Why the x_E distribution triggered by a π^0 does not measure the fragmentation function

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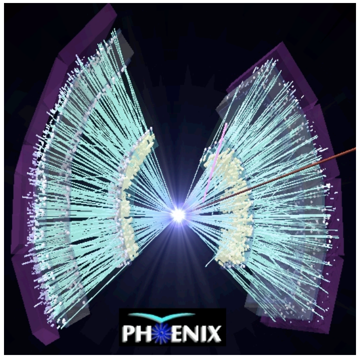

Hard Probes 2006
Asilomar, CA, USA
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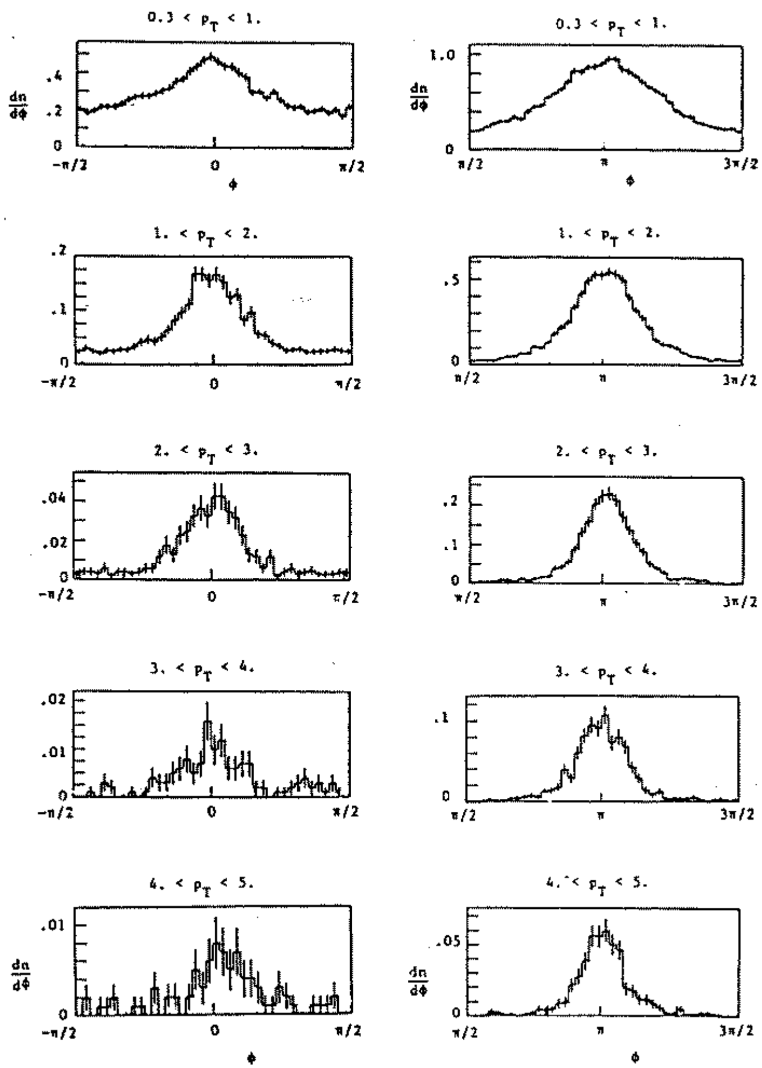
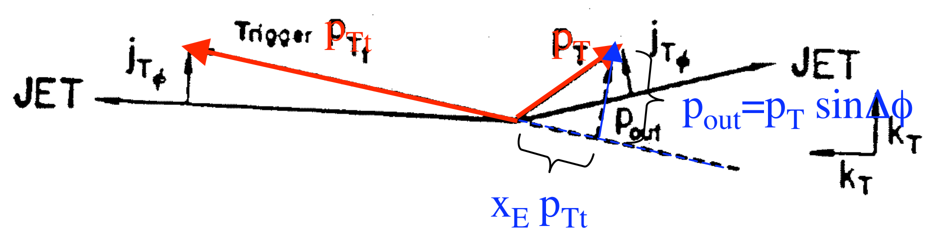
Part I
Introduction
derivation

How everything you want to know about JETS was measured with 2-particle correlations

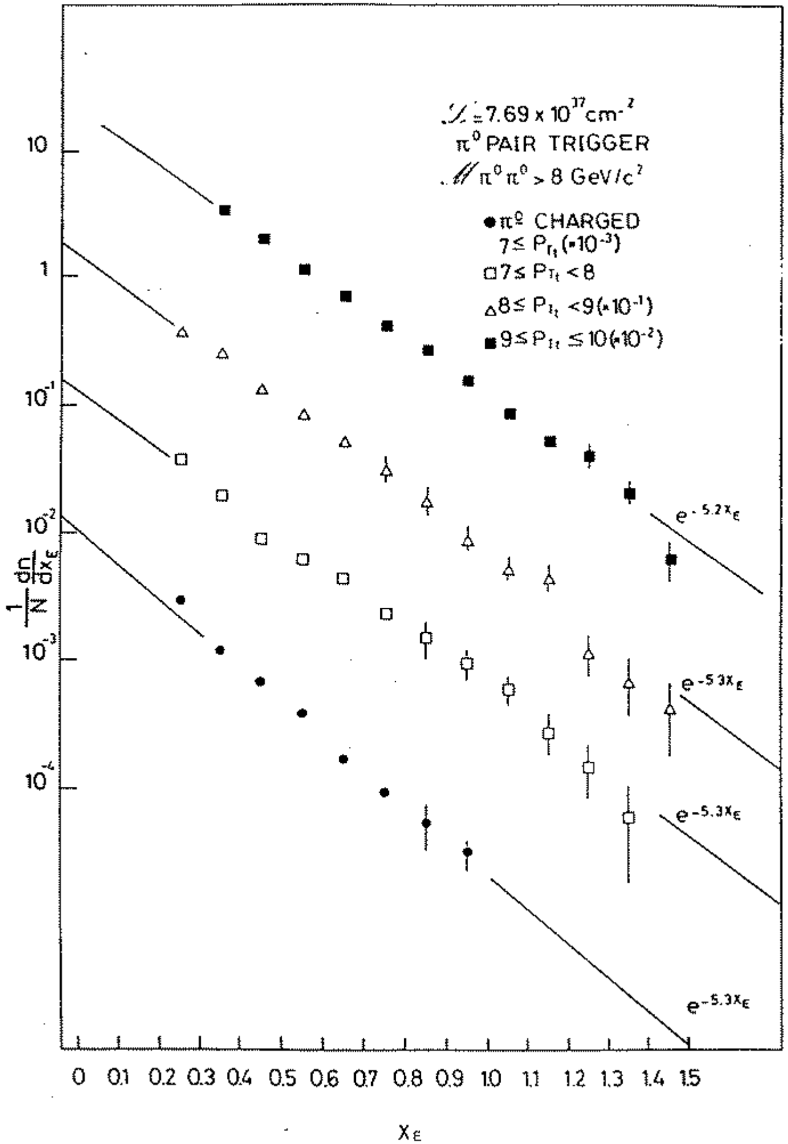


CCOR, A.L.S.Angelis, et al
Phys.Lett. **97B**, 163 (1980)
PhysicaScripta **19**, 116 (1979)



$p_{Tt} > 7$ GeV/c vs p_T

x_E distribution measures fragmentation fn.



$$x_E \sim z/\langle z_{trig}\rangle$$

$\langle z_{trig}\rangle = 0.85$ measured

$$\Rightarrow D^q_\pi(z) \sim e^{-6z}$$

- independent of p_{Tt}

See M. Jacob’s talk EPS 1979 Geneva

From RHIC97--HP04

Where did I (and everybody in HEP) get this idea?---from Feynman, Field and Fox

38 *R.P Feynman et al. / Large transverse momenta*

FFF Nucl.Phys. **B128**(1977) 1-65

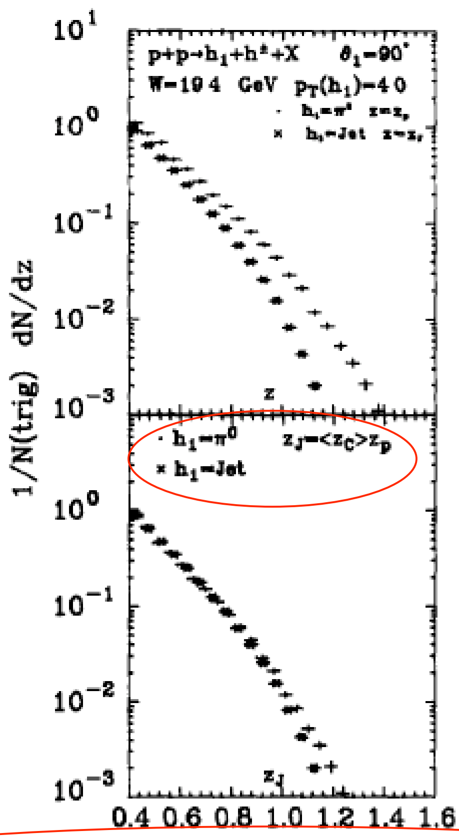


Fig. 23. Comparison of the π^0 and jet trigger away-side distribution of charged hadrons in pp collisions at $W = 19.4$ GeV, $\theta_1 = 90°$, and $p_\perp$ (trigger) = 4.0 GeV/c from the quark-quark scattering model. The upper figure shows the single-particle (π^0) trigger results plotted *versus* $z_p = -p_x(h^\pm)/p_\perp(\pi^0)$ and the jet trigger plotted *versus* $z_j = -p_x(h^\pm)/p_\perp(\text{jet})$ (see table 1). In the lower figure, we plot *both versus* z_J, where for the jet trigger $z_J = z_j$ but for the single-particle trigger $z_J = \langle z_c \rangle z_p$. The away hadrons are integrated over all rapidity Y and $|180° - \phi| \leqslant 45°$ and the theory is calculated using $\langle k_\perp \rangle_{h\to q} = 500$ MeV. • $h_1 = \pi^0$, × h_1 = jet.

"There is a simple relationship between experiments done with single-particle triggers and those performed with jet triggers. The only difference in the opposite side correlation is due to the fact that the 'quark', from which a single-particle trigger came, always has a higher $p_\perp$ than the trigger (by factor $1/z_{trig}$). The away-side correlations for a single-particle trigger at $p_\perp$ should be roughly the same as the away side correlations for a jet trigger at $p_\perp$ (jet)= $p_\perp$ (single particle)/ $\langle z_{trig} \rangle$".

As measured at the ISR by Darriulat, etc.

P. Darriulat, et al, Nucl.Phys. **B107** (1976) 429-456

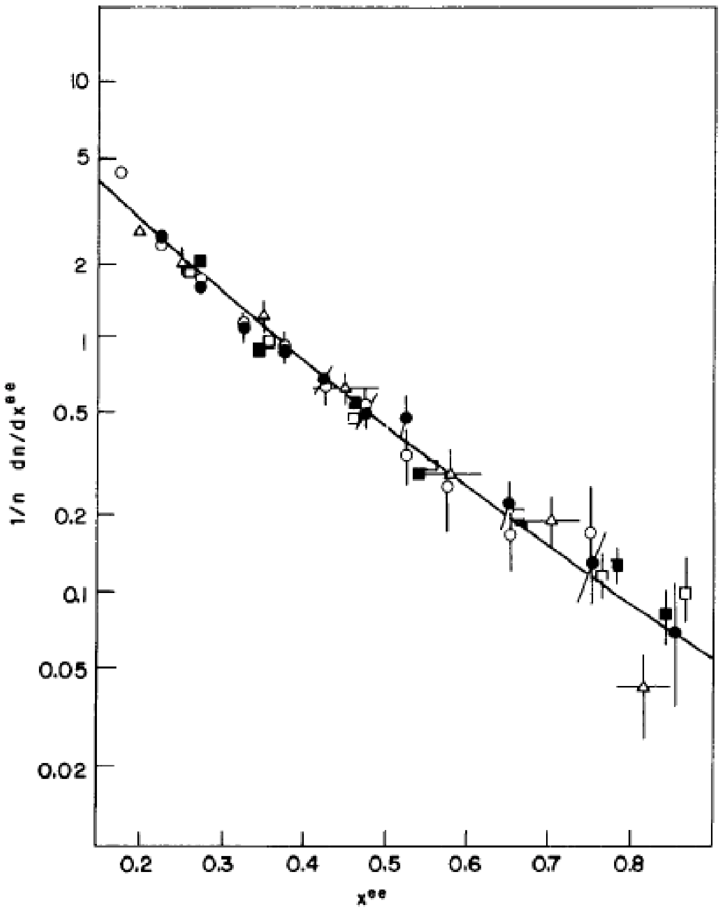


Figure 21 Jet fragmentation functions measured in different processes: ν-p interactions (*open triangles*, Van der Welde 1979); e^+e^- annihilations (*solid line*, Hanson et al 1975); and pp collisions (*full circles* CS, $p_T < 6$ GeV/c, *open circles* CS, $p_T > 6$ GeV/c, *full squares* CCOR, $p_T > 5$ GeV/c, *open squares* CCOR, $p_T > 7$ GeV/c).

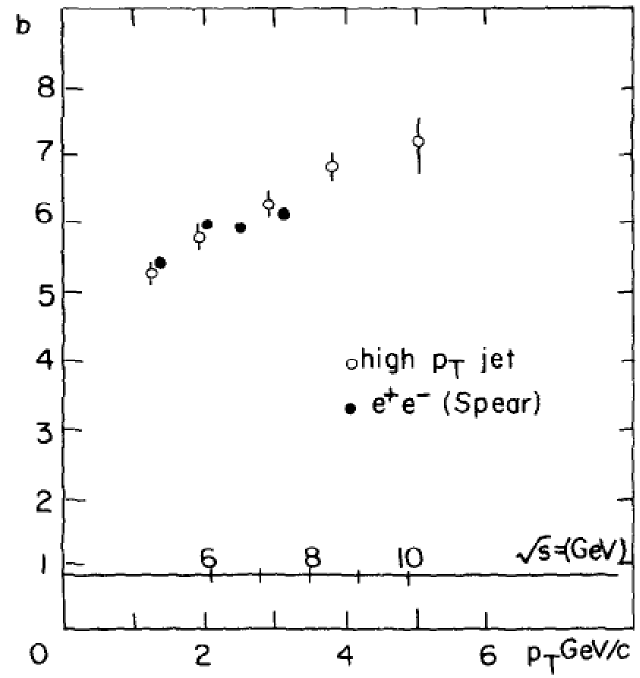


Figure 19 The slopes *b* obtained from exponential fits to the jet fragmentation function in the interval $0.2 < z < 0.8$ in e^+e^- annihilation (*full circles*) and LPTH data of the BS Collaboration (*open circles*).

Figures from P. Darriulat, ARNPS **30** (1980) 159-210 showing that Jet fragmentation functions in νp, e^+e^- and pp (CCOR) are the same with the same dependence of *b* (exponential slope) on "$\hat{s}$"

Formula for x_E distribution from hep-ex/0605039

$$\frac{d^2\sigma_\pi}{d\hat{p}_{T_t} dz_t} = \Sigma'_q(\hat{p}_{T\mathrm{t}}) \times D^q_\pi(z_\mathrm{t}) = \frac{z_t\, d^2\sigma_\pi}{dp_{T_t} dz_t}$$

Prob. that you make a jet with $\hat{p}_{Tt}$ which fragments to a π with $z_t = p_{Tt}/\hat{p}_{Tt}$

$$\Sigma'_q(\hat{p}_{T\mathrm{t}}) \equiv \hat{p}_{T\mathrm{t}}\ f'_q(\hat{p}_{T\mathrm{t}}) = \frac{d\sigma_q}{d\hat{p}_{T\mathrm{t}}} = \frac{A}{\hat{p}_{T_t}{}^{n-1}} = A\,\frac{z_t^{n-1}}{p_{T_t}{}^{n-1}}$$

$$\frac{d\sigma_\pi}{d\hat{p}_{T\mathrm{t}} dz_\mathrm{t} dz_\mathrm{a}} = \Sigma'_q(\hat{p}_{T\mathrm{t}}) D^q_\pi(z_\mathrm{t}) D^q_\pi(z_\mathrm{a})$$

Prob. that away jet with $\hat{p}_{Ta}$ fragments to a π with $z_a = p_{Ta}/\hat{p}_{Ta}$

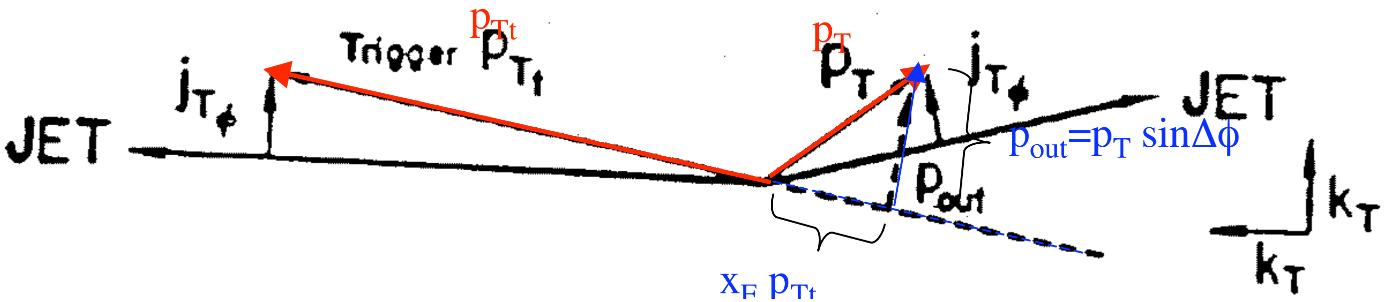
$$z_\mathrm{a} = \frac{p_{T\mathrm{a}}}{\hat{p}_{T\mathrm{a}}} = \frac{p_{T\mathrm{a}}}{\hat{x}_\mathrm{h}\hat{p}_{T\mathrm{t}}} = \frac{z_\mathrm{t} p_{T\mathrm{a}}}{\hat{x}_\mathrm{h} p_{T\mathrm{t}}}$$

(1)
$$\boxed{\frac{d\sigma_\pi}{dp_{T\mathrm{t}} dz_\mathrm{t} dp_{T\mathrm{a}}} = \frac{1}{\hat{x}_\mathrm{h} p_{T\mathrm{t}}} \Sigma'_q\left(\frac{p_{T\mathrm{t}}}{z_\mathrm{t}}\right) D^q_\pi(z_\mathrm{t}) D^q_\pi\left(\frac{z_\mathrm{t} p_{T\mathrm{a}}}{\hat{x}_\mathrm{h} p_{T\mathrm{t}}}\right)}$$

Appears to be sensitive to away jet Frag. Fn.

How we found this problem in PHENIX

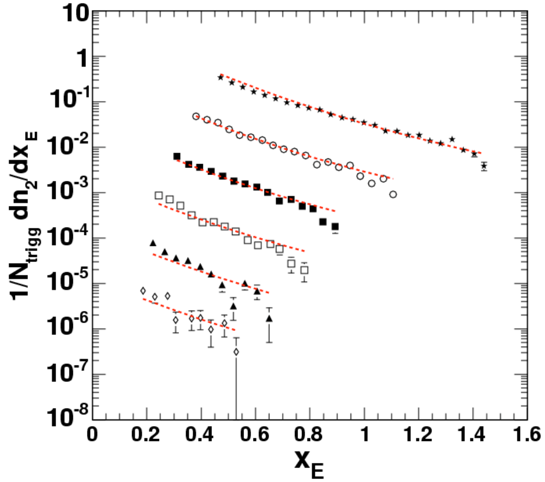
Following FFF and CCOR PLB**97**(1980)163-168 we were trying to measure the net transverse momentum of the di-jet ($\sqrt{2}$ x $\langle k_T \rangle$)



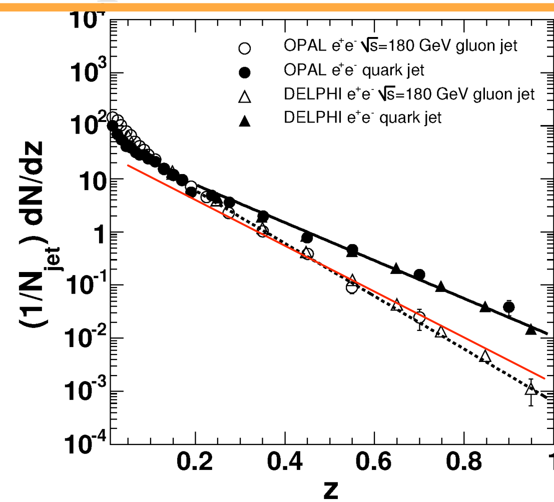
$$\frac{\langle z_t(k_T, x_h)\rangle \sqrt{\langle k_T^2 \rangle}}{\hat{x}_h(k_T, x_h)} = \frac{1}{x_h}\sqrt{\langle p_{out}^2\rangle - \left\langle j_{Ty}^2 \right\rangle (1 + x_h^2)} \qquad x_h \equiv \frac{p_{T_a}}{p_{T_t}} \qquad \hat{x}_h \equiv \frac{\hat{p}_{T_a}}{\hat{p}_{T_t}}$$

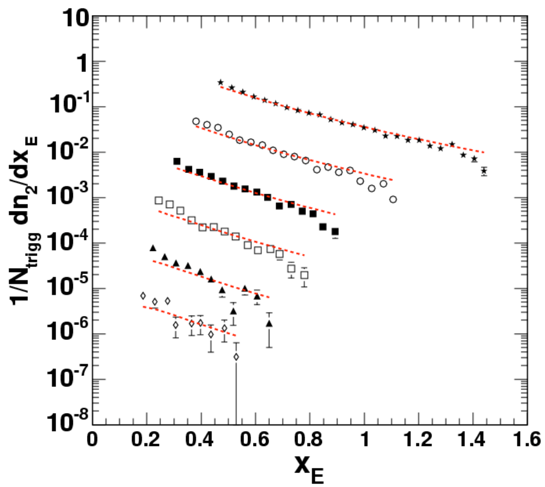
- j_T is parton fragmentation transverse momentum
- k_T is transverse momentum of a parton in a proton (2 protons)
- $x_E = -p_T \bullet p_{Tt}/|p_{Tt}|^2$ represents away jet fragmentation z
- p_{out} is component of away p_T perpendicular to trigger p_{Tt}
- We needed $\langle z_t \rangle$ to solve for k_T. Tried to get it from x_E dist.

Fit x_E distributions to form of D(z) used by LEP measurements by integrating Eq (1) numerically

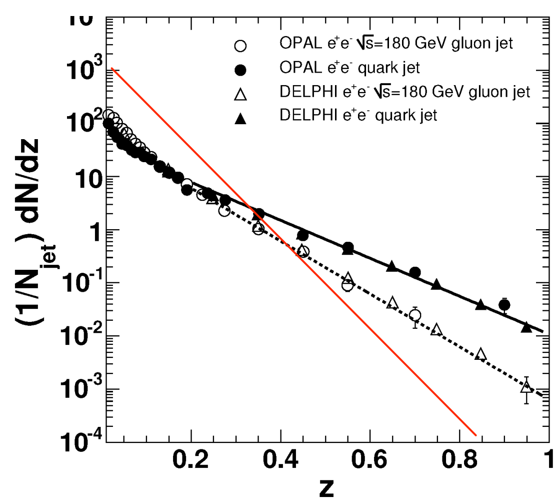


$D(z)=\exp(-10z)$





$D(z)=\exp(-20z)$



After many convergence difficulties, Jan Rak gets desperate and tries two vastly different frag. Functions$\Rightarrow$No effect on calculated x_E distributions---Mike, can you check this analytically?!

Amazingly, I could; and got a neat result

$$\boxed{\frac{d\sigma_\pi}{dp_{T\mathrm{t}}dz_{\mathrm{t}}dp_{T\mathrm{a}}} = \frac{1}{\hat{x}_{\mathrm{h}} p_{T\mathrm{t}}} \Sigma'_q\left(\frac{p_{T\mathrm{t}}}{z_{\mathrm{t}}}\right) D^q_\pi(z_{\mathrm{t}}) D^q_\pi\left(\frac{z_{\mathrm{t}} p_{T\mathrm{a}}}{\hat{x}_{\mathrm{h}} p_{T\mathrm{t}}}\right)} \qquad (1)$$

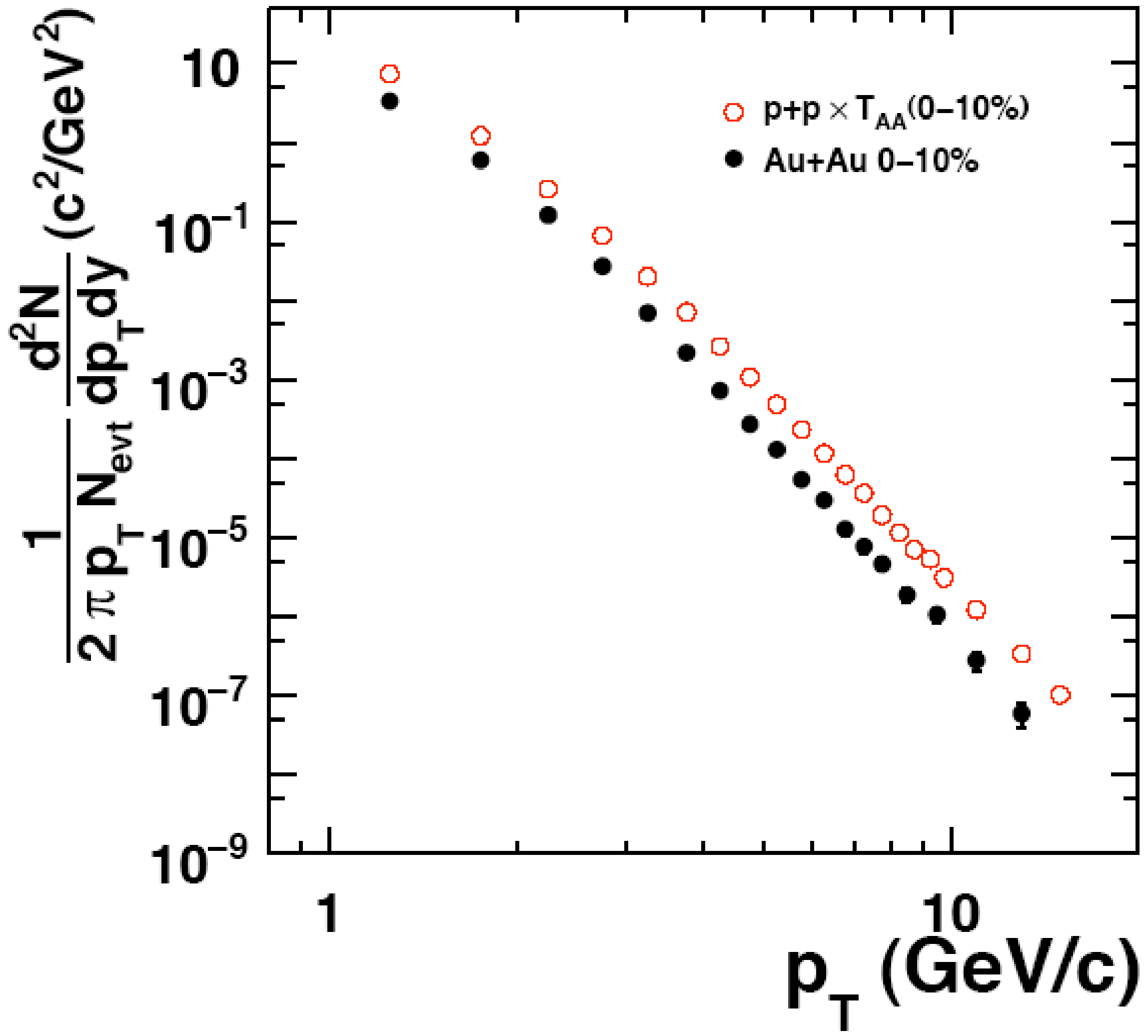
Take: $D(z) = B \exp(-bz) \qquad \Sigma'_q\left(\frac{p_{T\mathrm{t}}}{z_{\mathrm{t}}}\right) = A\left(\frac{p_{T\mathrm{t}}}{z_{\mathrm{t}}}\right)^{-(n-1)}$

$$(2) \quad \frac{d\sigma_\pi}{dp_{T\mathrm{t}}dp_{T\mathrm{a}}} = \frac{B^2}{\hat{x}_{\mathrm{h}}} \frac{A}{p^n_{T_t}} \int_{x_{T_t}}^{\hat{x}_{\mathrm{h}} \frac{p_{T\mathrm{t}}}{p_{T\mathrm{a}}}} dz_{\mathrm{t}}\, z_{\mathrm{t}}^{n-1} \exp -bz_{\mathrm{t}}\left(1 + \frac{p_{T\mathrm{a}}}{\hat{x}_{\mathrm{h}} p_{T\mathrm{t}}}\right)$$

$$\frac{d\sigma_\pi}{dp_{T\mathrm{t}}} = \frac{AB}{p^{n-1}_{T_t}} \int_{x_{T_t}}^{1} dz_{\mathrm{t}}\, z_{\mathrm{t}}^{n-2} \exp -bz_{\mathrm{t}}$$

Using: $\Gamma(a, x) \equiv \int_x^\infty t^{a-1}\, e^{-t}\, dt$ Where Γ(a,0)= Γ(a)=(a-1) Γ(a)

Inclusive invariant π^0 spectrum is beautiful power law for $p_T \geq 3$ GeV/c n=8.1±0.1



The final result

$$\frac{d^2\sigma_\pi}{dp_{T\rm t}dp_{T\rm a}} \approx \frac{\Gamma(n)}{b^n}\frac{B^2}{\hat{x}_{\rm h}}\frac{A}{p_{T_t}^n}\frac{1}{(1+\frac{p_{T\rm a}}{\hat{x}_{\rm h}p_{T\rm t}})^n}$$

$$\frac{d\sigma_\pi}{dp_{T\rm t}} \approx \frac{\Gamma(n-1)}{b^{n-1}}\frac{AB}{p_{T_t}^{n-1}} \qquad ,$$

$$\left.\frac{dP_\pi}{dp_{T\rm a}}\right|_{p_{T\rm t}} \approx \frac{B(n-1)}{bp_{T\rm t}}\frac{1}{\hat{x}_{\rm h}}\frac{1}{(1+\frac{p_{T\rm a}}{\hat{x}_{\rm h}p_{T\rm t}})^n} \qquad . \qquad (42)$$

In the collinear limit, where $p_{T\rm a}=x_{\rm E}p_{T\rm t}$:

$$\left.\frac{dP_\pi}{dx_{\rm E}}\right|_{p_{T\rm t}} \approx \frac{B(n-1)}{b}\frac{1}{\hat{x}_{\rm h}}\frac{1}{(1+\frac{x_E}{\hat{x}_{\rm h}})^n} \qquad . \qquad (43)$$

Where B/b≈<m>≈b is the mean charged multiplicity in the jet

Why dependence on the Frag. Fn. vanishes

- The only dependence on the fragmentation function is in the normalization constant B/b which equals <m>, the mean multiplicity in the away jet from the integral of the fragmentation function.

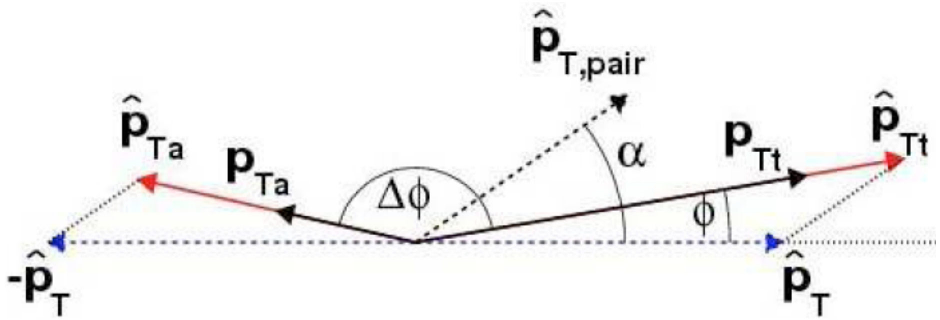
- The dominant term in the x_E distribution is the Hagedorn function $1/(1+x_E/\hat{x}_h)^n$ so that at fixed p_{Tt} the x_E distribution is predominantly a function only of x_E and thus exhibits x_E scaling, as observed.

- The reason that the x_E distribution is not sensitive to the shape of the fragmentation function is that the integral over z_t in (1, 2) for fixed p_{Tt} and p_{Ta} is actually an integral over jet transverse momentum $\hat{p}_{T_t}$. However since the trigger and away jets are always roughly equal and opposite in transverse momentum (in p+p), integrating over $\hat{p}_{T_t}$ simultaneously integrates over $\hat{p}_{T_a}$. The integral is over z_t, which appears in both trigger and away side fragmentation functions in (1).

Part II
Discussion
Application

A very interesting formula

$$\left.\frac{dP_\pi}{dx_{\rm E}}\right|_{p_{T{\rm t}}} \approx \langle m\rangle\,(n-1)\frac{1}{\hat{x}_{\rm h}}\frac{1}{(1+\frac{x_E}{\hat{x}_{\rm h}})^n}$$

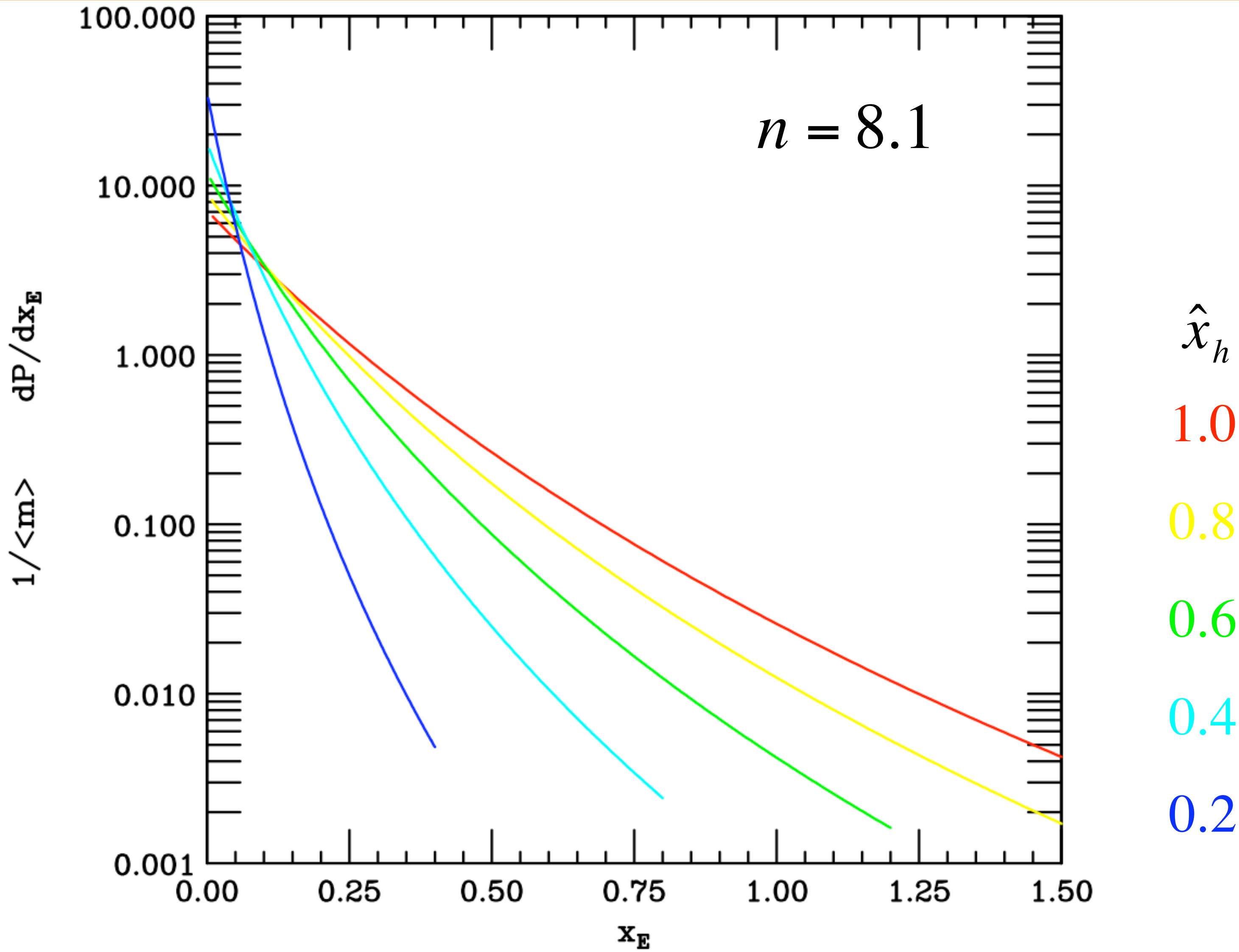


$$x_E = \frac{-p_{T_a}\cos\Delta\phi}{p_{T_t}} \simeq \frac{p_{T_a}}{p_{T_t}} \quad \longrightarrow \quad \hat{x}_h = \frac{\hat{p}_{T_a}}{\hat{p}_{T_t}}$$

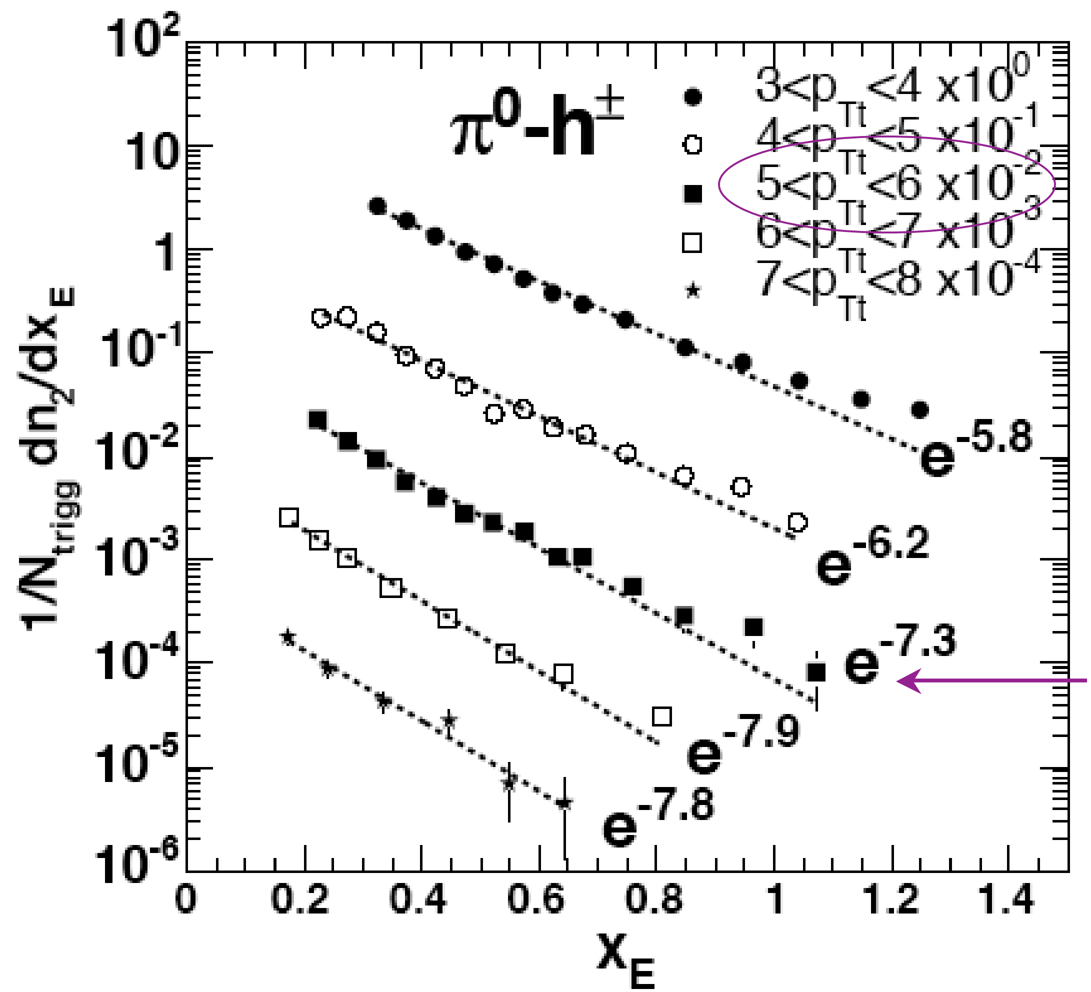
measured

Ratio of jet transverse momenta

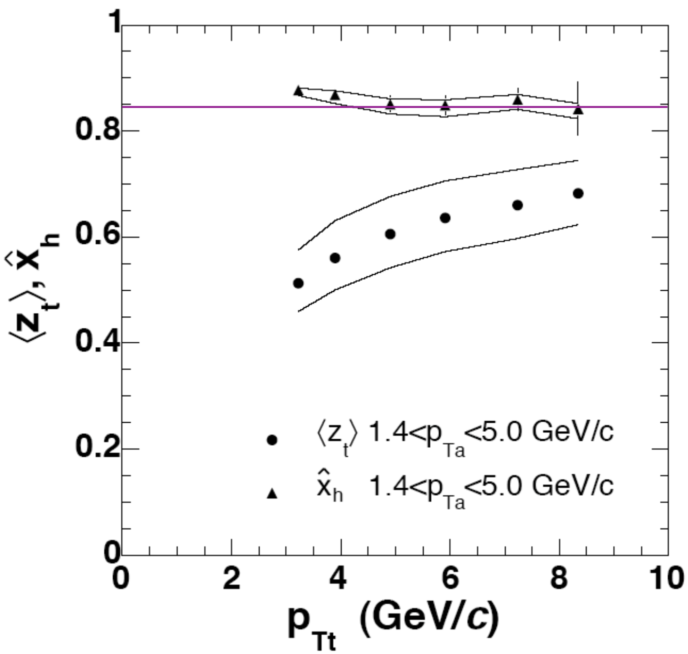
Shape of x_E distribution depends on $\hat{x}_h$ and *n* but not on *b*



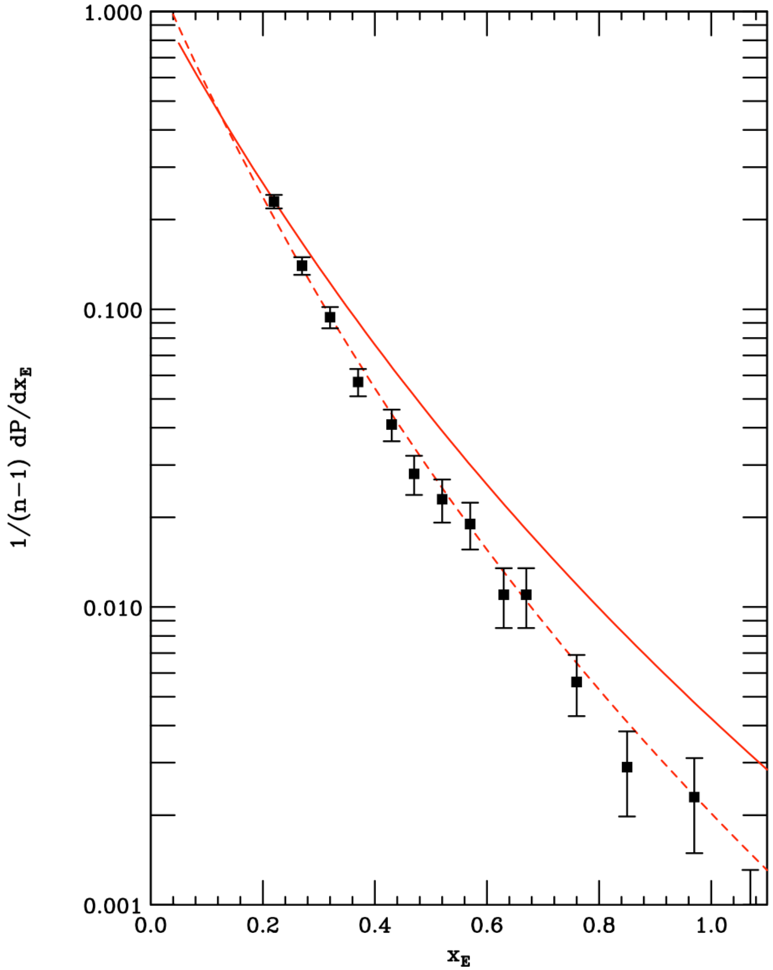
Does the formula work?



PHENIX p+p
hep-ex/0605039



It works for PHENIX p+p hep-ex/0605039



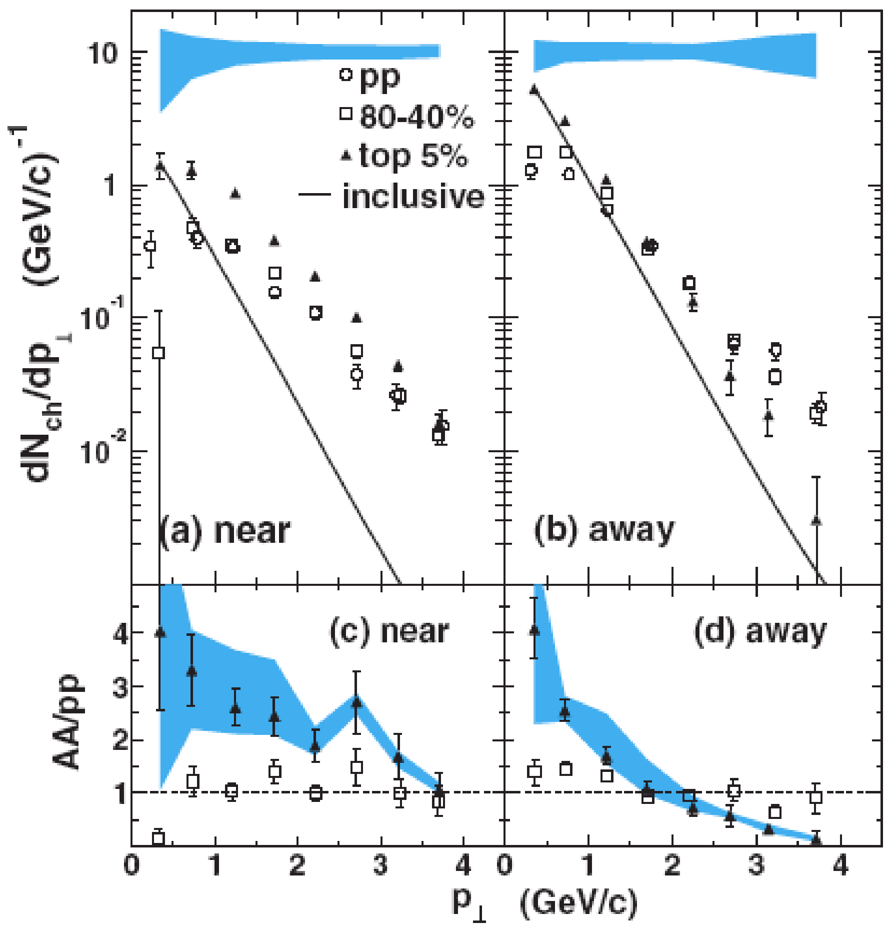
$$f(y) = \frac{1}{(1+y)^{8.1}}$$

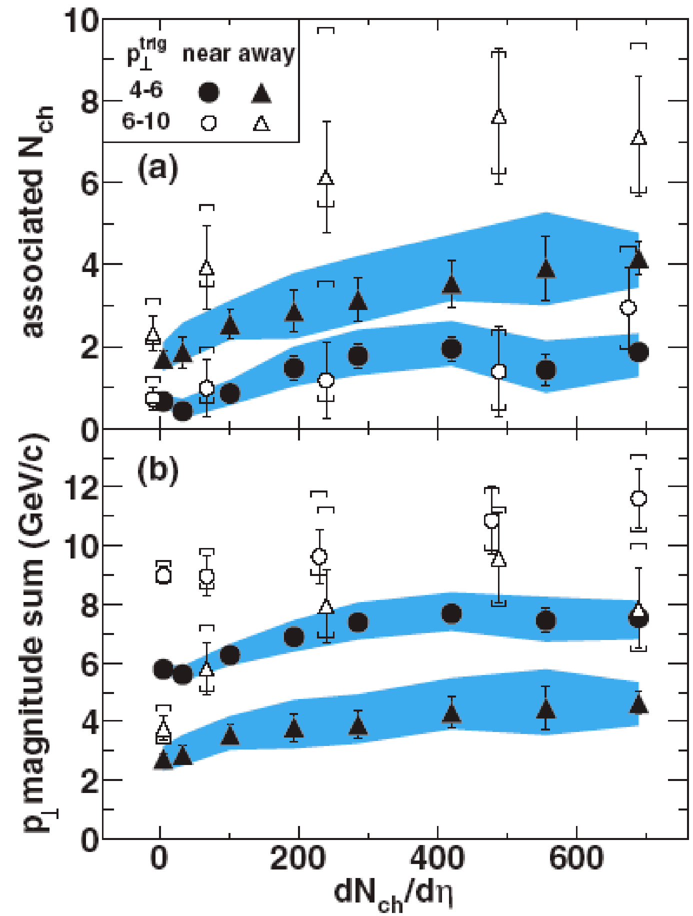
$x_E = y$ (solid line)

$x_E = \hat{x}_h\, y = 0.8\, y$ (dashed line)

nb: vertical scale labels on this and following similar plots should be multiplied by 10

Now Apply Eq. To (STAR) Au+Au data



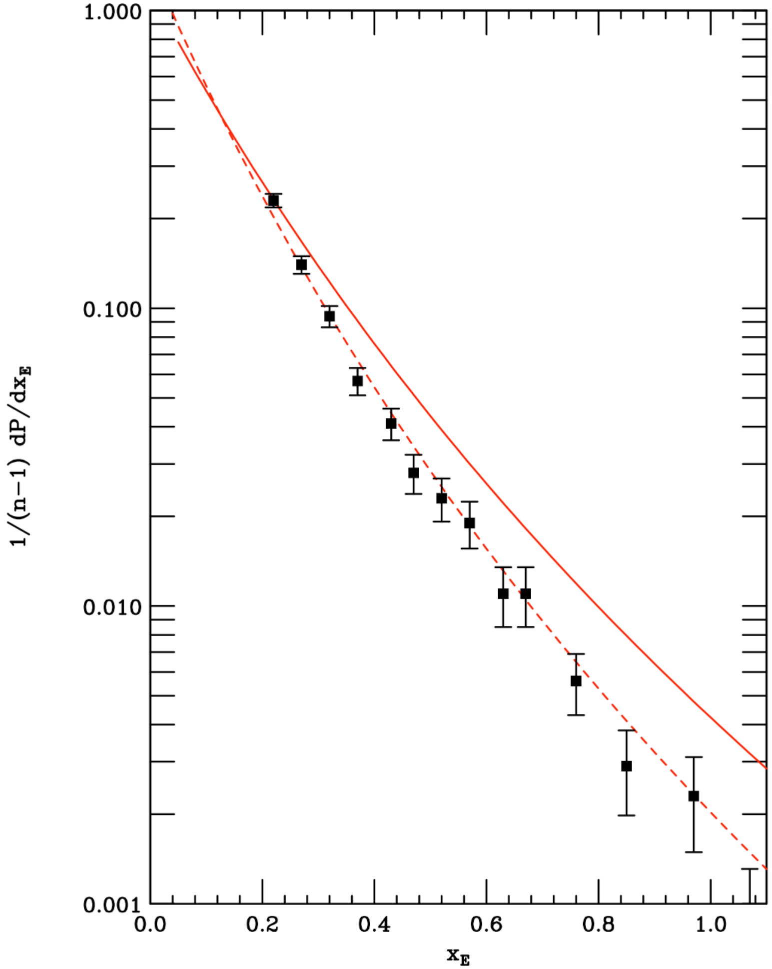


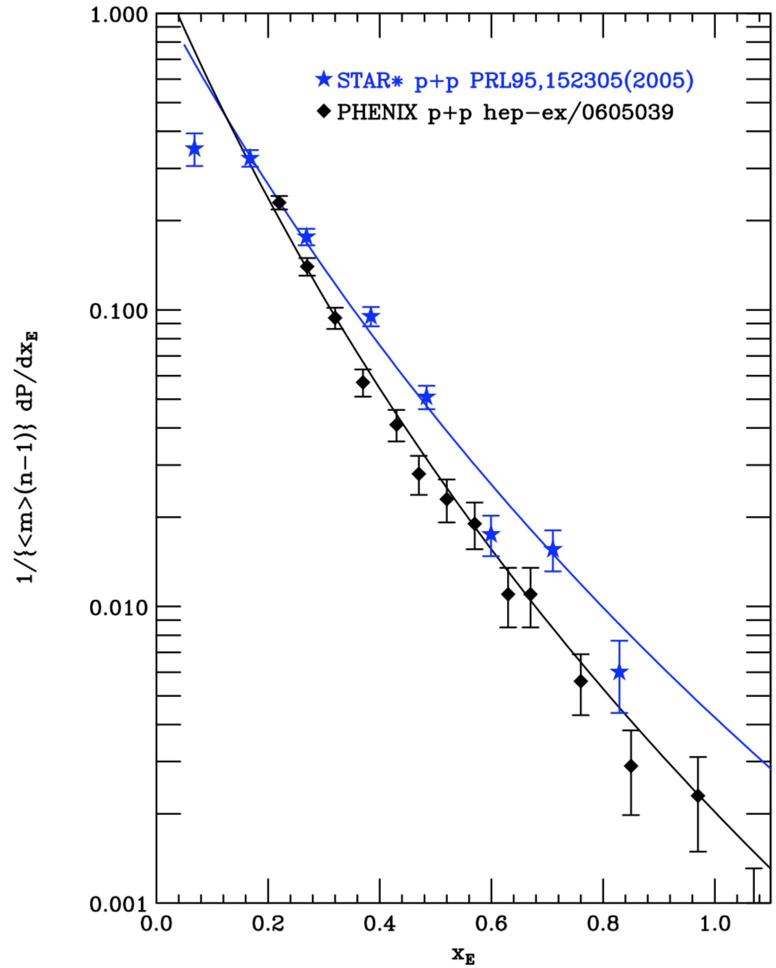
$4 < p_{Tt} < 6$ GeV/c $\langle p_{Tt}\rangle$=4.56 GeV/c

pp, AuAu $\sqrt{s_{NN}}$=200 GeV

STAR, J. Adams, Fuqiang Wang, et al PRL **95**, 152301 (2005)

It works for STAR p+p and:

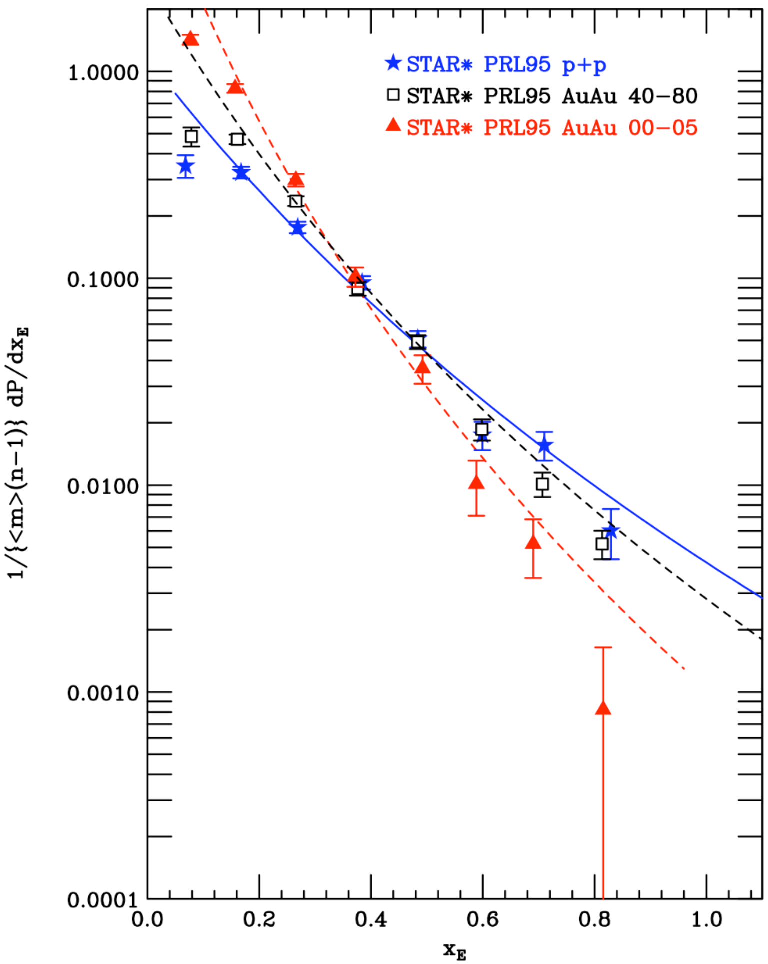




a) * means data normalized w.r. to hep-ex/0605039

b) $\hat{x}_h = 1.0$

Clear effect with centrality

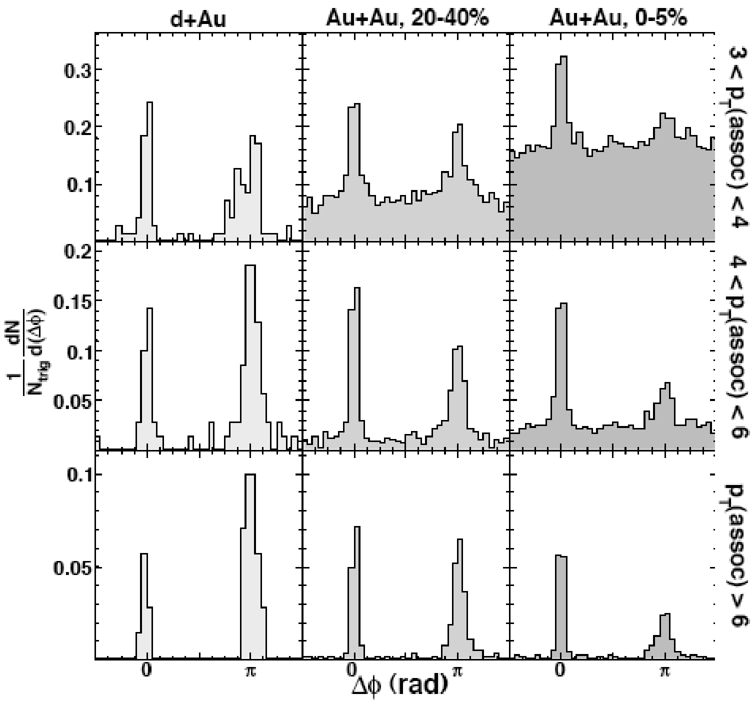


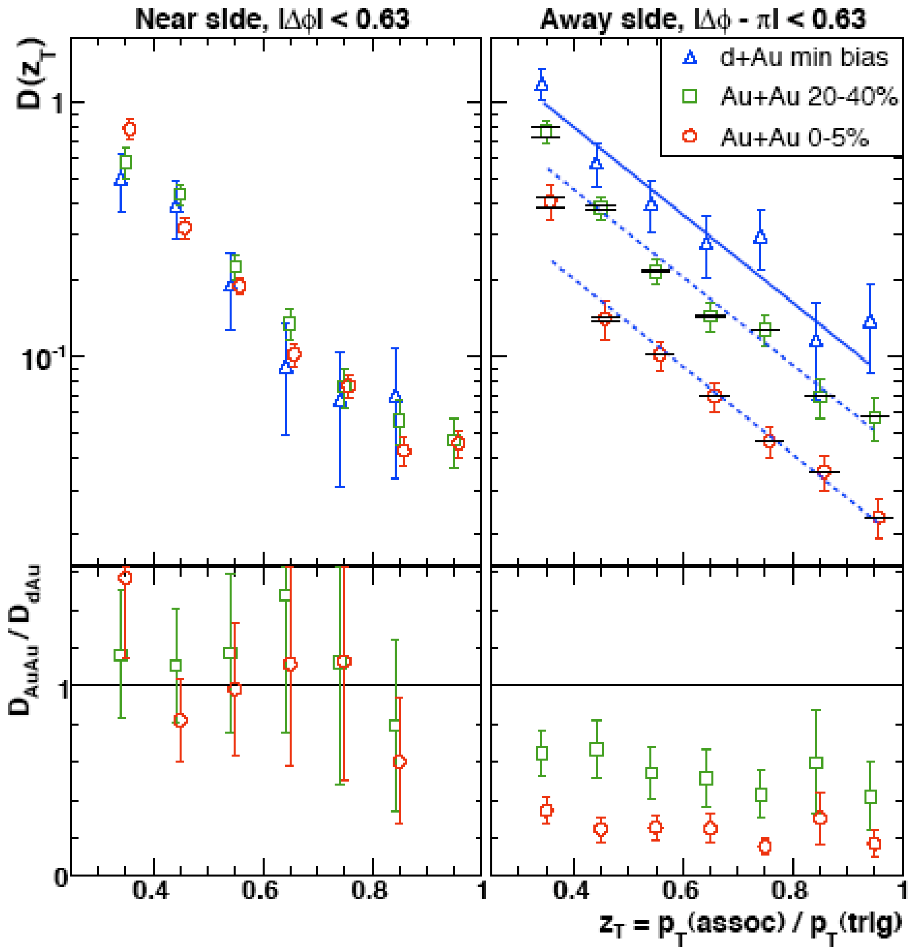
p+p	data*0.6	fit*1.0	$\hat{x}_h$ =1.0
AuAu40-80	data*0.6	fit*1.75	$\hat{x}_h$ =0.75
AuAu00-05	data*0.6	fit*4.0	$\hat{x}_h$ =0.48

- Away jet p_{Ta}/trigger jet p_{Tt} decreases with increasing centrality
- consistent with increase of energy loss with distance traversed in medium

STAR, J. Adams, Fuqiang Wang, et al PRL **95**, 152301 (2005)

New STAR data AuAu: nucl-ex/0604018

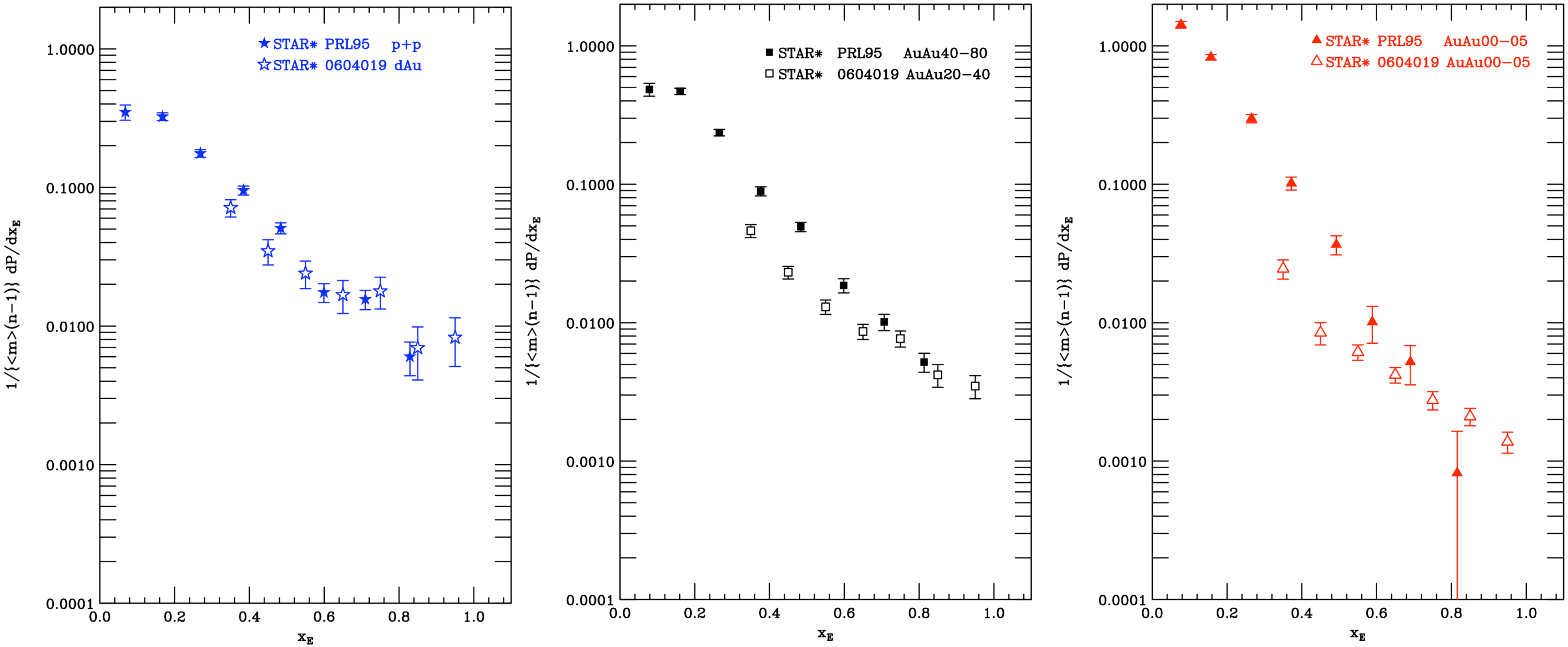




$8 < p_{Tt} < 15$ GeV/c $\langle p_{Tt}\rangle$=9.38 GeV/c

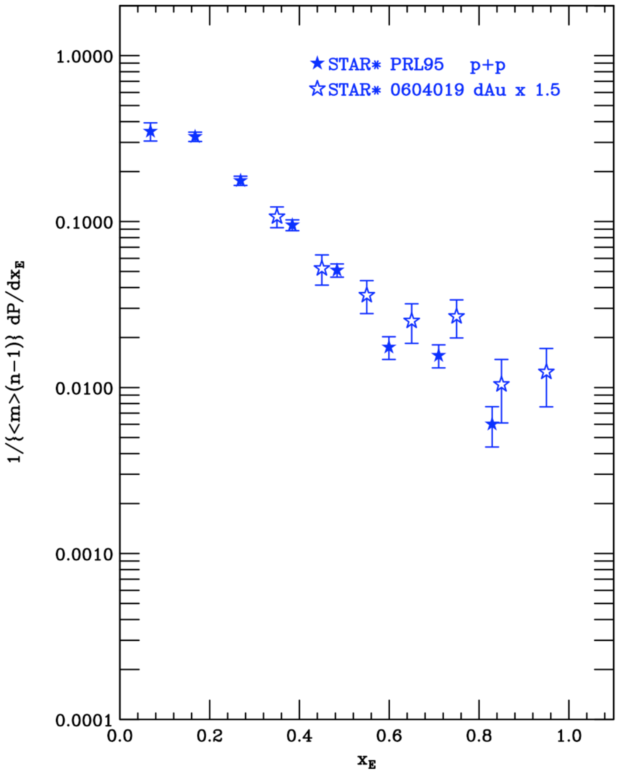
Thanks to Dan Magestro for table of data points

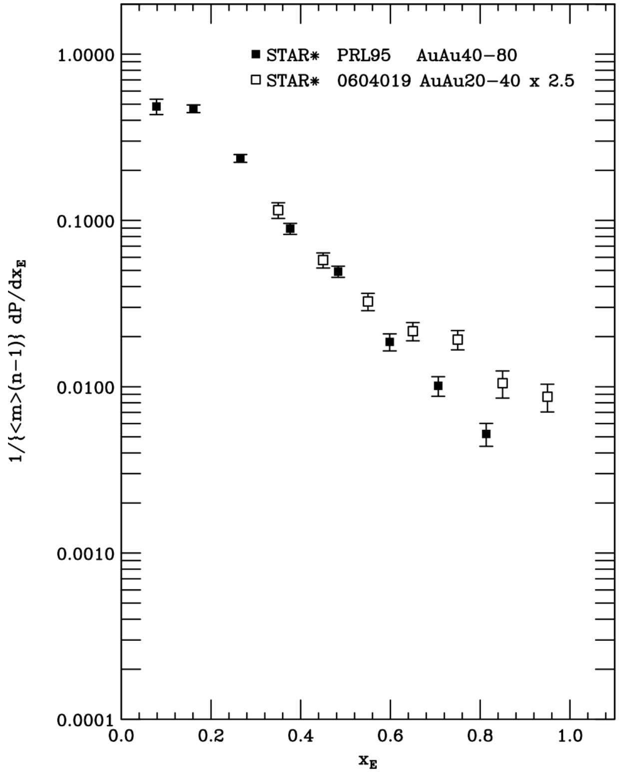
STAR(nucl-ex/0604018) differs from STAR (PRL95) in normalization and SHAPE

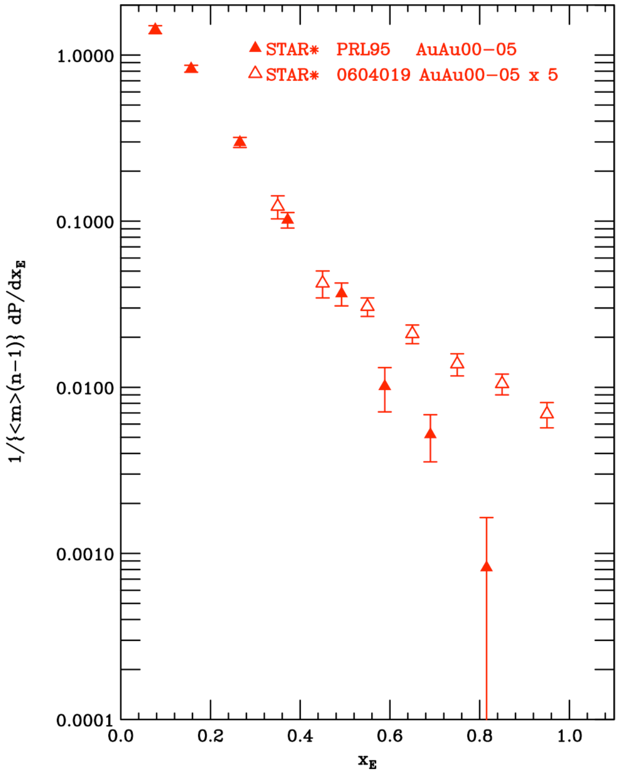


nb: vertical scale labels on these and following similar plots should be multiplied by 10
STAR* 0604019 label should be STAR* 0604018

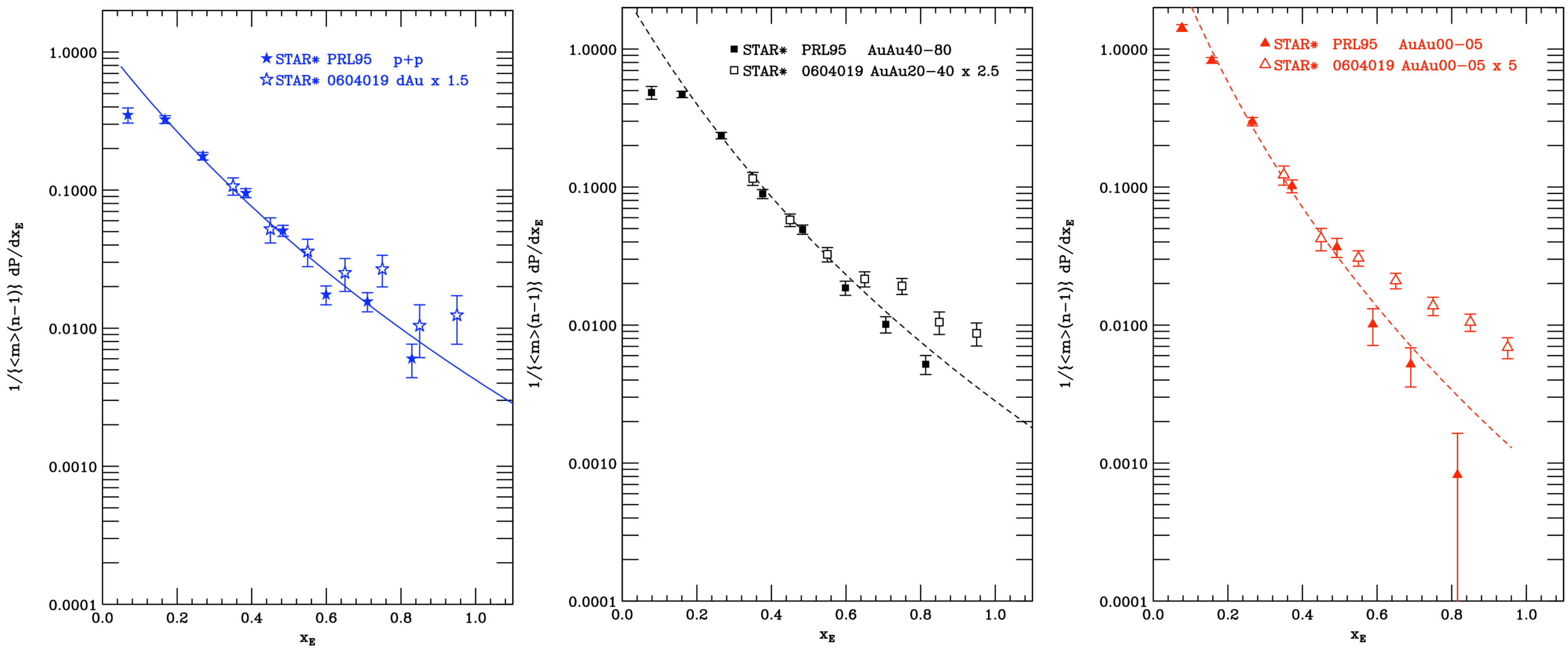
Normalize 064018 to PRL95 by eye



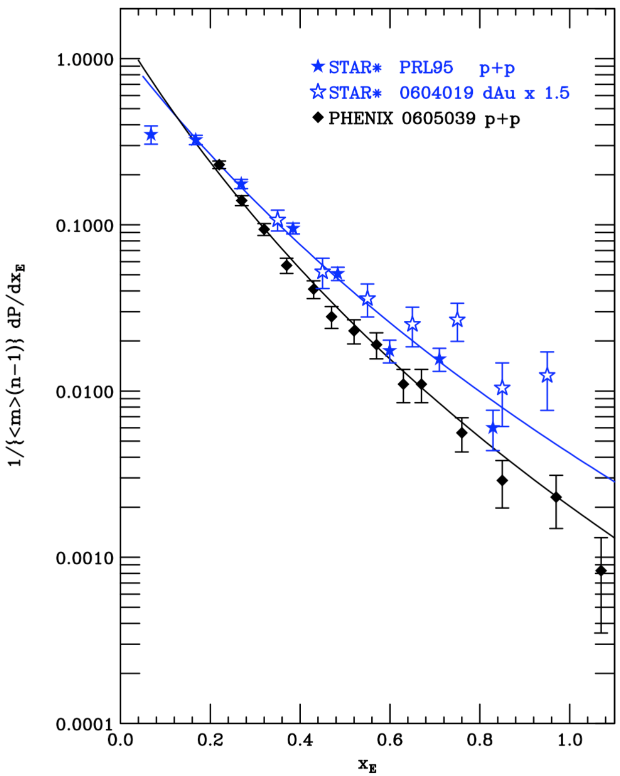


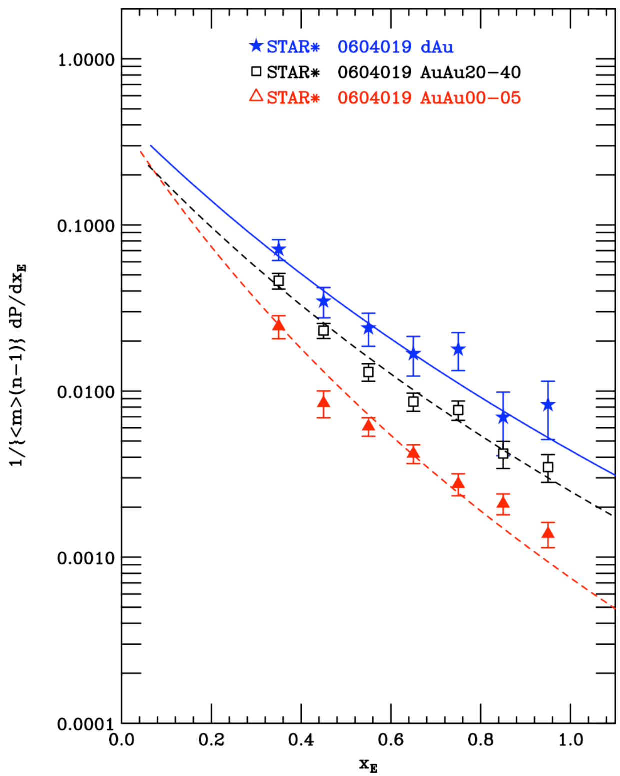


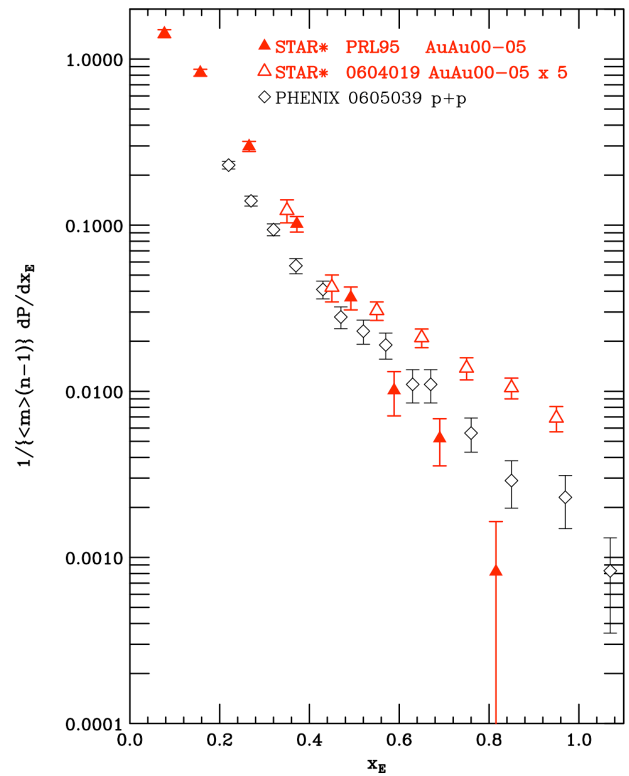
Normalized data with PRL95 curve



STAR 0604018 AuAu central flatter than PHENIX 0605039 p+p for x_E>0.5!





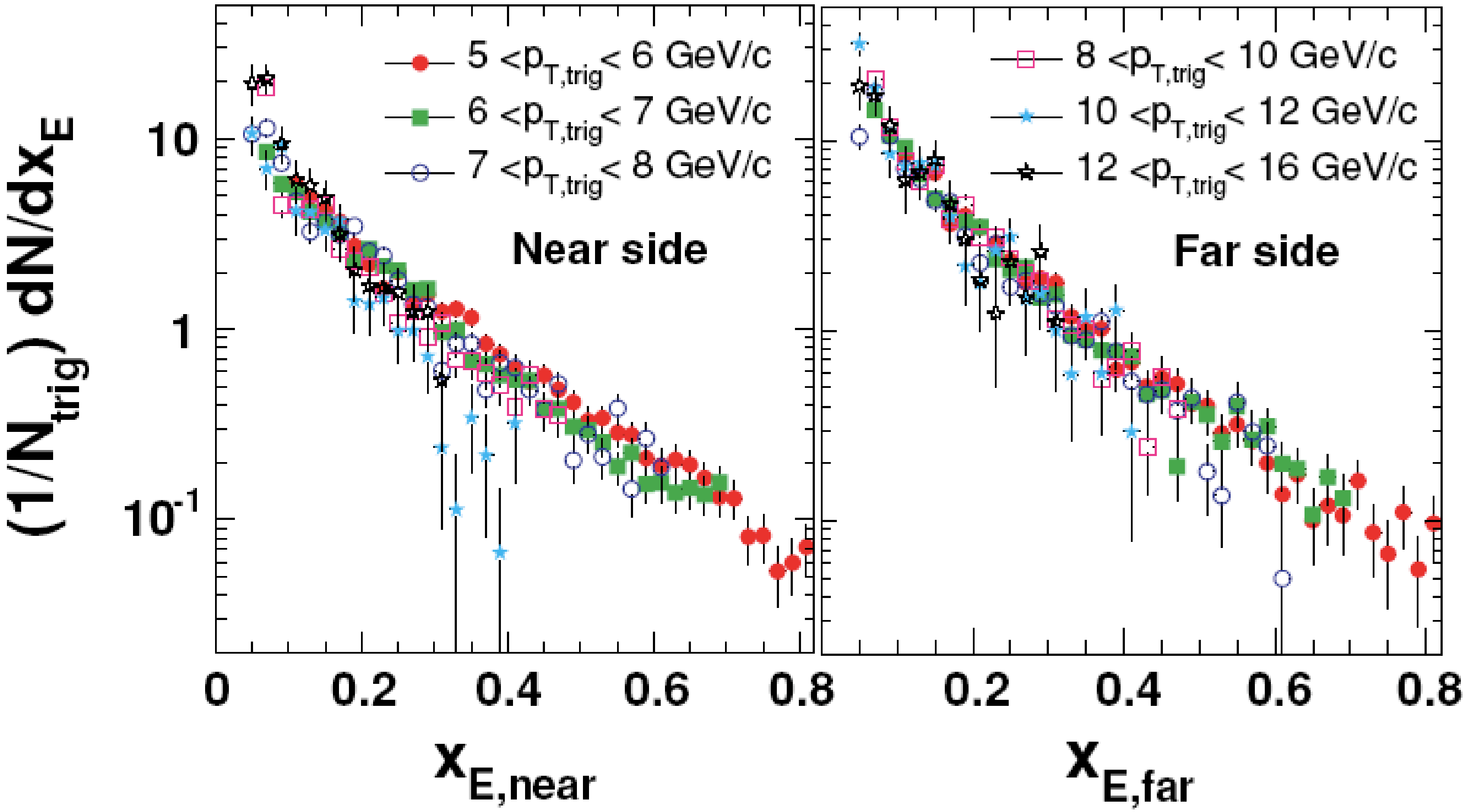


Norm (data)	Norm fit	hatx_h
Data*0.6	Fit*0.500	1.300
Data*0.6	Fit*0.350	1.200
Data*0.6	Fit*0.300	0.850

Can still fit, but curves too flat x_h>1, but still decreases with increasing centrality

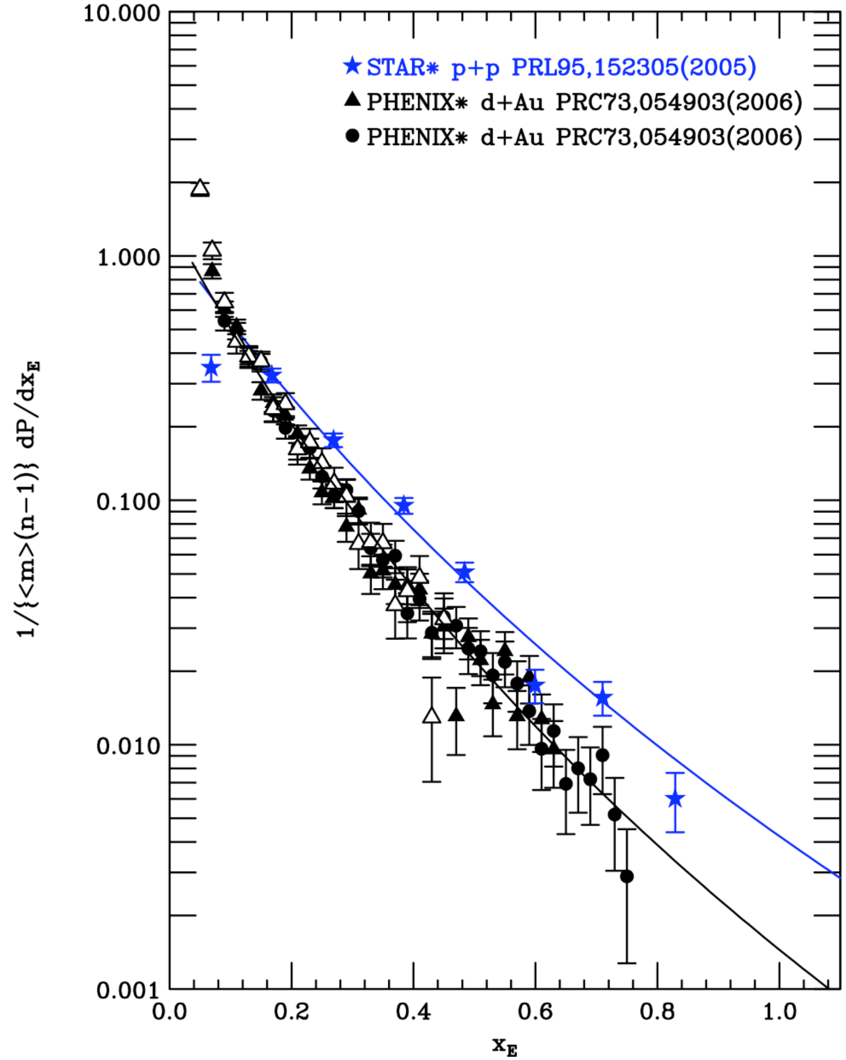
PHENIX d+Au results

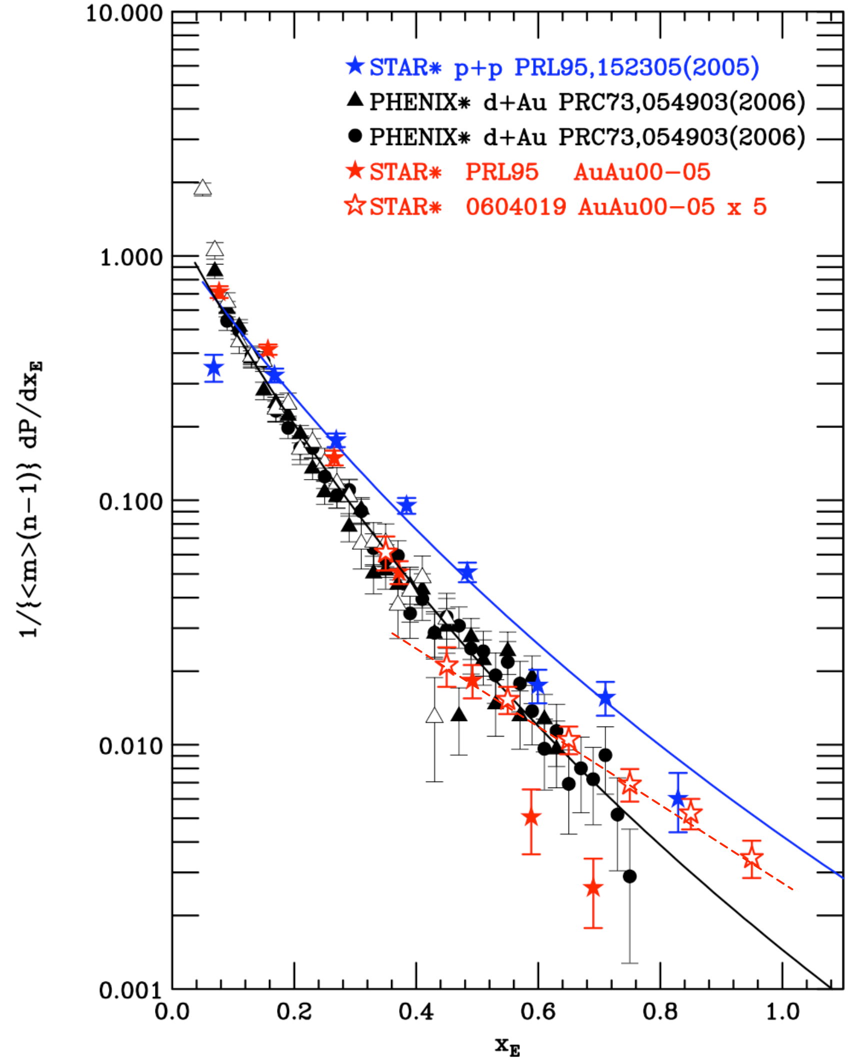
PHENIX, PRC 73, 054903 (2006)



- Beautiful p+p and dAu results with pTt in STAR punchthrough range.

PHENIX d+Au PRC73





$\hat{x}_h = 0.75$ $\hat{x}_h = 1.0$

- STAR 0604018 AuAu0-5 flatter than all published p+p and d+Au data ????

Conclusions

- Nice `fit' of $1/(1+y)^{n=8.1}$ with $x_E = \hat{x}_h\, y$ to PHENIX hep-ex/0605039 and PRC73; and STAR PRL95 x_E distributions. But STAR nucl-ex/0604018 d+Au much flatter than PHENIX d+Au PRC73,054903 (2006)

- Both STAR Au+Au measurements show a decrease in the ratio of the transverse momentum of the away jet relative to the trigger jet with increasing centrality. For both data sets $\hat{x}_h$ decreases by a factor of ~2 from p+p (dAu) to Au+Au central collisions. Much more info than I_{AA}.

- New STAR `punchthrough' data has much too flat shape, an apparent sharp break, and disagrees in normalization with STAR PRL95.

- Comparison of two STAR data sets would benefit by going lower in p_{Ta} (z_T) for the data of nucl-ex/0604018 to see whether slope is really steeper at low z_T, with dramatic break and (unreasonable in my opinion) flattening of the z_T distribution for $z_T \geq 0.5$