Elliptic flow from Lee-Yang zeroes

Workshop on Transverse Dynamics at RHIC
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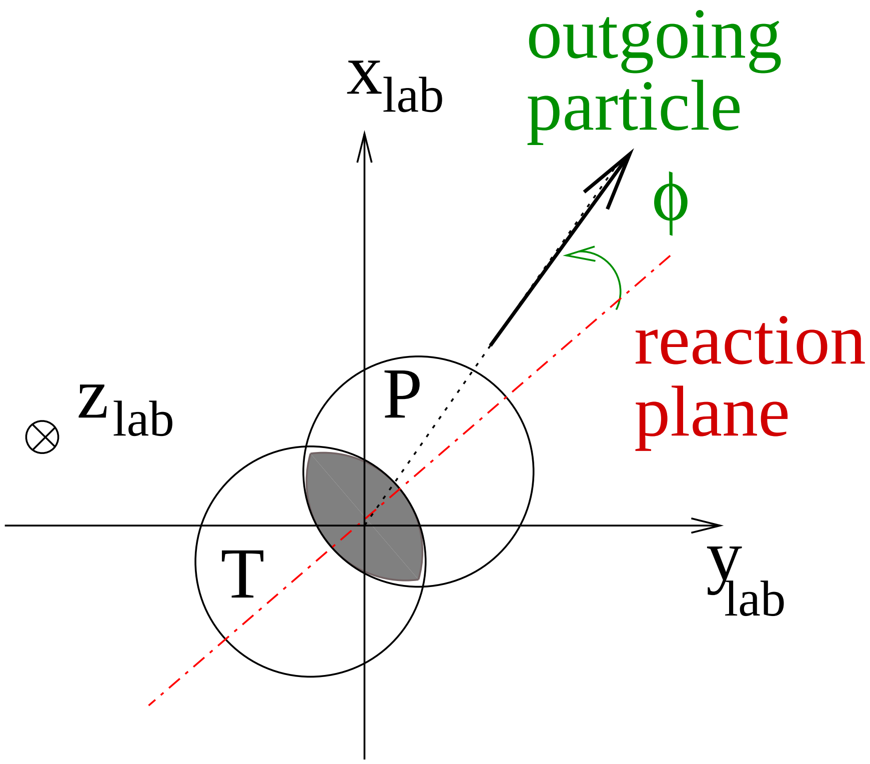
(work in progress)

Outline

- Why we need a new method to analyze elliptic flow at RHIC
- The new method: theoretical basis
  - Cumulants: genuine correlations between 2, 4,.. particles
  - Large orders in the cumulant expansion
  - Sensitivity to nonflow effects
- The new method: practical implementation
  - Method to obtain v_2
  - Statistical errors
  - Monte-Carlo results
- Summary and perspectives

Definitions: directed flow, elliptic flow



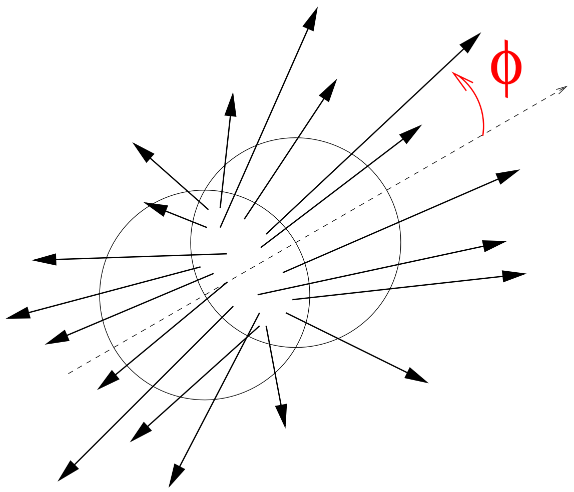
$$\frac{\mathrm{d}N}{\mathrm{d}\phi} \propto \underbrace{1}_{\text{isotropic}} + 2\,\underbrace{v_1}_{\text{directed}} \cos\phi + 2\,\underbrace{v_2}_{\text{elliptic}} \cos 2\phi + \ldots$$

$$\begin{aligned} v_n &= \langle \cos n\phi \rangle = \langle e^{in\phi} \rangle \\ &= \text{Correlation to the reaction plane} \\ &\equiv \text{“anisotropic flow”} \end{aligned}$$

$v_2 \sim 6\% \gg v_1$ at RHIC: v_1 yet unobserved.

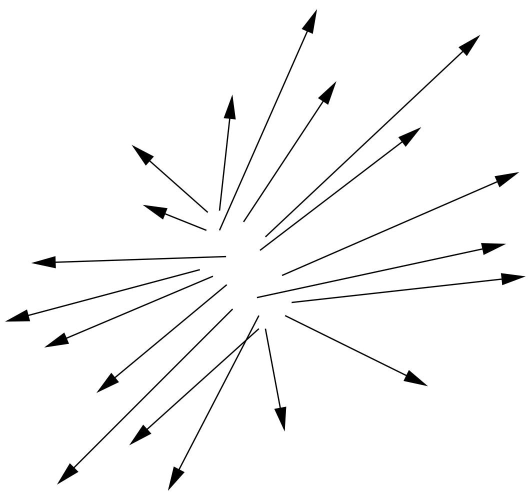
Elliptic flow is collective motion

Seen by Theory



$v_2 = \langle \cos 2\phi \rangle > 0$. Trivial. Not even a correlation

Seen by an experimentalist



- In a given event, all momenta have a preferred, unknown direction → collective motion
- This direction fluctuates randomly from one event to the other

Methods for measuring elliptic flow

- Two-particle correlations: assuming all correlations are due to flow

  Plain two-particle method: PHENIX

  Wang et al, Phys.Rev.C44:1091 (1991)

  Event-plane method: STAR, PHOBOS

  Danielewicz and Odyniec, Phys.Lett.B157:146 (1985)
  Ollitrault, nucl-ex/9711003

  Poskanzer and Voloshin, nucl-ex/9805001

- Cumulants:

  Improved methods of analysis using genuine 4-, 6-particle correlations.

  Borghini, Dinh, Ollitrault, nucl-th/0007063

  nucl-th/0105040

  Applied by

  STAR nucl-ex/0206001

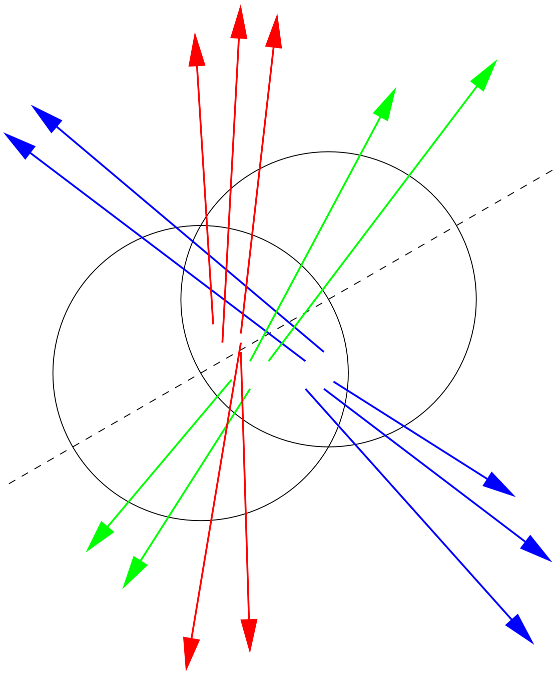
  NA49 nucl-ex/0303001

- Now comes the ultimate method: analysis of genuine collective motion

v_2 at RHIC: a controversy

Kovchegov Tuchin hep-ph/0203013
nucl-th/0207037

Minijets produce bunches of collinear particles:



Observed 2-, 4-particle correlations have nothing to do with the geometry of the system. It's all "nonflow".

Motivation of our work: Can we see experimentally the difference between a correlation involving a small number (< 10) of particles and a truly collective (> 100) motion?

Tools for the flow analysis
I. Generating function

Q is defined for each event as

sum over particles seen in the event

$$\downarrow$$

$$Q \equiv \sum_{j=1}^{M} \cos 2\phi_j$$

$$\uparrow$$

azimuthal angles measured in the laboratory

Next define $G(z)$, with z complex, as

$$G(z) = \left\langle e^{zQ} \right\rangle$$

$$\uparrow$$

average over events

(note that the $\sin(2\phi_j)$ are not needed here)

Tools for the flow analysis
II. Moments and cumulants

Moments

$$G(z) = \sum_{k=0}^{+\infty} \frac{z^k}{k!} \left\langle Q^k \right\rangle$$

The moment of order k involves k-particle averages:

$$\langle Q^2 \rangle = \left\langle \sum_{i,j} \cos 2\phi_i \cos 2\phi_j \right\rangle$$

Cumulants

$$\ln G(z) = \sum_{k=0}^{+\infty} \frac{z^k}{k!} c_k$$

The cumulant of order k involves k-particle correlations:

$$\begin{aligned} c_2 &= \left\langle \sum_{i,j} \cos 2\phi_i \cos 2\phi_j \right\rangle \\ &- \left\langle \sum_i \cos 2\phi_i \right\rangle \left\langle \sum_j \cos 2\phi_j \right\rangle \end{aligned}$$

Why cumulants

Because they are small when no elliptic flow.

If the system can be seen as a superposition of N independent subsystems

(i.e., no collective motion, independent NN collisions)

Then

$$Q = \sum_{i=1}^{N} Q_i$$
$$\left\langle Q^k \right\rangle \propto N^k$$

While

$$G(z) = \prod_{i=1}^{N} G_i(z)$$
$$\ln G(z) = \sum_{i=1}^{N} \ln G_i(z)$$
$$c_k \propto N$$

Moments scale like N^k, cumulants scale like N

Cumulants become smaller and smaller compared to moments as k increases.

Cumulants and flow

Collective flow $\rightarrow$ cumulants $\sim$ moments.

(no more reason why cumulants should be smaller)

Check this explicitly with a simple model:

For a fixed orientation of the reaction plane Φ_R, assume independent particles

$$\begin{aligned}
\langle \cos 2\phi_j | \Phi_R \rangle &= v_2 \cos 2\Phi_R \\
\left\langle e^{z \cos 2\phi_j} | \Phi_R \right\rangle &\simeq 1 + z v_2 \cos 2\Phi_R + \frac{z^2}{4} + \cdots \\
\left\langle e^{zQ} | \Phi_R \right\rangle &\simeq \exp\left(M\left(z v_2 \cos 2\Phi_R + z^2/4\right)\right)
\end{aligned}$$

Next, average over Φ_R:

$$\begin{aligned}
G(z) &= \int_0^{2\pi} \frac{d\Phi_R}{2\pi} \left\langle e^{zQ} | \Phi_R \right\rangle \\
&= e^{Mz^2/4} I_0(M z\, v_2)
\end{aligned}$$

For $v_2 > 0$, the Bessel function contributes to all orders in the cumulant expansion.

Large-order behaviour

In order to minimize nonflow effects, we must construct the cumulants c_k for large k.

$$\ln G(z) = \sum_{k=1}^{+\infty} \frac{z^k}{k!} c_k$$

Asymptotic behaviour of c_k for $k \to +\infty$
$\leftrightarrow$ Radius of convergence of the series
$\leftrightarrow$ First singularity of $\ln G(z)$ for complex z
$\leftrightarrow$ First zero of $G(z)$ in the complex plane.

Very generally, studying collective motion amounts to finding the zero of the generating function $G(z)$ which is closest to the origin in the complex plane of z.

Where are the Lee-Yang zeroes?

An analogy:

Lee Yang Phys.Rev.87 (1952) 404

A theory of phase transitions from the zeroes of the grand partition function

$$G(z) = \sum_{N=0}^{+\infty} Z_N e^{zN}$$

The zeroes lie on the imaginary axis.
Here, N is replaced by Q:

$$\begin{aligned} G(z) &= \langle e^{zQ} \rangle \\ &= \int_{-\infty}^{+\infty} p(Q)\, e^{zQ}\, dQ, \end{aligned}$$

The zeroes still lie on the imaginary axis!

Example:

$$G(z) = e^{Mz^2/4} I_0(M z\, v_2)$$

The first zero lies at

$$M\, z v_2 \simeq 2.404\, i,$$

Determine z experimentally, then obtain v_2 from this formula.

Sensitivity to nonflow effects

Recall how the cumulant of order k scales with the multiplicity M:

- Nonflow: $c_k \propto M$
- Only flow: $c_k \propto (M v_2)^k$

One always neglects the "nonflow" contribution, i.e., one uses the second equation to obtain v_2. If $v_2 = 0$, the nonflow term generates a spurious v_2:

$$(M v_2^{\rm spurious})^k \sim M$$

- 2-particle correlations: $v_2^{\rm spurious} \propto M^{-1/2}$
- 4-particle cumulants: $v_2^{\rm spurious} \propto M^{-3/4}$
- Lee-Yang zeroes: $v_2^{\rm spurious} \propto M^{-1}$

The new method minimizes nonflow effects.

Practical implementation

$G(z)$ is obtained from an average over a finite number of collisions, and the detector is not perfectly isotropic:
$\rightarrow$ The zeroes of $G(z)$ don't lie exactly on the imaginary axis.
$\rightarrow$ Rather than the first zero, we determine the first minimum of $|G(z)|$ on the imaginary axis.

Until now, we have only used the $\cos 2\phi_j$. In order to use also the information on $\sin 2\phi_j$,

- We repeat the whole procedure with $\cos 2(\phi_j - \theta)$ where θ is a fixed angle $\rightarrow$ we obtain an estimate of v_2 for each value of θ;
- We average the result over the values of θ.

Statistical errors

Even if there is no v_2, a spurious v_2 appears due to statistical fluctuations.

In practice, $G(z)$ is evaluated only on the imaginary axis, and for a finite number of events $N_{\rm evts}$

$$G(iy) = \frac{1}{N_{\rm evts}} \sum_{j=1}^{N_{\rm evts}} \left\langle e^{iyQ_j} \right\rangle .$$

The typical statistical error on $G(iy)$ is naturally $1/\sqrt{N_{\rm evts}}$.

This is to be compared to the expected value

$$G(iy) = e^{-My^2/4}$$

A spurious minimum may appear as soon as

$$e^{-My^2/4} \sim 1/\sqrt{N_{\rm evts}}$$

The resulting spurious flow is

$$v_2^{\rm spurious} = \frac{2.404}{My} \sim \frac{2.404}{\sqrt{2M \ln N_{\rm evts}}}$$

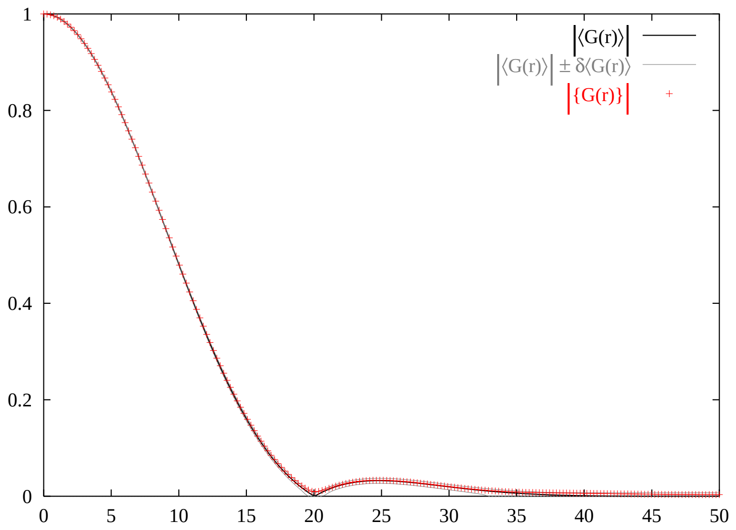
Typically 3% at STAR.

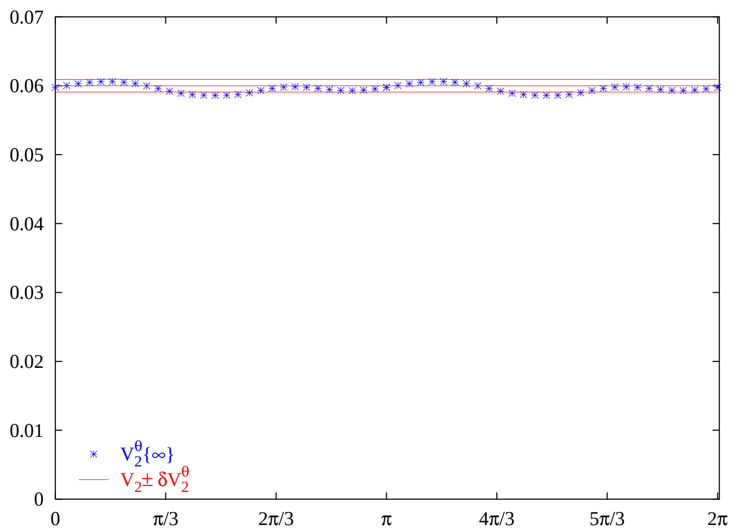
Increasing $N_{\rm evts}$ does not help, increasing M does.

A numerical simulation

Realistic situation corresponding to a Au-Au collision at RHIC with impact parameter approx. 7 fm, as seen by the STAR experiment:

- $N_{\rm coll} = 20000$ collisions
- $M = 300$ particles detected in each collision
- Input $v_2 = 6\%$.





Perspectives

- For the first time, it is now possible to extract genuine collective behavior in heavy ion collisions.
- It can be done globally for the whole event, and also differentially as a function of p_T, y (not shown in this talk)
- In practice, the strongest limitation comes from statistical errors when the flow is small
  - OK: v_1 and v_2 at GSI and AGS, v_2 at top SPS energy (NA49), at RHIC (STAR, Phobos) and LHC (Alice)
  - not OK: v_1 at SPS and RHIC, v_2 at lower SPS energies
- Could be extended to other observables in order to look for critical fluctuations