~~New PHENIX results on T and μ_B at freezeout~~
Net charge Fluctuations, Negative Binomials, Cumulants, Lattice QCD, a theorem from Quantitative Finance and efficiency vs. acceptance corrections

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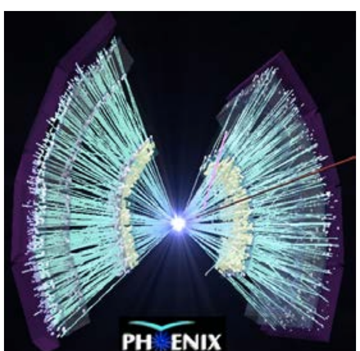
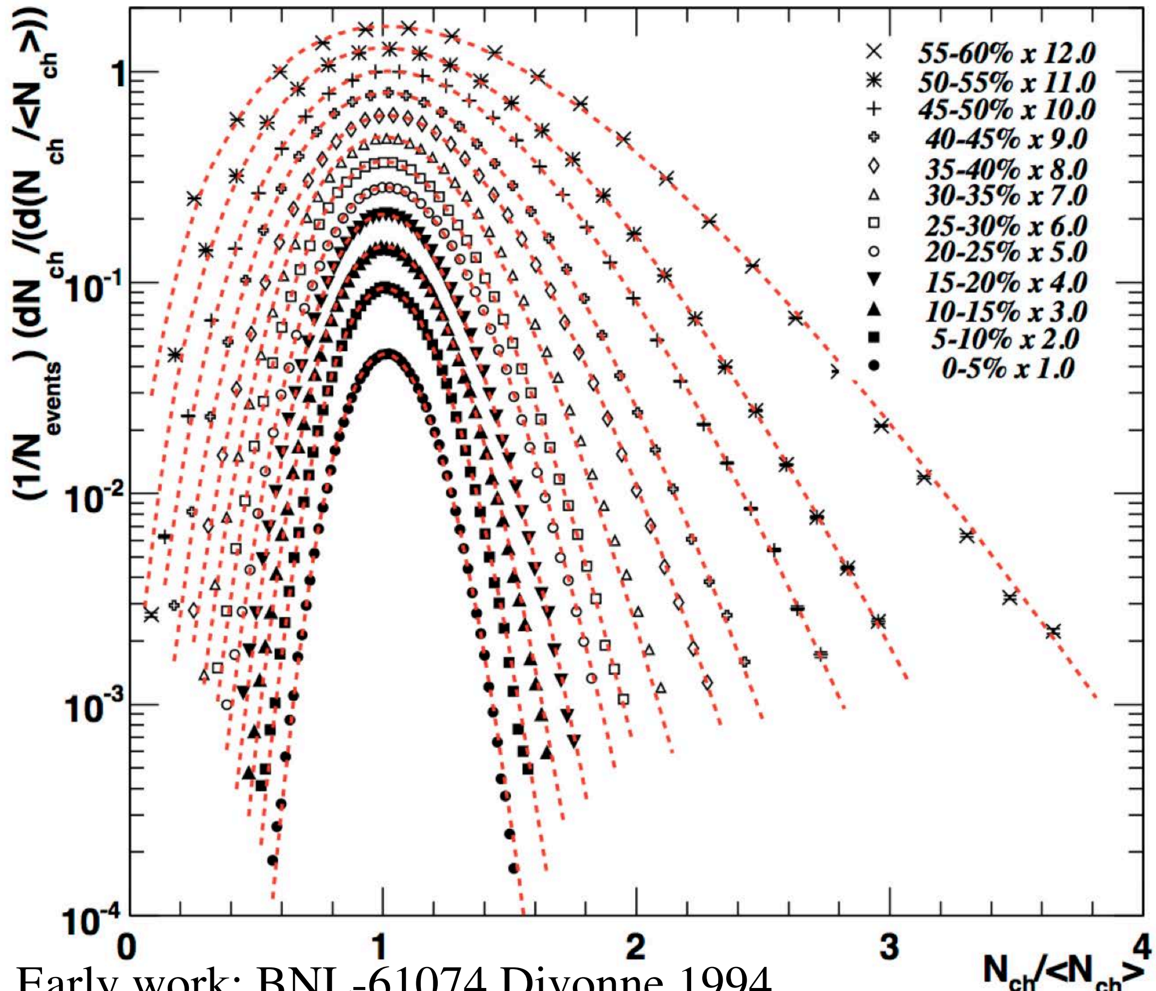


The 2015 RHIC/AGS Annual User's Meeting
"The Perfect Liquid at RHIC: A Decade of Discovery"
BNL, Upton, NY USA June 10, 2015

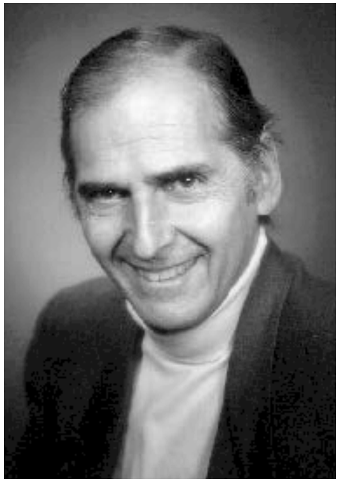
How did I get into this?
1) Early work E802 1993–
2) Talk at Erice in 2011 criticizing physics by press release

From one of Jeff Mitchell's talks 2001: "Multiplicity Fluctuations"

PHENIXAuAu Multiplicity N_{ch} PRC **78**, (2008) 044902



It's not a Gaussian… it's a Gamma distribution!



Early work: BNL-61074 Divonne 1994
http://www.osti.gov/scitech/servlets/purl/10108142

Also: It's not Poisson, it's negative binomial

Moments and Distributions

- The moments of a distribution P(x) are defined as

$$\mu'_k \equiv \langle x^k \rangle \equiv \int_{-\infty}^{\infty} x^k P(x)dx \to \sum_{i=1}^{n} x_i^k P(x_i)$$

where $\mu'_1 \equiv \mu = \langle x \rangle$ and $\sigma^2 = \mu_2 \equiv \langle (x-\mu)^2 \rangle$ is the variance

- Cumulants are moments with all combinations of lower order moments subtracted.

- Combinations of moments and cumulants which are sensitive to fluctuations (thus correlations) will be used. For instance, the second "normalized binomial cumulant" A. H. Mueller PRD **4**,151 (1971)

$$K_2 = \frac{\sigma^2}{\mu^2} - \frac{1}{\mu}$$

vanishes for a Poisson distribution (no correlations).

- Most people use the normalized variance σ^2/μ which is 1 for a Poisson. It has its purpose, but not what everybody thinks.

Bayes Rule and Conditional Probability

Bayes rule is one of the most powerful yet seemingly simple rules in probability. Let A and B be two possible outcomes with probabilities $P(A)$ and $P(B)$. Bayes Rule defines the conditional probabilities, where $P(A.and.B)$ is the probability for both outcomes to occur:

$$P(A.and.B) = P(A) \times P(B)|_A = P(B) \times P(A)|_B \qquad .$$

The apriori or prior probabilities $P(A)$ and $P(B)$ are very different from the conditional probabilities $P(A)|_B$, the conditional probability of A given that B has occurred, and $P(B)|_A$, the conditional probability of B given that A has occurred. However the conditional probabilities are simply related to each other:

$$P(A)|_B = \frac{P(A) \times P(B)|_A}{P(B)} = P(B)|_A \times \frac{P(A)}{P(B)} \qquad .$$

An interesting example of the application of Bayes rule is given in my book.

Also don't forget that if A and B are statistically independent, then

$$\begin{aligned} P(A)|_B &= P(A) \\ P(B)|_A &= P(B) \\ \text{so that} \qquad P(A.and.B) &= P(A) \times P(B) \qquad . \end{aligned}$$

Binomial Distribution

- A **Binomial** distribution is the result of repeated independent trials, each with the same two possible outcomes: success, with probability *p*, and failure, with probability $q=1-p$. The probability for *m* successes on *n* trials ($m,n \geq 0$) is:

$$P(m)|_n = \frac{n!}{m!(n-m)!} p^m (1-p)^{n-m}$$

  - The moments are:

$$\mu = \langle m \rangle = np \qquad \sigma_m^2 = np(1-p)$$

$$\frac{\sigma^2}{\mu^2} = \frac{1}{\mu} - \frac{1}{n} \qquad \frac{\sigma^2}{\mu} = 1 - p \leq 1$$

- Example: distributing a total number of particles *n* onto a limited acceptance. Note that if $p \rightarrow 0$ with $\mu=np$=constant we get a

Poisson Distribution

- A **Poisson** distribution is the limit of the Binomial Distribution for a large number of independent trials, *n*, with small probability of success *p* such that the expectation value of the number of successes $\mu = \langle m \rangle = np$ remains constant, i.e. the probability of *m* counts when you expect μ.

$$P(m)|_\mu = \frac{\mu^m e^{-\mu}}{m!}$$

- Moments: $\langle m \rangle = \mu \qquad \sigma_m^2 = \mu$

$$\frac{\sigma^2}{\mu^2} = \frac{1}{\mu} \qquad \frac{\sigma^2}{\mu} = 1 \qquad \frac{\sigma^2}{\mu^2} - \frac{1}{\mu} = 0$$

- Example: The Poisson Distribution is intimately linked to the exponential law of Radioactive Decay of Nuclei, the time distribution of nuclear disintegration counts, giving rise to the common usage of the term "statistical fluctuations" to describe the Poisson statistics of such counts. The only assumptions are that the decay probability/time of a nucleus is constant, is the same for all nuclei and is independent of the decay of other nuclei.

Negative Binomial Distribution NBD

- For statisticians, the **Negative Binomial Distribution** represents the first departure from statistical independence of rare events, i.e. the presence of correlations. There is a second parameter $1/k$, which represents the correlation: NBD → Poisson as $k \to \infty$, $1/k \to 0$

$$P(m)|_\mu = \frac{(m+k-1)!}{m!(k-1)!} \frac{\left(\frac{\mu}{k}\right)^m}{\left(1+\frac{\mu}{k}\right)^{m+k}}$$

- Moments: $\langle m \rangle = \mu \qquad \frac{\sigma^2}{\mu^2} = \frac{1}{\mu} + \frac{1}{k} \qquad \frac{\sigma^2}{\mu} = 1 + \frac{\mu}{k}$

- The n-th convolution of NBD is an NBD with $k \to nk$, $\mu \to n\mu$ such that μ/k remains constant. Hence constant σ^2/μ vs N_{part} means multiplicity added by each participant is independent.

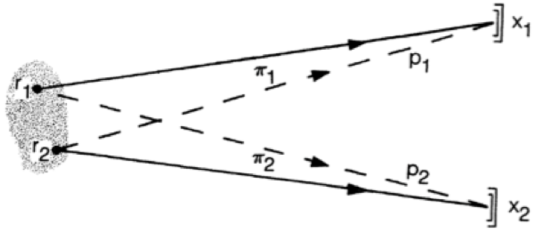
- Example: Multiplicity Distributions in p+p and A+A are NBD. There are both long-range and short-range correlations in rapidity.

Short range multiplicity correlations do not vanish in A+A collisions!

- Short range multiplicity correlations in p-p collisons come largely from hadron decays such as $\rho \rightarrow \pi\, \pi$, $\Lambda \rightarrow \pi^- p$, etc., with correlation length $\xi \sim 1$ unit of rapidity

- In A+A collisions the chance of getting two particles from the same ρ meson is reduced by $\sim 1/N_{part}$ so that **the only remaining correlations are Bose-Einstein Correlations---**when two identical Bosons, e.g. $\pi^+ \pi^+$, occupy nearly the same coordinates in phase space so that constructive interference occurs due to the symmetry of the wave function from Bose statistics---a quantum mechanical effect, which remains at the same strength in A+A collisions:the amplitudes from the two different points add giving a large effect also called Hanbury-Brown Twiss (HBT).

See W.A.Zajc, et al, PRC 29 (1984) 2173



HBT effects in 2-particle Correlations

- The normalized two-particle short range rapidity correlation $R_2(y_1,y_2)$ is defined as

$$R_2(y_1,y_2) \equiv \frac{C_2(y_1,y_2)}{\rho_1(y_1)\rho_1(y_2)} \equiv \frac{\rho_2(y_1,y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1 = R(0,0)\, e^{-|y_1-y_2|/\xi} \quad , \tag{8}$$

where $\rho_1(y)$ and $\rho_2(y_1,y_2)$ are the inclusive densities for a single particle (at rapidity y) or 2 particles (at rapidities y_1 and y_2), $C_2(y_1,y_2) = \rho_2(y_1,y_2) - \rho_1(y_1)\rho_1(y_2)$ is the Mueller correlation function for 2 particles (which is zero for the case of no correlation), and ξ is the two-particle short-range rapidity correlation length[3] for an exponential parameterization.

$$\boxed{K_2(\delta\eta) = 2R(0,0)\frac{(\delta\eta/\xi - 1 + e^{-\delta\eta/\xi})}{(\delta\eta/\xi)^2}}$$

for NBD: $k(\delta\eta) = 1/K_2(\delta\eta)$

The rapidity correlation length $\xi = 0.2$ for Si+Au E802, PRC56(1977) 1544 is from HBT.

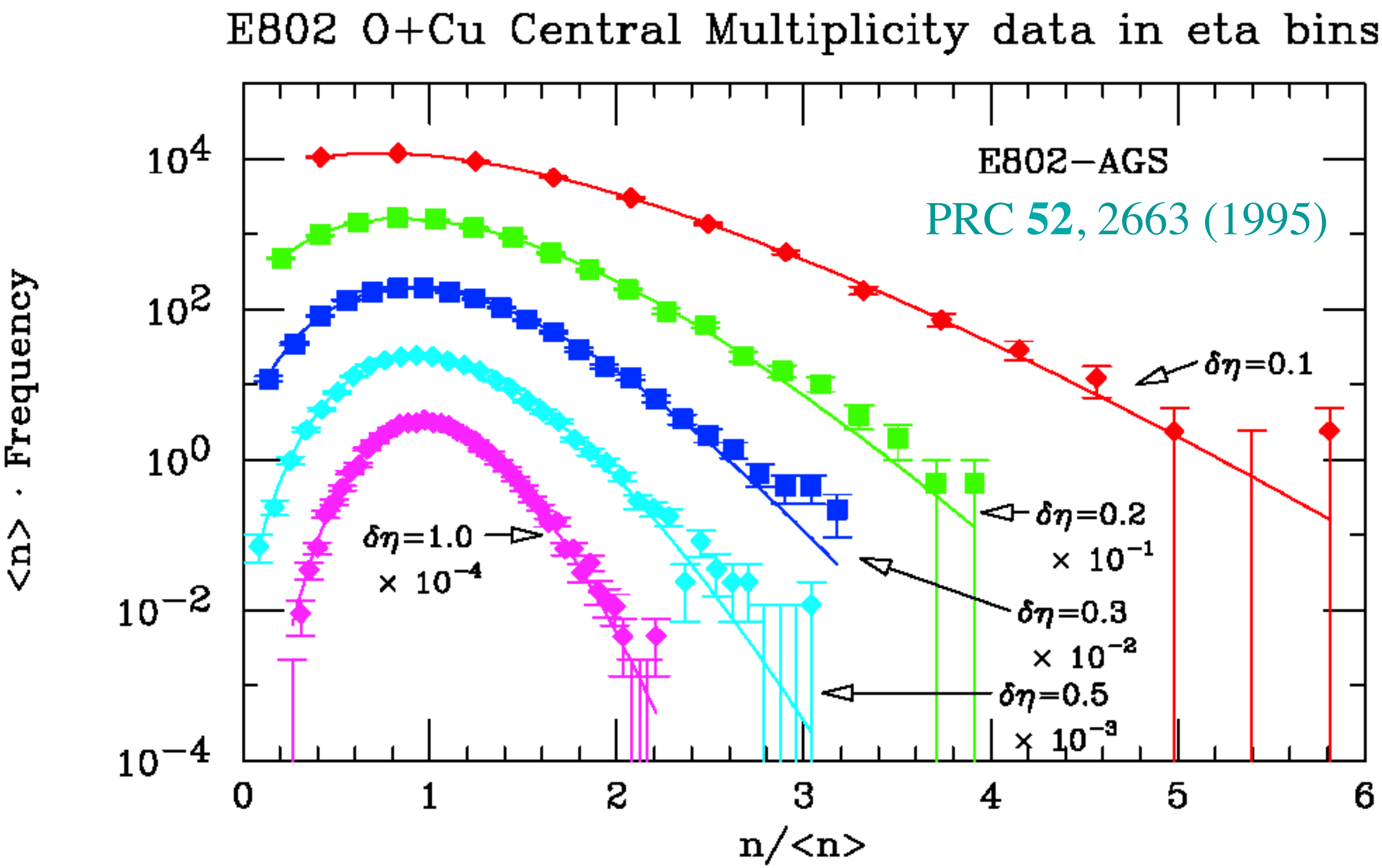
if $\delta\eta \ll \xi$, $k \rightarrow 1/R(0,0)$=constant if $\delta\eta \gg \xi$, $k/\delta\eta \approx k/\mu \rightarrow$ constant

- For HBT analyses of two particles with $\mathbf{p}_1$ and $\mathbf{p}_2$, $C^{HBT}_2(\mathbf{q}) = R_2(\mathbf{p}_1 - \mathbf{p}_2)+1$ and the random (un-correlated) distribution is taken from particles with $\mathbf{p}_1$ and $\mathbf{p}_2$ on different events. The HBT correlation function is taken as a Gaussian not an exponential as in (8) and is written:

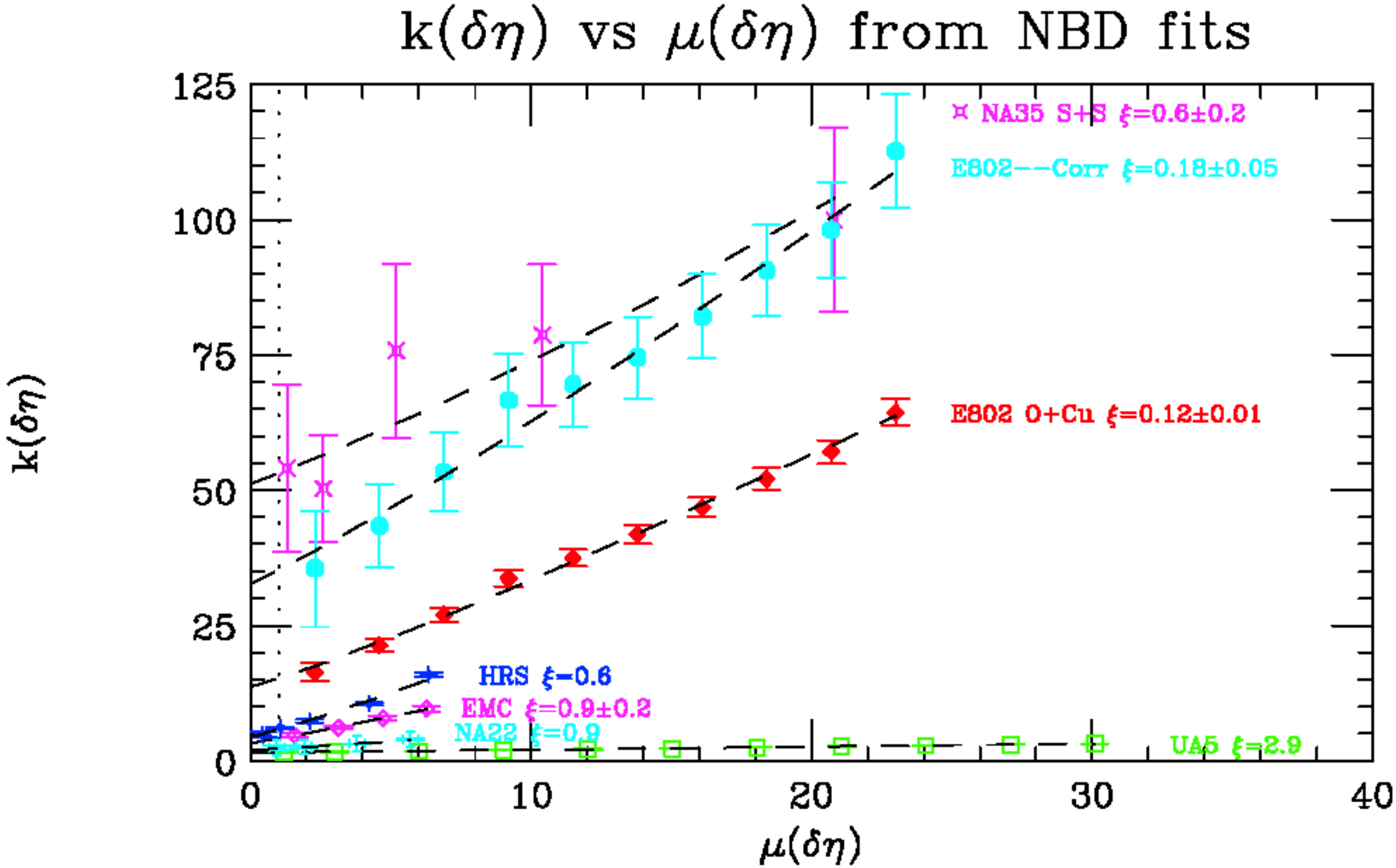
$$\boxed{C_2^{HBT} = 1 + \lambda \exp-(R_{side}^2 q_{side}^2 + R_{out}^2 q_{out}^2 + R_{long}^2 q_{long}^2)}$$

NBD-p+p discoveryUA5 PLB **160**, 193,199 (1985); **167**, 476 (1986)

NBD in O+Cu central collisions at AGS vs Δη central collisions defined by zero spectators (ZDC) Correlations due to to B-E don't vanish



k(δη) vs μ "linear" with non-zero intercept in p+p, μ-p, e^+- e^- and Light Ion reactions.



Also see MJT PLB **347**, 431(1995)

- This killed "intermittency" but dont ask, see E802 PRC**52**,2663 (1995)

Hagedorn liked my Talk at Divonne les Bains where I first showed the previous plot

Early work: BNL-61074 Divonne 1994

http://www.osti.gov/scitech/servlets/purl/10108142

THANKS TO ALL OF YOU FOR EVERYTHING !

Best wishes and friendly greetings; yours
salutations amicales et bonnes voeux;bien à vous
gute Wünsche und frendschaftliche Grüsse Ihr

Rolf Hagedorn
(Rolf Hagedorn)

Dear Dr. Tannenbaum,

Thank you so much for coming to the workshop and presenting the amazing results of the E802 Collaboration. Your analysis is brilliant and leaves – as far as its consistency goes – no questions open. As for the physics there is the puzzle of why the $p\bar{p}$ behaves so differently from nuclei? Is the washing-out of correlations by the multitude of interactions sufficient?

Long Range Correlations: Binomial Split of NBD Carruthers and Shih PLB 165 (1985)209

If a population n is distributed as NBD(μ, k) and then divided randomly into 2 subpopulations with probabilities p and $q=1-p$, then the distribution on p is NBD ($p\mu$, k) and on q is NBD ($q\mu$, k) BUT the two sub-intervals are not statistically independent.

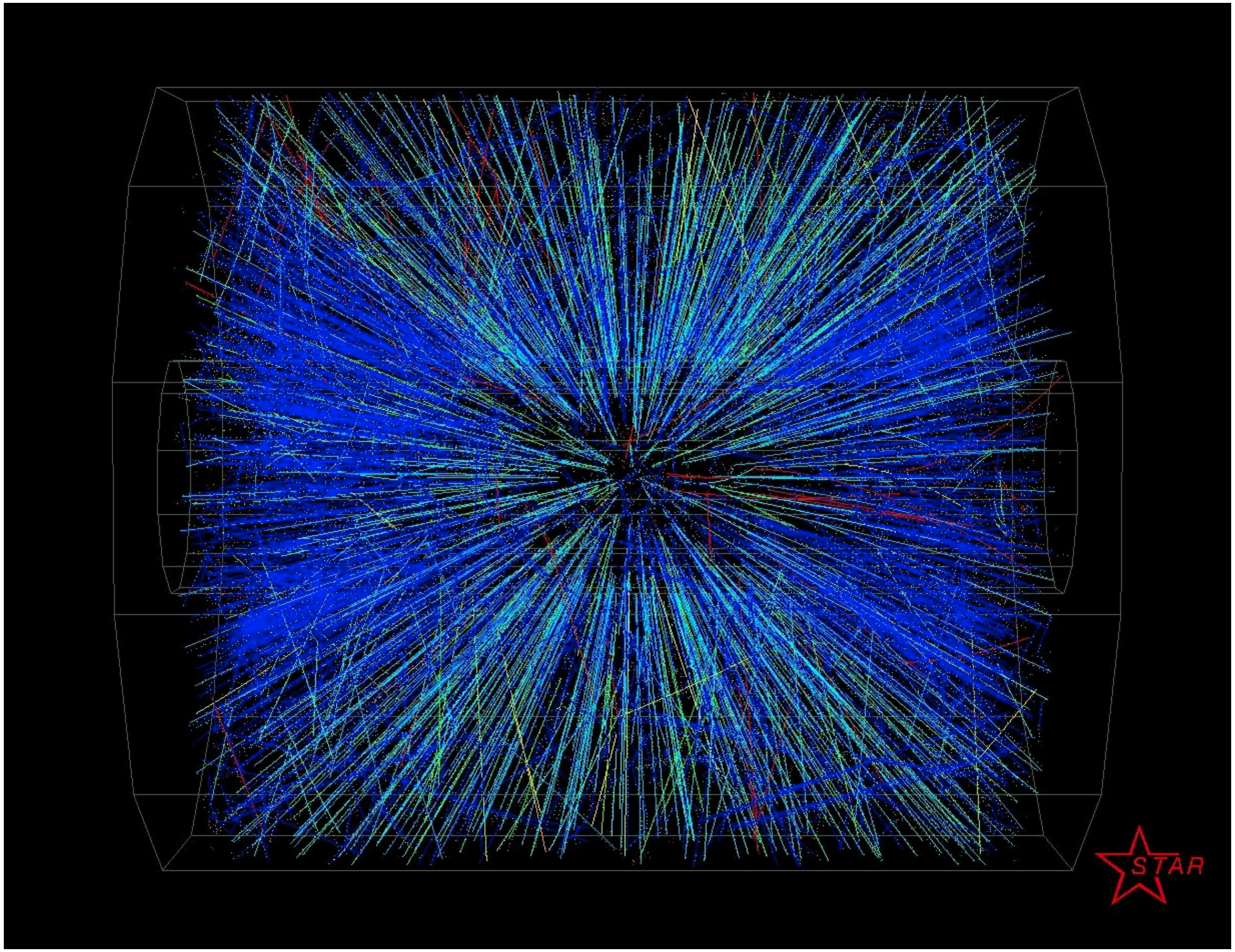
Given a sample with result $\mathbf{m}$ on interval p, the conditional probability distribution on the interval $q=1-p$ *is* NBD($\langle m_q(\mathbf{m})\rangle$, $k_q(\mathbf{m})$), where

$\langle m\rangle=p\mu \quad \langle m_q\rangle=q\mu \qquad\qquad k_q(\mathbf{m})=k+\mathbf{m}$

$\langle m_q(\mathbf{m})\rangle=\langle m_q\rangle(k+\mathbf{m})/(k+\langle m\rangle) \qquad \langle m_q(\mathbf{m})\rangle/k_q(\mathbf{m})=\langle m_q\rangle/(k+\langle m\rangle)$

This long range correlation was known in p-p collisions and I was told that this is what gave Ekspong the idea to try the NBD.

STAR first event 2001
Long Range Rapidity correlations in A+A



Large multiplicity on left side η<0 also has large multiplicity η>0

From Erice ISSP2011, see arXiv:1406.1100

The QGP was discovered at RHIC, announced on April 19, 2005 (230th anniversary of Paul Revere's Ride) as 'the perfect fluid', published NPA750,757(2005)1-171,1-283 with properties quite different from the 'new state of matter claimed' by the CERN fixed target heavy ion program on February 10, 2000 ("unpublished")

http://www.nationalcenter.org/PaulRevere'sRide.html

Mortadella-NYTimes 2/10/2000

"All the News That's Fit to Print"

The New York Times

Late Edition

NEW YORK, THURSDAY, FEBRUARY 10, 2000

75 CENTS

Spread of Attacks on Web Sites Is Slowing Traffic on the Internet

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EFFORT TO HELP NOMINEE

DIALLO TESTIMONY: A MAN CRIED 'GUN!'

Democrats Drawn to McCain Are Unsettling Republicans

Mayor Unfairly Using Religion, First Lady Says

More Hostages Freed

Lobbyists for McCain

A Trade Truce, Over Lunch

Particle Physicists Getting Closer To the Bang That Started It All

In Britain's Health Service, Sick Itself, Cancer Care Is Dismal

INSIDE

After my European Baloney statement at Erice with the CERN research director, Sergio Bertolucci, in the audience (FYI he agreed with me)

I got sandbagged by a Press Release from RHIC (actually LBL not BNL)

Hot off the presses-LBL Press release June 24, 2011
Lattice and Experiment Compared-a first?

Sourendu Gupta, et al., Science 332,1525 (2011)-LBL press release

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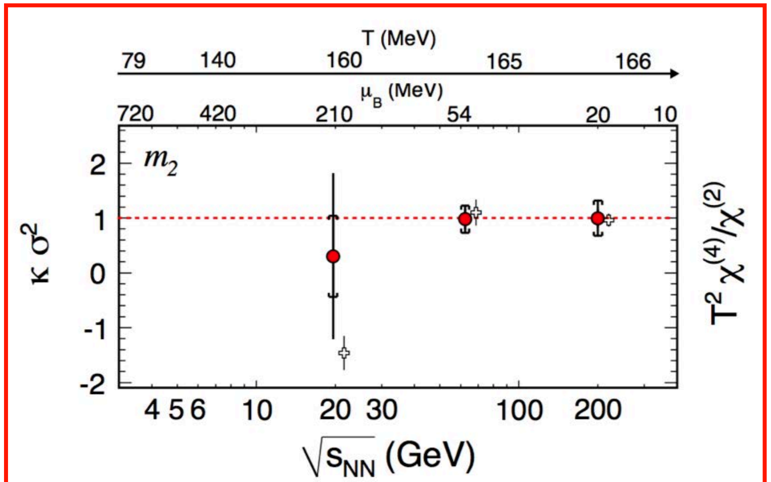
When Matter Melts

By comparing theory with data from STAR, Berkeley Lab scientists and their colleagues map phase changes in the quark-gluon plasma

June 23, 2011

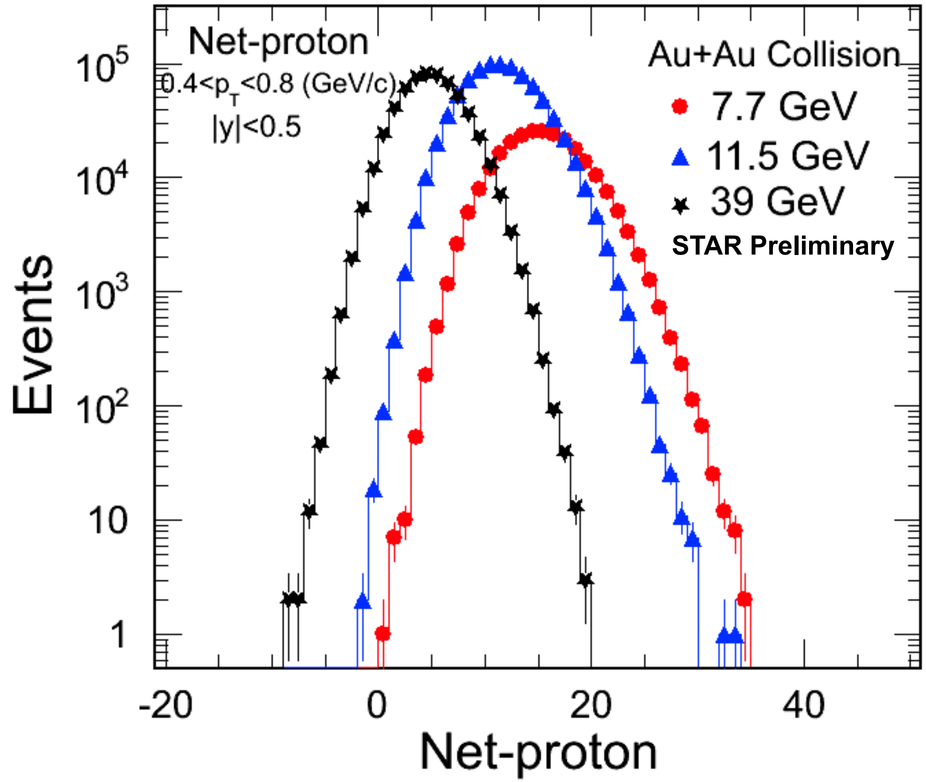
Theory:Lattice shows huge deviation of $T^2\,\chi^{(4)}/\,\chi^{(2)}$ from 1 near 20 GeV, suggesting critical fluctuations. Expt $\kappa\sigma^2$: maybe but with big errors.



I had to do lots of work to address this issue in my second lecture to understand whether this physics by press-release (not published in PRL) was also Baloney

Hot off the presses-LBL Press release June 24,2011
Higher Moments of Net-Proton Distributions

- 1^{st} moment: mean = $\mu = \langle x \rangle$
- 2^{nd} cumulant: variance $\kappa_2 = \sigma^2 = \langle(x-\mu)^2\rangle$
- 3^{rd} cumulant: $\kappa_3 = \sigma^3 = \langle(x-\mu)^3\rangle$
- 3^{rd} standardized cumulant: skewness = $S = \kappa_3/\kappa_2^{3/2} = \langle(x-\mu)^3\rangle/\sigma^3$
- 4^{th} cumulant: $\kappa_4 = \langle(x-\mu)^4\rangle - 3\kappa_2^2$
- 4^{th} standardized cumulant: kurtosis = $\kappa = \kappa_4/\kappa_2^2 = \{\langle(x-\mu)^4\rangle/\sigma^4\} - 3$
- Calculate moments from the event-by-event net proton distribution.
  - ✓ Have similar plots for net-charge and net-kaon distributions.



MJT-If you know the distribution, you know all the moments, but statistical mechanics and Lattice Gauge use Taylor expansions, hence moments/cumulants

Statistical Mechanics uses derivatives of the free energy to find susceptibilities

- Theoretical analyses tend to be made in terms of a Taylor expansion of the free energy $F=-T \ln Z$ around the critical temperature T_c where Z is the partition function or sum over states, $Z \approx \exp -[(E-\Sigma_i \mu_i Q_i)/kT]$ and μ_i chemical potentials associated with conserved charges Q_i

- The terms of the Taylor expansion are called susceptibilities or χ

- The only connection of this method to mathematical statistics is that the Cumulant generating function is also a Taylor expansion of the ln of an exponential:

$$g_x(t) = \ln \langle e^{tx} \rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!} \qquad \kappa_m = \left. \frac{d^m g_x(t)}{dt^m} \right|_{t=0}$$

If you measure the distribution, then you know all the cumulants

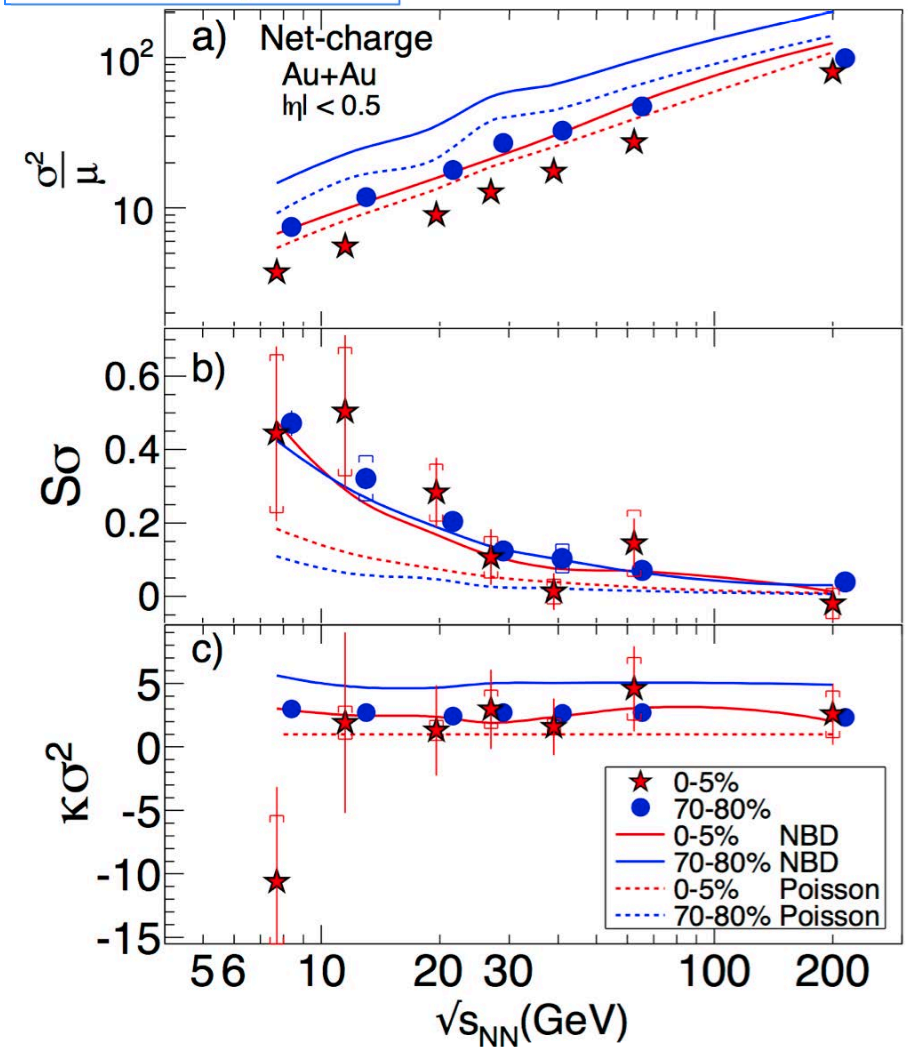
Cumulants for Poisson, Binomial and Negative Binomial Distributions			
Cumulant	Poisson	Binomial	Negative Binomial
$\kappa_1 = \mu$	μ	np	μ
$\kappa_2 = \mu_2 = \sigma^2$	μ	$\mu(1-p)$	$\mu(1+\mu/k)$
$\kappa_3 = \mu_3$	μ	$\sigma^2(1-2p)$	$\sigma^2(1+2\mu/k)$
$\kappa_4 = \mu_4 - 3\kappa_2^2$	μ	$\sigma^2(1-6p+6p^2)$	$\sigma^2(1+6\mu/k+6\mu^2/k^2)$
$S \equiv \kappa_3/\sigma^3$	$1/\sqrt{\mu}$	$(1-2p)/\sigma$	$(1+2\mu/k)/\sigma$
$\kappa \equiv \kappa_4/\kappa_2^2$	$1/\mu$	$(1-6p+6p^2)/\sigma^2$	$(1+6\mu/k+6\mu^2/k^2)/\sigma^2$
$S\sigma = \kappa_3/\kappa_2$	1	$(1-2p)$	$(1+2\mu/k)$
$\kappa\sigma^2 = \kappa_4/\kappa_2$	1	$(1-6p+6p^2)$	$(1+6\mu/k+6\mu^2/k^2)$

Thanks to Gary Westfall of STAR in a paper presented at Erice-International School of Nuclear Physics 2012, I found out that the cumulants of the difference of samples from two such distributions P(n-m) where $P^+(n)$ and $P^-(m)$ are both Poisson, Binomial or NBD with Cumulants κ_j^+ and κ_j^- respectively is the same as if they were statistically independent, so long as they are not 100% correlated. This is discussed for Skellam (Poisson P^+, P^-) in Wikipedia.

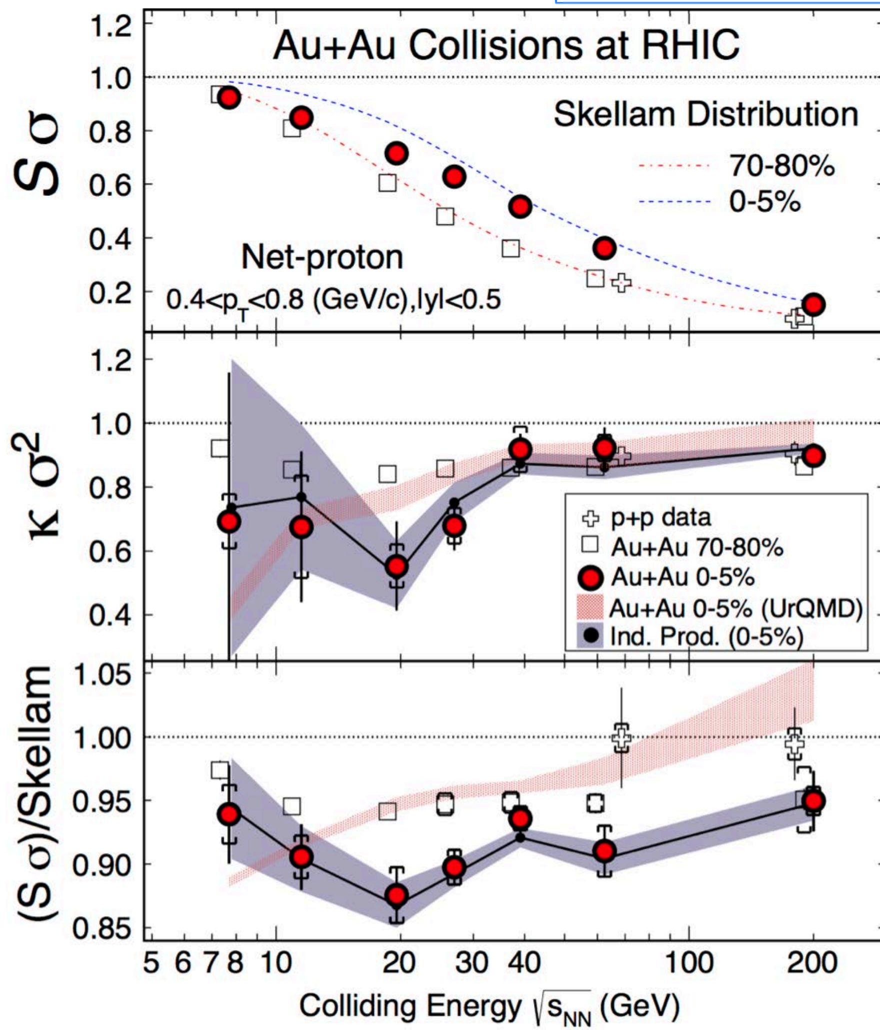
$$\kappa_j = \kappa_j^+ + (-1)^j \kappa_j^-$$

New STAR publications 2014

PRL 113(2014) 092301



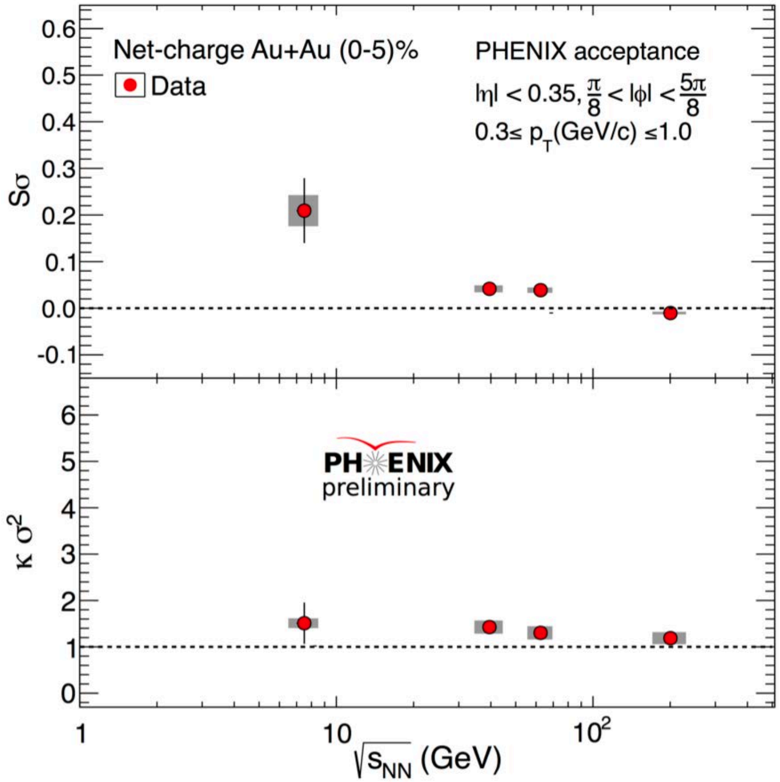
PRL 112(2014) 032302

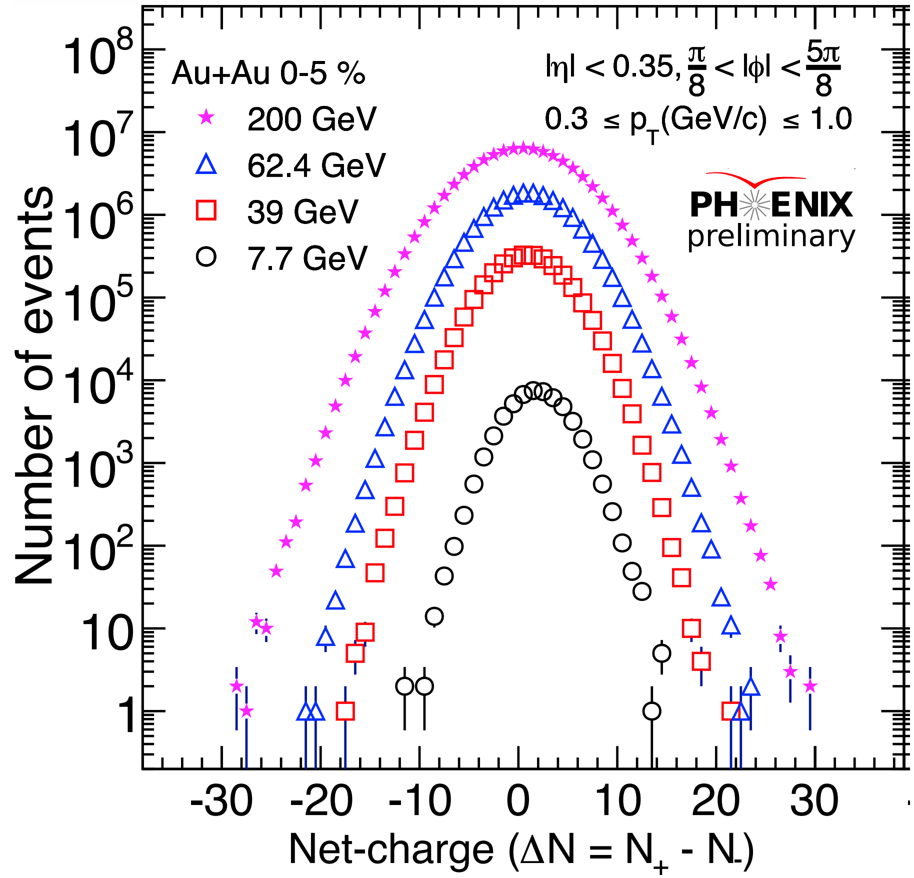


$S\sigma$ clearly favors NBD, not Poisson (!).
No non- monotonic behavior in $S\sigma$ or $\kappa\sigma^2$
but $\kappa\sigma^2$=-1.5 at $\sqrt{s_{NN}}$ =20 can't be ruled out

$\kappa\sigma^2$=-1.5 at $\sqrt{s_{NN}}$ =20 **can** be ruled out
$\kappa\sigma^2$ changes for $\sqrt{s_{NN}}$ ≤20 GeV but
antiprotons become negligible <0.1 p

PHENIX preliminary data net-charge not corrected for efficiency





The key difference of the final PHENIX and STAR results is that the error on all corrected cumulant ratios is 20-30% for PHENIX while for STAR the error on e.g. $\kappa\sigma^2$ is >100% but <1% for σ^2/μ!!!

Final result has $|\eta|<0.35$, $\delta\phi=\pi$, $0.3<p_T<2.0$ GeV/c + $\sqrt{s_{NN}}$ = 19.7 and 27 GeV and is corrected for efficiency

Efficiency Corrected Cumulants

It must be that statistical errors and efficiency corrections are a BIG issue in these measurements even though the correction is simply Binomial; and analytical for NBD N^+ and N^- distributions (**k unchanged, $\mu_t = \mu/p$ where p is the efficiency**) thanks to the NBD "integer value Levy process" cumulant theorem:

Tarnowsky, Westfall PLB 724 (2013) 51

Barndorff-Nielsen,Pollard,Shephard

http://www.economics.ox.ac.uk/materials/papers/4382/paper490.pdf

$$\boxed{\kappa_j = \kappa_j^+ + (-1)^j \kappa_j^-}$$

Efficiency-Corrected NBD Cumulant Ratios

$$\frac{\mu}{\sigma^2} = \frac{\kappa_1^+ - \kappa_1^-}{\kappa_2^+ + \kappa_2^-} = \frac{\mu_t^+ - \mu_t^-}{\mu_t^+[1 + (\frac{\mu_t^+}{k^+})] + \mu_t^-[1 + (\frac{\mu_t^-}{k^-})]}$$

$$\mu_t = \frac{\mu}{\varepsilon}$$

$$\frac{S\sigma^3}{\mu} = \frac{\kappa_3^+ - \kappa_3^-}{\kappa_1^+ - \kappa_1^-} = \frac{\mu_t^+[1 + 3(\frac{\mu_t^+}{k^+}) + 2(\frac{\mu_t^+}{k^+})^2] - \mu_t^-[1 + 3(\frac{\mu_t^-}{k^-}) + 2(\frac{\mu_t^-}{k^-})^2]}{\mu_t^+ - \mu_t^-}$$

$$S\sigma = \frac{\kappa_3^+ - \kappa_3^-}{\kappa_2^+ + \kappa_2^-} = \frac{\mu_t^+[1 + 3(\frac{\mu_t^+}{k^+}) + 2(\frac{\mu_t^+}{k^+})^2] - \mu_t^-[1 + 3(\frac{\mu_t^-}{k^-}) + 2(\frac{\mu_t^-}{k^-})^2]}{\mu_t^+[1 + \frac{\mu_t^+}{k^+}] + \mu_t^-[1 + \frac{\mu_t^-}{k^-}]}$$

$$\kappa\sigma^2 = \frac{\kappa_4^+ + \kappa_4^-}{\kappa_2^+ + \kappa_2^-} = \frac{\mu_t^+[1 + 7(\frac{\mu_t^+}{k^+}) + 12(\frac{\mu_t^+}{k^+})^2 + 6(\frac{\mu_t^+}{k^+})^3] + \mu_t^-[1 + 7(\frac{\mu_t^-}{k^-}) + 12(\frac{\mu_t^-}{k^-})^2 + 6(\frac{\mu_t^-}{k^-})^3]}{\mu_t^+[1 + \frac{\mu_t^+}{k^+}] + \mu_t^-[1 + \frac{\mu_t^-}{k^-}]}$$

The error on μ_t <<than the error on μ_t/k so is neglected. The errors are highly correlated for the sums of powers of μ_t/k in both the numerator and denominator. These correlations are handled by varying the $(\mu_t/k)^+$ and $(\mu_t/k)^-$ by $\pm 1\sigma$ independently and adding the variations in quadrature

Compare to Bzdak-Koch standard Binomial efficiency correction PRC 86 (2012) 044904

Efficiency corrected cumulants in terms of corrected double Factorial moments

$$N = \langle N_+ \rangle + \langle N_- \rangle$$

$$\kappa_1 = \langle N_+ \rangle - \langle N_- \rangle = \frac{\langle n_+ \rangle}{\epsilon_+} - \frac{\langle n_- \rangle}{\epsilon_-},$$

$$F_{ik} = \sum_{N_1=i}^{\infty} \sum_{N_2=k}^{\infty} P(N_1, N_2) \frac{N_1!}{(N_1-i)!} \frac{N_2!}{(N_2-k)!}$$

$$\kappa_2 = N - \kappa^2{}_1 + F_{02} - 2F_{11} + F_{20},$$

$$\kappa_3 = \kappa_1 + 2\,\kappa^3{}_1 - F_{03} - 3F_{02} + 3\,F_{12} + 3\,F_{20} - 3F_{21} + F_{30}$$
$$- 3\kappa_1(N + F_{02} - 2F_{11} + F_{20}),$$

$$\kappa_4 = N - 6\kappa^4{}_1 + F_{04} + 6\,F_{03} + 7\,F_{02} - 2F_{11} - 6F_{12} - 4F_{13}$$
$$+ 7F_{20} - 6F_{21} + 6F_{22} + 6F_{30} - 4F_{31} + F_{40}$$
$$+ 12\kappa^2{}_1(N + F_{02} - 2F_{11} + F_{20}) - 3(N + F_{02} - 2F_{11} + F_{20})^2$$
$$- 4\kappa_1(\kappa_1 - F_{03} - 3F_{02} + 3\,F_{12} + 3\,F_{20} - 3F_{21} + F_{30})$$

Here you can see the nice subtraction of the lower order moments; but new quantities, double Factorial Moments are introduced and very difficult to compute $P(13^+, 11^-)=?$ so you need to know both N^+ and N^- distributions and their correlations. Better to hope for integer Levy processes like Poisson or NBD and use the theorem. NBD only uses 4 quantities for the same calculation: μ_t^+ and μ_t^- $(\mu_t/k)^+$ and $(\mu_t/k)^-$

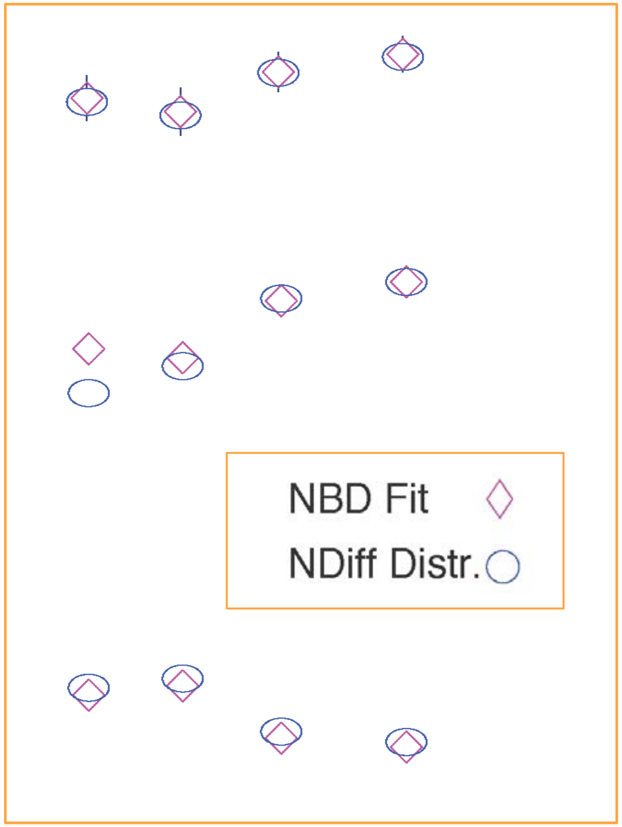
The errors of the cumulants and ratios by the direct method are also very complicated

A recent thorough treatment of both statistical errors and efficiency, with even more complicated formulas than Bzdak and Koch is given by Xiaofeng Luo, PRC **91** (2015) 034907
BUT to test the method:

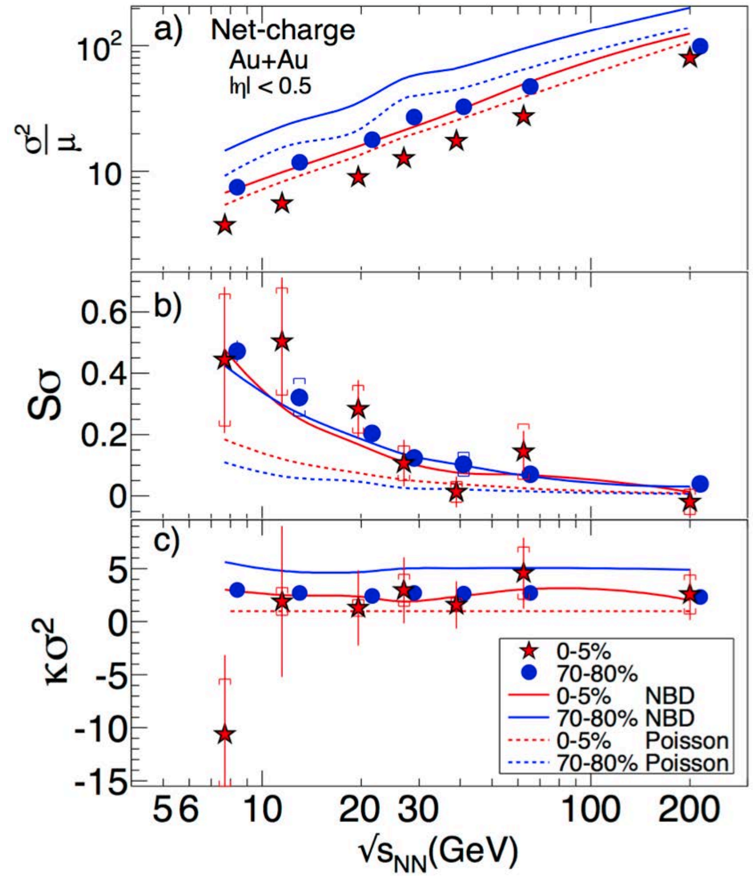
"By deriving the covariance between factorial moments, one can obtain the general error formula for the efficiency corrected moments based on the error propagation derived from the Delta theorem. The Skellam-distribution-based Monto Carlo simulation is used to test the Delta theorem and bootstrap error estimation methods."

I note, of course, that Skellam is the difference between two Poissons so satisfies the integer Levy process theorem!   I also note that Bzdak and Koch have not been idle PRC **91**(2015) 027901

Do NBD and direct Cumulants get the same answer at RHIC?---PHENIX YES! STAR close



PHENIX uncorrected cumulants
no particular order!!!



STAR corrected-red solid
line cf. red stars

Efficiency-Corrected NBD Cumulant Ratios

$$\mu_t = \frac{\mu}{\varepsilon}$$

$$\frac{\mu}{\sigma^2} = \frac{\kappa_1^+ - \kappa_1^-}{\kappa_2^+ + \kappa_2^-} = \frac{\mu_t^+ - \mu_t^-}{\mu_t^+[1 + (\frac{\mu_t^+}{k^+})] + \mu_t^-[1 + (\frac{\mu_t^-}{k^-})]}$$

$$\frac{S\sigma^3}{\mu} = \frac{\kappa_3^+ - \kappa_3^-}{\kappa_1^+ - \kappa_1^-} = \frac{\mu_t^+[1 + 3(\frac{\mu_t^+}{k^+}) + 2(\frac{\mu_t^+}{k^+})^2] - \mu_t^-[1 + 3(\frac{\mu_t^-}{k^-}) + 2(\frac{\mu_t^-}{k^-})^2]}{\mu_t^+ - \mu_t^-}$$

$$S\sigma = \frac{\kappa_3^+ - \kappa_3^-}{\kappa_2^+ + \kappa_2^-} = \frac{\mu_t^+[1 + 3(\frac{\mu_t^+}{k^+}) + 2(\frac{\mu_t^+}{k^+})^2] - \mu_t^-[1 + 3(\frac{\mu_t^-}{k^-}) + 2(\frac{\mu_t^-}{k^-})^2]}{\mu_t^+[1 + \frac{\mu_t^+}{k^+}] + \mu_t^-[1 + \frac{\mu_t^-}{k^-}]}$$

$$\kappa\sigma^2 = \frac{\kappa_4^+ + \kappa_4^-}{\kappa_2^+ + \kappa_2^-} = \frac{\mu_t^+[1 + 7(\frac{\mu_t^+}{k^+}) + 12(\frac{\mu_t^+}{k^+})^2 + 6(\frac{\mu_t^+}{k^+})^3] + \mu_t^-[1 + 7(\frac{\mu_t^-}{k^-}) + 12(\frac{\mu_t^-}{k^-})^2 + 6(\frac{\mu_t^-}{k^-})^3]}{\mu_t^+[1 + \frac{\mu_t^+}{k^+}] + \mu_t^-[1 + \frac{\mu_t^-}{k^-}]}$$

The error on μ_t <<than the error on μ_t/k so is neglected. The errors are highly correlated for the sums of powers of μ_t/k in both the numerator and denominator. These correlations are handled by varying the $(\mu_t/k)^+$ and $(\mu_t/k)^-$ by $\pm 1\sigma$ independently and adding the variations in quadrature

Are acceptance corrections possible?

$$\frac{S\sigma^3}{\mu} = \frac{\kappa_3^+ - \kappa_3^-}{\kappa_1^+ - \kappa_1^-} = \frac{\mu_t^+[1+3(\frac{\mu_t^+}{k^+})+2(\frac{\mu_t^+}{k^+})^2] - \mu_t^-[1+3(\frac{\mu_t^-}{k^-})+2(\frac{\mu_t^-}{k^-})^2]}{\mu_t^+ - \mu_t^-}$$

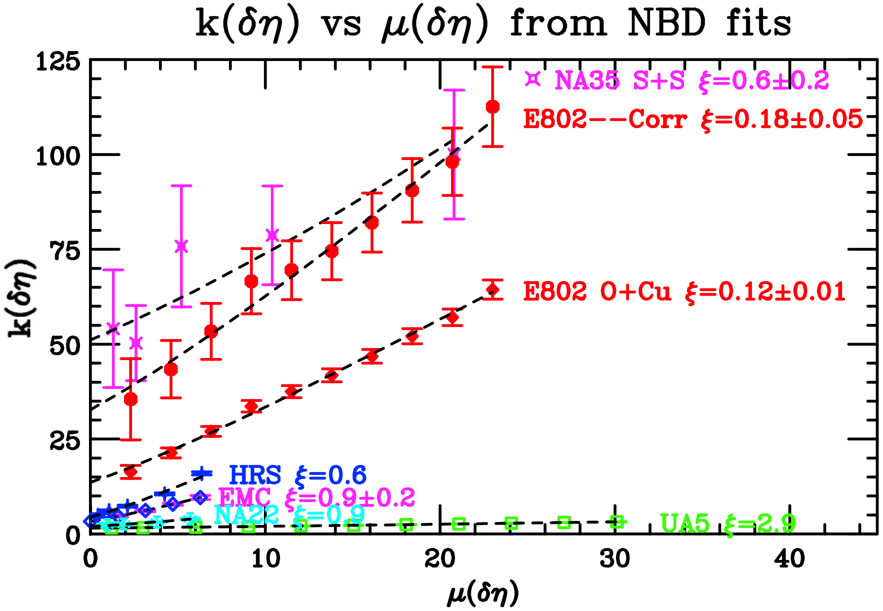
$$R_{32} - R_{12} = S\sigma - \frac{\mu}{\sigma^2} = \frac{\mu_t^+[3(\frac{\mu_t^+}{k^+})+2(\frac{\mu_t^+}{k^+})^2] - \mu_t^-[3(\frac{\mu_t^-}{k^-})+2(\frac{\mu_t^-}{k^-})^2]}{\mu_t^+[1+\frac{\mu_t^+}{k^+}] + \mu_t^-[1+\frac{\mu_t^-}{k^-}]}$$

Bzdak and Koch (and likely many others) have expressed concern about what is the "required acceptance" for an experimental result e.g. on the above quantities to compare with Lattice QCD calculations

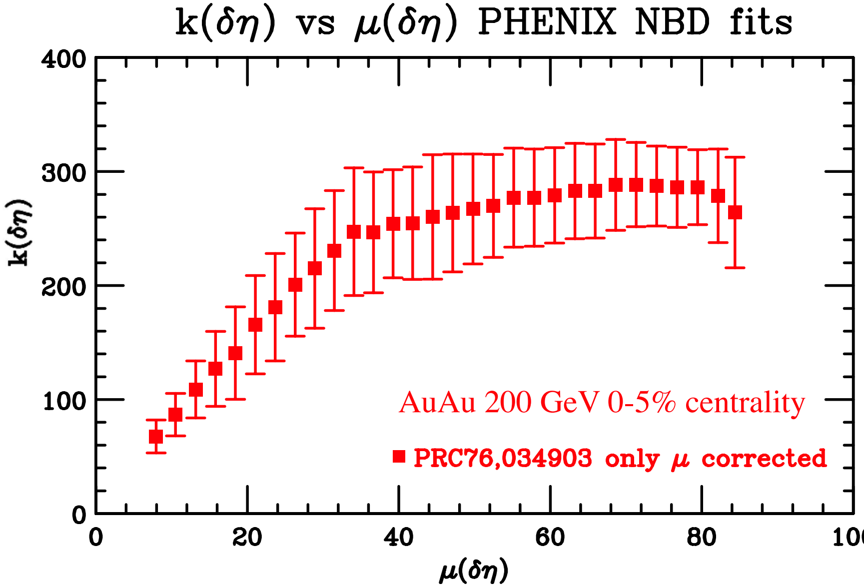
The good news from the above equations and those on the previous page is that if the ratios $(\mu_t/k)^+$ and $(\mu_t/k)^-$ don't change with the acceptance and if μ_t^+ and μ_t^- scale by the same amount with the acceptance (e.g. $dn/d\eta$ constant in rapidity and azimuth) then the above formulas remain unchanged. What does nature say?

Recall the earlier slide that Hagedorn liked, BUT

E802 PRC**52**,2663(1995)



PHENIX PRC**76**,0349033(2007)

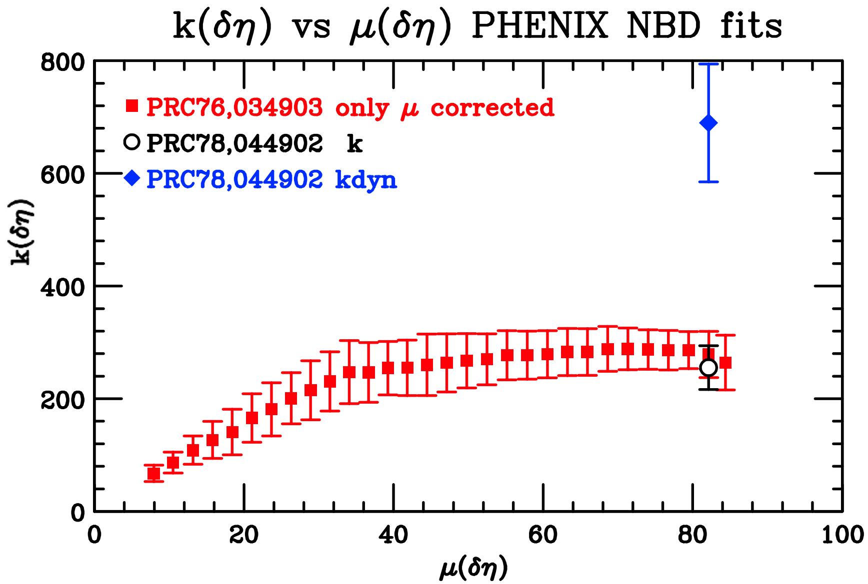


The nice examples of short range correlation with ξ, indicated in the E802 plot, change dramatically in the newer PHENIX Au+Au (200 GeV) measurement with the abrubt flattening of $k(\delta\eta)$ for $\mu(\delta\eta)>30$, $|\eta|>0.15$. This as far as I know is the only such measurement at RHIC or LHC. The E802 data has perfect centrality, all nucleons interact as measured in a ZDC, so the suggestion is that the flattening could be a long range correlation due to fluctuations in the number of participants in a centrality bin.

Cumulants are additive for independent processes -another NBD advantage

$$\frac{1}{k^{meas}(\delta\eta)} = K_2^{meas}(\delta\eta) = K_2^{dyn}(\delta\eta) + K_2^{bkg}(\delta\eta)$$

The two entries for E802 represent such a correction for background correlation from hits on adjacent wires.



In PRC78, PHENIX measured the effect of "geometry fluctuations" in 5% wide centrality bins and made a correction to $k_{dyn}=1/K_2^{dyn}$ which is shown for the 1 overlapping bin in the PRC76 and PRC78 measurements. (This would appear to return to the trend $k/\mu \approx$ constant vs the $\delta\eta$ interval and if true at all $\delta\eta$ would preserve the cumulant ratios vs the $\delta\eta$ acceptance!)??

Conclusions

•The NBD cumulant theorem brings a huge simplification to calculating the efficiency correction and statistical errors on net-charge cumulants.

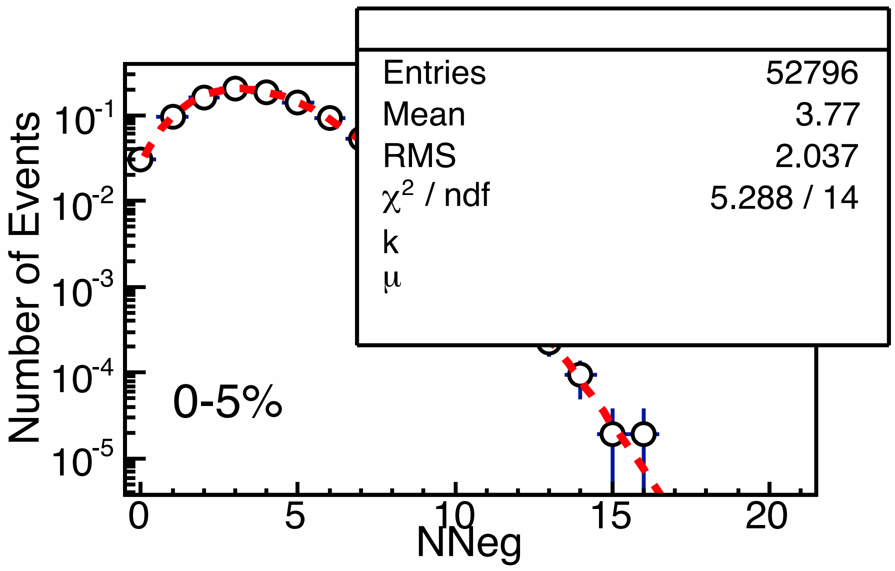
•Acceptance corrections are much more difficult because of short range correlations in $\delta\eta$ and $\delta\phi$, but in certain cases discussed above the cumulant ratios will remain constant independent of acceptance, so would be one possible resolution to the question of the “required acceptance” to compare experiments with Lattice QCD calculations

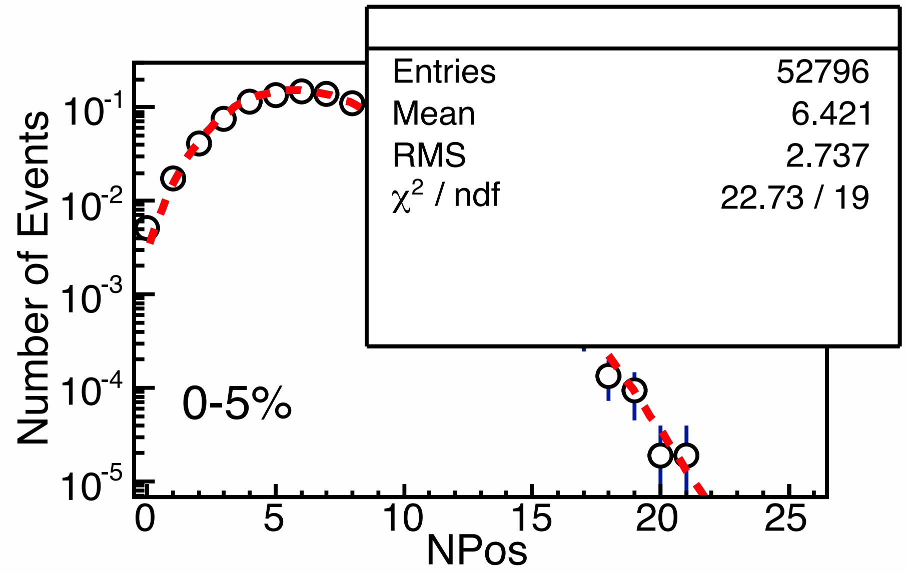
•Fortunately, the two above issues can be further investigated by both experiment and theory. For instance if the STAR NBD data for net charge were available, I could calculate the corrected values and the errors for $\kappa\sigma^2$, etc. Similarly STAR could make cuts in acceptance in their measurements to determine the variation in the results and whether or where the “required acceptance” is satisfied.

Extras

- NBD fit plots
- 4 generating functions
- k(δη) PRC76,0349033(2007)

PHENIX NBD fits

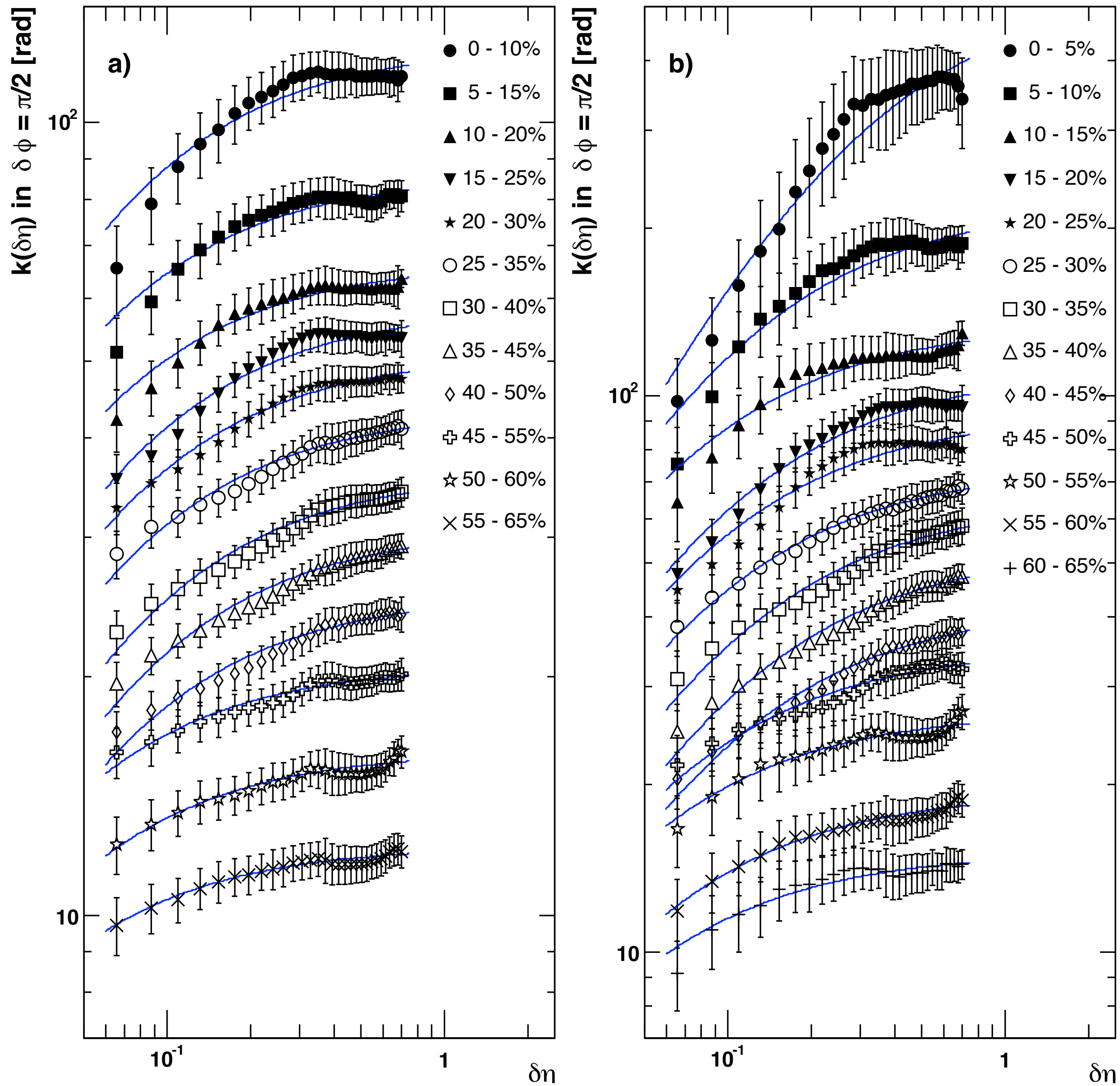




4 Generating functions

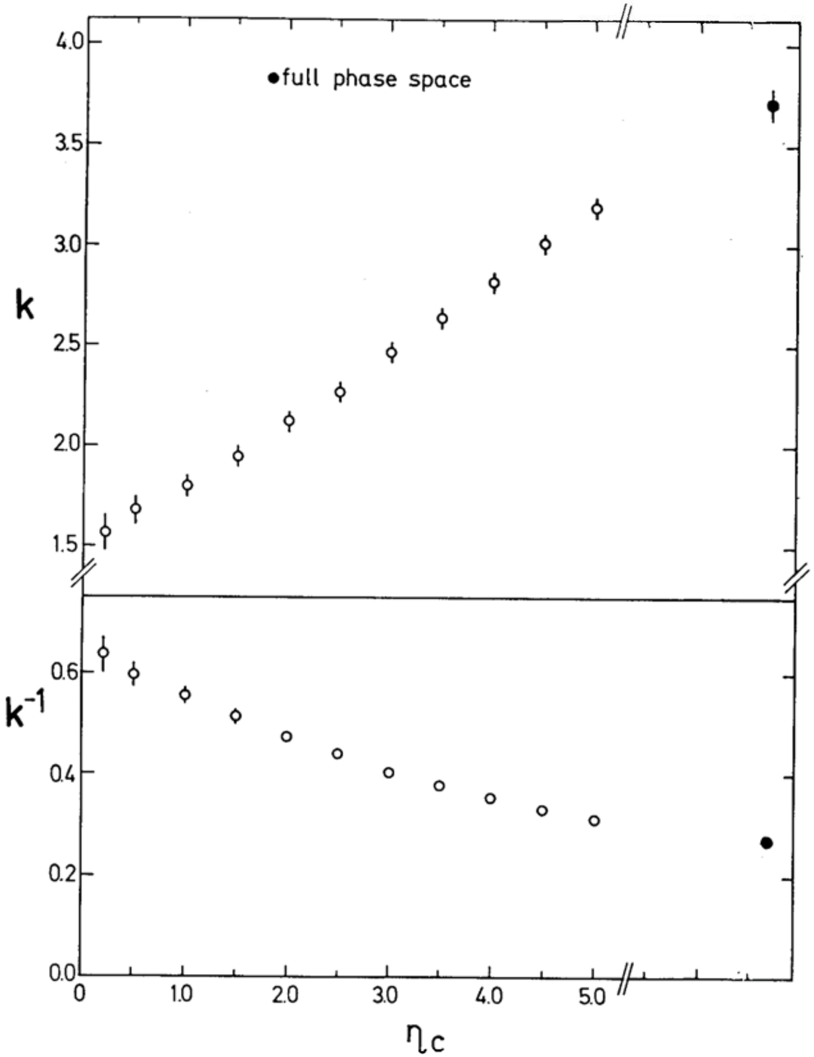
Moment generating fn	Cumulant generating function
$M'_x(t) = \left\langle e^{tx} \right\rangle$	$g_x(t) = \ln M'_x(t) = \ln \left\langle e^{tx} \right\rangle$
Factorial moment gen fn.	Factorial cumulant gen fn.
$M_x(t) = \left\langle (1+t)^x \right\rangle$	$g_x(t) = \ln \left\langle (1+t)^x \right\rangle$

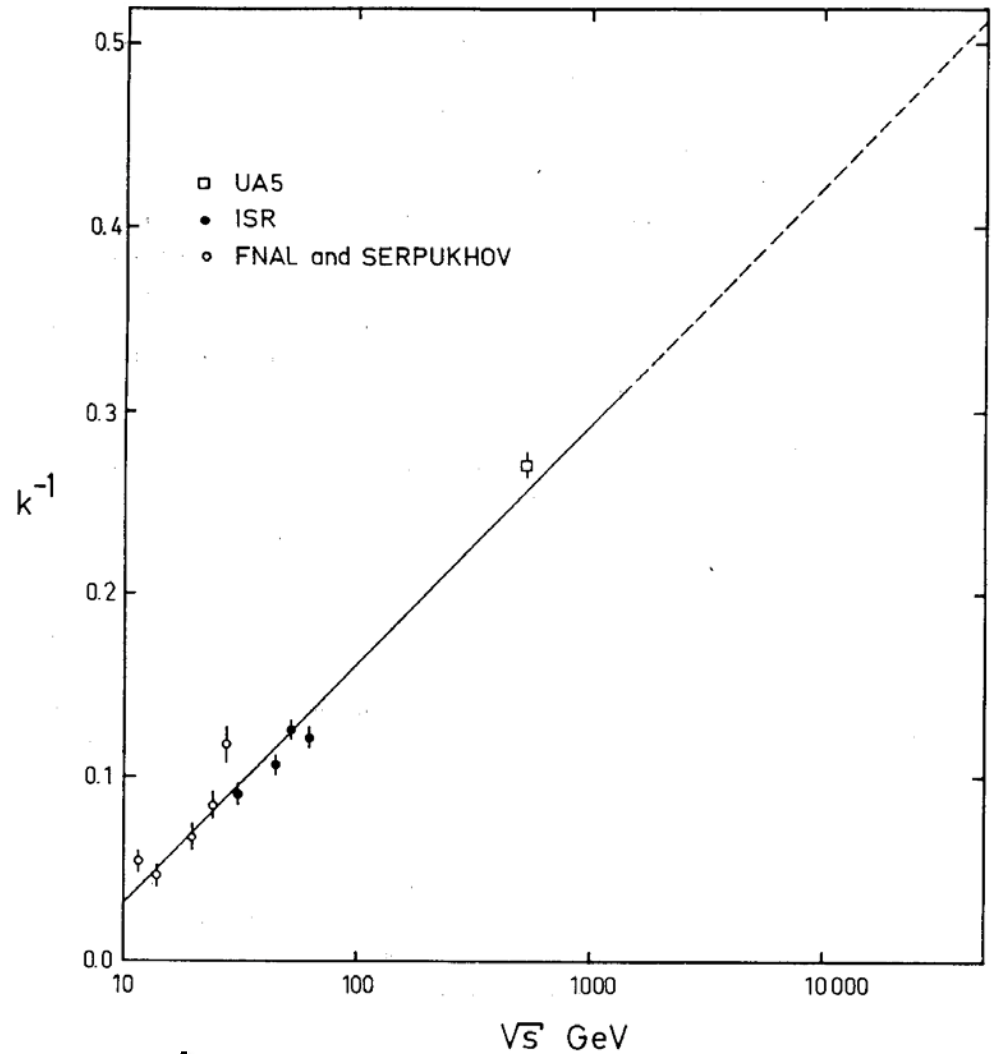
PHENIX k(δη) PRC76,0349033(2007)



END

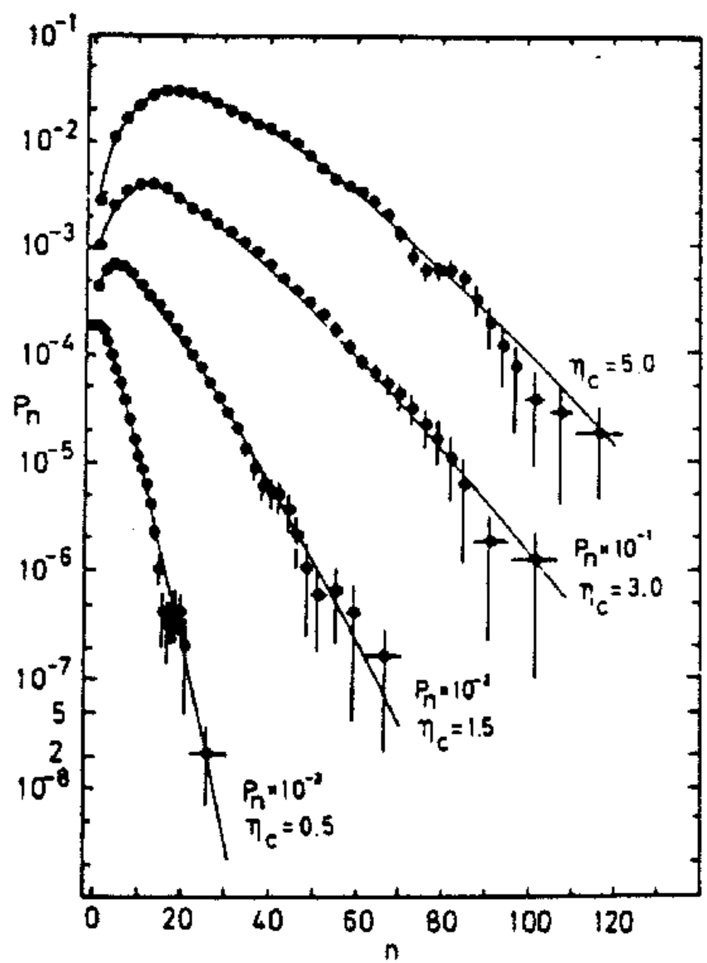
k vs $\Delta\eta=2\eta_c$ and $\sqrt{s}$

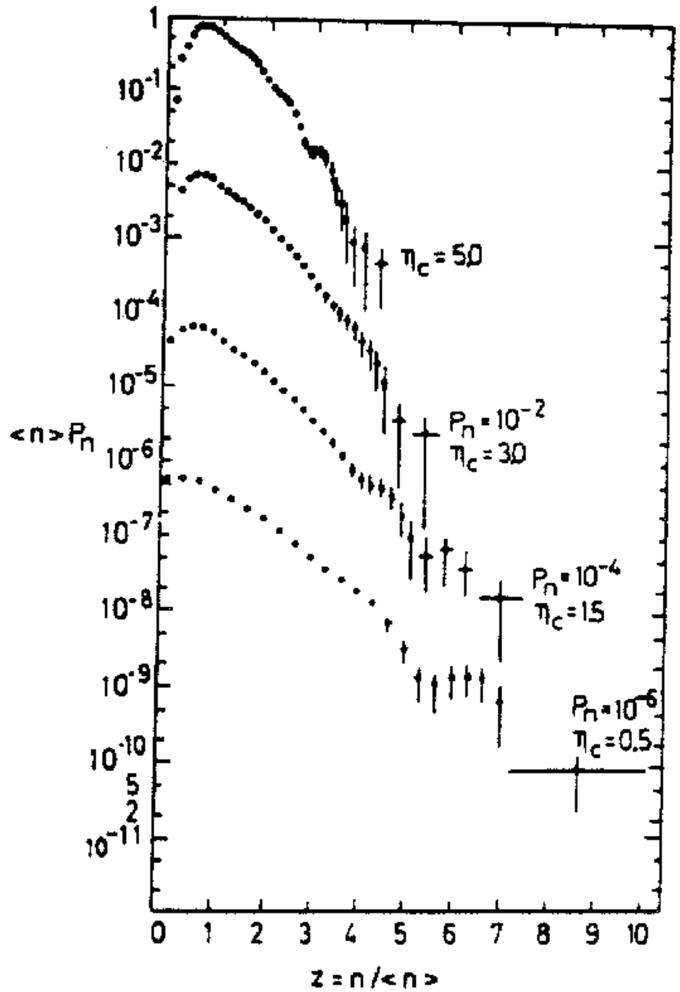




- Distributions are never poisson at any $\sqrt{s}$ and $\Delta\eta$
- Something fishy with NA49 p+p result

UA5--Multiplicity Distributions in (small) intervals |η|<η$_c$ around mid-rapidity are NBD





UA5 PLB **160**, 193,199 (1985); **167**, 476 (1986)

$\sqrt{s}$=540 GeV

Distributions are Negative Binomial, NOT POISSON: implies correlations

Proposed Phase diagrams Nuclear matter

Pawlowski-QM2014