Low-x physics, saturation and Forward Drell-Yan production

A.H. Mueller*
Columbia University

The idea of saturation; the dipole picture

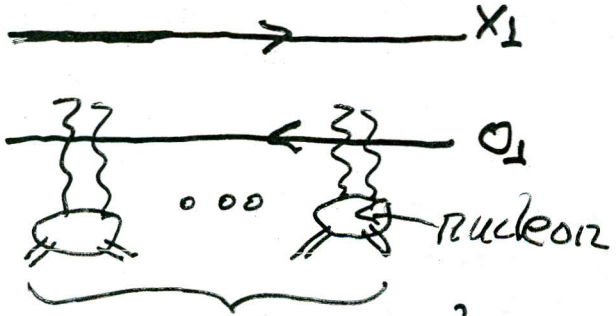
Creating dipoles

Physical processes in Forward deuteron (proton) - nucleus scattering

With F. Dominguez, S. Munier, B. Xiao

The Idea of Saturation; The Dipole Picture

A. Dipole scattering in the McLerran-Venugopalan model



$$S(x_1, L) = e^{-\sigma_{dip}\rho L/2}$$

$$S^2(x_1, L) = e^{-L/\lambda}$$

$$\lambda = \frac{1}{\rho\,\sigma_{dip}}$$

One also writes

$$S(x_1, L) = e^{-x_1^2 \bar{Q}_s^2/4}$$

when $x_1 > 2/\bar{Q}_s$ strong scatt.

when $x_1 < 2/\bar{Q}_s$ weak scatt.

$\bar{Q}_s^2$ = quark saturation momentum $= \frac{C_F}{N_c} Q_s^2$

$$Q_s^2 = \frac{4\pi^2 \alpha N_c}{N_c^2 - 1} \rho L \, x G_N(x, 1/x_1^2)$$

At $x \simeq 10^{-2}$ and $b_1 \simeq 0$ $Q_s^2 \simeq 1\,GeV^2$ For large nucleus

The number of gluons per unit area in the nucleus is, when $x_1 \lesssim 2/Q_s$,

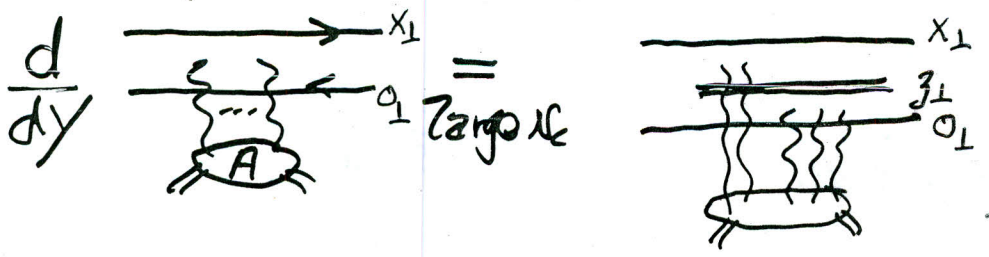
$$\#g/area \simeq \rho L x G_N = x G_A(x, b_1, 1/x_1)$$

The occupation is

$$f_g = \frac{(2\pi)^3}{N_c^2-1} \times G_A \frac{1}{\pi Q_s^2} = \frac{(2\pi)^3}{N_c^2-1} \frac{dX G_A}{\underbrace{d^2p_\perp}_{\pi Q_s^2} d^2b_\perp \underbrace{dp_z dz}_{1}}$$

$f_g \simeq 2/\alpha_s N_c$ at the saturation momentum

B. Adding evolution

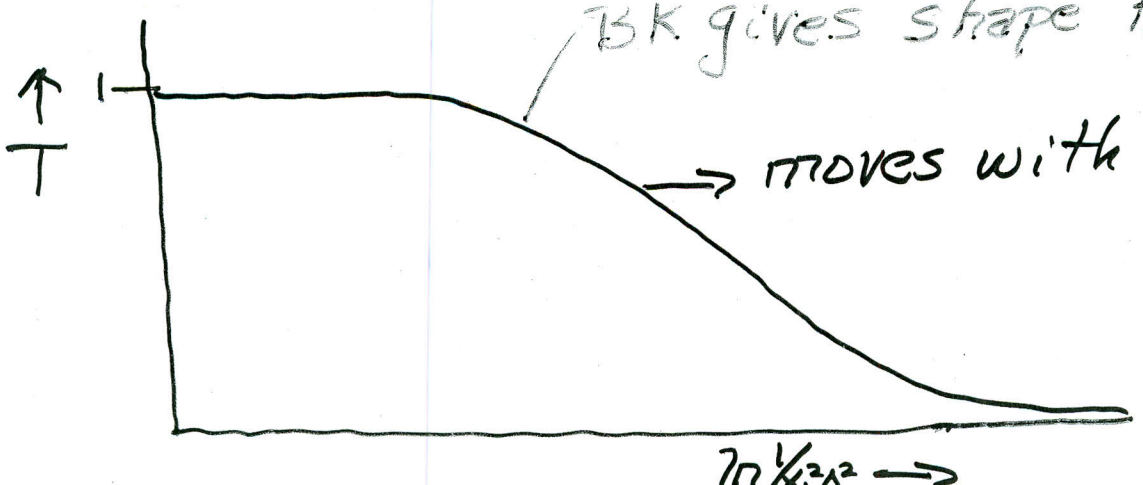


BFKL

$$\frac{dT(x_\perp,Y)}{dY} = \frac{\alpha N_c}{2\pi^2}\int d^2z_\perp \frac{x_\perp^2}{z_\perp^2(x_\perp-z_\perp)^2}\Big[T(x_\perp-z_\perp,Y)+T(z_\perp,Y)-T(x_\perp,Y) - \underbrace{T^{(2)}(x_\perp-z_\perp,z_\perp,Y)}_{T(x_\perp-z_\perp,Y)\cdot T(z_\perp,Y)}\Big]$$

is Balitsky-Kovchegov equation.

$T^{(2)}$ Factorizes for large nuclei when Y not too large. In general $T^{(2)}$ Factorizes For large occupancy on an event by event basis.

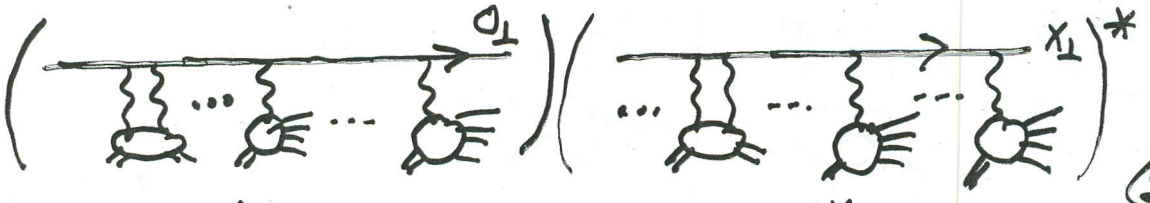


Creating Dipoles

Dipoles of a controllable size occur in $F_L(x,Q^2)$ and in onia, but are hard to build otherwise. However, <u>to some level</u>, they also occur in "easily" accessible processes.

A. $p_\perp$-broadening in MV

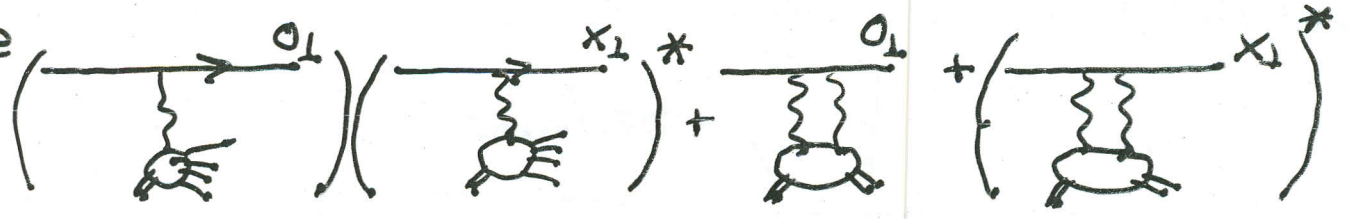
Send quark through nucleus and measure the transverse momentum it picks up.

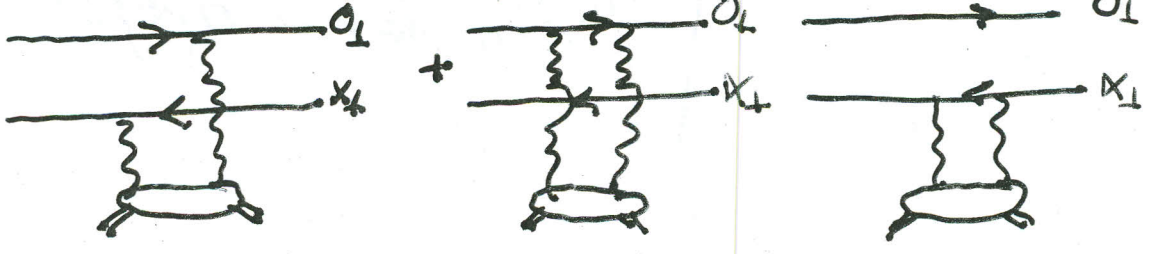
$\int$ 

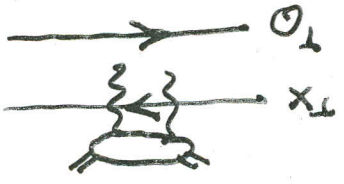
$\frac{e^{ik_\perp\cdot x_\perp}}{(2\pi)^2}\, d^2x_\perp$

$A \qquad\qquad A^*$

$$= \frac{1}{N}\frac{dN}{d^2k_\perp} = \int \frac{d^2x_\perp}{(2\pi)^2}\, e^{ik_\perp\cdot x_\perp} A^* A$$

Use 

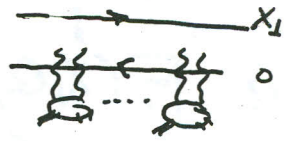
$=$ 

$=$ 

$= S^{(1)}(x_\perp)$

Put in multiple scatterong by Glauber

$$\int S(x_\perp)\frac{d^2x_\perp}{(2\pi)^2}e^{ik_\perp\cdot x_\perp} = \frac{1}{N}\frac{dN}{d^2k_\perp} =$$

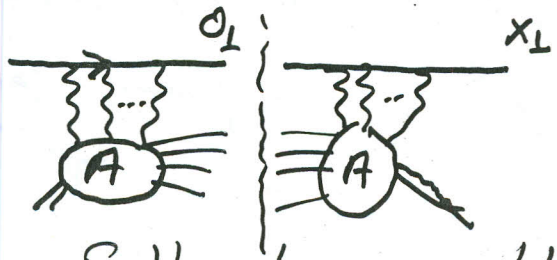


$k_\perp$-broadening given in terms of F dipole scattering

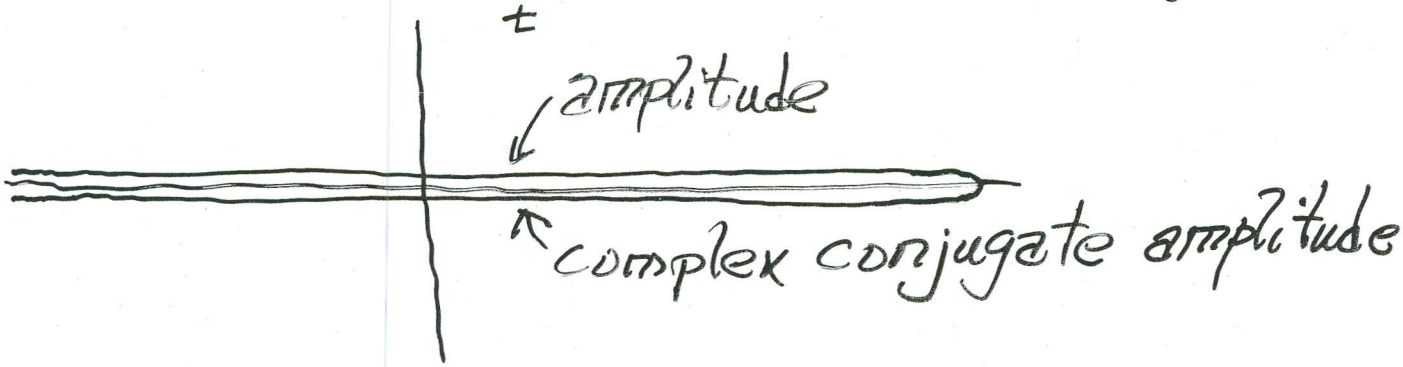
B. Bring in evolution

In general

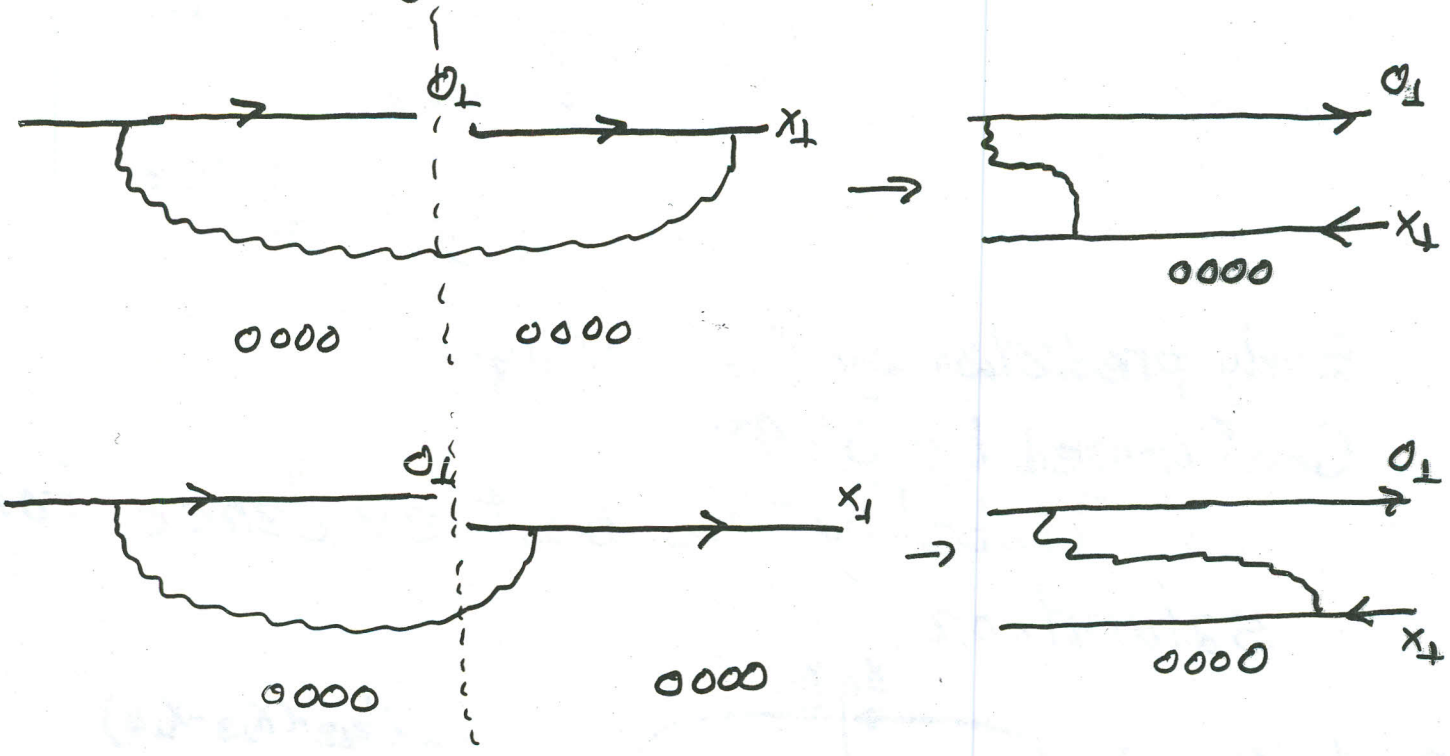
$$\int\frac{d^2k_\perp}{(2\pi)^2}e^{ik_\perp\cdot x_\perp}$$



cannot be usefully be written in terms of a time-ordered product. Need two time formalism (or subtle discontinuity)



However, at leading log level one can identify

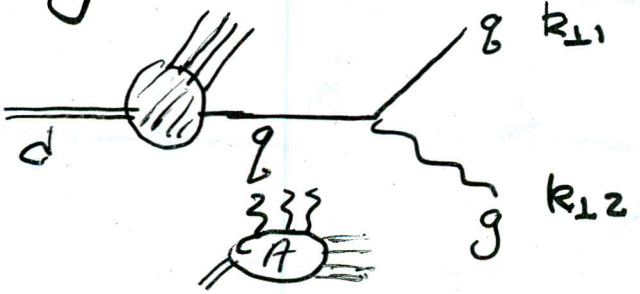


1-1 correspondence between dipole graphs and the corrections in the original process. OK For $(\alpha N_c Y)^n$ terms (leading logs). Kovchegov Appears to break down at next to leading level.

Should be ok to use dipole picture and BK equation to study $p_\perp$-broadening. But, higher order corrections still an open question.

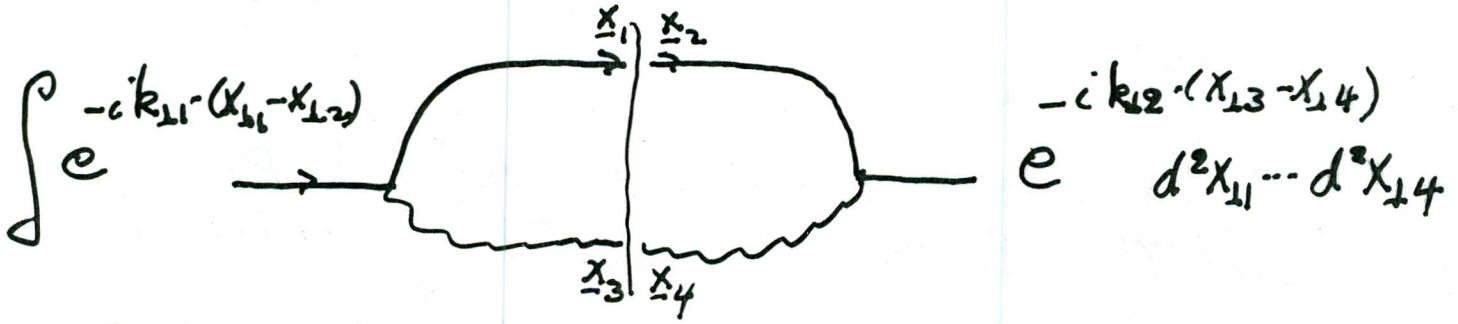
Physical Processes in Forward deuteron(proton)-Nucleus Scattering

A. $g \to g + g$

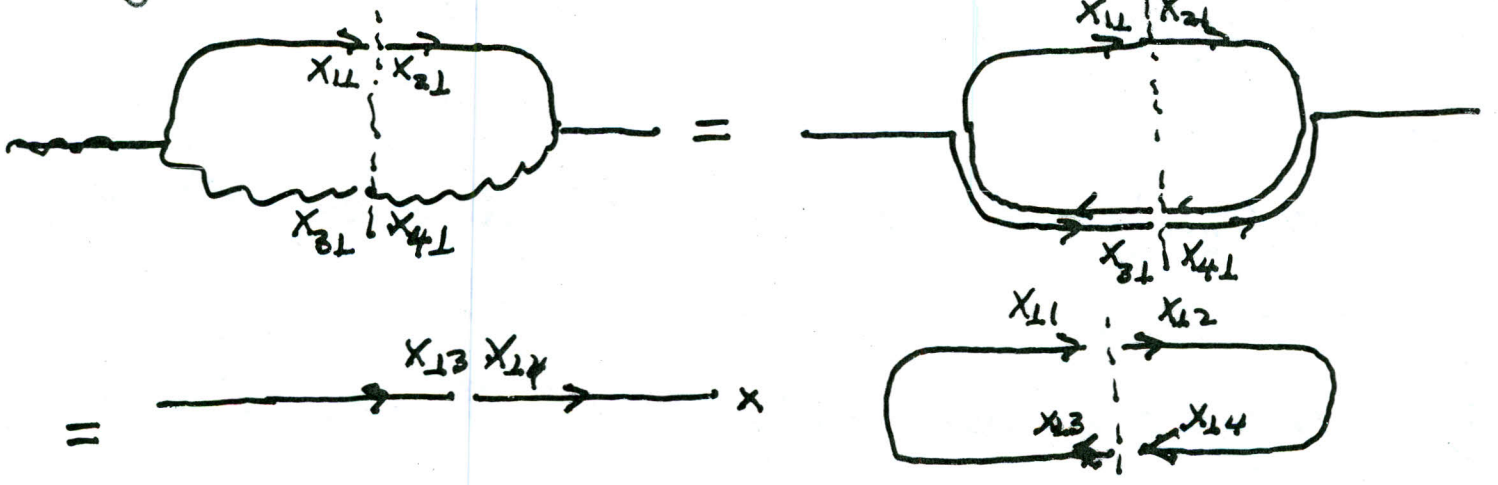


Early prediction by Cyrille Marquet
Confirmed by STAR.

Probably the best evidence for saturation.

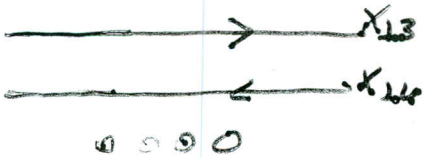


In large N_c limit

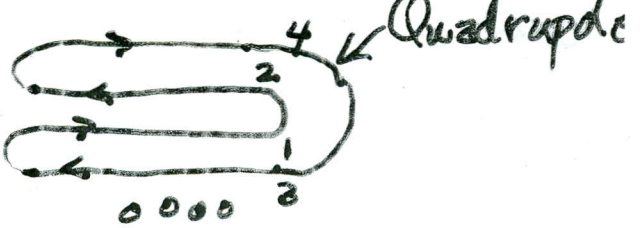


MV level

=

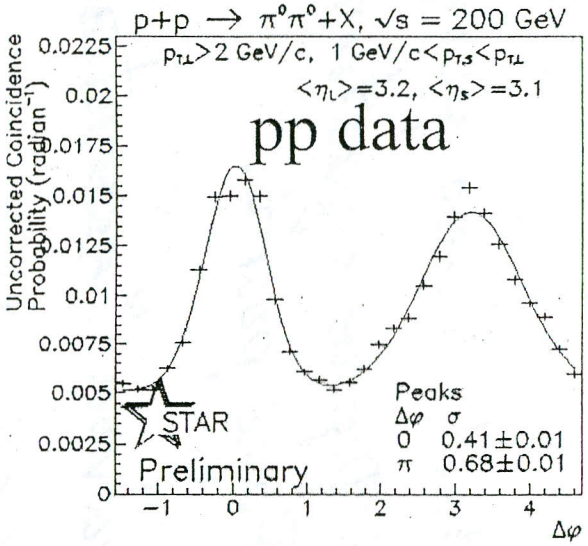


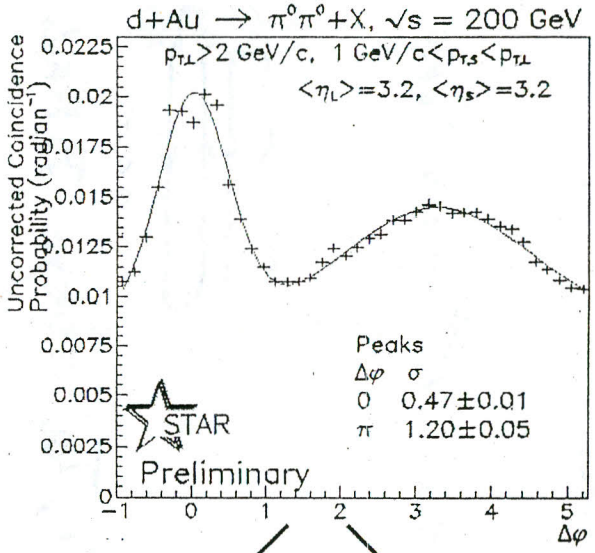
$S_D(x_{\perp 3}-x_{\perp 4})$



$S_Q(x_{\perp 1}-x_{\perp 2}, x_{\perp 1}-x_{\perp 3}, x_{\perp 2}-x_{\perp 4})$

Forward-Forward: Centrality dependence





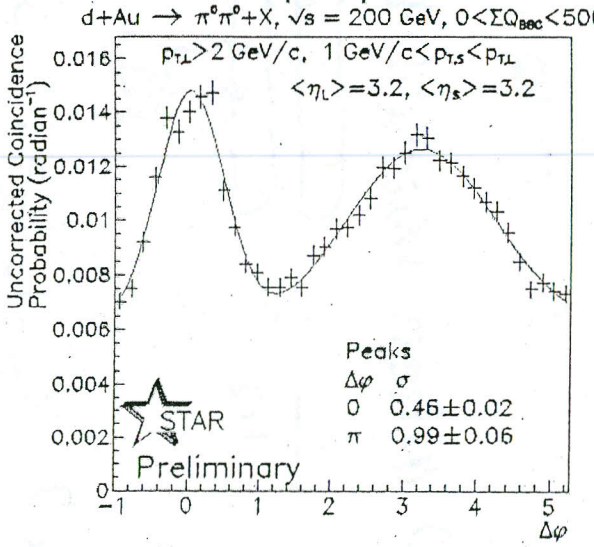
dAu all data

Away-side peaks evident in peripheral dAu and pp.

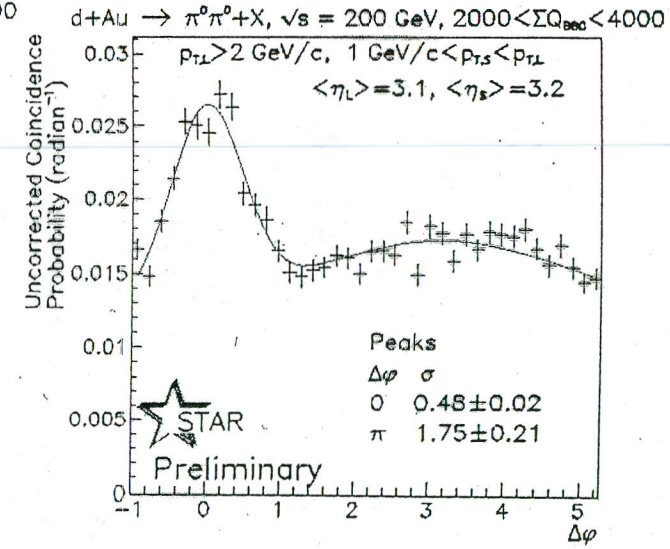
Near side peaks unchanged in dAu for peripheral to central.

Azimuthal decorrelations show significant dependence on centrality.

dAu peripheral



dAu central



At MV level S_Q can be evaluated. Dominguez Marquet Xiao Yuan

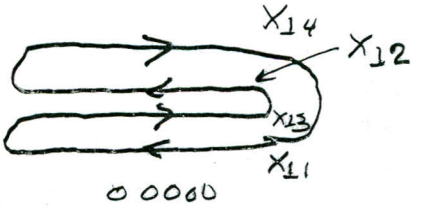
Evolution complicated. Schematically,

$$\frac{dS_Q}{dy} = \int P_{Q\to DD}(S_D S_D - S_Q) + \int P_{Q\to QD}(S_Q S_D - S_Q)$$

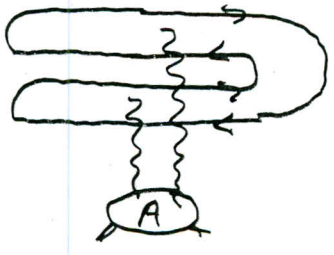
$P_{Q\to DD}$ and $P_{Q\to QD}$ not positive definite.

$P_{Q\to DD} + P_{Q\to QD} \geq 0.$

$S_Q = 1 - T_Q$



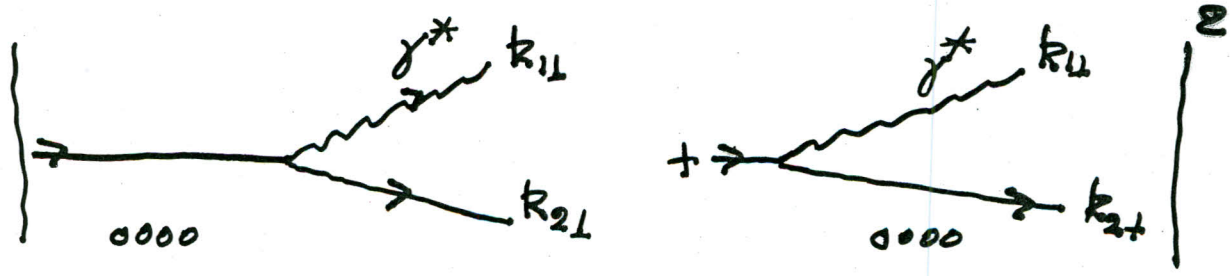
(i) When $|x_{1i} - x_{1j}|\, Q_s \ll 1$, T_Q obeys BFKL equation.

$T_Q =$ 

(ii) $T_D = 1, T_Q = 1$ is stable fixed point of Quadrupole and Dipole equations. $T_D = 0$, $T_Q = 0$ is unstable fixed point.

Quadrupole evolve "essentially" like dipole, but difficult to evaluate numerically.

B. $q \to q + \gamma^* \to q + \mu^+ + \mu^-$



Interactions with nucleons, and evolution only occur on the quark line. Very similar to $p_\perp$-broadening.

(i) When $k_{1\perp} \gg Q_s$ should see recoil $k_{2\perp} \simeq -k_{1\perp}$. Shape in $(k_{1\perp} + k_{2\perp})$ calculable in terms of nuclear scattering, small-x evolution and Sudakov. (Lomingues, Munier, A.M., Xiao in progress)

(ii) When $k_{1\perp} \sim Q_s$ should not see recoil of γ^*. Transition from seeing recoil to the disappearance of recoil governed by BK equation. This is only such process!

$$\frac{d\sigma^{d \to \gamma^* + q + X}}{d^2k_{1\perp} d^2k_{1\perp 2} dy_1 dy_2} = \underbrace{\frac{d\sigma^{d \to \gamma^* + g + X}}{d^2k_{1\perp} dy_1 dy_2}}_{\substack{k_{2\perp} \text{ integrated} \\ \text{process}}} \cdot \underbrace{\frac{1}{N} \frac{dN}{d^2(k_{1\perp 2} - k_{1\perp})}}_{\int \frac{d^2x_\perp}{(2\pi)^2} e^{i(k_{1\perp 2} - k_{1\perp}) \cdot x_\perp} S(x_\perp)}$$