Comments on e-by-e Sensitivity

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•GENERAL DEFINITIONS

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\langle \bar{x} \rangle = \langle x \rangle \equiv \mu$$

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$\langle x^2 \rangle = \sigma_x^2 + \langle x \rangle^2$$

$$\langle \bar{x}^2 \rangle = \sigma_{\bar{x}}^2 + \langle \bar{x} \rangle^2$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{n} \text{ for stat. ind. emission}$$

•Two Measures of Sensitivity—a Theorem

1)

$$(\sigma_{\bar{x}} - \frac{1}{\sqrt{n}}\sigma_x)/\frac{1}{\sqrt{n}}\sigma_x = (\frac{\sigma_{\bar{x}}}{\mu} - \frac{1}{\sqrt{n}}\frac{\sigma_x}{\mu})/\frac{1}{\sqrt{n}}\frac{\sigma_x}{\mu} = F$$

2)

$$(\frac{\sigma_{\bar{x}}^2}{\mu^2} - \frac{1}{n}\frac{\sigma_x^2}{\mu^2})/\frac{1}{n}\frac{\sigma_x^2}{\mu^2}$$

I prefer 2, but-a simple theorem

$$\frac{\Delta\sigma^2}{\sigma^2} = 2\frac{\Delta\sigma}{\sigma} = 2F$$

Event by Event Distribution

$$M_{p_T} = \overline{p_T}_{(n)} = \frac{1}{n}\sum_{i=1}^{n} p_{T_i} = \frac{1}{n} E_{Tc}$$

Analytical formula for statistically independent emission

For statistical independent emission an analytical formula for the distribution in M_{p_T} can be obtained. It depends on the 4 semi-inclusive parameters $\langle n \rangle$, $1/k$, b and p which are derived from the quoted means and standard deviations of the semi-inclusive p_T and multiplicity distributions for central Pb+Pb collisions for NA49. The result is in excellent agreement with the NA49 measurement.

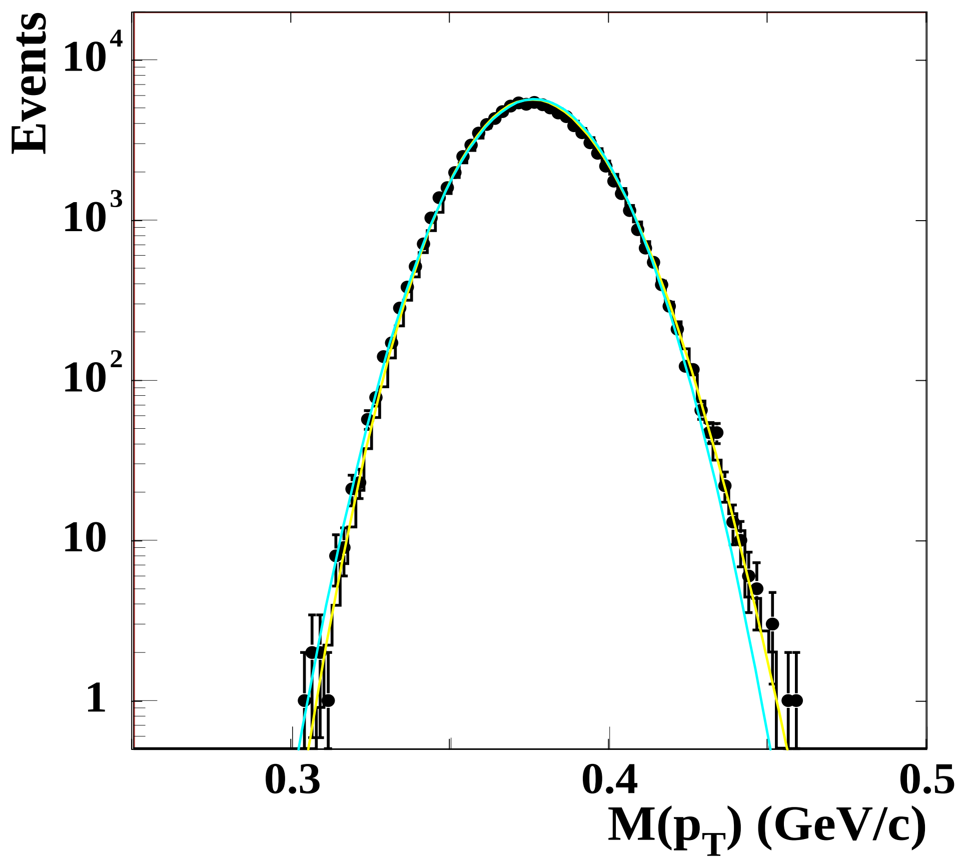


Figure 1: Full distribution in M_{p_T} (light line) compared to NA49 measurement (filled points) and mixed event distribution (histogram).

Hint, it's a Gamma Distribution

see M. J. Tannenbaum, Phys. Lett. B498, 29 (2001)

• The Φ Issue

$$\Phi = F \times \sigma_x$$

In words, Φ multiplies the fractional difference between the e-by-e σ from random by the INCLUSIVE σ.

Suppose the e-by-e σ is $\sim 1\%$ of the inclusive σ and you find a 1% difference in the e-by-e σ from random, why would you claim that this is equivalent to $\Phi = 1\%$ of the inclusive σ, which in this example equals the ENTIRE e-by-e σ ???

•What the e-by-e average tells you that you can't learn from the INCLUSIVE average

♡ e-by-e separates two or several classes of events with different μ and σ

A) Same μ different σ

B) Different μ same σ

B') Continuously varying T From a recent paper [R. Korus, *et al.* Phys. Rev. C**64**, 054908 (2001)] T varies with a mean $\langle T\rangle$ and standard deviation σ_T.

Note that in all cases the mean and standard deviation of the compound distribution must equal the measured inclusive values

♡ For B and B' the e-by-e effect is seen at the level of the moments

$$\frac{\sigma^2_{\bar{x}_T}}{\mu^2_T} = \frac{1}{n}\frac{\sigma^2_{x_T}}{\mu^2_T} + (1 - \frac{1}{n})\frac{\sigma^2_T}{\langle T\rangle^2} \qquad . \tag{1}$$

If you divide by random, you get an additional factor of n:

$$F = \frac{p}{2}(\langle n\rangle - 1)\frac{\sigma^2_T}{\langle T\rangle^2} \qquad , \tag{2}$$

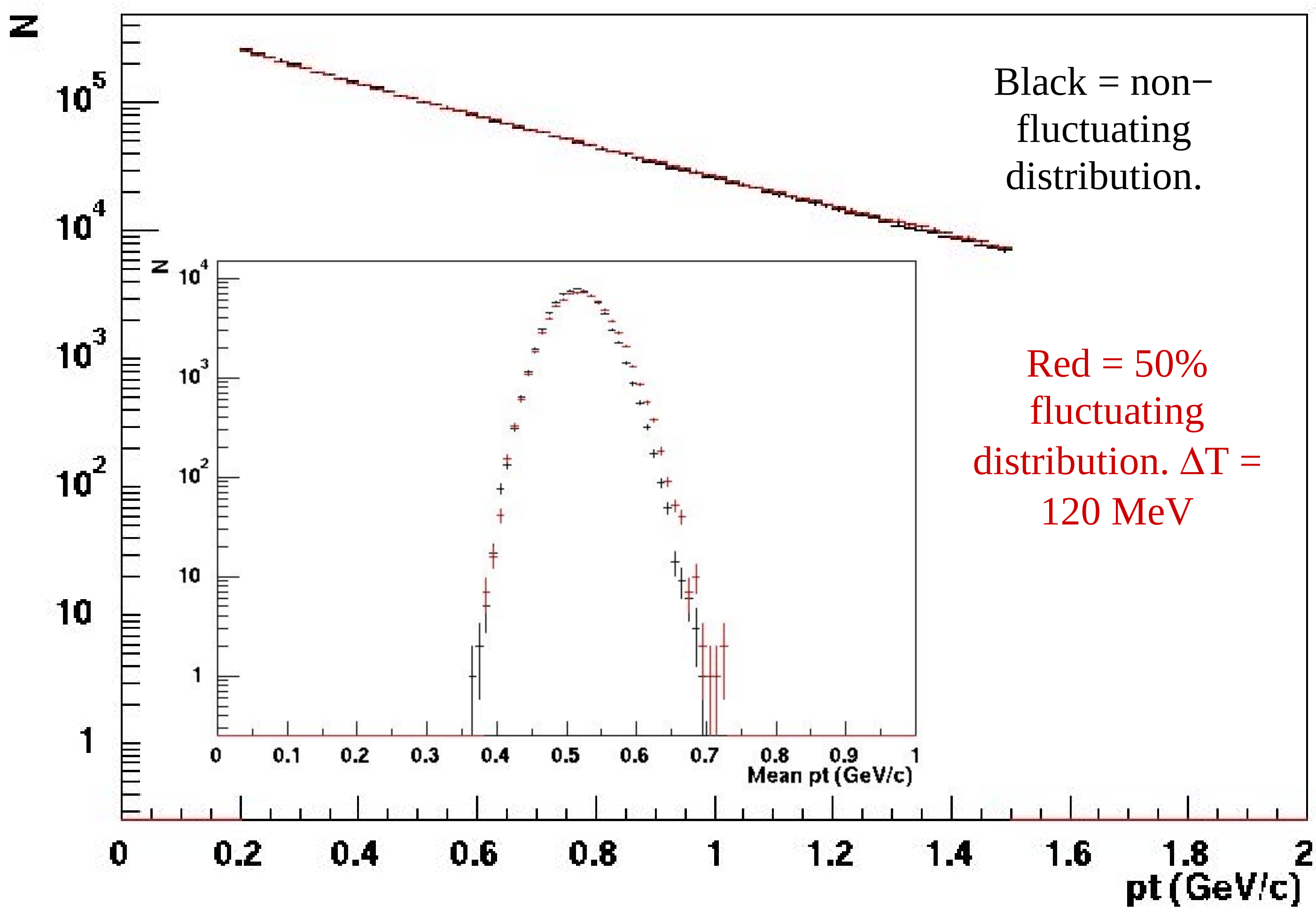
where p is the parameter of the inclusive Γ distribution.

♡ For A the e-by-e effect cannot be seen at the level of the moments The standard deviation $\sigma_{\bar{x}_c}$, of the compound distribution scales as $1/n$, exactly like the simple distribution, so that the e-by-e effect can not be seen from the variance. However the detailed shape of the e-by-e distributions are different for the simple and compound cases.

$$\frac{\sigma^2_{\bar{x}_c}}{\mu^2} = \frac{1}{n}\frac{\sigma^2_{x_c}}{\mu^2}$$

A) Distribution for same μ different σ

"Same mean, different variance" model demo



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