Analysis of two-particle jet correlations with a scaling formula

M. J. Tannenbaum
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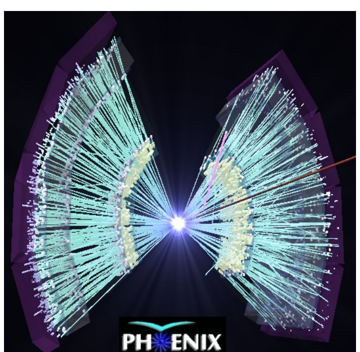

DNP 2007
Newport News, VA
October 13, 2007

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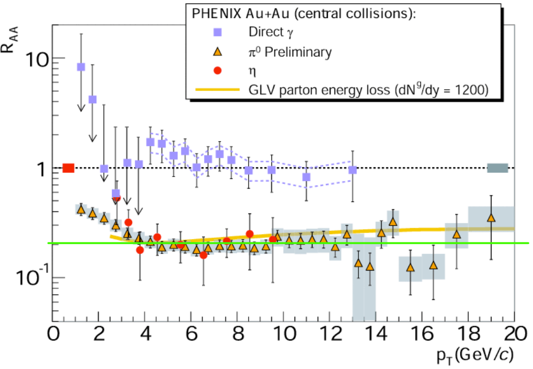

DNP 2007
Newport News, VA
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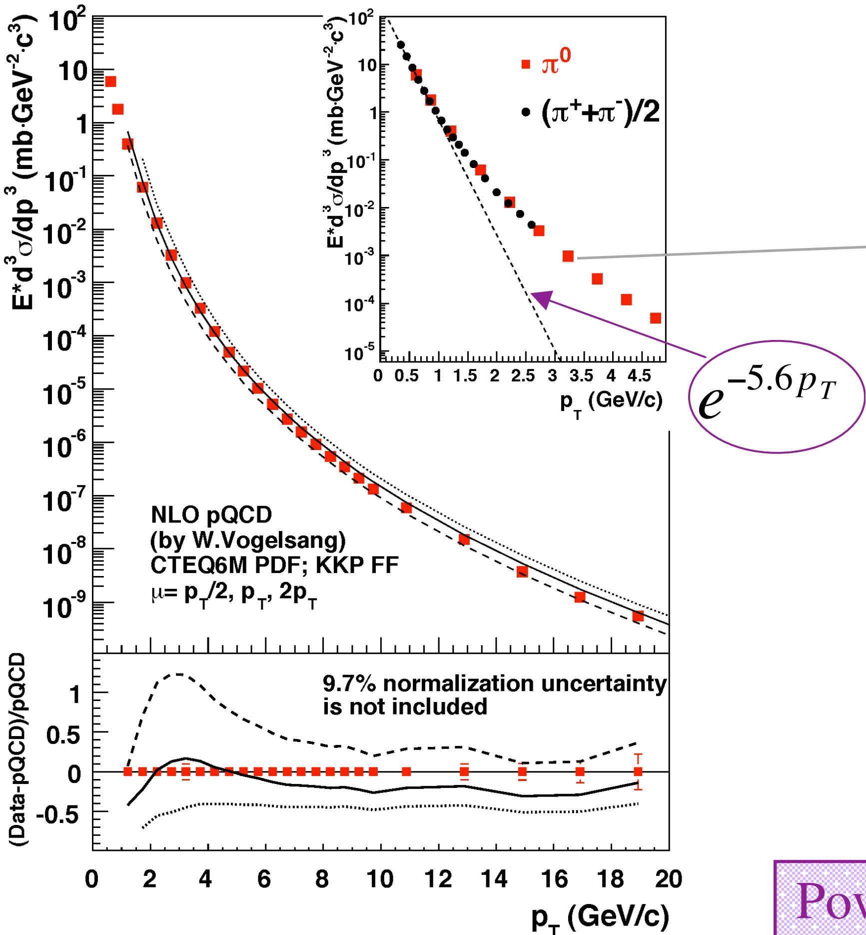
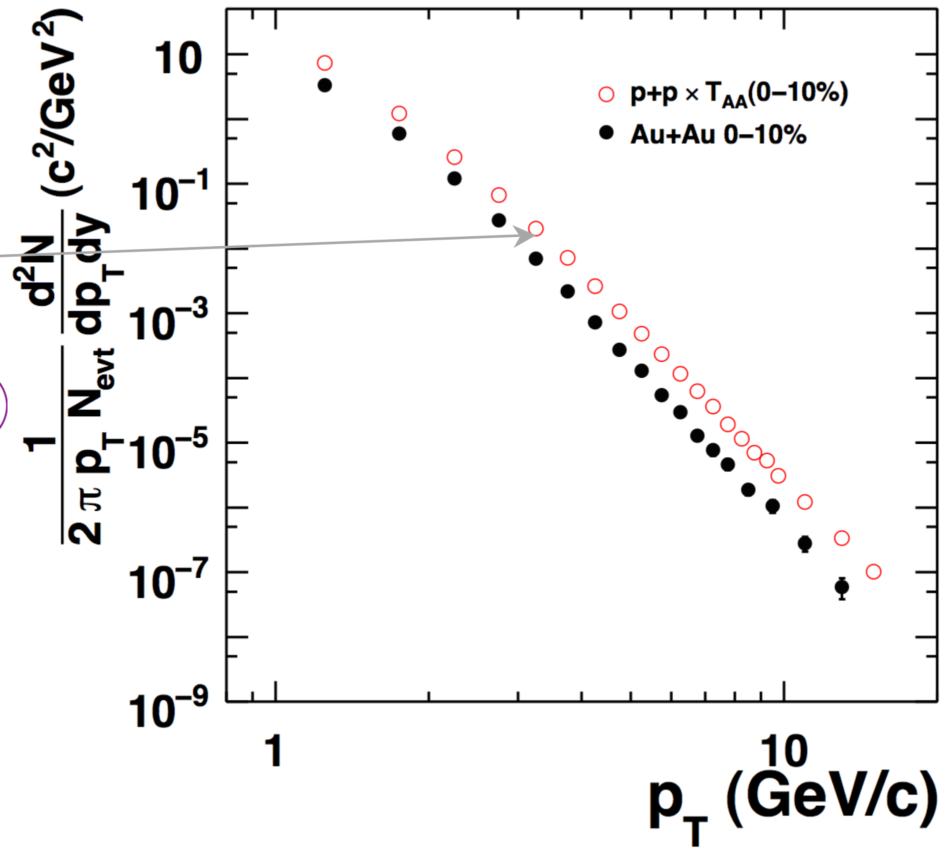
Inclusive invariant π^0 spectrum in p-p and AuAu is beautiful power law for $p_T \geq 3$ GeV/c n=8.1

PHENIX, PRD**76**(2007)051006(R)

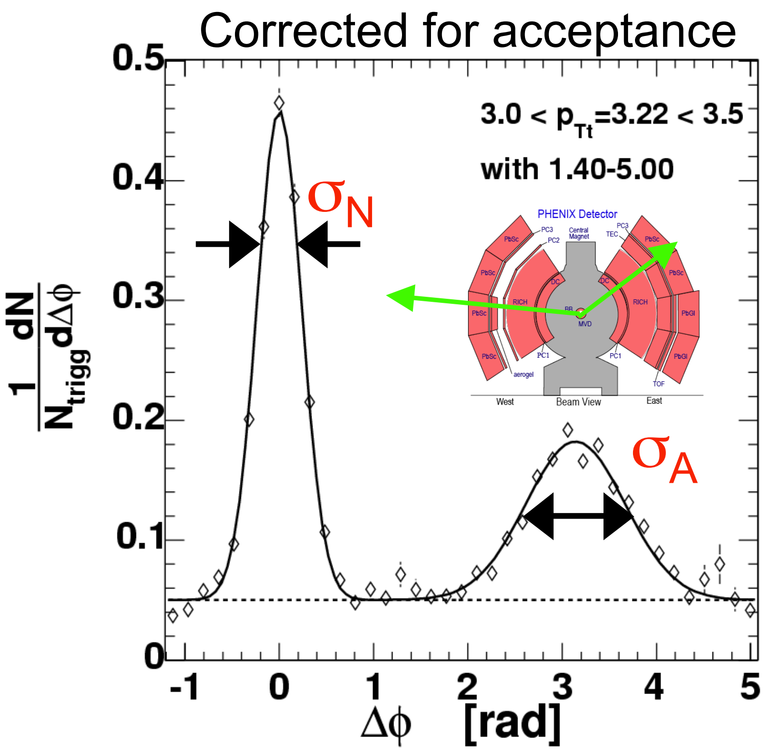


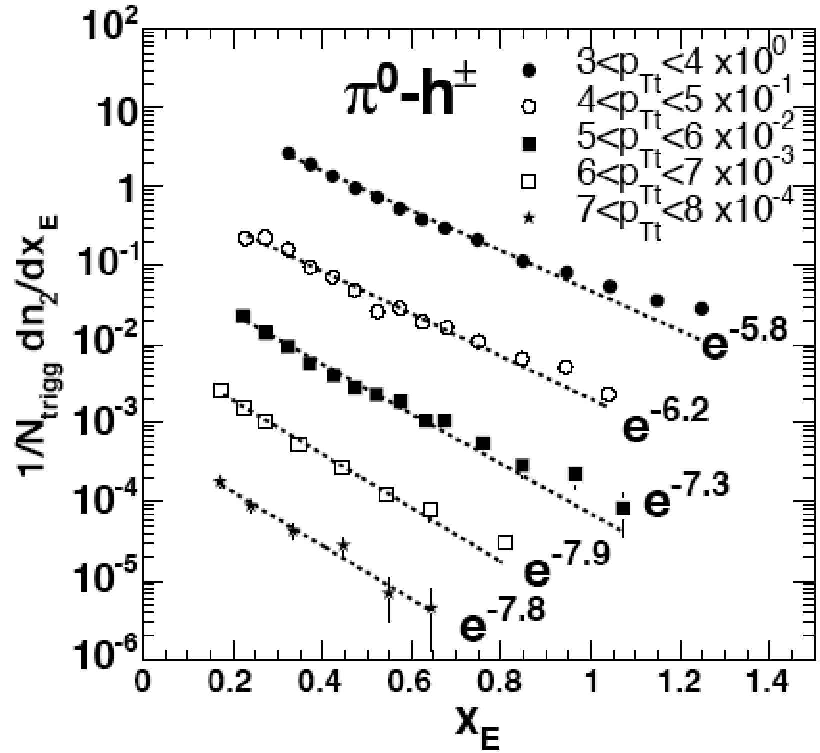
PHENIX, PRC**76**(2007)034904



Power Law ⇒ Hard Scattering (pQCD)

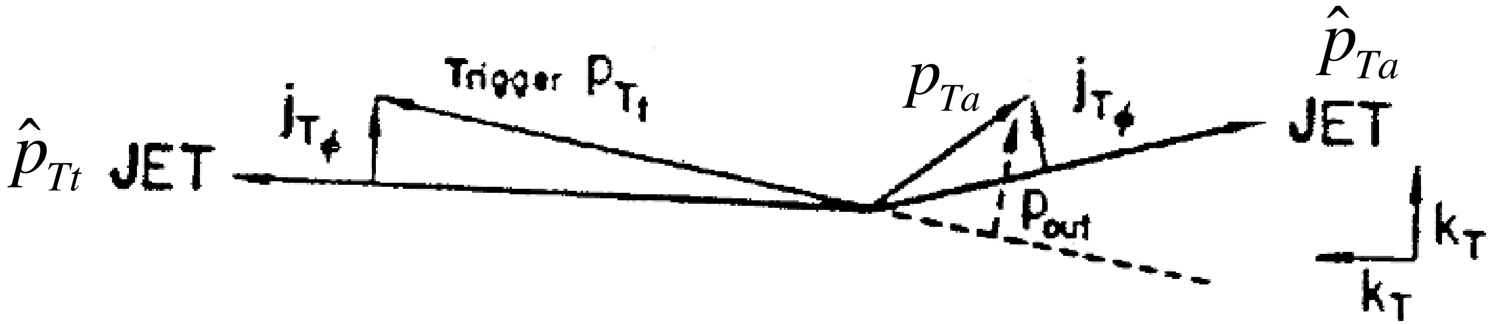
PHENIX π^o - $h^\pm$ correlation functions p+p $\sqrt{s}$=200 GeV: PRD **74**, 072002 (2006)





Trigger on a particle e.g. π^0 with transverse momentum p_{Tt}. Measure azimuthal angular distribution w.r.t the trigger azimuth of associated (charged) particles with transverse momentum p_{Ta}. The strong same and away side peaks in p-p collisions indicate di-jet origin from hard-scattering of partons. For the away distribution calculate the conditional yield in the peak as a function of $x_E \sim p_{Ta}/p_{Tt}$

Kinematics



$z = p_T / \hat{p}_T$ is the jet fragmentation variable: z_t and z_a

$D^q_\pi(z) = Be^{-bz}$ is a typical Fragmentation Function, b~ 8-11 at RHIC

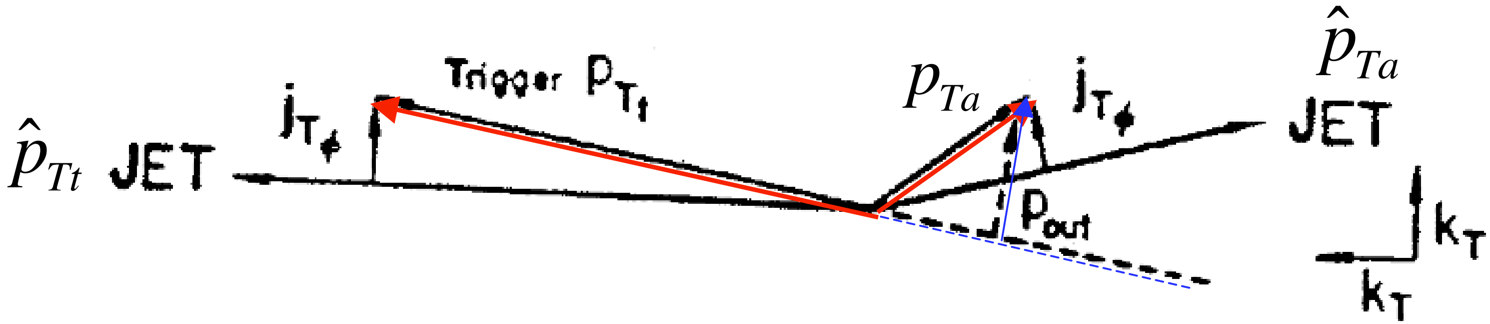
Due to the steeply falling spectrum, the trigger π^0 are biased towards large z_t, $\langle z_t \rangle \approx (n-1)/b$ while unbiased $\langle z \rangle \approx 1/b$

$$x_E = \left|\frac{\vec{p}_{T_a} \cdot \vec{p}_{T_t}}{p^2_{T_t}}\right| = \frac{-p_{T_a}\cos\Delta\phi}{p_{T_t}} \approx \frac{p_{T_a}}{p_{T_t}} = \frac{p_{T_a}/\hat{p}_{T_t}}{p_{T_t}/\hat{p}_{T_t}} \approx \frac{z_a}{\langle z_t \rangle}$$

From Feynman, Field and Fox: the x_E distribution corrected for $\langle z_t \rangle$ measures the unbiased fragmentation function

$$\frac{dP^{\mathrm{FFF}}}{dx_E} \approx \langle z_t \rangle \, B \exp -b \langle z_t \rangle \, x_E$$

Kinematics



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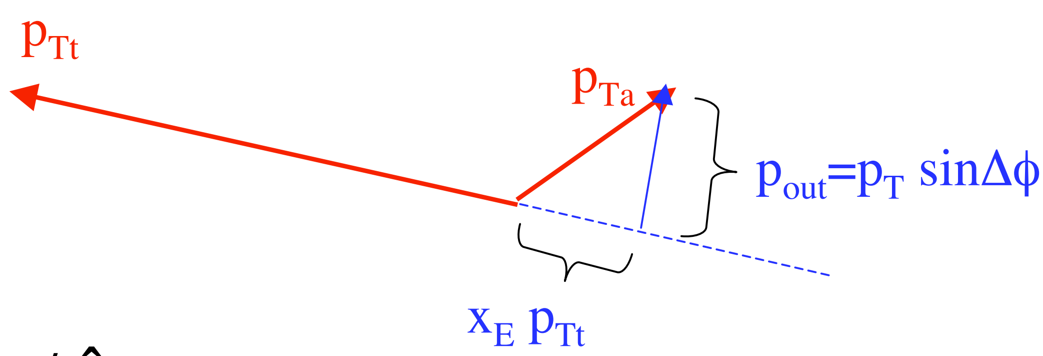

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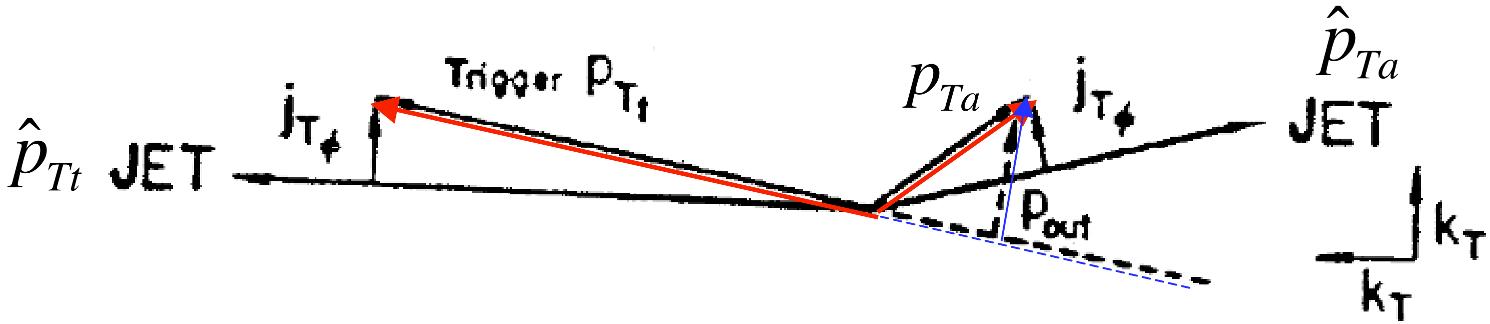

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From Feynman, Field and Fox
Nucl Phys B128 (1977) 1--65

38 *R.P Feynman et al. / Large transverse momenta*

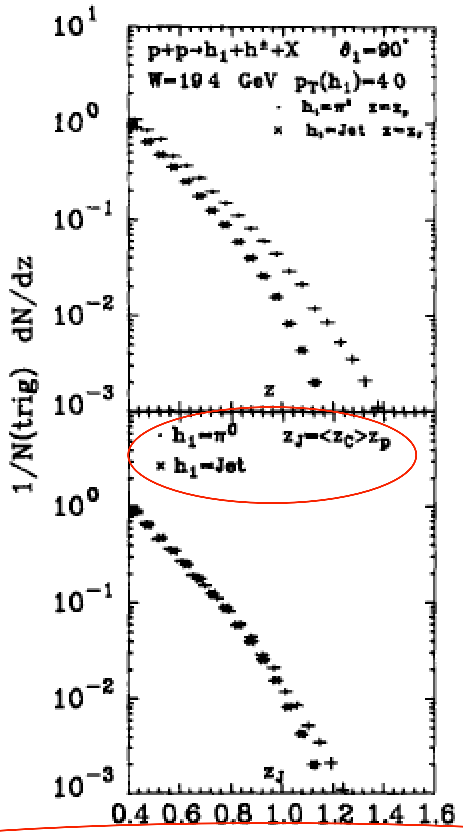
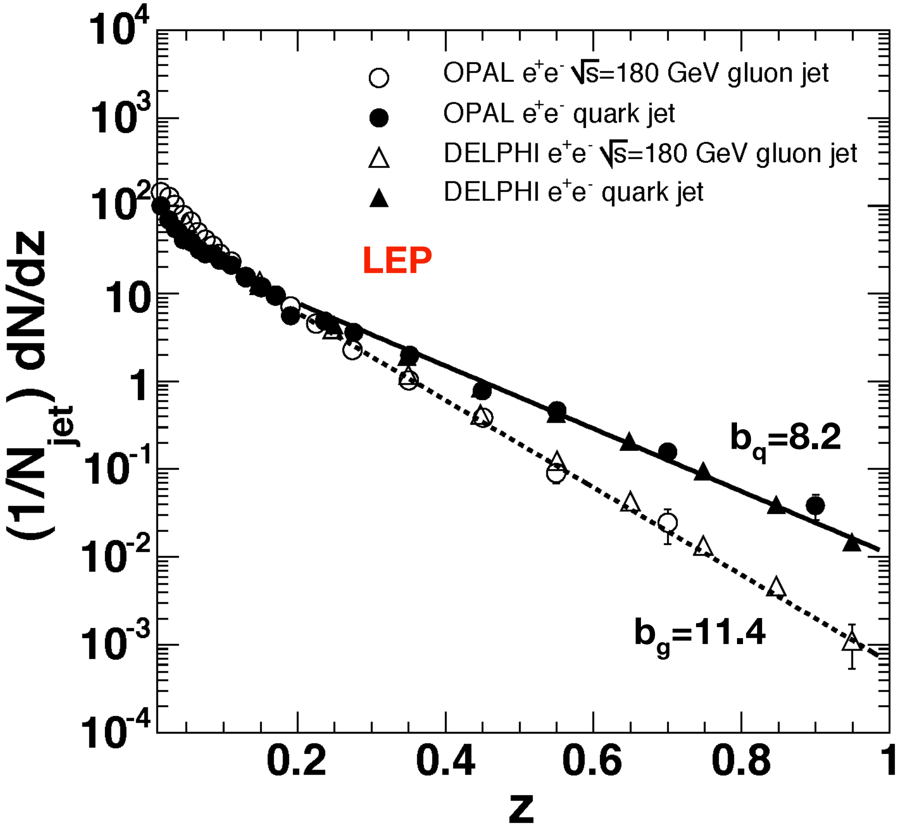
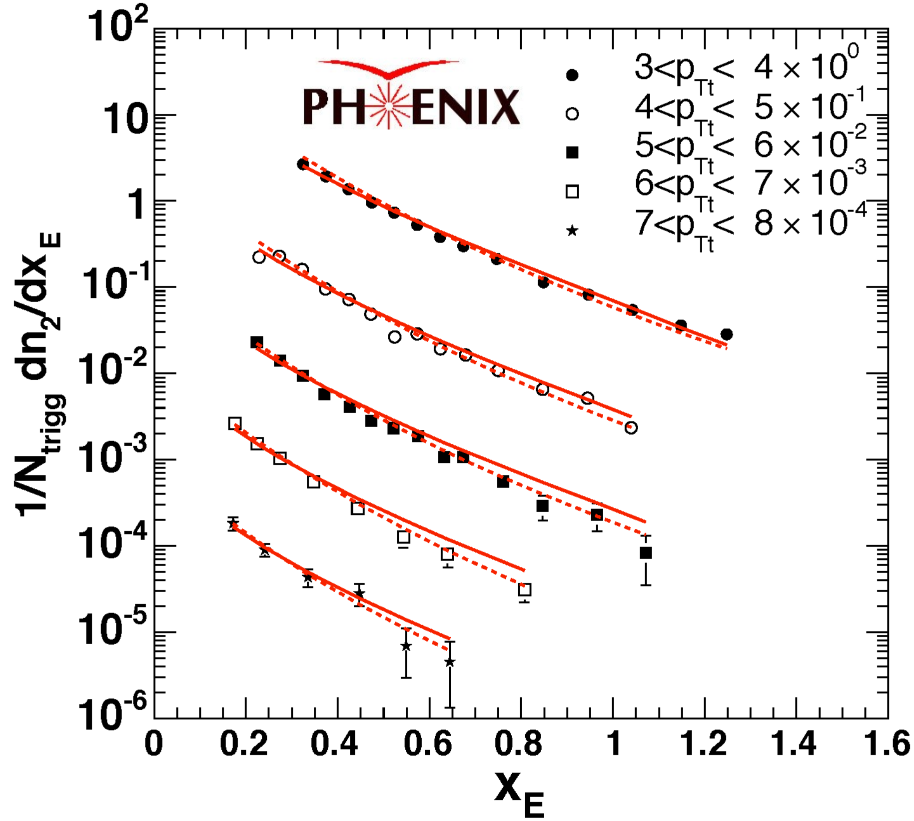


Fig. 23. Comparison of the π^0 and jet trigger away-side distribution of charged hadrons in pp collisions at $W = 19.4$ GeV, $\theta_1 = 90°$, and $p_\perp$ (trigger) = 4.0 GeV/c from the quark-quark scattering model. The upper figure shows the single-particle (π^0) trigger results plotted *versus* $z_p = -p_x(h^\pm)/p_\perp(\pi^0)$ and the jet trigger plotted *versus* $z_J = -p_x(h^\pm)/p_\perp(\text{jet})$ (see table 1). In the lower figure, we plot *both versus* z_J, where for the jet trigger $z_J = z_J$ but for the single-particle trigger $z_J = \langle z_c \rangle z_p$. The away hadrons are integrated over all rapidity Y and $|180° - \phi| \leq 45°$ and the theory is calculated using $\langle k_\perp \rangle_{h\to q} = 500$ MeV. ● $h_1 = \pi^0$, × h_1 = jet.

"There is a simple relationship between experiments done with single-particle triggers and those performed with jet triggers. The only difference in the opposite side correlation is due to the fact that the 'quark', from which a single-particle trigger came, always has a higher $p_\perp$ than the trigger (by factor $1/z_{trig}$). The away-side correlations for a single-particle trigger at $p_\perp$ should be roughly the same as the away side correlations for a jet trigger at $p_\perp$ (jet)= $p_\perp$ (single particle)/ $\langle z_{trig} \rangle$".

PHENIX-compared measured x_E distribution in p-p to numerical integral using LEP fragmentation functions

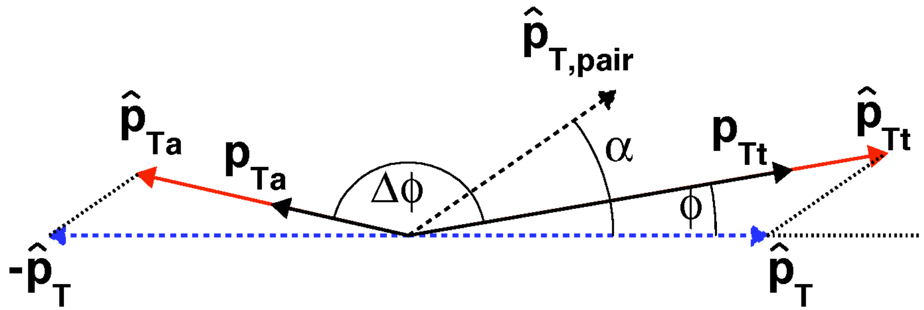




PHENIX PRD **74** (2006) 072002. The x_E distribution triggered by a leading fragment (π^0) is not sensitive to the shape of the fragmentation function!!! Disagrees with FFF!!

A very interesting new formula for the x_E distribution was derived by PHENIX in PRD74

$$\left.\frac{dP_\pi}{dx_E}\right|_{p_{T_t}} \approx \langle m\rangle (n-1)\frac{1}{\hat{x}_h}\frac{1}{(1+\frac{x_E}{\hat{x}_h})^n}$$



Relates ratio of particle p_T

$$x_E = \frac{-p_{T_a}\cos\Delta\phi}{p_{T_t}} \simeq \frac{p_{T_a}}{p_{T_t}}$$

measured

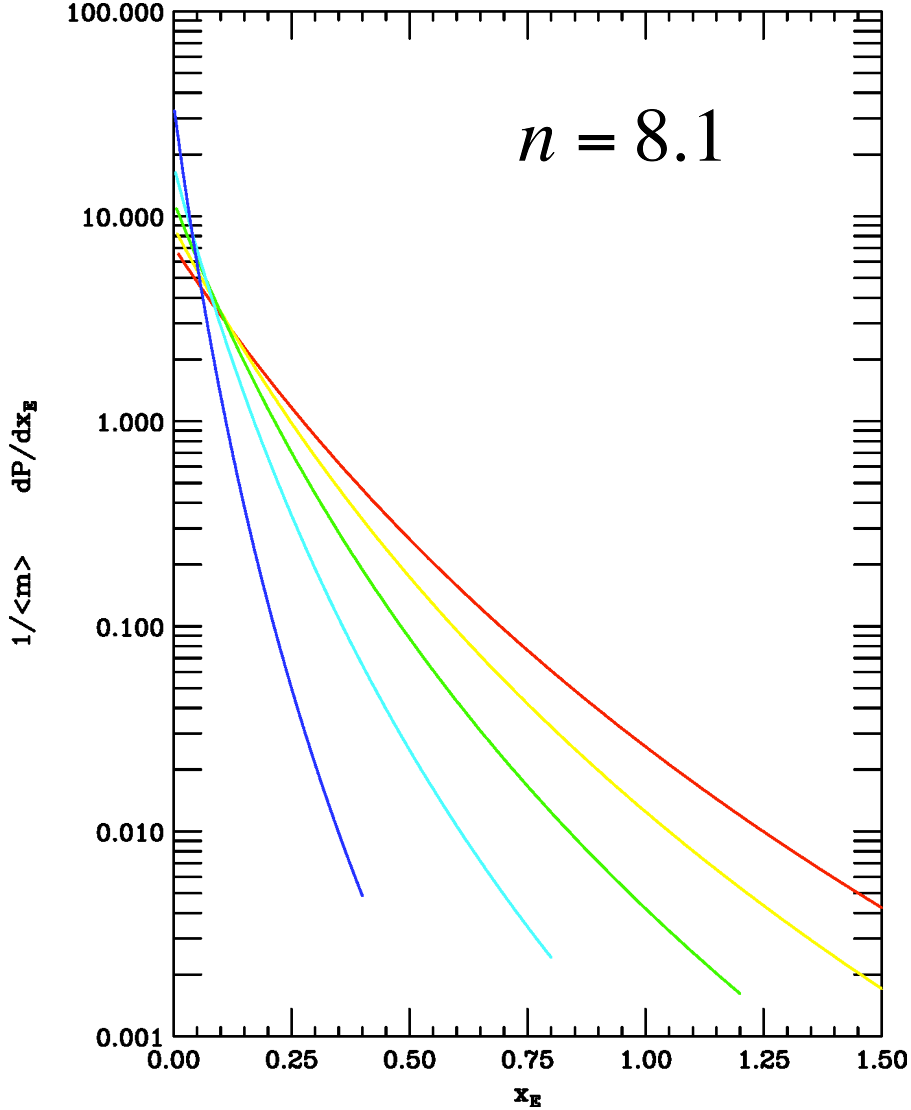
Ratio of jet transverse momenta

$$\hat{x}_h = \frac{\hat{p}_{T_a}}{\hat{p}_{T_t}}$$

Can be determined

If formula works, we can also use it in Au+Au to determine the relative energy loss of the away jet to the trigger jet (surface biased by large *n*)

Shape of x_E distribution depends on $\hat{x}_h$ and n but not on b



$$\left.\frac{dP_\pi}{dx_E}\right|_{p_{T_t}} \approx \langle m \rangle (n-1)\frac{1}{\hat{x}_h}\frac{1}{(1+\frac{x_E}{\hat{x}_h})^n}$$

$\hat{x}_h$

1.0

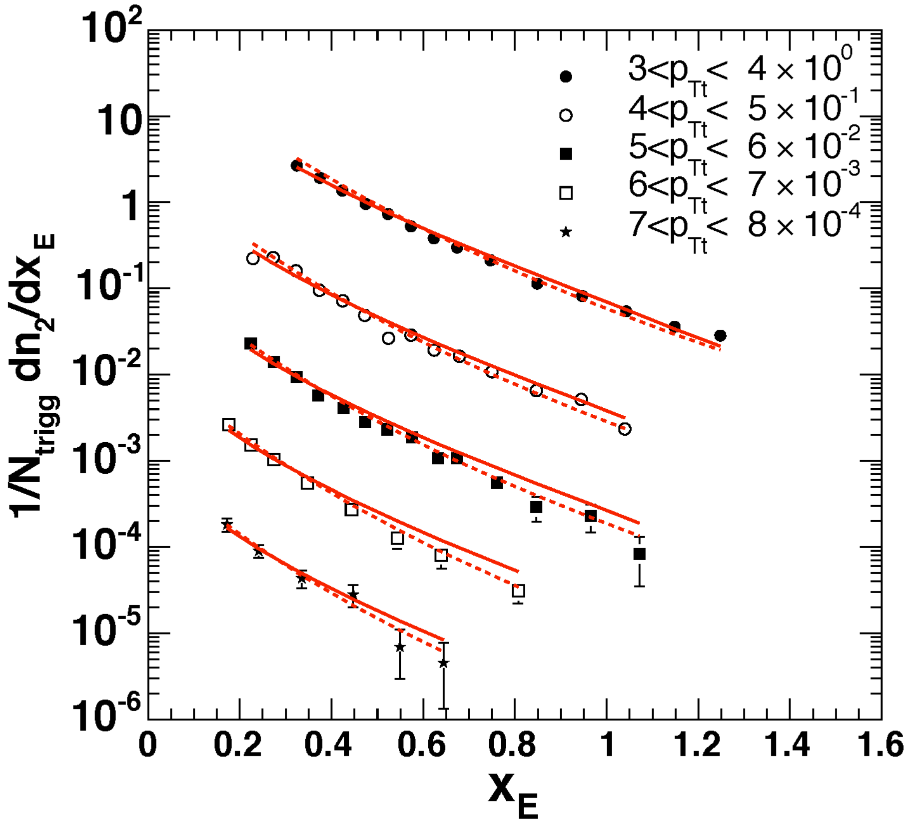
0.8

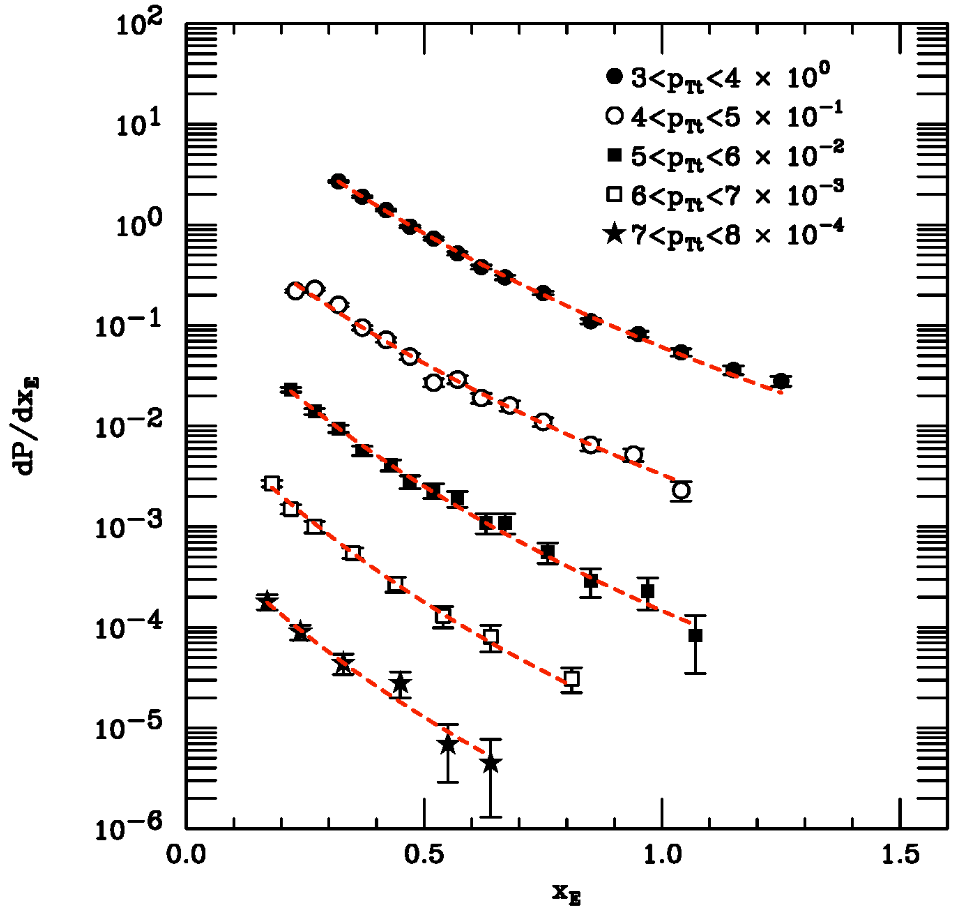
0.6

0.4

0.2

Fit works for PHENIX p+p PRD **74**, 072002

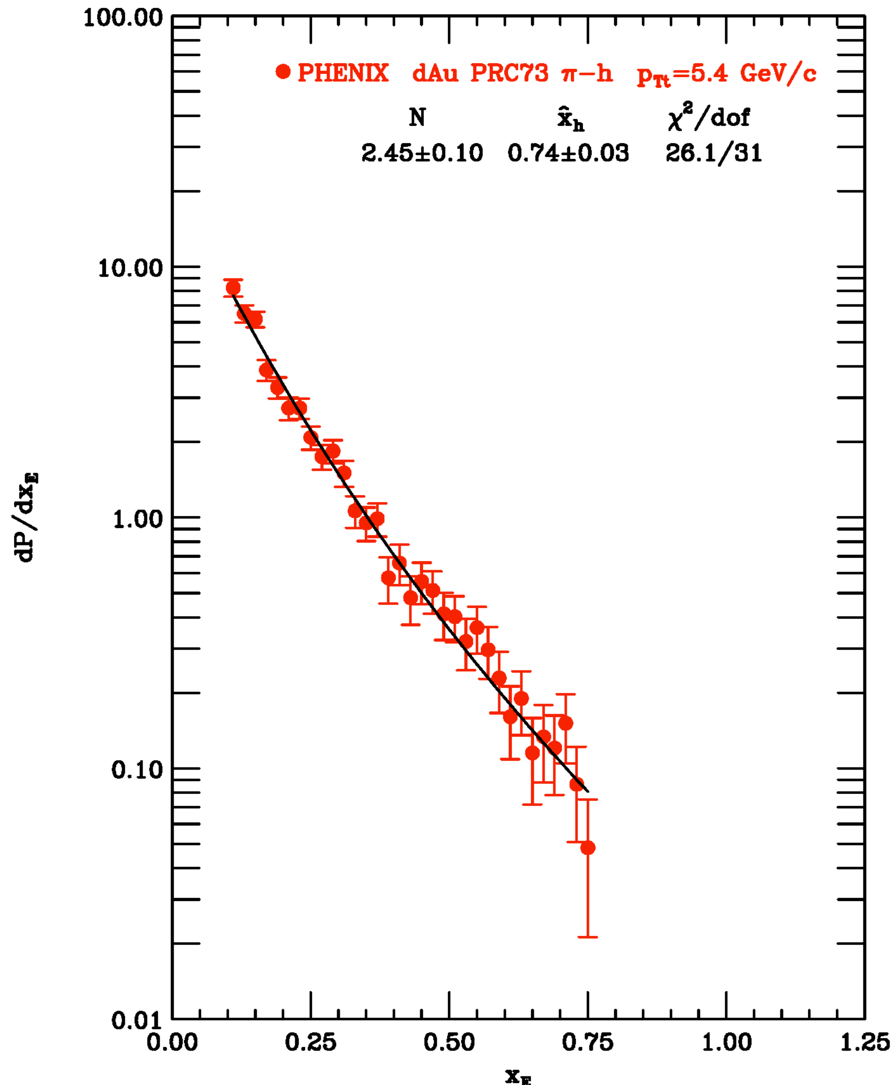


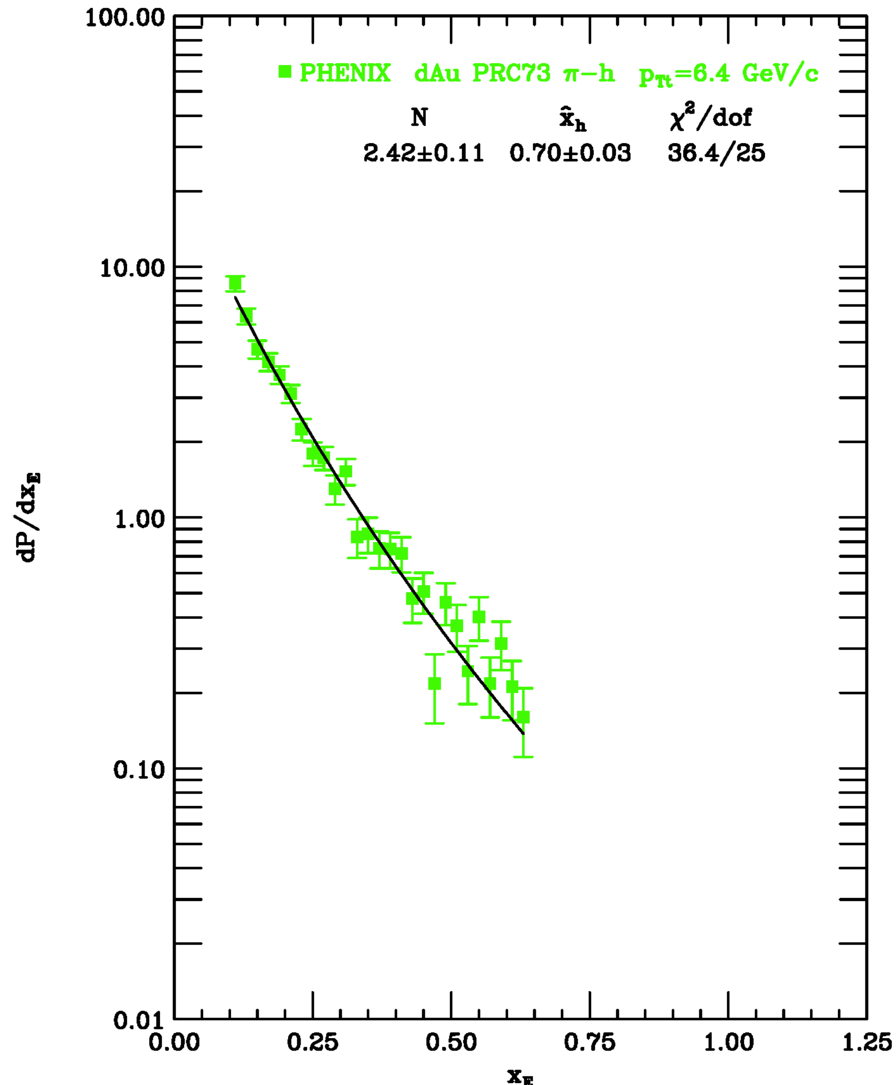


Calculation from Fragmentation Fn.

New fits. Very nice!

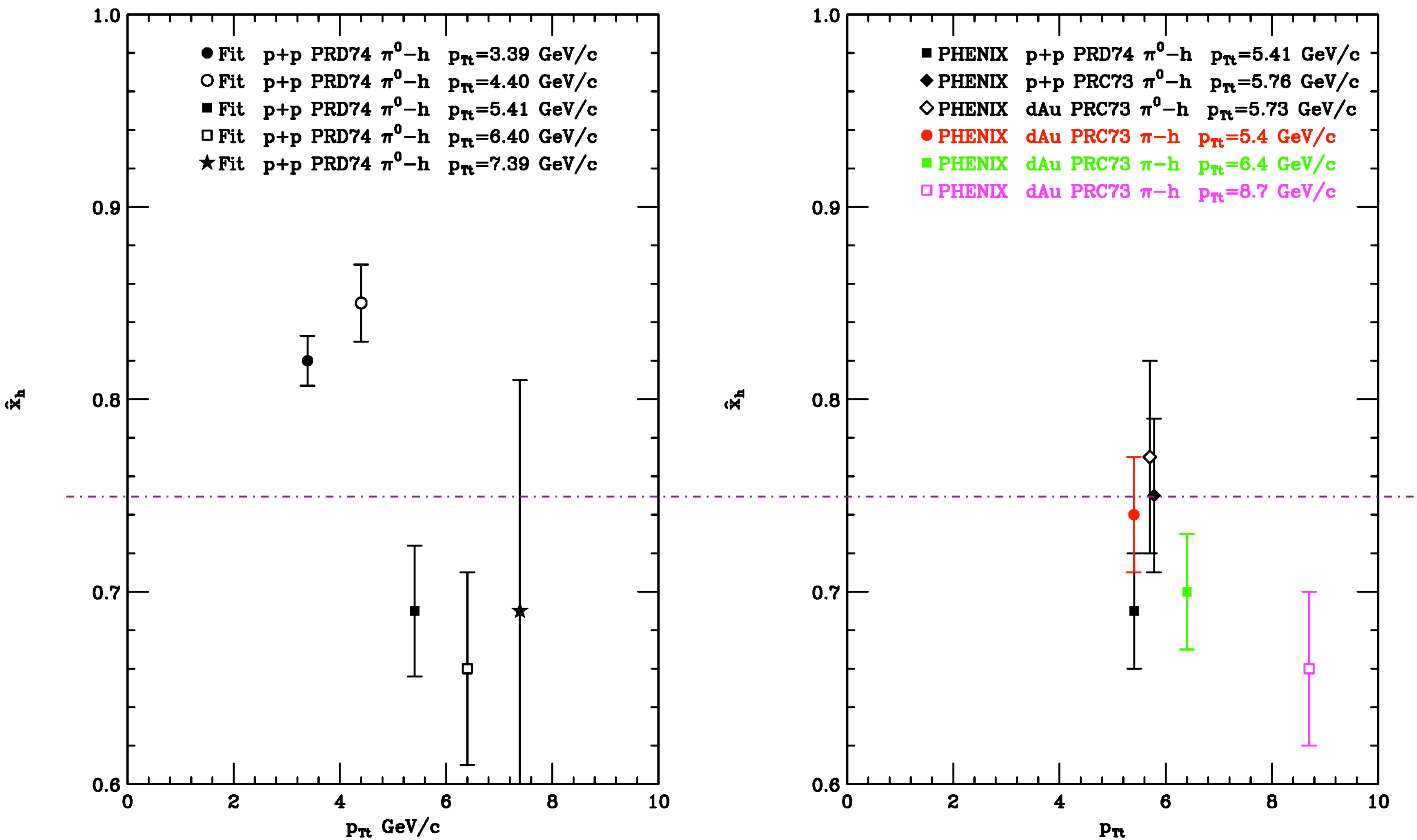
PHENIX $\pi^{\pm}$-h d+Au PRC73 (2006) 054903



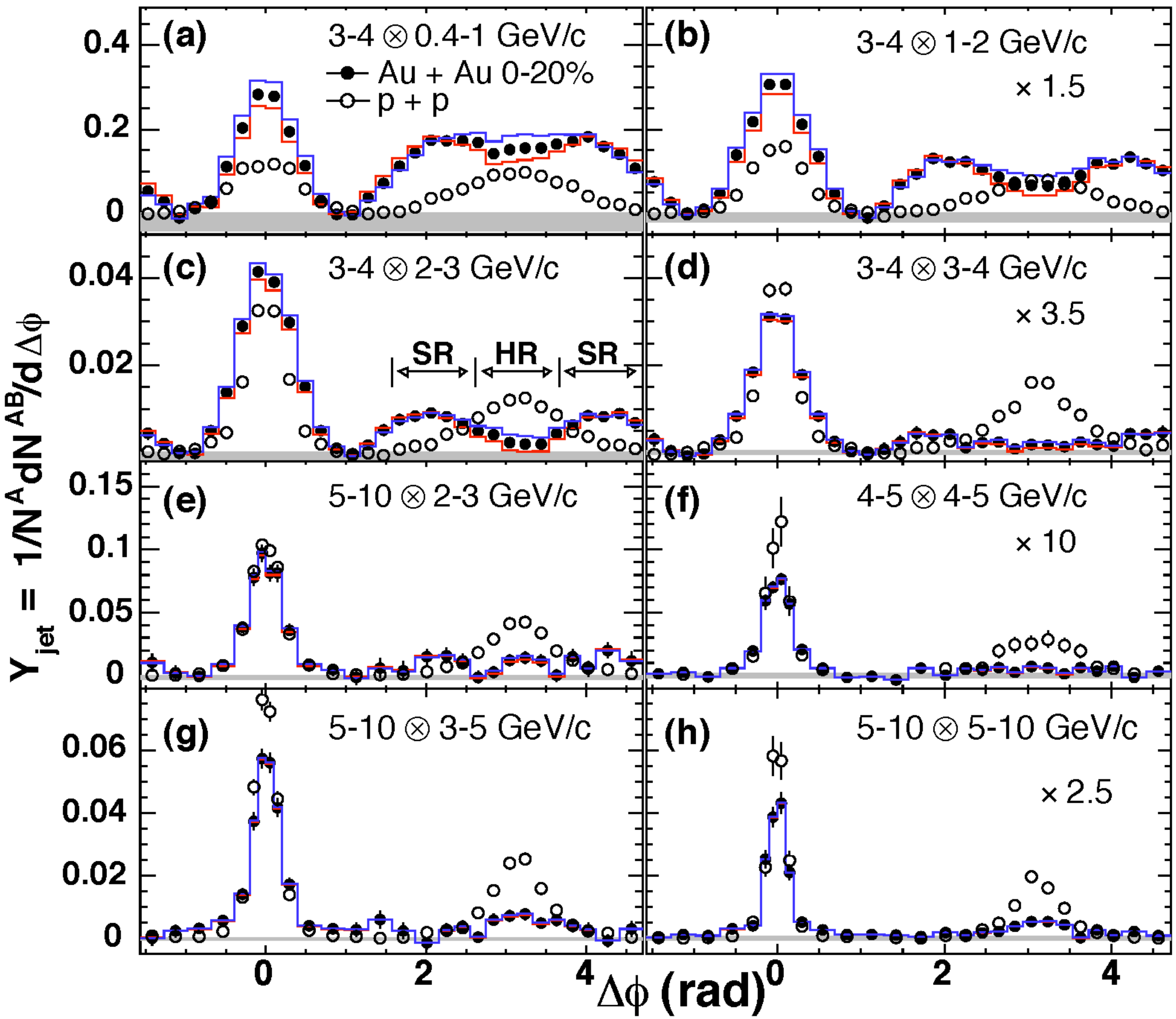


Excellent χ^2 in most cases

$\hat{x}_h \sim 0.75$ due to k_T smearing in p-p, dAu

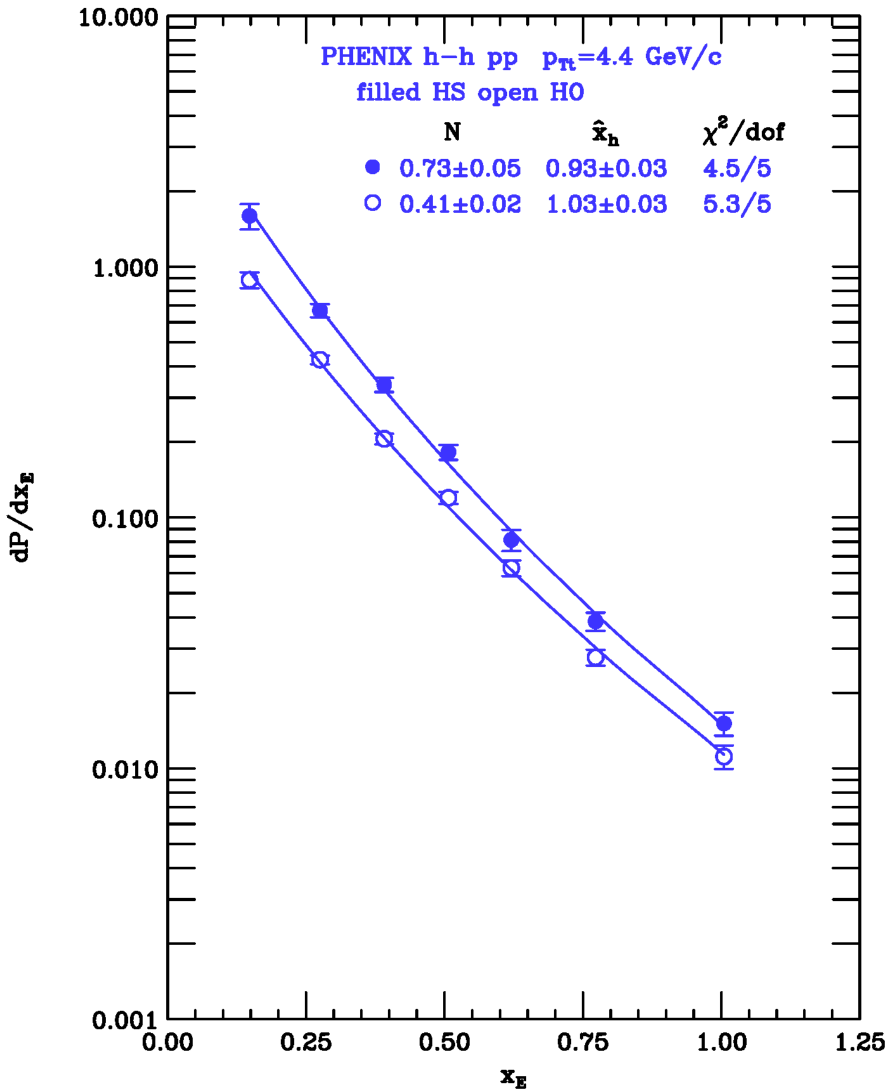


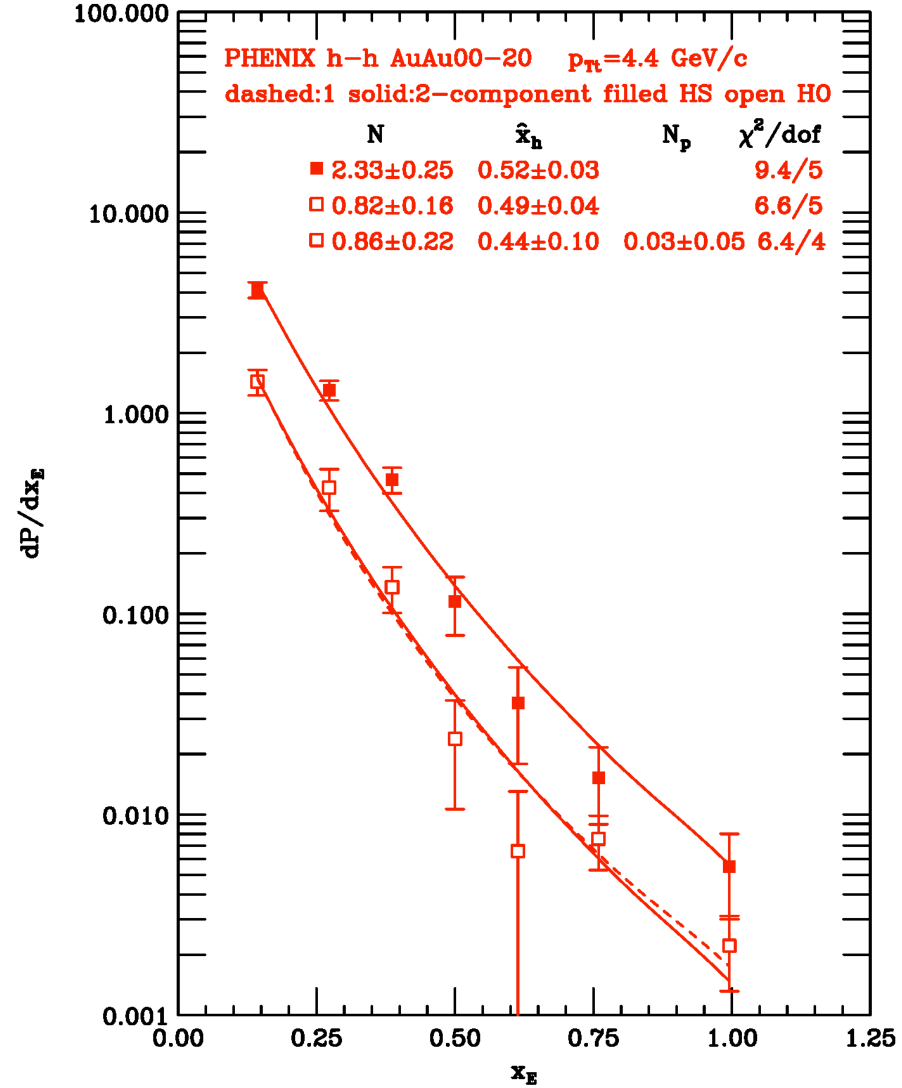
New PHENIX AuAu arXiv:0705.3238



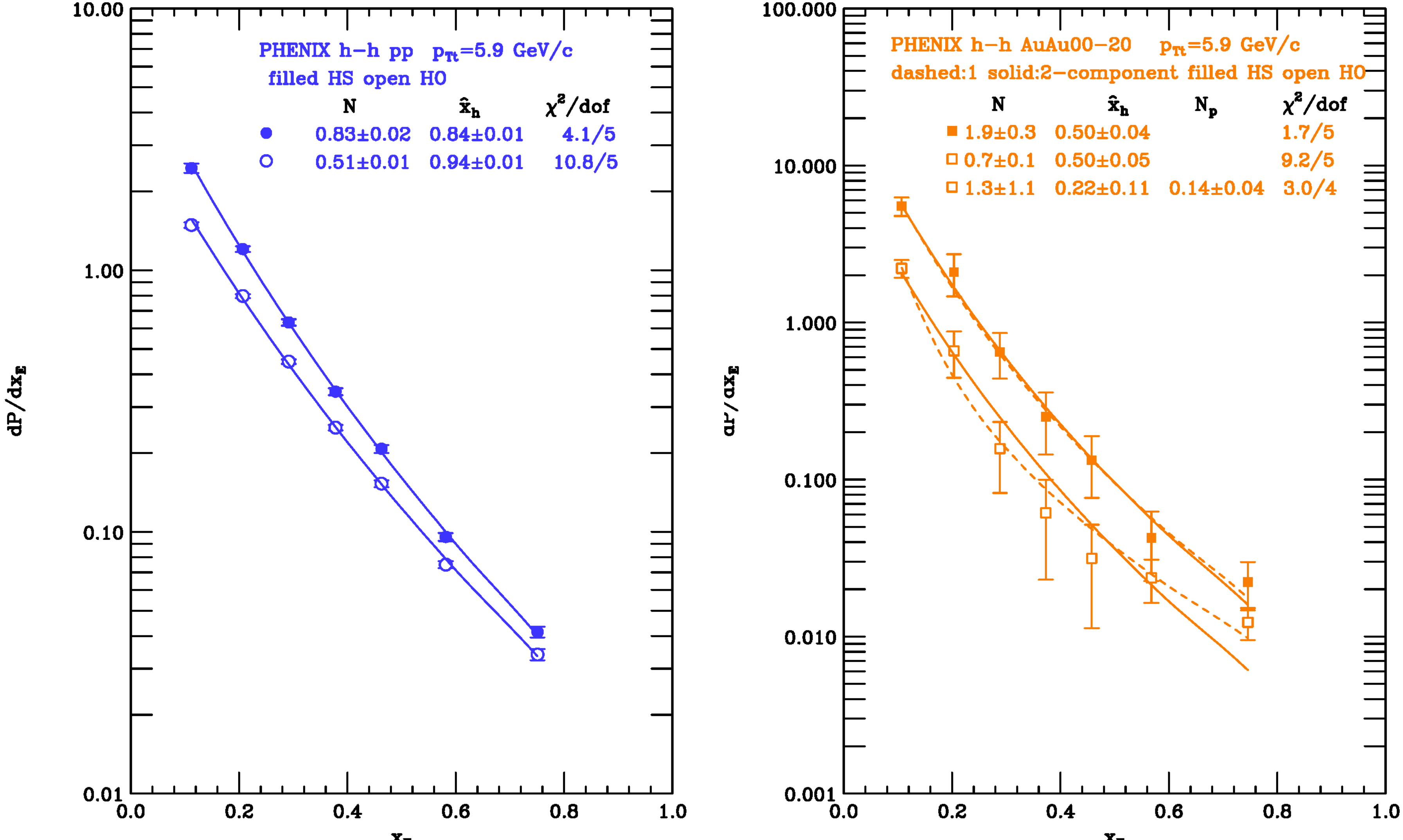
Define Head region (HR) and Shoulder regions (SR) for wide away side correlation.

Fit H+S and HO (head only) $4<p_{Tt}<5$ GeV/c



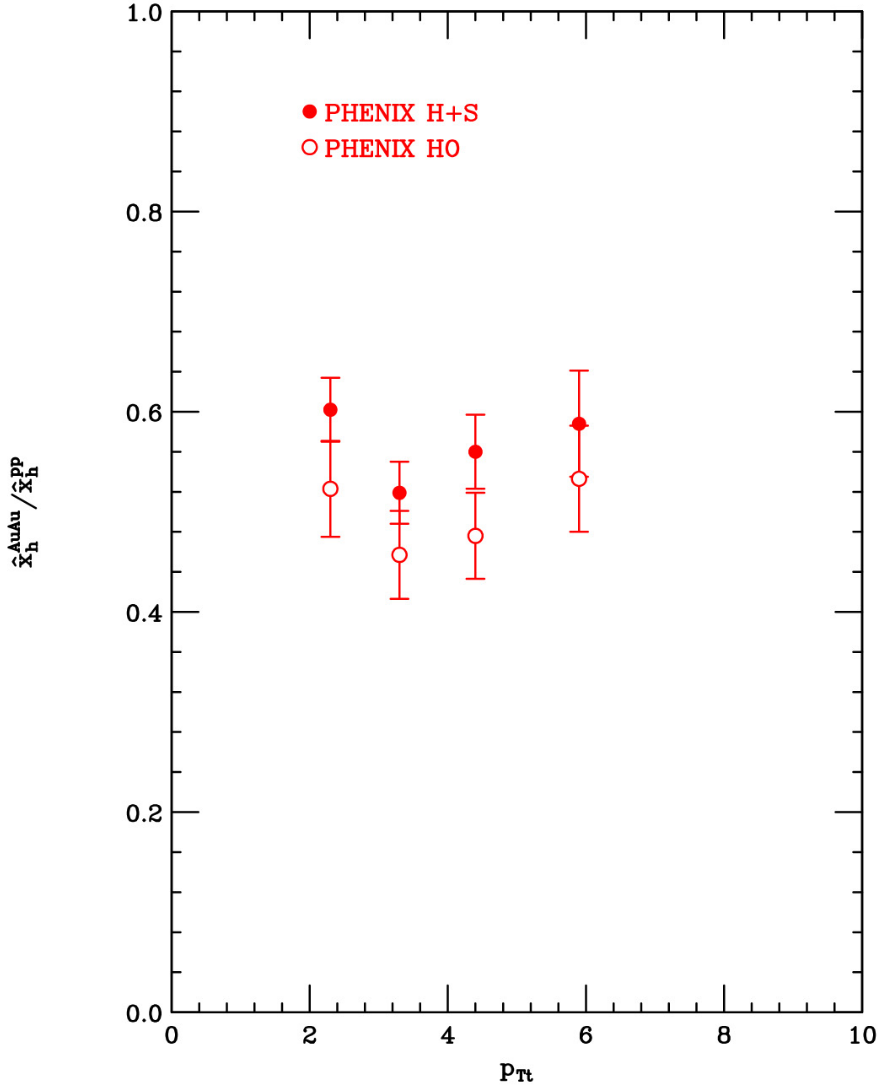


5< p_{Tt} <10 GeV/c

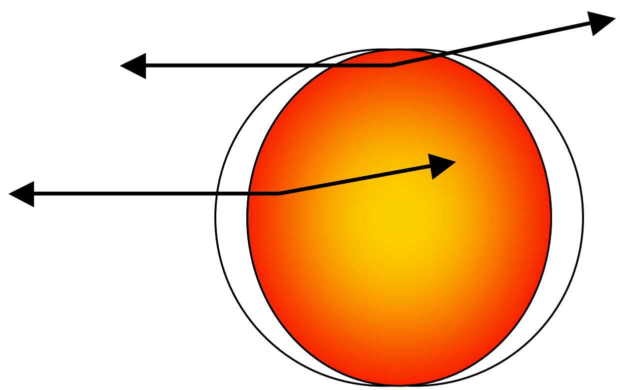


Statistical two-component distribution (⇒punch-through) for Head-Only

Formula works in Au+Au: Away-side x_E distribution is steeper in Au+Au than p-p indicating energy loss

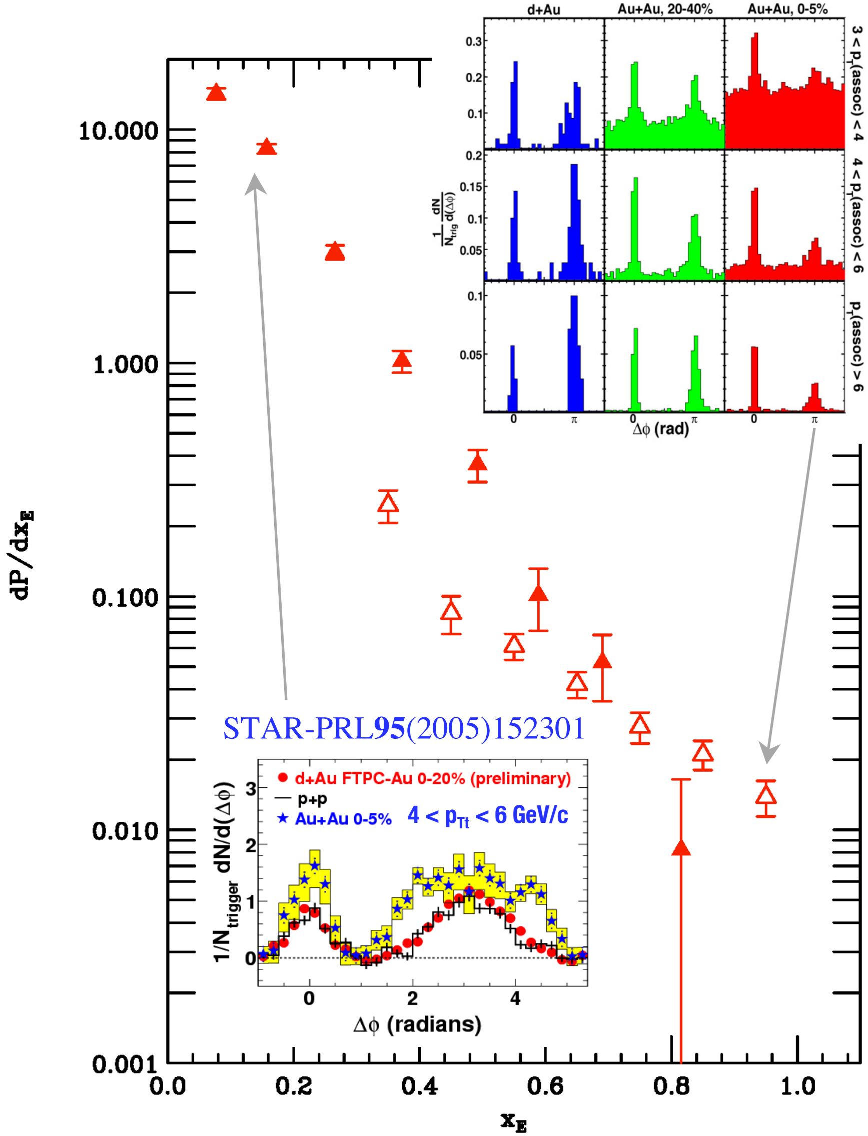


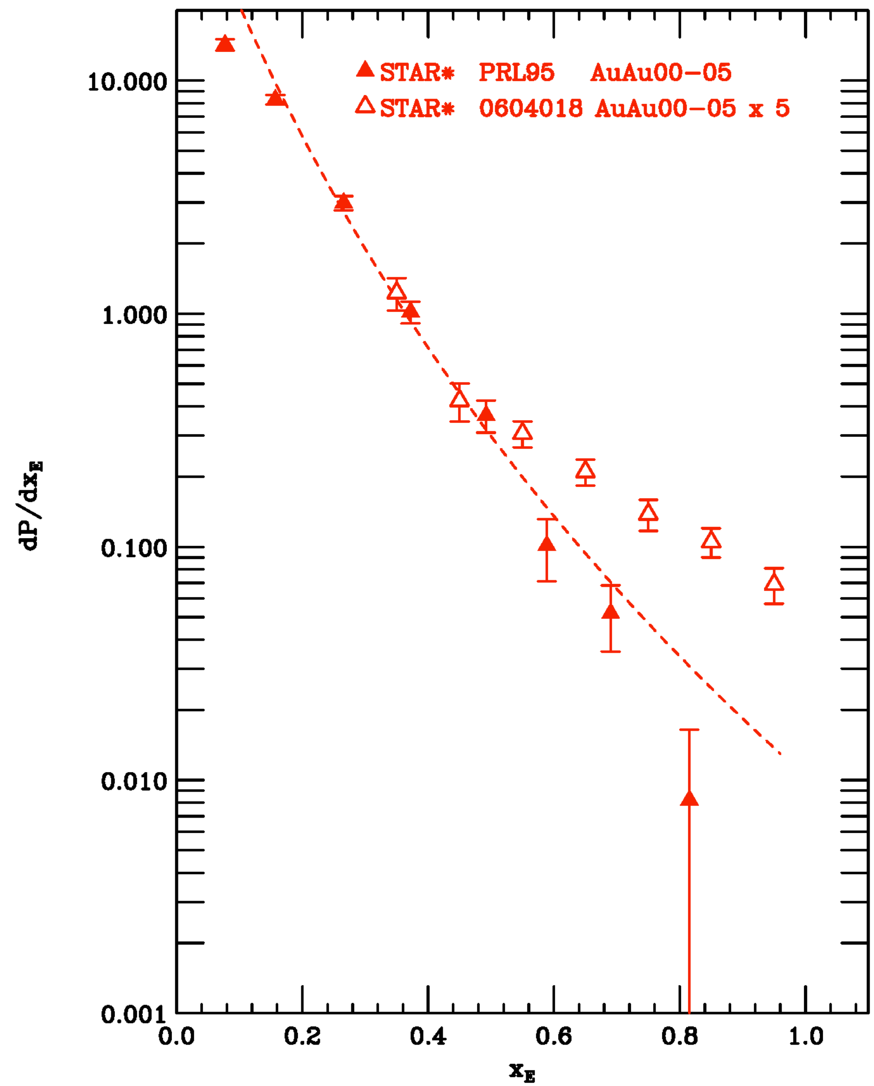
Since the trigger jet is surface biased, the away jet must cross through nearly the entire medium except in the case of tangential emission. The decrease of $\hat{x}_h = \hat{p}_{Ta} / \hat{p}_{Tt}$ in Au+Au central collisions relative to p-p by a factor of ~0.5-0.6 indicates that the away jet has lost energy by traversing the medium and gives a quantitative measurement.



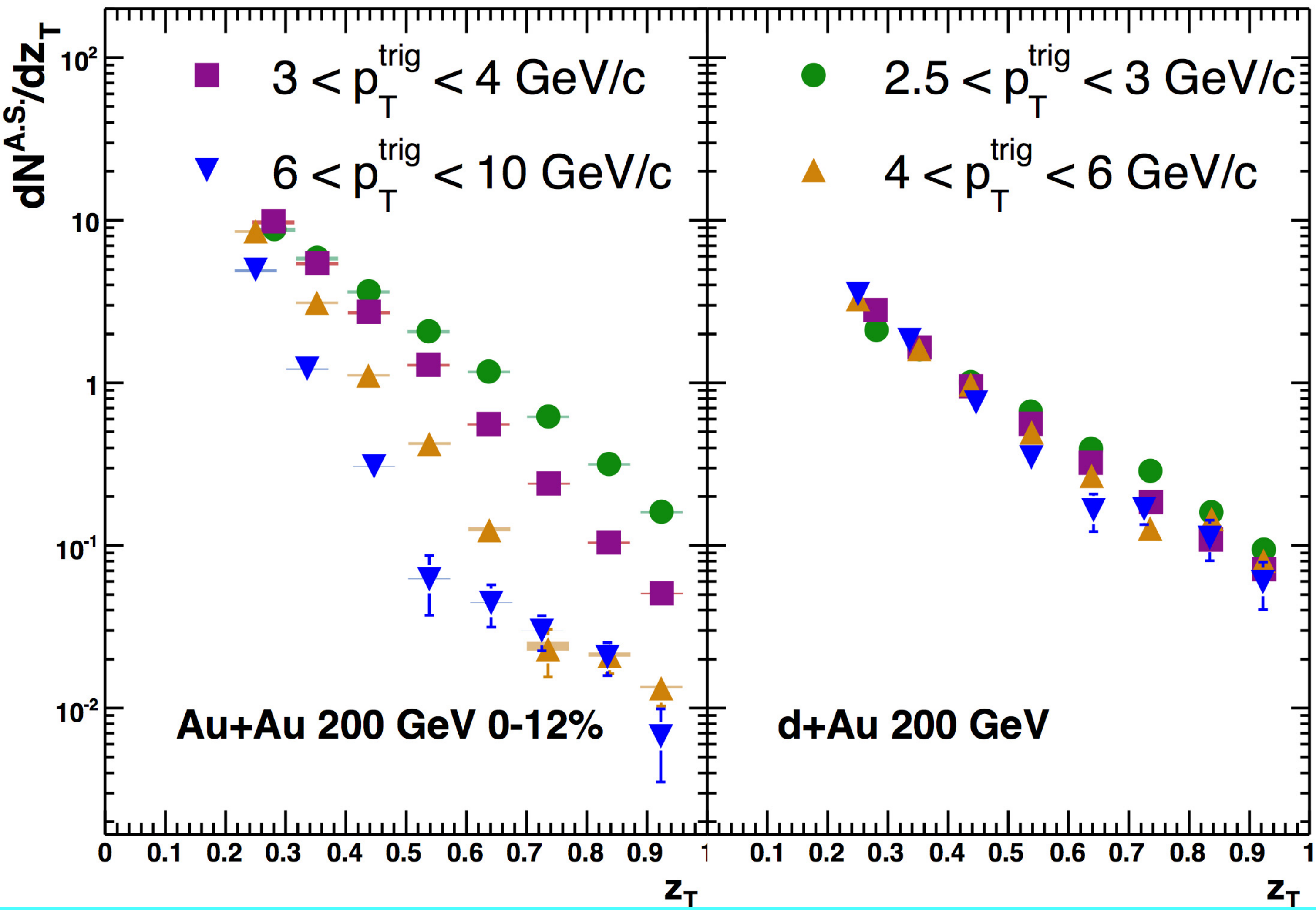
STAR dAu, AuAu

STAR-PRL **97** (2006) 162301
$8 < p_{Tt} < 15$ GeV/c



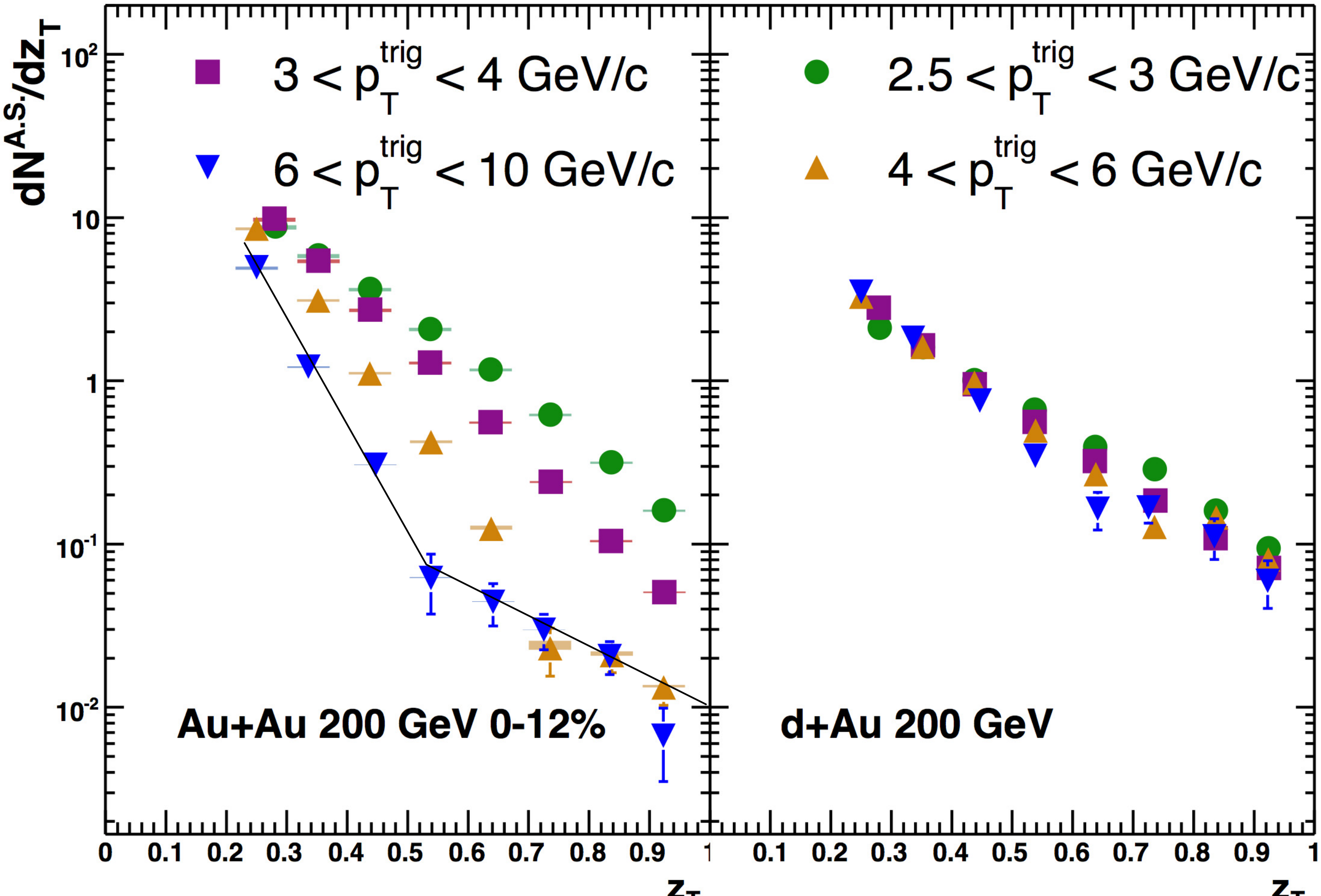


Newest STAR-Horner-QM2006-JPG34 (2007) S995



Two-component distribution (punch-through) is now clear for $6 < p_{Tt} < 10$ GeV/c

Newest STAR-Horner-QM2006-JPG34 (2007) S995



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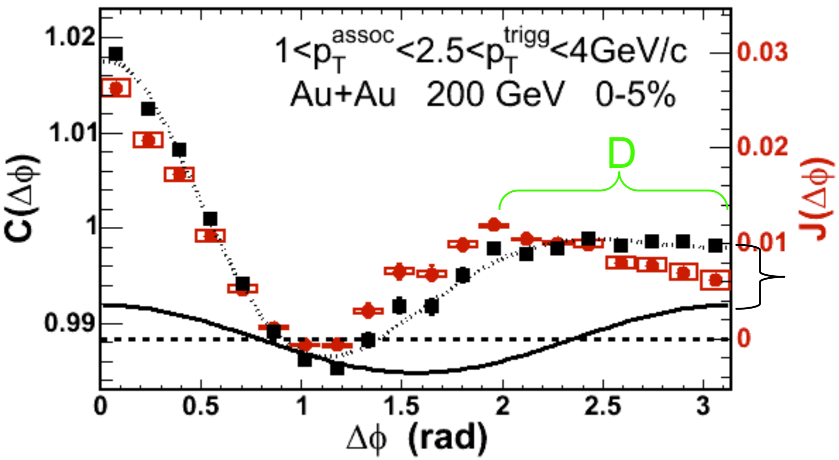
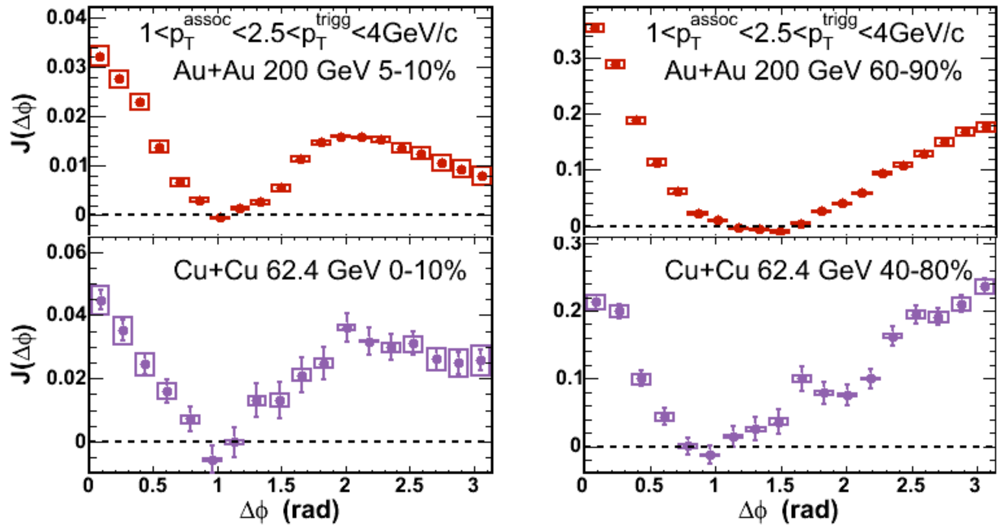
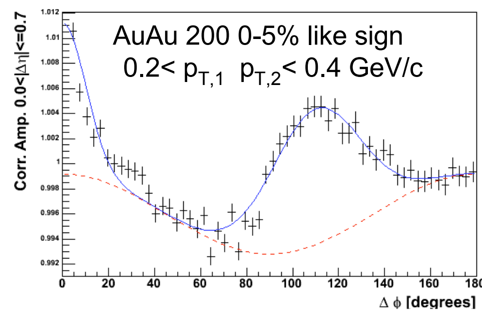
The End

Away side correlations in Au+Au much wider than in p-p

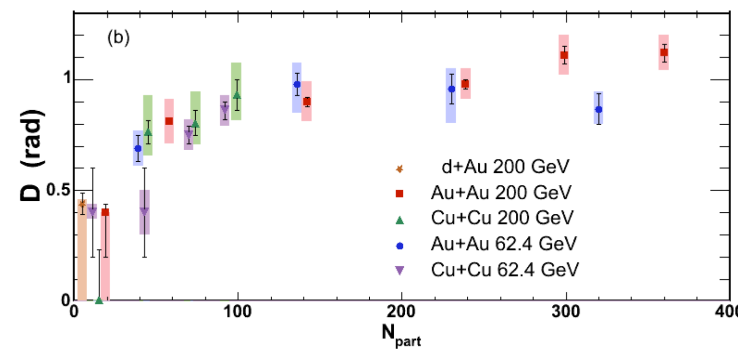




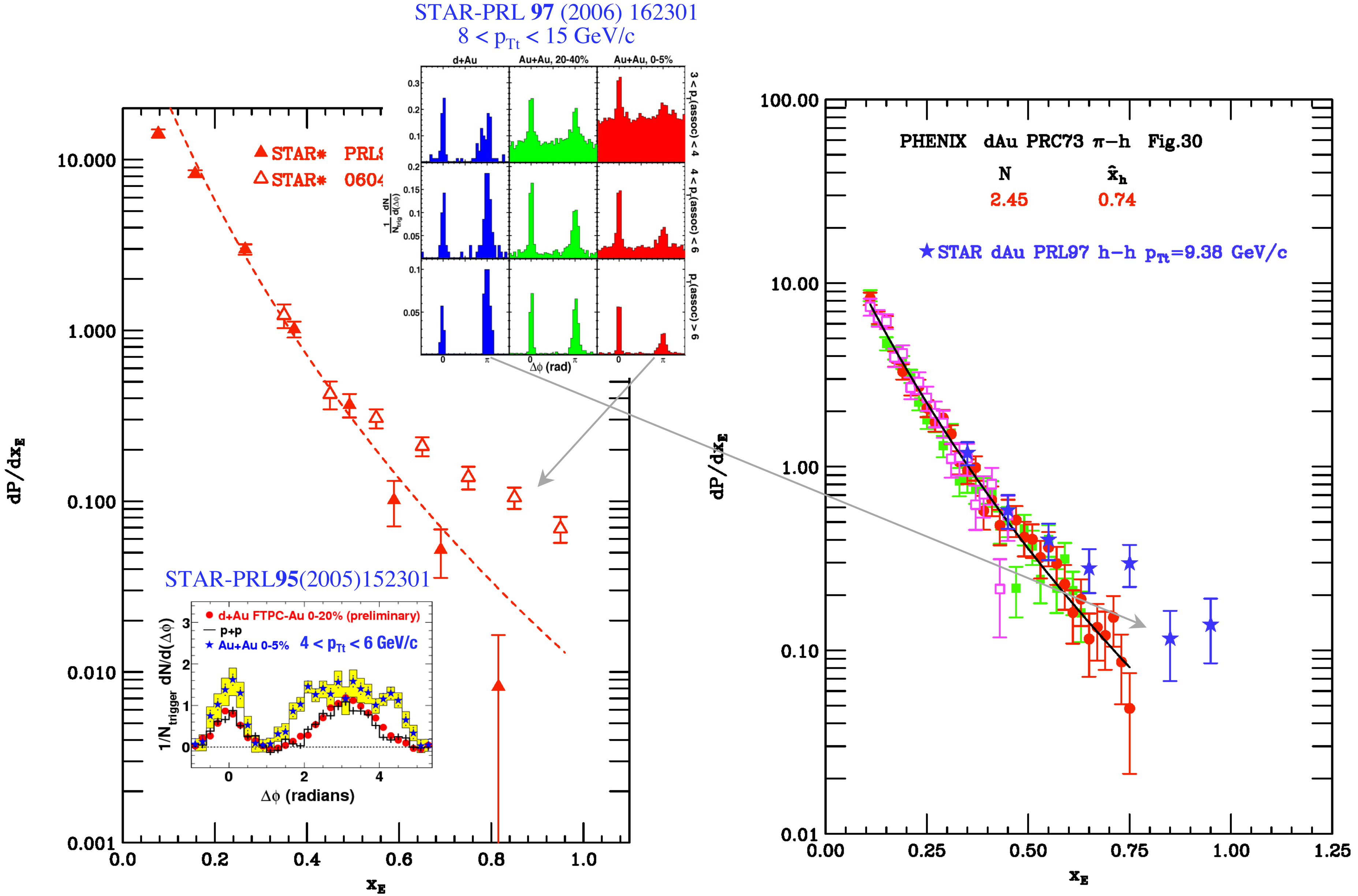
Away side distribution much wider in A+A than p-p in correlation fn. C(Δφ) Subtraction of v2 (flow?) effect→ J(Δφ) causes a dip at 180º which gives 2 peaks at π±D~1 radian independent of system and centrality for N_{part} >100. This is also seen for (auto) correlations of low p_T particles. Is this the medium reaction to the passage of a color-charged parton? Stay tuned, much more study needed.



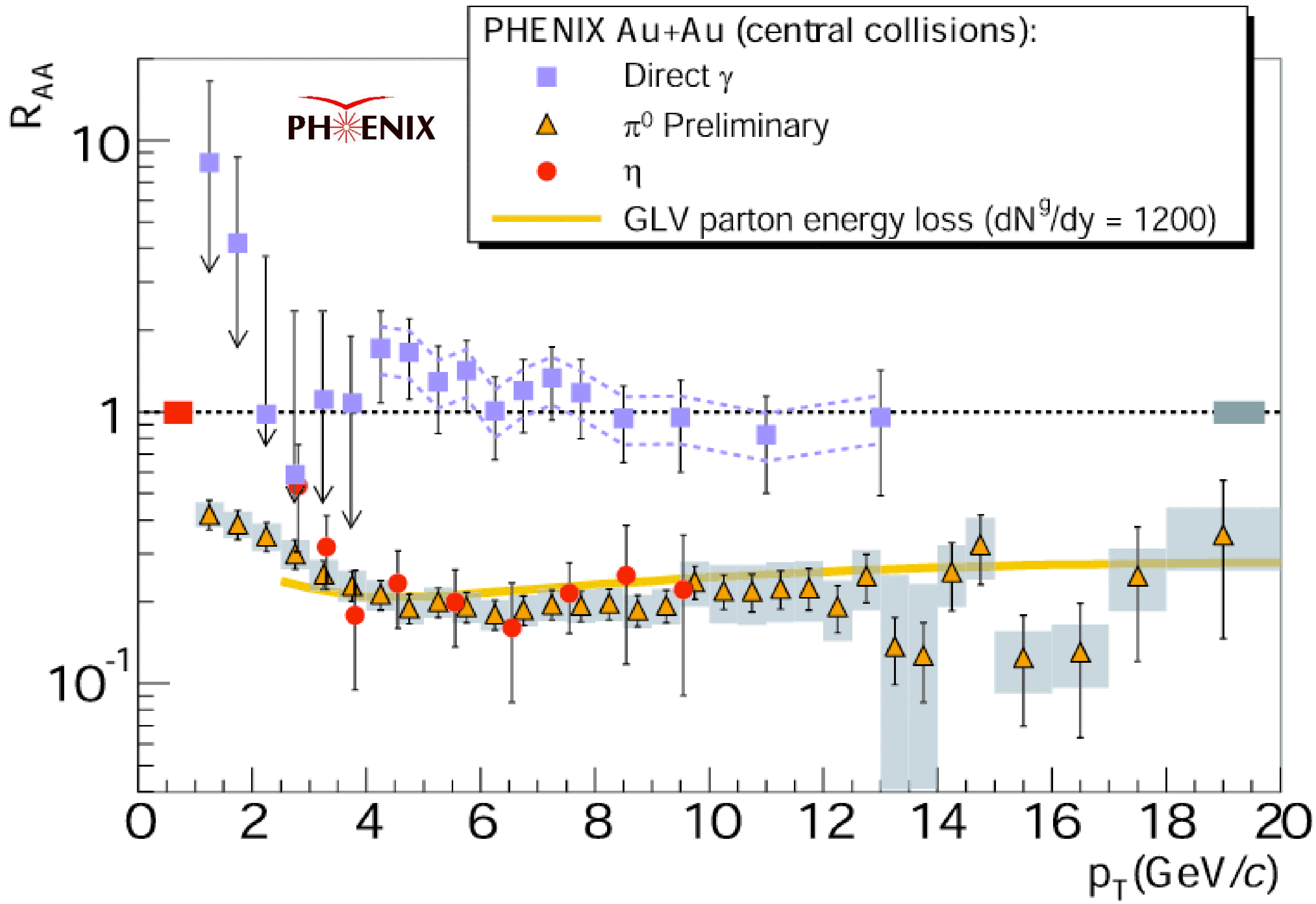
PHENIX AuAu PRL **98** (2007) 232302



STAR dAu, AuAu

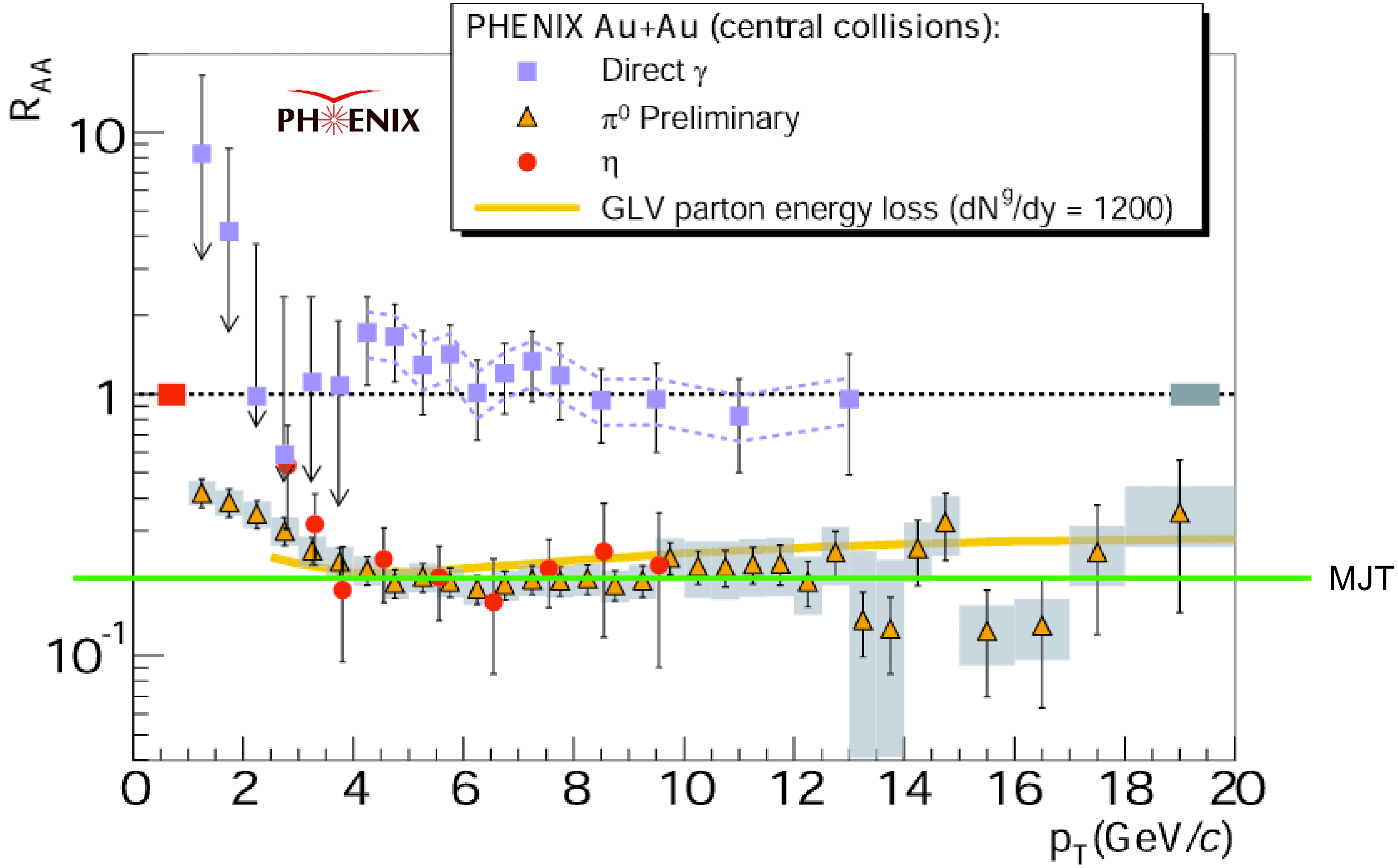


Status of R_{AA} in AuAu at $\sqrt{s_{NN}}$=200 GeV QM05



Direct γ are not suppressed. π^0 and η suppressed even at high p_T
Implies a strong medium effect (energy loss) since γ not affected.
Suppression is flat at high p_T. Are data flatter than theory?

Status of R_{AA} in AuAu at $\sqrt{s_{NN}}$=200 GeV QM05



Direct γ are not suppressed. π^0 and η suppressed even at high p_T
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Suppression is flat at high p_T. Are data flatter than theory?

Normalization of Fragmentation Functions

For an exponential fragmentation function,

$$D(z) = Be^{-bz} \qquad ,$$

the mean multiplicity of fragments in the jet is:

$$\langle m \rangle = \int_0^1 D(z)dz = \frac{B}{b}(1 - e^{-b})$$

and these fragments carry the total momentum of the jet:

$$\int_0^1 zD(z)dz = \frac{B}{b^2}(1 - e^{-b}(1+b)) \equiv 1 \qquad ,$$

where the $\langle z \rangle$ per fragment is:

$$\langle z \rangle = \frac{\int_0^1 zD(z)dz}{\int_0^1 D(z)dz} = \frac{1}{\langle m \rangle} \qquad .$$

The results are:

$$B = \frac{b^2}{1 - e^{-b}(1+b)} \approx b^2$$

$$\langle m \rangle = \frac{b(1 - e^{-b})}{1 - e^{-b}(1+b)} \approx b \qquad ,$$

$$\langle z \rangle = \frac{1 - e^{-b}(1+b)}{b(1 - e^{-b})} \approx \frac{1}{b} \qquad .$$

I assumed b is the same for π^0 and all charged. Then using B/b=b which normalizes total momentum to 1, I get the correct jet cross section. Obviously the total momentum for π^0 is ~1/3 so B for pure π^0 ~1/3 b^2

As measured at the ISR by Darriulat, etc.

P. Darriulat, et al, Nucl.Phys. **B107** (1976) 429-456

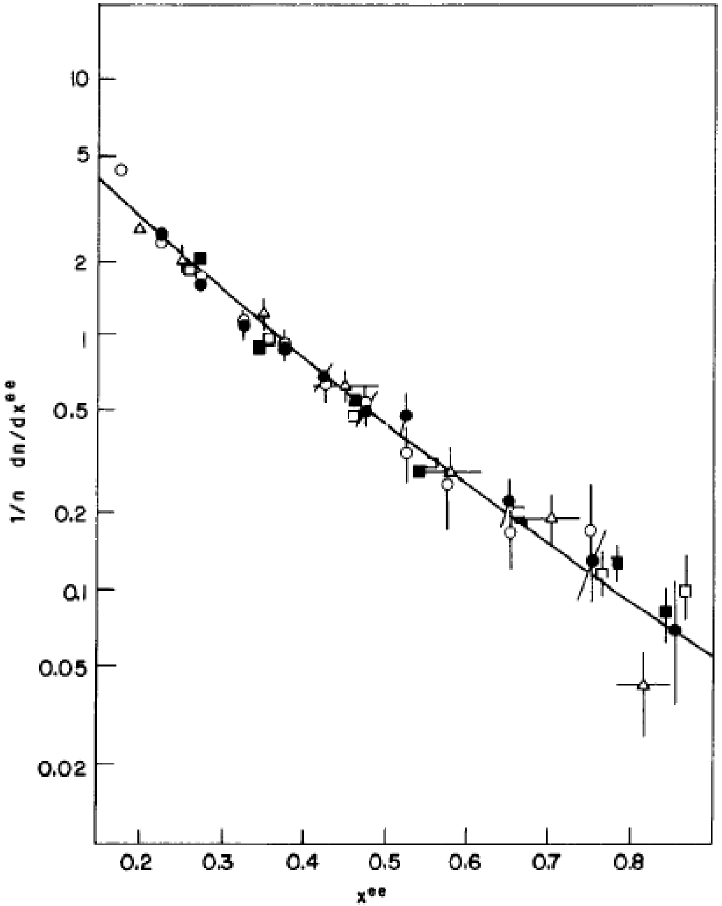

Figure 21 Jet fragmentation functions measured in different processes: ν-p interactions (*open triangles*, Van der Welde 1979); e^+e^- annihilations (*solid line*, Hanson et al 1975); and pp collisions (*full circles* CS, $p_T < 6$ GeV/c, *open circles* CS, $p_T > 6$ GeV/c, *full squares* CCOR, $p_T > 5$ GeV/c, *open squares* CCOR, $p_T > 7$ GeV/c).

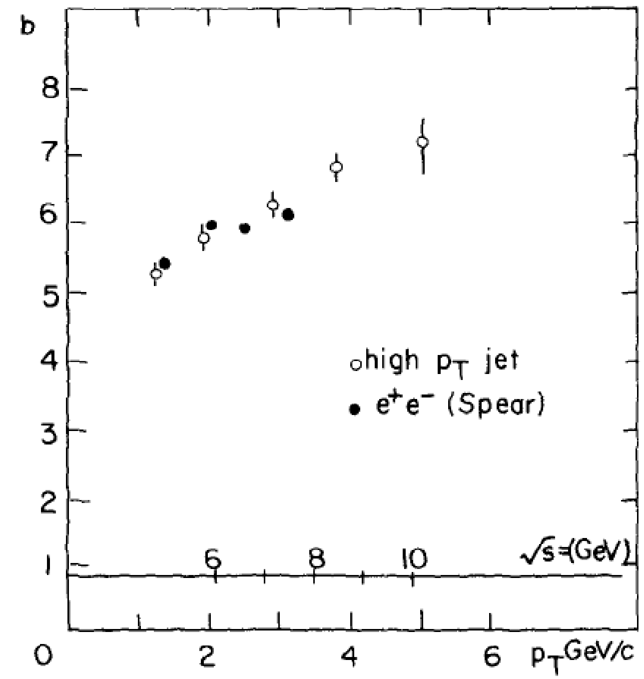

Figure 19 The slopes *b* obtained from exponential fits to the jet fragmentation function in the interval $0.2 < z < 0.8$ in e^+e^- annihilation (*full circles*) and LPTH data of the BS Collaboration (*open circles*).

Figures from P. Darriulat, ARNPS **30** (1980) 159-210 showing that Jet fragmentation functions in νp, e^+e^- and pp (CCOR) are the same with the same dependence of *b* (exponential slope) on "$\hat{s}$"

The leading-particle effect a.k.a. trigger bias

- Due to the steeply falling power-law spectrum of the scattered partons, the inclusive particle p_T spectrum is dominated by fragments biased towards large z. This was unfortunately called trigger bias by M. Jacob and P. Landshoff, Phys. Rep. **48C**, 286 (1978) although it has nothing to do with a trigger.

$$\frac{d^2\sigma_\pi(\hat{p}_{T_t}, z_t)}{d\hat{p}_{T_t} dz_t} = \frac{d\sigma_q}{d\hat{p}_{T_t}} \times D^q_\pi(z_t)$$

Fragment spectrum given $\hat{p}_{T_t}$

$$= \frac{A}{\hat{p}^{n-1}_{T_t}} \times D^q_\pi(z_t)$$

Power law spectrum of parton $\hat{p}_{T_t}$

let $\hat{p}_{T_t} = p_{T_t}/z_t \qquad d\hat{p}_{T_t}/dp_{T_t}|_{z_t} = 1/z_t$

$$\frac{d^2\sigma_\pi(p_{T_t}, z_t)}{dp_{T_t} dz_t} = \frac{1}{z_t}\frac{A}{(p_{T_t}/z_t)^{n-1}} \times D^q_\pi(z_t)$$

$$= \frac{A}{p^{n-1}_{T_t}} \times z_t^{n-2} D^q_\pi(z_t)$$

Fragment spectrum given p_{Tt} is weighted to high z_t by z_t^{n-2}

where $z_{t\min}|_{p_{T_t}} = x_{T_t} \qquad D^q_\pi(z_t) = Be^{-bz_t}$ (<z>=1/b)

Continuing as in PRD **74**, 072002 (2006)

We can integrate over the trigger jet z_t and find the inclusive pion cross section:

$$\frac{1}{p_{T_t}}\frac{d\sigma_\pi}{dp_{T_t}} = \frac{AB}{p_{T_t}^n}\int_{x_{T_t}}^1 dz_t z_t^{n-2} \exp -bz_t \qquad , \tag{8}$$

which can be written as:

$$\frac{1}{p_{T_t}}\frac{d\sigma_\pi}{dp_{T_t}} = \frac{AB}{p_{T_t}^n}\frac{1}{b^{n-1}}\left[\Gamma(n-1, bx_{T_t}) - \Gamma(n-1, b)\right] \qquad , \tag{9}$$

where

$$\Gamma(a,x) \equiv \int_x^\infty t^{a-1}\, e^{-t}\, dt \tag{10}$$

is the Complementary or upper Incomplete Gamma function, and $\Gamma(a,0) = \Gamma(a)$ is the Gamma function, where $\Gamma(a) = (a-1)!$ for a an integer.

A reasonable approximation for small x_T values is obtained by taking the lower limit of Eq. 8 to zero and the upper limit to infinity, with the result that:

$$\boxed{\frac{1}{p_{T_t}}\frac{d\sigma_\pi}{dp_{T_t}} \approx \frac{\Gamma(n-1)}{b^{n-1}}\frac{AB}{p_{T_t}^n}}$$

Bjorken parent-child relation: parton and particle invariant p_T spectra have same power n

$$\boxed{\langle z_t(p_{T_t})\rangle = \frac{\int_{x_{T_t}}^1 dz_t z_t^{n-1} \exp -bz_t}{\int_{x_{T_t}}^1 dz_t z_t^{n-2} \exp -bz_t} = \frac{1}{b}\frac{\left[\Gamma(n, bx_{T_t}) - \Gamma(n,b)\right]}{\left[\Gamma(n-1, bx_{T_t}) - \Gamma(n-1,b)\right]} \approx \frac{n-1}{b}}$$

Inclusive high p_T particle has n-1 times larger <z> than unbiased fragmentation, <z>=1/b

2 particle Correlations

$$\frac{d^2\sigma_\pi(\hat{p}_{T_t}, z_t)}{d\hat{p}_{T_t} dz_t} = \frac{d\sigma_q}{d\hat{p}_{T_t}} \times D^q_\pi(z_t)$$

Prob. that you make a jet with $\hat{p}_{Tt}$ which fragments to a π with $z_t = p_{Tt}/\hat{p}_{Tt}$

Also detect fragment with $z_a = p_{T_a}/\hat{p}_{T_a}$
from away jet with $\hat{p}_{T_a}/\hat{p}_{T_t} \equiv \hat{x}_h$

$$\frac{d^3\sigma_\pi(\hat{p}_{T_t}, z_t, z_a)}{d\hat{p}_{T_t} dz_t dz_a} = \frac{d\sigma_q}{d\hat{p}_{T_t}} \times D^q_\pi(z_t) \times D^q_\pi(z_a)$$

Prob. that away jet with $\hat{p}_{Ta}$ fragments to a π with $z_a = p_{Ta}/\hat{p}_{Ta}$

$$z_a = \frac{p_{T_a}}{\hat{p}_{T_a}} = \frac{p_{T_a}}{\hat{x}_{\rm h}\, \hat{p}_{T_t}} = \frac{z_t\, p_{T_a}}{\hat{x}_{\rm h}\, p_{T_t}}$$

(1)
$$\boxed{\frac{d\sigma_\pi}{dp_{T_t} dz_t dp_{T_a}} = \frac{1}{\hat{x}_{\rm h}\, p_{T_t}} \frac{d\sigma_q}{d(p_{T_t}/z_t)} D^q_\pi(z_t) D^q_\pi\left(\frac{z_t p_{T_a}}{\hat{x}_{\rm h} p_{T_t}}\right)}$$

Appears to be sensitive to away jet Frag. Fn.

Amazingly, I got a neat analytical result

$$\boxed{\frac{d^3\sigma_\pi}{dp_{T_t}dz_t dp_{T_a}} = \frac{1}{\hat{x}_{\rm h}\, p_{T_t}} \frac{d\sigma_q}{d(p_{T_t}/z_t)} D_q^\pi(z_t) D_q^\pi\left(\frac{z_t p_{T_a}}{\hat{x}_{\rm h} p_{T_t}}\right)} \qquad (1)$$

Take: $D(z) = B\exp(-bz)$ $\qquad \frac{d\sigma_q}{d\hat{p}_{T_t}} = \frac{A}{\hat{p}_{T_t}^{n-1}} = A\frac{{z_t}^{n-1}}{p_{T_t}^{n-1}}$

$$(2) \qquad \frac{d^2\sigma_\pi}{dp_{T_t}dp_{T_a}} = \frac{B^2}{\hat{x}_h} \frac{A}{p_{T_t}^n} \int_{x_{T_t}}^{\hat{x}_h \frac{p_{T_t}}{p_{T_a}}} dz_t z_t^{n-1} \exp[-bz_t(1 + \frac{p_{T_a}}{\hat{x}_h p_{T_t}})]$$

$$\frac{d\sigma_\pi}{dp_{T_t}} = \frac{AB}{p_{T_t}^{n-1}} \int_{x_{T_t}}^{1} dz_t z_t^{n-2} \exp -bz_t$$

Using: $\Gamma(a,x) \equiv \int_x^\infty t^{a-1} e^{-t}\, dt$ $\qquad$ Where Γ(a,0)= Γ(a)=(a-1) Γ(a)

The final result

$$\frac{d^2\sigma_\pi}{dp_{T_t}dp_{T_a}} \approx \frac{\Gamma(n)}{b^n}\frac{B^2}{\hat{x}_h}\frac{A}{p_{T_t}^n}\frac{1}{(1+\frac{p_{T_a}}{\hat{x}_h p_{T_t}})^n}$$

$$\frac{d\sigma_\pi}{dp_{T_t}} \approx \frac{\Gamma(n-1)}{b^{n-1}}\frac{AB}{p_{T_t}^{n-1}}$$

$$\left.\frac{dP_\pi}{dp_{T_a}}\right|_{p_{T_t}} \approx \frac{B(n-1)}{b p_{T_t}}\frac{1}{\hat{x}_h}\frac{1}{(1+\frac{p_{T_a}}{\hat{x}_h p_{T_t}})^n}$$

In the collinear limit, where $p_{T_a} = x_E p_{T_t}$:

$$\left.\frac{dP_\pi}{dx_E}\right|_{p_{T_t}} \approx \frac{B(n-1)}{b}\frac{1}{\hat{x}_h}\frac{1}{(1+\frac{x_E}{\hat{x}_h})^n}$$

Where B/b≈<m>≈b is the mean charged multiplicity in the jet

Why dependence on the Frag. Fn. vanishes

- The only dependence on the fragmentation function is in the normalization constant B/b which equals <m>, the mean multiplicity in the away jet from the integral of the fragmentation function.

- The dominant term in the x_E distribution is the Hagedorn function $1/(1+x_E/\hat{x}_h)^n$ so that at fixed p_{Tt} the x_E distribution is predominantly a function only of x_E and thus exhibits x_E scaling, as observed.

- The reason that the x_E distribution is not sensitive to the shape of the fragmentation function is that the integral over z_t in (1, 2) for fixed p_{Tt} and p_{Ta} is actually an integral over jet transverse momentum $\hat{p}_{T_t}$. However since the trigger and away jets are always roughly equal and opposite in transverse momentum (in p+p), integrating over $\hat{p}_{T_t}$ simultaneously integrates over $\hat{p}_{T_a}$. The integral is over z_t, which appears in both trigger and away side fragmentation functions in (1).