# Jets and hard-scattering in p-p collisions: Lessons from ISR/FNAL/SPS

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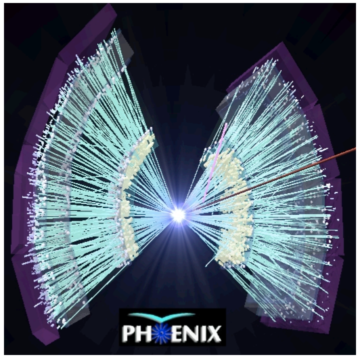


Colloquium
University of Jyväskylä, Finland
March 23, 2007

# $E_T$/Jets/hard-scattering Lessons from ISR/FNAL/SPS or why nobody of a certain age believes "proof by Monte Carlo"

[e.g. see M. D. Corcoran, Phys. Rev. D**32** (1985)592-603]

# Bjorken Scaling in Deeply Inelastic Scattering and the Parton Model---1968

♡ The discovery that the DIS structure function

$$F_2(Q^2,\nu) = F_2(\frac{Q^2}{\nu}) \qquad (1)$$

**"SCALED"** i.e just depended on the ratio

$$x = \frac{Q^2}{2M\nu} \qquad (2)$$

independently of $Q^2$ $(\sim 1/r^2)$

♡ as originally suggested by **Bjorken** Phys. Rev. **179**, 1547 (1969)

♡ Led to the concept of a proton composed of point-like **partons**. Phys. Rev. **185**, 1975 (1969)

$$\nu = \frac{Q^2}{2Mx}$$

- The probability for a parton to carry a fraction $x$ of the proton's momentum is measured by $F_2(x)$

S.M.Berman, J.D.Bjorken and J.B.Kogut, Phys. Rev. **D4**, 3388 (1971)

- BBK calculated for p+p collisions, the inclusive reaction

$$A+B \rightarrow C + X \qquad \text{when particle } C \text{ has } p_T \gg 1 \text{ GeV/c}$$

  - The charged partons of DIS **must scatter electromagnetically** *"which may be viewed as a* **lower bound** *on the real cross section at large $p_T$."*

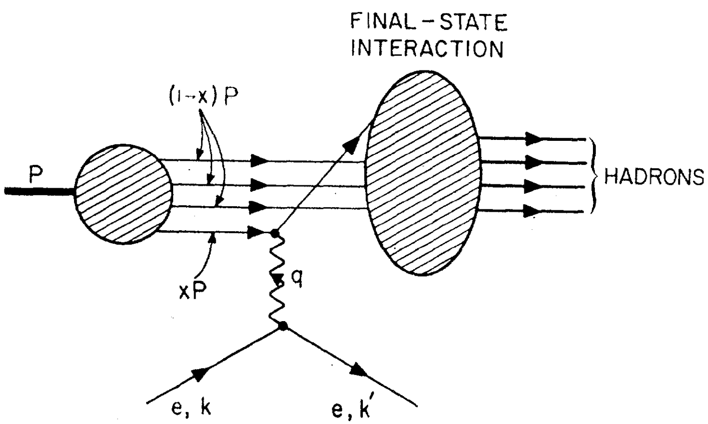
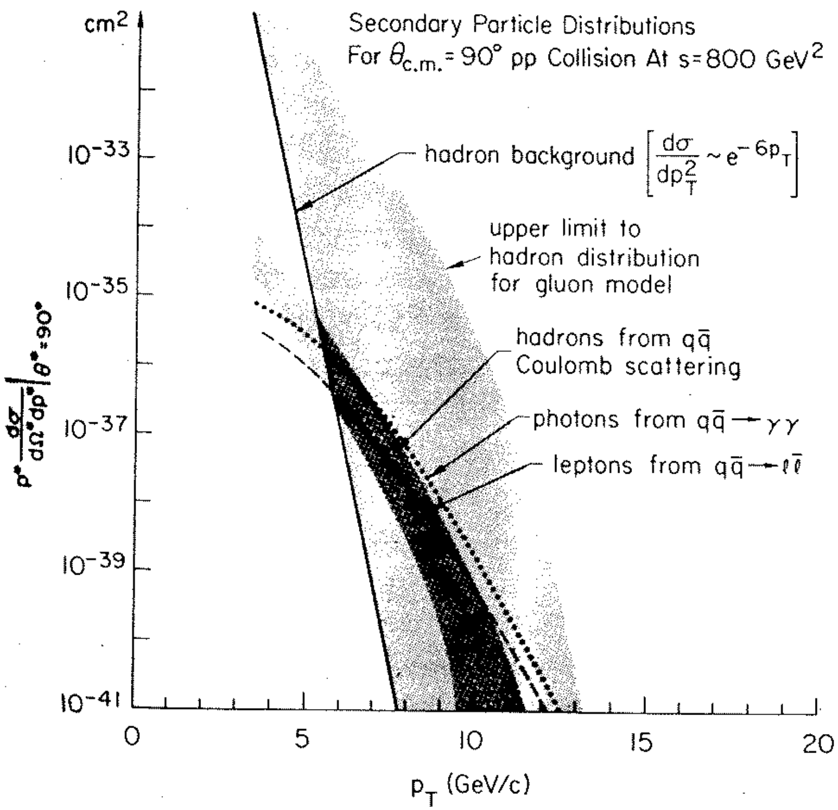



FIG. 1. Kinematics of lepton-nucleon scattering in the parton model.



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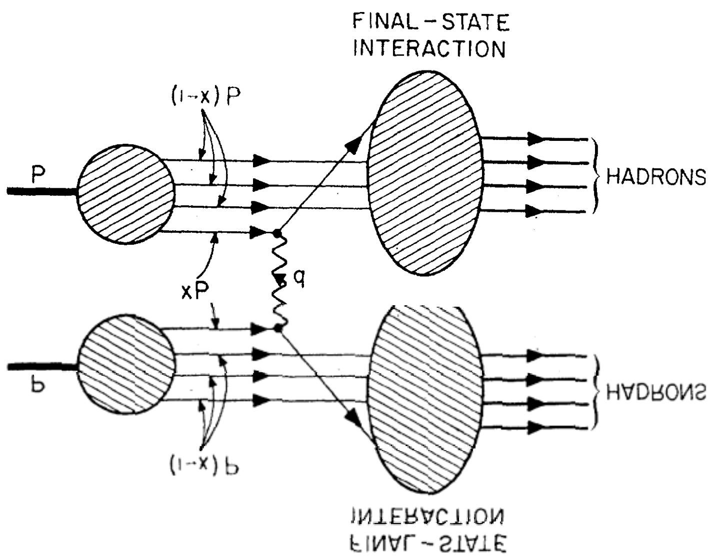
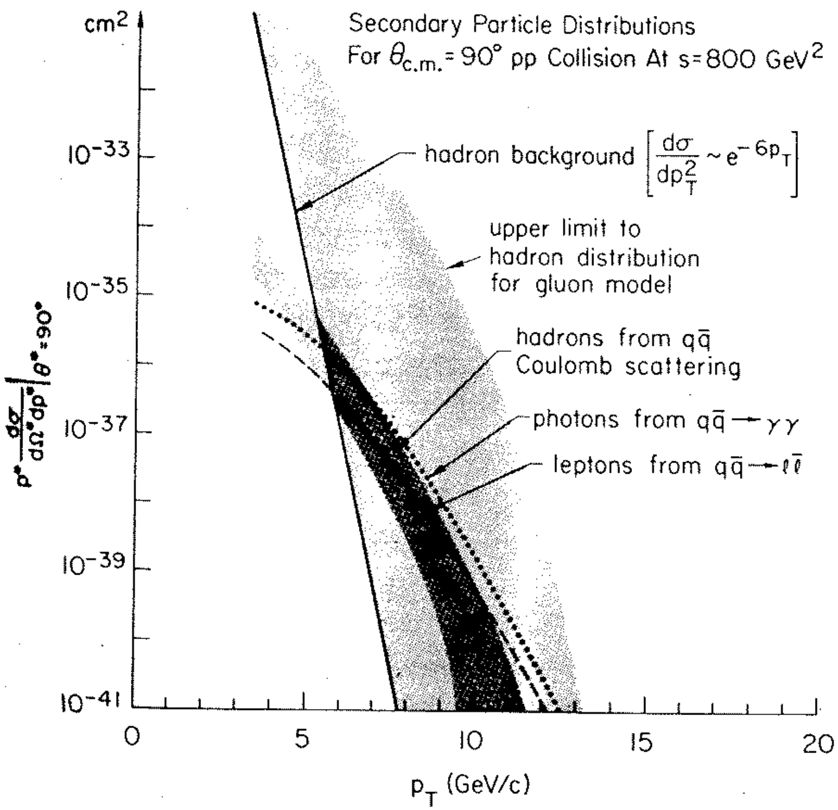


# BBK 1971-continued: the era of SCALING

♡ BBK propose a **General Form** for high $p_T$ cross sections, for the **EM** scattering, which must exist:

$$E\frac{d^3\sigma}{dp^3} = \frac{4\pi\alpha^2}{p_T^4}\mathcal{F}(x_1 = \frac{-\hat{u}}{\hat{s}}, x_2 = \frac{-\hat{t}}{\hat{s}}) \qquad (4)$$

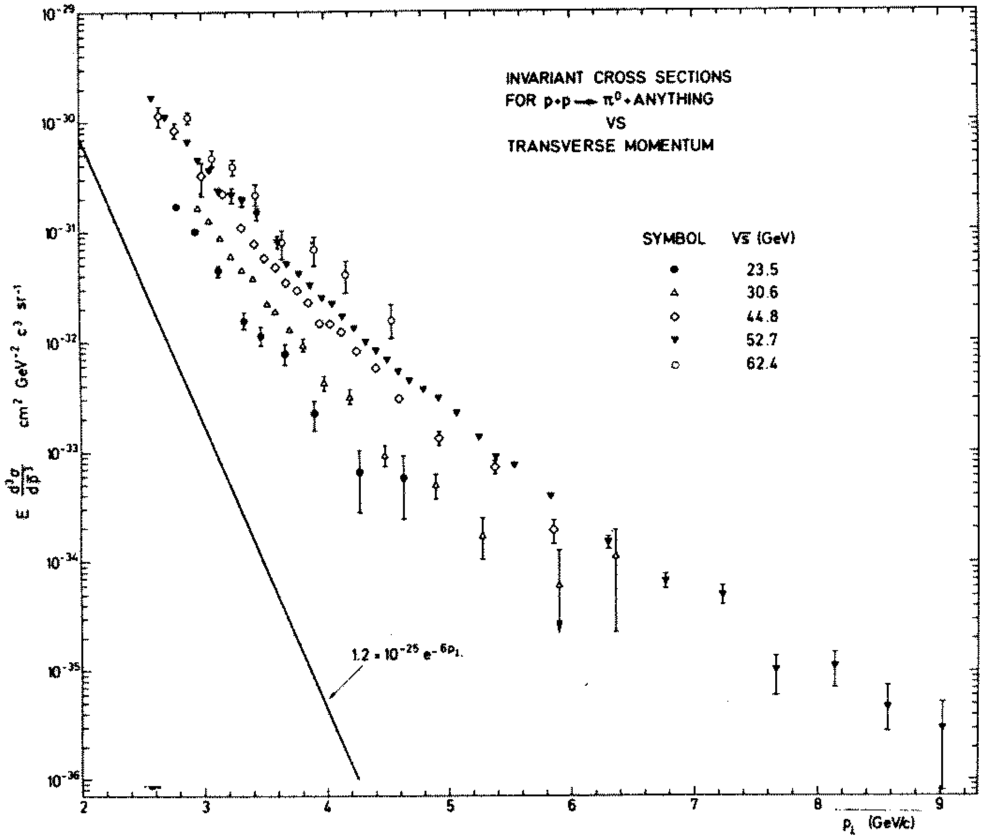
♡ The two factors are a $1/p_T^4$ term, characteristic of single photon exchange and a form factor $\mathcal{F}$

♡ Note that $x_{1,2}$ are not $x_{\rm BJ}$

- The point is that $\mathcal{F}$ **scales**, i.e. is only a function of the ratio of momenta.

♡ Vector ($J = 1$) Gluon exchange gives the same form as Eq. 4 but much larger.

# CCR at the CERN-ISR
# Discovery of high $p_T$ production in p-p



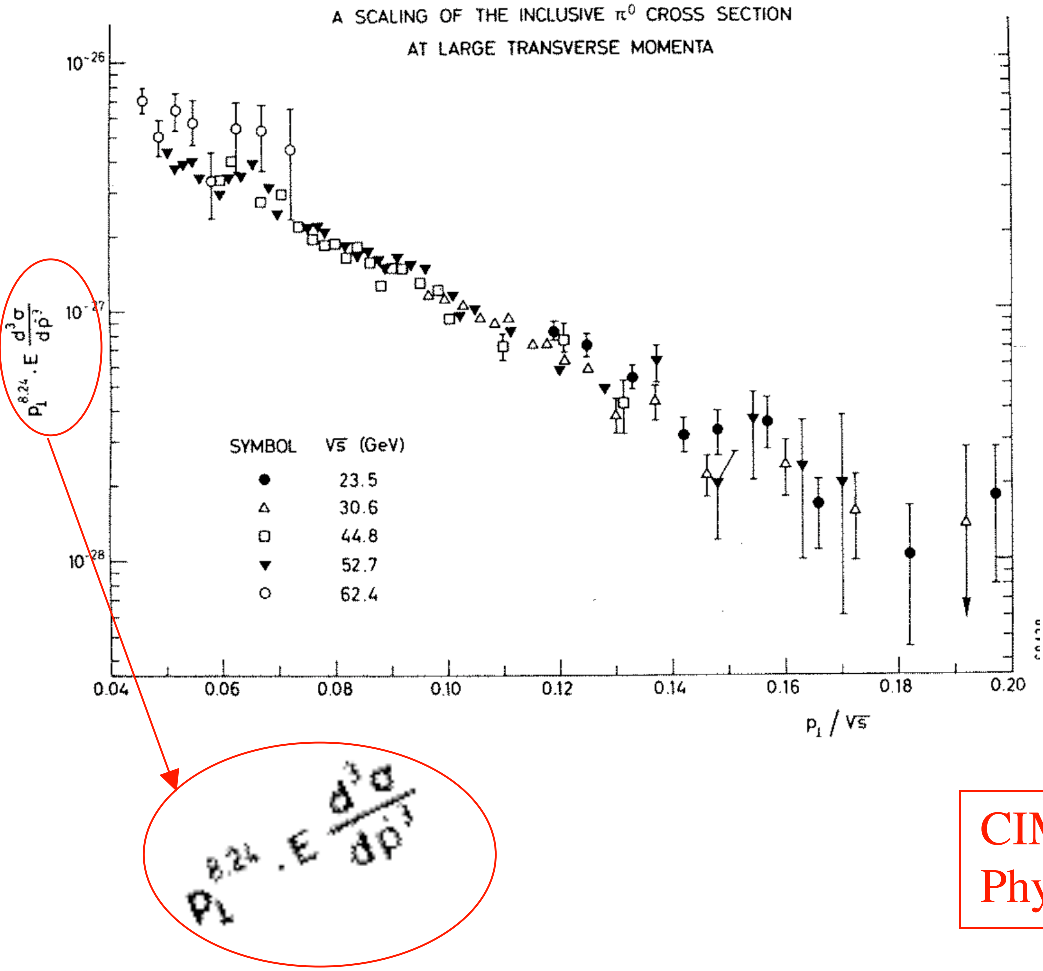

F.W. Busser, *et al.*,
CERN, Columbia, Rockefeller Collaboration
Phys. Lett. **46B**, 471 (1973)

Bj scaling → BBK scaling →
Blankenbecler, Brodsky, Gunion Scaling PL **42B**, 461 (1972)

$$E\frac{d^3\sigma}{dp^3} = \frac{1}{p_T^n}F(\frac{p_T}{\sqrt{s}})$$

- $e^{-6p_T}$ breaks to a power law at high $p_T$ with characteristic √s dependence
- Large rate indicates that partons interact strongly (>> EM) with other.
- Data follow BBK-BBG scaling but with n=8!, not n=4 as expected for QED

# $x_T$ scaling with n=8, not 4 Inspires Constituent Interchange Model




$$E\frac{d^3\sigma}{dp^3} = \frac{1}{p_T^n} F\left(\frac{p_T}{\sqrt{s}}\right)$$

$x_T = 2p_T/\sqrt{s}$

n=4 for QED or vector gluon

n=8 for quark-meson scattering by the exchange of a quark

CIM-Blankenbecler, Brodsky, Gunion, Phys.Lett.**42B**,461(1972)

# Constituent Interchange Model

Blankenbecler, Brodsky, Gunion, *Inclusive Processes at High Transverse Momentum*, Phys. Lett. **42B**, 461(1972)

♡ Inspired by the *dramatic features* of pion inclusive reactions revealed by "the recent measurements at CERN ISR of single-particle inclusive scattering at 90° and large transverse momentum", Blankenbecler, Brodsky and Gunion propose a new general scaling form:

$$E\frac{d^3\sigma}{dp^3} = \frac{1}{p_T^n} F\left(\frac{p_T}{\sqrt{s}}\right) \tag{5}$$

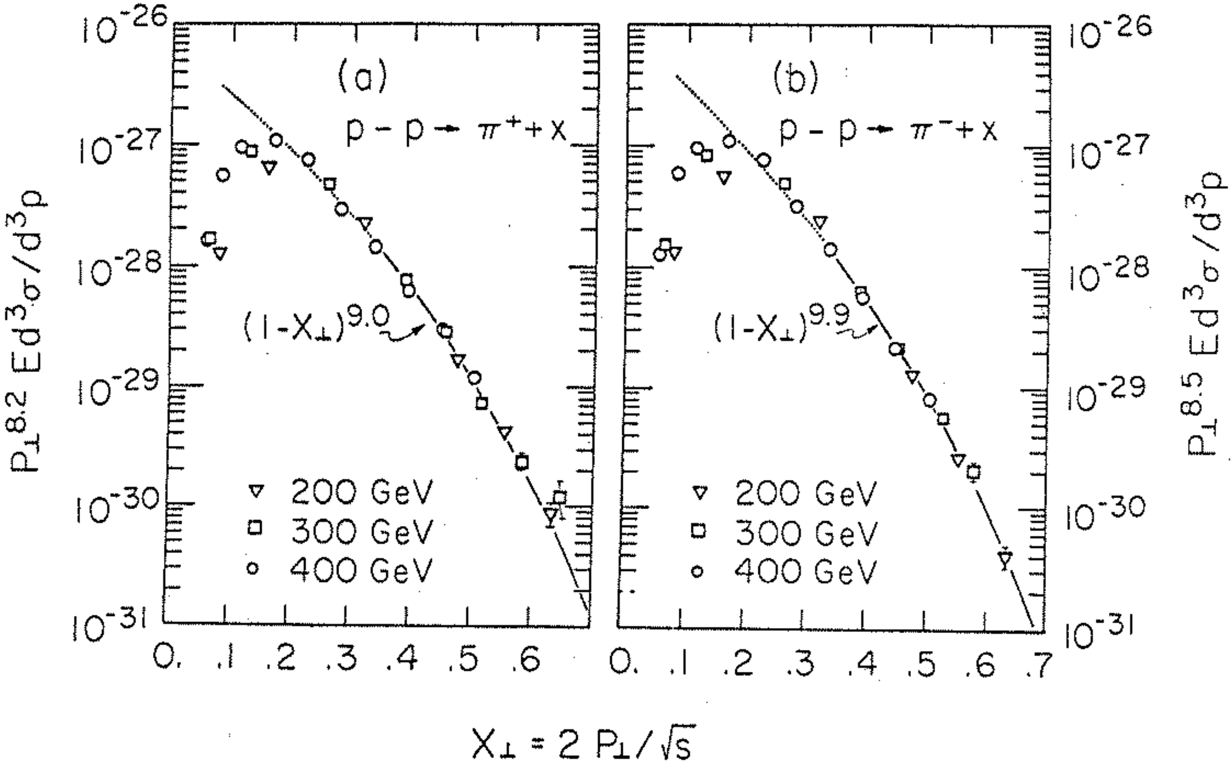
♡ $n$ gives the form of the force-law between constituents

♡ $n = 4$ for QED or Vector Gluon

♡ Perhaps more importantly, BBG predict $n$=**8** for the case of quark-meson scattering by the exchange of a quark, **C.I.M.**, as apparently observed.

# State of the Art
# Fermilab 1977

D. Antreasyan, J. Cronin, et al., PRL **38**, 112 (1977)




Beautiful $x_T$ scaling at all 3 fixed target energies with n=8

Totally Misleading--Not CIM or QCD but $k_T$

# First prediction using 'QCD' 1975

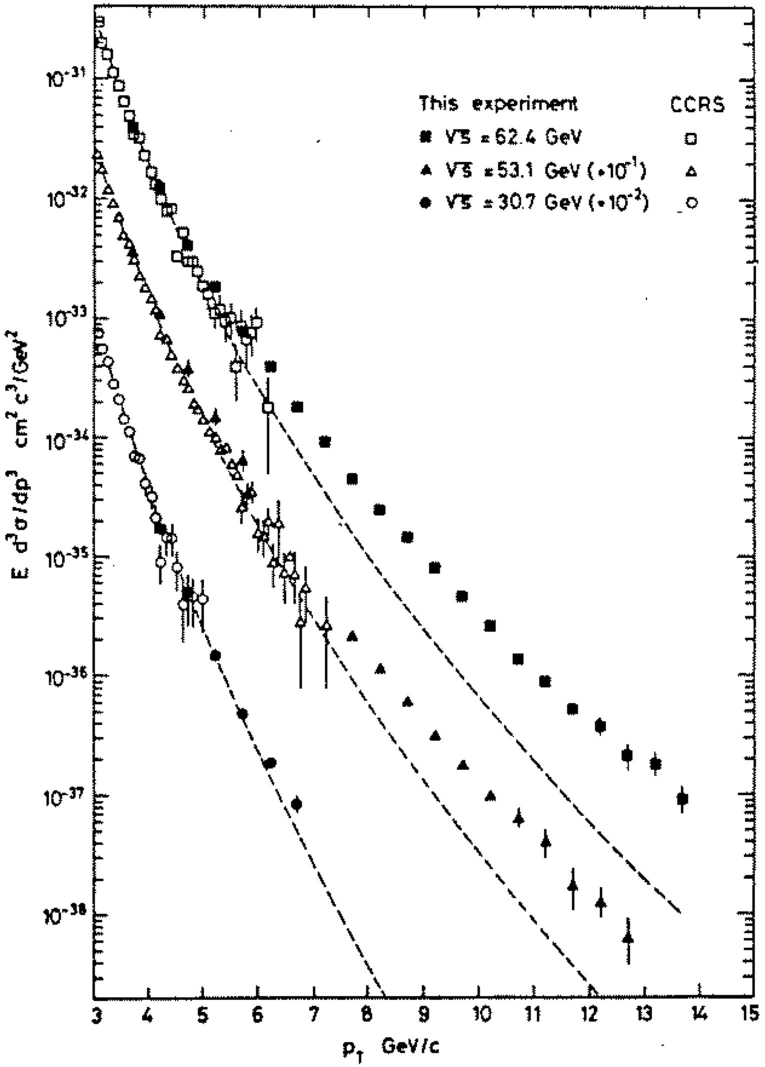
R. F. Cahalan, K. A. Geer, J. Kogut and Leonard Susskind, Phys. Rev. **D11**, 1199 (1975)

*"Asymptotic freedom and the "absence" of vector-gluon exchange in wide-angle hadronic collisions"*

- **Abstract:** The naive, pointlike parton model of Berman, Bjorken and Kogut is generalized to scale-invariant and asymptotically free field theories. The asymptotically free field generalization is studied in detail. Although such theories contain vector fields, single vector-gluon exchange contributes insignificantly to wide-angle hadronic collisions. This follows from (1) the smallness of the invariant charge at small distances and (2) the *breakdown of naive scaling* in these theories. These effects should explain the apparent absence of vector exchange in inclusive and exclusive hadronic collisions at large momentum transfers observed at Fermilab and at the CERN ISR.
- An interesting **Acknowledgement:** ... Two of us (J. K. and L. S. also thank S. Brodsky for *emphasizing to us repeatedly* that the present data on wide-angle hadron scattering *show no evidence for vector exchange.*
- Nobody's perfect, they get *one* thing right! They introduce the "effective index" $n(x_T, \sqrt{s})$ to account for 'scale breaking':

$$E\frac{d^3\sigma}{dp^3} = \frac{1}{p_T^{n(x_T,\sqrt{s})}} F(\frac{p_T}{\sqrt{s}}) = \frac{1}{\sqrt{s}^{n(x_T,\sqrt{s})}} G(\frac{p_T}{\sqrt{s}})$$

# CCOR 1978--Discovery of "REALLY high $p_T$>7 GeV/c" at ISR



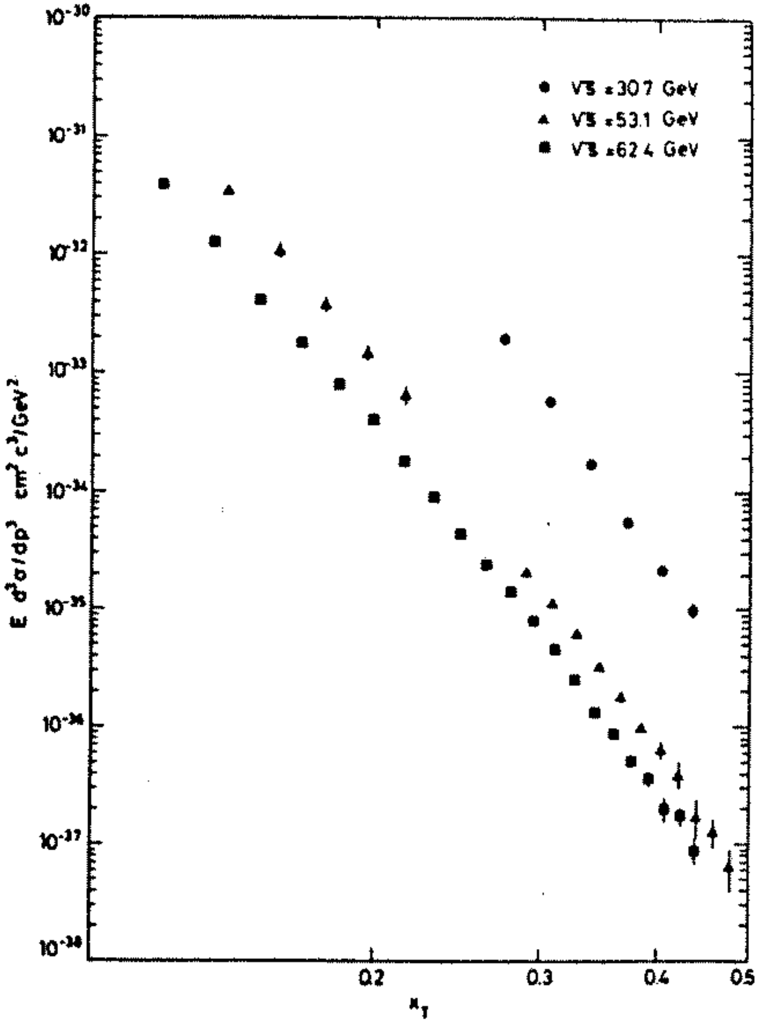

CCOR A.L.S. Angelis, et al, Phys.Lett. **79B**, 505 (1978)

See also A.G. Clark, et al Phys.Lett **74B**, 267 (1978)

- Agrees with CCR, CCRS (Busser) data for $p_T < 7$ GeV/c.
- Disagrees with CCRS fit pT > 7 GeV/c
- New fit is:

$$\heartsuit\quad Ed^3\sigma/dp^3 \simeq p_T^{-5.1\pm0.4}(1-x_T)^{12.1\pm0.6}$$
$$7.5 \leq p_T \leq 14.0 \text{ GeV/c},$$
$$53.1 \leq \sqrt{s} \leq 62.4 \text{ GeV}$$
(including *all* systematic errors).

# $n(x_T, \sqrt{s})$ WORKS $n \to 5 = 4^{++}$

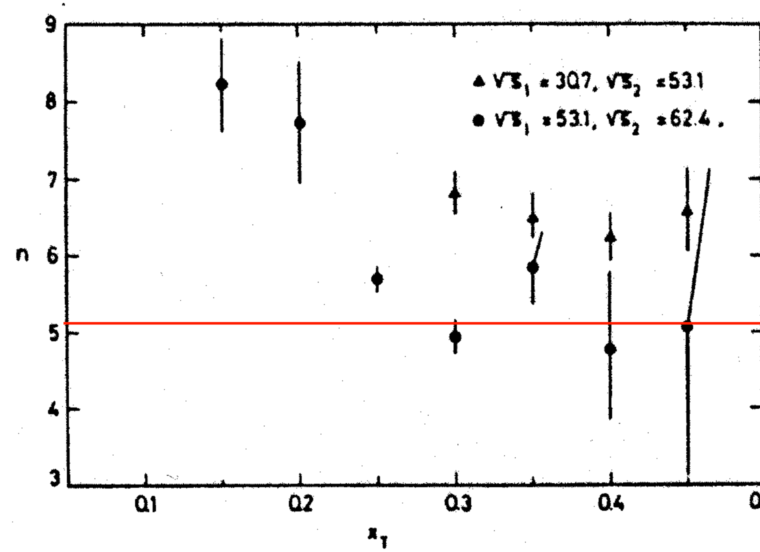



Same data $Ed^3\sigma/dp^3(x_T)$ ln-ln plot

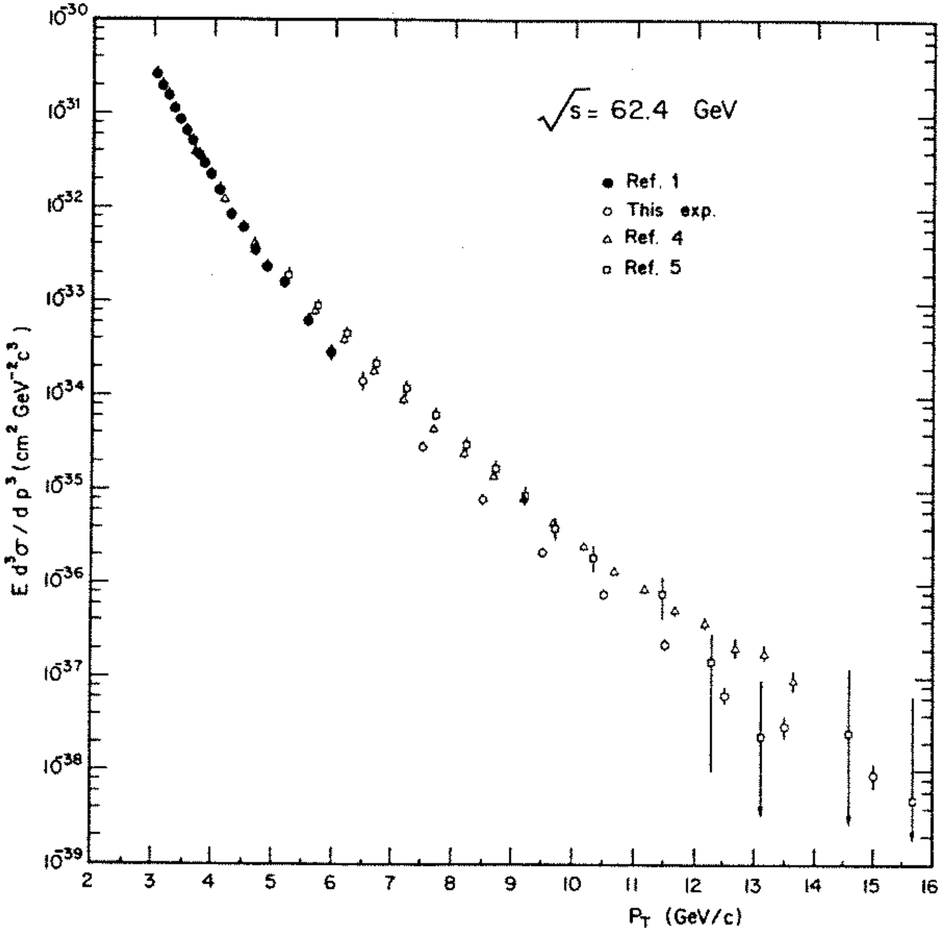
QCD: Cahalan, Geer, Kogut, Susskind, PR**D11**, 1199 (1975)

$$E\frac{d^3\sigma}{dp^3} = \frac{1}{\sqrt{s}^{\,n(x_T,\sqrt{s})}}\,G(x_T)$$

$$\left(\frac{\sqrt{s_1}}{\sqrt{s_2}}\right)^{n(x_T,\sqrt{s})} = \frac{E\frac{d^3\sigma}{dp^3}(x_T,\sqrt{s_2})}{E\frac{d^3\sigma}{dp^3}(x_T,\sqrt{s_1})}$$

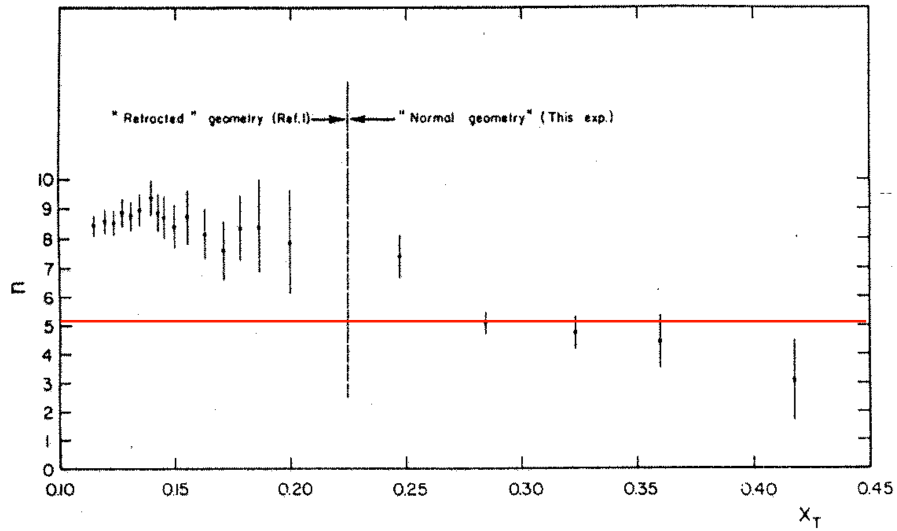


# ISR Expt's more interested in $n(x_T,\sqrt{s})$ than absolute cross section




Athens BNL CERN Syracuse Collaboration,
C.Kourkoumelis, et al
Phys.Lett. **84B**, 279 (1979)

But $n(x_T,\sqrt{s})$ agrees




cross sections vary by factor of 2

# Status of ISR single particle measurements 1978

♡ Hard-scattering was visible both at ISR and FNAL (Fixed Target) energies by single particle inclusive at large $p_T \geq$ 2-3 GeV/c.

♡ Scaling and dimensional arguments for plotting data revealed the systematics and underlying physics.

♡ The theorists had the basic underlying physics correct; but many (inconvenient) details remained to be worked out, several by experiment.

♡ $k_T$, the transverse momentum imbalance of outgoing partons (due to initial state radiation), was discovered by experiment.

$k_T$ is what made $n=4^{++} \rightarrow n=8$

# Status of QCD Theory in 1978

- The first modern QCD calculation and prediction for high $p_T$ single particle inclusive cross sections including non-scaling and initial state radiation was done in 1978 by J. F. Owens, E. Reya, M.Gluck, PRD **18**, 1501 (1978), "*Detailed quantum-chromodynamic predictions for high-$p_T$ processes*," and J.F. Owens, J. D. Kimel, PRD **18**, 3313 (1978), "*Parton-transverse-momentum effects and the quantum-chromodynamic description of high-$p_T$ processes*".

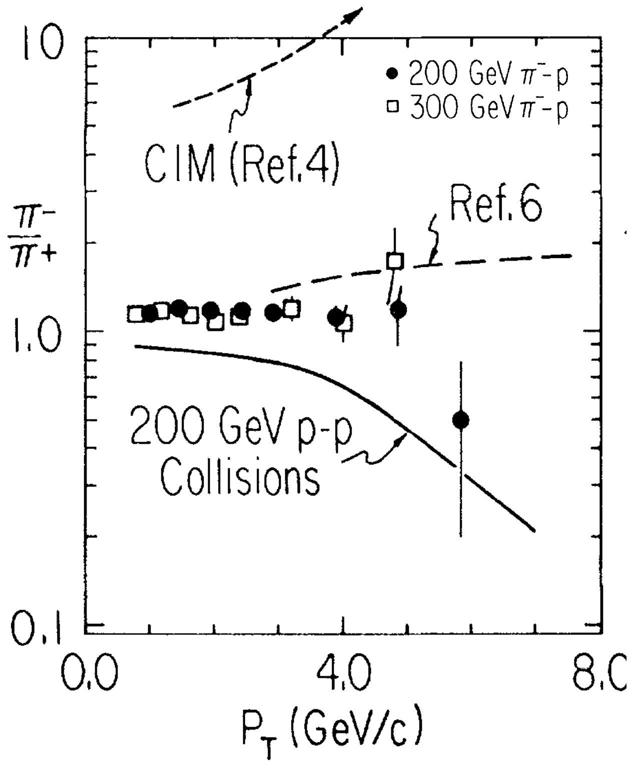
- This work was closely followed and corroborated by Feynman, Field, Fox PRD **18**, 3320 (1978), "*Quantum-chromodynamic approach for the large-transverse-momentum production of particles and jets.*"

- Unfortunately jets in $4\pi$ Calorimeters at ISR energies or lower are invisible below $\sqrt{\hat{s}} \approx E_T \leq 25$ GeV, which led to considerable confusion in the period 1980-1982.

# p.s. Fermilab experiment (1980) kills CIM

H. J. Frisch, et al., PRL **44**, 511 (1980) $\pi^- + p \rightarrow \pi^\pm + X$

If quark-meson scattering by exchange of a quark dominates then $\pi^-$ should dominate $\pi^+$ at large $p_T$




Statement in PLB**637**, 58 (2006):"We find that high-$p_T$ hadrons are produced by different mechanisms at fixed-target and collider energies. For pions, higher-twist subprocesses where the pion is produced directly dominate at fixed target energy," is contradicted by this measurement

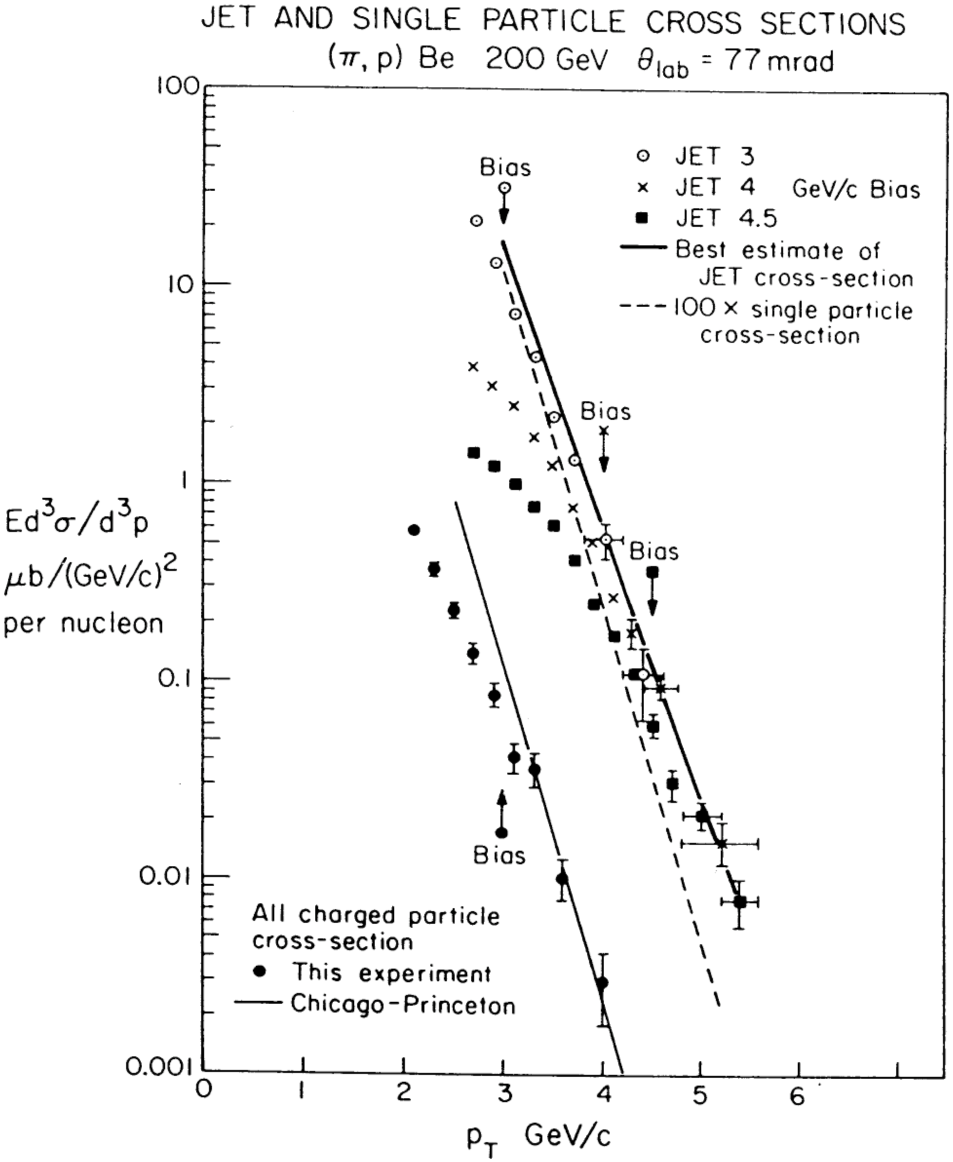
# QCD and Jets are now a cornerstone of the standard model

- Incredibly at the famous Snowmass conference in July 1982, many if not most people were skeptical
- The International HEP conference in Paris, three weeks later, changed everything.

# Why nobody (in the U.S.) believed in jets

- In 1972-73, soon after hard-scattering was discovered in p-p collisions, Bjorken PRD**8** (1973) 4098 and Willis (ISABELLE Physics Prospects-BNL-17522) proposed 4π hadron calorimeters to search for jets from fragmentation of scattered partons with large $p_T$ realizing that a substantial increase in rate would be expected in measuring the entire jet at a given $p_T$ rather than just the leading fragment. (Bjorken's parent-child effect)
- It took until 1980 to get a full azimuth $\Delta\eta\sim\pm0.88$ ($\Delta\Theta\sim\pm45^o$) calorimeter but meanwhile experiments were done with smaller back-to-back calorimeters each with aperture $\Delta\Phi\sim\pm45^o$ $\Delta\eta\sim\pm0.55$ and many new trigger biases were discovered, for instance, jets wider than the calorimeter aperture would deposit less energy than narrow jets of the same $p_T$ and be suppressed by the steeply falling spectrum⇒jet structure is dominated by the calorimeter geometry [e.g. see M. Dris NIM **158** (1979) 89]

# (In)famous FNAL E260 found "Jets" (1977)

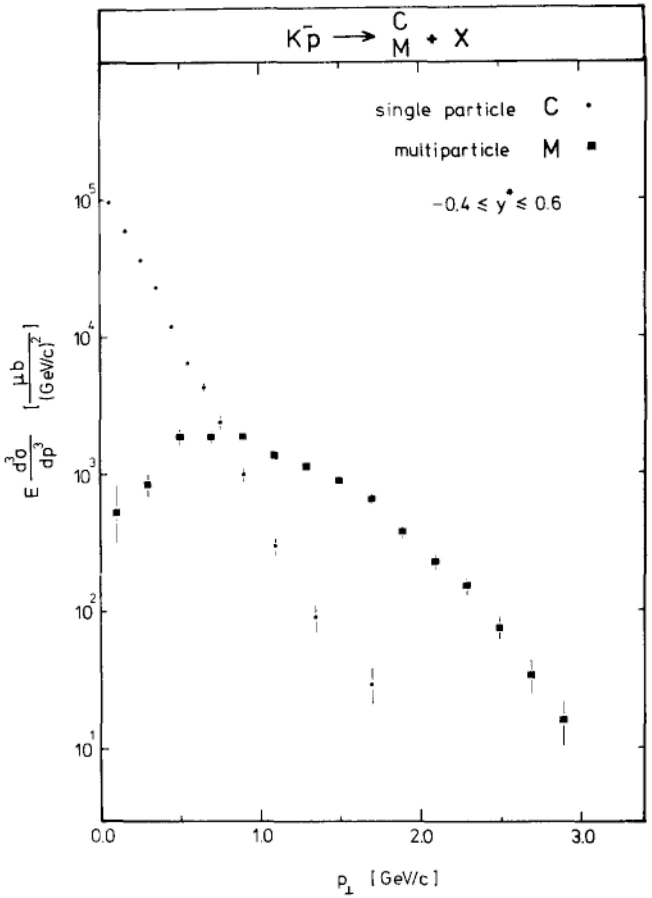


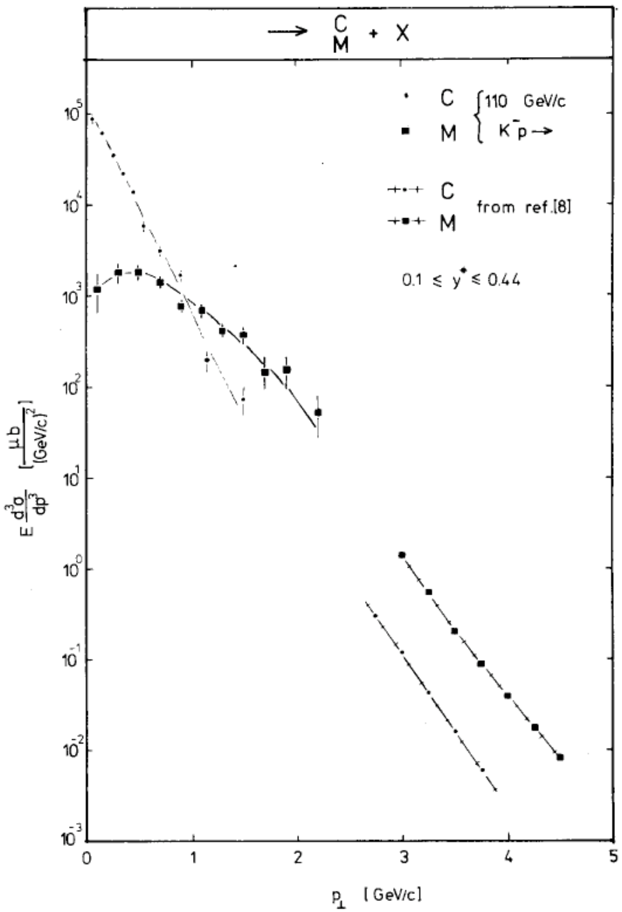

- In each of 2 back to back calorimeters with $\Delta\Phi \sim \pm 45^{o}$ $\Delta\eta \sim \pm 0.36$ (same as PHENIX) the invariant cross section of several particles with a vector sum $p_T$ is much larger than a single particle of the same $p_T$. The authors took this as evidence for the exactly back-to-back in azimuth jets of constituent scattering⇒Never let an interested theorist collaborate on an experiment.

**C.Bromberg et al E260, PRL 38 (1977)1447, NPB134 (1978) 189**

# But, experiments with different apertures got different results

- The first 4π experiment was a bubble chamber(!) 110 GeV/c $K^-$ on p [M. Deutschmann, et al, ABCCLVW collab, NPB**155** (1979)307]

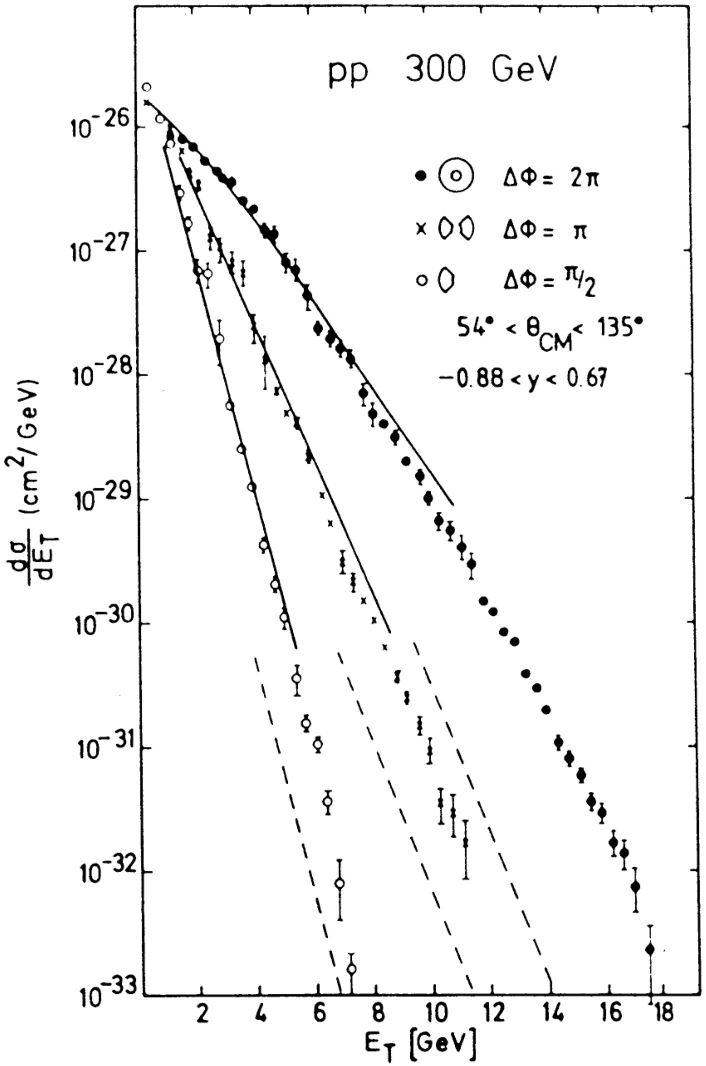



- multiparticle cross section for $p_T$> 1.5 GeV/c >> single particle
- Data extrapolate nicely to those of E260 [8] in slope and magnitude.
- But ``principal axis'' analysis of the data shows "the vast majority of events with large $p_T$ multiparticle systems DO NOT exhibit jet-like structure."

# NA5-the coup-de-grâce to jets (1980)

- Full azimuth calorimeter $-0.88<\eta^*<0.67$ ($\rightarrow$ NA35, NA49)



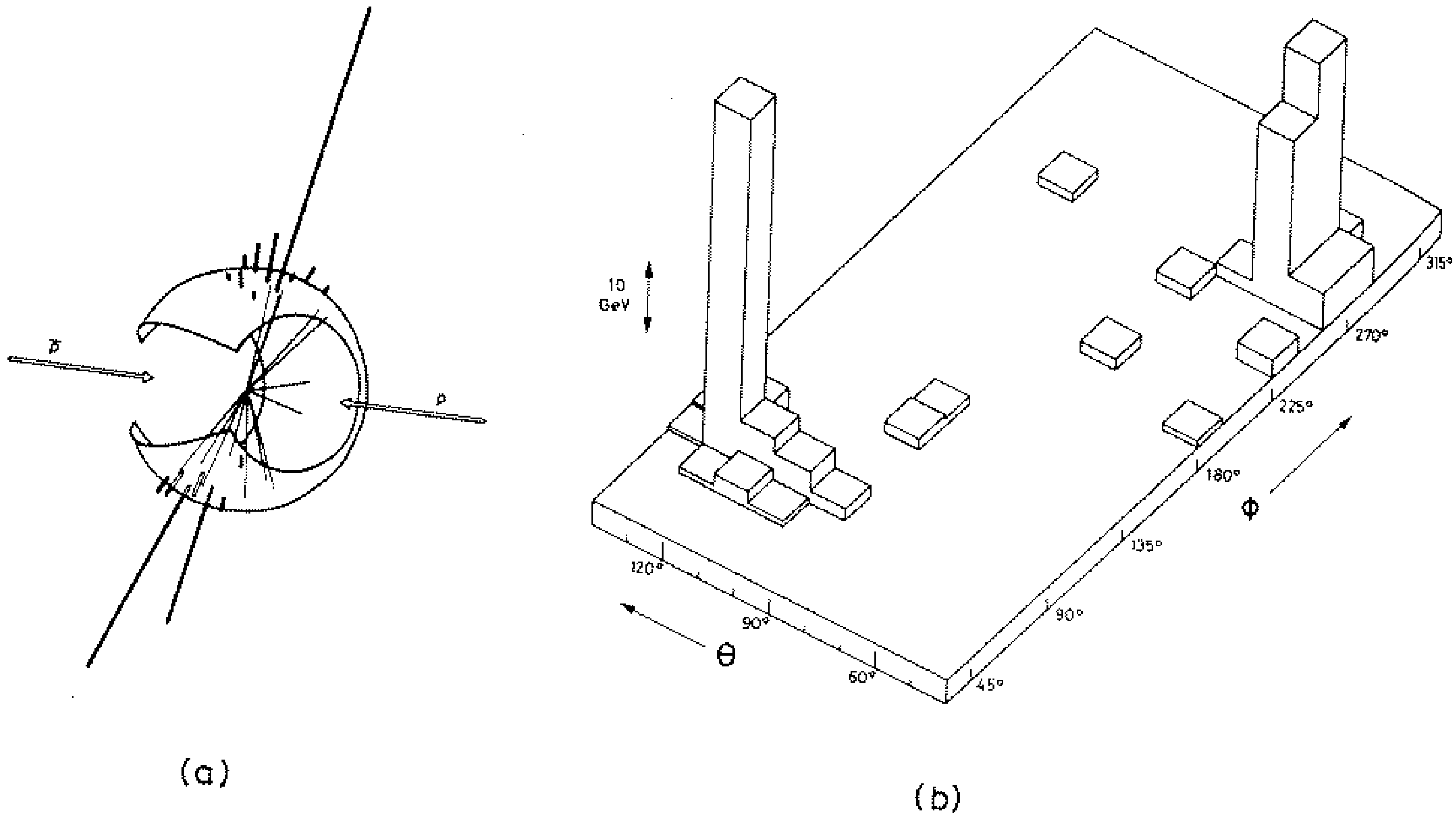

- plus triggered in two smaller apertures corresponding to E260.
- No jets in full azimuth data
- All data way above QCD predictions--------
- The large $E_T$ observed is the result of "a large number of particles with a rather small transverse momentum"--the first $E_T$ measurement in the present terminology.

K. Pretzl, Proc 20th ICHEP (1980)
C. DeMarzo et al NA5, PLB**112**(1982)173

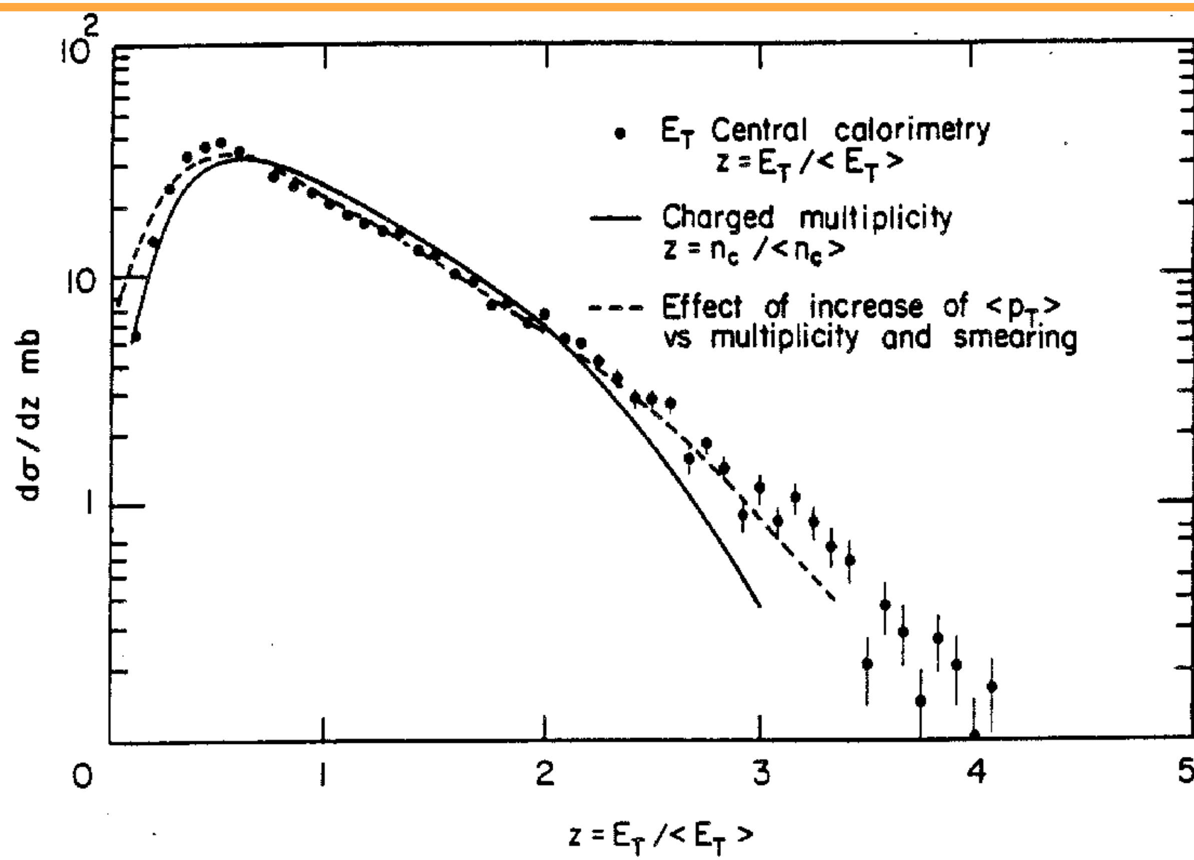
For more on $E_T$ see MJT IJMPA **4** (1989)3377

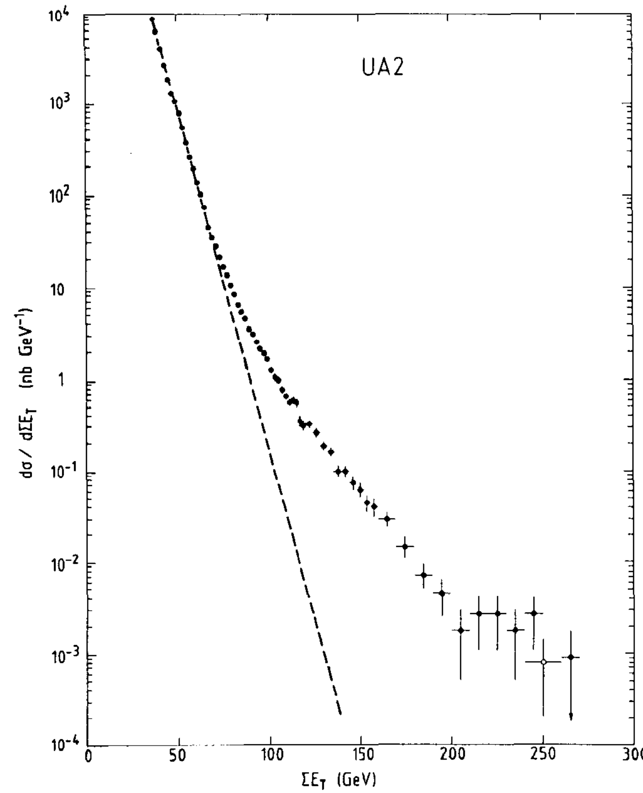
# Back to-THE UA2 Jet-Paris 1982

From 1980--1982 most high energy physicists doubted jets existed because of the famous NA5 $E_T$ spectrum which showed NO JETS. This one event from UA2 in 1982 changed everybody's opinion.



# UA1-Carlo himself explained $E_T$ (no jet) dist. before seeing UA2 plot-explanation is correct

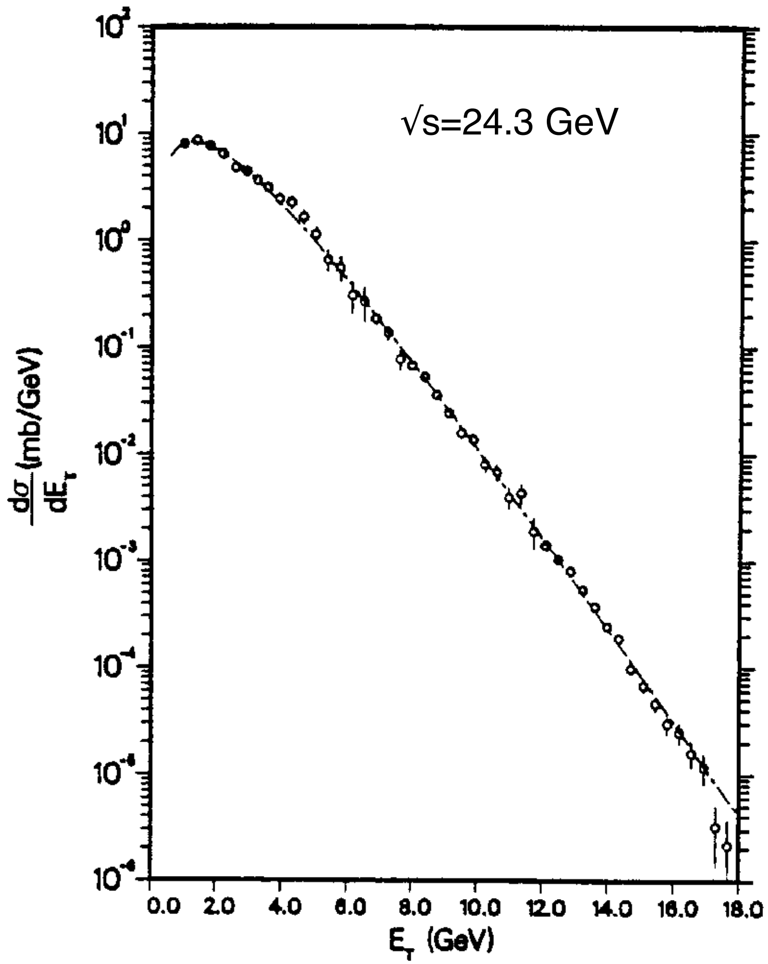


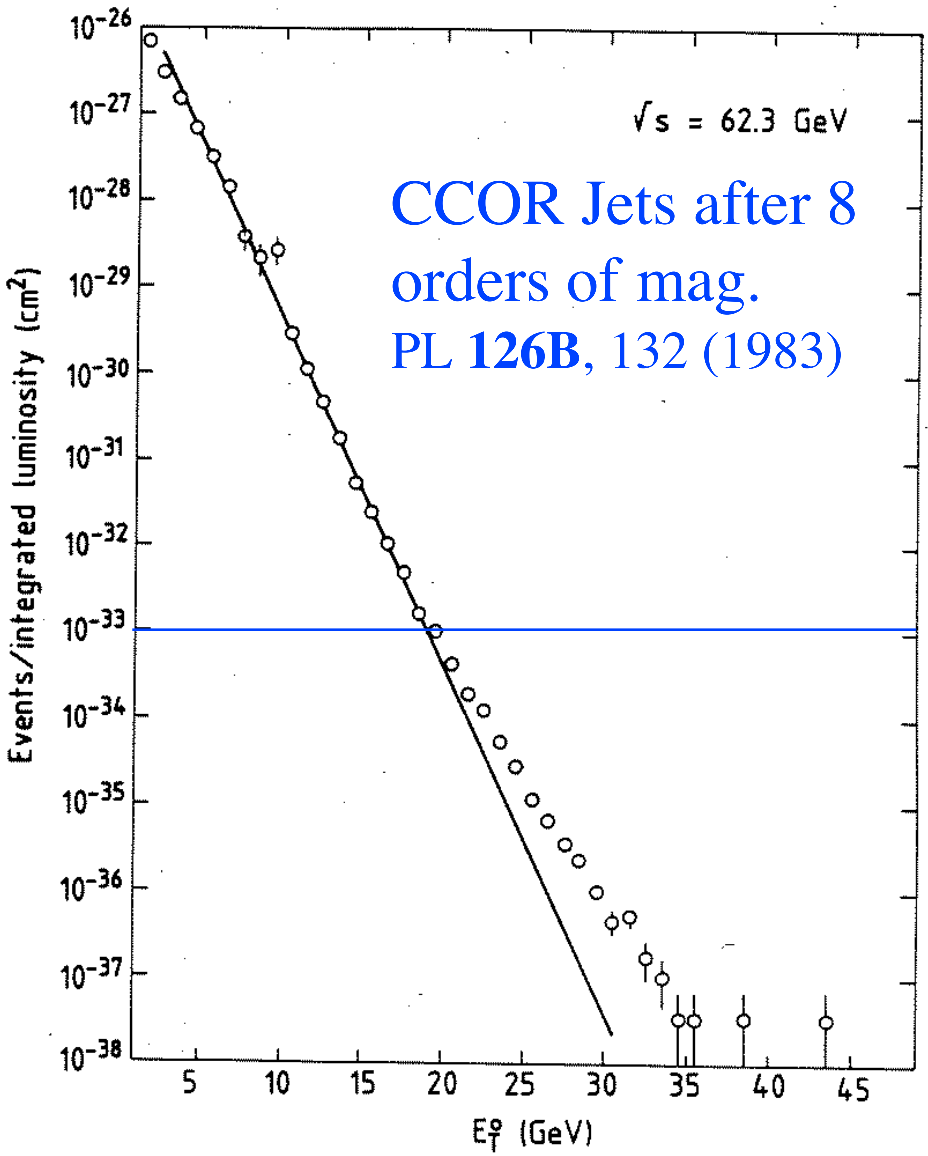

UA1 (1982) Paris-withdrawn (C.Rubbia) $\sqrt{s}$=540 GeV. No Jets because $E_T$ is like multiplicity (n), composed of many soft particles near $<p_T>$ ! CERN-EP-82/122.

OOPS UA2 discovers jets ~5-6 orders of magnitude down in $E_T$ distribution!

# Also Paris 1982-Jets in $E_T$ distribution

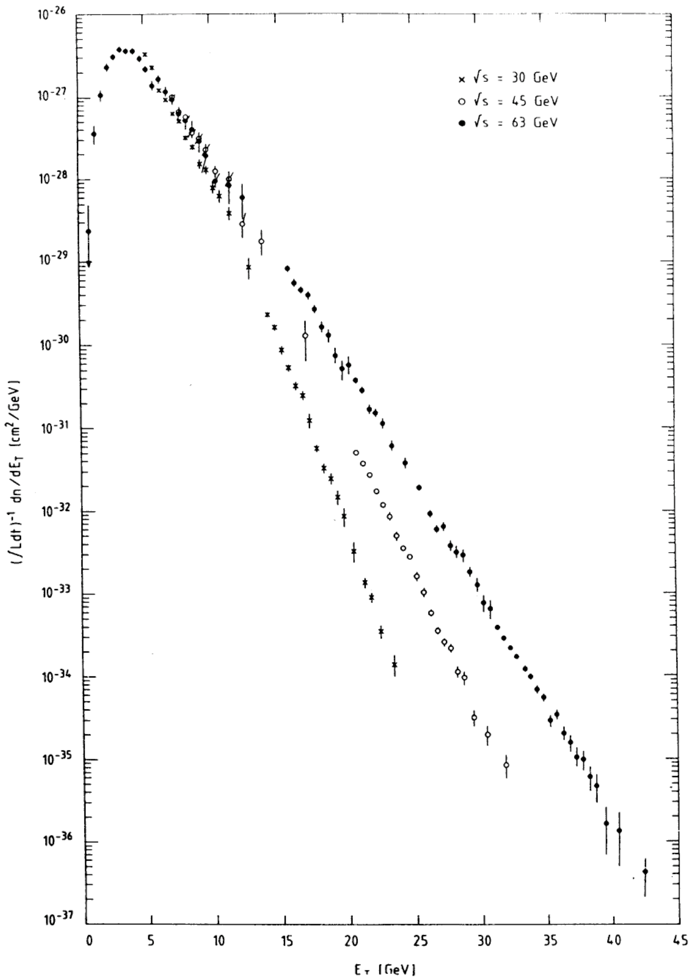


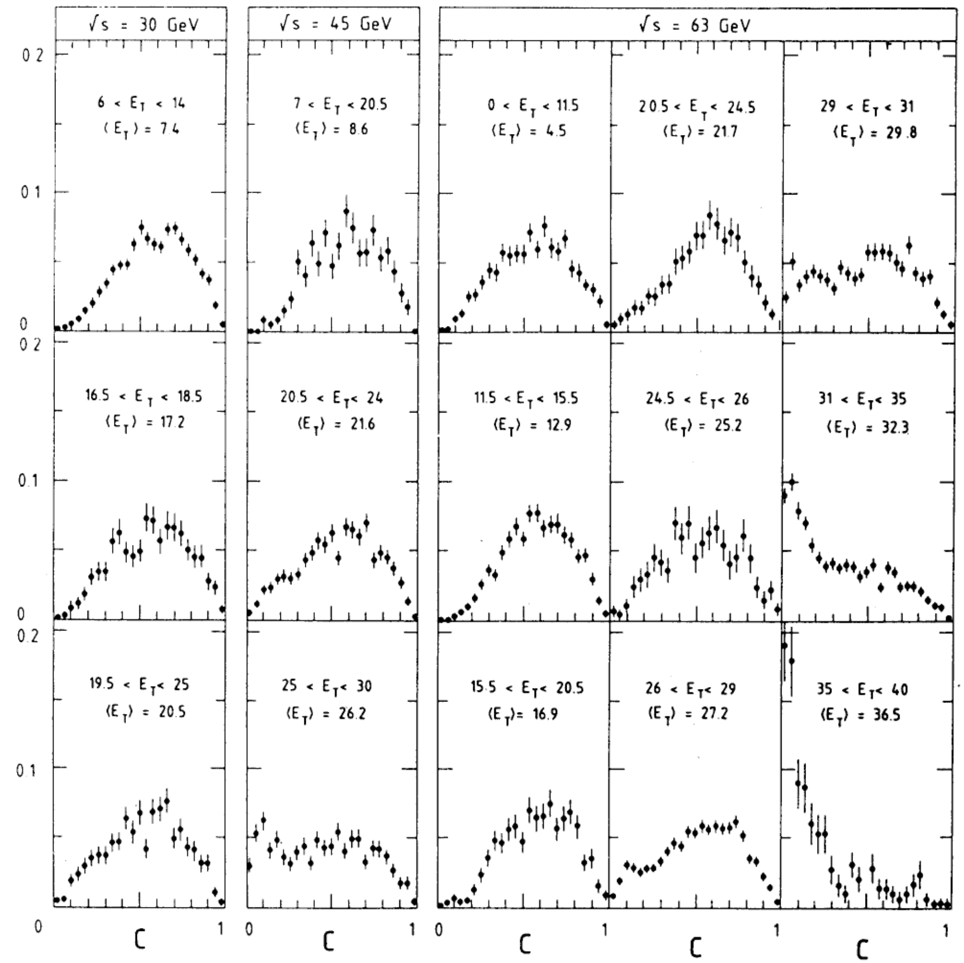

NA5-1980 ICHEP-No Jets
7 orders of magnitude




CCOR Jets after 8 orders of mag.
PL **126B**, 132 (1983)

# Jets in ET with unbiased trigger-circularity




Low circularity, i.e. pencil-like di-jet structure seen only for

$$\sqrt{\hat{s}} \approx E_T > 25\text{-}30 \text{ GeV}$$

# LO-QCD in 1 slide

## Cross Section in p-p collisions c.m. energy $\sqrt{s}$

The overall p-p reaction cross section
is the sum over constituent reactions

$$a + b \to c + d$$

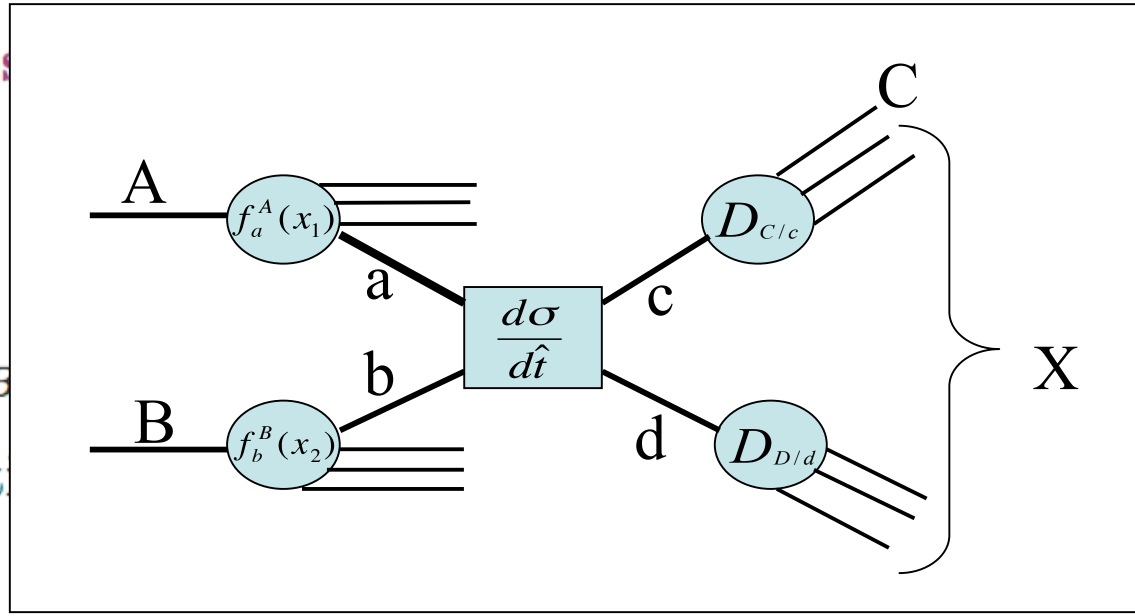
$f_a^A(x_1)$, $f_b^B(x_2)$, are structure functions, the differential probabilities
for constituents $a$ and $b$ to carry momentum fractions $x_1$ and $x_2$
of their respective protons, e.g. $u(x_1)$,

$$\frac{d^3\sigma}{dx_1 dx_2 d\cos\theta^*} = \frac{1}{s}\sum_{ab} f_a^A(x_1) f_b^B(x_2) \frac{\pi\alpha_s^2(Q^2)}{2x_1x_2}\Sigma^{ab}(\cos\theta^*)$$

$\Sigma^{ab}(\cos\theta^*)$, the characteristic subprocess angular distributions
and $\alpha_s(Q^2) = \frac{12\pi}{25\ln(Q^2/\Lambda^2)}$ are predicted by QCD

# LO-QCD in 1 slide

Cros ... $\sqrt{s}$




$f_a^A(x_1)$, $f_b^B$ ... robabilities
for const ... $_1$ and $x_2$

$$\frac{d^3\sigma}{dx_1 dx_2 d\cos\theta^*} = \frac{1}{s}\sum_{ab} f_a^A(x_1) f_b^B(x_2) \frac{\pi\alpha_s^2(Q^2)}{2x_1x_2} \Sigma^{ab}(\cos\theta^*)$$

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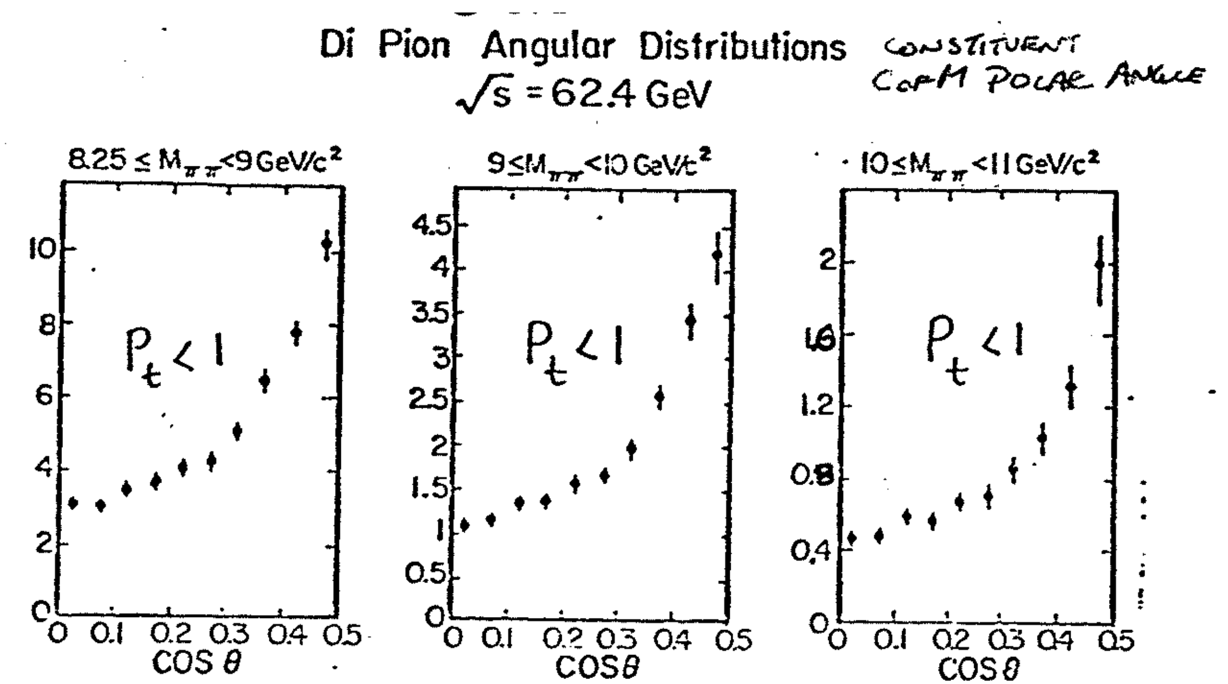
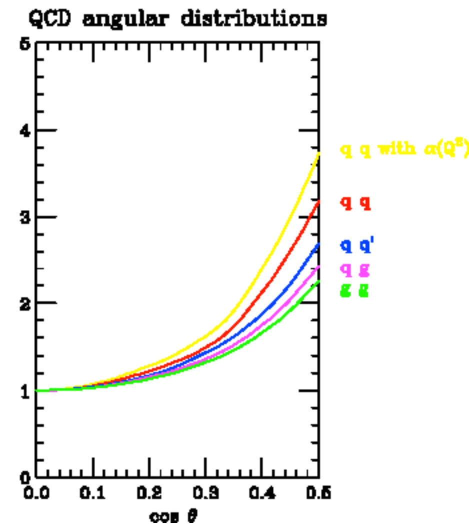
# Also Paris1982-first measurement of QCD subprocess angular distribution using $\pi^0$-$\pi^0$ correlations

DATA: CCOR NPB 209, 284 (1982)




QCD




$$\frac{d^3\sigma}{dx_1 dx_2 d\cos\theta^*} = \frac{1}{s}\sum_{ab} f_a^A(x_1) f_b^B(x_2) \frac{\pi\alpha_s^2(Q^2)}{2x_1x_2}\Sigma^{ab}(\cos\theta^*)$$

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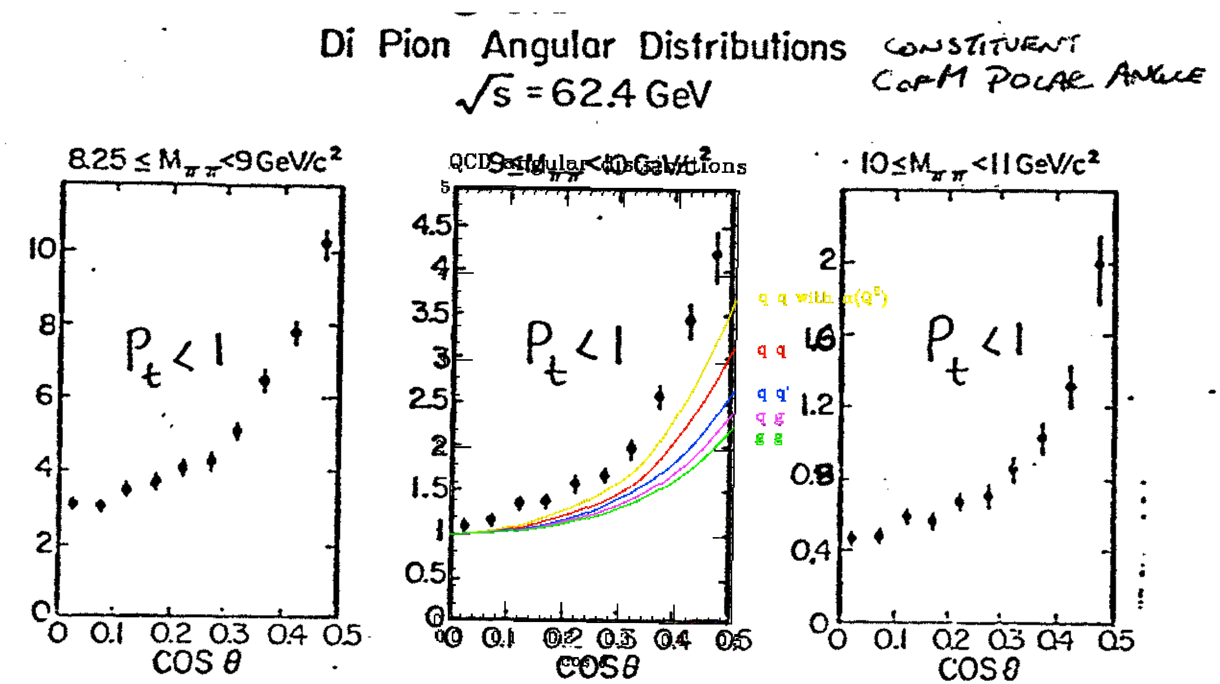
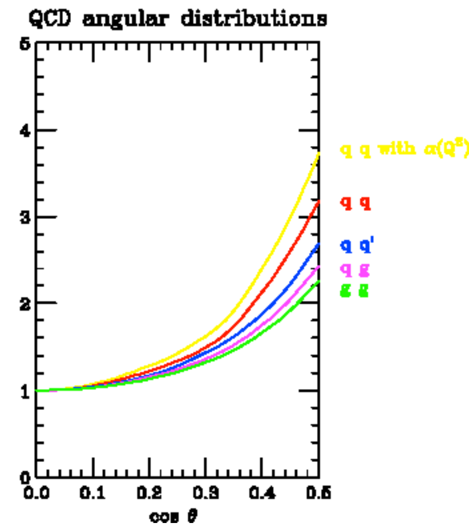



QCD




$$\frac{d^3\sigma}{dx_1 dx_2 d\cos\theta^*} = \frac{1}{s}\sum_{ab} f_a^A(x_1) f_b^B(x_2) \frac{\pi\alpha_s^2(Q^2)}{2x_1x_2}\Sigma^{ab}(\cos\theta^*)$$

$\Sigma^{ab}(\cos\theta^*)$, the characteristic subprocess angular distributions and $\alpha_s(Q^2) = \frac{12\pi}{25\ln(Q^2/\Lambda^2)}$ are predicted by QCD

# Eventually this was measured with di-jets

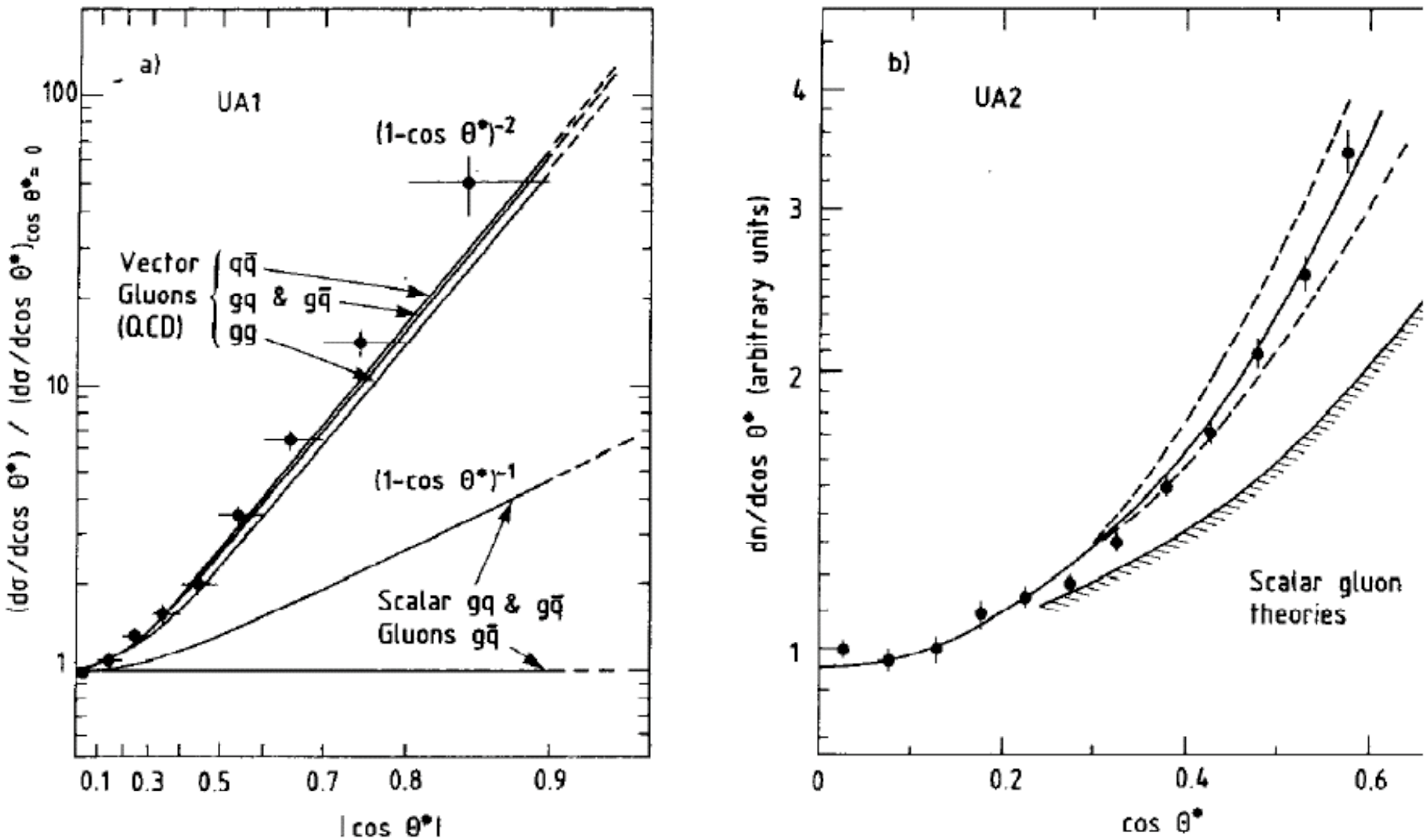
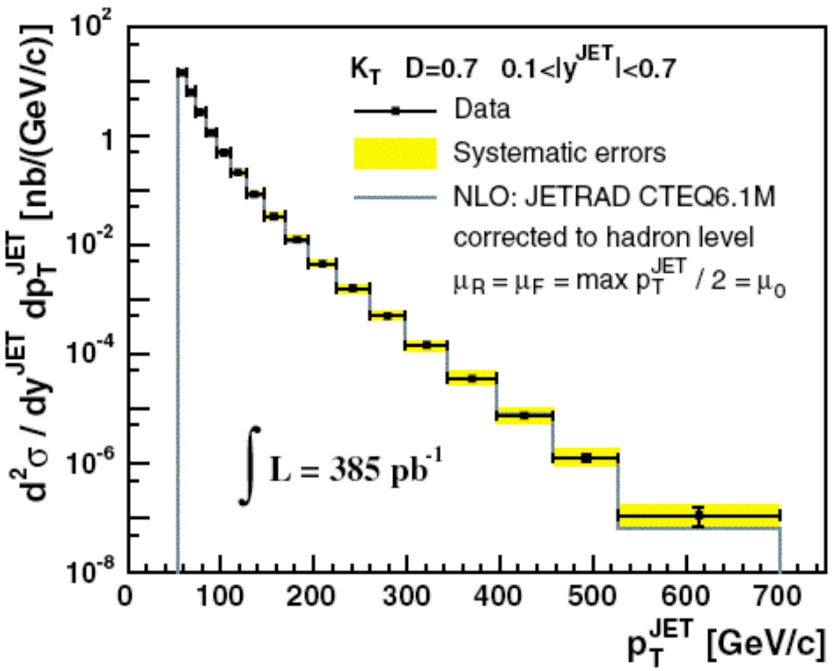
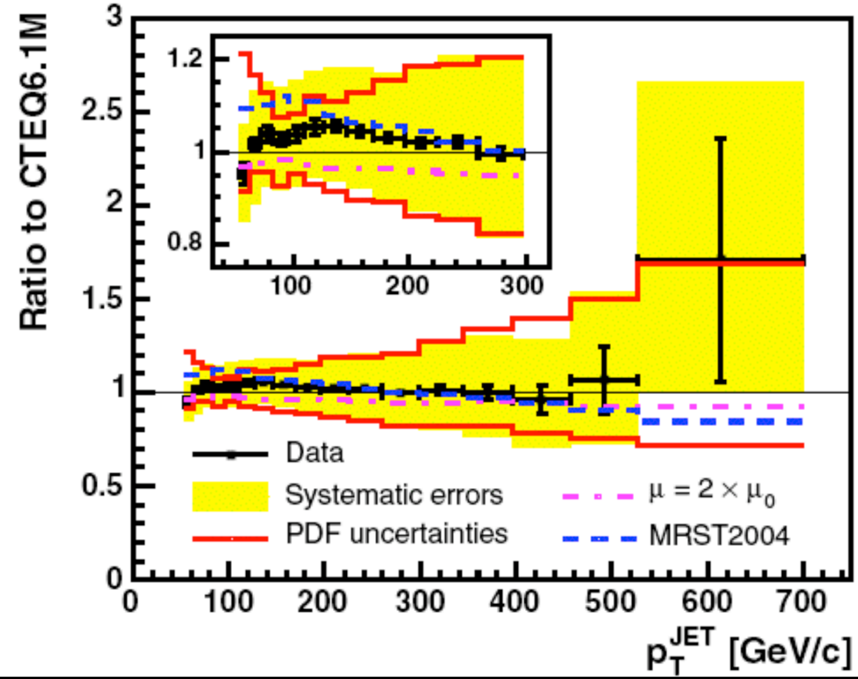



*Figure 10* (*a*) Distribution of $\cos\theta^*$ for hard parton scattering as measured in the UA1 experiment (42). T normalization is defined by setting the value at $\cos\theta^* = 0$ equal to 1. (*b*) Distribution of $\cos\theta^*$ for ha parton scattering as measured in the UA2 experiment (43). All the different QCD processes (except for $\rightarrow q'\bar{q}'$), separately normalized to the data, lie in the area between the two dashed curves. The full line is t overall QCD prediction, normalized to the data.

see L. Di Lella ARNPS **35** (1985) 107--134

# Jet measurements of QCD in pp collisions are now standard after a ~30 year learning curve



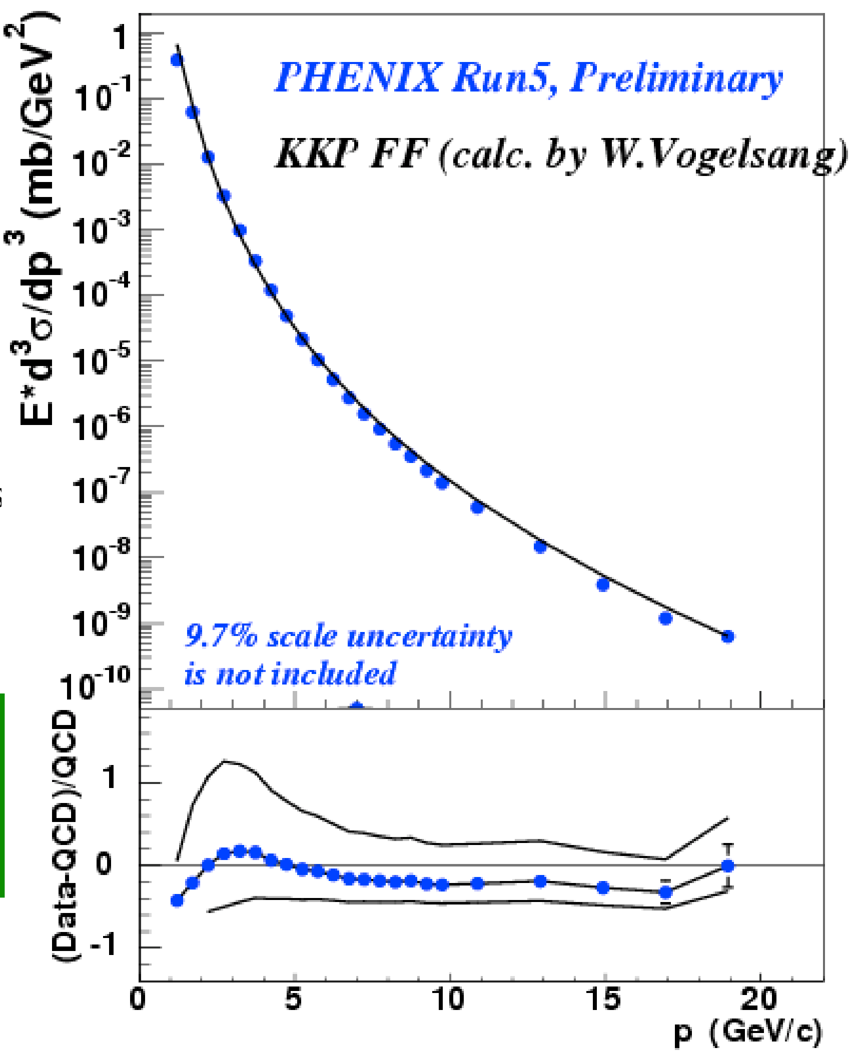

The measured crosssection is in agreement with NLO pQCD predictions after the necessary nonperturbative parton-to-hadron corrections are taken into account. i.e. Make sure to read the fine print!

A. Abulencia, et al, CDF PRL 96 (2006) 122001-$k_T$ algorithm

**At RHIC, inclusive single particles provide a precision pQCD probe, well calibrated in pp, dAu… collisions**

# $\pi^0$'s in p+p $\sqrt{s}$=200 GeV: Data vs. pQCD

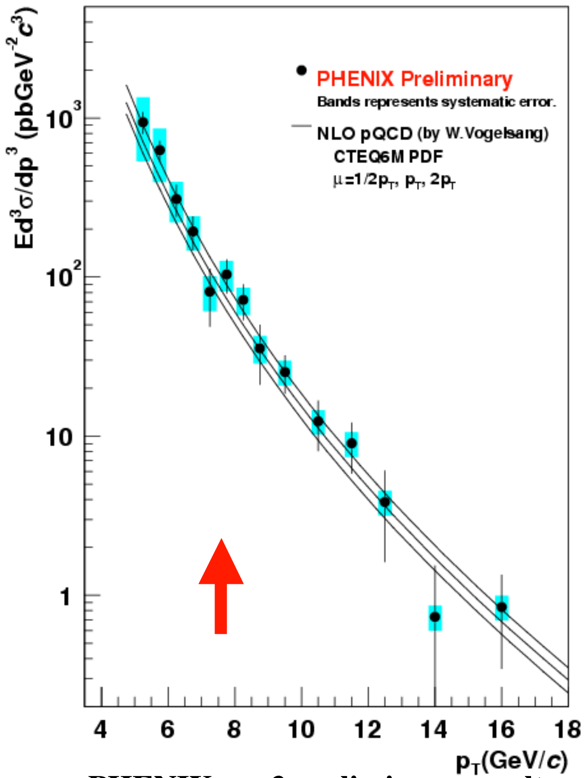
- Result from run2 published-a classic
  - ✓ PRL91 (2003) 241803
- New result from run5
  - ✓ preliminary
- Comparison of $\pi^0$ cross section
  - ✓ Next-to-leading order(NLO) pQCD
    - CTEQ6M + KKP or Kretzer
    - Matrix calculation by Aversa, et. al.
    - Renormalization and factorization scales are set to be equal and set to
      $1/2p_T$, $p_T$, $2p_T$
    - Calculated by W.Vogelsang

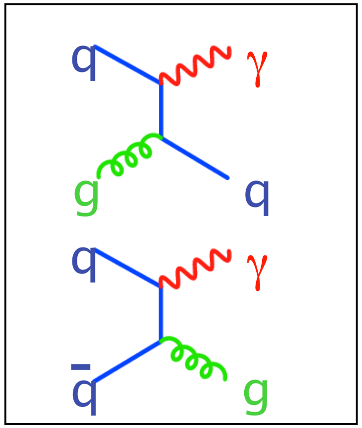



NLO-pQCD described very well down even to $p_T \sim 1$ GeV/$c$

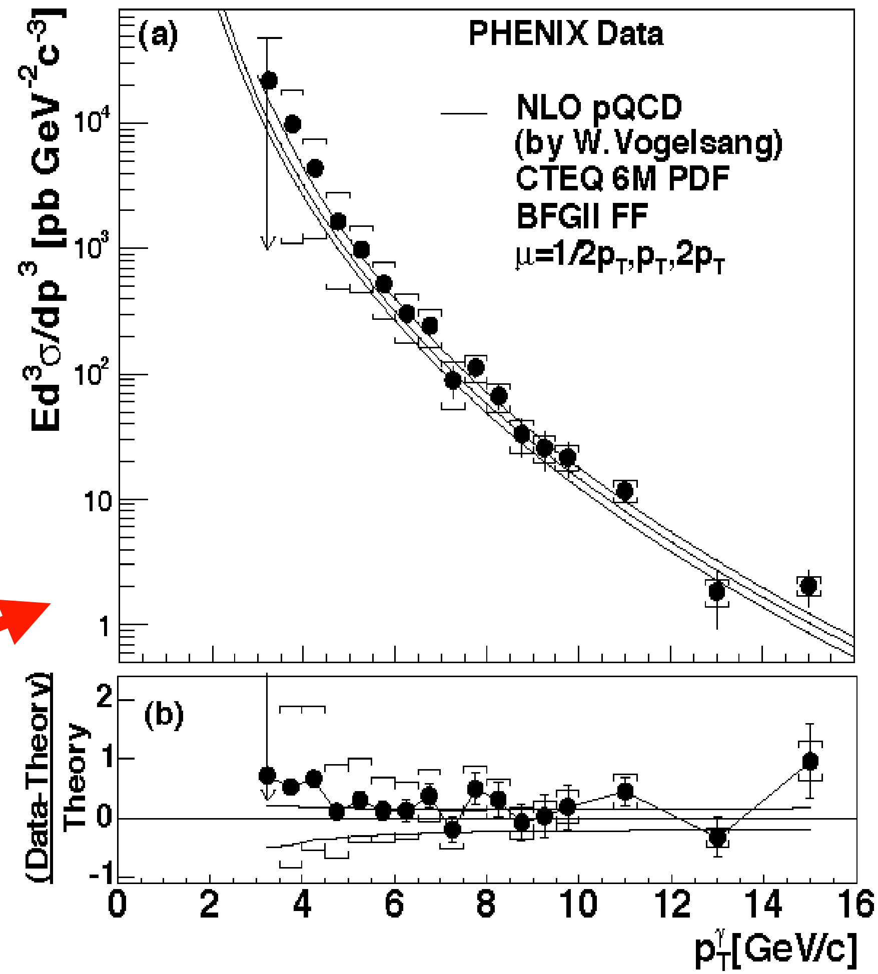
Inclusive invariant $\pi^0$ spectrum is pure power law for $p_T \geq 3$ GeV/c
n=8.1±0.1

# PHENIX Direct γ's in p+p: Data vs. pQCD




Run 3

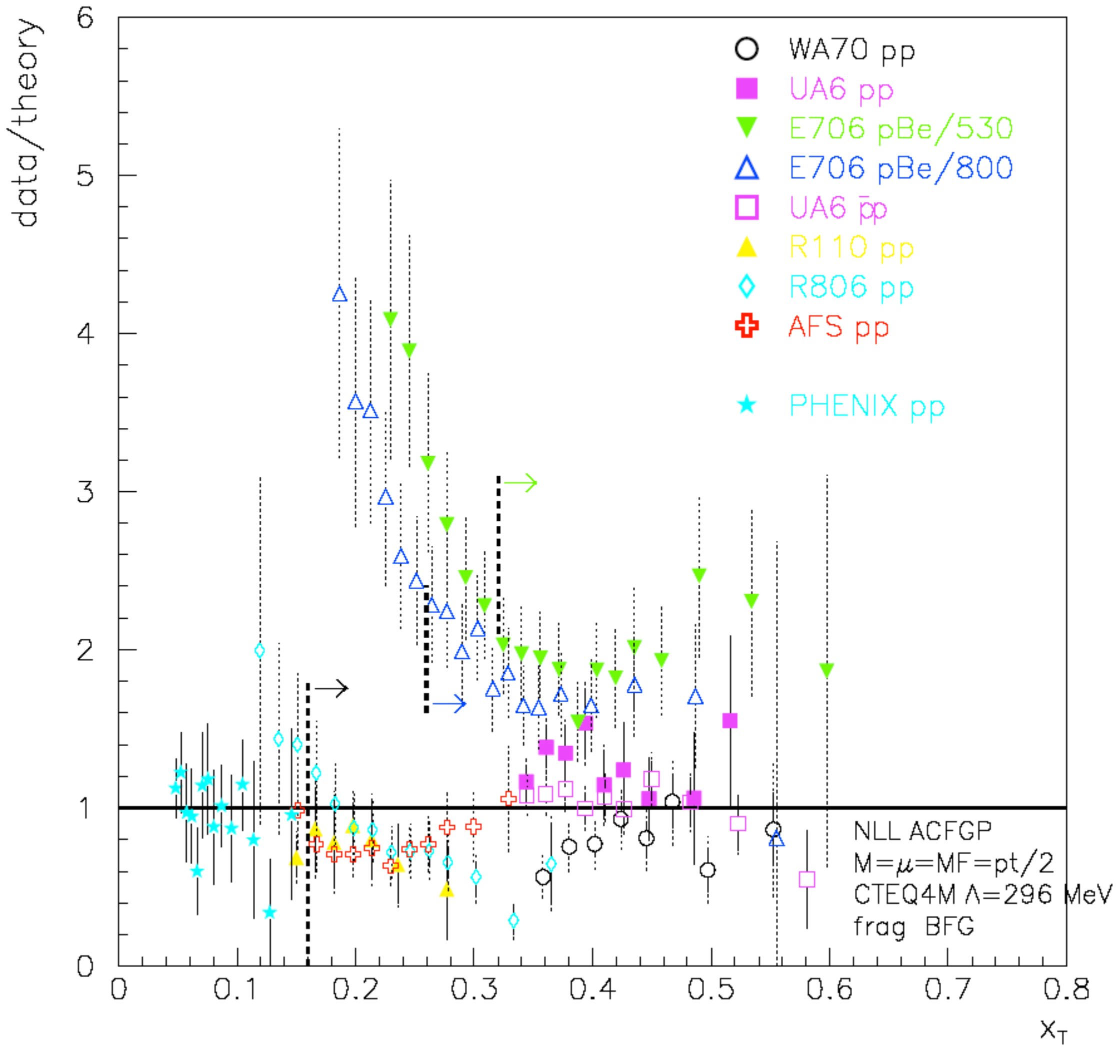



- **PHENIX run3 preliminary result.**
  - ✓ **Recent Update down to 3GeV/c**
  - ✓ PRL **98** ,012002 (2007)
- **NLO-pQCD calculation**
  - ✓ **Private communication with W.Vogelsang**
  - ✓ **CTEQ6M PDF.**
  - ✓ **Sum of direct γ + Bremsstrahlung γ**
  - ✓ **3 scales (1/2pT,1pT,2 pT)**

Preliminary results for $p_T$> 5GeV/c; published results $p_T$>3 GeV/c

# Comparison with other data and pQCD

Aurenche et al  Eur. Phys. JC**9** (1999)107




Talk by Monique Werlen at
RHIC&AGS users meeting 2005

PHENIX data clarifies longstanding data/theory puzzle

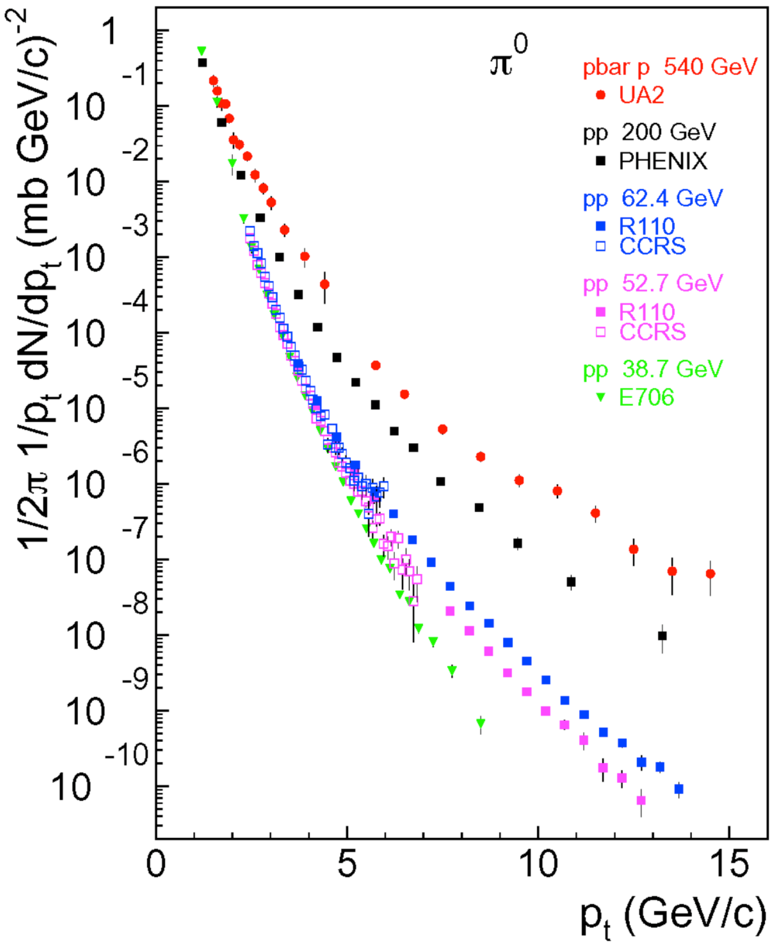
# QCD follows $x_T$ scaling-very powerful tool

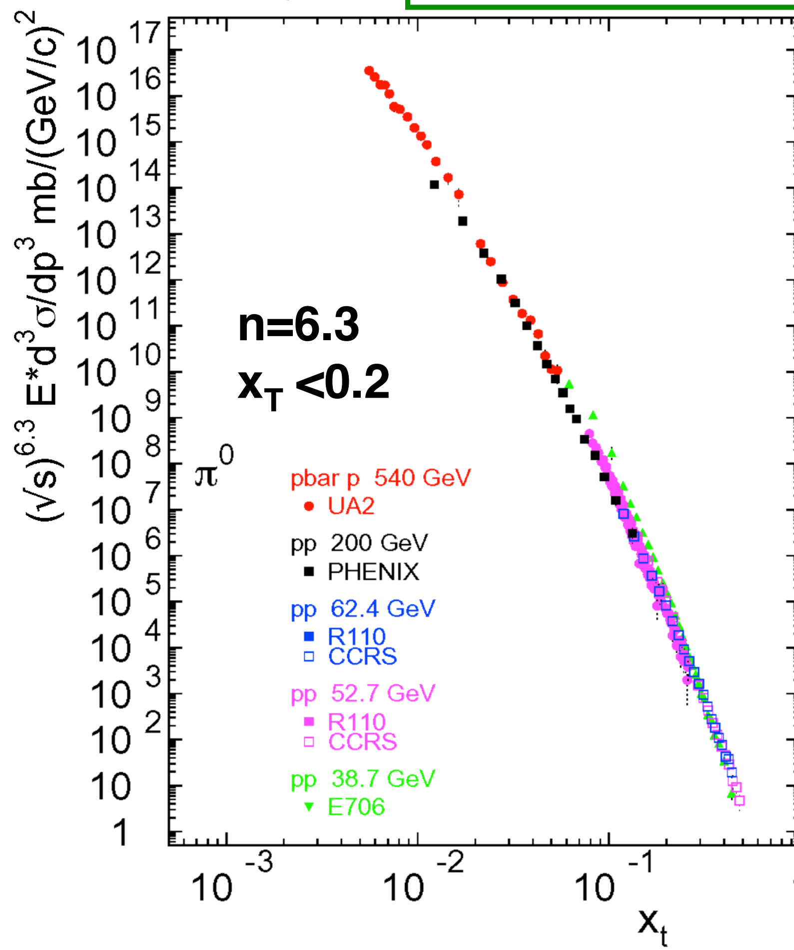
$$E\frac{d^3\sigma}{d^3p} = \frac{1}{p_T^n}F\left(\frac{p_T}{\sqrt{s}}\right) = \frac{1}{\sqrt{s}^n}G(x_T)$$

$$x_T = \frac{2p_T}{\sqrt{s}}$$

LOQCD, QED: n=4

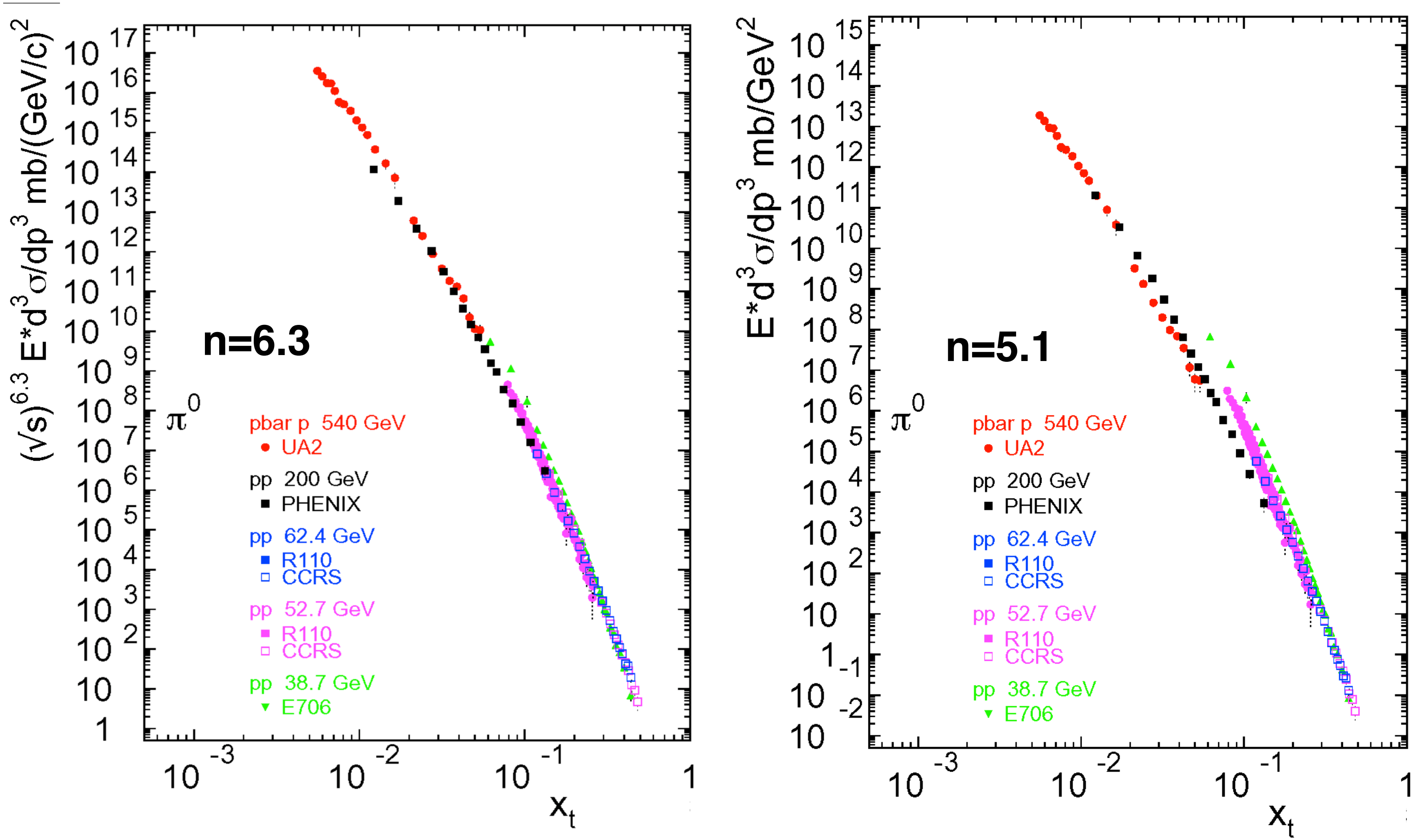
QCD $n(x_T, \sqrt{s}) = 4^{++}$



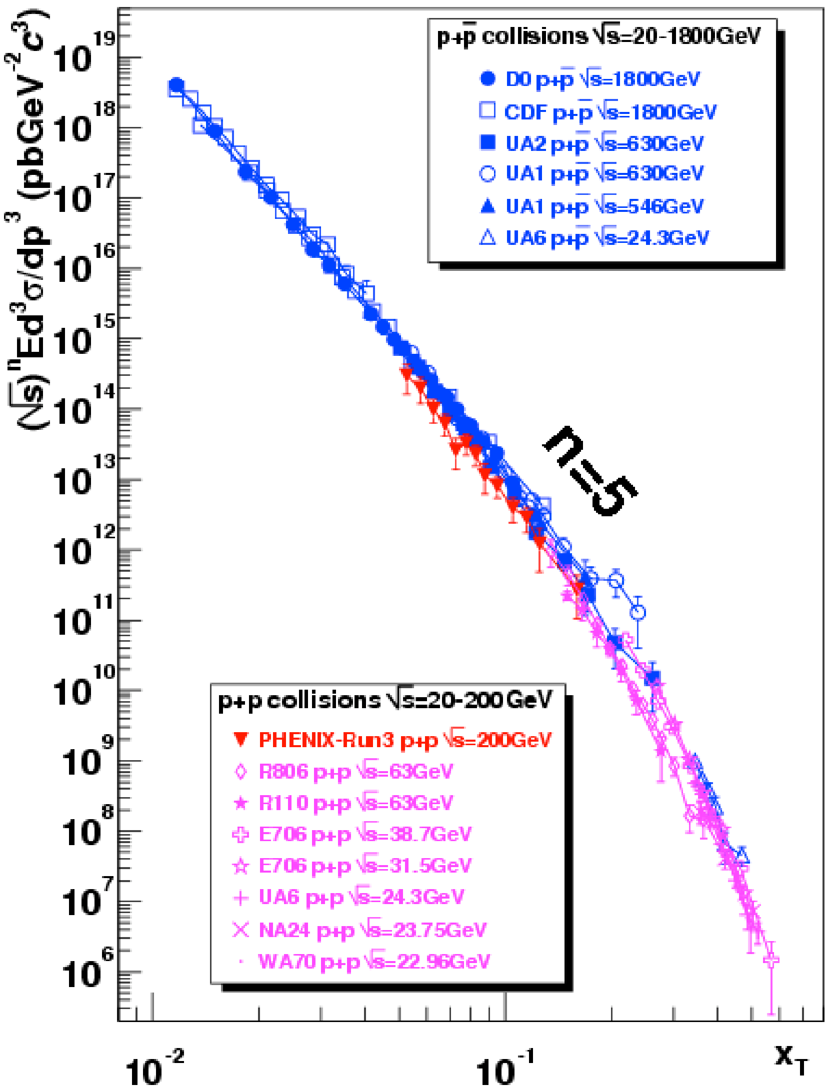




Structure and Fragmentation fns., which `scale', i.e. are functions only of ratios of momenta, are in F (G)

# Recall $\pi^0$: n=5.1 works better for $x_T > 0.2$

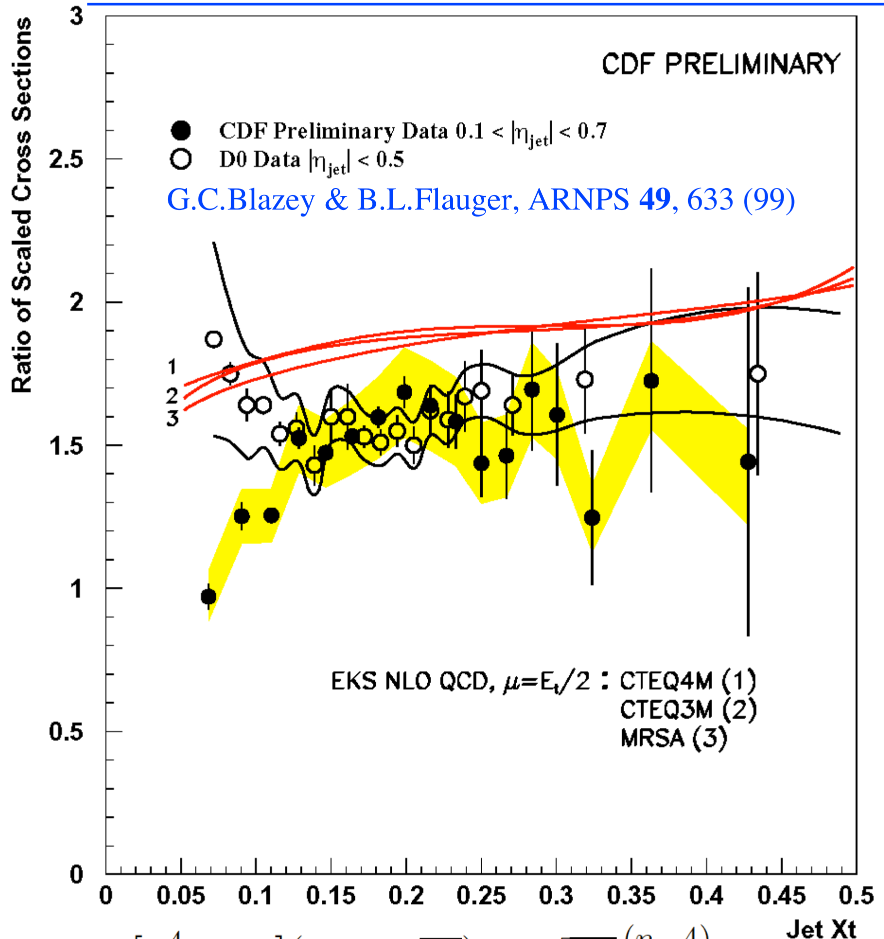


# $x_T$ scaling: a) Direct-$\gamma$ b) Jets




Direct $\gamma$ n≈5

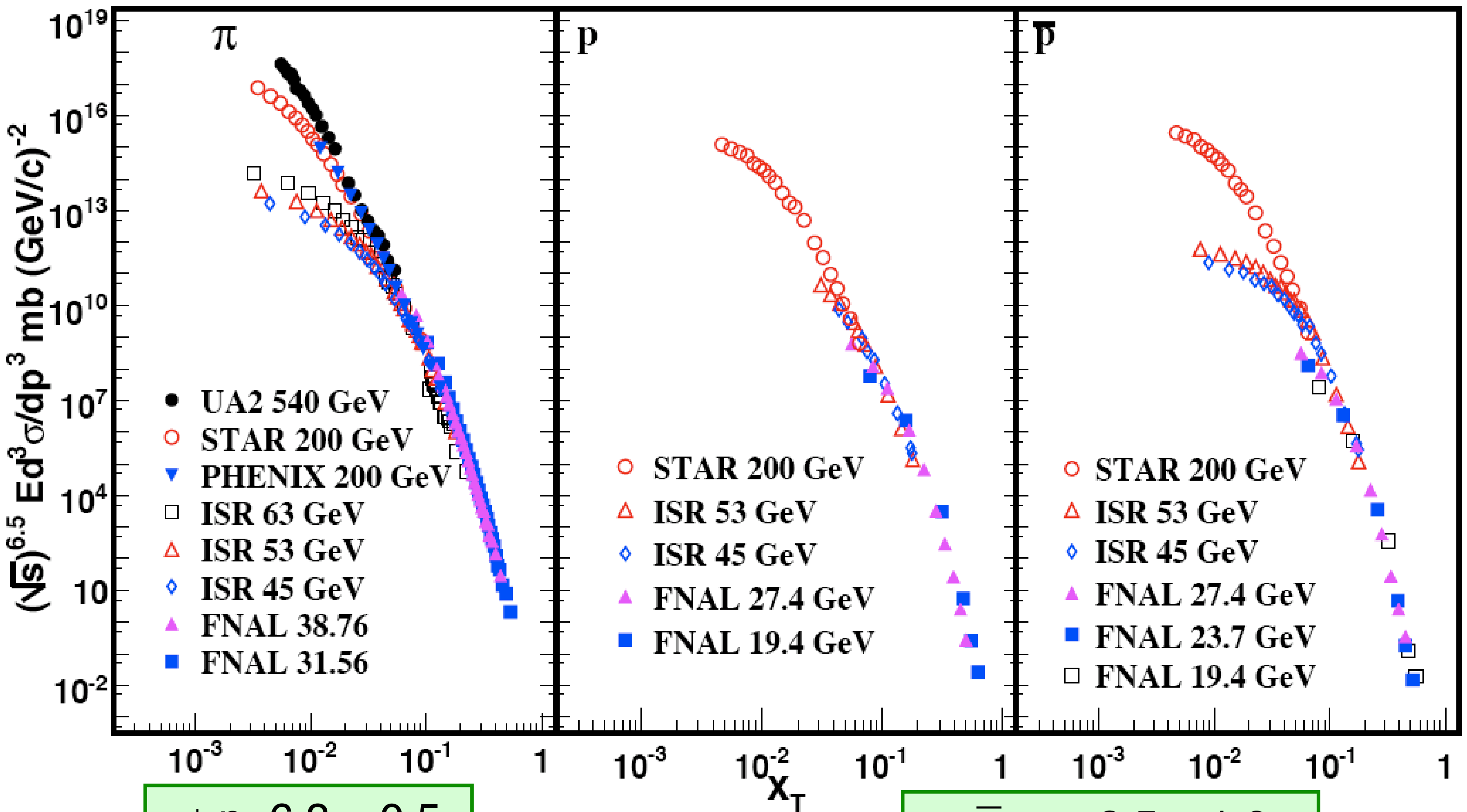
Jets-Ratio of Scaled Cross Sections 630/1800




$$\frac{[p_T^4\,\sigma_{\rm inv}](x_T,\sqrt{s_1})}{[p_T^4\,\sigma_{\rm inv}](x_T,\sqrt{s_2})} = \sqrt{\frac{s_2}{s_1}}^{(n-4)}$$

Jets n≈4.5

# New: STAR $x_T$ scaling result for (anti-)protons in p+p collisons $\sqrt{s_{NN}}$=200 GeV nucl-ex/0601033




$\pi^{\pm}$ n=6.8 ± 0.5

$p, \bar{p}$ n =6.5 ± 1.0

Agrees with PHENIX $\pi^0$ n=6.3 ± 0.5 PRC **69**, 034910 (2004)

Difficult to reconcile with Brodsky, et al PLB**637**, 58 (2006)

# Correlations

# The leading-particle effect a.k.a. trigger bias

- Due to the steeply falling power-law spectrum of the scattered partons, the inclusive particle $p_T$ spectrum is dominated by fragments biased towards large z. This was unfortunately called trigger bias by M. Jacob and P. Landshoff, Phys. Rep. **48C**, 286 (1978) although it has nothing to do with a trigger.

$$\frac{d^2\sigma_\pi(\hat{p}_{T_t}, z_t)}{d\hat{p}_{T_t} dz_t} = \frac{d\sigma_q}{d\hat{p}_{T_t}} \times D^q_\pi(z_t)$$

Fragment spectrum given $\hat{p}_{T_t}$

$$= \frac{A}{\hat{p}^{n-1}_{T_t}} \times D^q_\pi(z_t)$$

Power law spectrum of parton $\hat{p}_{T_t}$

$$\text{let } \hat{p}_{T_t} = p_{T_t}/z_t \qquad d\hat{p}_{T_t}/dp_{T_t}|_{z_t} = 1/z_t$$

$$\frac{d^2\sigma_\pi(p_{T_t}, z_t)}{dp_{T_t} dz_t} = \frac{1}{z_t} \frac{A}{(p_{T_t}/z_t)^{n-1}} \times D^q_\pi(z_t)$$

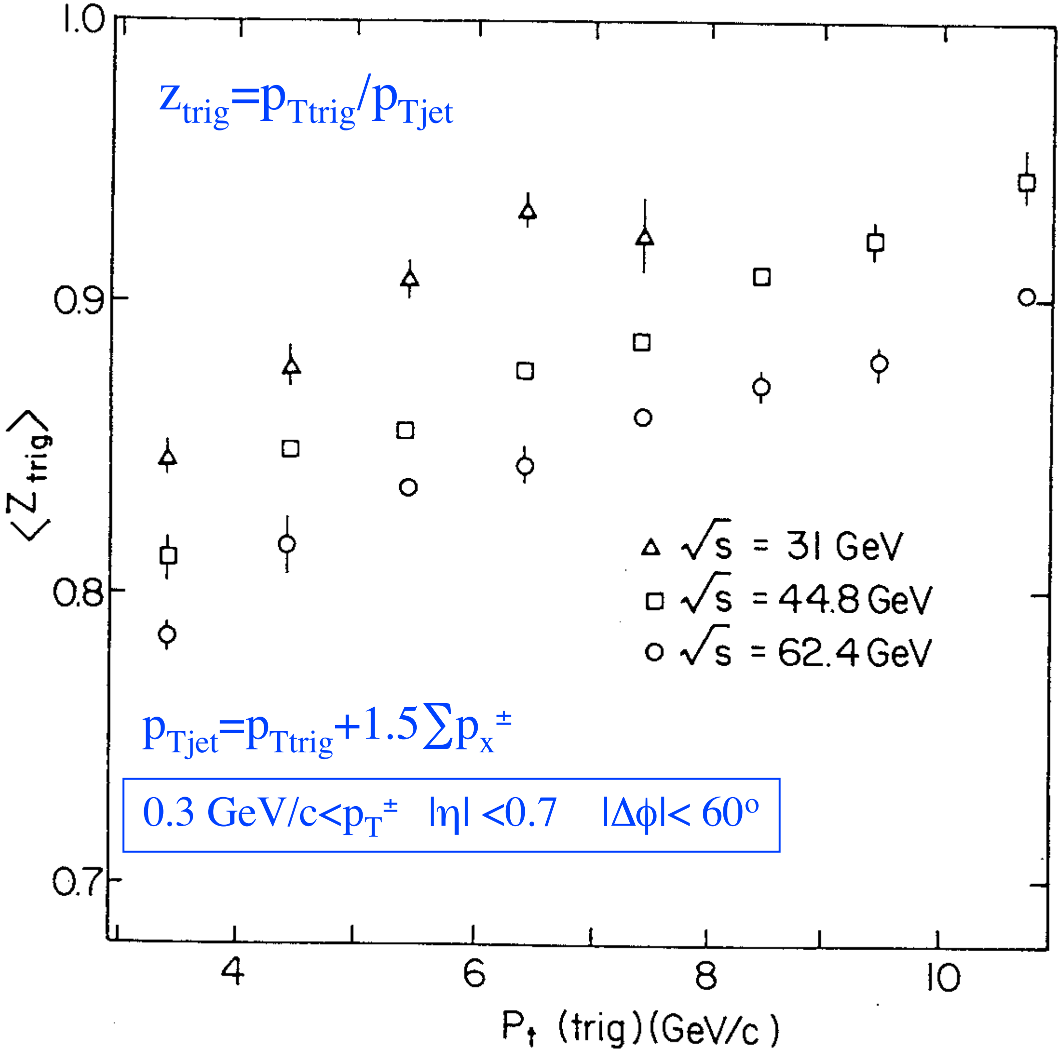
$$= \frac{A}{p^{n-1}_{T_t}} \times z_t^{n-2} D^q_\pi(z_t)$$

Fragment spectrum given $p_{Tt}$ is weighted to high $z_t$ by $z_t^{n-2}$

$$\text{where } z_{t\min}|_{p_{T_t}} = x_{T_t} \qquad D^q_\pi(z_t) = B e^{-6z_t}$$

# $\langle z_{trig} \rangle$ measured at ISR

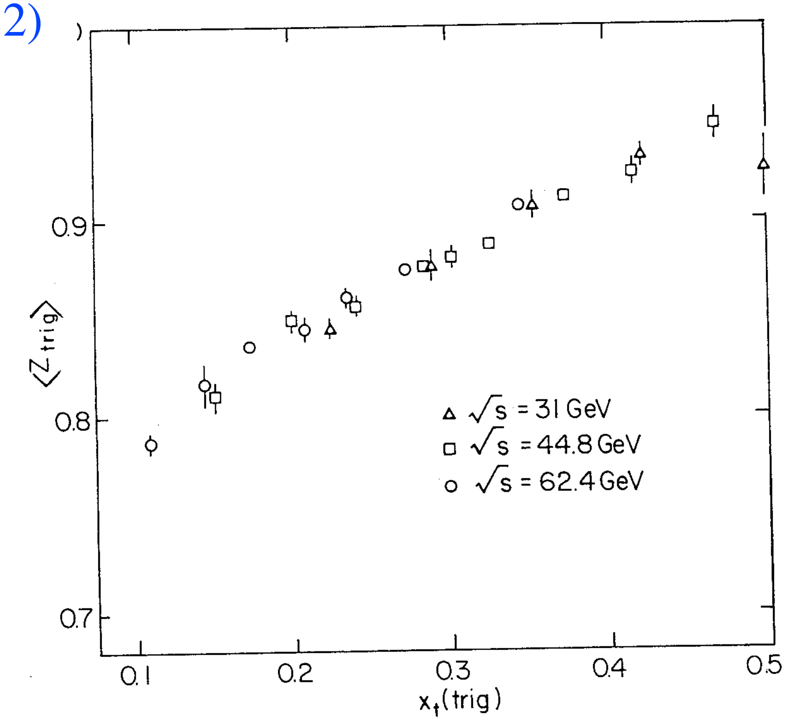
DATA: CCOR NPB 209, 284 (1982)




$z_{trig}=p_{Ttrig}/p_{Tjet}$

$p_{Tjet}=p_{Ttrig}+1.5\sum p_x^{\pm}$

0.3 GeV/c$<p_T^{\pm}$ $|\eta|<0.7$ $|\Delta\phi|<60^o$




- $\langle z_{trig} \rangle \sim$ 0.8-0.9 at ISR, n~ 11
- $\langle z_{trig} \rangle$ $x_T$ scales

# 2 particle Correlations

$$\frac{d^2\sigma_\pi(\hat{p}_{T_t}, z_t)}{d\hat{p}_{T_t} dz_t} = \frac{d\sigma_q}{d\hat{p}_{T_t}} \times D^q_\pi(z_t)$$

Prob. that you make a jet with $\hat{p}_{Tt}$ which fragments to a π with $z_t=p_{Tt}/\hat{p}_{Tt}$

Also detect fragment with $z_a = p_{T_a}/\hat{p}_{T_a}$
from away jet with $\hat{p}_{T_a}/\hat{p}_{T_t} \equiv \hat{x}_h$

$$\frac{d^3\sigma_\pi(\hat{p}_{T_t}, z_t, z_a)}{d\hat{p}_{T_t} dz_t dz_a} = \frac{d\sigma_q}{d\hat{p}_{T_t}} \times D^q_\pi(z_t) \times D^q_\pi(z_a)$$

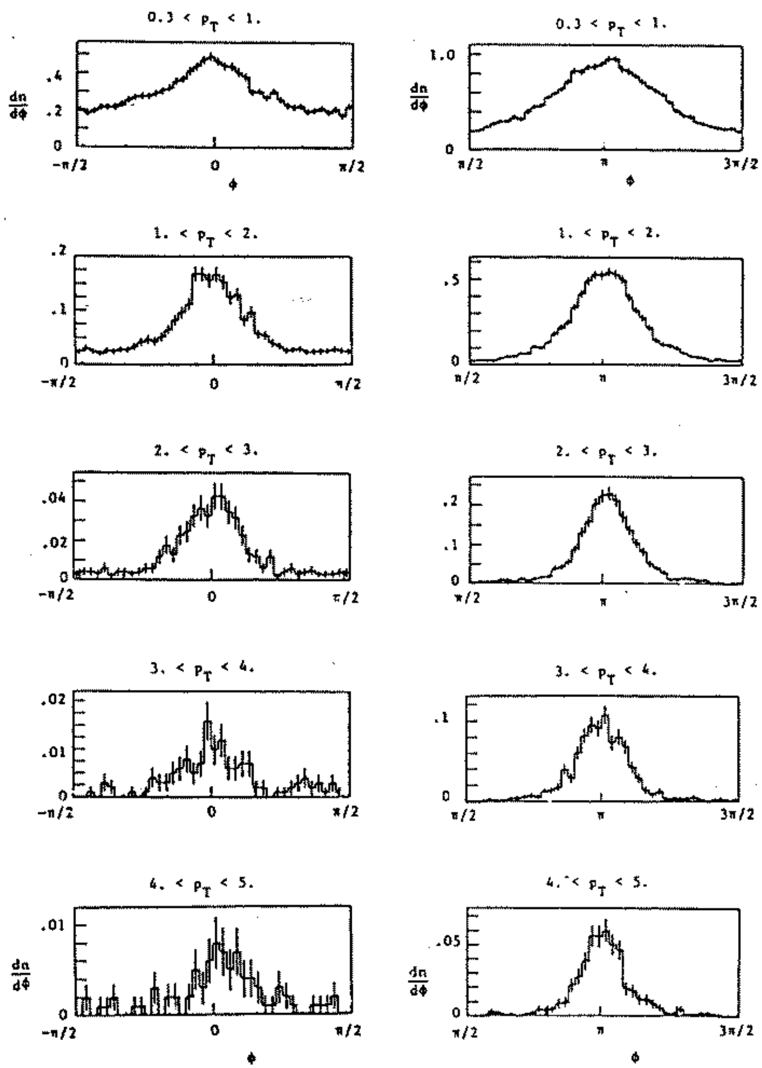
Prob. that away jet with $\hat{p}_{Ta}$ fragments to a π with $z_a=p_{Ta}/\hat{p}_{Ta}$

$$z_a = \frac{p_{T_a}}{\hat{p}_{T_a}} = \frac{p_{T_a}}{\hat{x}_{\rm h}\, \hat{p}_{T_t}} = \frac{z_t\, p_{T_a}}{\hat{x}_{\rm h}\, p_{T_t}}$$

(1)
$$\boxed{\frac{d\sigma_\pi}{dp_{T_t} dz_t dp_{T_a}} = \frac{1}{\hat{x}_{\rm h}\, p_{T_t}} \frac{d\sigma_q}{d(p_{T_t}/z_t)} D^q_\pi(z_t) D^q_\pi\left(\frac{z_t p_{T_a}}{\hat{x}_{\rm h} p_{T_t}}\right)}$$

Appears to be sensitive to away jet Frag. Fn.

# How everything you want to know about JETS was measured with 2-particle correlations




CCOR, A.L.S.Angelis, et al
Phys. Lett. **97B**, 163 (1980)
Physica Scripta **19**, 116 (1979)

$p_{Tt} > 7$ GeV/c vs $p_T$

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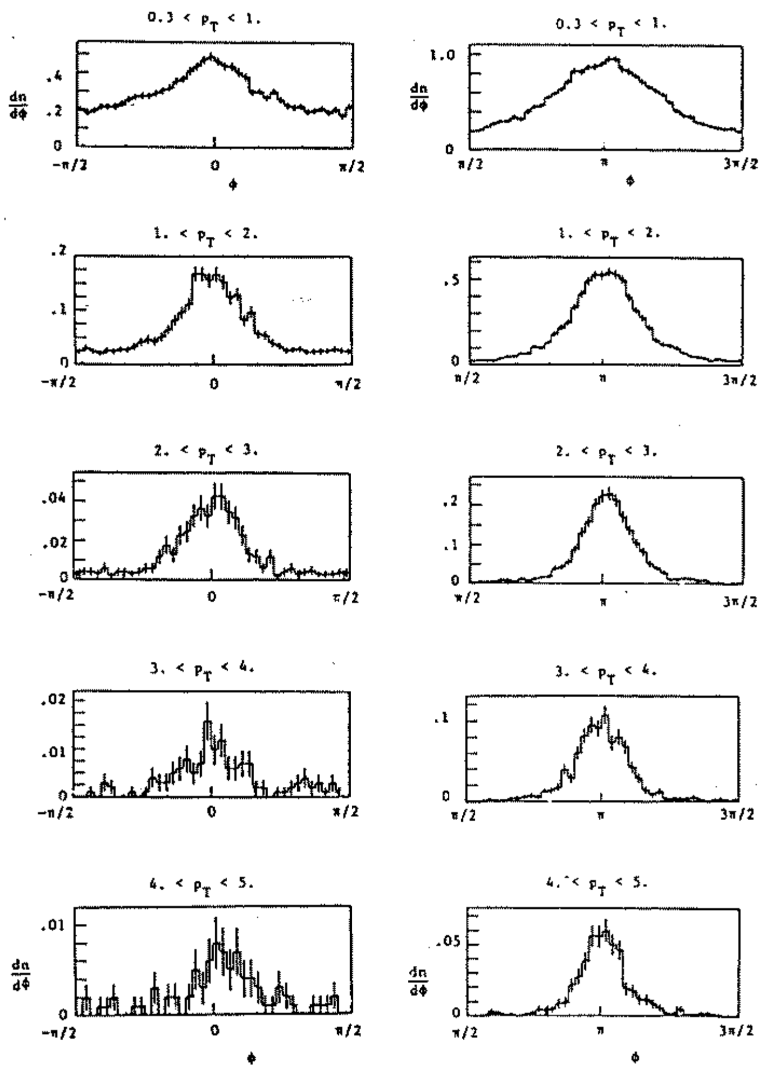


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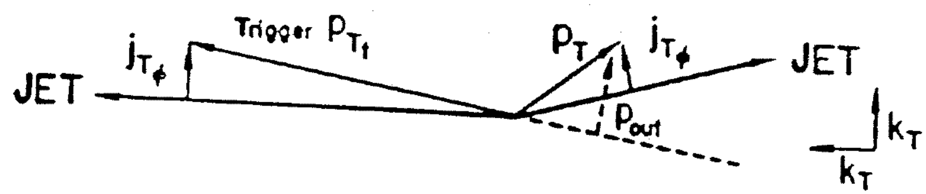


$p_{Tt} > 7$ GeV/c vs $p_T$

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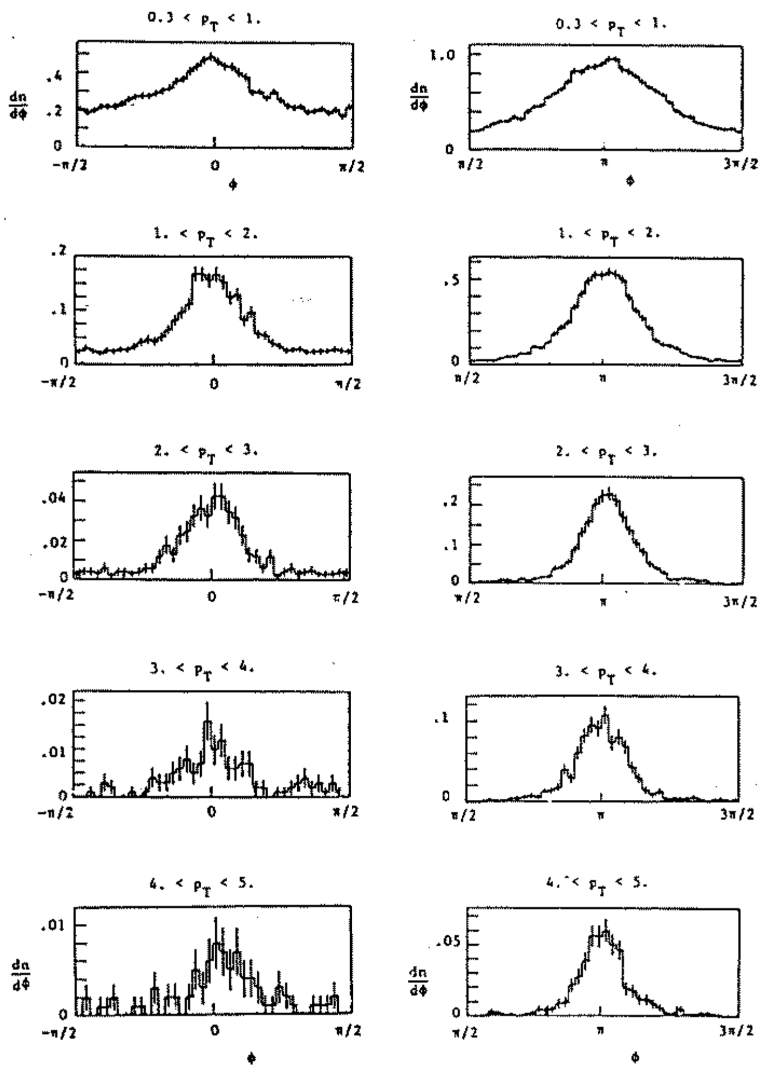


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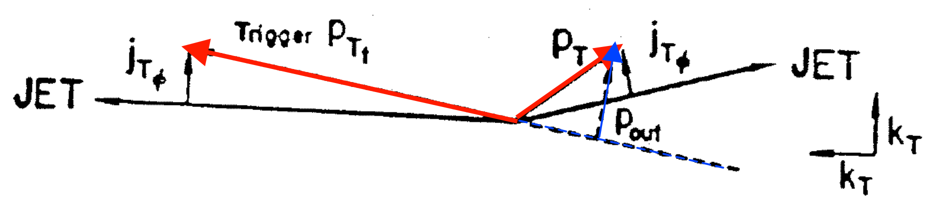


$p_{Tt} > 7$ GeV/c vs $p_T$

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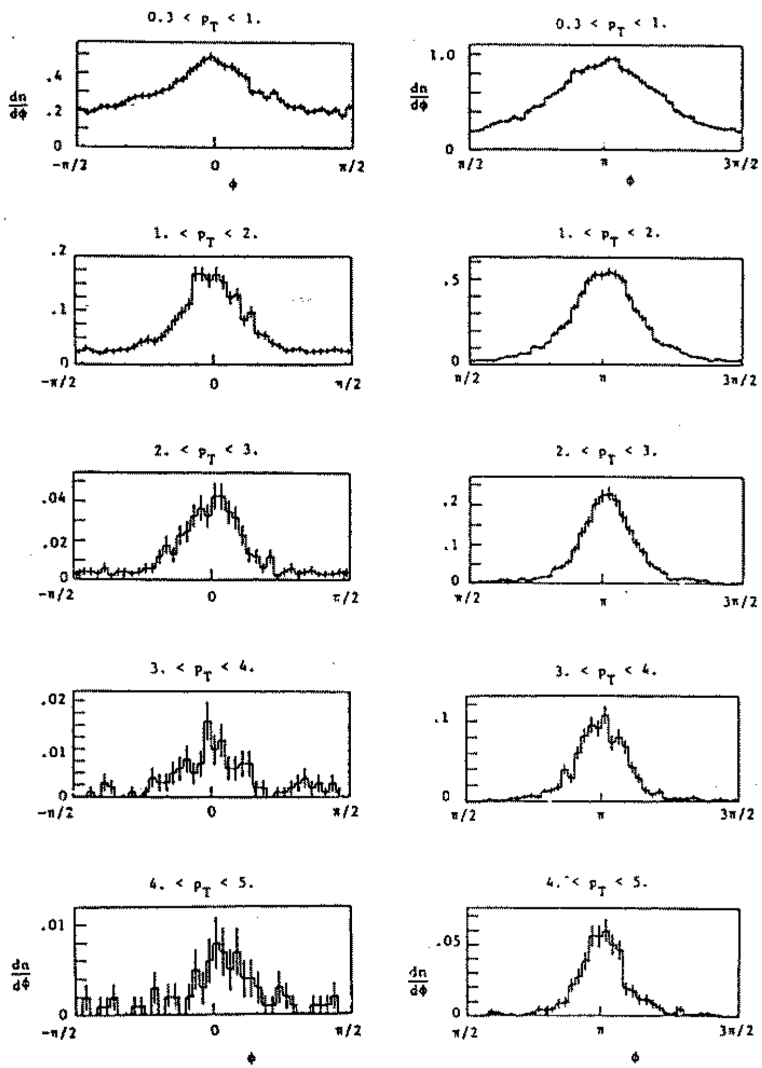
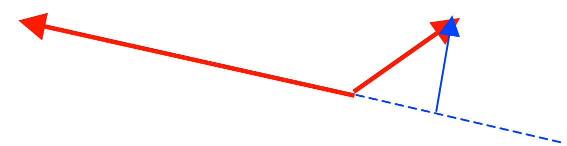



CCOR, A.L.S.Angelis, et al
Phys. Lett. **97B**, 163 (1980)
Physica Scripta **19**, 116 (1979)



$p_{Tt} > 7$ GeV/c vs $p_T$

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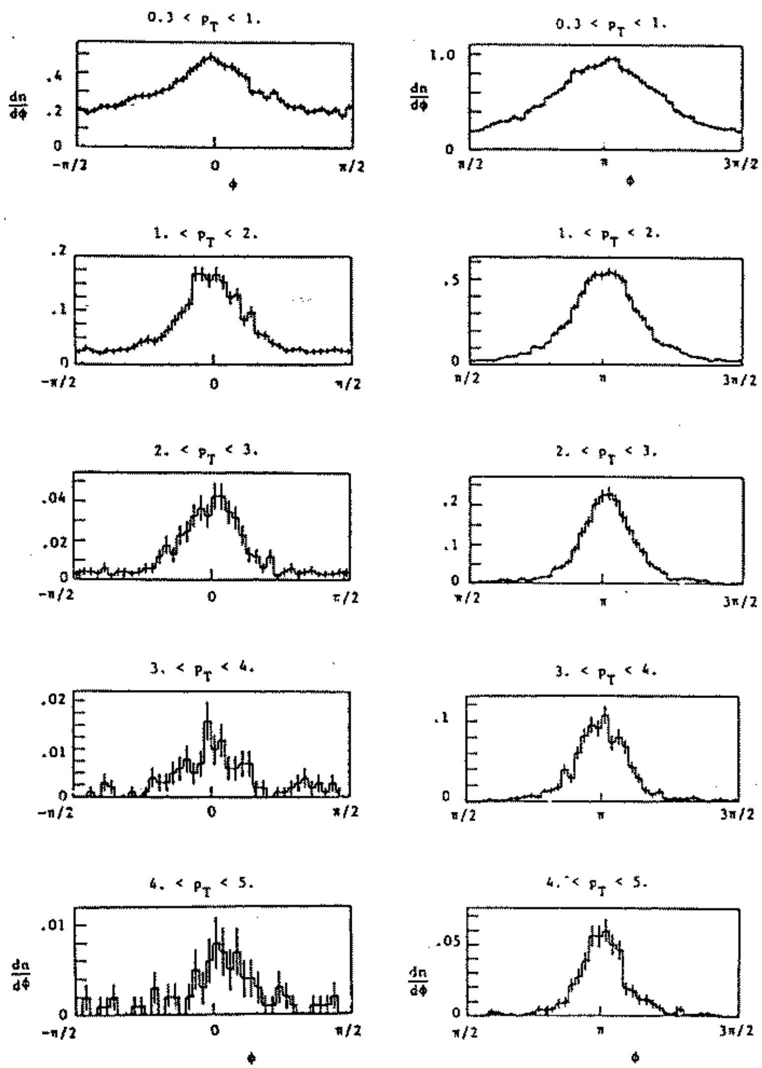
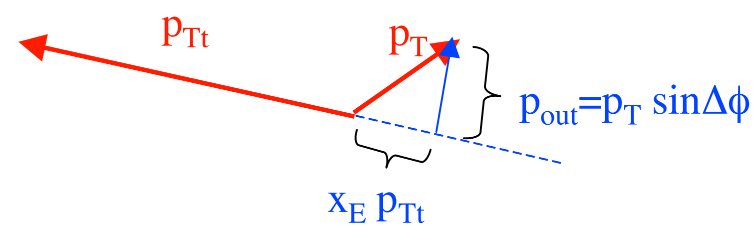
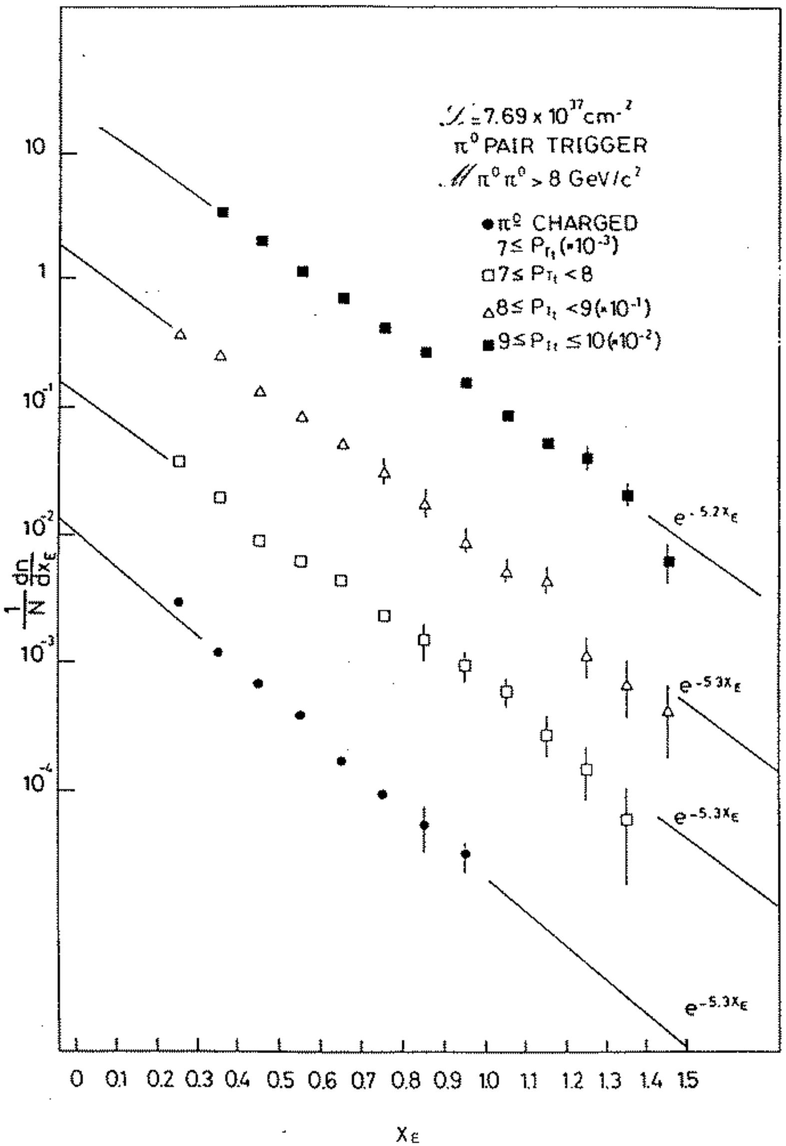



CCOR, A.L.S.Angelis, et al
Phys. Lett. **97B**, 163 (1980)
Physica Scripta **19**, 116 (1979)




$p_{Tt} > 7$ GeV/c vs $p_T$

# $x_E$ distribution measures fragmentation fn.




$$x_E \sim p_{Ta}/p_{Tt} \sim z/\langle z_{trig}\rangle$$

$\langle z_{trig}\rangle$=0.85 measured*

$$\Rightarrow D^q_\pi(z) \sim e^{-6z}$$

- independent of $p_{Tt}$

See M. Jacob's talk EPS 1979 Geneva

* but we did learn something new on this issue in PHENIX

# $k_T$ is not a parameter, it can be measured

- In leading order QCD or the Quark-Parton model, the net transverse momentum $\langle p_T \rangle_{\rm pair} = \sqrt{2} \times \langle k_T \rangle$, of a hard-scattering jet-pair, or a Drell-Yan pair, or a pair of high $p_T$ photons, or the $\gamma+$ Jet pair for direct photon production is zero. All the above pairs should be coplanar with the incident beam axis.
- However, early Drell-Yan and inclusive high $p_T$ particle studies showed that $k_T$ was measurable and non-zero.

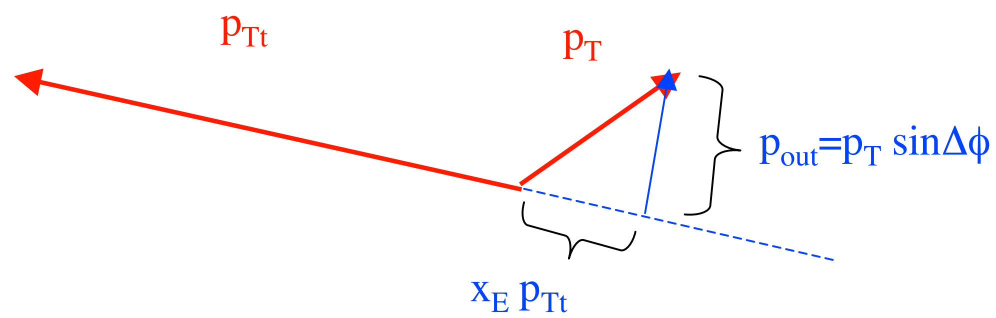
♡ The history of $k_T$ is worth reviewing as $k_T$ was predicted to be zero by theorists, but was discovered to be non-zero by experimentalists. The CCHK experiment [M. Della Negra, et al., Nucl. Phys. **B127**, 1 (1977)] discovered that back-to-back jets had considerable out of plane transverse momentum $p_{\rm out}$, and proposed that this was due to transverse momentum of partons inside a proton.

# Feynman Field & Fox to the rescue

♡ This was elaborated by Feynman, Field and Fox, [Nucl. Phys. **B128**, 1, (1997), Phys Rev. **D18**, 3320 (1978)] who introduced the $k_T$ phenomenology of a parton in a proton, which they discussed in terms of 'intrinsic transverse momentum' from confinement which would be constant as a function of $x$ and $Q^2$, and NLO effects due to hard gluon emission which would vary with $x$ and $Q^2$, but they used an constant 'effective' $k_T$ to 'explain' the available measurements.

♡ A subsequent ISR experiment, CCOR, showed that $k_T$ for jet-pairs was roughly the same as for Drell-Yan and increased similarly with $\sqrt{s}$ (and $p_T$) i.e. was not constant.

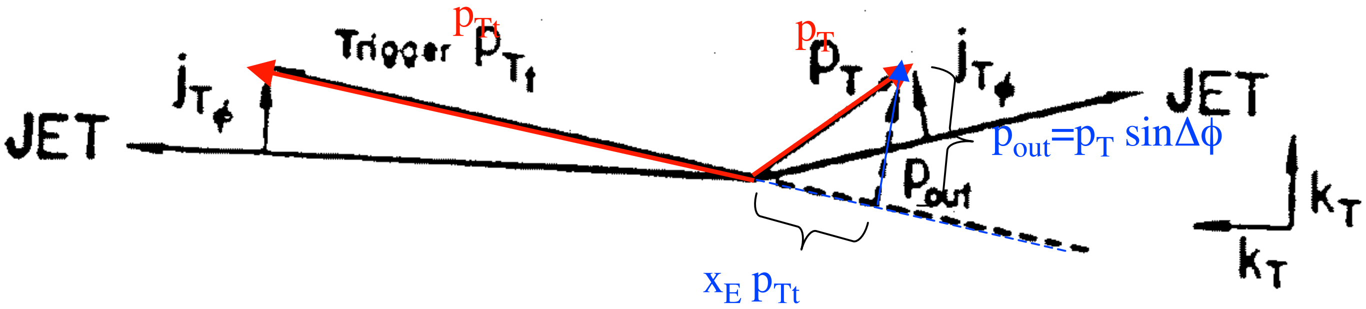
# $j_T$, $k_T$, $x_E$, $p_{out}$ definitions all in plane transverse to beam direction




$$\langle|p_{out}|\rangle^2 = x_E^2[2\langle|k_{Ty}|\rangle^2 + \langle|j_{Ty}|\rangle^2] + \langle|j_{Ty}|\rangle^2$$

- $j_T$ is parton fragmentation transverse momentum
- $k_T$ is transverse momentum of a parton in a proton (2 protons)
- $x_E = -p_T \bullet p_{Tt}/|p_{Tt}|^2$ represents away jet fragmentation z
- $p_{out}$ is component of away $p_T$ perpendicular to trigger $p_{Tt}$

# $j_T$, $k_T$, $x_E$, $p_{out}$ definitions all in plane transverse to beam direction

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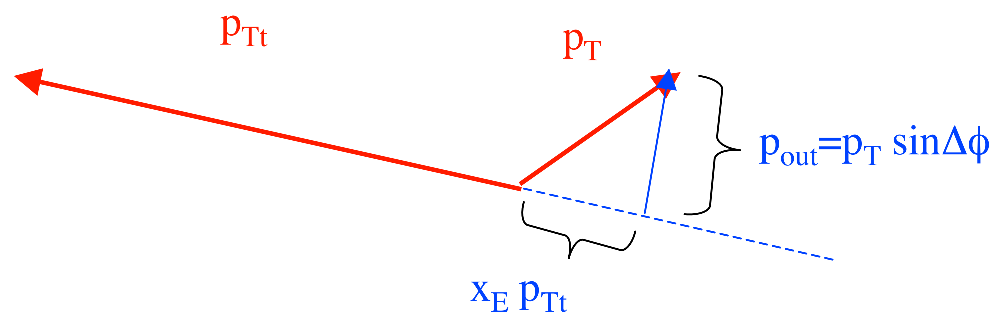


$$\langle|\mathbf{p}_{out}|\rangle^2 = x_E^2[2\langle|\mathbf{k}_{Ty}|\rangle^2 + \langle|\mathbf{j}_{Ty}|\rangle^2] + \langle|\mathbf{j}_{Ty}|\rangle^2$$

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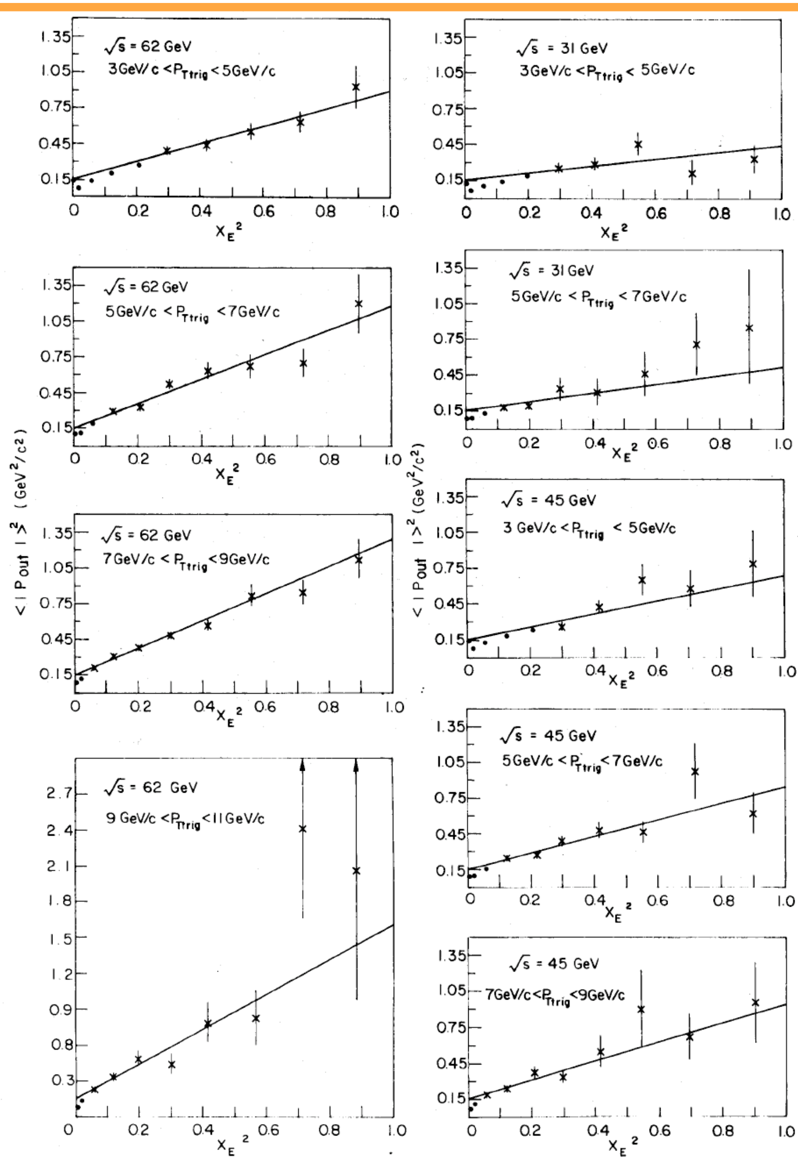
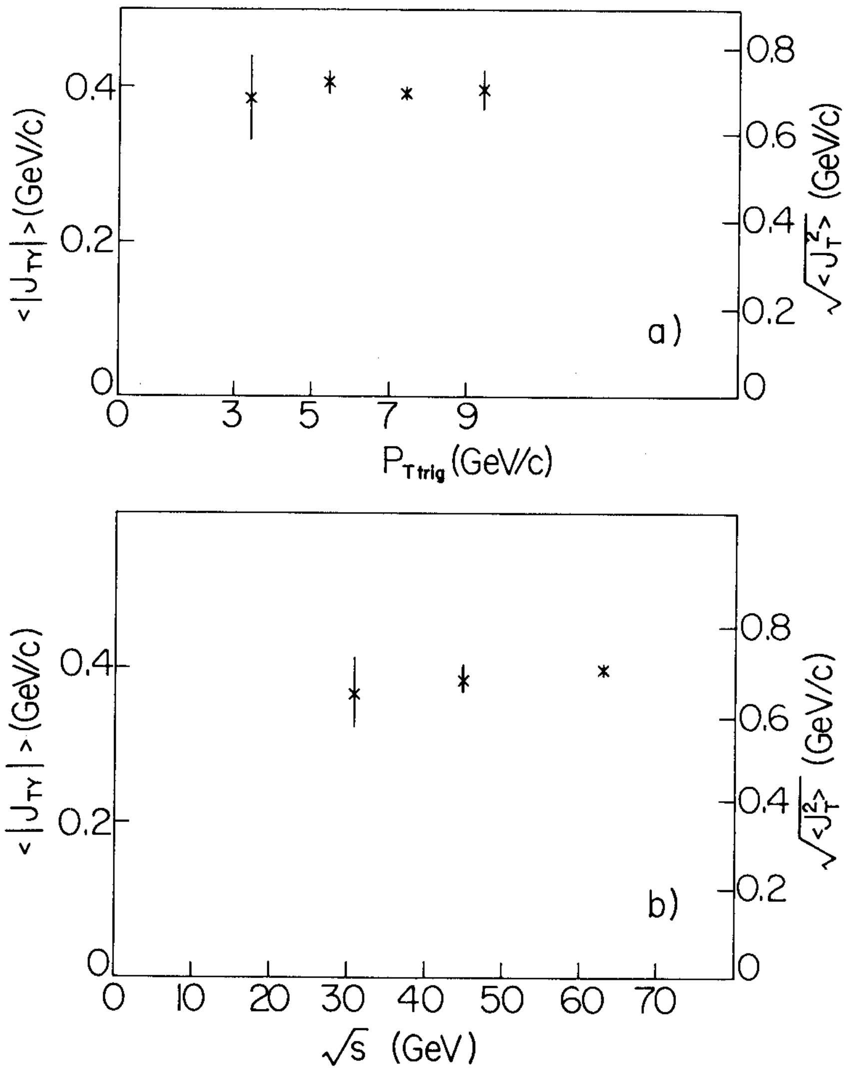
# CCOR $\langle|p_{out}|\rangle^2$ vs $x^2_E$




$$\langle|p_{out}|\rangle^2 = x_E^2[2\langle|k_{Ty}|\rangle^2 + \langle|j_{Ty}|\rangle^2] + \langle|j_{Ty}|\rangle^2$$

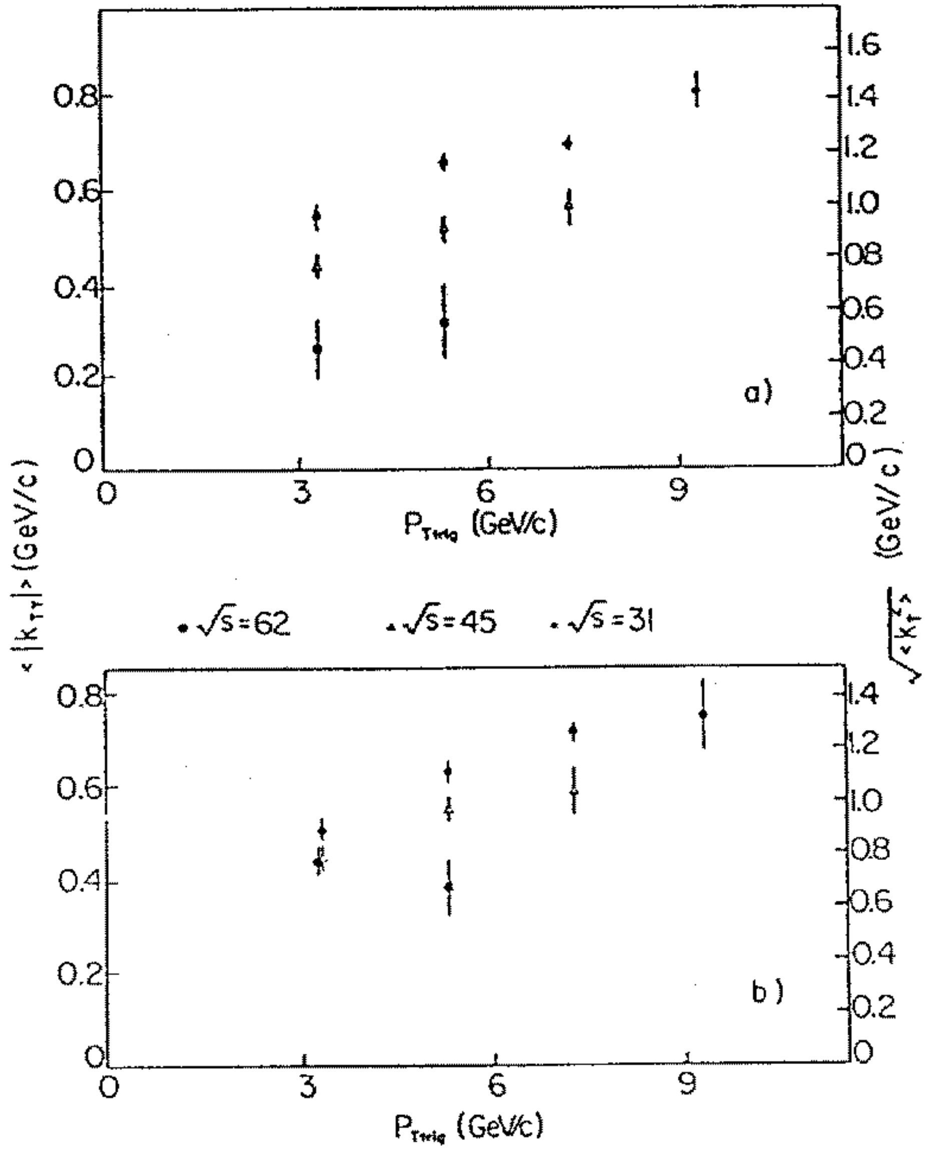
CCOR PLB**97**(1980)163-168

# $j_T$ is constant-independent of $p_{Tt}$ and $\sqrt{s}$ Characteristic of jet fragmentation



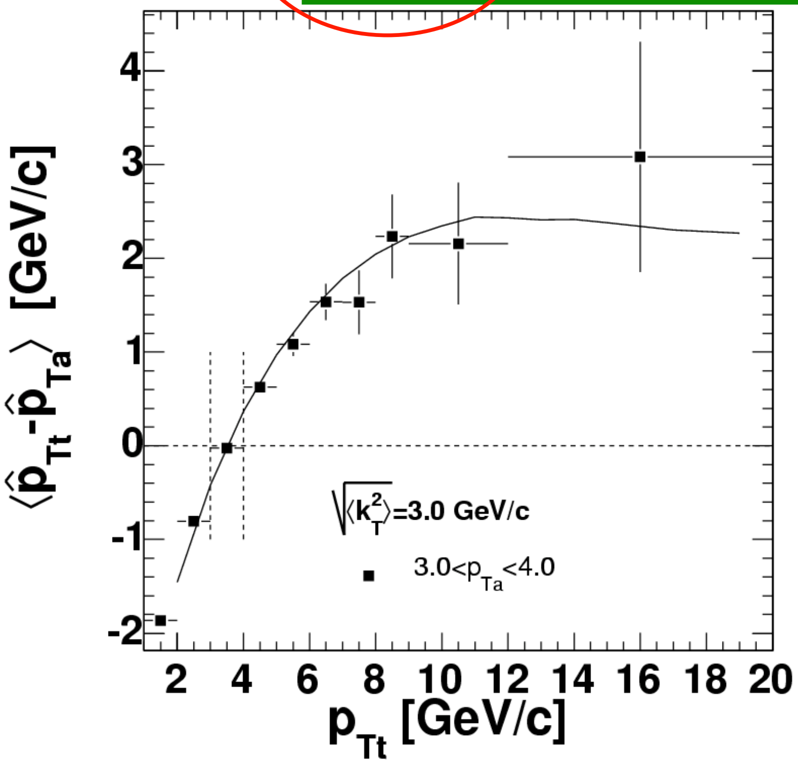

- it took the $e^+ e^-$ people several more years to get this correct--- because they didn't understand the seagull effect: ($j_T < p_T$)

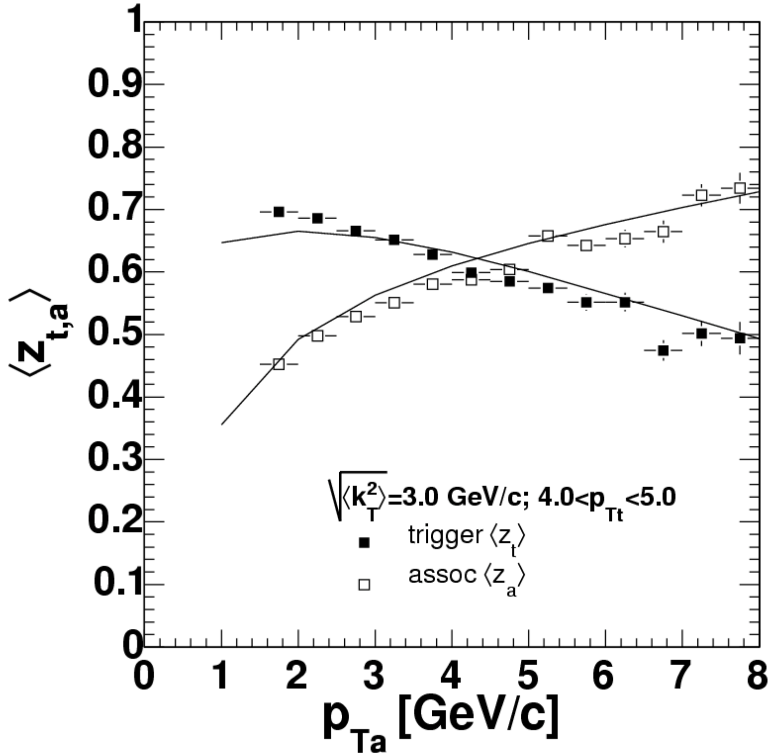
# $k_T$ varies with $p_{Tt}$ and $\sqrt{s}$--not intrinsic



# At RHIC extra complications due to larger $k_T$

$$\hat{x}_h^{-1}\langle z_t\rangle\sqrt{\langle k_T^2\rangle} = x_h^{-1}\sqrt{\langle p_{out}^2\rangle - \langle j_{Ty}^2\rangle(1+x_h^2)}$$



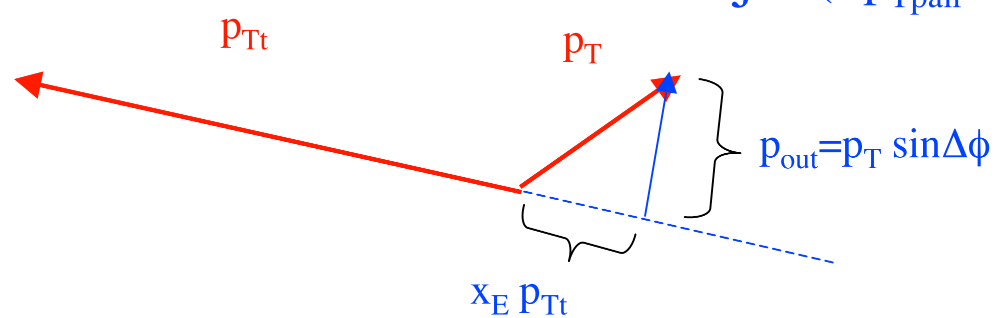




Larger Jet imbalance due to $k_T$ smearing

smaller $<z_t>$ due to smaller n
$<z_{t,a}>$ vary with $p_{Tt}$ $p_{Ta}$

# Additionally, PHENIX found a mistake

Following FFF and CCOR PLB**97**(1980)163-168 we were trying to measure the net transverse momentum of the di-jet ($\langle p_{T\text{pair}}\rangle = \sqrt{2} \times \langle k_T\rangle$)



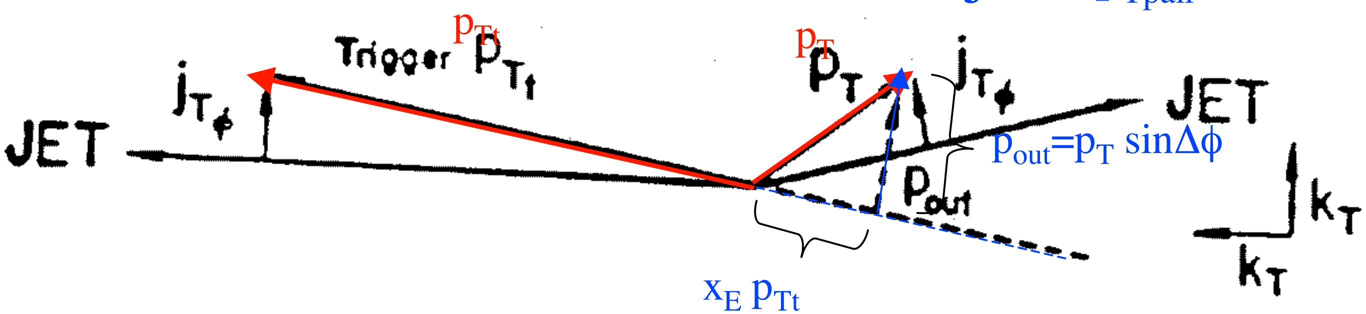

$$\frac{\langle z_t(k_T,x_h)\rangle\sqrt{\langle k_T^2\rangle}}{\hat{x}_h(k_T,x_h)} = \frac{1}{x_h}\sqrt{\langle p_{out}^2\rangle - \left\langle j_{Ty}^2\right\rangle(1+x_h^2)} \qquad x_h \equiv \frac{p_{T_a}}{p_{T_t}} \qquad \hat{x}_h \equiv \frac{\hat{p}_{T_a}}{\hat{p}_{T_t}}$$

- $j_T$ is parton fragmentation transverse momentum
- $k_T$ is transverse momentum of a parton in a proton (2 protons)
- $x_E=-p_T \bullet p_{Tt}/|p_{Tt}|^2$ represents away jet fragmentation z
- $p_{out}$ is component of away $p_T$ perpendicular to trigger $p_{Tt}$

We needed $\langle z_t\rangle$ to solve for $k_T$. Tried to get it from $x_E$ dist.

# Additionally, PHENIX found a mistake

Following FFF and CCOR PLB**97**(1980)163-168 we were trying to measure the net transverse momentum of the di-jet ($\langle p_{Tpair}\rangle = \sqrt{2} \times \langle k_T \rangle$)




$$\frac{\langle z_t(k_T,x_h)\rangle \sqrt{\langle k_T^2\rangle}}{\hat{x}_h(k_T,x_h)} = \frac{1}{x_h}\sqrt{\langle p_{out}^2\rangle - \left\langle j_{Ty}^2\right\rangle (1+x_h^2)} \qquad x_h \equiv \frac{p_{T_a}}{p_{T_t}} \qquad \hat{x}_h \equiv \frac{\hat{p}_{T_a}}{\hat{p}_{T_t}}$$

- $j_T$ is parton fragmentation transverse momentum
- $k_T$ is transverse momentum of a parton in a proton (2 protons)
- $x_E = -p_T \bullet p_{Tt} / |p_{Tt}|^2$ represents away jet fragmentation z
- $p_{out}$ is component of away $p_T$ perpendicular to trigger $p_{Tt}$

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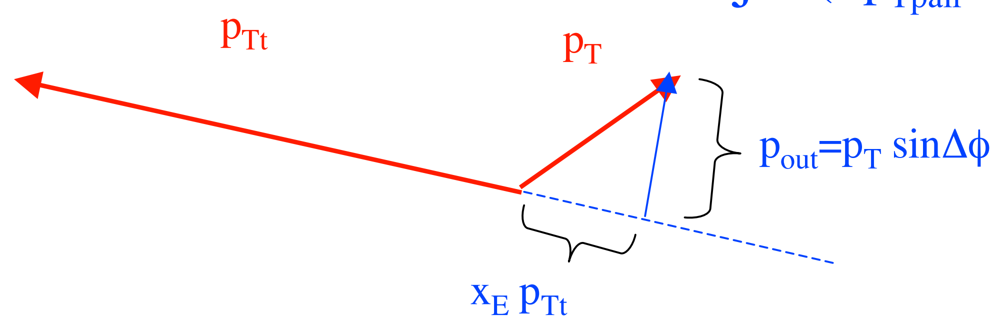


$$\frac{\langle z_t(k_T, x_h)\rangle \sqrt{\langle k_T^2\rangle}}{\hat{x}_h(k_T, x_h)} = \frac{1}{x_h}\sqrt{\langle p_{\mathrm{out}}^2\rangle - \left\langle j_{Ty}^2\right\rangle (1 + x_h^2)} \qquad x_h \equiv \frac{p_{T_a}}{p_{T_t}} \qquad \hat{x}_h \equiv \frac{\hat{p}_{T_a}}{\hat{p}_{T_t}}$$

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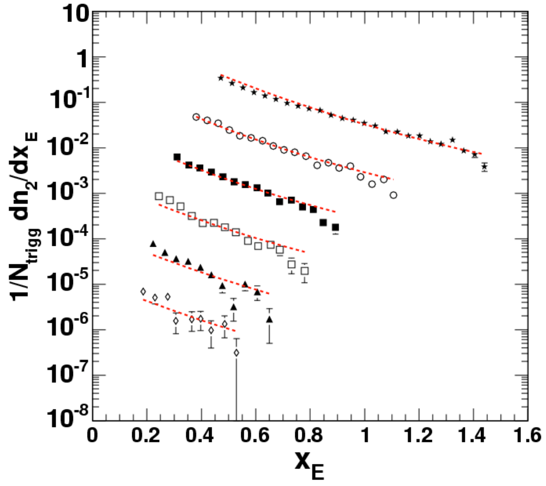
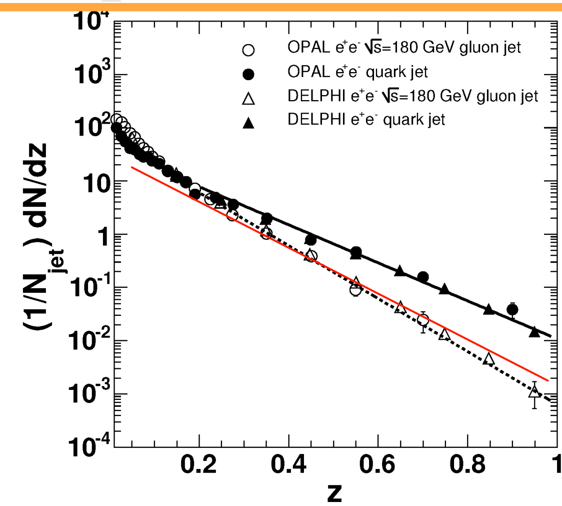
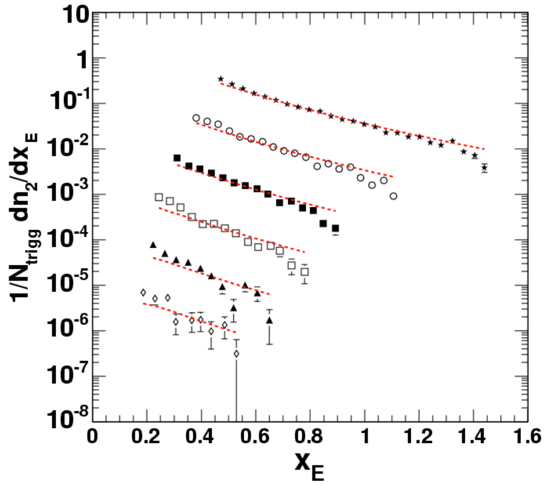
# Fit $x_E$ distributions to form of D(z) used by LEP measurements by integrating Eq (1) numerically

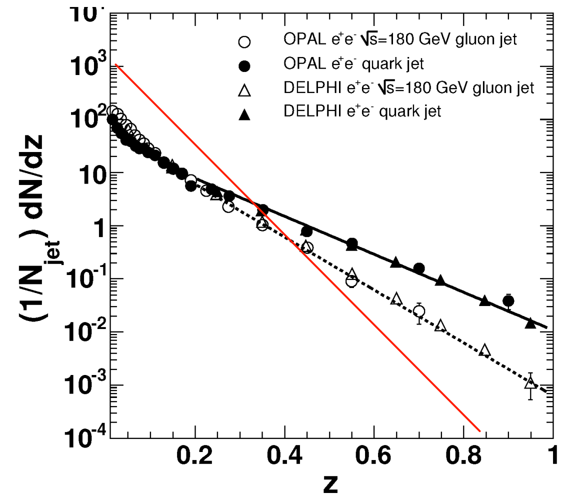



D(z)=exp(-10z)




D(z)=exp(-20z)




After many convergence difficulties, Jan Rak tried two vastly different fragmentation functions $\Rightarrow$no effect on calculated $x_E$ distributions---Mike, can you check this analytically?!

# Amazingly, I could; and got a neat result

$$\boxed{\frac{d\sigma_\pi}{dp_{T_t} dz_t dp_{T_a}} = \frac{1}{\hat{x}_{\rm h}\, p_{T_t}} \frac{d\sigma_q}{d(p_{T_t}/z_t)} D^q_\pi(z_t) D^q_\pi(\frac{z_t p_{T_a}}{\hat{x}_{\rm h} p_{T_t}})} \qquad (1)$$

Take: $D(z) = B\exp(-bz)$ $\qquad \dfrac{d\sigma_q}{d\hat{p}_{T_t}} = \dfrac{A}{\hat{p}_{T_t}^{\;n-1}} = A\,\dfrac{z_t^{n-1}}{{p_{T_t}}^{n-1}}$

$$(2)\quad \frac{d\sigma_\pi}{dp_{T\rm t}dp_{T\rm a}} = \frac{B^2}{\hat{x}_{\rm h}} \frac{A}{p^n_{T_t}} \int_{x_{T_t}}^{\hat{x}_{\rm h}\frac{p_{T\rm t}}{p_{T\rm a}}} dz_{\rm t}\, z_{\rm t}^{n-1} \exp -bz_{\rm t}(1+\frac{p_{T\rm a}}{\hat{x}_{\rm h} p_{T\rm t}})$$

$$\frac{d\sigma_\pi}{dp_{T\rm t}} = \frac{AB}{p_{T_t}^{n-1}} \int_{x_{T_t}}^{1} dz_{\rm t}\, z_{\rm t}^{n-2} \exp -bz_{\rm t}$$

Using: $\Gamma(a,x) \equiv \displaystyle\int_x^\infty t^{a-1}\, e^{-t}\, dt$ $\qquad$ Where Γ(a,0)= Γ(a)=(a-1) Γ(a)

# The final result

$$\frac{d^2\sigma_\pi}{dp_{T\mathrm{t}}dp_{T\mathrm{a}}} \approx \frac{\Gamma(n)}{b^n}\frac{B^2}{\hat{x}_{\mathrm{h}}}\frac{A}{p_{T_t}^n}\frac{1}{(1+\frac{p_{T\mathrm{a}}}{\hat{x}_{\mathrm{h}}p_{T\mathrm{t}}})^n}$$

$$\frac{d\sigma_\pi}{dp_{T\mathrm{t}}} \approx \frac{\Gamma(n-1)}{b^{n-1}}\frac{AB}{p_{T_t}^{n-1}} \quad ,$$

$$\left.\frac{dP_\pi}{dp_{T\mathrm{a}}}\right|_{p_{T\mathrm{t}}} \approx \frac{B(n-1)}{bp_{T\mathrm{t}}}\frac{1}{\hat{x}_{\mathrm{h}}}\frac{1}{(1+\frac{p_{T\mathrm{a}}}{\hat{x}_{\mathrm{h}}p_{T\mathrm{t}}})^n} \quad . \qquad (42)$$

In the collinear limit, where $p_{T\mathrm{a}}=x_{\mathrm{E}}p_{T\mathrm{t}}$ :

$$\left.\frac{dP_\pi}{dx_{\mathrm{E}}}\right|_{p_{T\mathrm{t}}} \approx \langle m\rangle\,(n-1)\frac{1}{\hat{x}_{\mathrm{h}}}\frac{1}{(1+\frac{x_E}{\hat{x}_{\mathrm{h}}})^n} \quad . \qquad (43)$$

Where B/b≈<m>≈b is the mean charged multiplicity in the jet

# The $x_E$ distribution triggered by a $\pi^0$ does not measure the Fragmentation Function!!!

- The only dependence on the fragmentation function is in the normalization constant B/b which equals <m>, the mean multiplicity in the away jet from the integral of the fragmentation function.

- The dominant term in the $x_E$ distribution is the Hagedorn function $1/(1+x_E/\hat{x}_h)^n$ so that at fixed $p_{Tt}$ the $x_E$ distribution is predominantly a function only of $x_E$ and thus exhibits $x_E$ scaling, as observed.

- The reason that the $x_E$ distribution is not sensitive to the shape of the fragmentation function is that the integral over $z_t$ in (1, 2) for fixed $p_{Tt}$ and $p_{Ta}$ is actually an integral over jet transverse momentum $\hat{p}_{T_t}$. However since the trigger and away jets are always roughly equal and opposite in transverse momentum (in p+p), integrating over $\hat{p}_{T_t}$ simultaneously integrates over $\hat{p}_{T_a}$. The integral is over $z_t$, which appears in both trigger and away side fragmentation functions in (1).

# NOTE:This contradicts a clear statement on this issue from Feynman, Field and Fox

38 *R.P Feynman et al. / Large transverse momenta*

FFF Nucl.Phys. **B128**(1977) 1-65

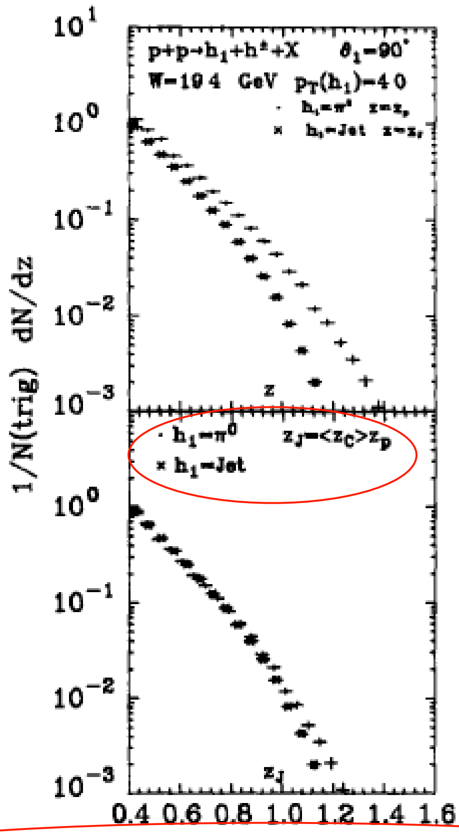



Fig. 23. Comparison of the $\pi^0$ and jet trigger away-side distribution of charged hadrons in pp collisions at $W = 19.4$ GeV, $\theta_1 = 90°$, and $p_\perp$ (trigger) = 4.0 GeV/$c$ from the quark-quark scattering model. The upper figure shows the single-particle ($\pi^0$) trigger results plotted *versus* $z_p = -p_x(h^\pm)/p_\perp(\pi^0)$ and the jet trigger plotted *versus* $z_J = -p_x(h^\pm)/p_\perp(\text{jet})$ (see table 1). In the lower figure, we plot *both versus* $z_J$, where for the jet trigger $z_J = z_J$ but for the single-particle trigger $z_J = \langle z_c \rangle z_p$. The away hadrons are integrated over all rapidity $Y$ and $|180° - \phi| \leqslant 45°$ and the theory is calculated using $\langle k_\perp \rangle_{h\to q} = 500$ MeV. • $h_1 = \pi^0$, × $h_1$ = jet.

"There is a simple relationship between experiments done with single-particle triggers and those performed with jet triggers. The only difference in the opposite side correlation is due to the fact that the 'quark', from which a single-particle trigger came, always has a higher $p_\perp$ than the trigger (by factor $1/z_{trig}$). The away-side correlations for a single-particle trigger at $p_\perp$ should be roughly the same as the away side correlations for a jet trigger at $p_\perp$ (jet)= $p_\perp$ (single particle)/ $\langle z_{trig} \rangle$".

# As shown at the ISR by Darriulat, etc, and believed by most High Energy Physicists

P. Darriulat, et al, Nucl.Phys. **B107** (1976) 429-456

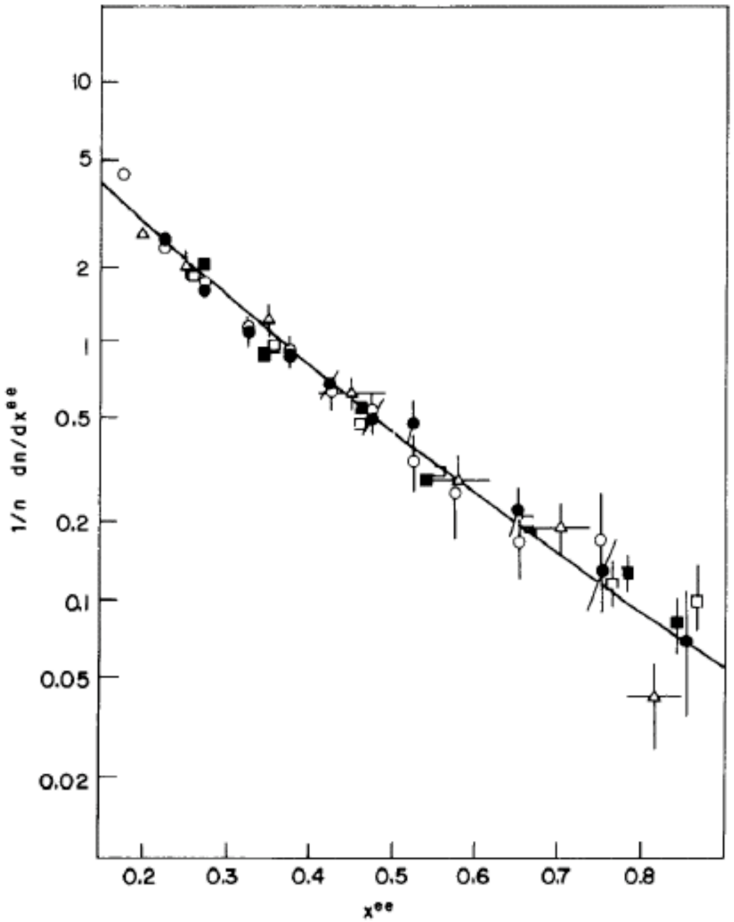



*Figure 21* Jet fragmentation functions measured in different processes: ν-p interactions (*open triangles*, Van der Welde 1979); $e^+e^-$ annihilations (*solid line*, Hanson et al 1975); and pp collisions (*full circles* CS, $p_T < 6$ GeV/c, *open circles* CS, $p_T > 6$ GeV/c, *full squares* CCOR, $p_T > 5$ GeV/c, *open squares* CCOR, $p_T > 7$ GeV/c).

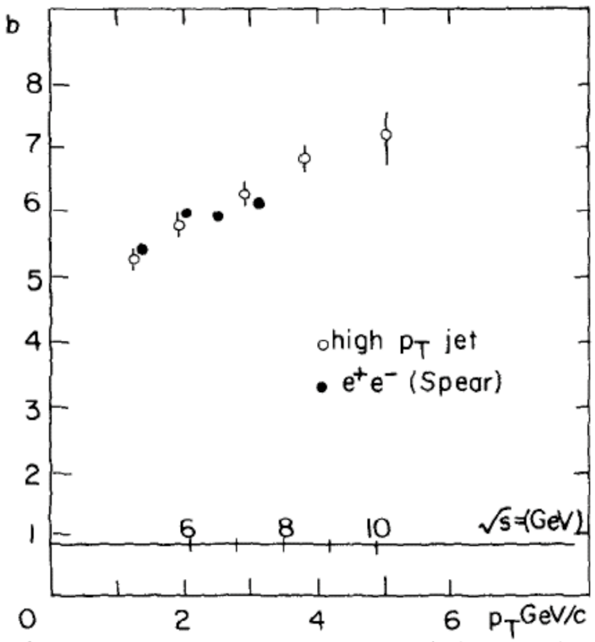
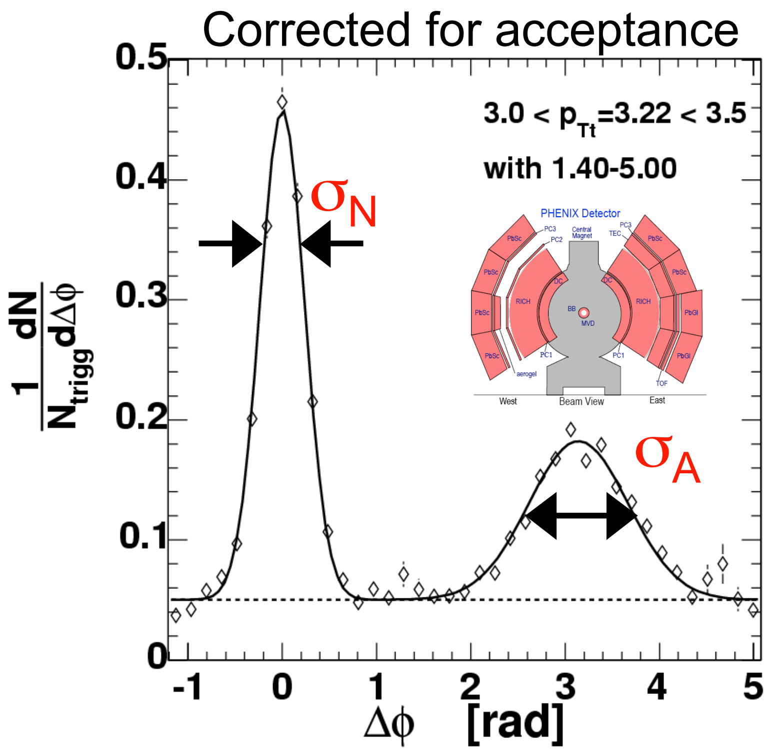
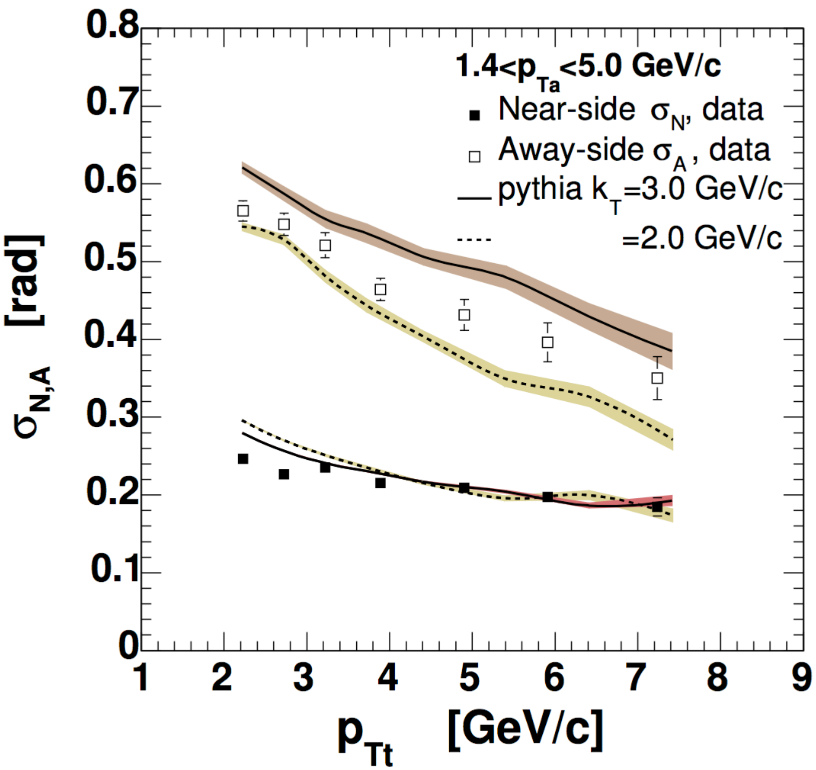



*Figure 19* The slopes *b* obtained from exponential fits to the jet fragmentation function in the interval $0.2 < z < 0.8$ in $e^+e^-$ annihilation (*full circles*) and LPTH data of the BS Collaboration (*open circles*).

Figures from P. Darriulat, ARNPS **30** (1980) 159-210 showing that Jet fragmentation functions in νp, $e^+e^-$ and pp (CCOR) are the same with the same dependence of *b* (exponential slope) on "$\hat{s}$"

# PHENIX $\pi^o$ - $h^\pm$ $j_T$, $k_T$, $x_E$ measurements p+p $\sqrt{s}$=200 GeV: PRD **74**, 072002 (2006)



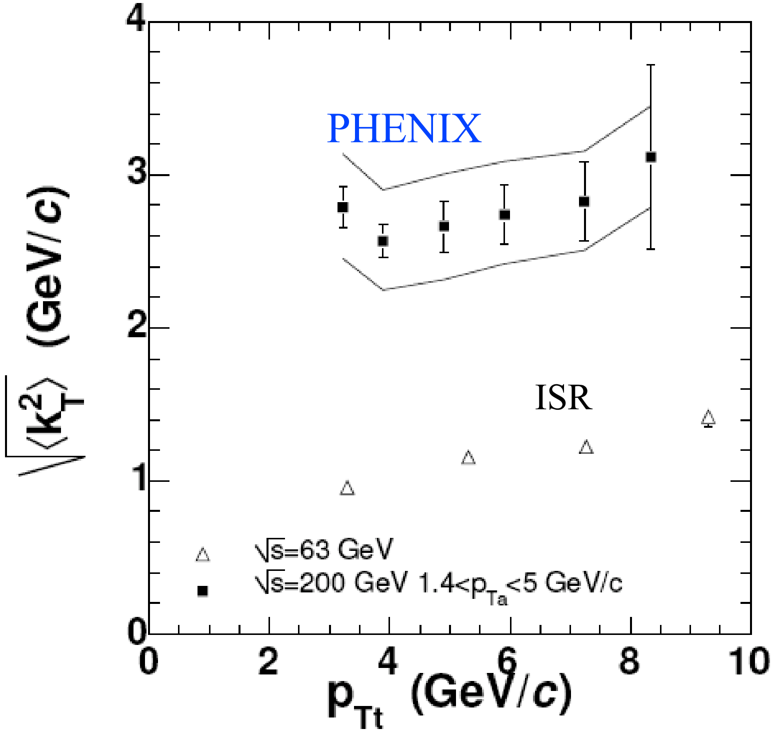

$\sigma_N \propto \langle j_T \rangle$ jet fragmentation transverse momentum-measure directly

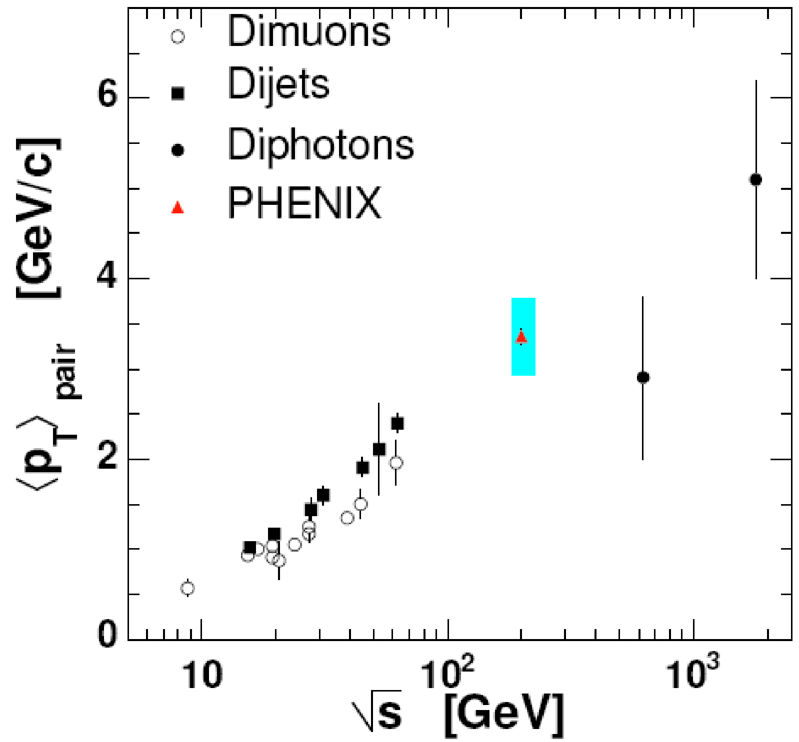
$$\boxed{\sqrt{\langle j_T^2 \rangle} = 585 \pm 6(\text{stat}) \pm 15(\text{sys})\ \text{MeV}/c}$$

$\sigma_F \propto \langle k_T \rangle$ parton transverse momentum-more complicated.

# Results RMS $k_T$ in p+p @ 200 GeV

$$\sqrt{\langle k_T^2 \rangle} = 2.68 \pm 0.07(\text{stat}) \pm 0.34(\text{sys})\ \text{GeV}/c$$

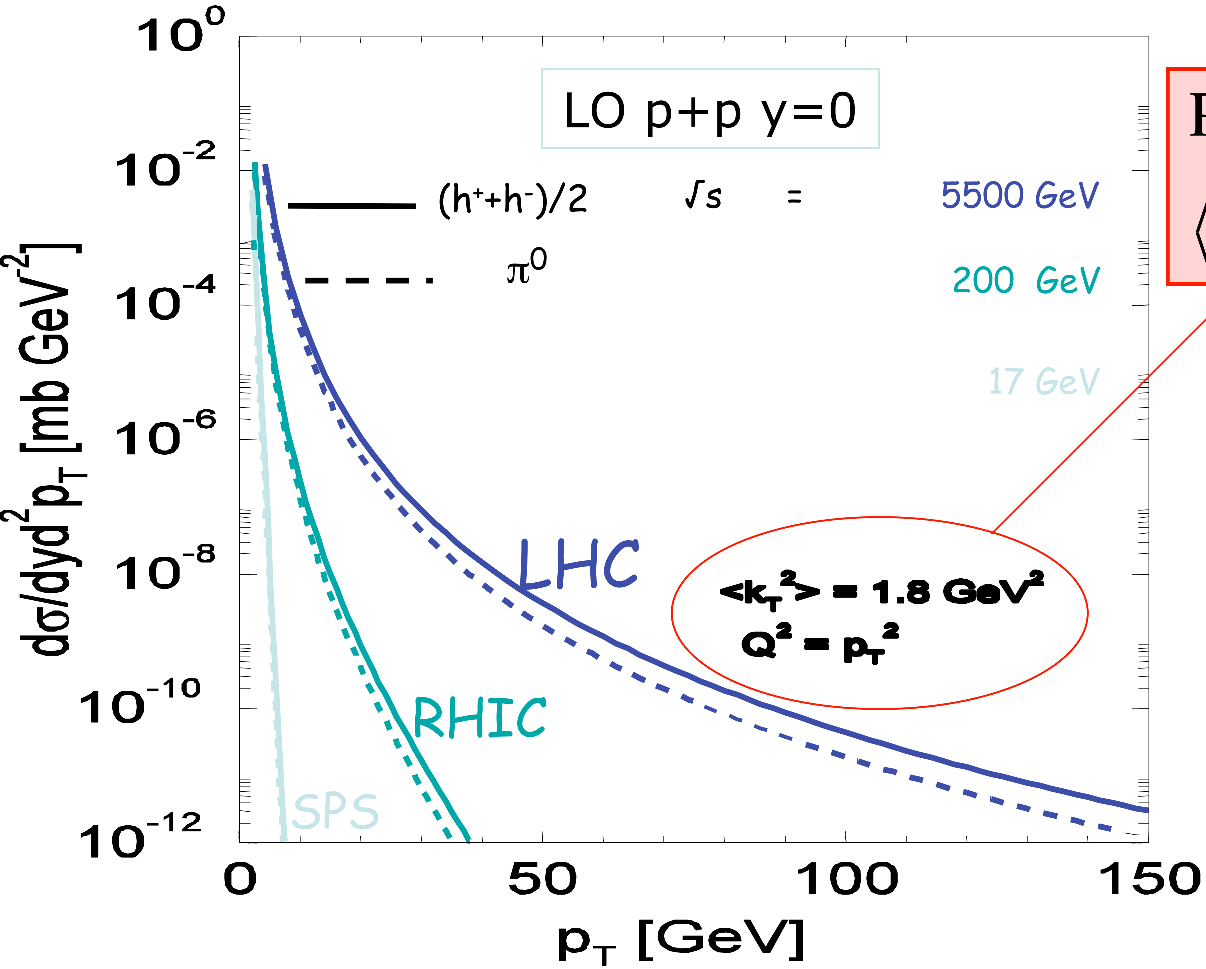



$$\langle p_T \rangle_{pair} = 3.36 \pm 0.09(\text{stat}) \pm 0.43(\text{sys})\ \text{GeV}/c$$

Main contribution to the systematic errors comes from unknown ratio gluon/quark jet => D(z) slope.

# LHC prediction from many talks Schutz, Gustaffson, Morsch ...




PHENIX $\sqrt{s}$=200 GeV

$$\left\langle k_T^2 \right\rangle = 7.2 \pm 1.8 \ \mathrm{GeV/c^2}$$

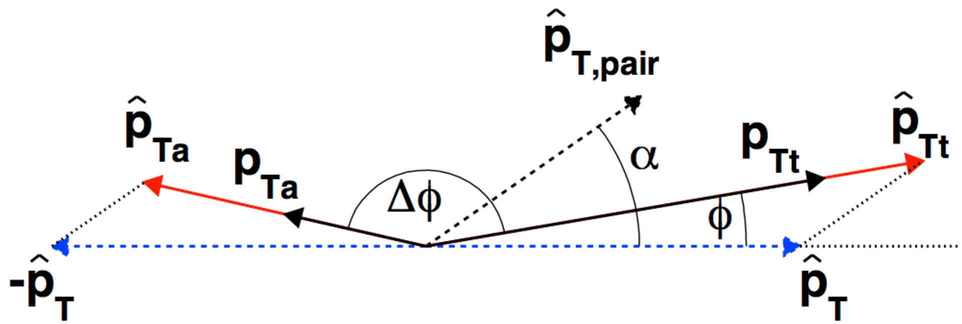
**Hmmm!**

# Discussion
# Application

# A very interesting formula

$$\left.\frac{dP_\pi}{dx_{\mathrm{E}}}\right|_{p_{T\mathrm{t}}} \approx \langle m \rangle (n-1) \frac{1}{\hat{x}_{\mathrm{h}}} \frac{1}{(1+\frac{x_E}{\hat{x}_{\mathrm{h}}})^n}$$

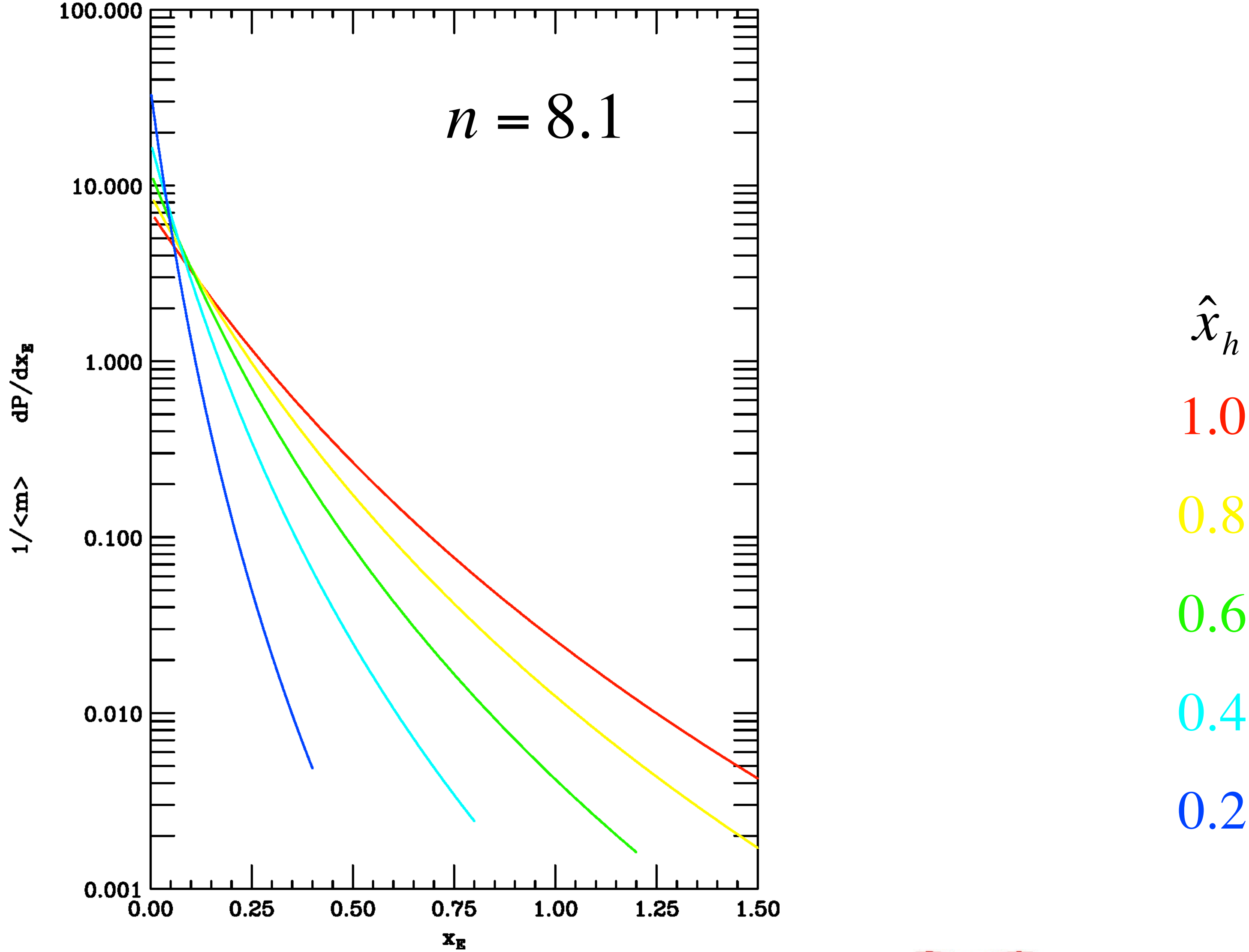
<m> is the mean multiplicity in the jet, n is the power of the $p_{Tt}$ spectrum
⇒ The $x_E$ spectrum scales in the variable $x_E / \hat{x}_h$



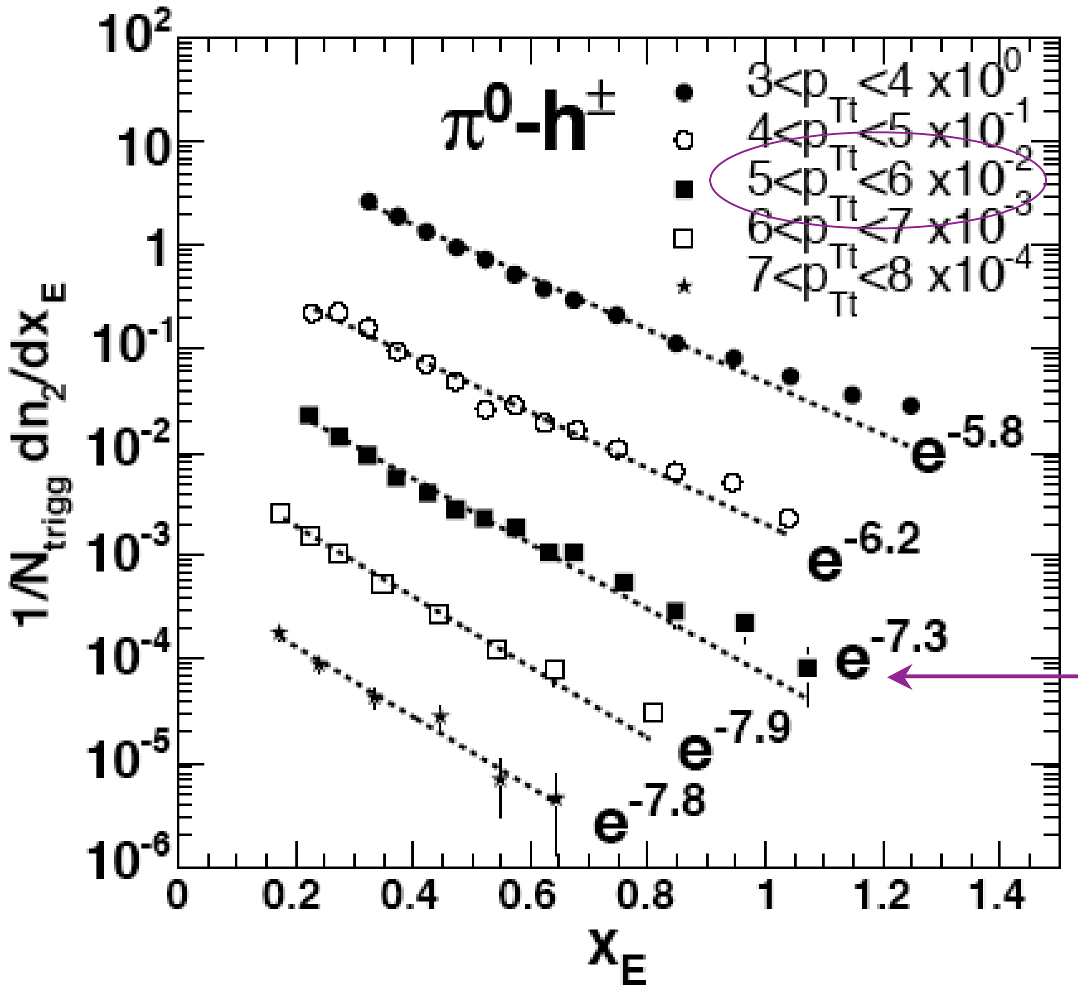

$$x_E = \frac{-p_{T_a} \cos \Delta\phi}{p_{T_t}} \simeq \frac{p_{T_a}}{p_{T_t}} \longrightarrow \hat{x}_h = \frac{\hat{p}_{T_a}}{\hat{p}_{T_t}}$$

Measured ratio of particle $p_{Ta}$, $p_{Tt}$ ⇒ Ratio of jet transverse momenta

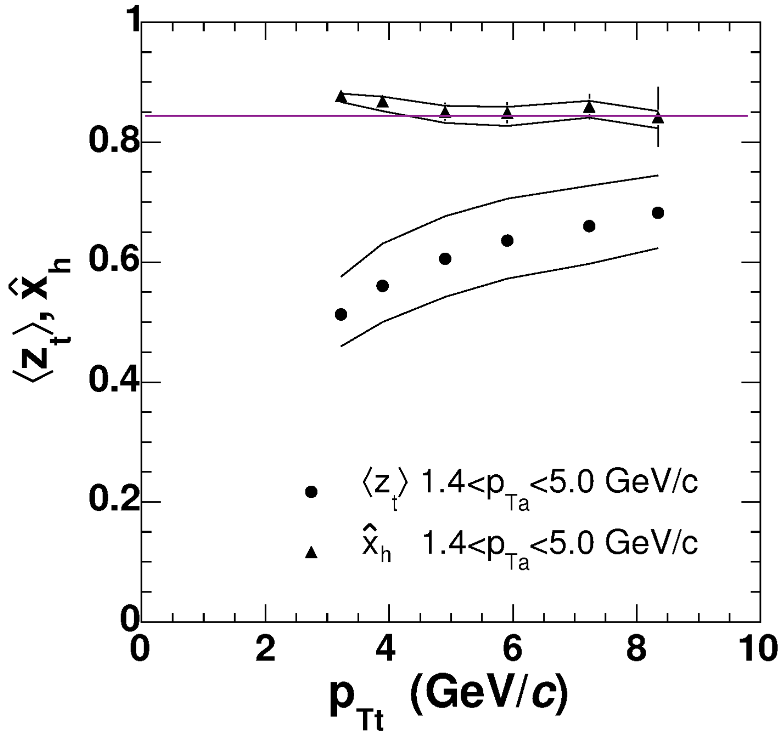
# Shape of $x_E$ distribution depends on $\hat{x}_h$ and $n$ but not on $b$



# Does the formula work?

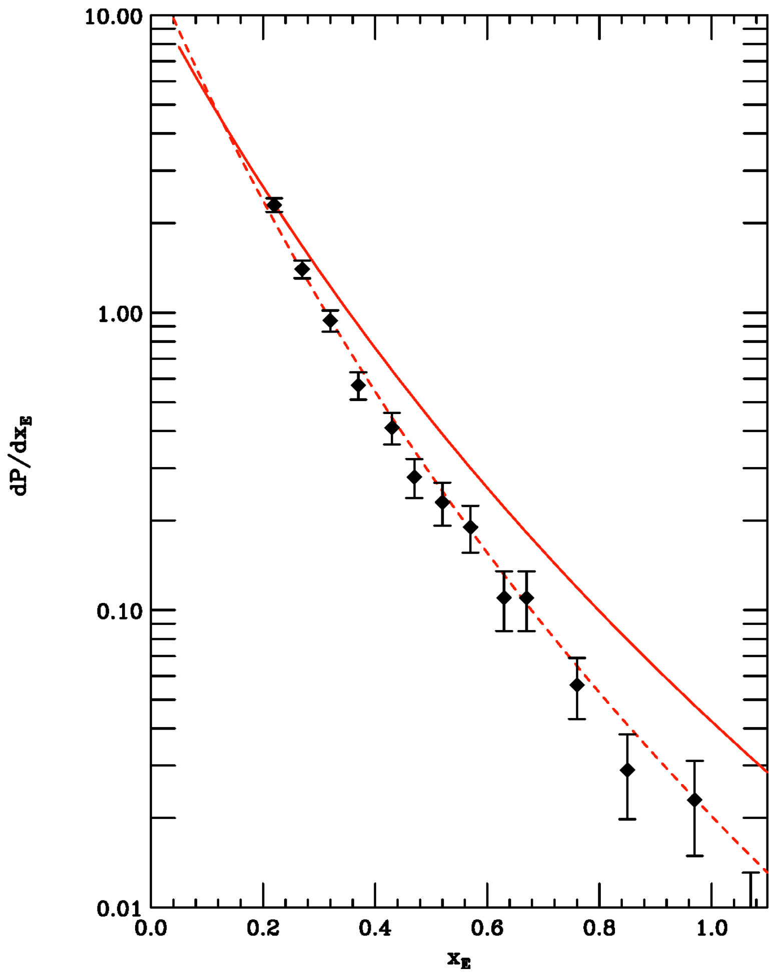



PHENIX p+p
PRD 74, 072002 (2006)




$\boxed{\hat{x}_{\rm h} \approx 0.8 \text{ due to } k_T \text{ smearing}}$

# It works for PHENIX p+p PRD **74**, 072002




$$f(y) = 1.63 \times \frac{7.1}{(1+y)^{8.1}}$$

$x_E = y$ (solid line)

$x_E = \hat{x}_h\, y = 0.8\, y$ (dashed line)

The formula should work just as well in Au+Au collisions to relate the measured $x_E$ distribution of fragments to the ratio of the transverse momenta of the parent jets. The only major assumptions are independent fragmentation of both jets with the same exponential fragmentation function and a power law parton $p_T$ distribution.

This is covered in my next talk.

END