Notes for Quantum Mechanics

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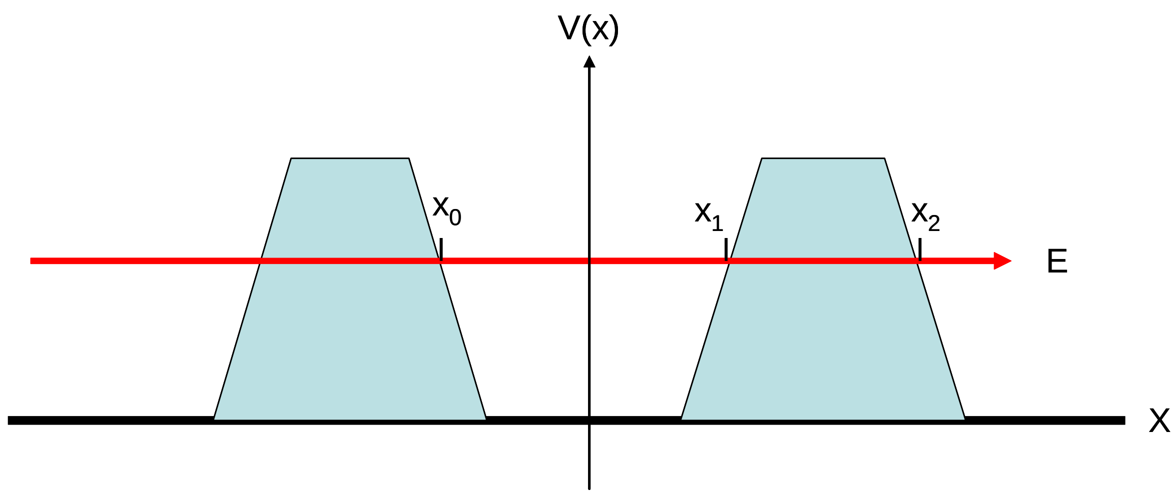
Lecture 22
Model Building

How strong is the strong interaction? Let's try to do something for real. We will build a theory of the strong interaction between two quarks using what we know. Then we might be able to use the model and some data to see if we might be able to measure something about the strength of the strong interaction. Before we begin let me give you the result of a mathematical approximation for more complicated potential wells.

More difficult potentials – *e.g.* the SHO

The is a standard approximation method called the WKB Approximation (for Wentzel, Kramers, Brillouin) to figure stuff out for more complex potentials. I will not actually derive it here but I will just give you the result. Let's take some arbitrary potential well. Notice that we have both a well and a tunneling situation, so some particle inside the well will eventually tunnel out. The WKB approximation will tell us both the tunneling probability, i.e. T and R=1-T and the eigenenergies in the potential well. x_0, x_1, and x_2 are the classical turning points where the energy is equal to the potential energy. This approximation uses some classical ideas, so is typically good when the quantum numbers are large.



WKB:

The transmission probability in the WKB approximation is T=exp(-2$\int_{x_1}^{x_2}\kappa\, dx$) where $\kappa=\sqrt{\frac{2m}{\hbar^2}(V(x)-E)}$ so

T=exp$\left(-2\int_{x_1}^{x_2} dx\sqrt{\frac{2m}{\hbar^2}(V(x)-E)}\right)$

where x_0, x_1, and x_2 are the classical turning points, so E= V(x), i.e. $V(x_0)=V(x_1)=V(x_2)=E$ .The energies allowed in the well will obey the condition

$\frac{1}{2\pi}\int_{x_0}^{x_1}\sqrt{2m(E-V(x))}\, dx=\left(n+\frac{1}{2}\right)\frac{\hbar}{2}$

Lets see these formula at work. First for a a couple of non-transmission problems where we can calculate the energy levels in the well. First the infinite potential well with walls at 0 and a? Here within the well V=0

$\frac{1}{2\pi}\int_0^a\sqrt{2\,\text{mE}}\ \text{dx}=\frac{\sqrt{2\,\text{mE}}}{2\pi}\text{a}=\left(n+\frac{1}{2}\right)\frac{\hbar}{2}$ and we get 2 mE = $\frac{4\pi^2}{a^2}\frac{\hbar^2}{4}\left(n+\frac{1}{2}\right)^2$ $E^{\text{WKB}}=\frac{\hbar^2\pi^2}{2\,\text{ma}^2}\left(n+\frac{1}{2}\right)^2$

The right answer is of course $E^{\text{real}}=\frac{\hbar^2\pi^2}{2\,\text{ma}^2}(n)^2$ which is not bad for n reasonably large. This tells us something - the WKB is a semi-classical approximation and is good for large quantum numbers. Note that in this case the n=1 state is off by more than 100% but for n=10, its good to about 10%.

Now lets do the SHO where the edges walls go to infinity. $V(x)=\frac{1}{2}\text{m}\omega^2x^2$ At the turning points $E=\frac{1}{2}\text{m}\omega^2x^2$ so

x_0 =-$\sqrt{\frac{2E}{\text{m}\omega^2}}$ x_1 =$\sqrt{\frac{2E}{\text{m}\omega^2}}$ so the energies must obey the condition $\frac{1}{2\pi}\int_{-\sqrt{\frac{2E}{\text{m}\omega^2}}}^{\sqrt{\frac{2E}{\text{m}\omega^2}}}\sqrt{2m\left(E-\frac{1}{2}\text{m}\omega^2x^2\right)}\, dx=\left(n+\frac{1}{2}\right)\frac{\hbar}{2}$

after some work we get $E_n=\left(n+\frac{1}{2}\right)\hbar\omega$ which is exactly the right answer.

Now lets try the only tunneling problem we have done, the square potential barrier. Remember

$\frac{1}{T}=1+\frac{V^2}{4E(V-E)}\sinh^2(2\kappa_2 a)$. Lets let E<<V so $\kappa_2=\sqrt{\frac{2m}{\hbar^2}(V-E)}$ where

we will assume that $2\kappa_2 a$ is large and since $\sinh(z)=\frac{1}{2}(e^z-e^{-z})$ we will assume that the e^{-z} term is negligible

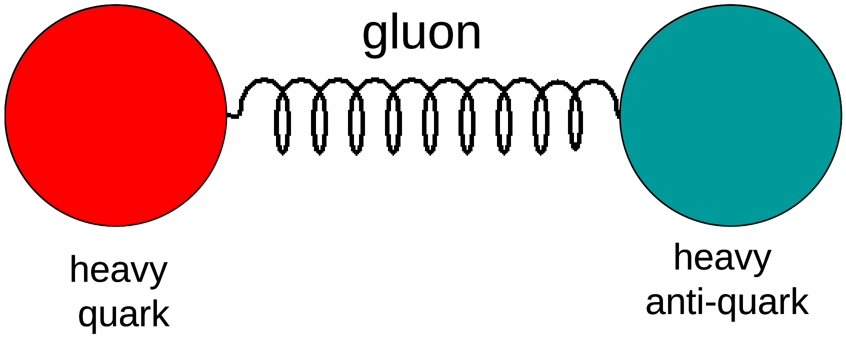
$T^{\text{real}}\sim 4\frac{\text{EV}}{V^2}\left[\frac{1}{2}\exp\left(2a\sqrt{\frac{2m}{\hbar^2}(V-E)}\right)\right]^{-2}\sim 16\frac{E}{V}\exp\left(-4a\sqrt{\frac{2m}{\hbar^2}(V-E)}\right)$ (I assume the 1 is negligible) and

$T^{\text{WKB}}=\exp\left(-2\int_{x_1}^{x_2}dx\sqrt{\frac{2m}{\hbar^2}(V(x)-E)}\right)=\exp\left(-2\int_{-a}^{a}dx\sqrt{\frac{2m}{\hbar^2}(V-E)}\right)=\exp\left(-2\sqrt{\frac{2m}{\hbar^2}(V-E)}\,(a+a)\right)=$

$\exp\left(-4a\sqrt{\frac{2m}{\hbar^2}(V-E)}\right)$ which is off by 16E/V

The 1 − *D* SHO model of *s* − wave (*i.e.* angular momentum = 0) heavy quark − antiquark mesons

Lets begin. We want to build a model for heavy mesons We will assume these are held together by a gluon which will have a SHO potential - it is a spring. We want to use heavy mesons because in our model these will be made of pairs of "heavy" quarks, which can be considered as non-relativistic. Light quarks move too fast. When we complete our model, we can figure out just how non-relativistic the quarks are. The quarks are 6, u,d,s,c,b,t The u and d quarks are light, less than 10 MeV/c^2 (remember a proton is 938 MeV/c^2) The t quark is so heavy that it never really stays together as a meson. The s,c, and b quarks are in the right ballpark at about 500 MeV, 1.5 GeV, and 5 GeV respectively.

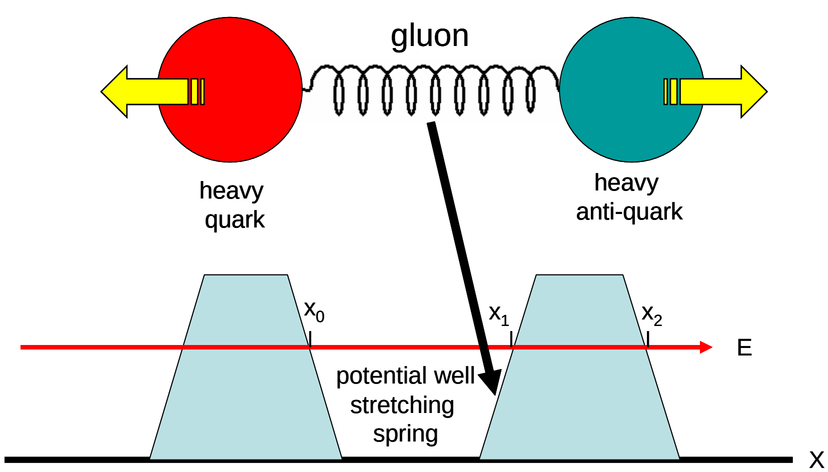


Model of a heavy quark meson held together by a gluon

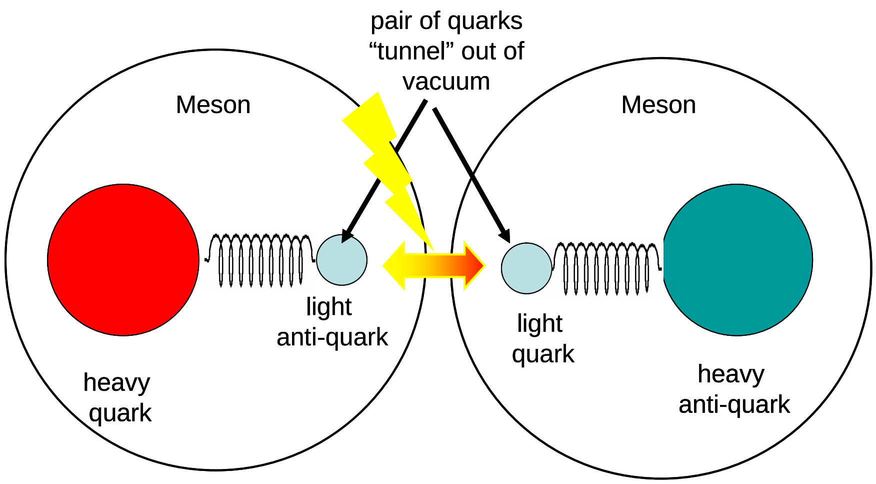
As an exercise - as yourself, how good you think this model will be. We will be predicting masses and decay times. How good do you expect our model to be? Note: it we get to a factor of 2 or so, its pretty good!

A note here on units. Energy is in eV (KeV, MeV, GeV). Assuming the formula $E = \text{mc}^2$ we will measure mass in MeV/c^2 where sometimes we get careless and leave off the c^2 - actually we are quoting the energy equivalent - i.e. mc^2. Distance is conveniently measured in fm = 10^{-15}m - a protons is about 1fm is size, and time in $\text{fm}/c = \frac{10^{-15}\,m}{3\,\text{x}10^8\,m/s} \sim 3\,\text{x}10^{-23}$ sec. Strong interaction decay times are $\sim 10^{-23}$ sec or about 1 fm/c. A convenient thing to know is $\hbar$c=197 MeV-fm (just remember 200 MeV-fm).

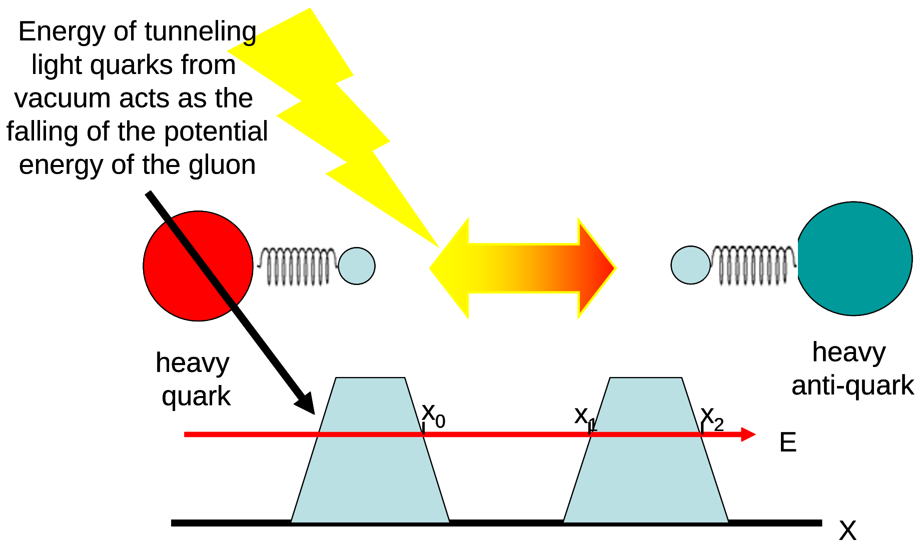
Now lets see if we can figure out a reasonable potential. While the particle is together (not decayed) it should have a SHO potential, $\frac{1}{2}\,\text{m}\omega^2\,x^2 = \frac{1}{2}\,\text{kx}^2$ where k is the spring constant. From classical mechanics remember that a spring with mass m and force constant k where F=-kx, will oscillate with frequency $\omega=\sqrt{\frac{k}{m}}$ This is why we write the potential the way we do**. Now k is a characteristic of the gluon. One of the important ideas of the model is that the gluon which holds the quarks together is always the same, i.e. k is the same for s, c, and b quarks. So we will demand that k be a constant.** So far we just have a potential well, and nothing to make the potential fall back to zero at large distances to give us the possibility of tunneling (or a decay).



When a quark anti-quark meson is pulled apart - it "pops" another quark anti-quark pair out of the vacuum (hence we never see individual quarks) to form two mesons



This will take energy from the gluon potential and give it a falling profile at large distance. Hence we have a potential with the possibility of tunneling.



What data can we use? We can look up on the web, from the Particle Data Group web page, the masses and widths of particles. We will concern ourselves with mesons which are pairs of s, c, or b mesons. The best measured are those which are pairs $s\,\bar{s}$, $c\,\overline{c}$, $b\,\overline{b}$ – called the ϕ (phi), ψ (psi), and Υ (capital upsilon). The ground states and several excited states. The excited states are often denotes by primes. Here they are.

"heavy" VECTOR MESONS

		Mass	dM	Width	lifetime	lifetime	major
	n	MeV	MeV	MeV	fm/c	sec	decay
							channel
phi	0	1019		4.26	46.24	4.6E-22	KK
	1	1680	661	150	1.3	1.3E-23	KK
K		494					
psi	0	3097		0.087	2264	2.3E-20	ee, μμ
	1	3686	589	0.277	711	7.1E-21	ee, μμ
	2	3770	84	23.6	8.3	8.3E-23	DD
	3	4040	270	52	3.8	3.8E-23	DD
	4	4159	119	78	2.5	2.5E-23	DD
	5	4415	256	43	4.6	4.6E-23	DD
D		1869					
Upsilon	0	9460		0.052	3788	3.8E-20	ee, μμ
	1	10023	563	0.044	4477	4.5E-20	ee, μμ
	2	10355	332	0.026	7577	7.6E-20	ee, μμ
	3	10580	225	20	9.9	9.9E-23	BB
	4	10865	285	110	1.8	1.8E-23	BB
	5	11019	154	79	2.5	2.5E-23	BB

We have taken the mass and width from the PDG for the relevant $s\,\overline{s}$, $c\,\overline{c}$, $b\,\overline{b}$ mesons.I have also included the masses of the lightest meson which includes a heavy quark and light quark ($K = s\,\overline{u}$, $D = c\,\overline{u}$, $B = b\,\overline{u}$), which serves as the major decay channel for which we will be concerned.dM is just the difference between the excited state and the one just below it.What is th width? Its the uncertainty in the mass when it is measured.We can use the Heisenberg uncertainty principle $\Delta E \Delta t \sim \hbar$ to get the lifetime! We can write this as $c\Delta t = \frac{\hbar c}{\Delta E}$ $i.e.$the lifetime in fm$/c$.I have also given you the lifetime in seconds just for reference. Now our model for these mesons is a pair of heavy quarks held together by a gluon, which acts like a spring $i.e.$it should exhibit a harmonic oscillator potential which give various energy states which we have labeled by n. Our model then already explains one of the characteristics of each of these families of particles, they have excited states, which I

From above we know that the WKB approximation gives us the exact solution for the energy levels in a SHO: $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$. What does this correspond to in the chart? We'll assume that the mass of a meson is the total energy at rest. This means that it is the sum of the mass of the heavy quark plus the energy in the spring. The E_n corresponds to the energy in the spring. (note I am going to be go into units such that c=1) Hence $M_{\text{meson}} = 2\, m_{\text{heavy quark}} + E_n = 2\, m_{\text{heavy quark}} + \left(n + \frac{1}{2}\right)\hbar\omega$. Now we don't really know $m_{\text{heavy quark}}$ so we will assume for now its just the mass of the lightest meson with one heavy quark and one light quark. After all the u and d quarks are ~10MeV, so for example we will assume $m_{c\text{ quark}} = M_D$. Now using this assumption, we can relate the spring constant of the gluon k to the frequency of the oscillation ω and hence the ΔE between the states $\hbar\omega$. $\hbar\omega = \hbar\sqrt{\frac{k}{m}}$. Turning this around and solving for $k = m\omega^2$ and making the units nice: $k = \frac{mc^2(\hbar\omega)^2}{(\hbar c)^2}$ where the mc^2 is the mass of the heavy quark taken as the mass of the lightest meson (K, D or B). We can now make a quick guess as to the value of k. Using the Δm=661 MeV as the difference between the n=0 and n=1 states of the ϕ, and $mc^2 = 494$ MeV the mass of the K we get k=494 MeV *661MeV/(197 MeV − fm$)^2 = 5545/\text{fm}^2$. This will be just a first guess. We will adjust it to get the best value of k to fit all the data. When the mass is calculated, I simply assume that it is $n\hbar\omega$ above the n=0 state - hence predictions are made about the excited states, taking the ground state as input. i.e $m = m_{0\,(\text{ground state})} + n\hbar\omega$.

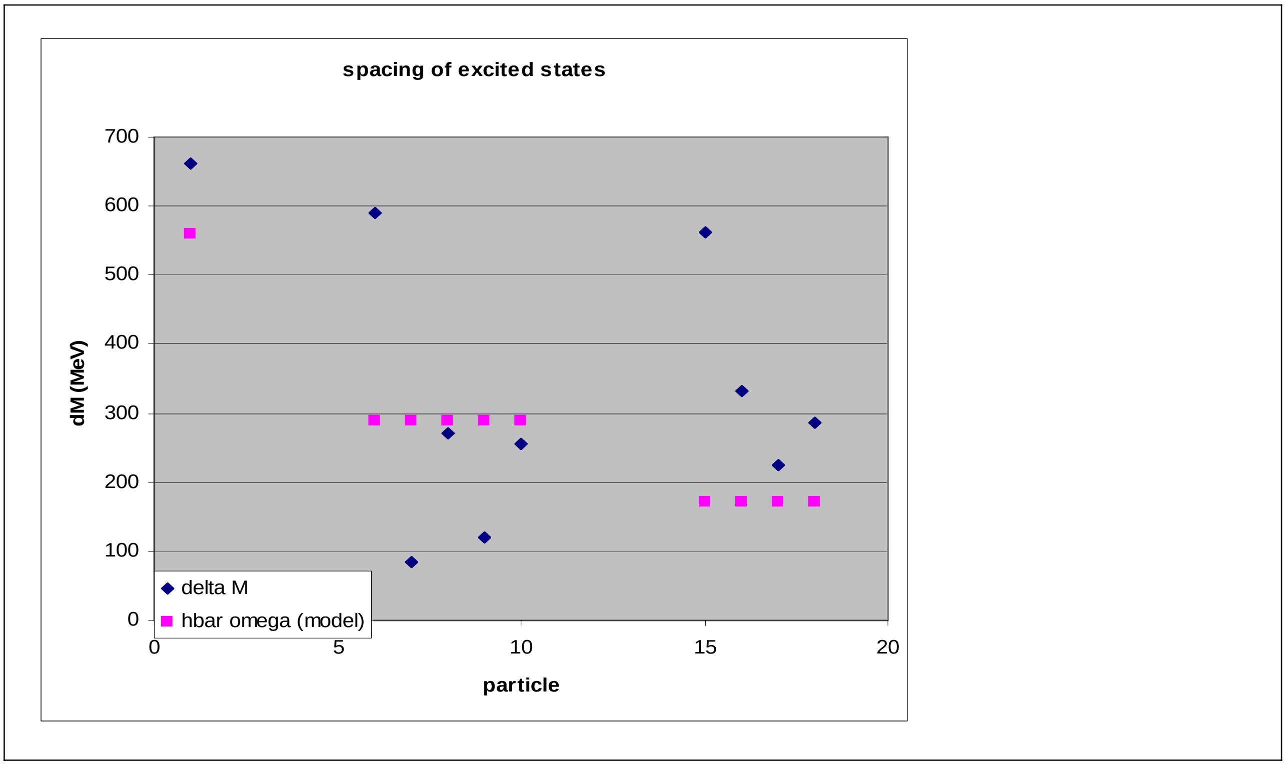
guess 1, $k = 5545$ from phi

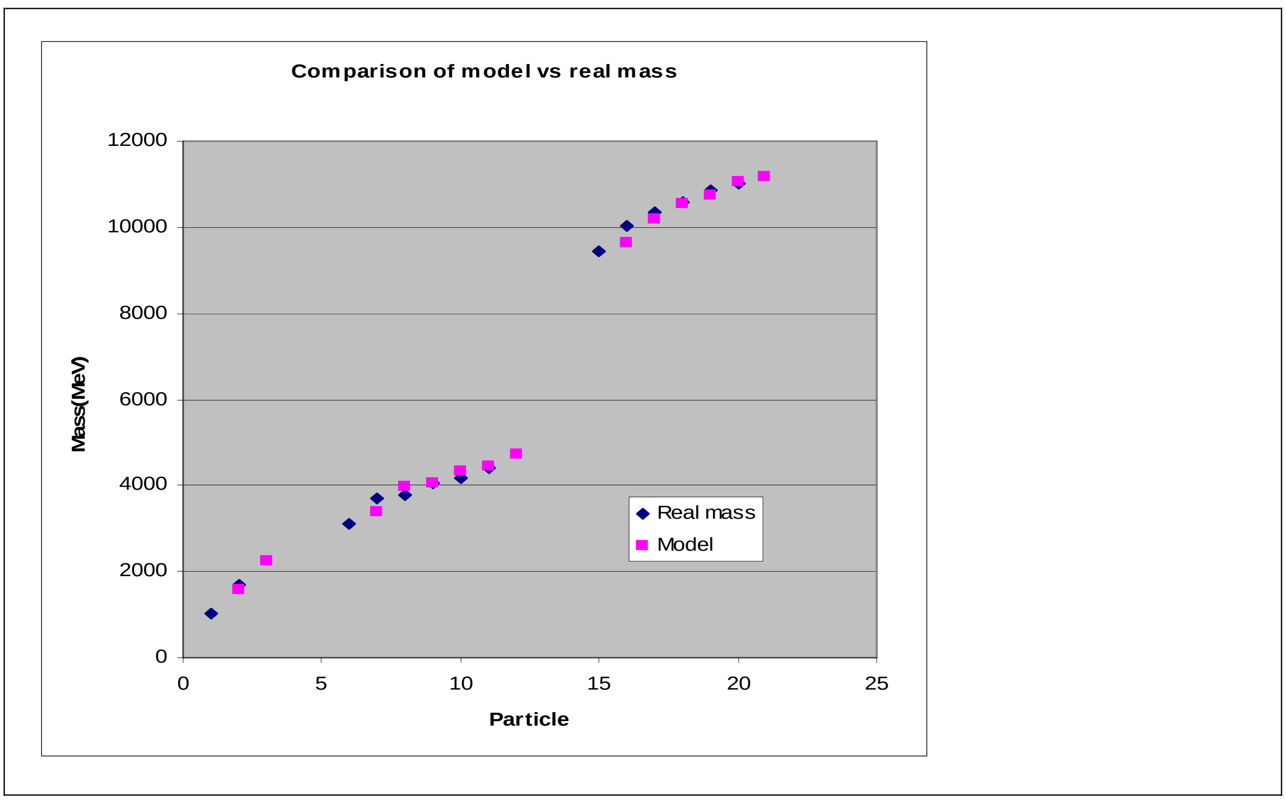
fixed									MeV/fm^2	
		MeV	MeV	MeV		Mev	%	%err	K	MeV
	n	mass	width	delta		m pred	error	dM		mc2
phi										
	0	1019	4.26						5545	494
	1	1680	150	661		1680	0	0	5545	494
guess	2					2340			5545	494
hbaromega				660						494
psi	0	3097	0.087						5545	1869
	1	3686	0.277	589		3436	-7	-42	5545	1869
	2	3770	23.6	84		4025	7	304	5545	1869
	3	4040	52	270		4109	2	26	5545	1869
	4	4159	78	119		4379	5	185	5545	1869
?	5	4415	43	256		4498	2	33	5545	1869
guess	6					4754			5545	1869
hbaromega				339						1869
Upsilon	0	9460	0.052						5545	5279
	1	10023	0.044	563		9662	-4	-64	5545	5279
	2	10355	0.026	332		10225	-1	-39	5545	5279
	3	10580	20	225		10557	0	-10	5545	5279
	4	10865	110	285		10782	-1	-29	5545	5279
?	5	11019	79	154		11067	0	31	5545	5279
guess	6					11221			5545	5279

First you might want to look compare the mass columns to the m pred column, where m pred is the predicted mass. First of course the mass for the phi(n=1) is exact. (There is no phi n=2, but I put a line there for use later as you will see.)Now look carefully at the %err column and the %err dM columns. the %error column is the error on the total mass. The %err dM is the error in the energy between the state and the state below it. We can see that particularly for the psi(n=2) state we are more than 300% off. Lets see if we can do a bit better. I played around with it and ended up setting k to 4000 $\mathrm{MeV}/\mathrm{fm}^2$. You can see its a little better, but not by much. A couple of the psi states are still off in the dM by more than 100%. Note though that the error in the mass is not bad. I also show graphically the real masses and dM's and the ones from the model. Note that $\hbar\omega$ changes for the different families since it is the spring constant k which is the same everywhere, the oscillation frequency depends on the mass of the heavy quark. We have $\hbar\omega = \sqrt{\frac{(\hbar c)^2 k}{mc^2}}$ So for the s quark $\hbar\omega$=561 MeV for the c quark $\hbar\omega$=288 MeV and for the b quark $\hbar\omega$=171 MeV.

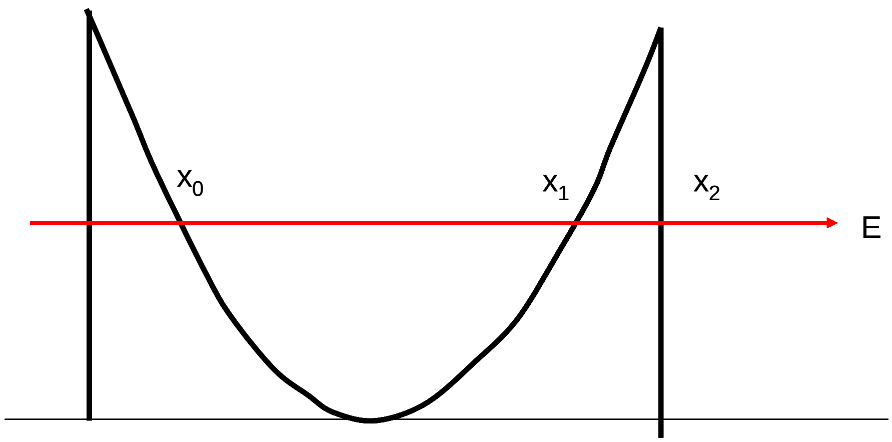
guess 2 $k = 4000$: optimum

fixed									MeV/fm^2	
		MeV	MeV	MeV		Mev	%	%err	K	MeV
	n	mass	width	delta		m pred	erroı	dM		mc2
phi										
	0	1019	4.26						4000	494
	1	1680	150	661		1580	-6	-15	4000	494
guess	2					2241			4000	494
hbaromega				561						494
psi	0	3097	0.087						4000	1869
	1	3686	0.277	589		3385	-8	-51	4000	1869
	2	3770	23.6	84		3974	5	243	4000	1869
	3	4040	52	270		4058	0	7	4000	1869
	4	4159	78	119		4328	4	142	4000	1869
?	5	4415	43	256		4447	1	13	4000	1869
guess	6					4703			4000	1869
hbaromega				288						1869
Upsilon	0	9460	0.052						4000	5279
	1	10023	0.044	563		9632	-4	-70	4000	5279
	2	10355	0.026	332		10195	-2	-48	4000	5279
	3	10580	20	225		10527	-1	-24	4000	5279
	4	10865	110	285		10751	-1	-40	4000	5279
?	5	11019	79	154		11036	0	11	4000	5279
guess	6					11190			4000	5279





Now lets see what our model says about the decay times. How do we figure out the decay time from what we can calculate - that is the transparency coefficient T? T is the probability of tunneling. We will assume that the quark is inside the potential and oscillates back and forth with frequency ω and bangs into the walls as it goes back and forth. Each time it its the wall, there is a probability of tunneling out (or decay). The frequency of hitting the wall is $f = 2\left(\frac{\omega}{2\pi}\right)$ - the 2 is because it can hit the a wall twice during one oscillation. So the probability of decay per second will be $P = \text{Tf} = \frac{\text{T}\omega}{\pi}$. The lifetime (or average time of decay) will be just $\tau = \frac{1}{P} = \frac{\pi}{\text{T}\omega}$ in seconds. We have found our model value of ω since we know the energy spacing $\hbar\omega$. We now need to calcuate T. Now from the WKB approx we have that T=$\exp\left(-2\int_{x_1}^{x_2} dx \sqrt{\frac{2m}{\hbar^2}(V(x)-E)}\right)$ where $V(x) = \frac{1}{2}\,\text{m}\omega^2$ m is the mass of the heavy quark, $x_1 = \sqrt{\frac{2E}{\text{m}\omega^2}}$ and $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$. x_2 is a bit more of a problem. It is not $\sqrt{\frac{2E}{\text{m}\omega^2}}$ since this assumes that the potential is SHO on the outside - remember the falling of the potential on the outside was caused by the popping of the two light quarks out of the vacuum. I have no idea what sort of shape that has. The best guess is just a sharp drop. So the potential really looks something like this



We will have to make a guess as to what x_2 is. That is why I let the model make a prediction as to the non-existent excited state above the highest experimental one (n=2 for the phi, and n=6 for psi and upsilon). The radius of this state which must be somewhere above the top of the potential must give us a limit on x_2 since at the very top of the potential $x_1 = x_2$. We must now evaluate T. The integral $\int_{x_1}^{x_2} dx \sqrt{\frac{2m}{\hbar^2}(V(x)-E)} = \int_{x_1}^{x_2} dx \sqrt{\frac{2m}{\hbar^2}\left(\frac{1}{2}\,\text{m}\omega^2 x^2 - \hbar\omega\left(n+\frac{1}{2}\right)\right)}$. We can actually do this analytically. Writing this as $\int_{x_1}^{x_2} dx \sqrt{b^2 x^2 - a^2})$ where b=$\sqrt{\frac{m^2\omega^2}{\hbar^2}} = \frac{\text{m}\omega}{\hbar} = \frac{\text{mc}^2\,\hbar\omega}{(\hbar\text{c})^2}$ and a=$\sqrt{\frac{2m}{\hbar}\,\omega\left(n+\frac{1}{2}\right)} = \sqrt{\frac{2\,\text{mc}^2}{(\hbar\text{c})^2}\,\hbar\omega\left(n+\frac{1}{2}\right)}$ we can use either mathematica, or math tables to get the answer.

Integrate[Sqrt[$b^2 * x^2 - a^2$], x]

$$\frac{1}{2} x \sqrt{b^2 x^2 - a^2} - \frac{a^2 \log\left(2 b x + 2 \sqrt{b^2 x^2 - a^2}\right)}{2 b}$$

Now $x_1 = \sqrt{\frac{2\left(n+\frac{1}{2}\right)\hbar}{\mathrm{m}\omega}} = \sqrt{\frac{2\left(n+\frac{1}{2}\right)(\hbar \mathrm{c})^2}{\mathrm{mc}^2\, \hbar\omega}}$

so we have an expression for a and b and the integral. We have one of the limits x_1 and we will have to make a guess at x_2. So we can now figure out T and hence the lifetime. We will calculate τ in fm / c so we use $\tau \mathrm{c} = \frac{\pi\hbar \mathrm{c}}{\mathrm{T}\hbar\omega}$. Before doing this I would like to make a few observations about the decays

If take a look at the lowest decay modes of both the psi and upsilon families, we see that they do not decay to mesons, but to leptons -e and μ both of which are rather light. The problem is that the bound state of the two heavy quarks are lighter than the sum of two decay mesons, e.g. $M_\psi < 2 M_{D.}$ This means that these states cannot decay to anything with a heavy quark in it. The only way this can happen is for the heavy quark to change identity to one of the lighter quarks, or to leptons. This can happen but not through the strong interaction. It means that for these decays, our picture is wrong. The mass spectrum may be OK since this does not depend on the possibility of decay, but the lifetime will not work. You can see that the lifetimes of these states are much longer than the others. We must then ignore the n=0,1 states for the psi and the n-=0,1,2 states for the upsilon when we use our model for lifetimes. There is another quirk. If you look at the heaviest state for these families, i.e. the n=5 state, we see that it has a *longer* lifetime then the state below it. This also makes no sense in our model since a higher state has a much shorter path to tunnel and should have a shorter lifetime. So lets ignore these states, i.e. the n=5 states for the psi and upsilon. What is left? We have the n=0,1 states for the phi, the n=2,3,4 states for the psi, and the n=3,4 states for the upsilon. As a first try for x_2 I set it for the radius x_1 of the non – existent n=2 state for the phi, and for the psi and upsilon, I use the radius of the n=5 state. The reasons for this are not very strong. Here is the spreadsheet.

Guess at x2

fixed									MeV/fm^2										
		MeV	MeV	MeV	fm	Mev	%	%err	K			fm	fm					fm	%
	n	mass	width	delta	tau	m pred	error	dM		a	b	x1	x2	v/c	int 1	int2	T gues	tau gue	error
phi																			
	0	1019	4.26		46.2				4000	2.7	7.13	0.374	0.837	0.34	-0.84	0.68	0.049	22.75	-51
	1	1680	150	661	1.3	1580	-6	-15	4000	4.6	7.13	0.649	0.837	0.59	-3.34	-2.88	0.397	2.78	112
guess	2					2241			4000	6.0	7.13	0.837		0.76					
hbaromega				561									0.837						
psi	0	3097	0.09		2264				4000	3.7	13.88	0.268	0.890	0.12	-1.00	3.30	0.000		
	1	3686	0.28	589	711	3385	-8	-51	4000	6.5	13.88	0.465	0.890	0.22	-3.84	-1.05	0.004		
	2	3770	23.6	84	8.3	3974	5	243	4000	8.3	13.88	0.600	0.890	0.28	-7.03	-5.34	0.034	63.33	659
	3	4040	52	270	3.8	4058	0	7	4000	9.9	13.88	0.710	0.890	0.33	-10.43	-9.56	0.175	12.30	225
	4	4159	78	119	2.5	4328	4	142	4000	11.2	13.88	0.805	0.890	0.37	-13.98	-13.69	0.555	3.87	53
?	5	4415	43	256	4.6	4447	1	13	4000	12.4	13.88	0.890	0.890	0.41	-17.64	#NUM!	#####	#NUM!	####
guess	6					4703			4000	13.4	13.88	0.968		0.45	-21.39	#NUM!			
hbaromega				288									0.890						
Upsilon	0	9460	0.05		3788				4000	4.8	23.33	0.207	0.687	0.06	-1.13	3.18			
	1	10023	0.04	563	4477	9632	-4	-70	4000	8.4	23.33	0.359	0.687	0.10	-4.23	-1.43			
	2	10355	0.03	332	7577	10195	-2	-48	4000	10.8	23.33	0.463	0.687	0.13	-7.68	-5.98			
	3	10580	20	225	9.9	10527	-1	-24	4000	12.8	23.33	0.548	0.687	0.15	-11.34	-10.47	0.173	20.84	112
	4	10865	110	285	1.8	10751	-1	-40	4000	14.5	23.33	0.621	0.687	0.17	-15.15	-14.85	0.552	6.54	265
?	5	11019	79	154	2.5	11036	0	11	4000	16.0	23.33	0.687	0.687	0.19	-19.07	-19.07	1.000	3.61	45
guess	6					11190			4000			0.747		0.21					

First, this is not the last iteration. There are now columns for a, b, x_1, my guess for x_2, for the integration at x_1 and x_2, T,the lifetime (tau guess) and the error on this number. You can see that the error on the lifetime is rather large in a couple of cases. We also are now able to calculate the velocity to see if it really is non-relativistic. We assume it goes a distance 2 x_1 at a frequency $\frac{\omega}{2\pi}$. This gives a time t=2π/ω and a velocity $\frac{\omega x_1}{\pi}$ or v = $\frac{\hbar\omega\, x_1\,(\text{in fm})}{\hbar c\,\pi}$. Lets try to do better.

best try

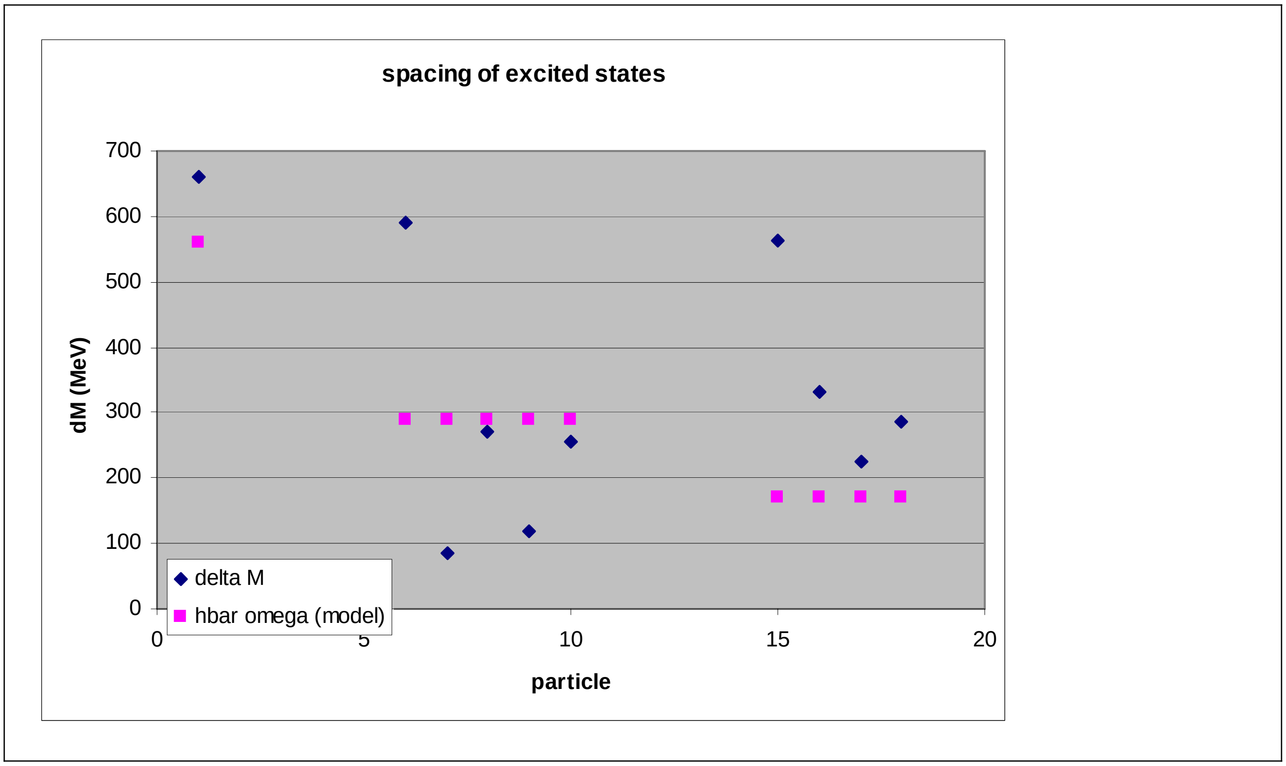
fixed										MeV/fm^2											
		MeV	MeV	MeV	fm	sec	Mev	%	%err	K	MeV			fm	fm					%	f
	n	mass	width	delta	tau	tau	m pred	errc	dM		mc2	a	b	x1	x2	v/c	int 1	int2	T gues	err	t
phi																					
	0	1019	4.26		46.24	4.6E-22				4000	494	2.7	7.1	0.374	0.806	0.34	-0.84	0.51	0.067	-181	
	1	1680	150	661	1.3	1.3E-23	1580	-6	-15	4000	494	4.6	7.1	0.649	0.806	0.59	-3.34	-2.99	0.496	41	
guess	2						2241			4000	494	6.0	7.1	0.837		0.76					
hbaromega				561							494				0.806						
psi	0	3097	0.087		2264	2.3E-20				4000	1869	3.7	13.9	0.268	0.806	0.12	-1.00	2.37	0.001		
	1	3686	0.277	589	711	7.1E-21	3385	-8	-51	4000	1869	6.5	13.9	0.465	0.806	0.22	-3.84	-1.87	0.020		
	2	3770	23.6	84	8.3	8.3E-23	3974	5	243	4000	1869	8.3	13.9	0.600	0.806	0.28	-7.03	-6.04	0.137	47	
	3	4040	52	270	3.8	3.8E-23	4058	0	7	4000	1869	9.9	13.9	0.710	0.806	0.33	-10.43	-10.10	0.513	10	
	4	4159	78	119	2.5	2.5E-23	4328	4	142	4000	1869	11.2	13.9	0.805	0.806	0.37	-13.98	-13.98	0.999	-18	
?	5	4415	43	256	4.6	4.6E-23	4447	1	13	4000	1869	12.4	13.9	0.890	0.806	0.41	-17.64	#####	#NUM!	####	
guess	6						4703			4000	1869	13.4	13.9	0.968		0.45	-21.39	#####			
hbaromega				288							1869				0.806						
Upsilon	0	9460	0.052		3788	3.8E-20				4000	5279	4.8	23.3	0.207	0.623	0.06	-1.13	2.25			
	1	10023	0.044	563	4477	4.5E-20	9632	-4	-70	4000	5279	8.4	23.3	0.359	0.623	0.10	-4.23	-2.25			
	2	10355	0.026	332	7577	7.6E-20	10195	-2	-48	4000	5279	10.8	23.3	0.463	0.623	0.13	-7.68	-6.68			
	3	10580	20	225	9.9	9.9E-23	10527	-1	-24	4000	5279	12.8	23.3	0.548	0.623	0.15	-11.34	-11.00	0.504	-38	
	4	10865	110	285	1.8	1.8E-23	10751	-1	-40	4000	5279	14.5	23.3	0.621	0.623	0.17	-15.15	-15.15	0.997	51	
?	5	11019	79	154	2.5	2.5E-23	11036	0	11	4000	5279	16.0	23.3	0.687	0.623	0.19	-19.07	#####	#NUM!	####	
guess	6						11190			4000	5279			0.747		0.21					
hbaromega				171							5279				0.623						

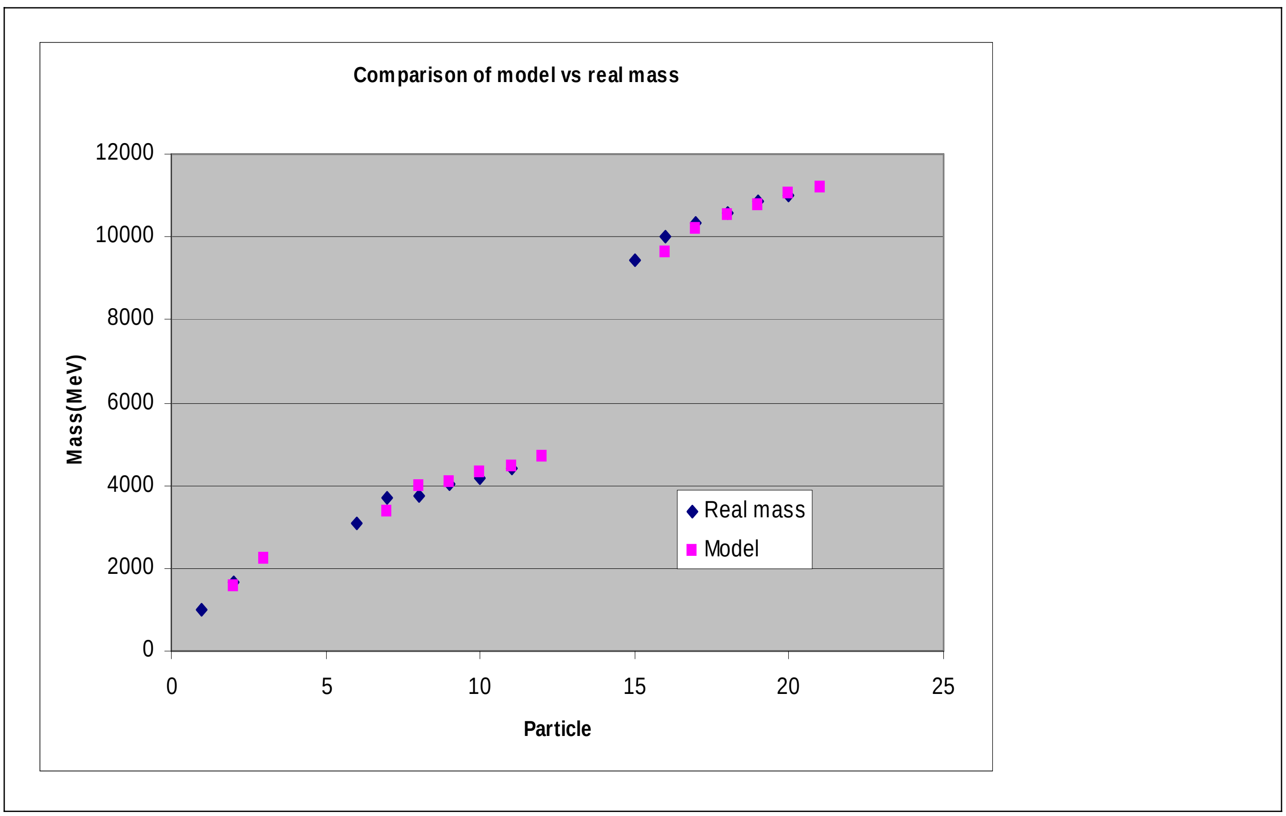
Here now is the final result. Perhaps you can find a better set of parameters. Remember you can adjust, k, and the three x_2's. I tried to keep the x_2's the same since it is the same potential well. Perhaps in the case of the b quark, the mass of the quark alters the depth of the potential well.

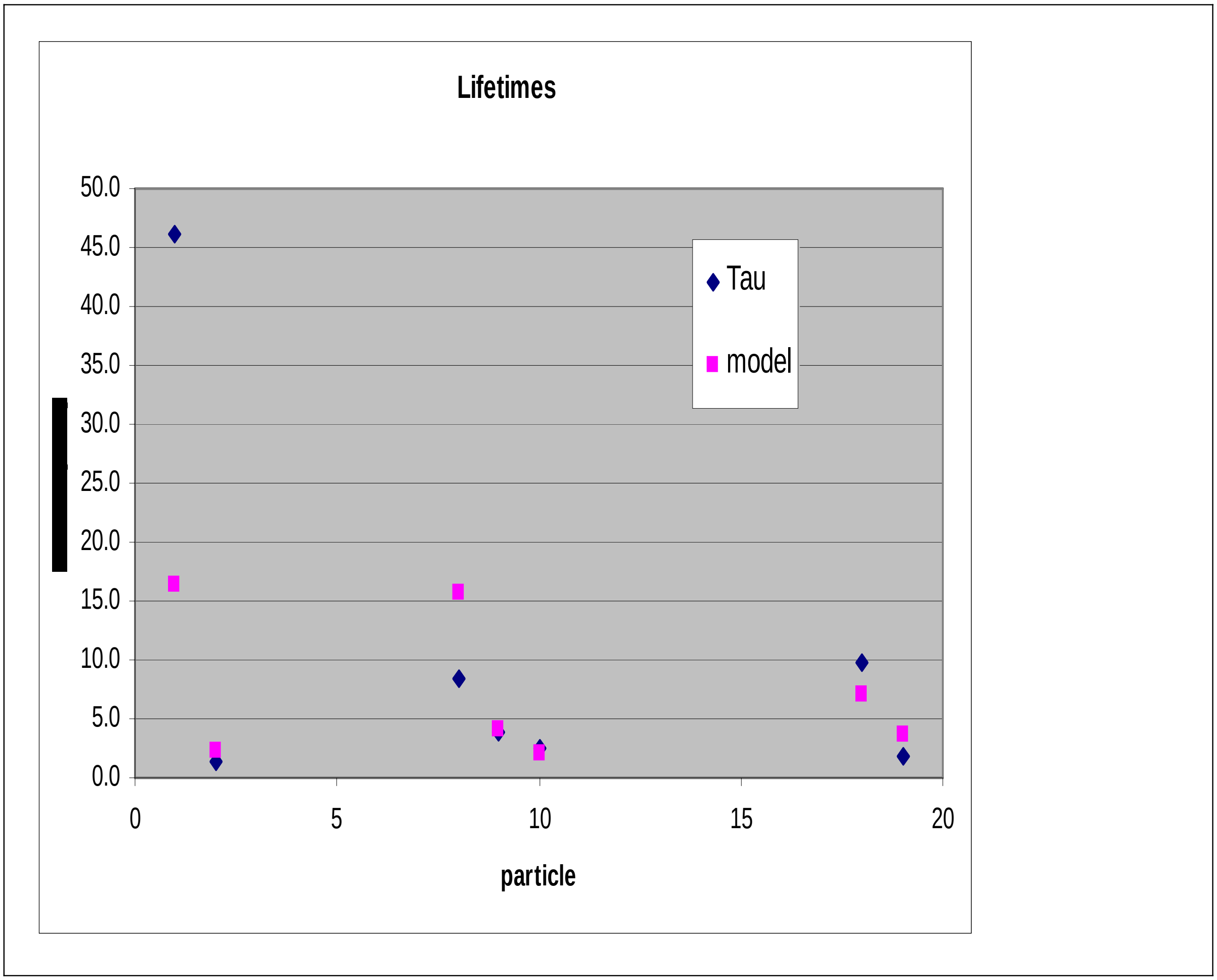
summary

fixed		MeV		%	er	fm		%
	n	mass	mass?	er	dM	tau	tau?	er
phi								
	0	1019				46.2	16.5	-64
	1	1680	1580	-6	-15	1.3	2.2	69
psi	0	3097				2264		
	1	3686	3385	-8	-51	711		
	2	3770	3974	5	243	8.3	15.7	88
	3	4040	4058	0	7	3.8	4.2	11
	4	4159	4328	4	142	2.5	2.1	-15
?	5	4415	4447	1	13	4.6	###	###
Upsilon	0	9460				3788		
	1	10023	9632	-4	-70	4477		
	2	10355	10195	-2	-48	7577		
	3	10580	10527	-1	-24	9.9	7.2	-27
	4	10865	10751	-1	-40	1.8	3.6	102
?	5	11019	11036	0	11	2.5	###	###

Here now is a summary. You can see the predictions for the lifetime under tau guess, and the error in the last column. I managed to get the answer to a factor of two! In the plots below I show plots of the comparison of model vs data.


spacing of excited states
dM (MeV)
700
600
500
400
300
200
100
0
0
5
10
15
20
particle
delta M
hbar omega (model)


Comparison of model vs real mass
Mass(MeV)
12000
10000
8000
6000
4000
2000
0
0
5
10
15
20
25
Particle
Real mass
Model



What have we learned? We have learned that k~4000 MeV / fm^2. Lets change this to MKS units.
$4000\ \frac{\text{MeV}}{\text{fm}^2}\left(\frac{1\ \text{fm}}{10^{-15}\ m}\right)^2 1.602\times 10^{-19}\ \frac{J}{\text{eV}}\ 10^6\ \frac{\text{ev}}{\text{MeV}} = 6.4\times 10^{20}\ N/m$. So if you stretched a gluon 1mm, it would have a force of about 6×10^{17} N or about 10^{17} lbs. How good do you think this number is. I bet its good to a factor of 10; whether its 10^{18}, 10^{17}, 10^{16} lbs - its a lot. That is why we call it the strong interaction.