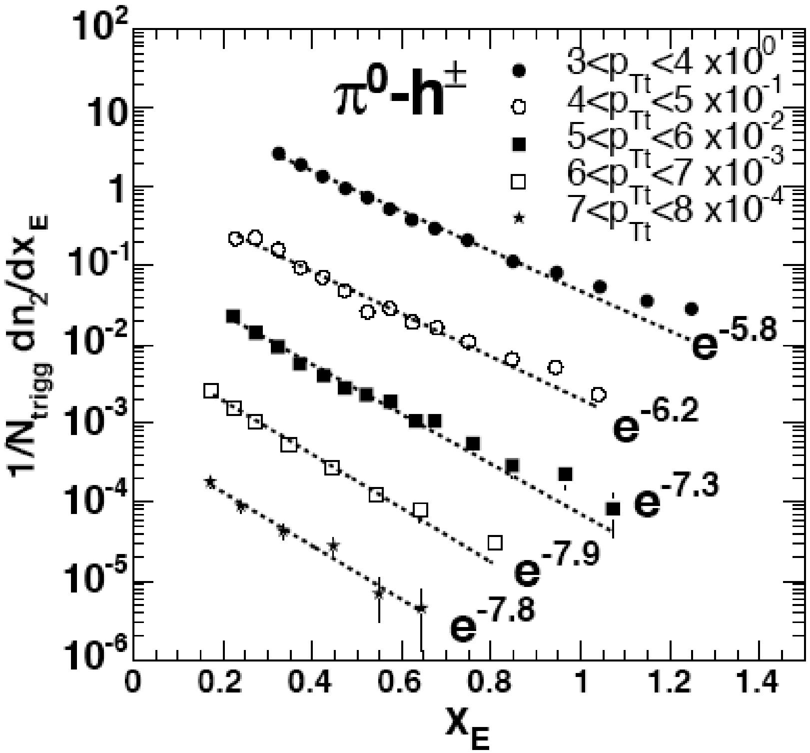
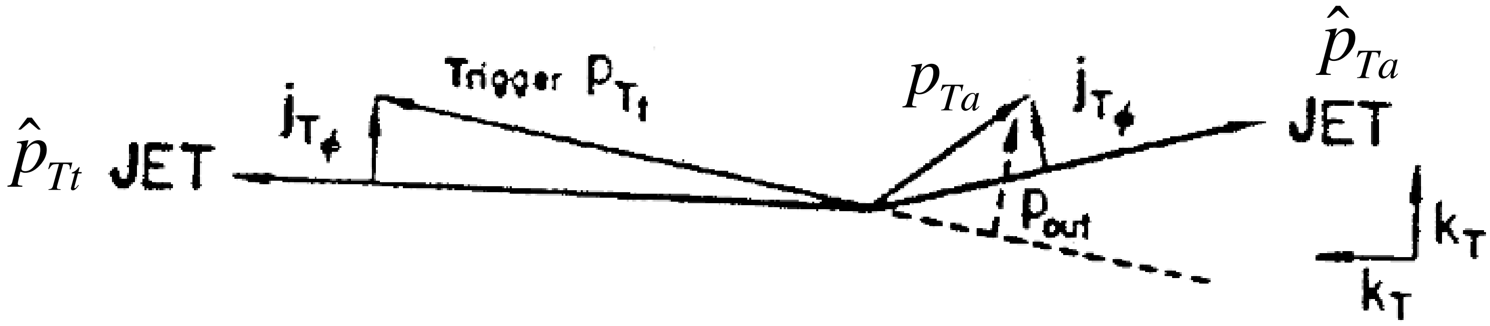
PHENIX π^o - $h^\pm$ correlation functions p+p $\sqrt{s}$=200 GeV: PRD **74**, 072002 (2006)





Trigger on a particle e.g. π^0 with transverse momentum p_{Tt}. Measure azimuthal angular distribution w.r.t the trigger azimuth of associated (charged) particles with transverse momentum p_{Ta}. The strong same and away side peaks in p-p collisions indicate di-jet origin from hard-scattering of partons. For the away distribution calculate the conditional yield in the peak as a function of $x_E \sim p_{Ta}/p_{Tt}$

Kinematics-Figure is from Moriond 1979



$z = p_T / \hat{p}_T$ is the jet fragmentation variable: z_t and z_a

$D^q_\pi(z) = Be^{-bz}$ is a typical Fragmentation Function, b~ 8-11 at RHIC

Due to the steeply falling spectrum, the trigger π^0 are biased towards large z_t, $\langle z_t \rangle \approx (n-1)/b$ while unbiased $\langle z \rangle \approx 1/b$

$$x_E = \left| \frac{\vec{p}_{T_a} \cdot \vec{p}_{T_t}}{p^2_{T_t}} \right| = \frac{-p_{T_a} \cos \Delta\phi}{p_{T_t}} \approx \frac{p_{T_a}}{p_{T_t}} = \frac{p_{T_a}/\hat{p}_{T_t}}{p_{T_t}/\hat{p}_{T_t}} \approx \frac{z_a}{\langle z_t \rangle}$$

M79

From Feynman, Field and Fox: the x_E distribution corrected for $<z_t>$ measures the unbiased fragmentation function

$$\frac{dP^{\mathrm{FFF}}}{dx_E} \approx \langle z_t \rangle \, B \exp -b \langle z_t \rangle \, x_E$$

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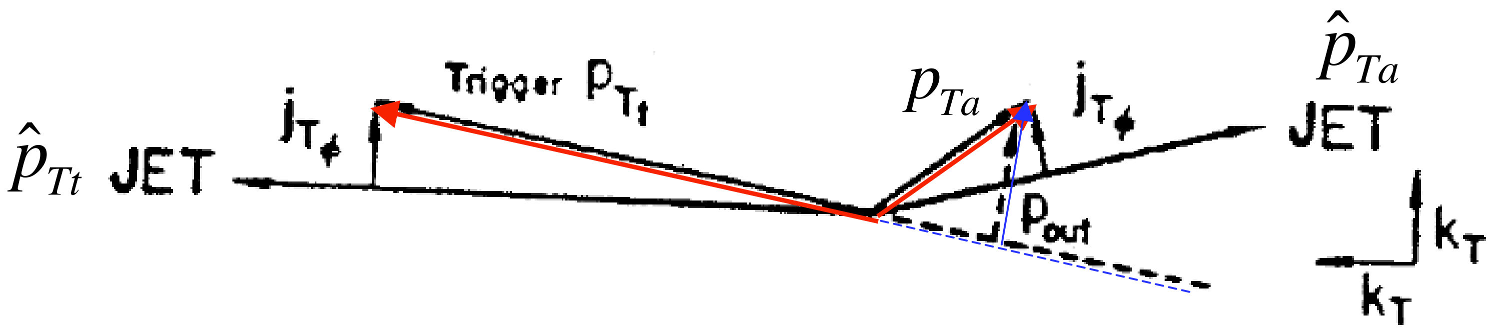

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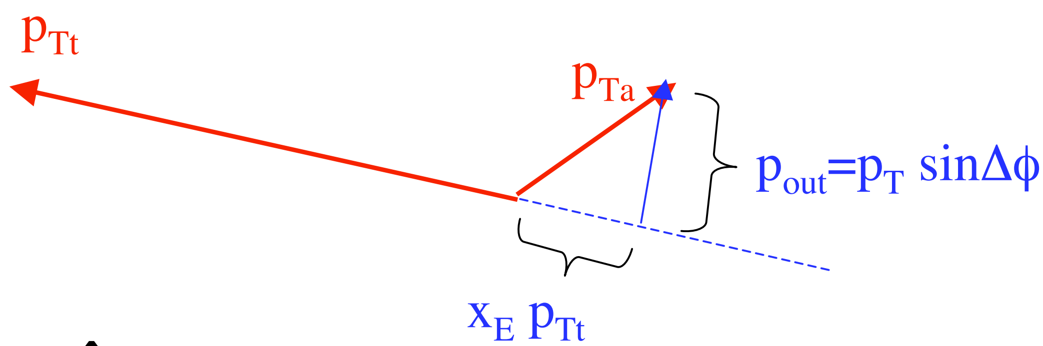

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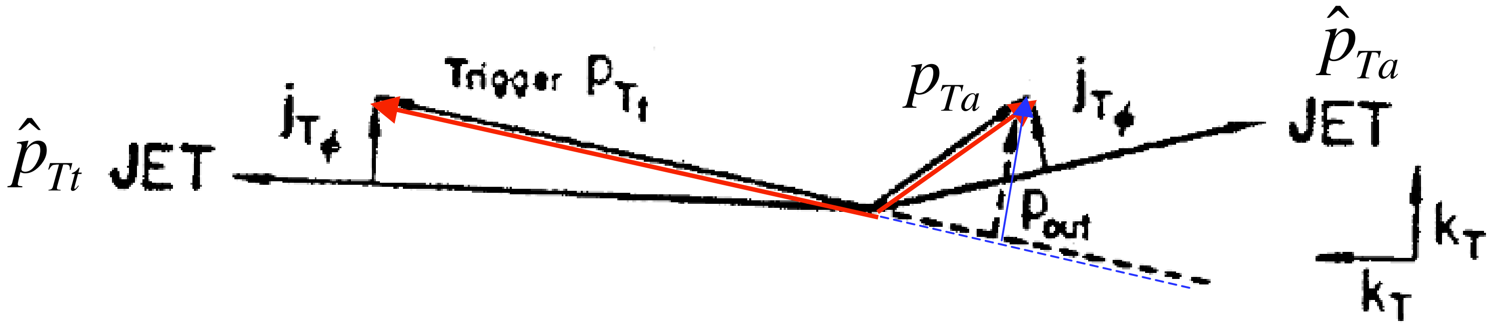

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From Feynman, Field and Fox Nucl Phys B128 (1977) 1--65

38 *R.P Feynman et al. / Large transverse momenta*

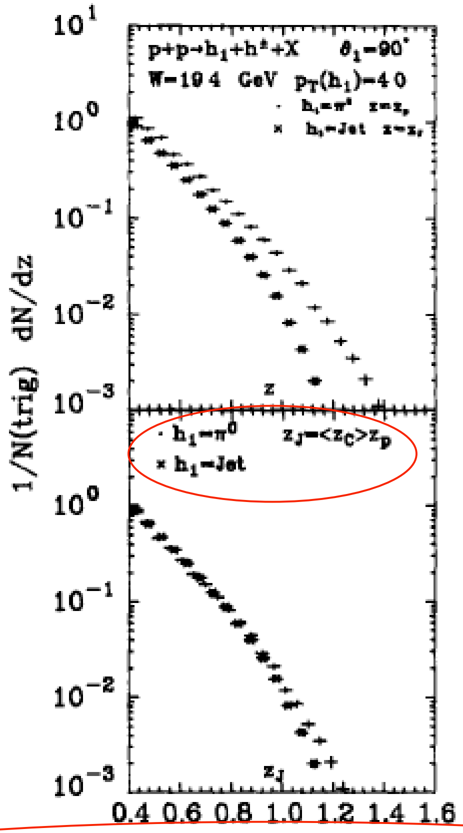
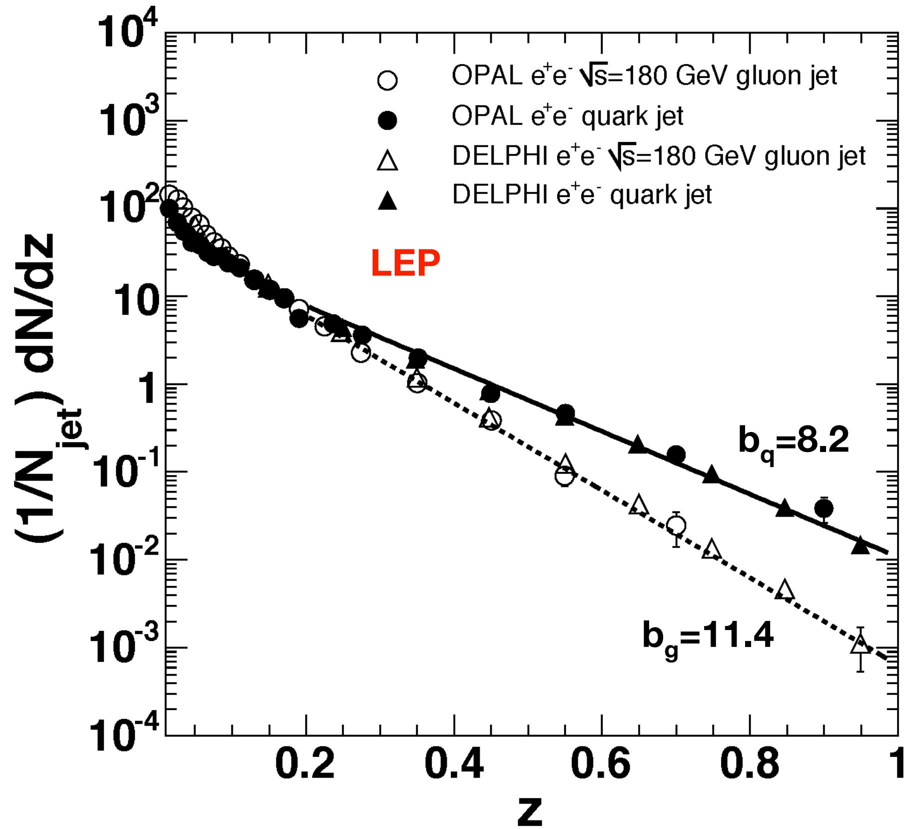
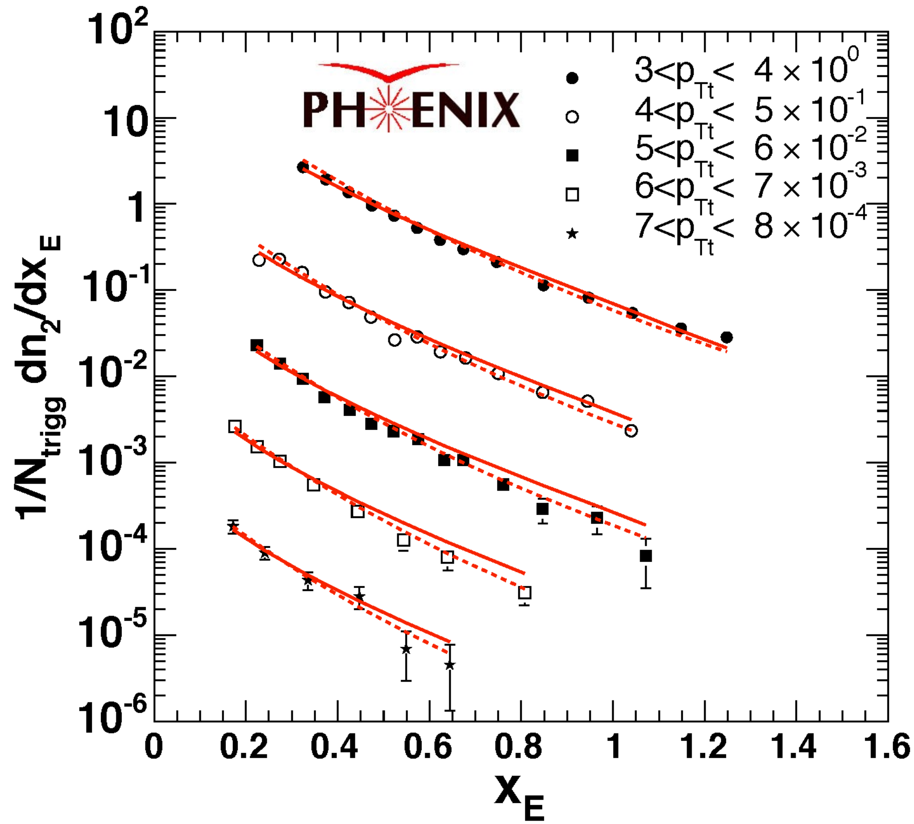


Fig. 23. Comparison of the π^0 and jet trigger away-side distribution of charged hadrons in pp collisions at $W = 19.4$ GeV, $\theta_1 = 90°$, and $p_\perp$ (trigger) = 4.0 GeV/c from the quark-quark scattering model. The upper figure shows the single-particle (π^0) trigger results plotted *versus* $z_p = -p_x(h^\pm)/p_\perp(\pi^0)$ and the jet trigger plotted *versus* $z_J = -p_x(h^\pm)/p_\perp(\text{jet})$ (see table 1). In the lower figure, we plot *both versus* z_J, where for the jet trigger $z_J = z_J$ but for the single-particle trigger $z_J = \langle z_c \rangle z_p$. The away hadrons are integrated over all rapidity Y and $|180° - \phi| \leqslant 45°$ and the theory is calculated using $\langle k_\perp \rangle_{h\to q} = 500$ MeV. • $h_1 = \pi^0$, × h_1 = jet.

"There is a simple relationship between experiments done with single-particle triggers and those performed with jet triggers. The only difference in the opposite side correlation is due to the fact that the 'quark', from which a single-particle trigger came, always has a higher $p_\perp$ than the trigger (by factor $1/z_{trig}$). The away-side correlations for a single-particle trigger at $p_\perp$ should be roughly the same as the away side correlations for a jet trigger at $p_\perp$ (jet)= $p_\perp$ (single particle)/ $\langle z_{trig} \rangle$".

PHENIX-compared measured x_E distribution in p-p to numerical integral using LEP fragmentation functions

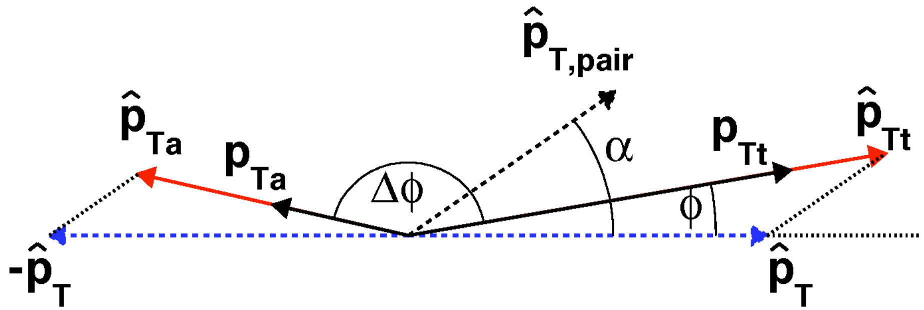




PHENIX PRD **74** (2006) 072002. The x_E distribution triggered by a leading fragment (π^0) is not sensitive to the shape of the fragmentation function!!! Disagrees with FFF!!

A very interesting new formula for the x_E distribution was derived by PHENIX in PRD74

$$\left.\frac{dP_\pi}{dx_E}\right|_{p_{T_t}} \approx \langle m\rangle (n-1)\frac{1}{\hat{x}_h}\frac{1}{(1+\frac{x_E}{\hat{x}_h})^n}$$



Relates ratio of particle p_T

$$x_E = \frac{-p_{T_a}\cos\Delta\phi}{p_{T_t}} \simeq \frac{p_{T_a}}{p_{T_t}}$$

measured

Ratio of jet transverse momenta

$$\hat{x}_h = \frac{\hat{p}_{T_a}}{\hat{p}_{T_t}}$$

Can be determined

If formula works, we can also use it in Au+Au to determine the relative energy loss of the away jet to the trigger jet (surface biased by large *n*)

Exponential Frag. Fn. and power law crucial

$$\frac{d^2\sigma_\pi(\hat{p}_{T_t}, z_t)}{d\hat{p}_{T_t} dz_t} = \frac{d\sigma_q}{d\hat{p}_{T_t}} \times D^q_\pi(z_t) = \boxed{\frac{A}{\hat{p}^{n-1}_{T_t}}} \times D^q_\pi(z_t)$$

Fragment spectrum given $\hat{p}_{T_t}$

Power law spectrum of parton $\hat{p}_{T_t}$

Let $\hat{p}_{T_t} = p_{T_t}/z_t \qquad d\hat{p}_{T_t}/dp_{T_t}|_{z_t} = 1/z_t$

$$\frac{d^2\sigma_\pi(p_{T_t}, z_t)}{dp_{T_t} dz_t} = \frac{A}{p^{n-1}_{T_t}} \times z_t^{n-2} D^q_\pi(z_t)$$

Fragment spectrum given p_{Tt} is weighted to high z_t by z_t^{n-2}

where $z_{t\min}|_{p_{T_t}} = x_{T_t}$

$$\boxed{D^q_\pi(z_t) = B e^{-bz_t}}$$

$$\boxed{\frac{1}{p_{T_t}} \frac{d\sigma_\pi}{dp_{T_t}} = \frac{AB}{p^n_{T_t}} \int_{x_{T_t}}^1 dz_t \; z_t^{n-2} \exp -bz_t}$$

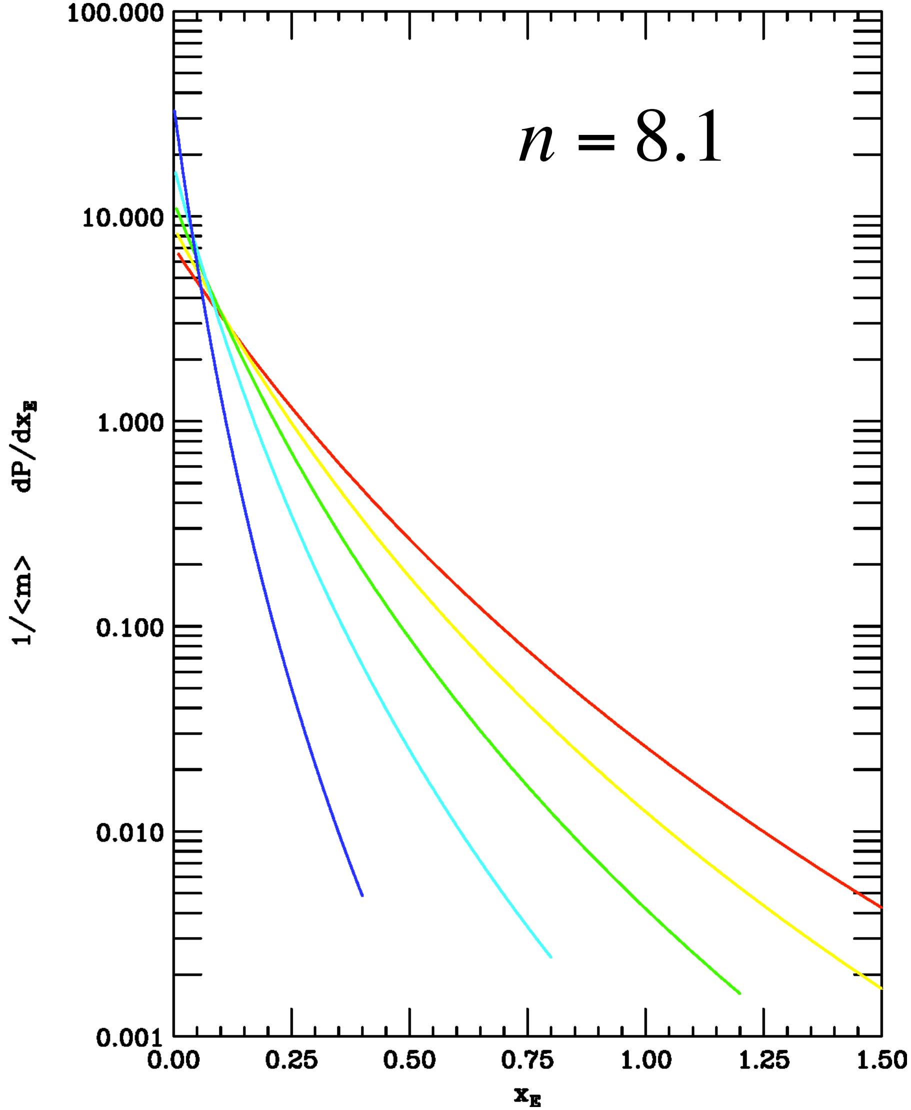
Incomplete gamma function

Good approximation $x_{T_t} \rightarrow 0$ upper limit $\rightarrow \infty$

$$\boxed{\frac{1}{p_{T_t}} \frac{d\sigma_\pi}{dp_{T_t}} \approx \frac{\Gamma(n-1)}{b^{n-1}} \frac{AB}{p^n_{T_t}}}$$

Bjorken parent-child relation: parton and particle invariant p_T spectra have same power n, etc.

Shape of x_E distribution depends on $\hat{x}_h$ and n but not on b-i.e. FFF failed



$$\left.\frac{dP_\pi}{dx_E}\right|_{p_{T_t}} \approx \langle m\rangle (n-1)\frac{1}{\hat{x}_h}\frac{1}{(1+\frac{x_E}{\hat{x}_h})^n}$$

$\hat{x}_h$

1.0

0.8

0.6

0.4

0.2

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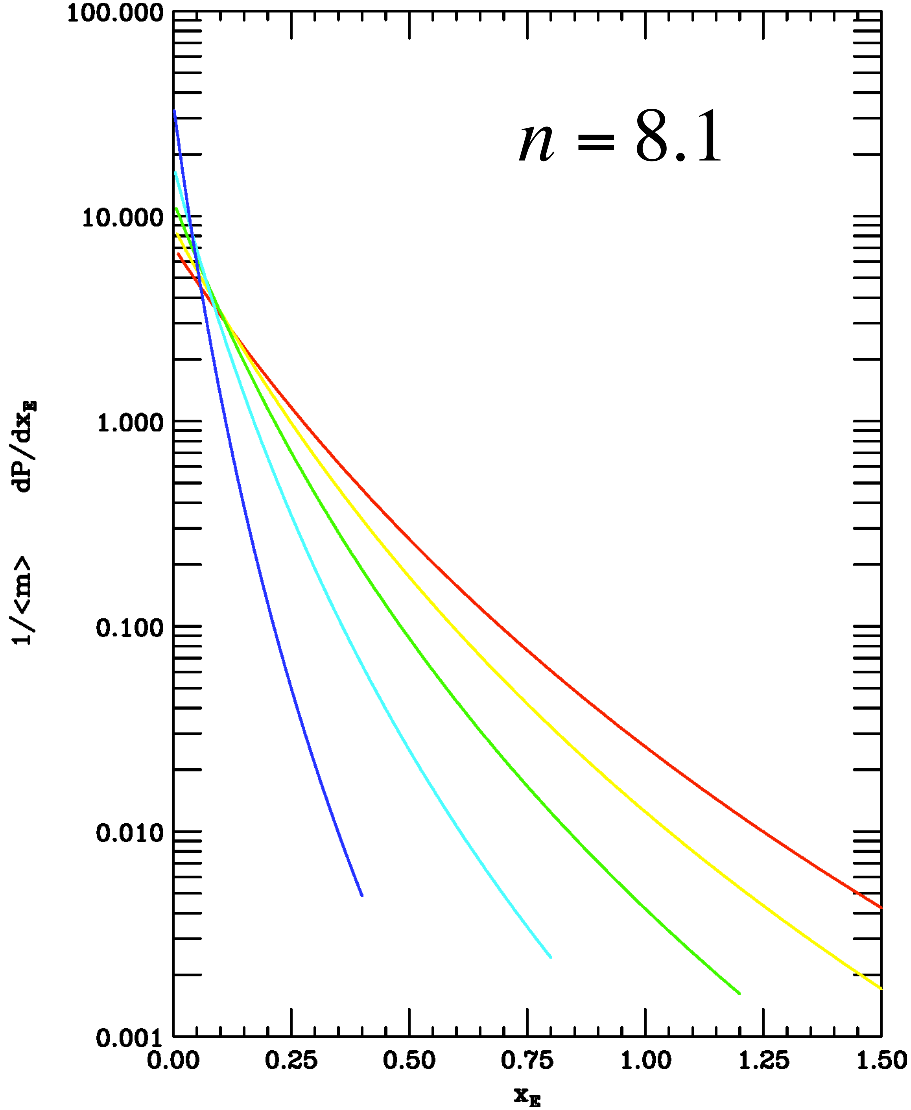


$$\left.\frac{dP_\pi}{dx_E}\right|_{p_{T_t}} \approx N(n-1)\frac{1}{\hat{x}_h}\frac{1}{(1+\frac{x_E}{\hat{x}_h})^n}$$

$\hat{x}_h$

1.0

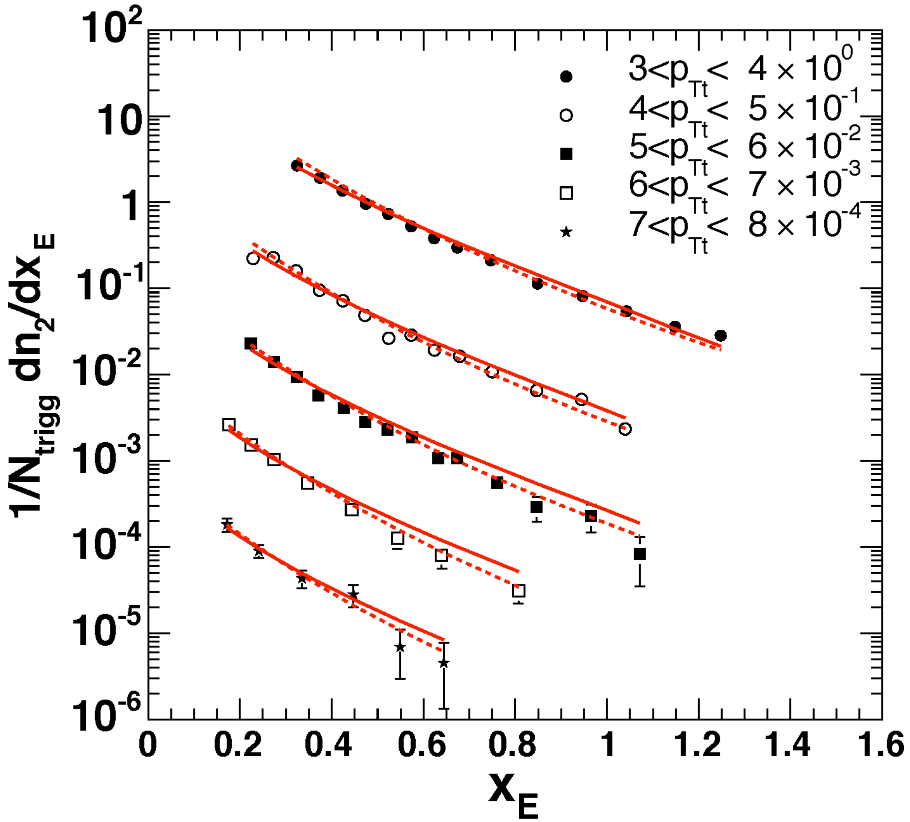
0.8

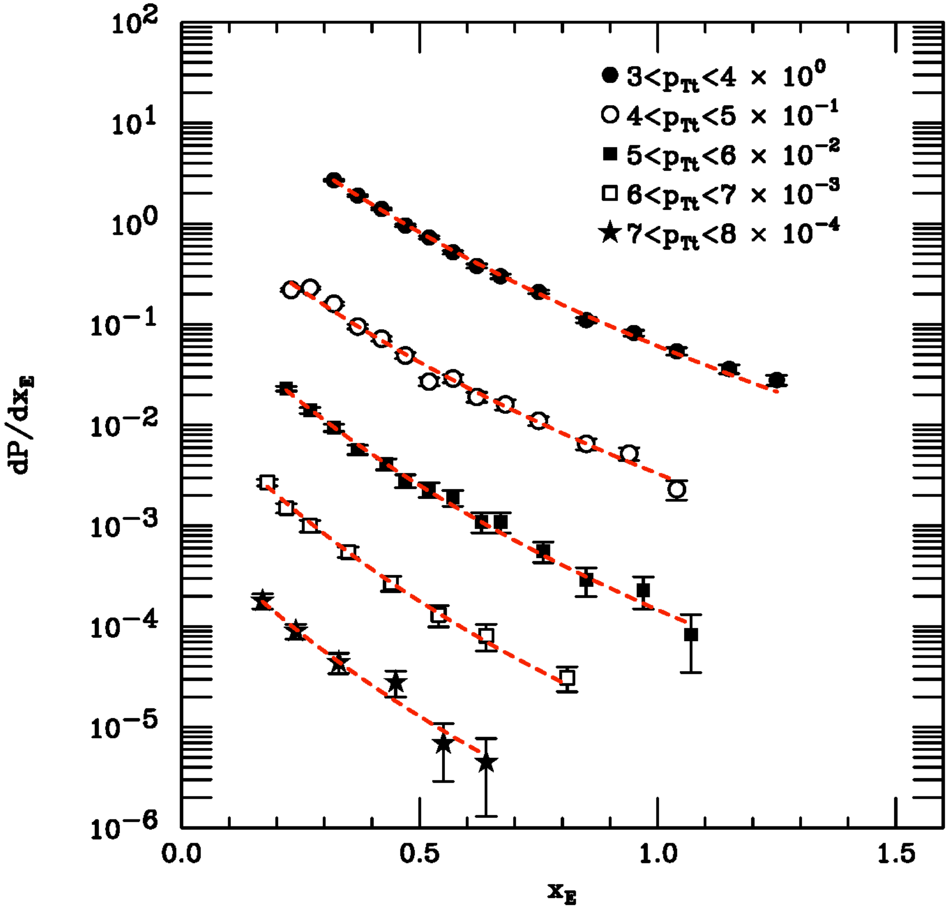
0.6

0.4

0.2

Fit works for PHENIX p+p PRD **74**, 072002

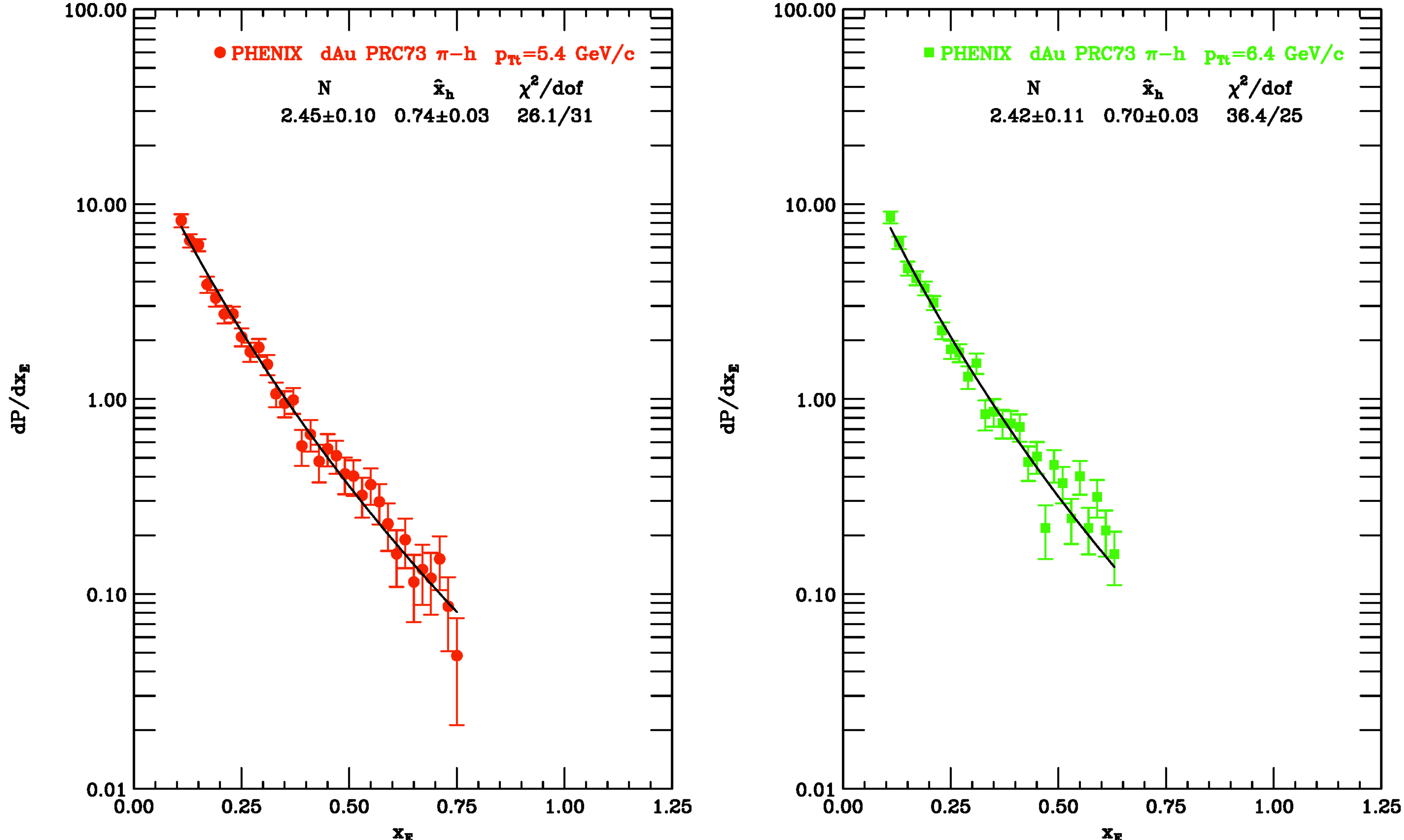




Calculation from Fragmentation Fn.

New fits. Very nice!

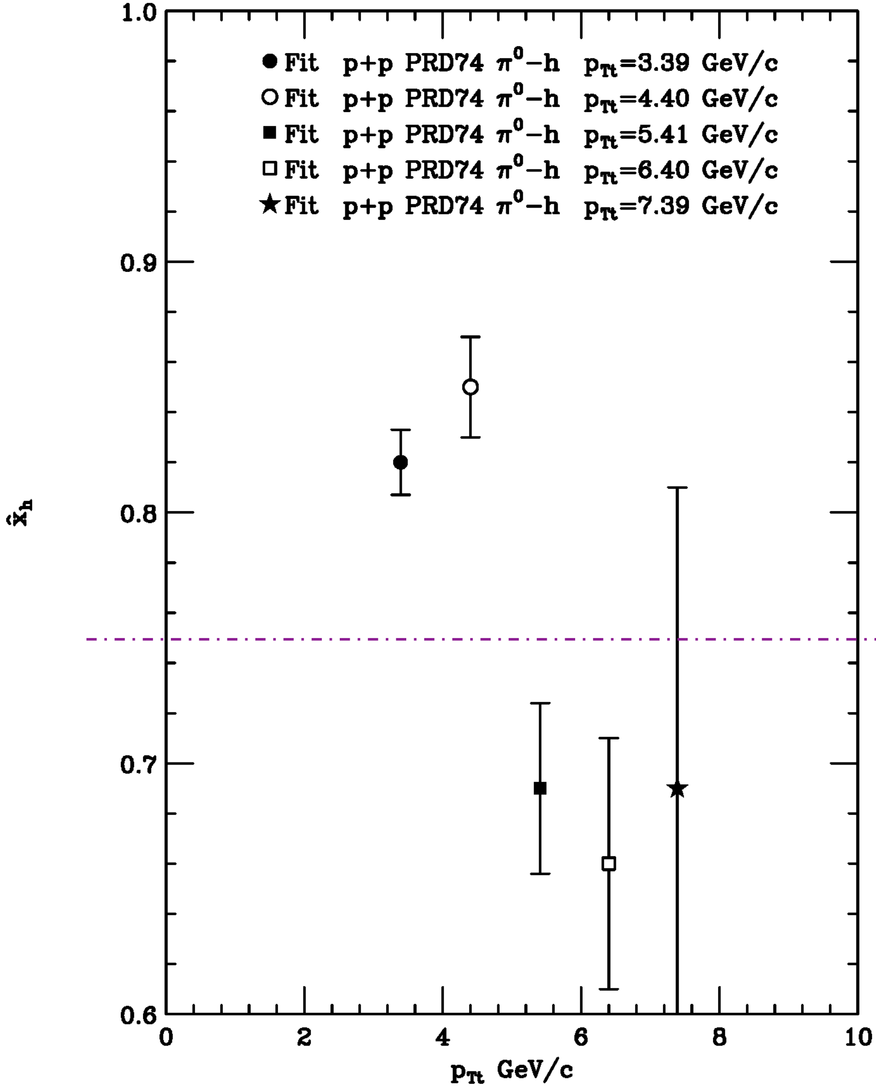
PHENIX $\pi^{\pm}$-h d+Au PRC73 (2006) 054903

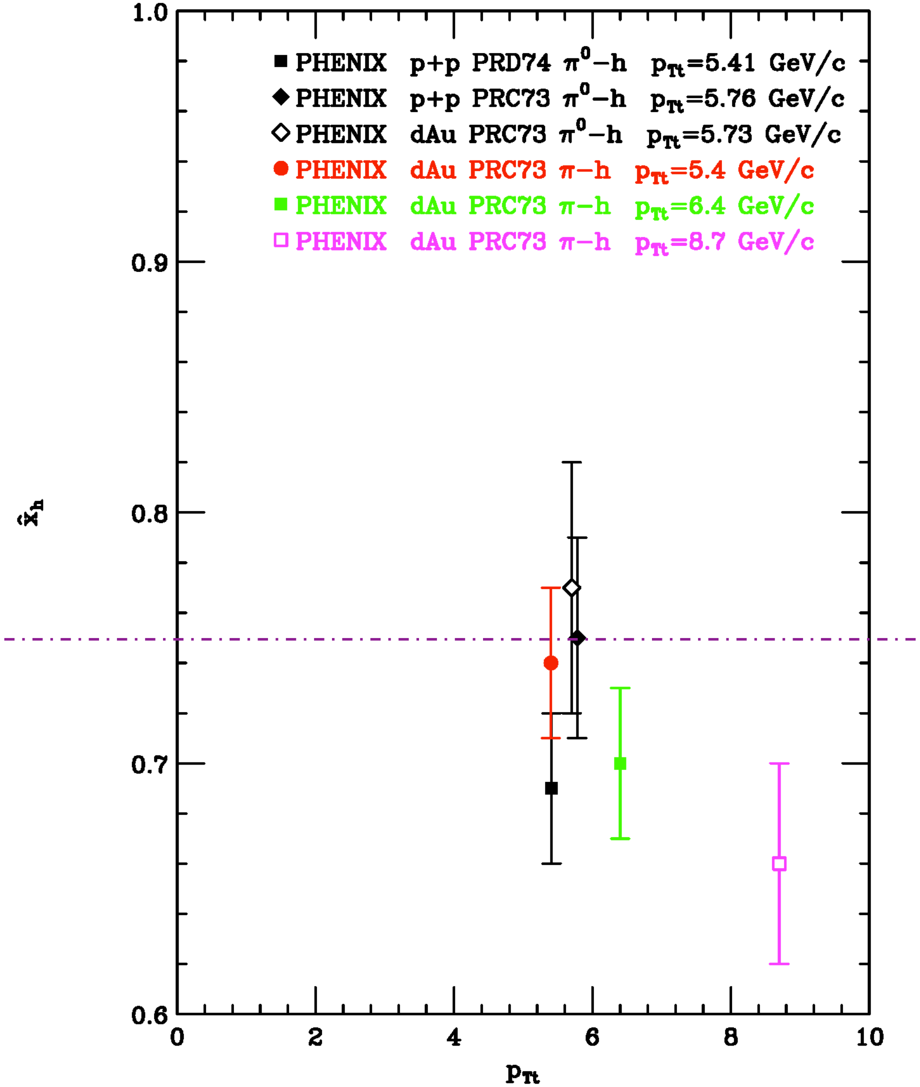


Excellent χ^2 in most cases

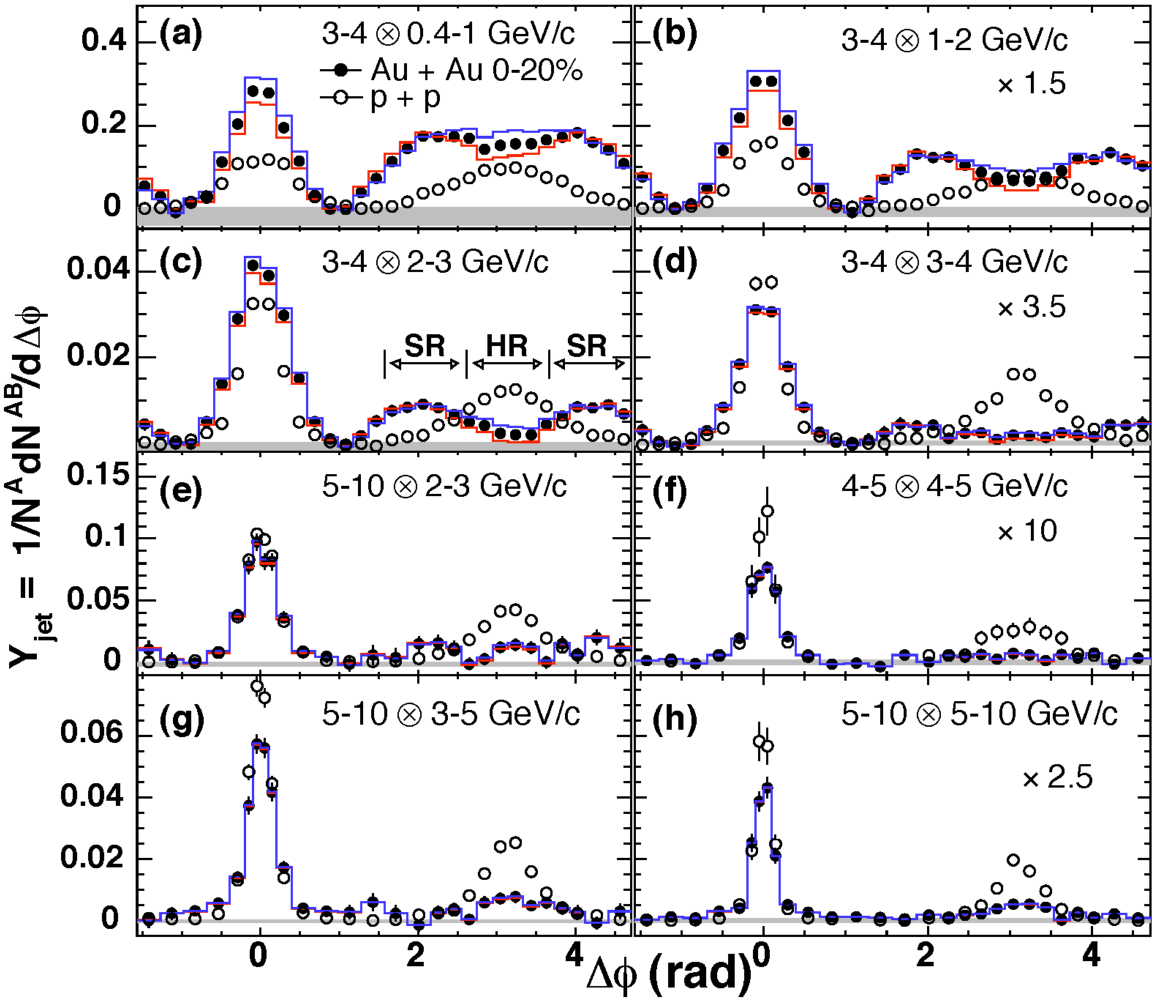
$\hat{x}_h \sim 0.75$ due to k_T smearing in p-p, dAu

k_T and k_T smearing was a big topic at Moriond 1979





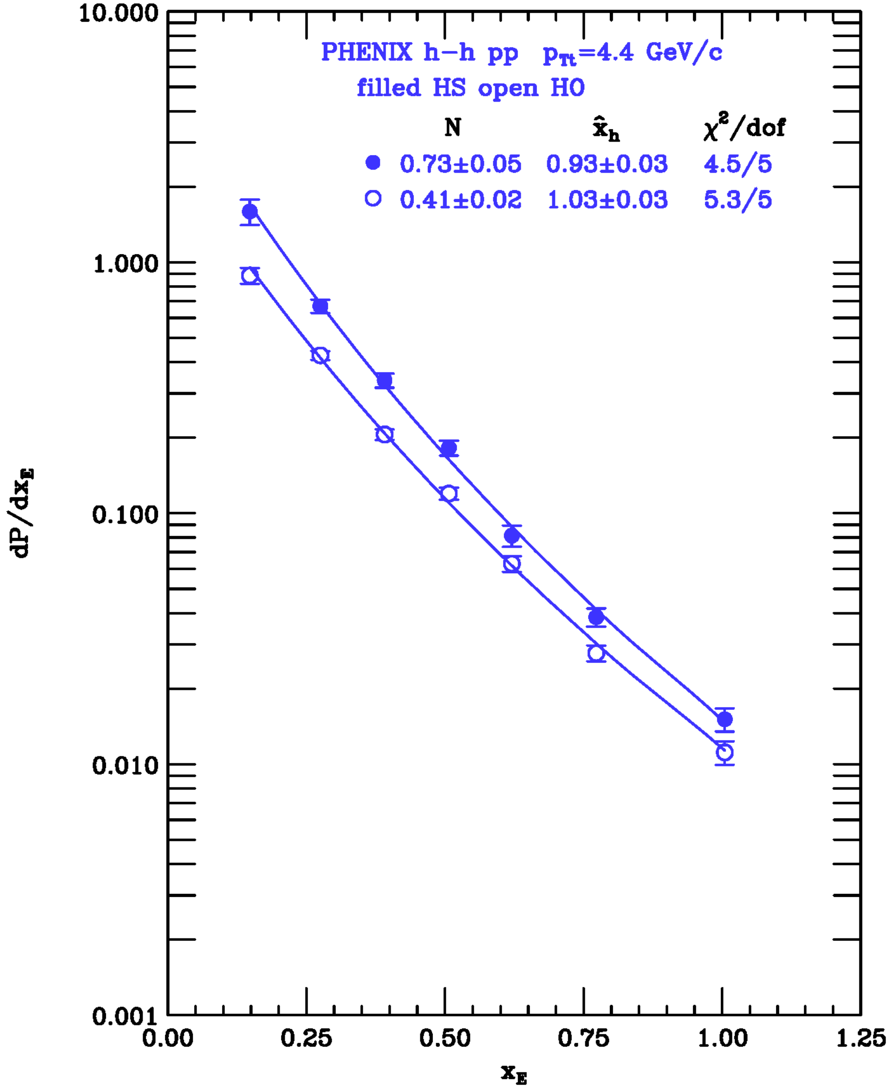
New PHENIX AuAu PRC 77,011901(R)(2008)

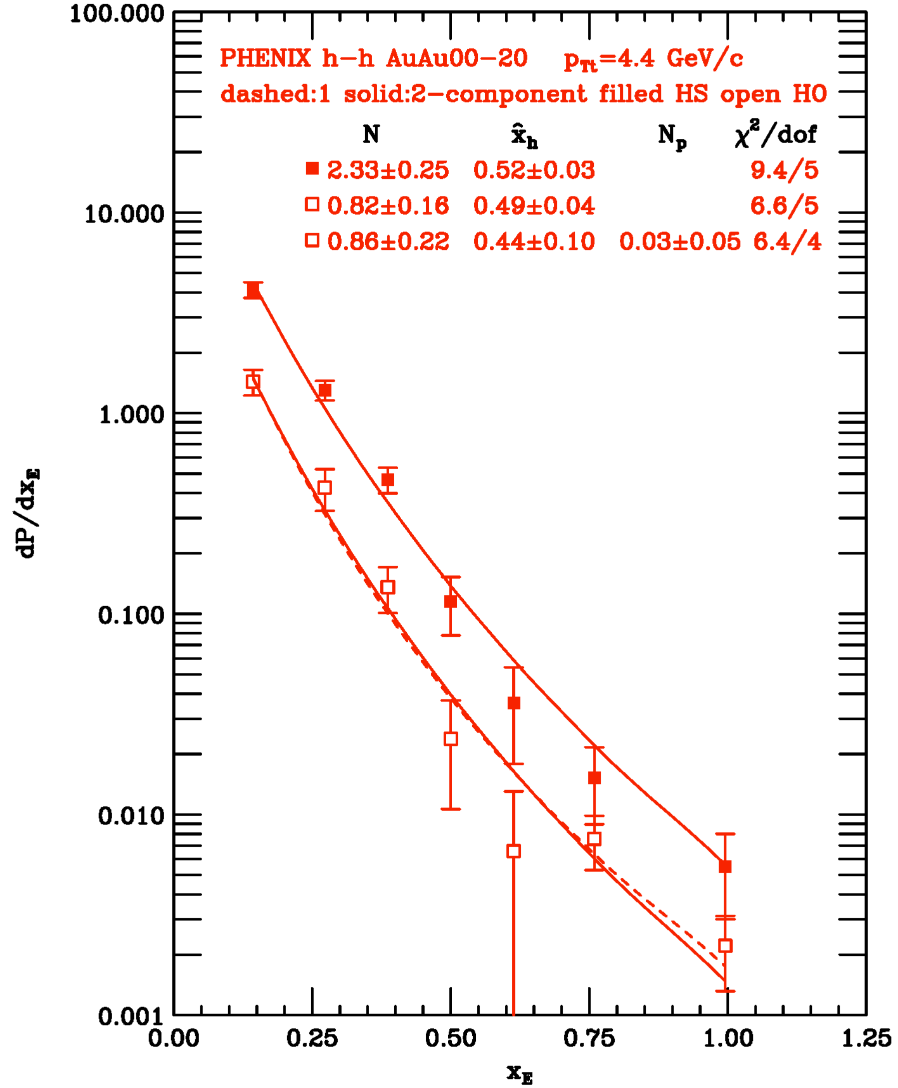


Away side correlation in Au+Au is generally wider than p-p with complicated structure

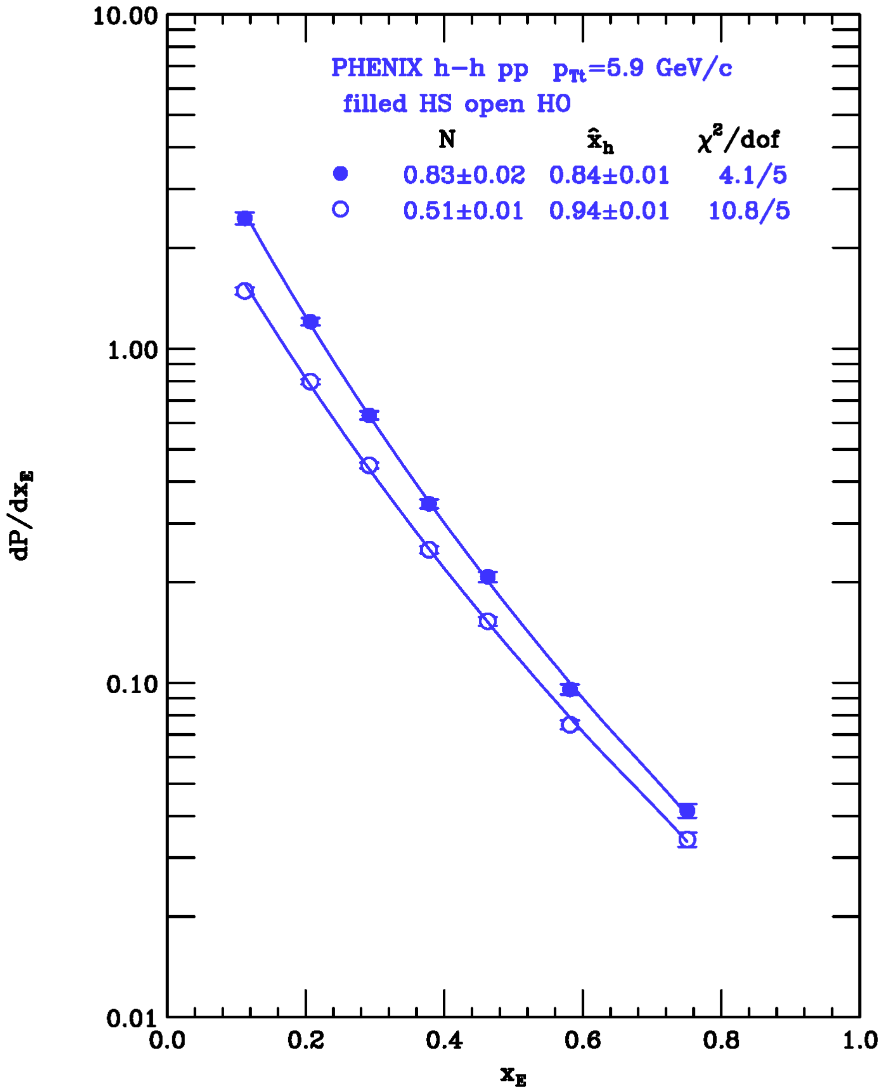
Define Head region (HR) and Shoulder regions (SR) for wide away side correlation.

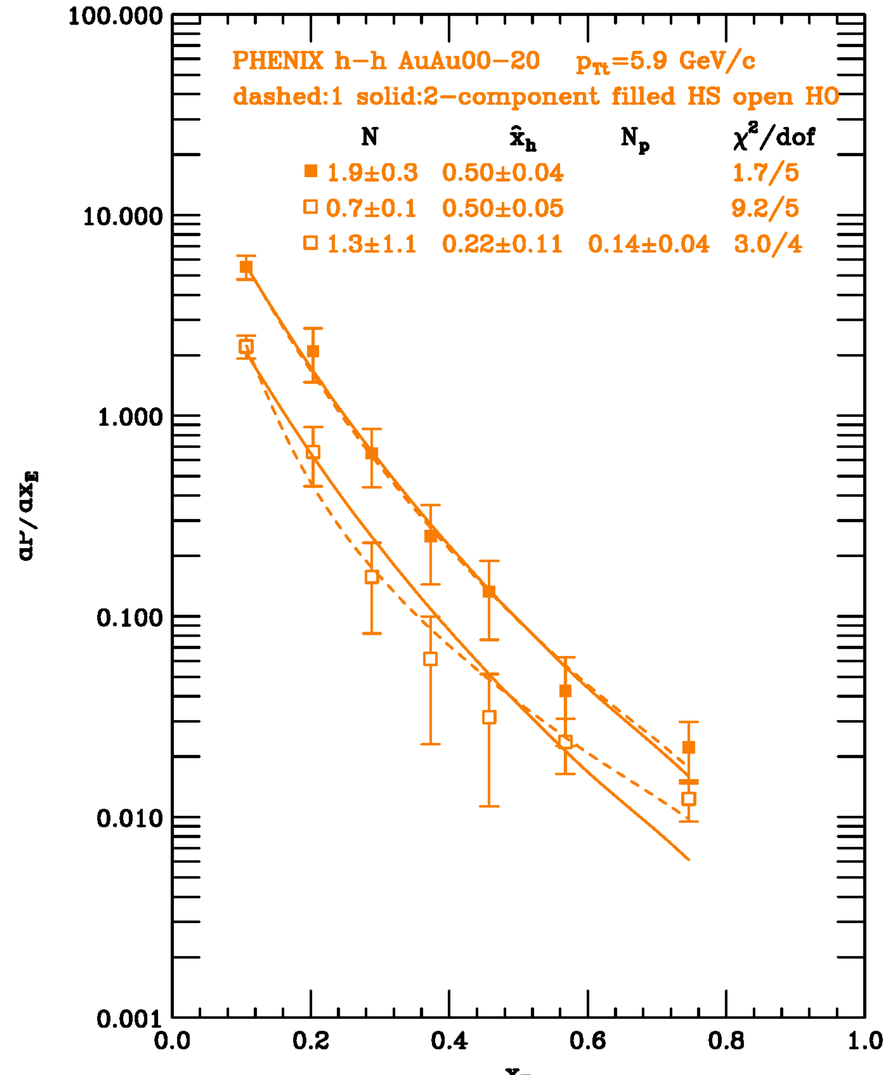
Fit H+S and HO (head only) $4<p_{Tt}<5$ GeV/c





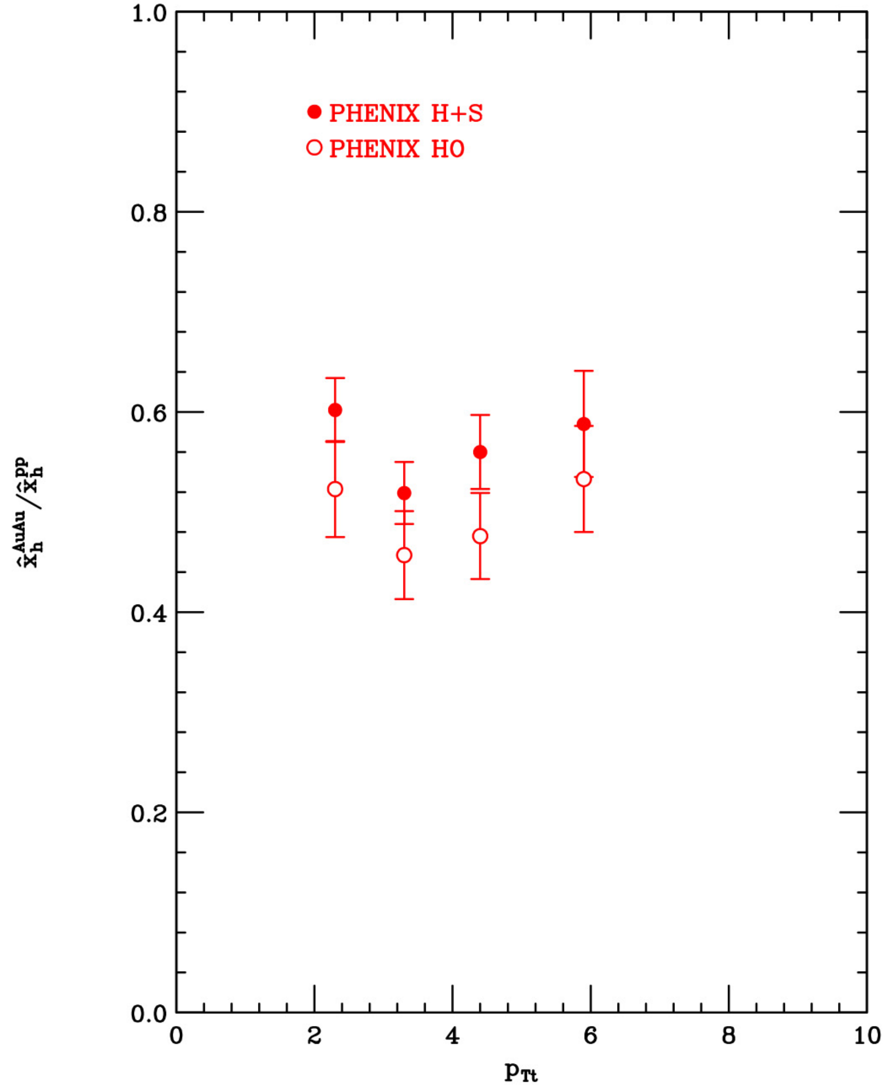
$5 < p_{Tt} < 10$ GeV/c



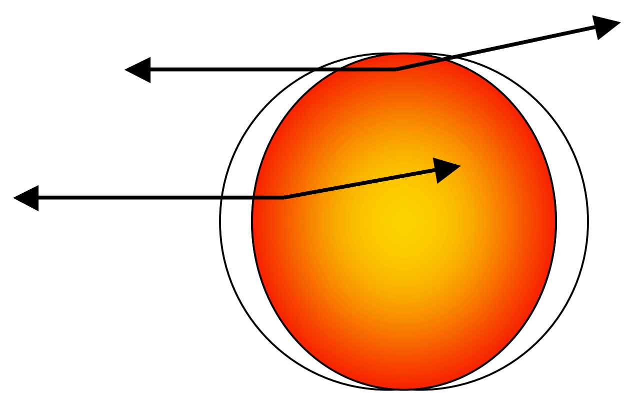


Statistical two-component distribution (⇒punch-through) for Head-Only

Formula works in Au+Au: Away-side x_E distribution is steeper in Au+Au than p-p indicating energy loss

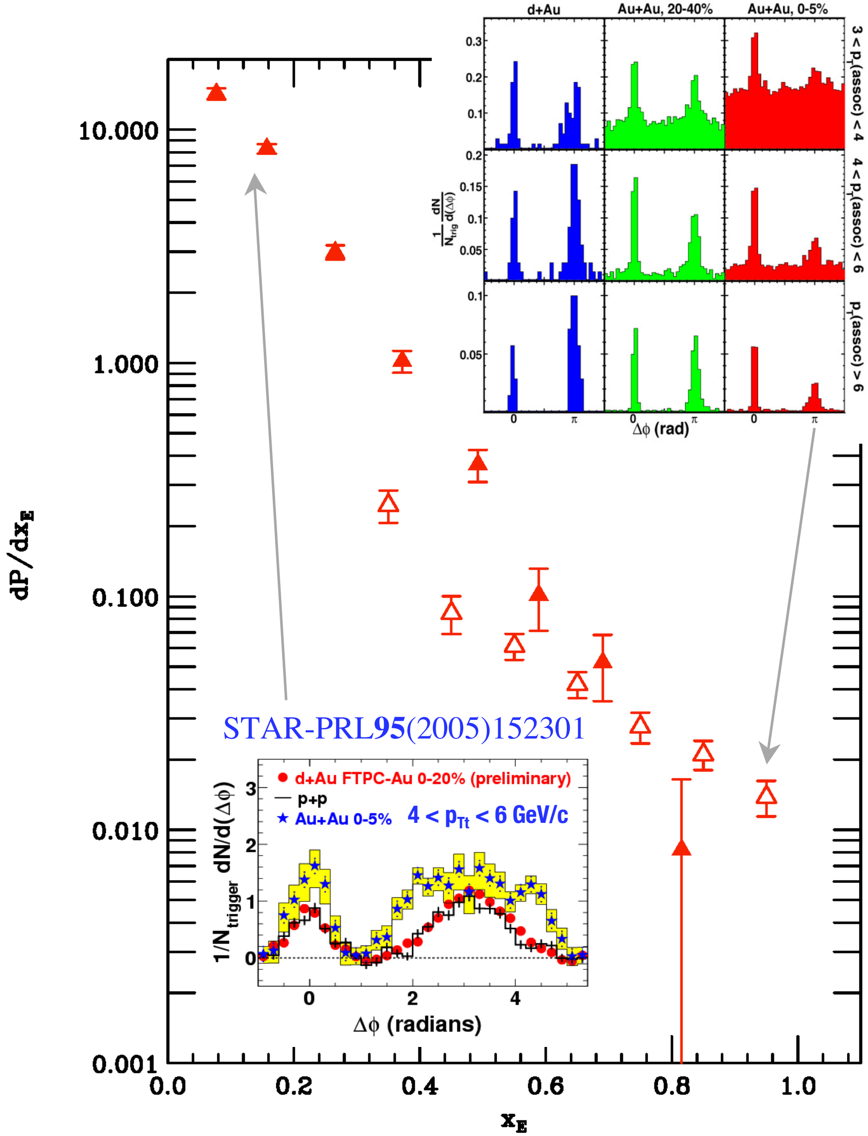


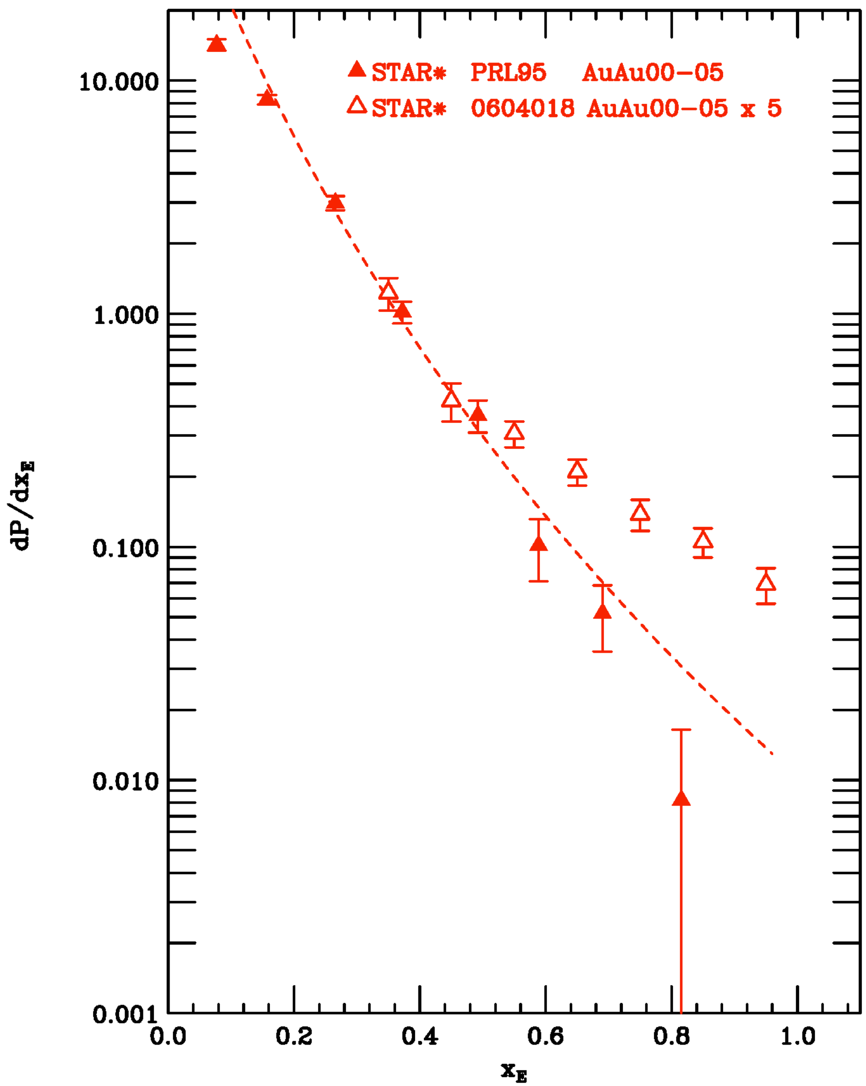
Since the trigger jet is surface biased, the away jet must cross through nearly the entire medium except in the case of tangential emission. The decrease of $\hat{x}_h = \hat{p}_{Ta}/\hat{p}_{Tt}$ in Au+Au central collisions relative to p-p by a factor of ~0.5-0.6 indicates that the away jet has lost energy by traversing the medium and gives a quantitative measurement.



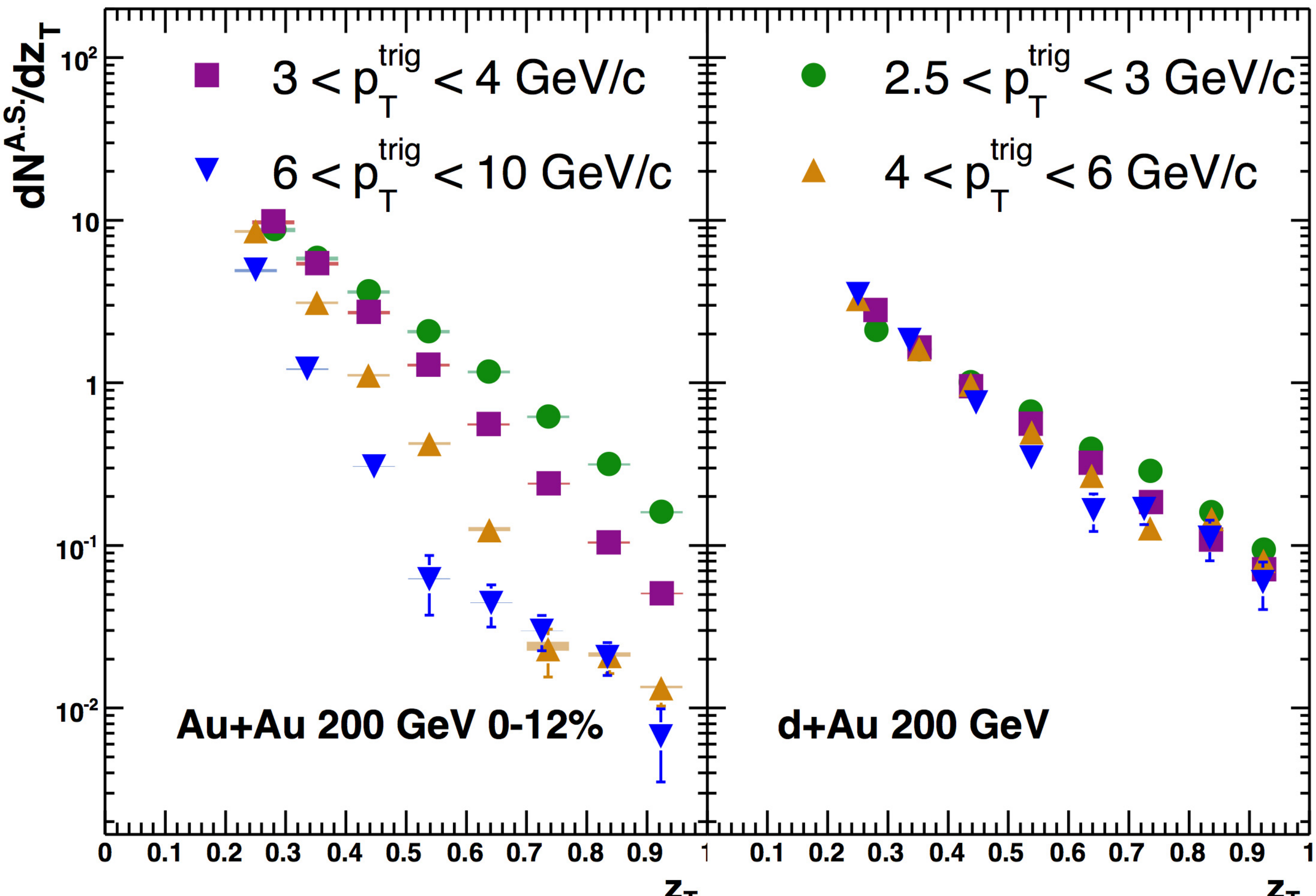
STAR dAu, AuAu

STAR-PRL **97** (2006) 162301
$8 < p_{Tt} < 15$ GeV/c



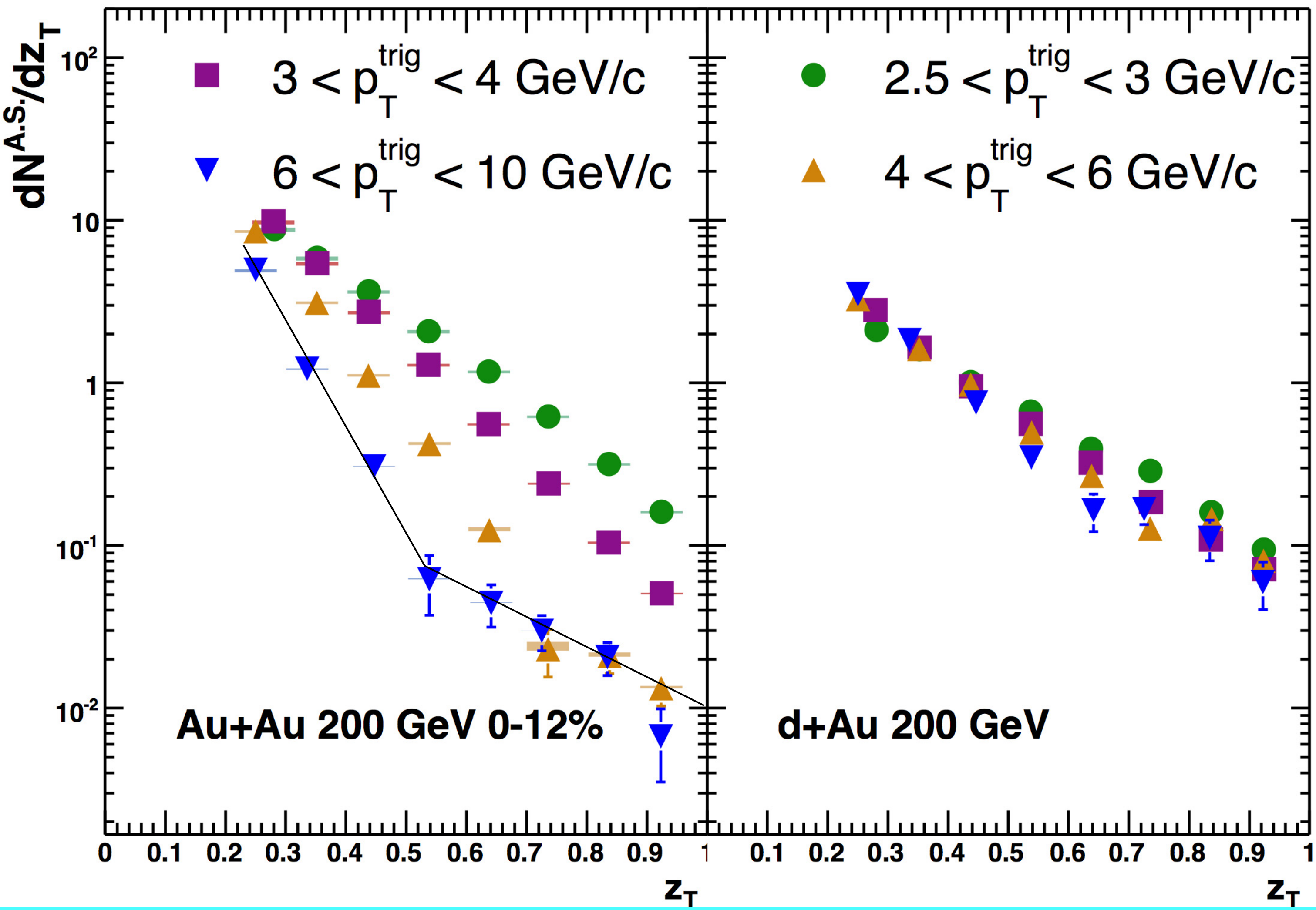


Newest STAR-Horner-QM2006-JPG34 (2007) S995



Two-component distribution (punch-through) is now clear for $6 < p_{Tt} < 10$ GeV/c

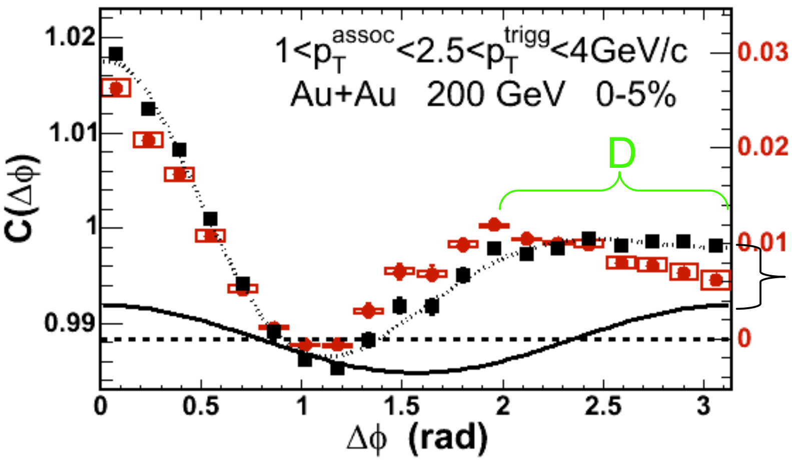
Newest STAR-Horner-QM2006-JPG34 (2007) S995

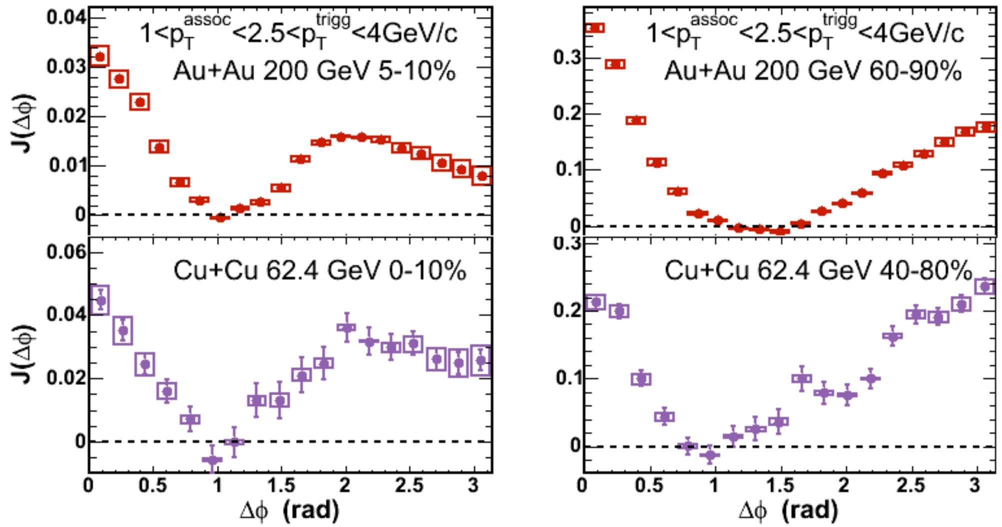


Two-component distribution (punch-through) is now clear for $6 < p_{Tt} < 10$ GeV/c

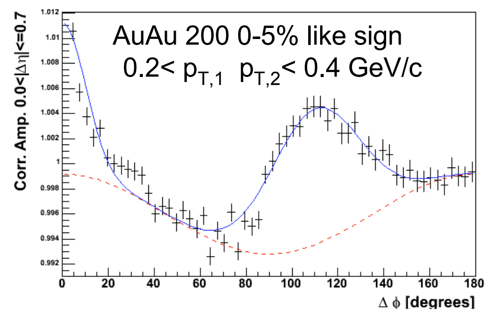
The End

Away side correlations in Au+Au much wider than in p-p

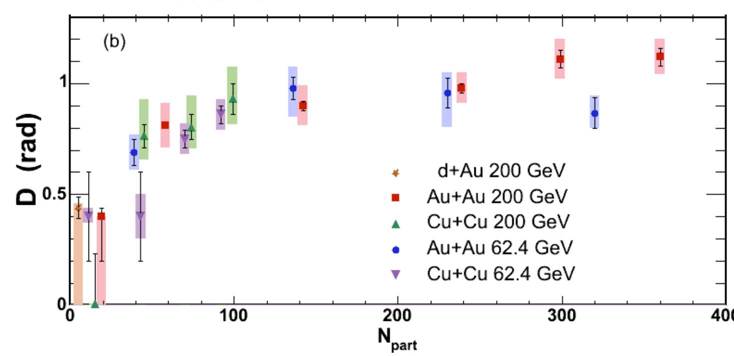




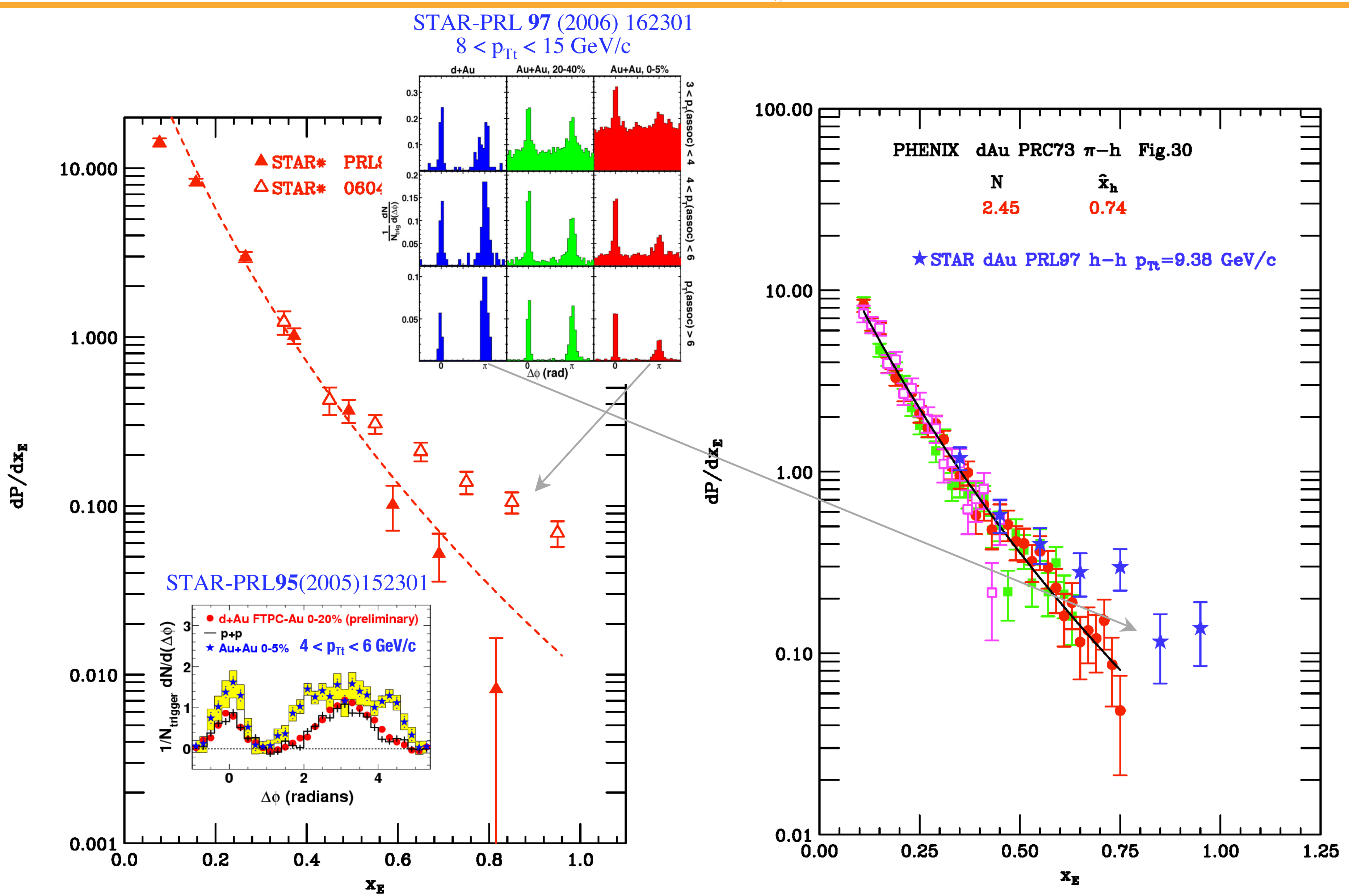
Away side distribution much wider in A+A than p-p in correlation fn. C(Δφ) Subtraction of v2 (flow?) effect→ J(Δφ) causes a dip at 180° which gives 2 peaks at π±D~1 radian independent of system and centrality for N_{part} >100. This is also seen for (auto) correlations of low p_T particles. Is this the medium reaction to the passage of a color-charged parton? Stay tuned, much more study needed.



PHENIX AuAu PRL **98** (2007) 232302



STAR dAu, AuAu



The leading-particle effect a.k.a. trigger bias

- Due to the steeply falling power-law spectrum of the scattered partons, the inclusive particle p_T spectrum is dominated by fragments biased towards large z. This was unfortunately called trigger bias by M. Jacob and P. Landshoff, Phys. Rep. **48C**, 286 (1978) although it has nothing to do with a trigger.

$$\frac{d^2\sigma_\pi(\hat{p}_{T_t}, z_t)}{d\hat{p}_{T_t} dz_t} = \frac{d\sigma_q}{d\hat{p}_{T_t}} \times D^q_\pi(z_t)$$

Fragment spectrum given $\hat{p}_{T_t}$

$$= \frac{A}{\hat{p}_{T_t}^{n-1}} \times D^q_\pi(z_t)$$

Power law spectrum of parton $\hat{p}_{T_t}$

let $\hat{p}_{T_t} = p_{T_t}/z_t \qquad d\hat{p}_{T_t}/dp_{T_t}|_{z_t} = 1/z_t$

$$\frac{d^2\sigma_\pi(p_{T_t}, z_t)}{dp_{T_t} dz_t} = \frac{1}{z_t}\frac{A}{(p_{T_t}/z_t)^{n-1}} \times D^q_\pi(z_t)$$

$$= \frac{A}{p_{T_t}^{n-1}} \times z_t^{n-2} D^q_\pi(z_t)$$

Fragment spectrum given p_{Tt} is weighted to high z_t by z_t^{n-2}

where $z_{t\min}|_{p_{T_t}} = x_{T_t} \qquad D^q_\pi(z_t) = Be^{-bz_t}$ (<z>=1/b)

Continuing as in PRD **74**, 072002 (2006)

We can integrate over the trigger jet z_t and find the inclusive pion cross section:

$$\frac{1}{p_{T_t}}\frac{d\sigma_\pi}{dp_{T_t}} = \frac{AB}{p_{T_t}^n}\int_{x_{T_t}}^1 dz_t z_t^{n-2} \exp -bz_t \quad , \tag{8}$$

which can be written as:

$$\frac{1}{p_{T_t}}\frac{d\sigma_\pi}{dp_{T_t}} = \frac{AB}{p_{T_t}^n}\frac{1}{b^{n-1}}\left[\Gamma(n-1, bx_{T_t}) - \Gamma(n-1,b)\right] \quad , \tag{9}$$

where

$$\Gamma(a,x) \equiv \int_x^\infty t^{a-1}\, e^{-t}\, dt \tag{10}$$

is the Complementary or upper Incomplete Gamma function, and $\Gamma(a,0) = \Gamma(a)$ is the Gamma function, where $\Gamma(a) = (a-1)!$ for a an integer.

A reasonable approximation for small x_T values is obtained by taking the lower limit of Eq. 8 to zero and the upper limit to infinity, with the result that:

$$\boxed{\frac{1}{p_{T_t}}\frac{d\sigma_\pi}{dp_{T_t}} \approx \frac{\Gamma(n-1)}{b^{n-1}}\frac{AB}{p_{T_t}^n}}$$

Bjorken parent-child relation: parton and particle invariant p_T spectra have same power n

$$\boxed{\langle z_t(p_{T_t})\rangle = \frac{\int_{x_{T_t}}^1 dz_t z_t^{n-1}\exp -bz_t}{\int_{x_{T_t}}^1 dz_t z_t^{n-2}\exp -bz_t} = \frac{1}{b}\frac{[\Gamma(n, bx_{T_t}) - \Gamma(n,b)]}{[\Gamma(n-1, bx_{T_t}) - \Gamma(n-1,b)]} \approx \frac{n-1}{b}}$$

Inclusive high p_T particle has n-1 times larger <z> than unbiased fragmentation, <z>=1/b

2 particle Correlations

$$\frac{d^2\sigma_\pi(\hat{p}_{T_t}, z_t)}{d\hat{p}_{T_t} dz_t} = \frac{d\sigma_q}{d\hat{p}_{T_t}} \times D^q_\pi(z_t)$$

Prob. that you make a jet with $\hat{p}_{Tt}$ which fragments to a π with $z_t=p_{Tt}/\hat{p}_{Tt}$

Also detect fragment with $z_a = p_{T_a}/\hat{p}_{T_a}$
from away jet with $\hat{p}_{T_a}/\hat{p}_{T_t} \equiv \hat{x}_h$

$$\frac{d^3\sigma_\pi(\hat{p}_{T_t}, z_t, z_a)}{d\hat{p}_{T_t} dz_t dz_a} = \frac{d\sigma_q}{d\hat{p}_{T_t}} \times D^q_\pi(z_t) \times D^q_\pi(z_a)$$

Prob. that away jet with $\hat{p}_{Ta}$ fragments to a π with $z_a=p_{Ta}/\hat{p}_{Ta}$

$$z_a = \frac{p_{T_a}}{\hat{p}_{T_a}} = \frac{p_{T_a}}{\hat{x}_{\rm h}\,\hat{p}_{T_t}} = \frac{z_t\,p_{T_a}}{\hat{x}_{\rm h}\,p_{T_t}}$$

$$\boxed{\frac{d\sigma_\pi}{dp_{T_t} dz_t dp_{T_a}} = \frac{1}{\hat{x}_{\rm h}\,p_{T_t}} \frac{d\sigma_q}{d(p_{T_t}/z_t)} D^q_\pi(z_t) D^q_\pi\left(\frac{z_t p_{T_a}}{\hat{x}_{\rm h} p_{T_t}}\right)} \quad (1)$$

Appears to be sensitive to away jet Frag. Fn.

Amazingly, I got a neat analytical result

$$\boxed{\frac{d^3\sigma_\pi}{dp_{T_t}dz_t dp_{T_a}} = \frac{1}{\hat{x}_{\mathrm{h}}\, p_{T_t}} \frac{d\sigma_q}{d(p_{T_t}/z_t)} D_q^\pi(z_t) D_q^\pi(\frac{z_t p_{T_a}}{\hat{x}_{\mathrm{h}} p_{T_t}})} \quad (1)$$

Take: $D(z) = B\exp(-bz)$ $\quad \frac{d\sigma_q}{d\hat{p}_{T_t}} = \frac{A}{\hat{p}_{T_t}^{n-1}} = A\frac{{z_t}^{n-1}}{p_{T_t}^{n-1}}$

$$(2) \quad \frac{d^2\sigma_\pi}{dp_{T_t}dp_{T_a}} = \frac{B^2}{\hat{x}_h}\frac{A}{p_{T_t}^n}\int_{x_{T_t}}^{\hat{x}_h \frac{p_{T_t}}{p_{T_a}}} dz_t z_t^{n-1} \exp[-bz_t(1+\frac{p_{T_a}}{\hat{x}_h p_{T_t}})]$$

$$\frac{d\sigma_\pi}{dp_{T_t}} = \frac{AB}{p_{T_t}^{n-1}}\int_{x_{T_t}}^1 dz_t z_t^{n-2} \exp -bz_t$$

Using: $\Gamma(a,x) \equiv \int_x^\infty t^{a-1}\, e^{-t}\, dt$ $\quad$ Where Γ(a,0)= Γ(a)=(a-1) Γ(a)

The final result

$$\frac{d^2\sigma_\pi}{dp_{T_t}dp_{T_a}} \approx \frac{\Gamma(n)}{b^n}\frac{B^2}{\hat{x}_h}\frac{A}{p_{T_t}^n}\frac{1}{(1+\frac{p_{T_a}}{\hat{x}_h p_{T_t}})^n}$$

$$\frac{d\sigma_\pi}{dp_{T_t}} \approx \frac{\Gamma(n-1)}{b^{n-1}}\frac{AB}{p_{T_t}^{n-1}}$$

$$\left.\frac{dP_\pi}{dp_{T_a}}\right|_{p_{T_t}} \approx \frac{B(n-1)}{b p_{T_t}}\frac{1}{\hat{x}_h}\frac{1}{(1+\frac{p_{T_a}}{\hat{x}_h p_{T_t}})^n}$$

In the collinear limit, where $p_{T_a} = x_E p_{T_t}$:

$$\left.\frac{dP_\pi}{dx_E}\right|_{p_{T_t}} \approx \frac{B(n-1)}{b}\frac{1}{\hat{x}_h}\frac{1}{(1+\frac{x_E}{\hat{x}_h})^n}$$

Where B/b≈<m>≈b is the mean charged multiplicity in the jet

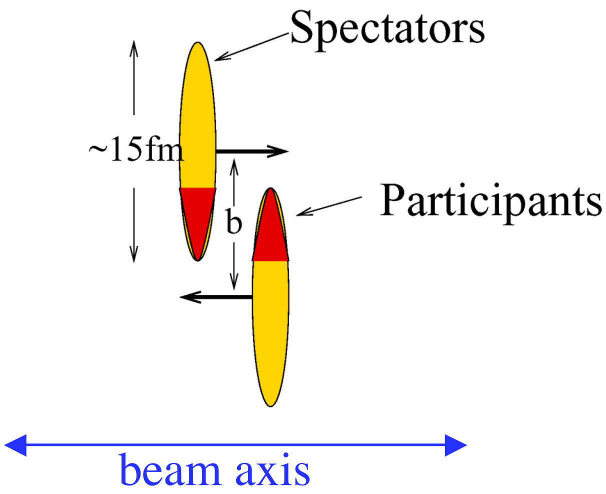
Why dependence on the Frag. Fn. vanishes

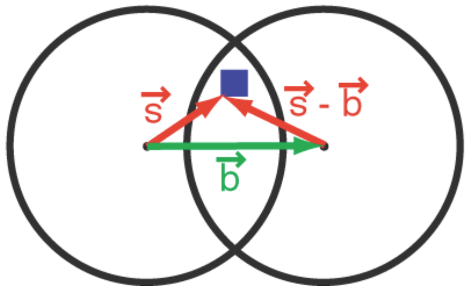
- The only dependence on the fragmentation function is in the normalization constant B/b which equals <m>, the mean multiplicity in the away jet from the integral of the fragmentation function.

- The dominant term in the x_E distribution is the Hagedorn function $1/(1+x_E/\hat{x}_h)^n$ so that at fixed p_{Tt} the x_E distribution is predominantly a function only of x_E and thus exhibits x_E scaling, as observed.

- The reason that the x_E distribution is not sensitive to the shape of the fragmentation function is that the integral over z_t in (1, 2) for fixed p_{Tt} and p_{Ta} is actually an integral over jet transverse momentum $\hat{p}_{T_t}$. However since the trigger and away jets are always roughly equal and opposite in transverse momentum (in p+p), integrating over $\hat{p}_{T_t}$ simultaneously integrates over $\hat{p}_{T_a}$. The integral is over z_t, which appears in both trigger and away side fragmentation functions in (1).

High p_T in A+B collisions---T_{AB} Scaling





- For point-like processes, the cross section in p+A or A+B collisions compared to p-p is simply proportional to the relative number of pointlike encounters
  - ✓ A for p+A, AB for A+B for the total rate
  - ✓ T_{AB} the overlap integral of the nuclear profile functions, as a function of impact parameter b

As measured at the ISR by Darriulat, etc.

P. Darriulat, et al, Nucl.Phys. **B107** (1976) 429-456

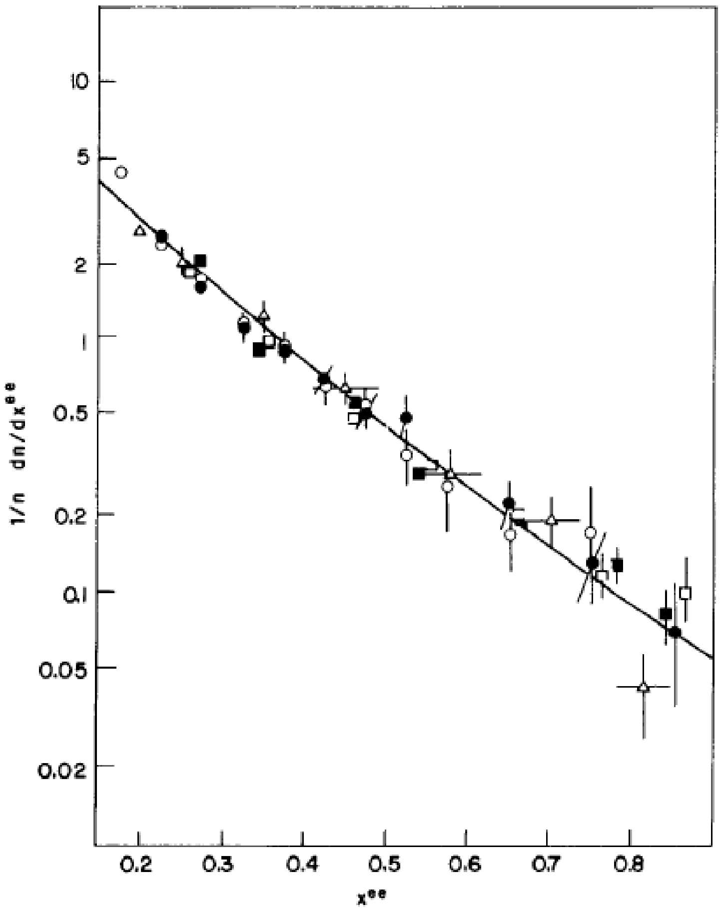


Figure 21 Jet fragmentation functions measured in different processes: ν-p interactions (*open triangles*, Van der Welde 1979); e^+e^- annihilations (*solid line*, Hanson et al 1975); and pp collisions (*full circles* CS, $p_T < 6$ GeV/c, *open circles* CS, $p_T > 6$ GeV/c, *full squares* CCOR, $p_T > 5$ GeV/c, *open squares* CCOR, $p_T > 7$ GeV/c).

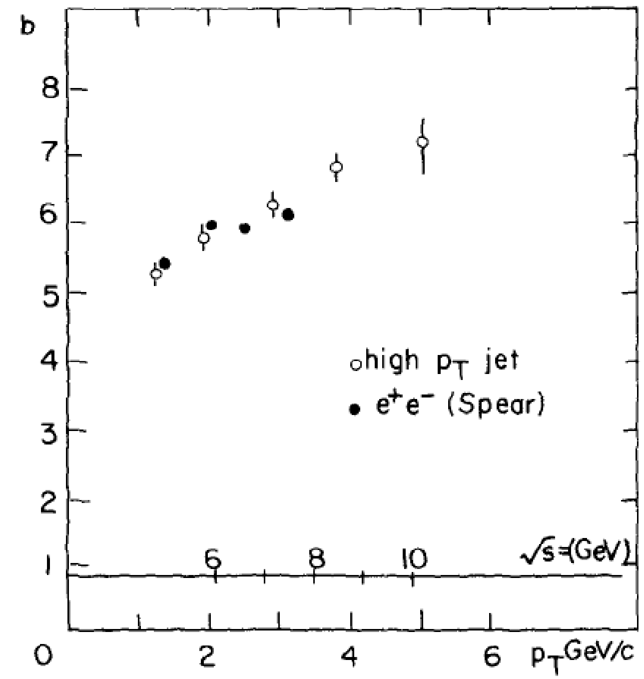


Figure 19 The slopes *b* obtained from exponential fits to the jet fragmentation function in the interval $0.2 < z < 0.8$ in e^+e^- annihilation (*full circles*) and LPTH data of the BS Collaboration (*open circles*).

Figures from P. Darriulat, ARNPS **30** (1980) 159-210 showing that Jet fragmentation functions in νp, e^+e^- and pp (CCOR) are the same with the same dependence of *b* (exponential slope) on "$\hat{s}$"

Three things are dramatically different in Relativistic Heavy Ion Physics than in p-p physics

- the multiplicity is ~A~200 times larger in AA central collisions than in p-p ⇒huge energy in jet cone: 300 GeV for R=1 at $\sqrt{s_{NN}}$=200 GeV
- huge azimuthal anisotropies which don't exist in p-p which are interesting in themselves, and are useful, but sometimes troublesome.
- space-time issues both in momentum space and coordinate space are important in RHI : for instance what is the spatial extent of parton fragmentation, is there a formation time/distance?