Statistical errors, efficiency and acceptance corrections in cumulants of measured net-charge (N^+ - N^-) distributions, a theorem from Quantitative Finance and NBD fits to the PHENIX N^+ and N^- distributions (New PHENIX results on T and μ_B at freezeout)

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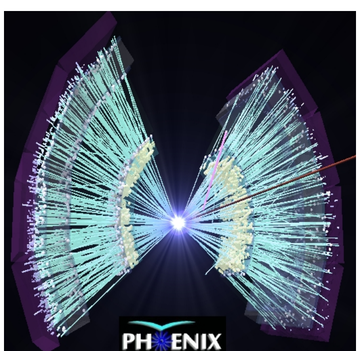
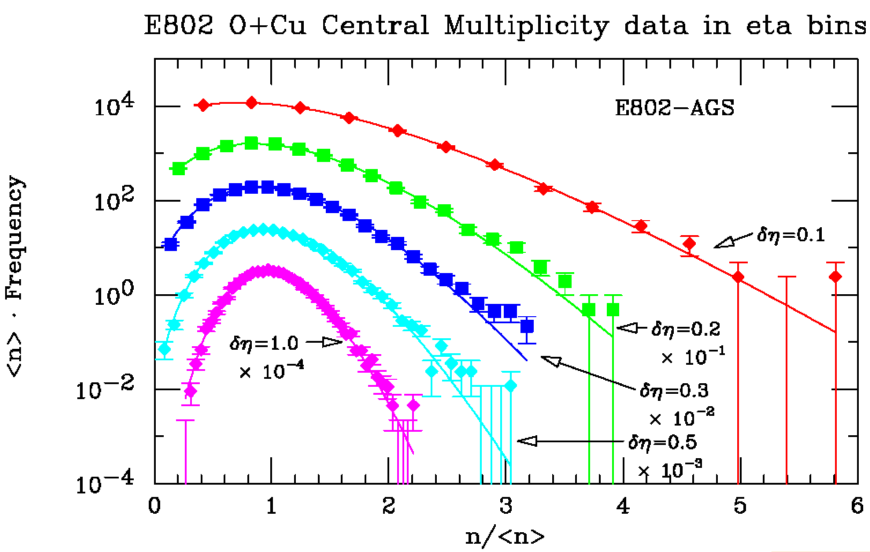


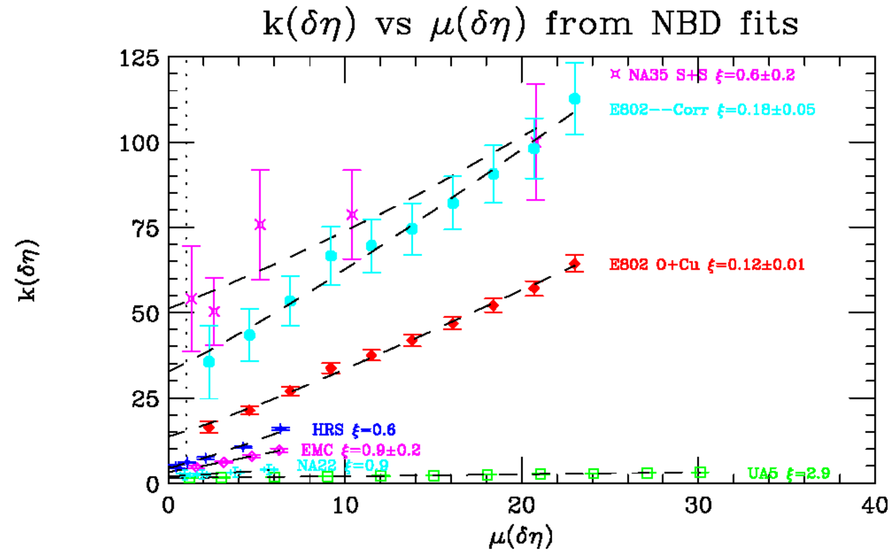
Zimanyi School '15
Budapest Hungary
December 2015

NBD-p+p discoveryUA5 PLB **160**, 193,199 (1985); **167**, 476 (1986)

NBD in O+Cu central collisions at AGS vs Δη central collisions defined by zero spectators (ZDC) Correlations due to to B-E don't vanish

E802 PRC **52**, 2663 (1995)





$$k(\delta\eta) \sim \frac{(\delta\eta/\xi)^2}{(\delta\eta/\xi - 1 + e^{-\delta\eta/\xi})}$$

Negative Binomial correlation= 1/k

Poisson, no correlation

$$\frac{\sigma^2}{\mu^2} - \frac{1}{\mu} = 0$$

$$\frac{\sigma^2}{\mu^2} - \frac{1}{\mu} = \frac{1}{k}$$

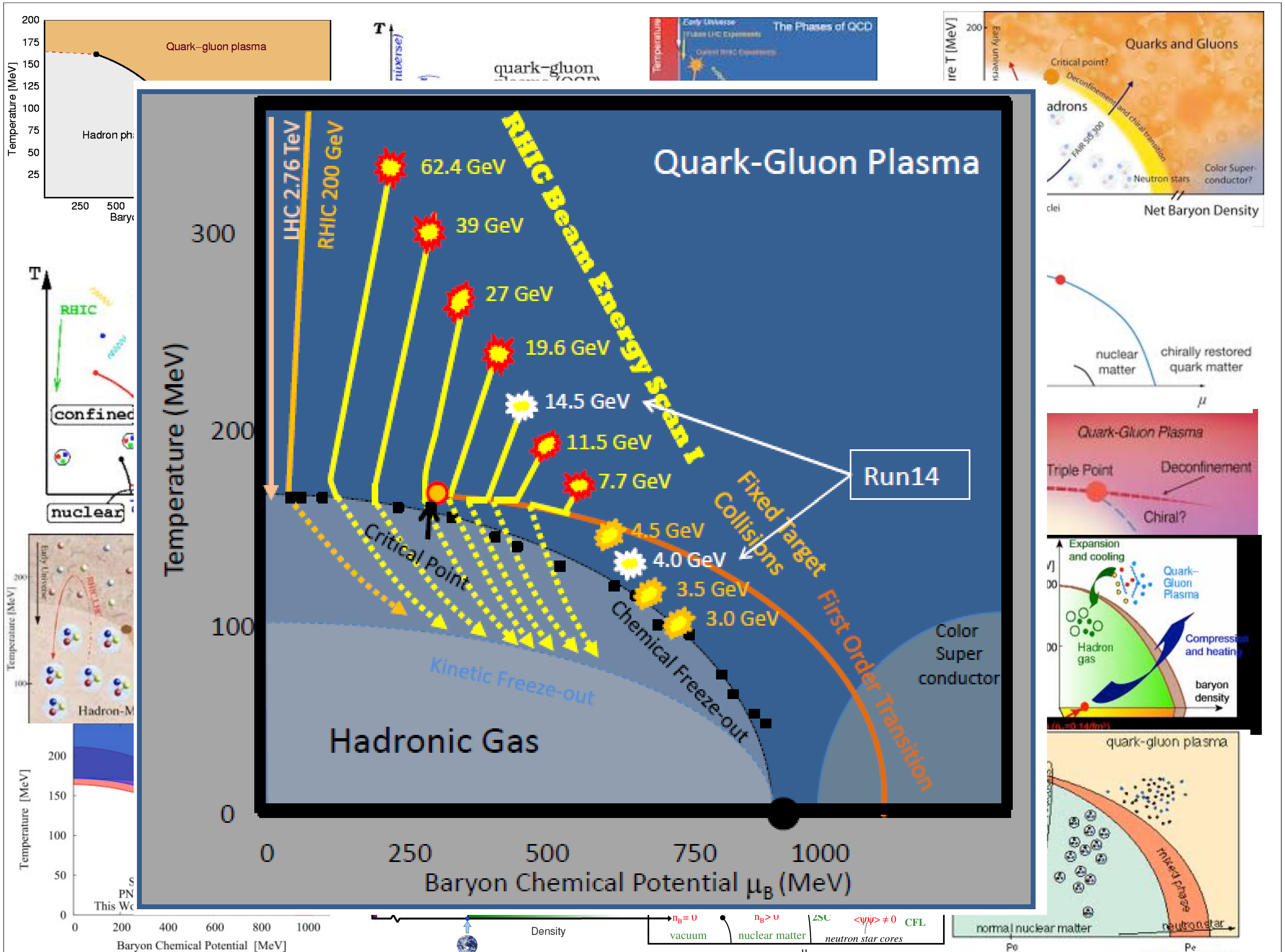
The rapidity correlation length ξ = 0.2 for O+Cu is from B-E.

E802, PRC56(1977) 1544

BEAM Energy Scan
Search for Critical Endpoint in Nuclear Matter Phase diagram Helped by Lattice QCD

Proposed Phase diagrams Nuclear matter

Pawlowski-QM2014



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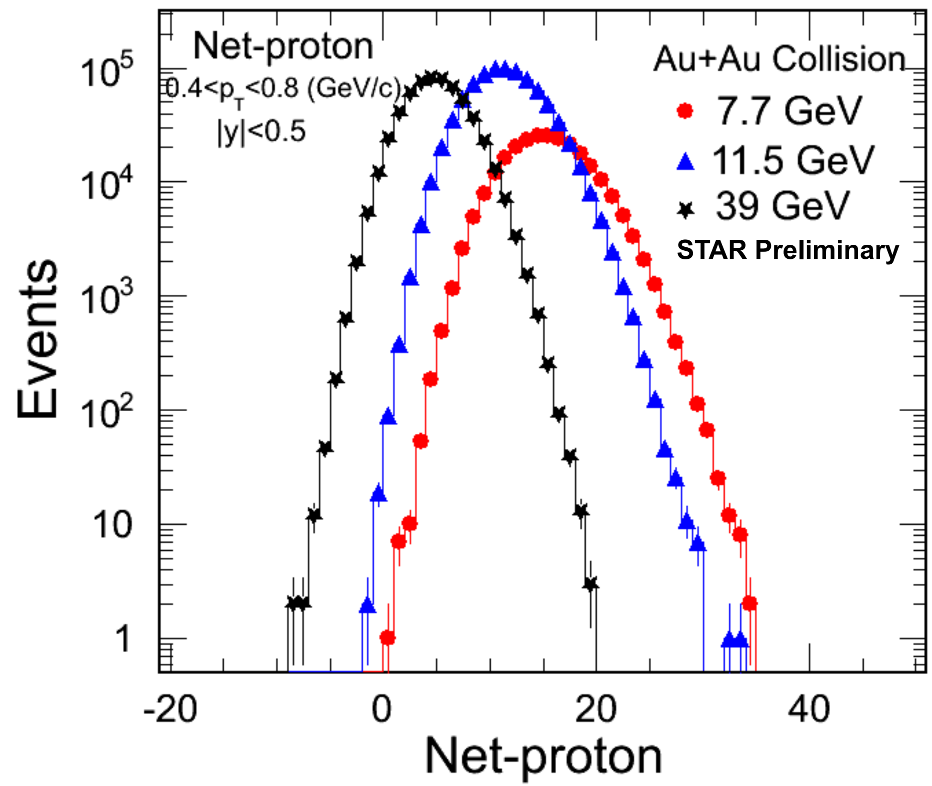
Hot off the presses-LBL Press release June 24,2011 Higher Moments of Net-Proton Distributions

- 1^{st} moment: mean = $\mu=\langle x\rangle$
- 2^{nd} cumulant: variance $\kappa_2=\sigma^2=\langle(x-\mu)^2\rangle$
- 3^{rd} cumulant: $\kappa_3=\mu_3=\langle(x-\mu)^3\rangle$
- 3^{rd} standardized cumulant: skewness = $S=\kappa_3/\kappa_2^{3/2}=\langle(x-\mu)^3\rangle/\sigma^3$
- 4^{th} cumulant: $\kappa_4=\langle(x-\mu)^4\rangle-3\kappa_2^2$
- 4^{th} standardized cumulant: kurtosis = $\kappa=\kappa_4/\kappa_2^2=\{\langle(x-\mu)^4\rangle/\sigma^4\}-3$
- Calculate moments from the event-by-event net proton distribution.
  - ✓ Have similar plots for net-charge and net-kaon distributions.



MJT-If you know the distribution, you know all the moments, but statistical mechanics and Lattice QCD use Taylor expansions, hence moments/cumulants

Statistical Mechanics uses derivatives of the free energy to find susceptibilities

- Theoretical analyses tend to be made in terms of a Taylor expansion of the free energy $F=-T \ln Z$ around the critical temperature T_c where Z is the partition function or sum over states, $Z \approx \exp -[(E-\Sigma_i \mu_i Q_i)/kT]$ and μ_i chemical potentials associated with conserved charges Q_i

- The terms of the Taylor expansion are called susceptibilities or $\chi_{(m)}$ which are proportional to the correlation length, e.g. $\chi_{(3)} \sim \xi^6$, $\chi_{(4)} \sim \xi^8$

- The only connection of this method to mathematical statistics is that the Cumulant generating function is also a Taylor expansion of the ln of an exponential:

$$g_x(t) = \ln \langle e^{tx} \rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!} \qquad \kappa_m = \left. \frac{d^m g_x(t)}{dt^m} \right|_{t=0}$$

If you measure the distribution, then you know all the cumulants

Cumulants for Poisson, Binomial and Negative Binomial Distributions

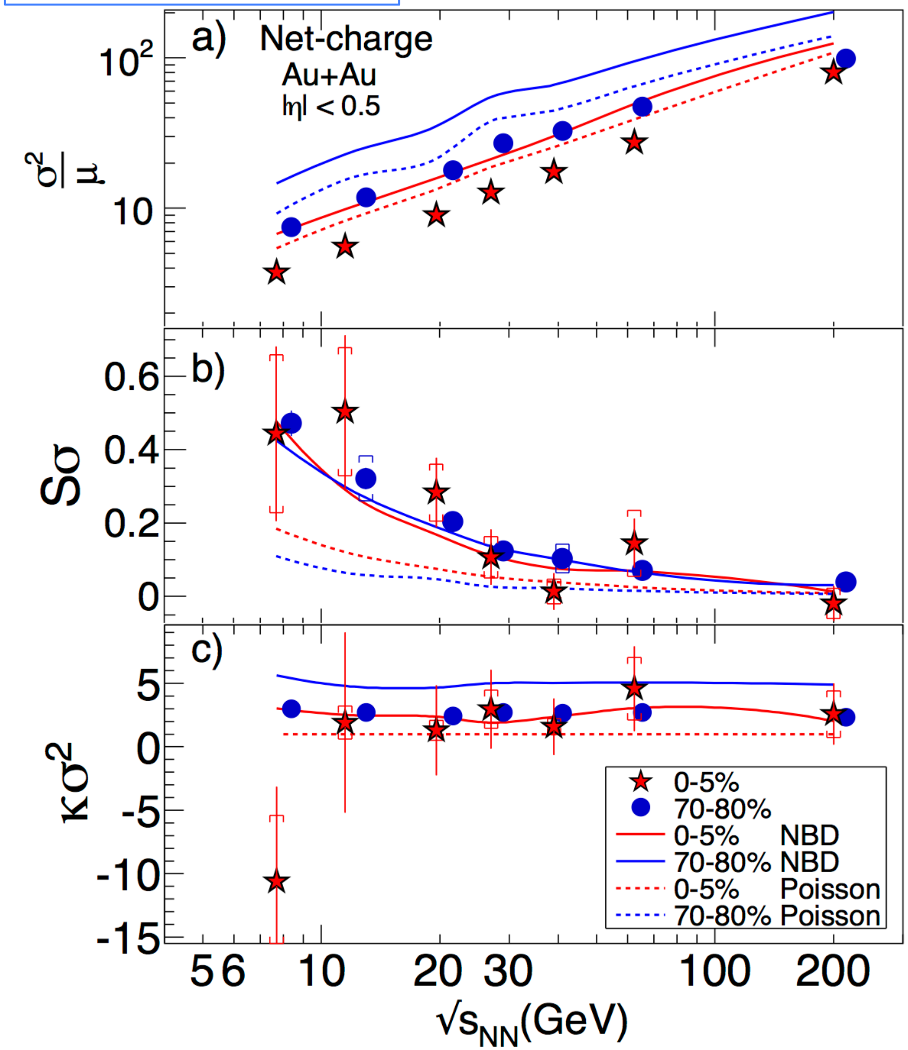
Cumulant	Poisson	Binomial	Negative Binomial
$\kappa_1 = \mu$	μ	np	μ
$\kappa_2 = \mu_2 = \sigma^2$	μ	$\mu(1-p)$	$\mu(1+\mu/k)$
$\kappa_3 = \mu_3$	μ	$\sigma^2(1-2p)$	$\sigma^2(1+2\mu/k)$
$\kappa_4 = \mu_4 - 3\kappa_2^2$	μ	$\sigma^2(1-6p+6p^2)$	$\sigma^2(1+6\mu/k+6\mu^2/k^2)$
$S \equiv \kappa_3/\sigma^3$	$1/\sqrt{\mu}$	$(1-2p)/\sigma$	$(1+2\mu/k)/\sigma$
$\kappa \equiv \kappa_4/\kappa_2^2$	$1/\mu$	$(1-6p+6p^2)/\sigma^2$	$(1+6\mu/k+6\mu^2/k^2)/\sigma^2$
$S\sigma = \kappa_3/\kappa_2$	1	$(1-2p)$	$(1+2\mu/k)$
$\kappa\sigma^2 = \kappa_4/\kappa_2$	1	$(1-6p+6p^2)$	$(1+6\mu/k+6\mu^2/k^2)$

Thanks to Gary Westfall of STAR in a paper presented at Erice-International School of Nuclear Physics 2012, I found out that the cumulants of the difference of samples from two such distributions P(n-m) where $P^+(n)$ and $P^-(m)$ are both Poisson, Binomial or NBD with Cumulants κ_j^+ and κ_j^- respectively is the same as if they were statistically independent, so long as they are not 100% correlated. I call this the NBD Cumulant Theorem

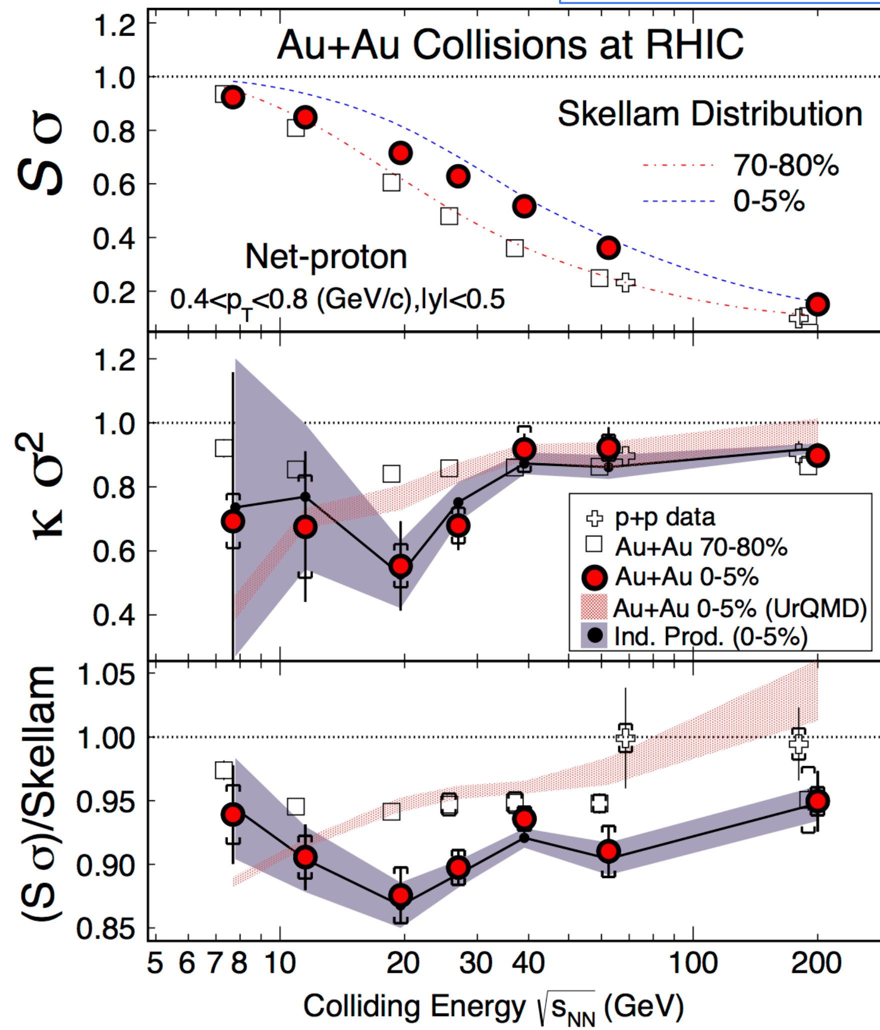
$$\kappa_j = \kappa_j^+ + (-1)^j \kappa_j^-$$

STAR publications 2014

PRL 113(2014) 092301



PRL 112(2014) 032302

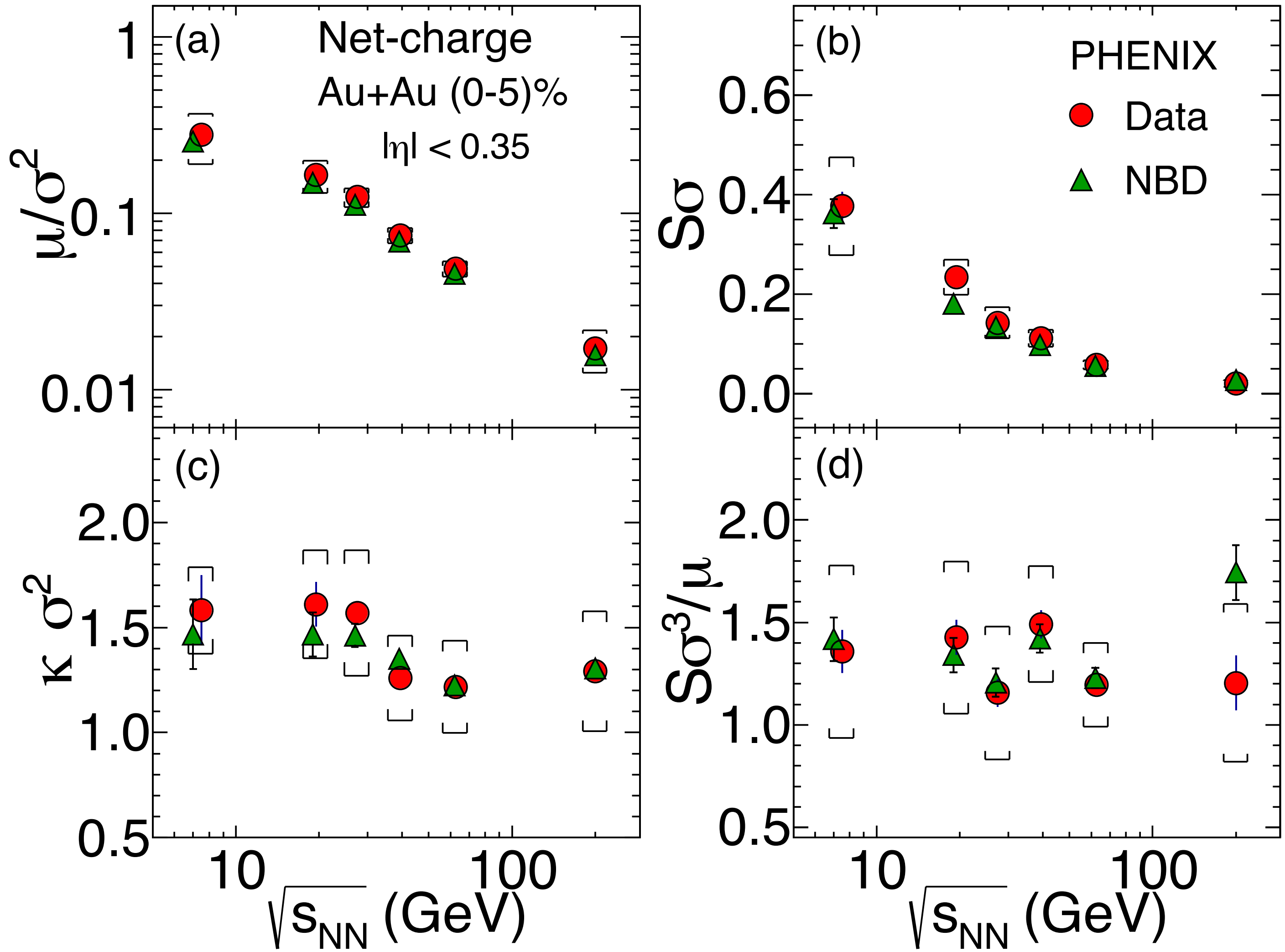


$S\sigma$ clearly favors NBD, not Poisson (!).
No non- monotonic behavior in $S\sigma$ or $\kappa\sigma^2$
but $\kappa\sigma^2$=-1.5 at $\sqrt{s_{NN}}$ =20 can't be ruled out

$\kappa\sigma^2$=-1.5 at $\sqrt{s_{NN}}$ =20 **can** be ruled out
$\kappa\sigma^2$ changes for $\sqrt{s_{NN}}$ ≤20 GeV but
antiprotons become negligible <0.1 p

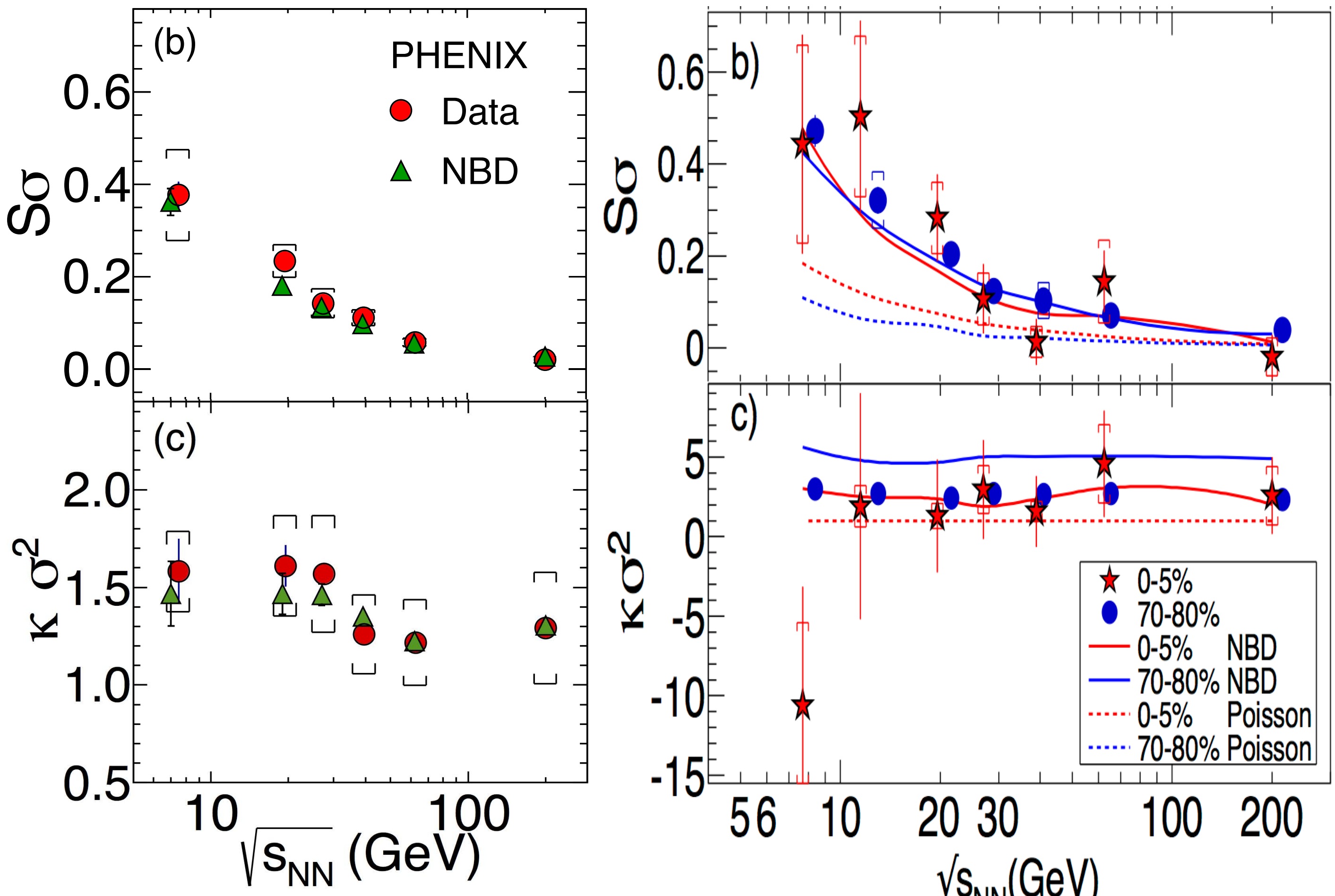
New PHENIX central cumulant ratios vs $\sqrt{s_{NN}}$

PHENIX arXiv:1506.07834

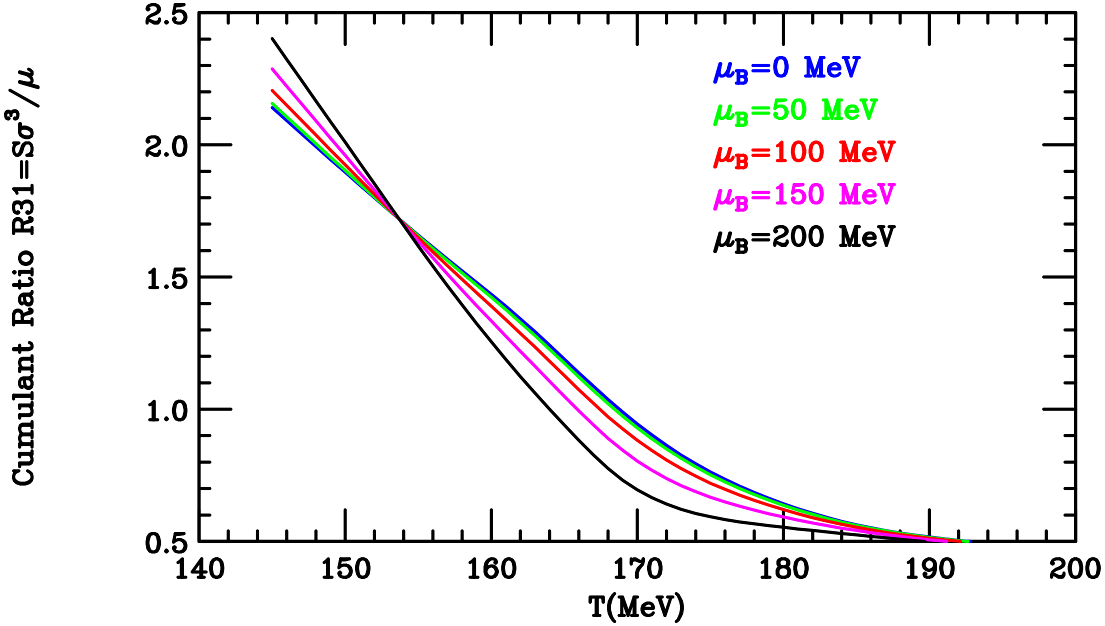


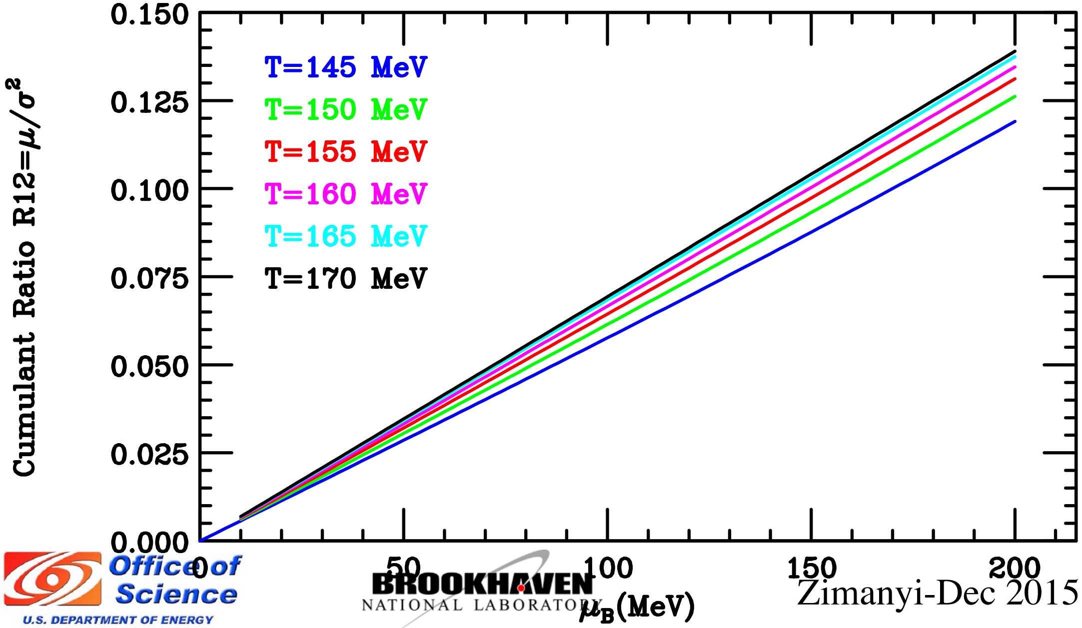
Note that the ``data'' • calculations from the $\Delta N_{ch}=N^+ - N^-$ distributions agree with the NBD fits to the N^+ and N^- distribution and the NBD Cumulant Theorem.

PHENIX and STAR comparison!!!



The key difference of the PHENIX and STAR results is that the error on all corrected cumulant ratios is 20-30% for PHENIX while for STAR the error on e.g. $S\sigma$ is ~ 50%, on $\kappa\sigma^2$ is >100% but <1% for σ^2/μ!!! (which turns out to be important)

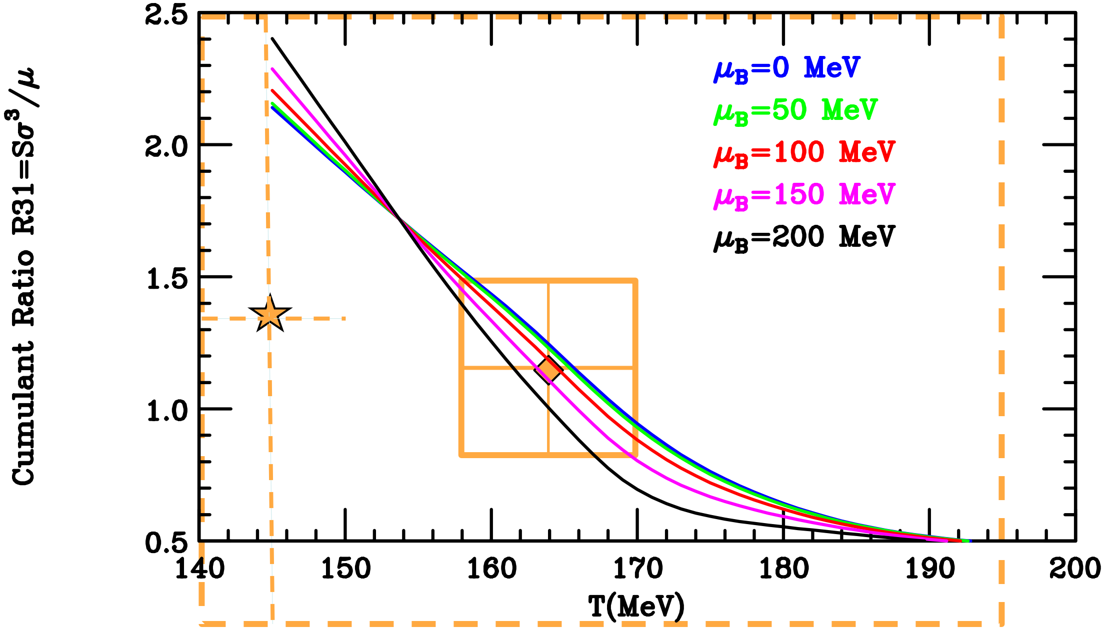


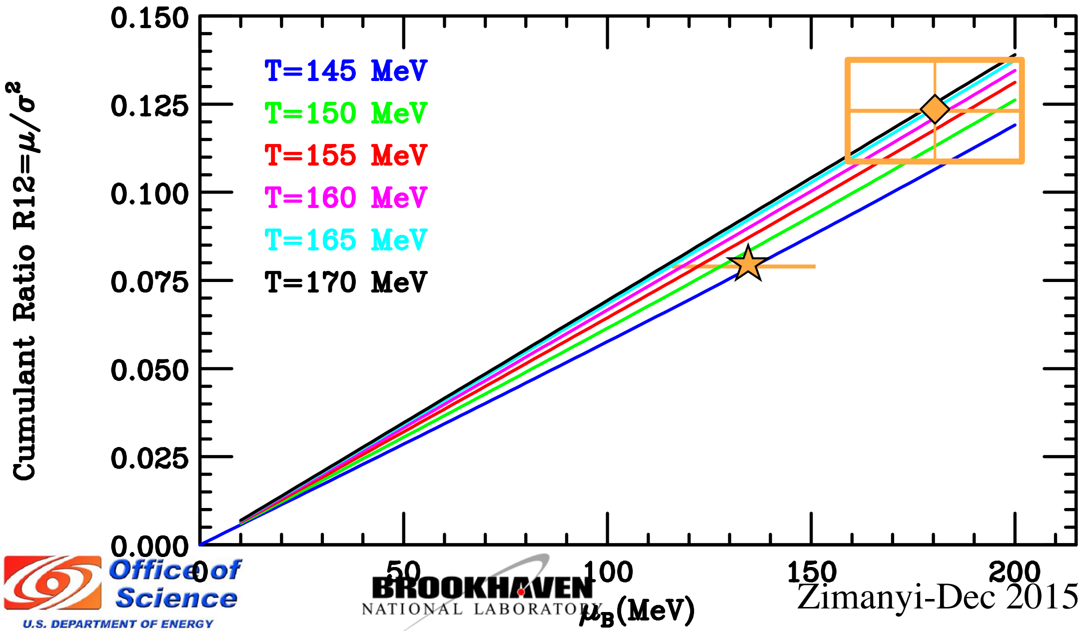


BNL Lattice QCD group predictions for cumulant ratios κ_3/κ_1=R31 and κ_1/κ_2=R12 vs T_f and μ_B at freezout (when QGP hadronizes)

PoS(CPOD2014)005.

PRL **109** (2012) 192302

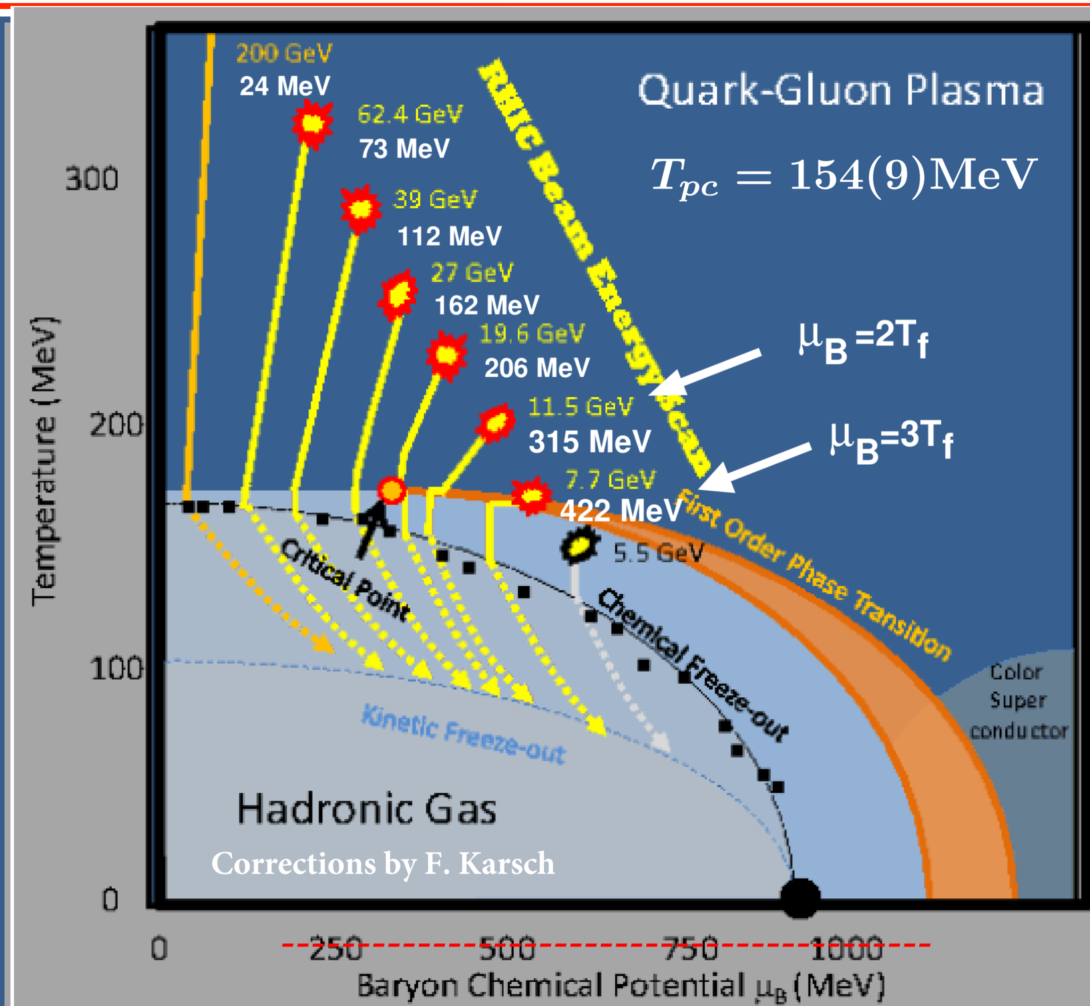




STAR☆ measurement of R31= κ_3 / κ_1 has such a huge error that the central ☆ could go anywhere in the dashed region, while R12 has such a small error that it is constrained to the region of the horizontal line by the assumption 140<T_f<150MeV
PRL **113** (2014) 052301

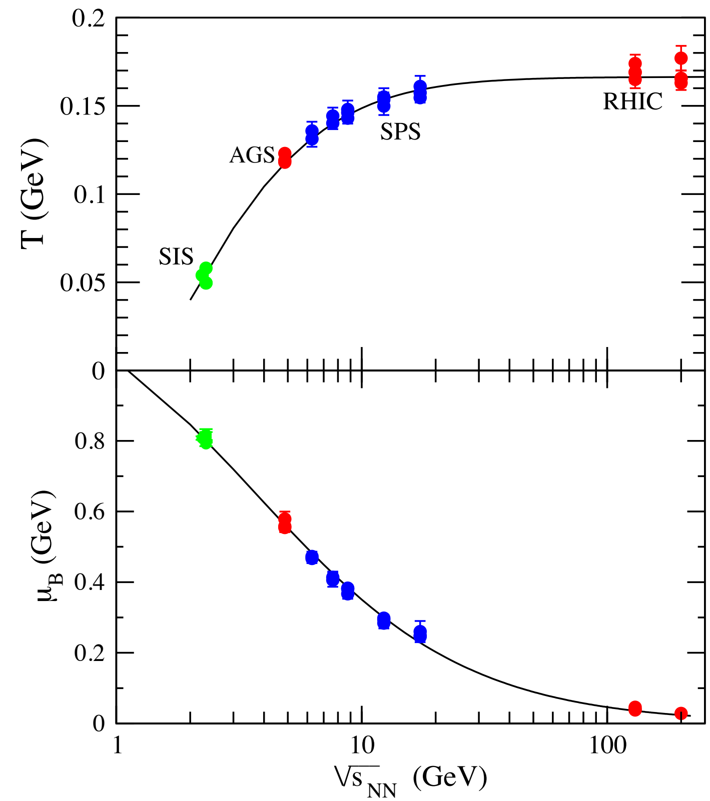
PHENIX ◇ measurement with comparable errors on R31 and R12 enables both T_f and μ_B to be determined from the Lattice QCD calculations: Pos(CPOD2014)005

STAR's opinion of PHASE diagram 2014



T=160 MeV
μ_B=300 MeV
$\sqrt{s_{NN}}$~27 GeV

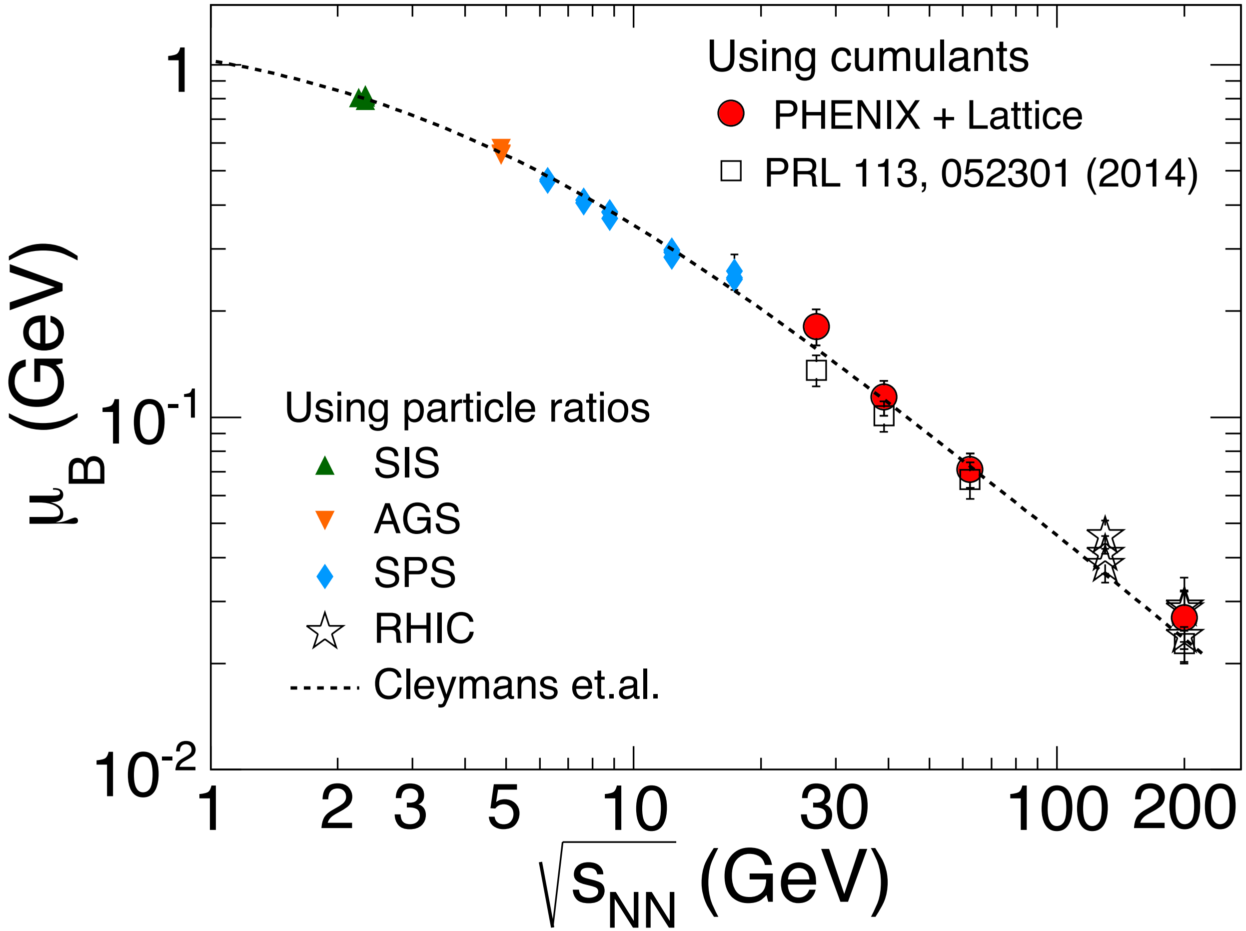
$$\frac{\bar{p}}{p} \approx 0.02$$



Cleymans,Oeschler *et al.*
PRC73(2006)034905

$$\frac{\bar{p}}{p} = \frac{e^{-(E+\mu_B)/T}}{e^{-(E-\mu_B)/T}} = e^{-(2\mu_B)/T}$$

NEW: Experiment +Theory=Physical Quantity



Experimental result on net-charge cumulants + Lattice QCD calculation gives both freezeout T_f+Baryon Chemical Potential μ_B without particle identification!! I think this is a first and it also agrees with the best accepted calculations from baryon/anti-baryon ratios, PRC73(2006)034905

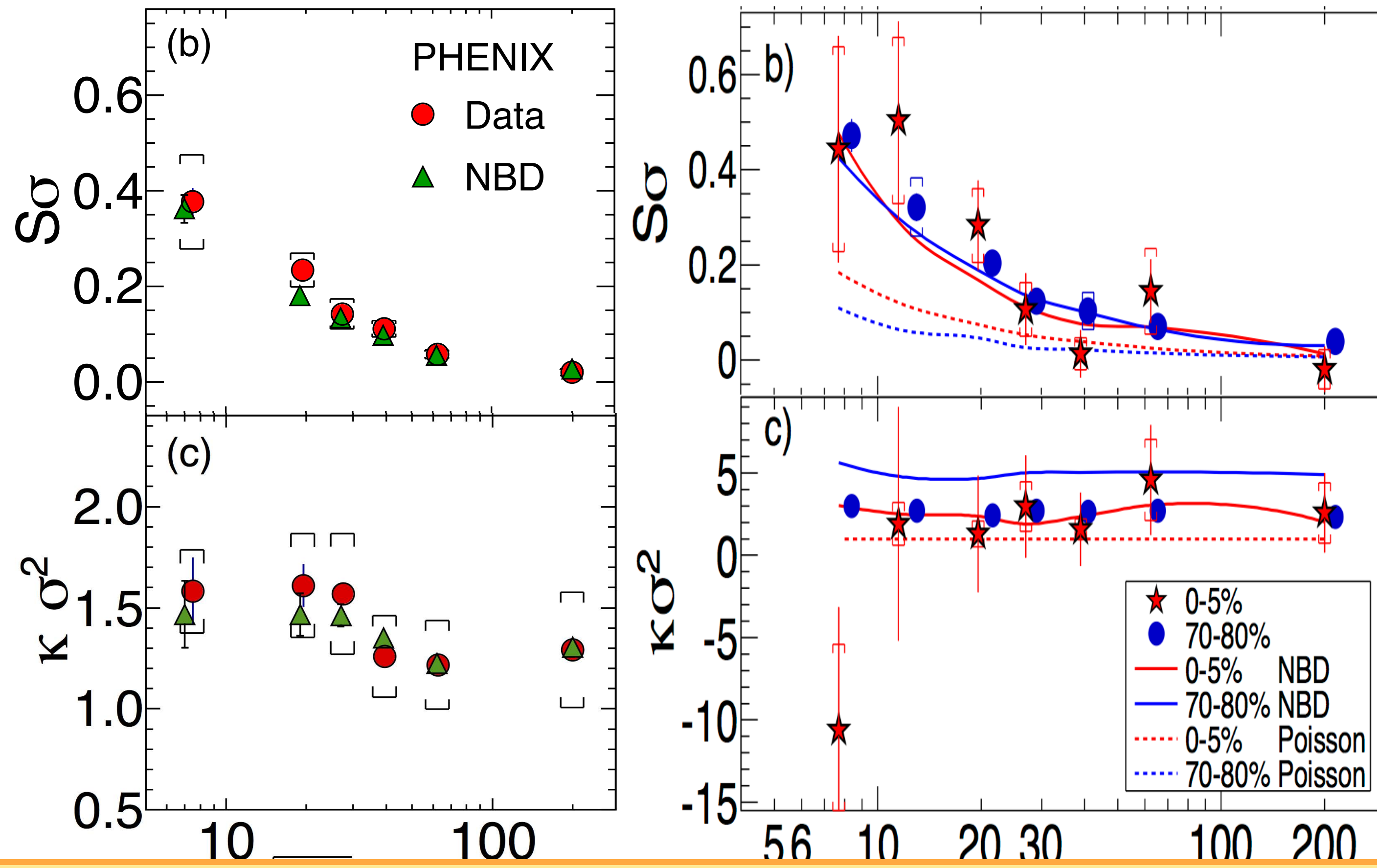
PHENIX T_f μ_B from net-ch measurement cf. Calculation using STAR net-ch and net-p data

$\sqrt{s_{NN}}$ (GeV)	T_f (MeV)	μ_B (MeV)	μ_B (MeV)**
27	164 ± 6	181 ± 21	136 ± 13.8
39	158 ± 5	114 ± 13	101 ± 10
62.4	163 ± 5	71 ± 8	66.6 ± 7.9
200	163 ± 8	27 ± 5	22.8 ± 2.6

****S. Borsanyi et al., Phys. Rev. Lett. 113, 052301 (2014) used T_f=145 ± 5 MeV from STAR net-proton data averaged over above 4 $\sqrt{s_{NN}}$, μ_B from STAR net-charge R_{12}**

PHENIX and STAR comparison!!!

PHENIX arXiv:1506.07834



STAR arXiv: 1402.1558

The key difference of the PHENIX and STAR results is that the error on all corrected cumulant ratios is 20-30% for PHENIX while for STAR the error on e.g. Sσ is ~ 50%, on $\kappa\sigma^2$ is >100% but <1% for σ^2/μ!!! WHY?

Why are STAR errors on R31 so large?

It must be that statistical errors and efficiency corrections are a BIG issue in these measurements even though the correction is simply Binomial; and analytical for NBD N^+ and N^- distributions (**k unchanged, $\mu_t=\mu/p$ where p is the efficiency**). So use the NBD "integer value Levy process" cumulant theorem:

Tarnowsky, Westfall PLB 724 (2013) 51

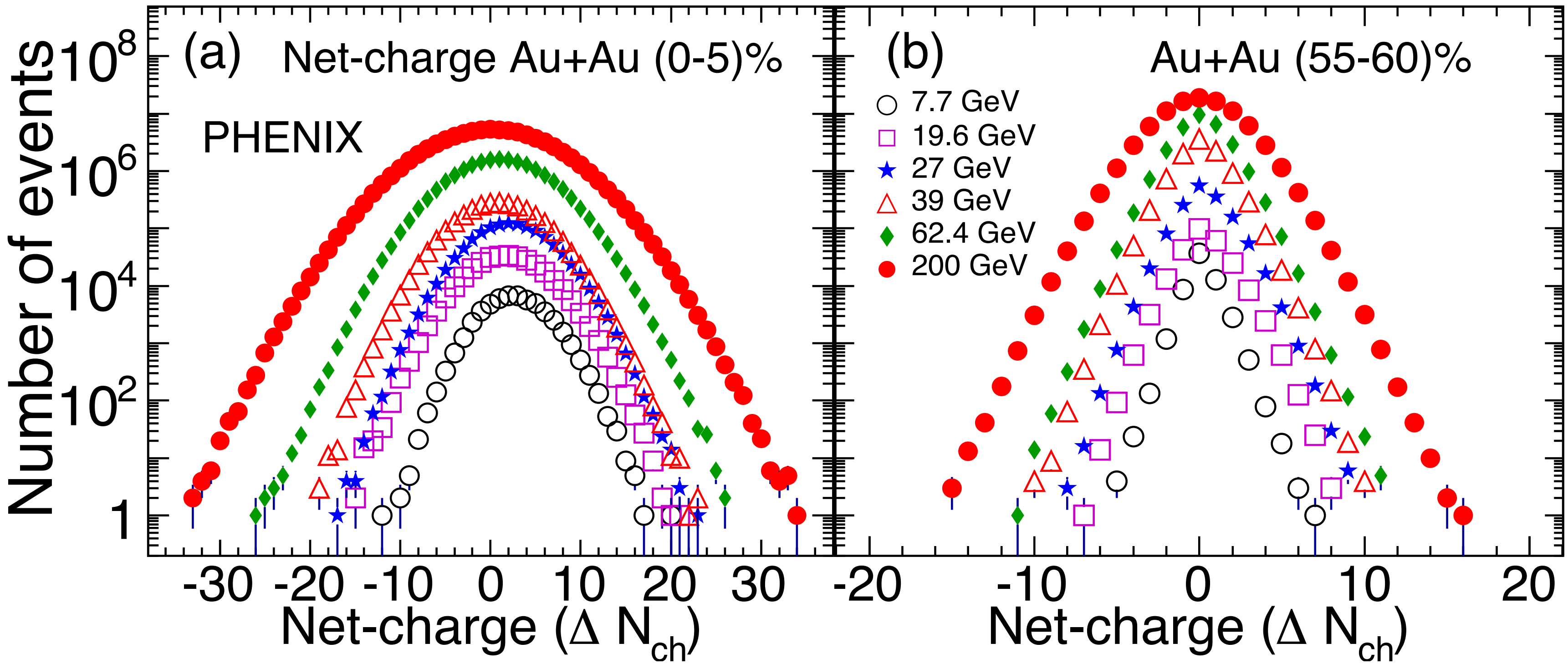
Barndorff-Nielsen,Pollard,Shephard

http://www.economics.ox.ac.uk/materials/papers/4382/paper490.pdf

$$\kappa_j = \kappa_j^+ + (-1)^j \kappa_j^-$$

From PHENIX net-charge fluctuations

PHENIX arXiv:1506.07834



$\Delta N_{ch}=N^+ - N^-$ distribution in $|\eta|<0.35$, $\delta\phi=\pi$, $0.3<p_T<2.0$ GeV/c
Not corrected for detection efficiency $\varepsilon\approx0.70$ in acceptance

The raw moments of the uncorrected distributions can be easily calculated

$$\mu'_k \equiv \langle x^k \rangle \equiv \sum_{i=1}^{n} x_i^k E(x_i) / \sum_{i=1}^{n} E(x_i)$$

$\mu'_1 \equiv \mu = \langle x \rangle$ and x_i is a bin in the $\Delta N_{\rm ch}$ plot with $E(x_i)$ events.

Statistical errors--the complications begin

$$\mu'_k \equiv \langle x^k \rangle \equiv \sum_{i=1}^{n} x_i^k E(x_i) / \sum_{i=1}^{n} E(x_i)$$

The statistical errors for every μ'_k can be calculated from the statistical errors of each data point $E(x_i) \pm \sigma_{E(x_i)}$. Even though the $\sigma_{E(x_i)}$ on each point are independent, the errors on each μ'_k are not independent because the same $\sigma_{E(x_i)}$ appears in all the moments.

Next one computes the cumulants κ_i from the raw (aka) non-central moments:

$$\begin{aligned}
\mu = \kappa_1 &= \mu'_1 \\
\sigma^2 = \mu_2 \equiv \langle (x-\mu)^2 \rangle = \kappa_2 &= \mu'_2 - \mu'^2_1 \\
\mu_3 = \kappa_3 &= \mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1 \\
\mu_4 - 3\mu_2^2 = \kappa_4 &= \mu'_4 - 4\mu'_3\mu'_1 - 3\mu'^2_2 + 12\mu'_2\mu'^2_1 - 6\mu'^4_1
\end{aligned}$$

Next correction---Efficiency

A certain random fraction of the tracks that fall on the acceptance are not detected because of inefficiency---a clearly random, thus binomial effect. This is further complicated if the N^+ and N^- measurements have different efficiencies.

Long Range Correlations: Binomial Split of NBD

Carruthers and Shih PLB 165 (1985)209

If a population n is distributed as NBD(μ_t, k) and then divided randomly into 2 subpopulations with probabilities p and $q=1-p$, then the distribution on p is NBD ($p\mu_t$, k) and on q is NBD ($q\mu_t$, k) BUT the two sub-intervals are not statistically independent. Also k does not change!!

So if you measure $\mu=p\mu_t$ with effieicncy p the true value is $\mu_t=\mu/p$

Bzdak-Koch standard Binomial efficiency correction PRC 86 (2012) 044904

Efficiency corrected cumulants in terms of corrected double Factorial moments

$$N = \langle N_+ \rangle + \langle N_- \rangle$$

$$F_{ik} = \sum_{N_1=i}^{\infty} \sum_{N_2=k}^{\infty} P(N_1, N_2) \frac{N_1!}{(N_1-i)!} \frac{N_2!}{(N_2-k)!}$$

$$\kappa_1 = \langle N_+ \rangle - \langle N_- \rangle = \frac{\langle n_+ \rangle}{\epsilon_+} - \frac{\langle n_- \rangle}{\epsilon_-},$$

$$\kappa_2 = N - \kappa^2{}_1 + F_{02} - 2F_{11} + F_{20},$$

$$\kappa_3 = \kappa_1 + 2\,\kappa^3{}_1 - F_{03} - 3F_{02} + 3\,F_{12} + 3\,F_{20} - 3F_{21} + F_{30}$$
$$- 3\kappa_1(N + F_{02} - 2F_{11} + F_{20}),$$

$$\kappa_4 = N - 6\kappa^4{}_1 + F_{04} + 6\,F_{03} + 7\,F_{02} - 2F_{11} - 6F_{12} - 4F_{13}$$
$$+ 7F_{20} - 6F_{21} + 6F_{22} + 6F_{30} - 4F_{31} + F_{40}$$
$$+ 12\kappa^2{}_1(N + F_{02} - 2F_{11} + F_{20}) - 3(N + F_{02} - 2F_{11} + F_{20})^2$$
$$- 4\kappa_1(\kappa_1 - F_{03} - 3F_{02} + 3\,F_{12} + 3\,F_{20} - 3F_{21} + F_{30})$$

Here you can see the nice subtraction of the lower order moments; but new quantities, double Factorial Moments are introduced and very difficult to compute $P(13^+, 11^-)$=? so you need to know both N_+ and N_- distributions and their correlations. The F_{ik} can be calculated from the data by making a 3d Lego plot with base axes N_+ and N_- and height $P(N_+, N_-)$ which costs statistical error but other methods, e.g Monte Carlo, are used.

If you measure the distribution, then you know all the corrected cumulants

Cumulants for Poisson, Negative Binomial Distributions Measured with **efficiency** p corrected to true value, explicit in μ_t and k

Measured Cumulant	Corrected Poisson	Corrected Negative Binomial Expanded
$\kappa_1 = \mu$	$\mu_t \equiv \mu/p$	$\mu_t \equiv \mu/p$
$\kappa_2 = \mu_2 = \sigma^2$	μ_t	$\mu_t(1+\mu_t/k) \equiv \sigma_t^2$
$\kappa_3 = \mu_3$	μ_t	$\mu_t(1+3\mu_t/k+2\mu_t^2/k^2)$
$\kappa_4 = \mu_4 - 3\kappa_2^2$	μ_t	$\mu_t(1+7\mu_t/k+12\mu_t^2/k^2+6\mu_t^3/k^3)$
$S \equiv \kappa_3/\sigma^3$	$1/\sqrt{\mu_t}$	$(1+2\mu_t/k)/\sqrt{\mu_t(1+\mu_t/k)}$
$\kappa \equiv \kappa_4/\kappa_2^2$	$1/\mu_t$	$(1+6\mu_t/k+6\mu_t^2/k^2)/[\mu_t(1+\mu_t/k)]$
$S\sigma = \kappa_3/\kappa_2$	1	$(1+2\mu_t/k)$
$\kappa\sigma^2 = \kappa_4/\kappa_2$	1	$(1+6\mu_t/k+6\mu_t^2/k^2)$
$\mu/\sigma^2 = \kappa_1/\kappa_2$	1	$1/(1+\mu_t/k)$
$S\sigma^3/\mu = \kappa_3/\kappa_1$	1	$(1+3\mu_t/k+2\mu_t^2/k^2)$

Use the NBD Cumulant Theorem allowing ε=p to be different for N^+ and N^-

$$\kappa_j = \kappa_j^+ + (-1)^j \kappa_j^-$$

Efficiency-Corrected NBD Cumulant Ratios

$$\frac{\mu}{\sigma^2} = \frac{\kappa_1^+ - \kappa_1^-}{\kappa_2^+ + \kappa_2^-} = \frac{\mu_t^+ - \mu_t^-}{\mu_t^+[1 + (\frac{\mu_t^+}{k^+})] + \mu_t^-[1 + (\frac{\mu_t^-}{k^-})]}$$

$$\mu_t = \frac{\mu}{\varepsilon}$$

$$\frac{S\sigma^3}{\mu} = \frac{\kappa_3^+ - \kappa_3^-}{\kappa_1^+ - \kappa_1^-} = \frac{\mu_t^+[1 + 3(\frac{\mu_t^+}{k^+}) + 2(\frac{\mu_t^+}{k^+})^2] - \mu_t^-[1 + 3(\frac{\mu_t^-}{k^-}) + 2(\frac{\mu_t^-}{k^-})^2]}{\mu_t^+ - \mu_t^-}$$

$$S\sigma = \frac{\kappa_3^+ - \kappa_3^-}{\kappa_2^+ + \kappa_2^-} = \frac{\mu_t^+[1 + 3(\frac{\mu_t^+}{k^+}) + 2(\frac{\mu_t^+}{k^+})^2] - \mu_t^-[1 + 3(\frac{\mu_t^-}{k^-}) + 2(\frac{\mu_t^-}{k^-})^2]}{\mu_t^+[1 + \frac{\mu_t^+}{k^+}] + \mu_t^-[1 + \frac{\mu_t^-}{k^-}]}$$

$$\kappa\sigma^2 = \frac{\kappa_4^+ + \kappa_4^-}{\kappa_2^+ + \kappa_2^-} = \frac{\mu_t^+[1 + 7(\frac{\mu_t^+}{k^+}) + 12(\frac{\mu_t^+}{k^+})^2 + 6(\frac{\mu_t^+}{k^+})^3] + \mu_t^-[1 + 7(\frac{\mu_t^-}{k^-}) + 12(\frac{\mu_t^-}{k^-})^2 + 6(\frac{\mu_t^-}{k^-})^3]}{\mu_t^+[1 + \frac{\mu_t^+}{k^+}] + \mu_t^-[1 + \frac{\mu_t^-}{k^-}]}$$

The NBD only uses 4 quantities for this calculation: μ_t^+ and μ_t^- $(\mu_t/k)^+$ and $(\mu_t/k)^-$
The error on μ_t <<than the error on μ_t/k so is neglected. The errors are highly correlated for the sums of powers of μ_t/k in both the numerator and denominator. These correlations are handled by varying the $(\mu_t/k)^+$ and $(\mu_t/k)^-$ by $\pm 1\sigma$ independently and adding the variations in quadrature

The errors of the cumulants and ratios by the direct method remain very complicated

A recent thorough treatment of both statistical errors and efficiency, with even more complicated formulas than Bzdak and Koch is given by Xiaofeng Luo, PRC **91** (2015) 034907 BUT to test the method:

"By deriving the covariance between factorial moments, one can obtain the general error formula for the efficiency corrected moments based on the error propagation derived from the Delta theorem. The Skellam-distribution-based Monto Carlo simulation is used to test the Delta theorem and bootstrap error estimation methods."

I note, of course, that Skellam is the difference between two Poissons so satisfies the integer Levy process theorem! I also note that Bzdak and Koch have not been idle PRC **91**(2015) 027901

Are acceptance corrections possible?

$$\frac{S\sigma^3}{\mu} = \frac{\kappa_3^+ - \kappa_3^-}{\kappa_1^+ - \kappa_1^-} = \frac{\mu_t^+[1 + 3(\frac{\mu_t^+}{k^+}) + 2(\frac{\mu_t^+}{k^+})^2] - \mu_t^-[1 + 3(\frac{\mu_t^-}{k^-}) + 2(\frac{\mu_t^-}{k^-})^2]}{\mu_t^+ - \mu_t^-}$$

$$R_{32} - R_{12} = S\sigma - \frac{\mu}{\sigma^2} = \frac{\mu_t^+[3(\frac{\mu_t^+}{k^+}) + 2(\frac{\mu_t^+}{k^+})^2] - \mu_t^-[3(\frac{\mu_t^-}{k^-}) + 2(\frac{\mu_t^-}{k^-})^2]}{\mu_t^+[1 + \frac{\mu_t^+}{k^+}] + \mu_t^-[1 + \frac{\mu_t^-}{k^-}]}$$

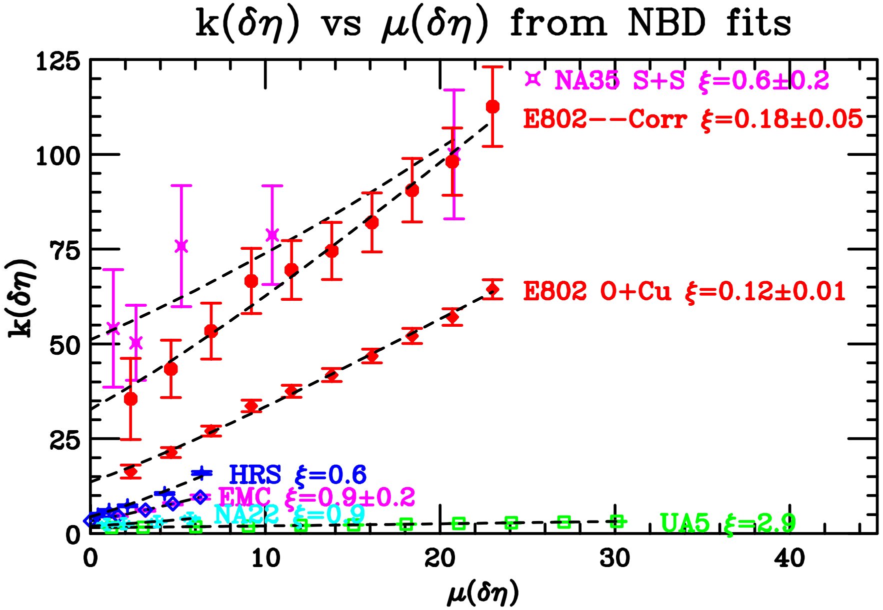
Bzdak and Koch (and likely many others) have expressed concern about what is the “required acceptance” for an experimental result e.g. on the above quantities to compare with Lattice QCD calculations

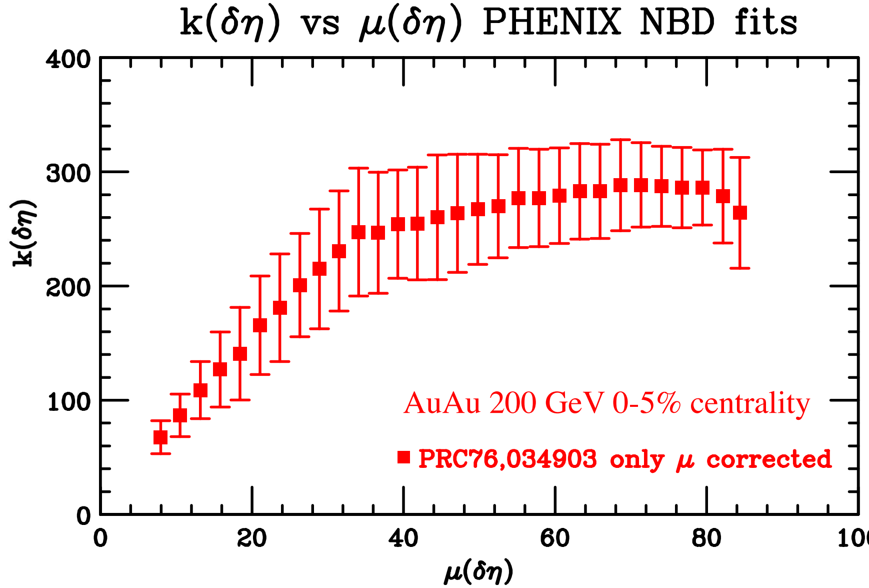
The good news from the above equations and those on the previous pages is that if the ratios $(\mu_t/k)^+$ and $(\mu_t/k)^-$ don’t change with the acceptance and if μ_t^+ and μ_t^- scale by the same amount with the acceptance (e.g. dn/dη constant in rapidity and azimuth) then the above formulas remain unchanged. What does nature say?

Recall the NBD slide from E802

E802 PRC**52**,2663(1995)

PHENIX PRC**76**,0349033(2007)



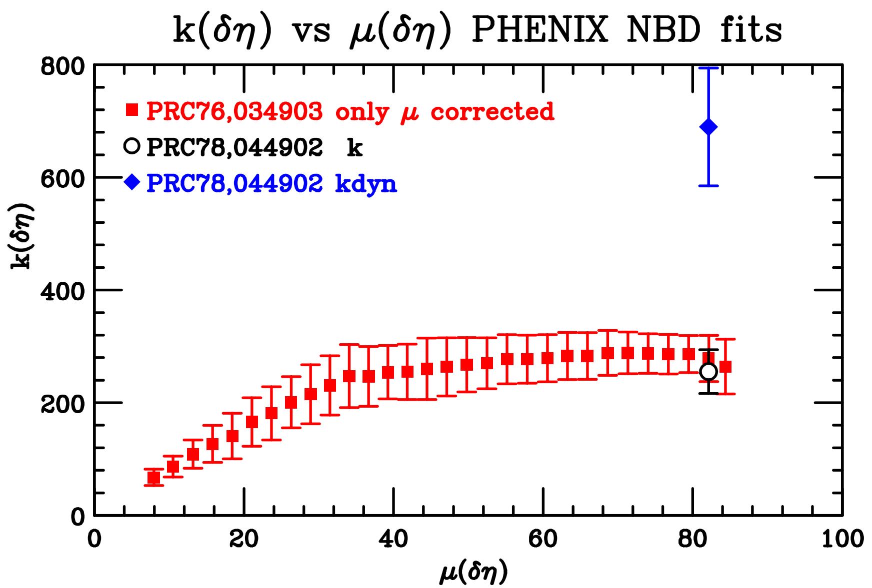


The nice examples of short range correlation with ξ, indicated in the E802 plot, change dramatically in the newer PHENIX Au+Au (200 GeV) measurement with the abrubt flattening of k(δη) for μ(δη)>30, |η|>0.15. This as far as I know is the only such measurement at RHIC or LHC. The E802 data has perfect centrality, all nucleons interact as measured in a ZDC, so the suggestion is that the flattening could be a long range correlation due to fluctuations in the number of participants in a centrality bin.

Cumulants are additive for independent processes -another NBD advantage

$$\frac{1}{k^{meas}(\delta\eta)} = K_2^{meas}(\delta\eta) = K_2^{dyn}(\delta\eta) + K_2^{bkg}(\delta\eta)$$

The two entries for E802 represent such a correction for background correlation from hits on adjacent wires.



In PRC78, PHENIX measured the effect of "geometry fluctuations" in 5% wide centrality bins and made a correction to $k_{dyn}=1/K_2^{dyn}$ which is shown for the 1 overlapping bin in the PRC76 and PRC78 measurements. (This would appear to return to the trend $k/\mu \approx$ constant vs the $\delta\eta$ interval and if true at all $\delta\eta$ would preserve the cumulant ratios vs the $\delta\eta$ acceptance!)??

Conclusions

- The NBD cumulant theorem brings a huge simplification to calculating the efficiency correction and statistical errors on net-charge cumulants.

- Acceptance corrections are much more difficult because of short range correlations in $\delta\eta$ and $\delta\phi$, but in certain cases discussed above the cumulant ratios will remain constant independent of acceptance, so would be one possible resolution to the question of the "required acceptance" to compare experiments with Lattice QCD calculations

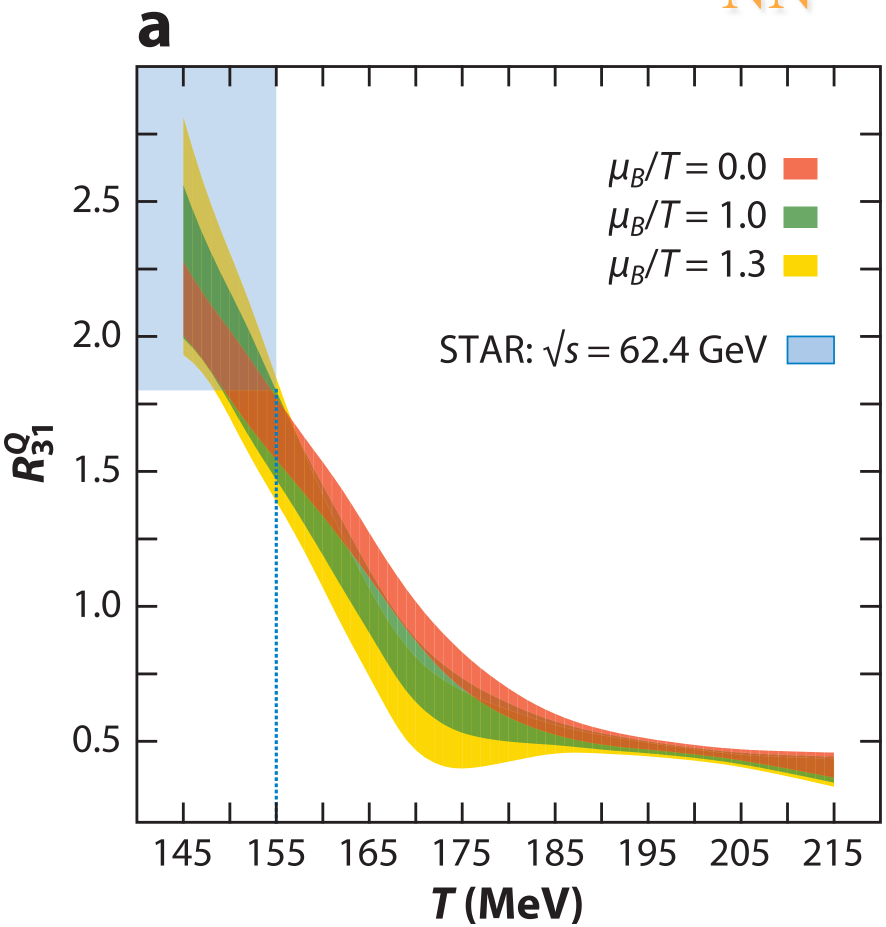
- Fortunately, the two above issues can be further investigated by both experiment and theory. For instance if the STAR NBD data for net charge were available, I could calculate the corrected values and the errors for $\kappa\sigma^2$, etc. Similarly STAR could make cuts in acceptance in their measurements to determine the variation in the results and whether or where the "required acceptance" is satisfied.

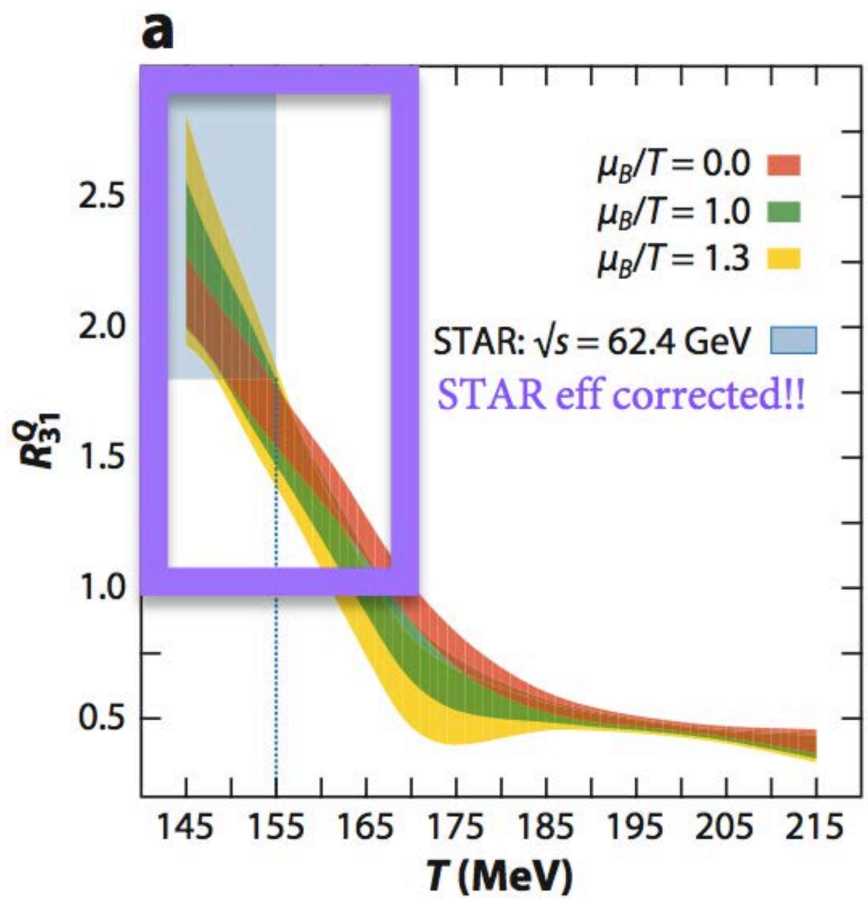
END

Extras

- Theory OOPS
- 4 generating functions
- NBD fit plots
- k(δη) PRC76,0349033(2007)
- other goodies

T_f difference for STAR raw vs corrected $\sqrt{s_{NN}}$=62.4 GeV





T_f <155 MeV 1std

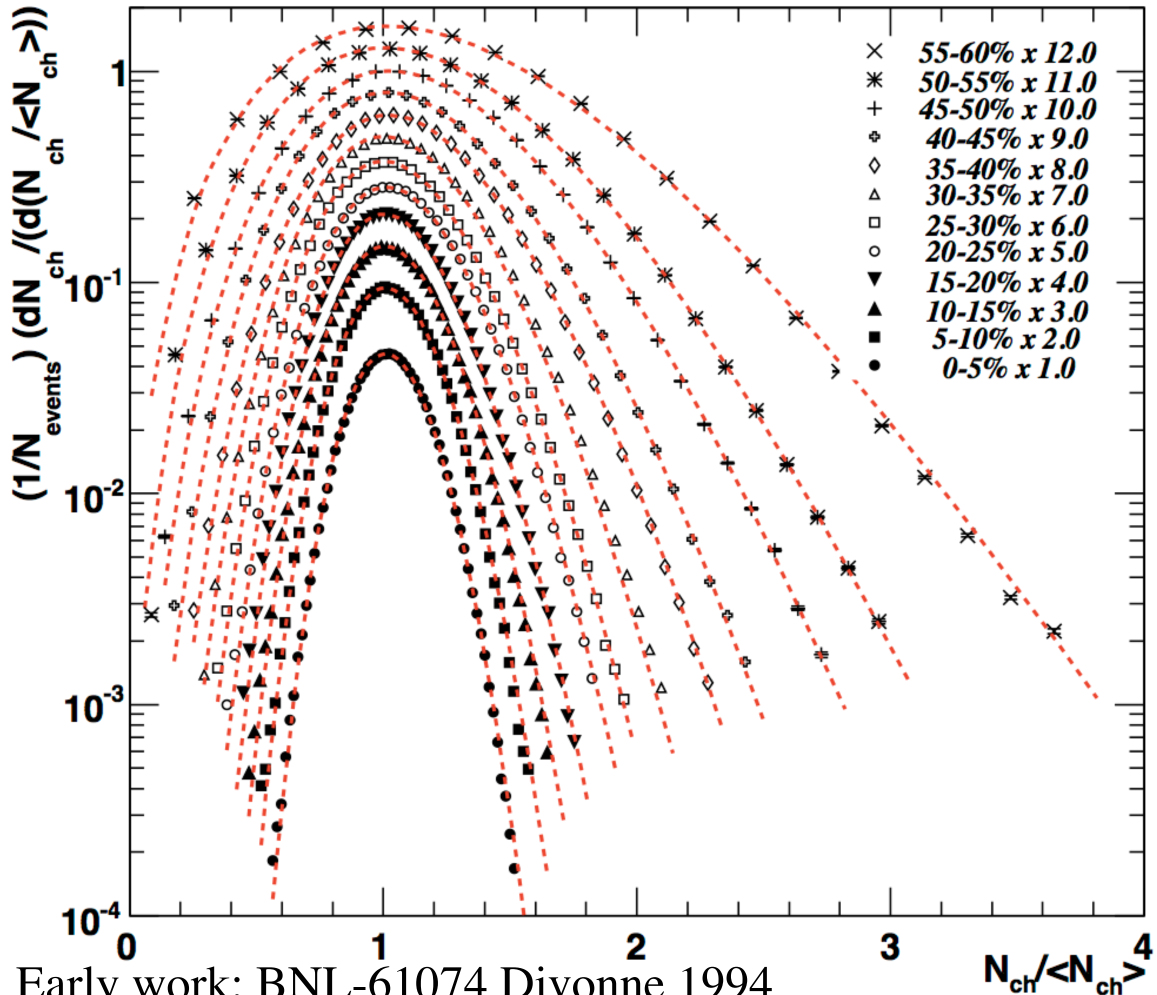
T_f <170 MeV 1std

4 Generating functions

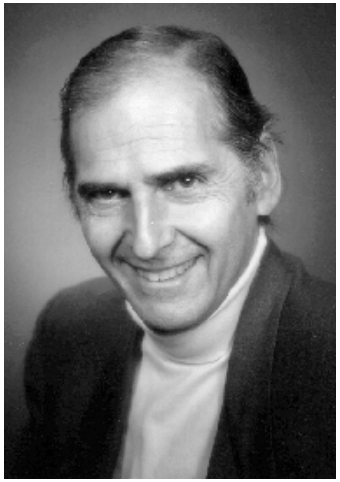
Moment generating fn	Cumulant generating function
$M'_x(t) = \langle e^{tx} \rangle$	$g_x(t) = \ln M'_x(t) = \ln \langle e^{tx} \rangle$
Factorial moment gen fn.	Factorial cumulant gen fn.
$M_x(t) = \langle (1+t)^x \rangle$	$g_x(t) = \ln \langle (1+t)^x \rangle$

From one of Jeff Mitchell's talks 2001: "Multiplicity Fluctuations"

PHENIXAuAu Multiplicity N_{ch} PRC **78**, (2008) 044902



It's not a Gaussian… it's a Gamma distribution!

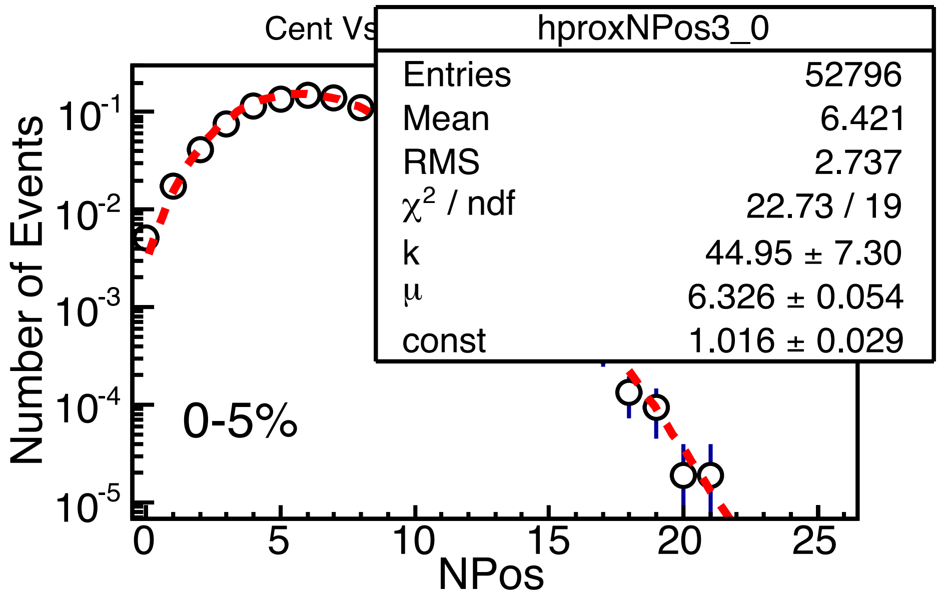


Early work: BNL-61074 Divonne 1994

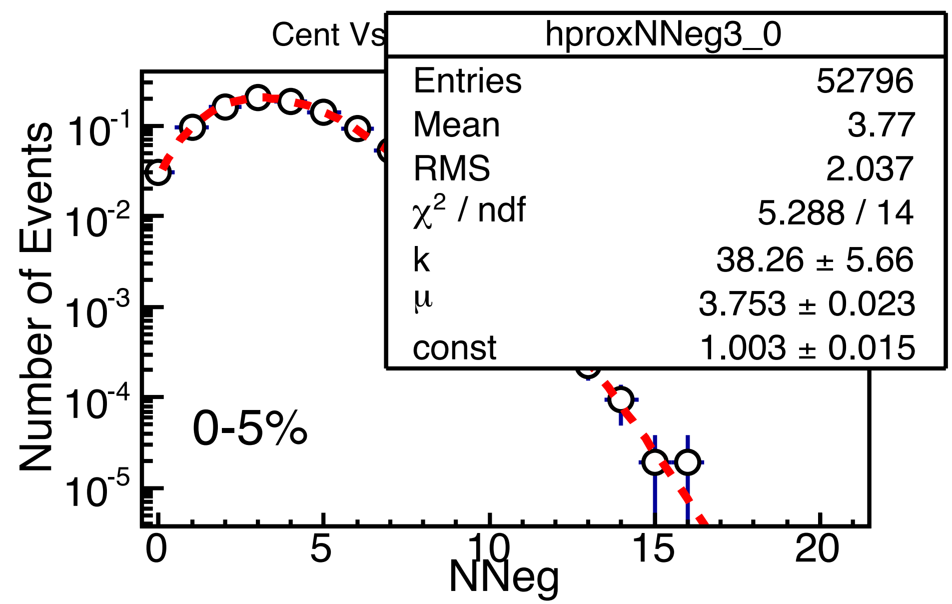
http://www.osti.gov/scitech/servlets/purl/10108142

Also: It's not Poisson, it's negative binomial

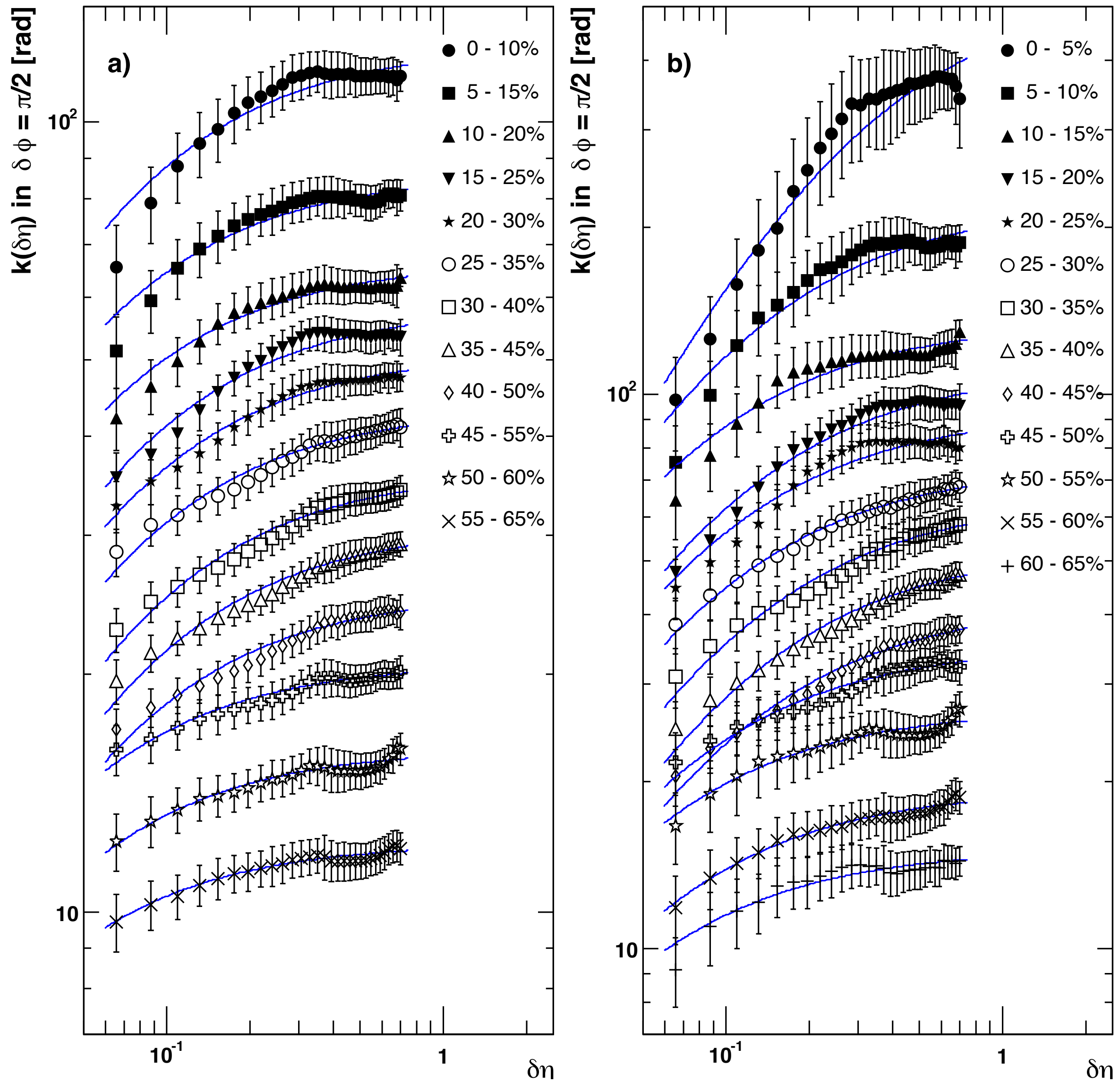
The N⁺, N⁻ (and N⁺+N⁻) distributions are NBD



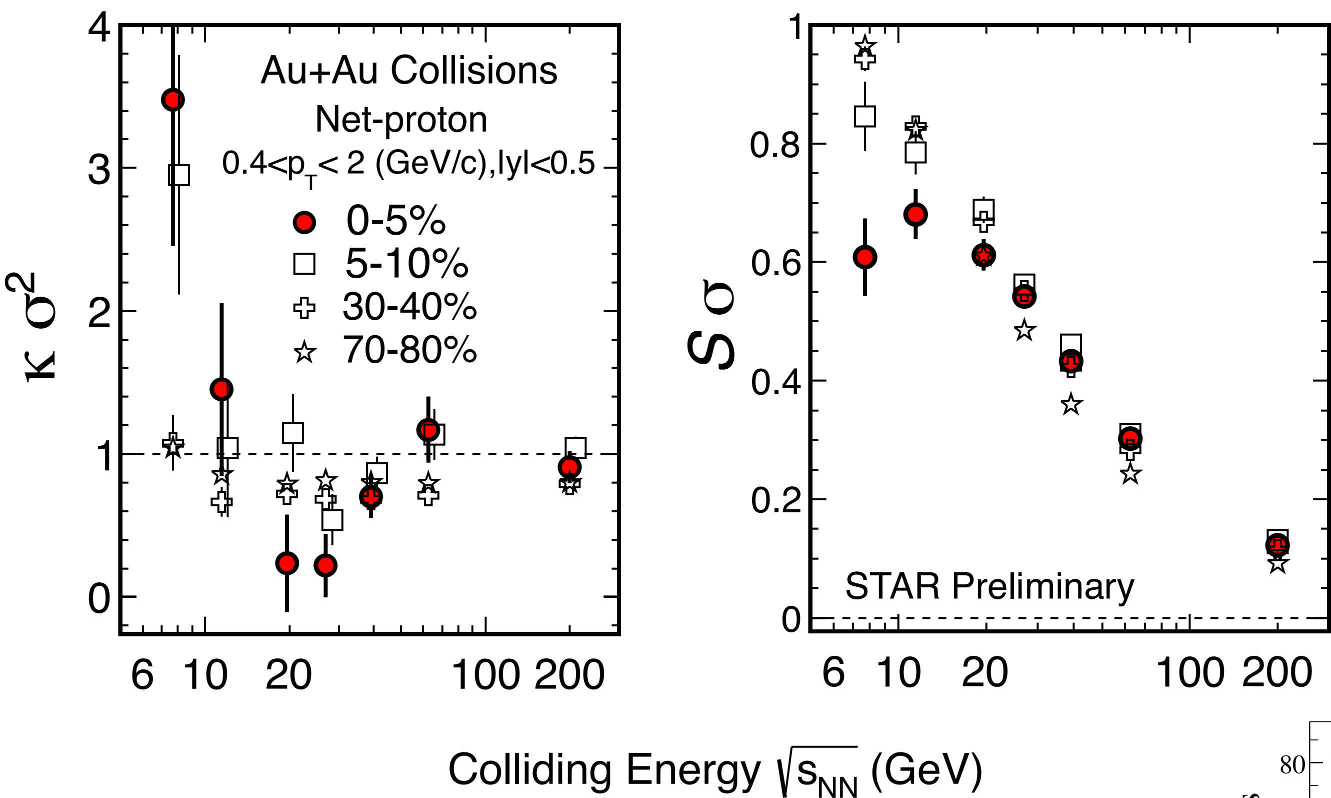
PHENIX: centrality 0-5%
$\sqrt{s_{NN}}$=7.7 GeV



PHENIX k(δη) PRC76,0349033(2007)



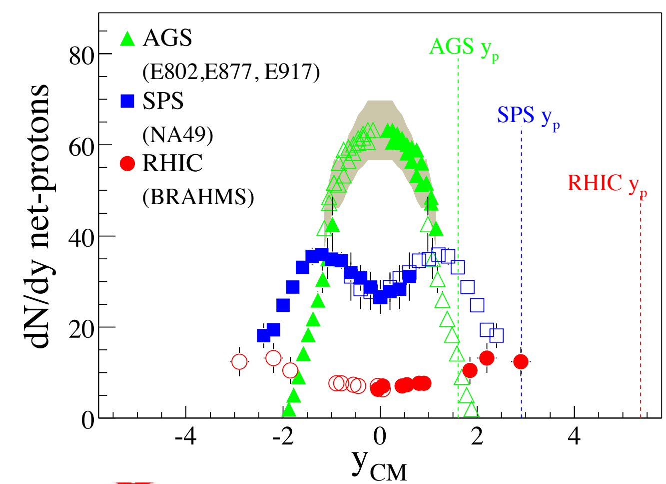
New STAR net-p Preliminary



Binomial p	Sσ	κσ^2
p	1-2p	1-6p+6p^2
0.01	0.98	0.941
0.02	0.96	0.882
0.03	0.94	0.825
0.04	0.92	0.770
0.05	0.9	0.715
0.06	0.88	0.662
0.07	0.86	0.609
0.08	0.84	0.558
0.09	0.82	0.509
0.1	0.8	0.460
0.15	0.7	0.235
0.2	0.6	0.040
0.25	0.5	-0.125
0.3	0.4	-0.260
0.35	0.3	-0.365
0.4	0.2	-0.440
0.45	0.1	-0.485
0.5	0	-0.500

X.Luo, arXiv:1503.02558

How can adding tracks >0.8 GeV/c make such changes in $\kappa\sigma^2$ but not in $S\sigma$



Taylor expansion of the pressure

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_B,\mu_S,\mu_Q)$$

$$= \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

generalized susceptibilities: $\chi_{ijk}^{BQS} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\right|_{\mu=0}$

conserved charge fluctuations: $\chi_n^X(T,\mu_B,\ldots) = \frac{\partial^n P/T^4}{\partial(\mu_X/T)^n}$

$X = B,\ Q,\ S$

$$\frac{M_X}{\sigma_X^2} = \frac{\chi_1^X(T,\mu)}{\chi_2^X(T,\mu)} \quad , \quad S_X\sigma_X = \frac{\chi_3^X(T,\mu)}{\chi_2^X(T,\mu)} \quad , \quad \kappa_X\sigma_X^2 = \frac{\chi_4^X(T,\mu)}{\chi_2^X(T,\mu)}$$

Hot off the presses-LBL Press release June 24, 2011
Lattice and Experiment Compared-a first?

Sourendu Gupta, et al., Science 332,1525 (2011)-LBL press release

When Matter Melts « Berkeley Lab News Center http://newscenter.lbl.gov/news-releases/2011/06/23/when-matter-melts/

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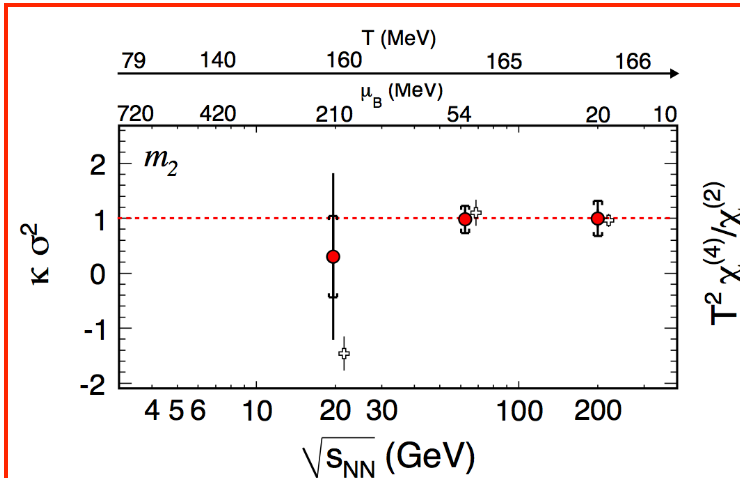
When Matter Melts

By comparing theory with data from STAR, Berkeley Lab scientists and their colleagues map phase changes in the quark-gluon plasma

June 23, 2011

Theory:Lattice shows huge deviation of $T^2\ \chi^{(4)}/\ \chi^{(2)}$ from 1 near 20 GeV, suggesting critical fluctuations. Expt $\kappa\sigma^2$: maybe but with big errors.



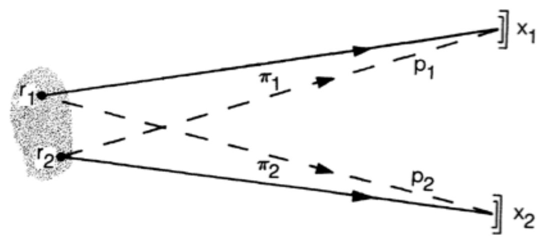
I had to do lots of work to address this issue in my Erice lecture to understand whether this physics by press-release (not published in PRL) was also Baloney According to Karsch and later measurements, it was!!!

Short range multiplicity correlations do not vanish in A+A collisions!

- Short range multiplicity correlations in p-p collisons come largely from hadron decays such as $\rho\rightarrow\pi\,\pi$, $\Lambda\rightarrow\pi^-\,p$, etc., with correlation length ξ~1 unit of rapidity

- In A+A collisions the chance of getting two particles from the same ρ meson is reduced by~$1/N_{part}$ so that **the only remaining correlations are Bose-Einstein Correlations---**when two identical Bosons, e.g. $\pi^+\,\pi^+$, occupy nearly the same coordinates in phase space so that constructive interference occurs due to the symmetry of the wave function from Bose statistics---a quantum mechanical effect, which remains at the same strength in A+A collisions:the amplitudes from the two different points add giving a large effect also called Hanbury-Brown Twiss (HBT).

See W.A.Zajc, et al, PRC 29 (1984) 2173



HBT effects in 2-particle Correlations

- The normalized two-particle short range rapidity correlation $R_2(y_1,y_2)$ is defined as

$$R_2(y_1,y_2) \equiv \frac{C_2(y_1,y_2)}{\rho_1(y_1)\rho_1(y_2)} \equiv \frac{\rho_2(y_1,y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1 = R(0,0)\, e^{-|y_1-y_2|/\xi} \quad , \qquad (8)$$

where $\rho_1(y)$ and $\rho_2(y_1,y_2)$ are the inclusive densities for a single particle (at rapidity y) or 2 particles (at rapidities y_1 and y_2), $C_2(y_1,y_2) = \rho_2(y_1,y_2) - \rho_1(y_1)\rho_1(y_2)$ is the Mueller correlation function for 2 particles (which is zero for the case of no correlation), and ξ is the two-particle short-range rapidity correlation length[3] for an exponential parameterization.

$$K_2(\delta\eta) = 2R(0,0)\frac{(\delta\eta/\xi - 1 + e^{-\delta\eta/\xi})}{(\delta\eta/\xi)^2}$$

for NBD: $k(\delta\eta) = 1/K_2(\delta\eta)$

The rapidity correlation length $\xi = 0.2$ for Si+Au E802, PRC56(1977) 1544 is from HBT.

if $\delta\eta \ll \xi$, $k \rightarrow 1/R(0,0)$=constant   if $\delta\eta \gg \xi$, $k/\delta\eta \approx k/\mu \rightarrow$constant

- For HBT analyses of two particles with $\mathbf{p}_1$ and $\mathbf{p}_2$, $C^{HBT}_2(\mathbf{q}) = R_2(\mathbf{p}_1 - \mathbf{p}_2)+1$ and the random (un-correlated) distribution is taken from particles with $\mathbf{p}_1$ and $\mathbf{p}_2$ on different events. The HBT correlation function is taken as a Gaussian not an exponential as in (8) and is written:

$$C_2^{HBT} = 1 + \lambda \exp-(R^2_{side}q^2_{side} + R^2_{out}q^2_{out} + R^2_{long}q^2_{long})$$