Constraints on transverse dynamics from emission angle dependent HBT measurements

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1. Motivation
2. Symmetry constraints
3. Hydrodynamical model calculations for RHIC and "LHC"
4. Conclusions

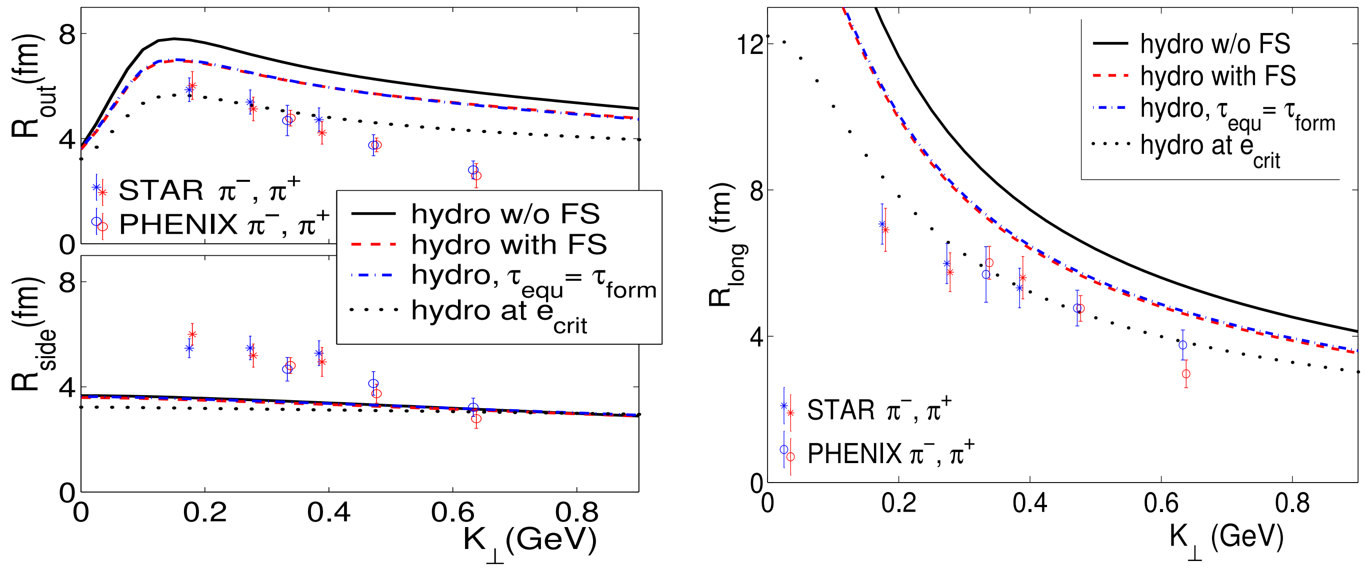
presented at: Tranverse Dynamics at RHIC, BNL, March 6-8, 2003

References:

U. Heinz, A. Hummel, M. Lisa, and U. Wiedemann, Phys. Rev. C 66, 044903 (2002)
U. Heinz and P.F. Kolb, Phys. Lett. B 542 (2002) 216
M. Lisa, U. Heinz and U. Wiedemann, Phys. Lett. B 489 (2000) 287

Motivation I: The RHIC HBT puzzle

Hydrodynamics gives excellent description of momentum distributions (spectra, radial and elliptic flow) but fails to reproduce HBT radii:

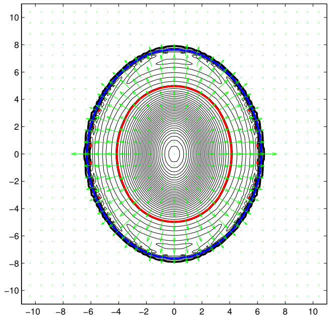


U.H., P. Kolb, hep-ph/0204061

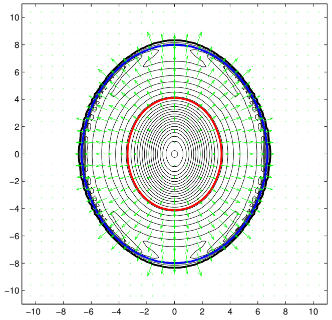
– need faster evolution and earlier freeze-out to reduce temporal contributions to R_l, R_o?

– what about too small R_s and too weak $K_\perp$-dependence of R_s and R_o?

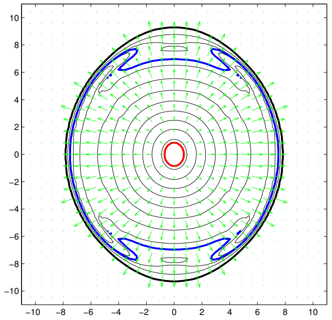
Motivation II: Explore changing source deformation



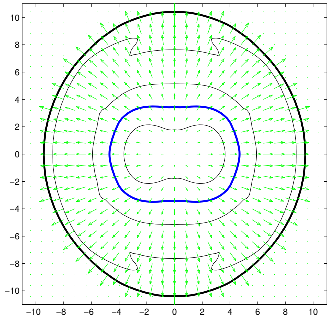
3.2 fm/c ($\epsilon_x = 0.160$, $\epsilon_p = 0.114$)



4.0 fm/c ($\epsilon_x = 0.127$, $\epsilon_p = 0.141$)



5.6 fm/c ($\epsilon_x = 0.067$, $\epsilon_p = 0.147$)



8.0 fm/c ($\epsilon_x = 0.003$, $\epsilon_p = 0.123$)

- deformation of source at freeze-out? can we constrain total expansion time in this way?
- enough time to change sign of $\epsilon_x = \frac{\langle\langle y^2 - x^2 \rangle\rangle}{\langle\langle y^2 + x^2 \rangle\rangle}$?
- HBT measures homogeneity regions, not entire source: so what should we expect to see, given that there is strong and anisotropic expansion flow?

Source size parameters and HBT radii:

- Spatial correlation tensor (widths of emission function):
$$S_{\mu\nu}(\boldsymbol{K}) = [\langle x_\mu x_\nu \rangle - \langle x_\mu \rangle \langle x_\nu \rangle](\boldsymbol{K}) \equiv \langle \tilde{x}_\mu \tilde{x}_\nu \rangle$$
where
$$\langle f(x) \rangle(\boldsymbol{K}) = \int d^4x \, f(x) \, S(x, \boldsymbol{K}) / \int d^4x \, S(x, \boldsymbol{K})$$

- Correlation radii:
$$C(\boldsymbol{q}, \boldsymbol{K}) = 1 + \exp\left[-\sum_{i,j=o,s,l} q_i q_j R^2_{ij}(\boldsymbol{K}) \right]$$

- Relation between correlator and source function:
$$C(\boldsymbol{q}, \boldsymbol{K}) = 1 + \left| \frac{\int d^4x \, S(x, \boldsymbol{K}) \, e^{i\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{\beta}t)}}{\int d^4x \, S(x, \boldsymbol{K})} \right|^2$$
where $\boldsymbol{\beta} = \boldsymbol{K}/K^0$ is the pair velocity.

Relation between source size parameters and HBT radii:

For a sufficiently Gaussian source one has (Wiedemann 1998)

$$\begin{aligned}
R_s^2 &= \tfrac{1}{2}(S_{11}+S_{22}) - \tfrac{1}{2}(S_{11}-S_{22})\cos(2\Phi) - S_{12}\sin(2\Phi) \\
R_o^2 &= \tfrac{1}{2}(S_{11}+S_{22}) + \tfrac{1}{2}(S_{11}-S_{22})\cos(2\Phi) + S_{12}\sin(2\Phi) \\
&\quad -2\beta_\perp(S_{01}\cos\Phi+S_{02}\sin\Phi) + \beta_\perp^2 S_{00}, \\
R_{os}^2 &= S_{12}\cos(2\Phi) - \tfrac{1}{2}(S_{11}-S_{22})\sin(2\Phi) + \beta_\perp(S_{01}\sin\Phi-S_{02}\cos\Phi), \\
R_l^2 &= S_{33} - 2\beta_l S_{03} + \beta_l^2 S_{00}, \\
R_{ol}^2 &= (S_{13}-\beta_l S_{01})\cos\Phi + (S_{23}-\beta_l S_{02})\sin\Phi - \beta_\perp S_{03} + \beta_l\beta_\perp S_{00}, \\
R_{sl}^2 &= (S_{23}-\beta_l S_{02})\cos\Phi - (S_{13}-\beta_l S_{01})\sin\Phi .
\end{aligned}$$

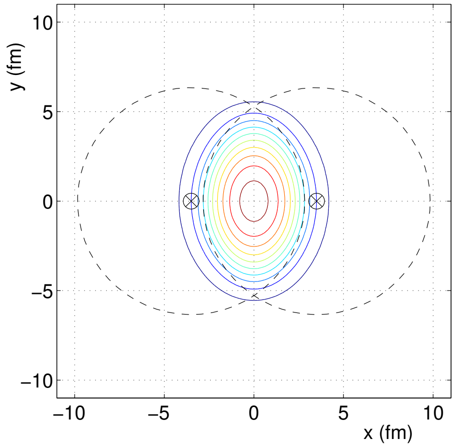
Note: explicit Φ-dependence due to rotation of coordinate system:

$$x_o = x\cos\Phi + y\sin\Phi, \quad x_s = -x\sin\Phi + y\cos\Phi.$$

implicit Φ-dependence due to $S_{\mu\nu}(\boldsymbol{K}) = S_{\mu\nu}(Y, K_\perp, \Phi)$.

Symmetries of the emission function:

I. Reflection symmetry w.r.t. the reaction plane (for spherical nuclei):



$$S(x,y,z,t;Y,K_\perp,\Phi) = S(x,-y,z,t;Y,K_\perp,-\Phi)$$

$$\Longrightarrow \boxed{S_{\mu\nu}(Y,K_\perp,\Phi) = \theta_1\, S_{\mu\nu}(Y,K_\perp,-\Phi)}$$

with $\theta_1 = (-1)^{\delta_{\mu 2}+\delta_{\nu 2}}$

II. Point reflection symmetry w.r.t.collision center (equal nuclei):

$$S(x,y,z,t;Y,K_\perp,\Phi) = S(-x,-y,-z,t;-Y,K_\perp,\Phi+\pi)$$

$$\Longrightarrow \boxed{S_{\mu\nu}(Y,K_\perp,\Phi) = \theta_2\, S_{\mu\nu}(-Y,K_\perp,\Phi+\pi)}$$

with $\theta_2 = (-1)^{\delta_{\mu 0}+\delta_{\nu 0}}$

Helps to restrict azimuthal Fourier decomposition at $Y=0$.

Fourier expansion of spatial correlation tensor:

$$S(\Phi) = \mathcal{C}_0 + 2\sum_{n=1}^{\infty} \left[\mathcal{C}_n \cos(n\Phi) + \mathcal{S}_n \sin(n\Phi)\right],$$

$$\mathcal{C}_n = \int_{-\pi}^{\pi} \frac{d\Phi}{2\pi} S(\Phi)\, \cos(n\Phi), \quad \mathcal{S}_n = \int_{-\pi}^{\pi} \frac{d\Phi}{2\pi} S(\Phi)\, \sin(n\Phi).$$

I $\Longrightarrow$

$\mathcal{S}_n = 0$ for all terms containing even powers of y $\mathcal{C}_n = 0$ for all terms containing odd powers of y

II $\Longrightarrow$

For terms containing even powers of t, $\mathcal{S}_n, \mathcal{C}_n$ are odd (even) functions of Y for odd (even) n. For terms containing odd powers of t, it's the other way around. The odd functions vanish at $Y = 0$.

Fourier expansion of spatial correlation tensor at $Y = 0$:

$S_{\mu\nu}$	θ_1	θ_2	Fourier expansion	Zeroes
$\frac{\langle \tilde{x}^2+\tilde{y}^2 \rangle}{2}$	1	1	$A_0 + 2\sum_{n\geq 2,\text{even}} A_n \cos(n\Phi)$	–
$\frac{\langle \tilde{x}^2-\tilde{y}^2 \rangle}{2}$	1	1	$B_0 + 2\sum_{n\geq 2,\text{even}} B_n \cos(n\Phi)$	–
$\langle \tilde{x}\tilde{y} \rangle$	-1	1	$2\sum_{n\geq 2,\text{even}} C_n \sin(n\Phi)$	$0^\circ, 90^\circ$
$\langle \tilde{t}^2 \rangle$	1	1	$D_0 + 2\sum_{n\geq 2,\text{even}} D_n \cos(n\Phi)$	–
$\langle \tilde{t}\tilde{x} \rangle$	1	-1	$2\sum_{n\geq 1,\text{odd}} E_n \cos(n\Phi)$	90°
$\langle \tilde{t}\tilde{y} \rangle$	-1	-1	$2\sum_{n\geq 1,\text{odd}} F_n \sin(n\Phi)$	0°
$\langle \tilde{t}\tilde{z} \rangle$	1	-1	$2\sum_{n\geq 1,\text{odd}} G_n \cos(n\Phi)$	90°
$\langle \tilde{x}\tilde{z} \rangle$	1	1	$H_0 + 2\sum_{n\geq 2,\text{even}} H_n \cos(n\Phi)$	–
$\langle \tilde{y}\tilde{z} \rangle$	-1	1	$2\sum_{n\geq 2,\text{even}} I_n \sin(n\Phi)$	$0^\circ, 90^\circ$
$\langle \tilde{z}^2 \rangle$	1	1	$J_0 + 2\sum_{n\geq 2,\text{even}} J_n \cos(n\Phi)$	–

Fourier expansion of HBT radii at $Y = 0$:

Insert the Fourier series of spatial correlation tensor components into Wiedemann relations and combine implicit with explicit Φ-dependence:

$$
\begin{aligned}
R_s^2 &= R_{s,0}^2 + 2\sum_{n=2,4,6,\dots} R_{s,n}^2 \cos(n\Phi),\\
R_o^2 &= R_{o,0}^2 + 2\sum_{n=2,4,6,\dots} R_{o,n}^2 \cos(n\Phi),\\
R_{os}^2 &= 2\sum_{n=2,4,6,\dots} R_{os,n}^2 \sin(n\Phi),\\
R_l^2 &= R_{l,0}^2 + 2\sum_{n=2,4,6,\dots} R_{l,n}^2 \cos(n\Phi),\\
R_{ol}^2 &= 2\sum_{n=1,3,5,\dots} R_{ol,n}^2 \cos(n\Phi),\\
R_{sl}^2 &= 2\sum_{n=1,3,5,\dots} R_{sl,n}^2 \sin(n\Phi).
\end{aligned}
$$

Note: These most general forms of the Fourier expansions for the HBT radii are preserved when averaging the correlation function over a finite, symmetric rapidity window around $Y = 0$. Only the relations between $R_{\alpha,n}^2$ and the Fourier components of $S_{\mu\nu}$ are modified.

Fourier components of HBT radii vs. source parameters:

($\langle \dots \rangle$ indicates average over finite, symmetric rapidity window around $Y = 0$.)

Φ-independent terms:

$$R^2_{s,0} = \langle A_0 - B_2 - C_2 \rangle,$$

$$R^2_{o,0} = \langle A_0 + B_2 + C_2 \rangle$$

$$-2\beta_\perp \langle E_1 + F_1 \rangle + \langle \beta_\perp^2 D_0 \rangle,$$

$$R^2_{l,0} = \langle J_0 \rangle - 2\langle \beta_l G_0 \rangle + \langle \beta_l^2 D_0 \rangle.$$

First harmonics:

$$R^2_{ol,1} = \tfrac{1}{2}\langle H_0 + H_2 + I_2 - \beta_l (E_0 + E_2 + F_2) \rangle$$

$$-\langle \beta_\perp G_1 - \beta_l D_1 \rangle,$$

$$R^2_{sl,1} = \tfrac{1}{2}\langle -H_0 + H_2 + I_2 - \beta_l (-E_0 + E_2 + F_2) \rangle$$

Second harmonics:

$$R^2_{s,2} = \langle A_2 \rangle - \tfrac{1}{2}\langle B_0 + B_4 + C_4 \rangle,$$

$$R^2_{o,2} = \langle A_2 \rangle + \tfrac{1}{2}\langle B_0 + B_4 + C_4 \rangle - \beta_\perp \langle E_1 + E_3 - F_1 + F_3 \rangle + \beta_\perp^2 \langle D_2 \rangle,$$

$$R^2_{os,2} = \tfrac{1}{2}\langle -B_0 + B_4 + C_4 \rangle + \tfrac{\beta_\perp}{2}\langle E_1 - E_3 - F_1 - F_3 \rangle,$$

$$R^2_{l,2} = \langle J_2 \rangle - 2\langle \beta_l G_2 \rangle + \beta_l^2 \langle D_2 \rangle.$$

Note: terms involving β_l vanish quadratically with the width ΔY of the rapidity window.

“Sum rule” (Wiedemann 1998)

If $D = \langle \tilde{t}^2 \rangle \approx$ independent of emission angle and higher harmonics $n \geq 3$ of $S_{\mu\nu}$ are small:

$$\boxed{R^2_{o,2} - R^2_{s,2} + 2R^2_{os,2} = 2(B_4{+}C_4) - 2\beta_\perp(E_3{+}F_3) + \beta_\perp^2 D_2 \approx 0}$$

If empirically confirmed for all $K_\perp \Longrightarrow D_2, E_3, F_3, B_4, C_4 \approx 0$

$$\begin{aligned}
\Longrightarrow R^2_{s,2} &= A_2 - \tfrac{1}{2}B_0, \\
R^2_{o,2} &= A_2 + \tfrac{1}{2}B_0 - \beta_\perp(E_1{-}F_1), \\
R^2_{os,2} &= \qquad -\tfrac{1}{2}B_0 + \tfrac{1}{2}\beta_\perp(E_1{-}F_1).
\end{aligned}$$

Note: $(E_1{-}F_1) \sim \langle(\tilde{x}{-}\tilde{y})\tilde{t}\rangle$ = first harmonic contribution to the correlation between emission points and times. Since this comes with factor $\beta_\perp$, one can perhaps isolate it at small $K_\perp$.

“Geometric relations”:

If dynamical space-momentum correlations (flow effects) are weak, all implicit Φ-dependences (i.e. all higher harmonics of $S_{\mu\nu}$) $\approx$ vanish

$\Longrightarrow$ Geometric relations:

$$\boxed{\begin{aligned}
&R^2_{s,0} = A_0,\\
&R^2_{o,0} - R^2_{s,0} = \beta_\perp^2 D_0,\\
&R^2_{l,0} = J_0,\\
&R^2_{ol,1} = -R^2_{sl,1} = \tfrac{1}{2} H_0,\\
&R^2_{o,2} = -R^2_{s,2} = -R^2_{os,2} = \tfrac{1}{2} B_0.
\end{aligned}}$$

In this case all five non-vanishing components of the spatial correlation tensor can be separated.

Angular binning and event-plane resolution correction:

$$C(\boldsymbol{q},\boldsymbol{K}) = \frac{N(\boldsymbol{q},\boldsymbol{K})}{D(\boldsymbol{q},\boldsymbol{K})} = \frac{\text{true pairs}}{\text{mixed pairs}}.$$

$$N(\boldsymbol{q},\Phi_j) = N_{\rm exp}(\boldsymbol{q},\Phi_j) + 2\sum_{n=1}^{n_{\rm bin}} \zeta_{n,m}(\Delta)\Big[N^{\rm exp}_{c,n}(\boldsymbol{q})\cos(n\Phi_j) + N^{\rm exp}_{s,n}(\boldsymbol{q})\sin(n\Phi_j)\Big],$$

$$D(\boldsymbol{q},\Phi_j) = D_{\rm exp}(\boldsymbol{q},\Phi_j) + 2\sum_{n=1}^{n_{\rm bin}} \zeta_{n,m}(\Delta)\Big[D^{\rm exp}_{c,n}(\boldsymbol{q})\cos(n\Phi_j) + D^{\rm exp}_{s,n}(\boldsymbol{q})\sin(n\Phi_j)\Big].$$

with correction factors

$$\zeta_{n,m}(\Delta) = \frac{n\Delta/2}{\sin(n\Delta/2)\langle\cos(n(\psi_m - \psi_R))\rangle} - 1$$

($\Delta =$ angular bin width)

Fourier decomposition of real and mixed pair data:

$$(N,D)^{\rm exp}_{c,n}(\boldsymbol{q}) = \langle (N,D)_{\rm exp}(\boldsymbol{q},\Phi)\cos(n\Phi)\rangle_\Phi = \frac{1}{n_{\rm bin}}\sum_{j=1}^{n_{\rm bin}}(N,D)_{\rm exp}(\boldsymbol{q},\Phi_j)\cos(n\Phi_j),$$

$$(N,D)^{\rm exp}_{s,n}(\boldsymbol{q}) = \langle (N,D)_{\rm exp}(\boldsymbol{q},\Phi)\sin(n\Phi)\rangle_\Phi = \frac{1}{n_{\rm bin}}\sum_{j=1}^{n_{\rm bin}}(N,D)_{\rm exp}(\boldsymbol{q},\Phi_j)\sin(n\Phi_j).$$

where $n_{\rm bin} = 2\pi/\Delta =$ number of Φ-bins.

Hydrodynamical simulations:

We studied two different source types — RHIC1 and IPES:

- RHIC1: $T_0^{\max} = 340\,\mathrm{MeV}$ @ $\tau_0 = 0.6\,\mathrm{fm}/c$, $T_{\rm dec} = 130\,\mathrm{MeV}$
- IPES: $T_0^{\max} = 2.0\,\mathrm{GeV}$ @ $\tau_0 = 0.1\,\mathrm{fm}/c$, $T_{\rm dec} = 100\,\mathrm{MeV}$

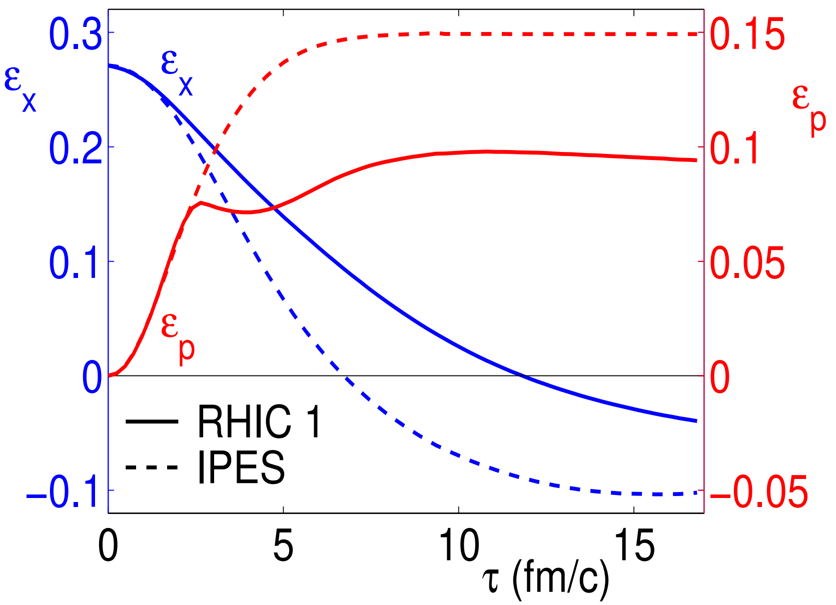
In both cases hadronization happens at $T_{\rm had} = 164\,\mathrm{MeV}$.

Emission function from Cooper-Frye prescription:

$$S(x,K) = \frac{1}{(2\pi)^3} \int_{\Sigma_{\rm f}} \frac{K\cdot d^3\sigma(x')\,\delta^4(x-x')}{\exp[K\cdot u(x')/T_{\rm f}] - 1}\,.$$

$\Sigma_{\rm f}$ and flow $u(x')$ on $\Sigma_{\rm f}$ extracted from hydrodynamics.
Only direct pions, no resonance contributions included, to minimize difference between R^2_{ij} extracted from $S_{\mu\nu}$ via Wiedemann relations and from Gaussian fit to measured $C(\boldsymbol{q},\boldsymbol{K})$.

Time evolution of anisotropies (Au+Au @ $b = 7\,\text{fm}$):



$$\epsilon_x = \frac{\langle\langle y^2 - x^2 \rangle\rangle}{\langle\langle y^2 + x^2 \rangle\rangle}$$ spatial anisotropy

$$\epsilon_p = \frac{\langle\langle T^{xx} - T^{yy} \rangle\rangle}{\langle\langle T^{xx} + T^{yy} \rangle\rangle}$$ momentum anisotropy

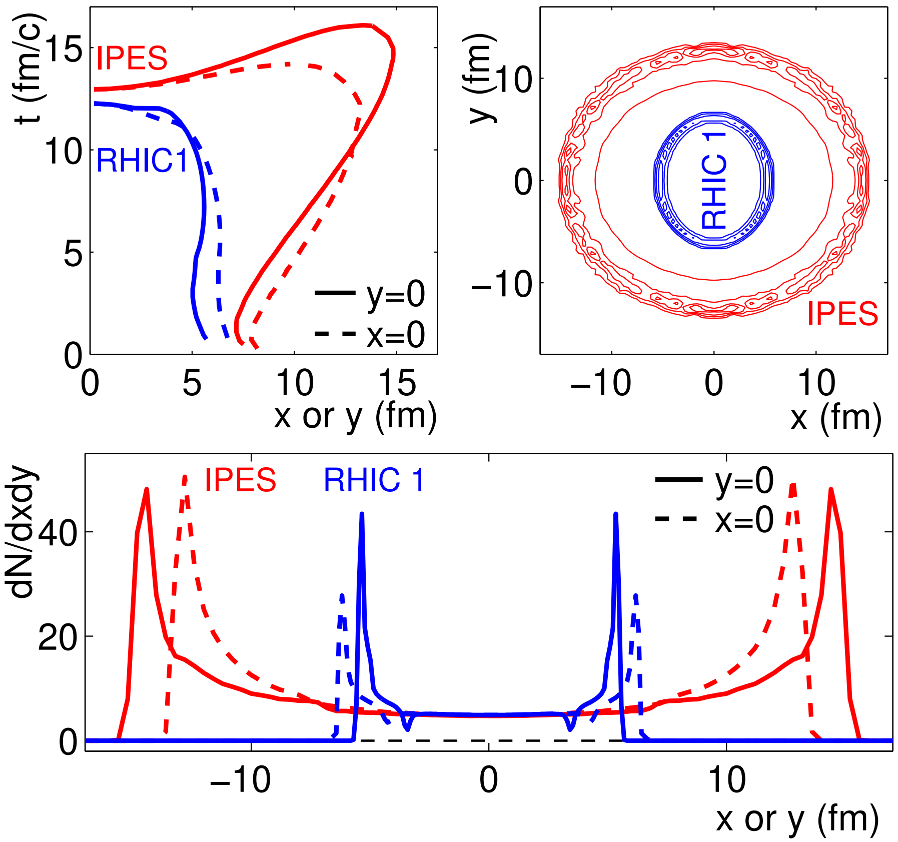
Momentum anisotropy saturates ≈ when spatial anisotropy passes through zero.

In both cases this happens before hadronization is complete.

Early "presaturation" (dip) for RHIC1 due to softening of EOS in phase transition region.

IPES blows up faster than RHIC1 source.

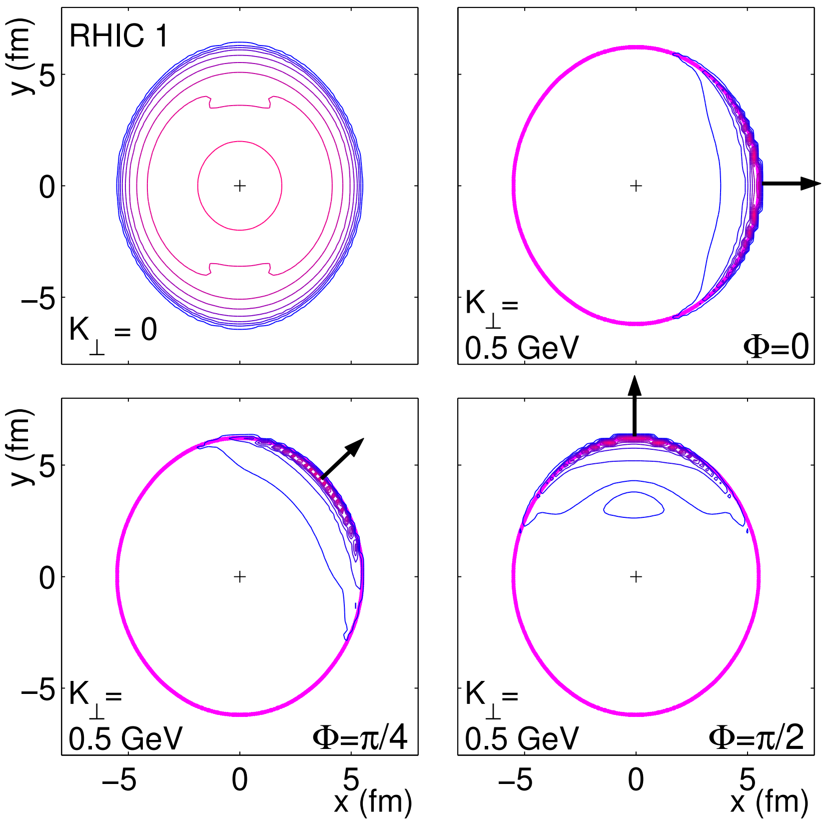
Freeze-out distributions:



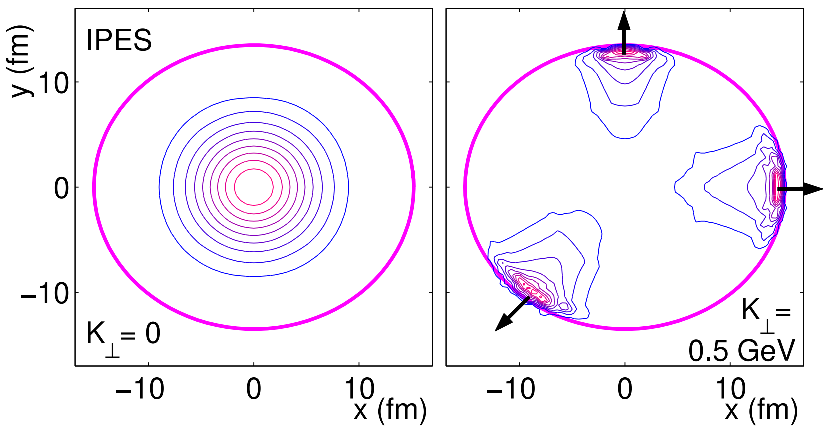
- RHIC1 source still out-of-plane elongated, IPES much larger and in-plane elongated
- onset of transverse "inside-out cascade" pattern at LHC energies
- emission strongly surface dominated; "opacity" strongest at RHIC energies

Homogeneity regions for pairs of given momentum:

RHIC1

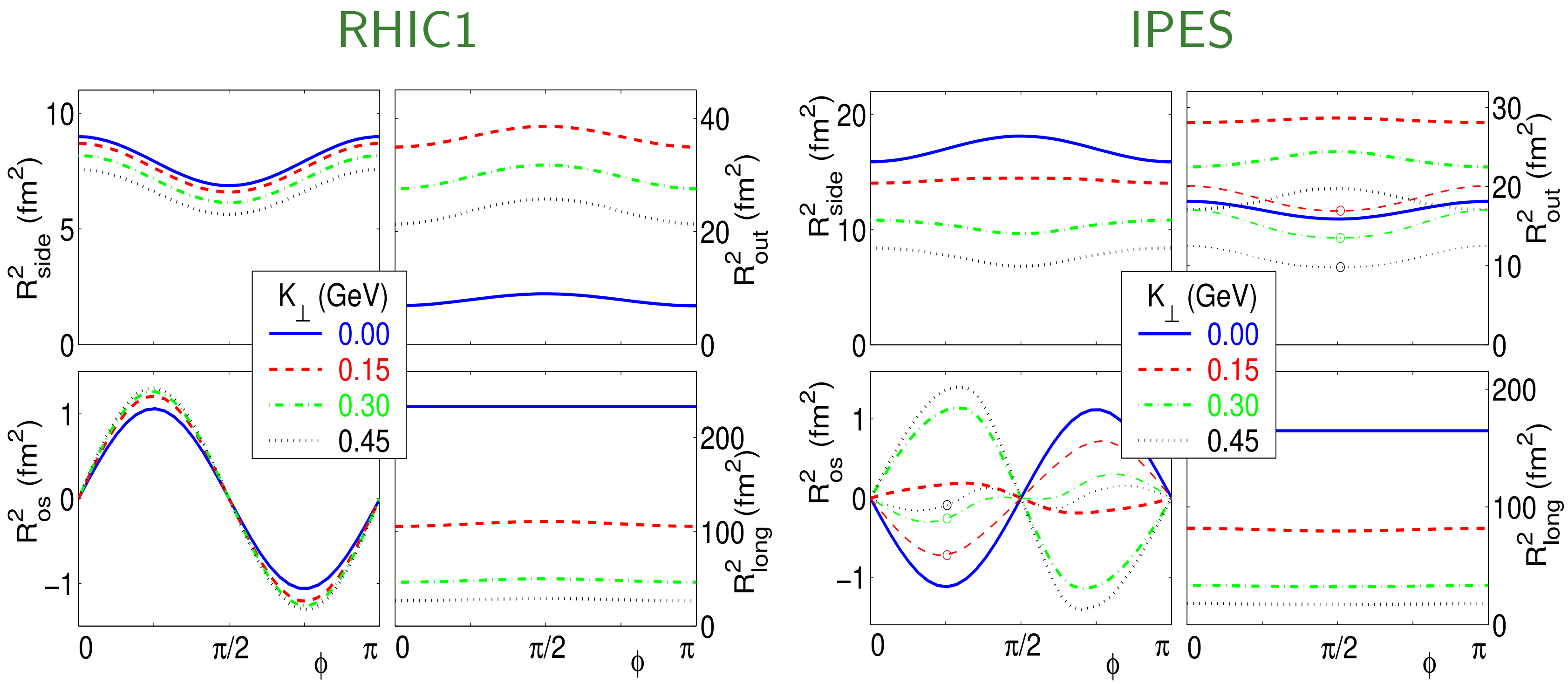


IPES



- transverse flow pushes emission region towards rim
- curvature of rim pushes back
- emission strongly surface dominated; "opacity" strongest at RHIC energies

Azimuthal HBT oscillations:



- RHIC1: oscillation pattern stable with $K_\perp$
- IPES: oscillations change sign at $K_\perp > 150\,\mathrm{MeV}$;
  at small $K_\perp$ oscillations of R_s^2 reflect out-of-plane geometric deformation,
  at large $K_\perp$ oscillation pattern of R_s^2 controlled by rim curvature

R^2_{os} oscillations and the "transverse tilt" of the source

Geometric and temporal contributions to R^2_{os}:

$$\begin{aligned} R^2_{os} &= R^{2,\mathrm{geom}}_{os} && + R^{2,\mathrm{temp}}_{os} \\ &= S_{12}\cos(2\Phi) - \tfrac{1}{2}(S_{11}-S_{22})\sin(2\Phi) && + \beta_\perp(S_{01}\sin\Phi - S_{02}\cos\Phi) \end{aligned}$$

The geometric contribution can be rewritten as

$$\boxed{R^{2,\mathrm{geom}}_{os} = \tfrac{1}{2}(S_{XX} - S_{YY})\sin(2\Theta)}$$

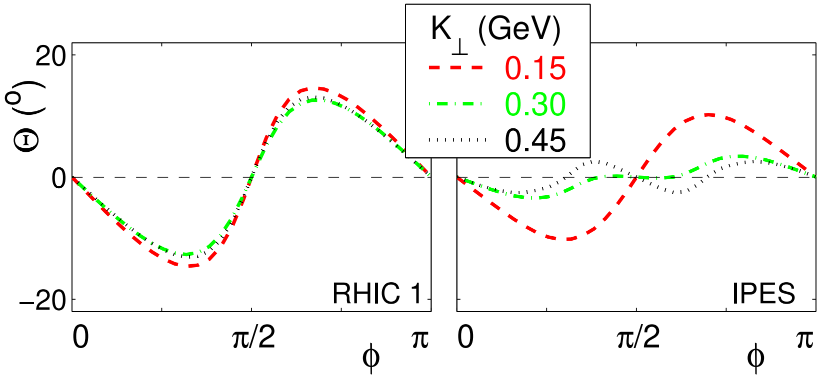
where X, Y are outward and sideward pointing major axes of the emission region and Θ = tilt angle between X and $\boldsymbol{K}_\perp$.

First quadrant:

RHIC1: $S_{XX} - S_{YY} < 0$, $R^{2,\mathrm{geom}}_{os} > 0$;

IPES: $S_{XX} - S_{YY} > 0$, $R^{2,\mathrm{geom}}_{os} < 0$;

$\Longrightarrow$ source always tilted clockwise ($\Theta < 0$)!



Space-time correlations at decoupling:

$S_{03}{=}S_{tz}$ vanishes by boost invariance.

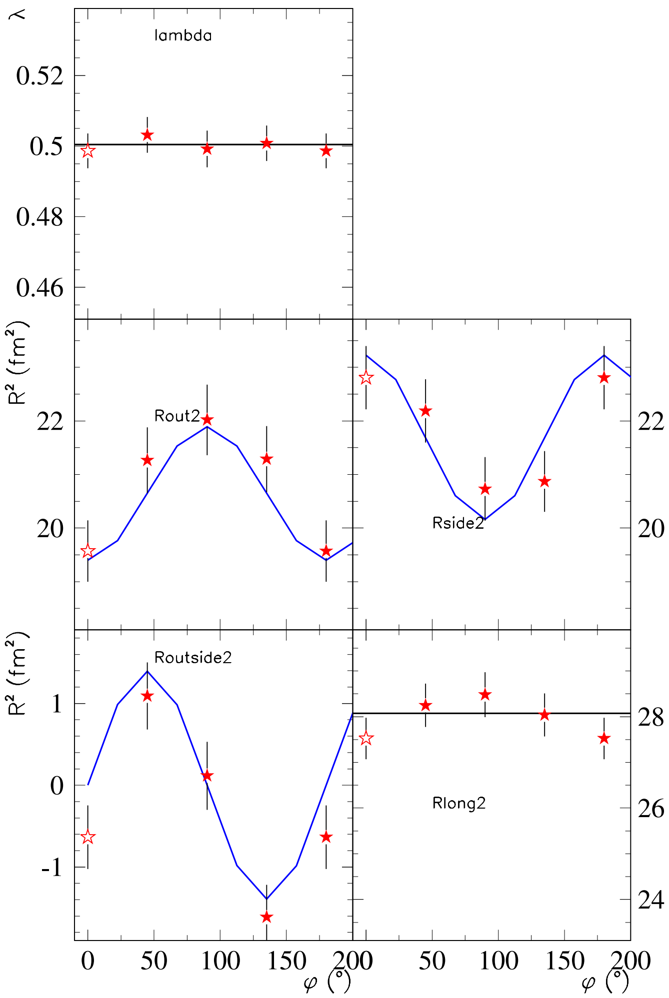
The contributions to HBT radii from $S_{01}{=}S_{tx}$ and $S_{02}{=}S_{ty}$

- grow with $K_\perp$
- reflect the changing elliptic deformation as time increases

$\Longrightarrow$ they are qualitatively similar for RHIC1 and IPES.

For RHIC1 they reinforce the geometric oscillations of R_o^2, R_{os}^2.
For IPES they act against them and cause a sign change
$\Longrightarrow$ the sign change of oscillations with increasing $K_\perp$ is a signature of the faster growth of the source in x than in y direction.

Preliminary RHIC data from the STAR Collaboration:



Au+Au at $\sqrt{s} = 130\,A$ GeV (M. Lopez-Noriega, Quark Matter 2002)

Data are qualitatively consistent with hydrodynamical picture and indicate (still slightly) out-of-plane extended source at RHIC.

Similar conclusion follows from fit with collectively expanding source ("blast wave") model by M. Lisa and R. Wells (see lines in data plots).

Oscillation amplitudes are in right ballpark, but quantitative comparison with hydro model negatively affected by "HBT puzzle".

Summary:

- Symmetries severely restrict structure of azimuthal Fourier series of HBT radii. The most general form of the latter, as given here, can be applied to any data collected in A+A collisions in a symmetric rapidity window around $Y = 0$.
- A constructive and easily applicable algorithm to fully and completely correct the data for finite angular binning and event-plane-resolution effects was derived.
- To separate temporal from geometric contributions to the HBT radii, the source must satisfy special properties. We showed how one can test for these properties, albeit not in a fully unambiguous (model-independent) way. Analogous to $b = 0$ case ($K_\perp$-dependence of HBT radii).
- At RHIC and LHC emission is very strongly surface dominated ("opacity").
- The freeze-out deformation of the source at freeze-out can be unambiguously extracted from the Φ-oscillations of R_s^2 at very small $K_\perp$.
- Azimuthal oscillations of $R_{os}^{2,\mathrm{geom}}$ reflect the transverse tilt of the emission region against the emission direction.
- Reduction (RHIC1) and sign change (IPES) of oscillation amplitudes of R_o^2, R_{os}^2 as $K_\perp$ increases are a signature of the faster expansion of the source in-plane than out-of-plane and the $x{-}t$ correlations induced thereby.