Photon Production at NLO in Hot QCD

Derek Teaney

SUNY Stony Brook and RBRC Fellow

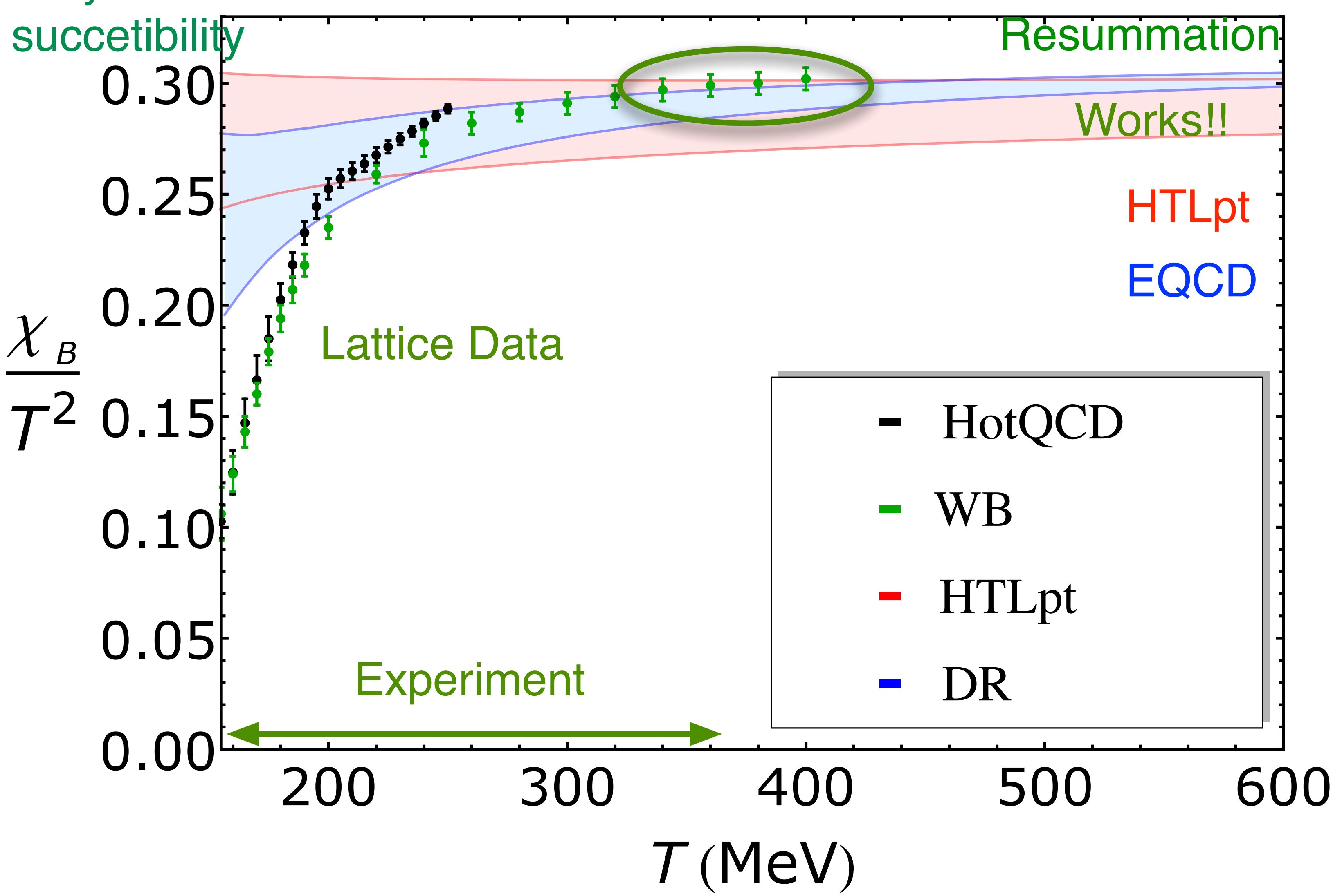
STONY BROOK
STATE UNIVERSITY OF NEW YORK

- Photons – In collaboration with Jacopo Ghiglieri, Juhee Hong, Aleksi Kurkela, Egang Lu, Guy Moore, arXiv:Almost.Done

Perturbation theory can work for thermodynamic quantities! Let's use it!

- HTLpt from Andersen, Su, Strickland. Dimensional Reduction/EQCD – the Finish Group

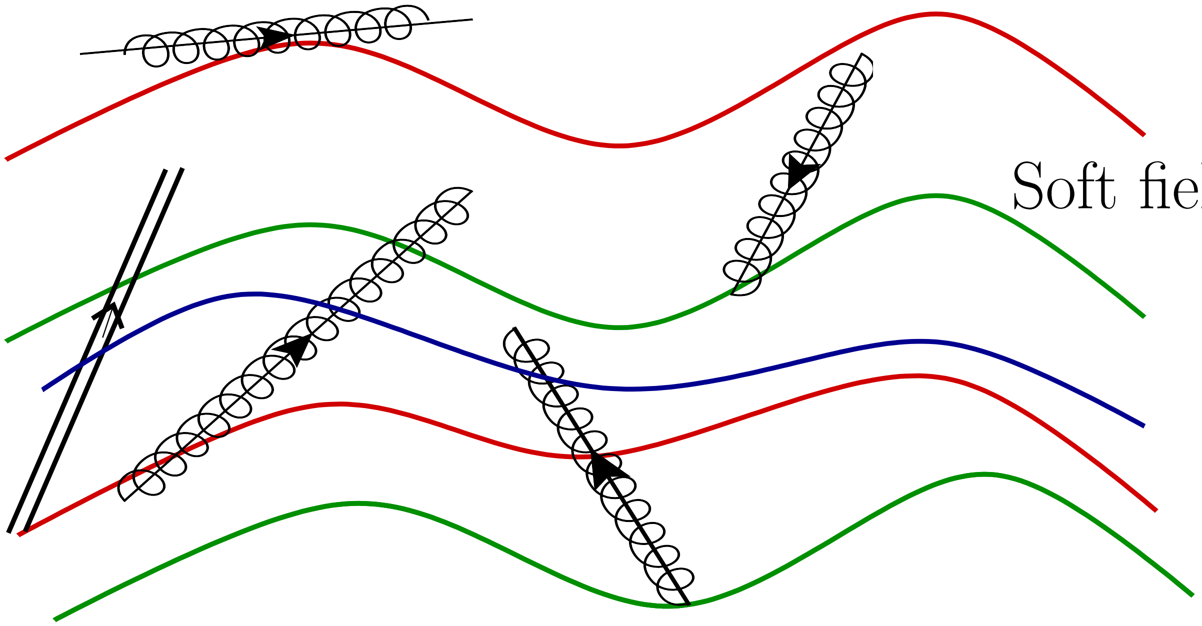
Baryon # succetibility



Want to compute transport with similar precision at high T

Basic picture of weakly coupled plasma

Hard particle modes, $P \sim T$



Soft field modes, $P \sim m_D$

1. Strong coupling – no quarks and gluons at scale T

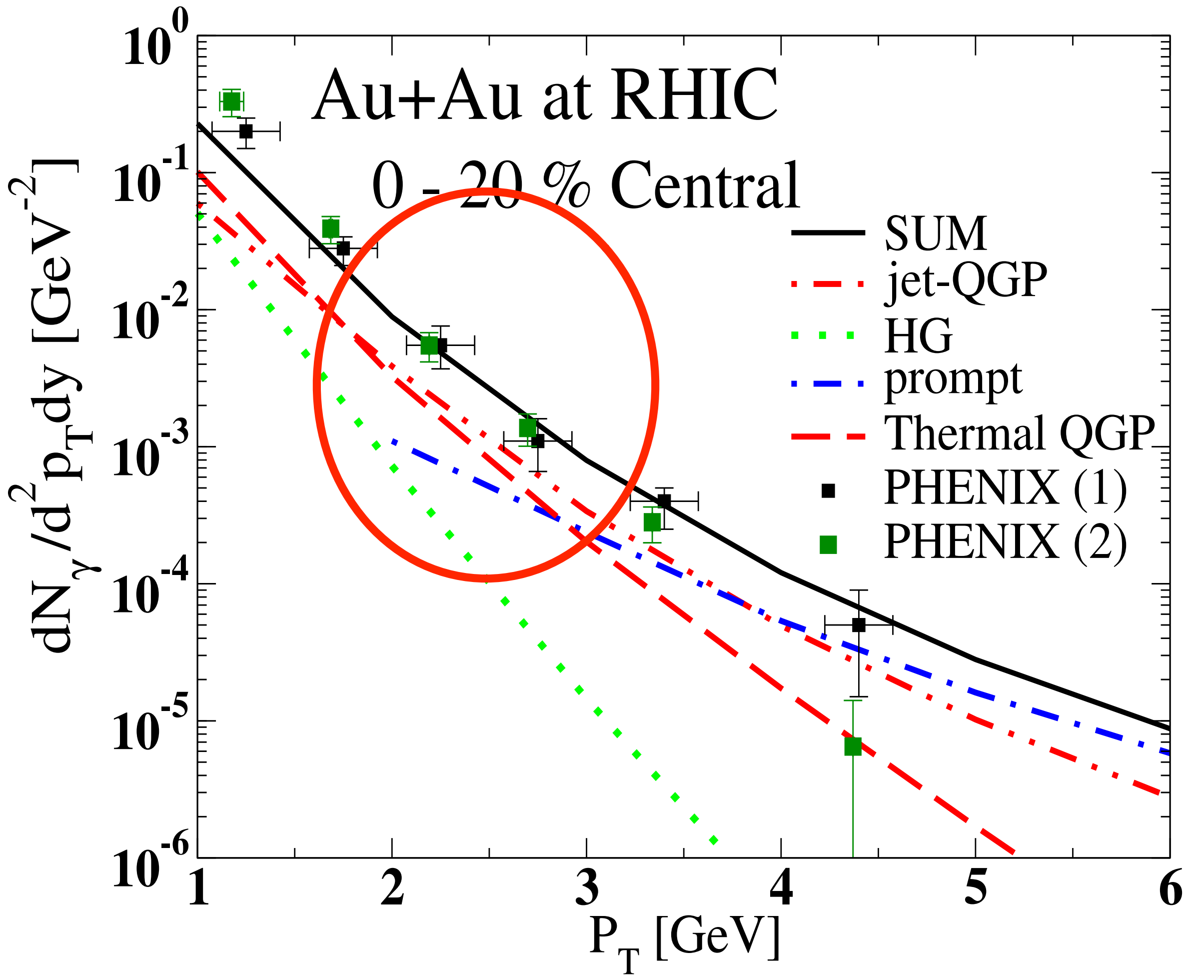
2. Weak coupling – quarks and gluons quasi-particles at scale gT

3. Intermediate coupling – no strict quark and gluon quasi-particles at scale gT

   - This is what these perturbative schemes are doing

Motivation

- This calculation uses LO order photon production rates (Turbide, Rapp, Gale)



We want to compute this rate at NLO

Thermal rate is dominant for a certain momentum range

Direct photons are measured, but this is not my real motivation . . .

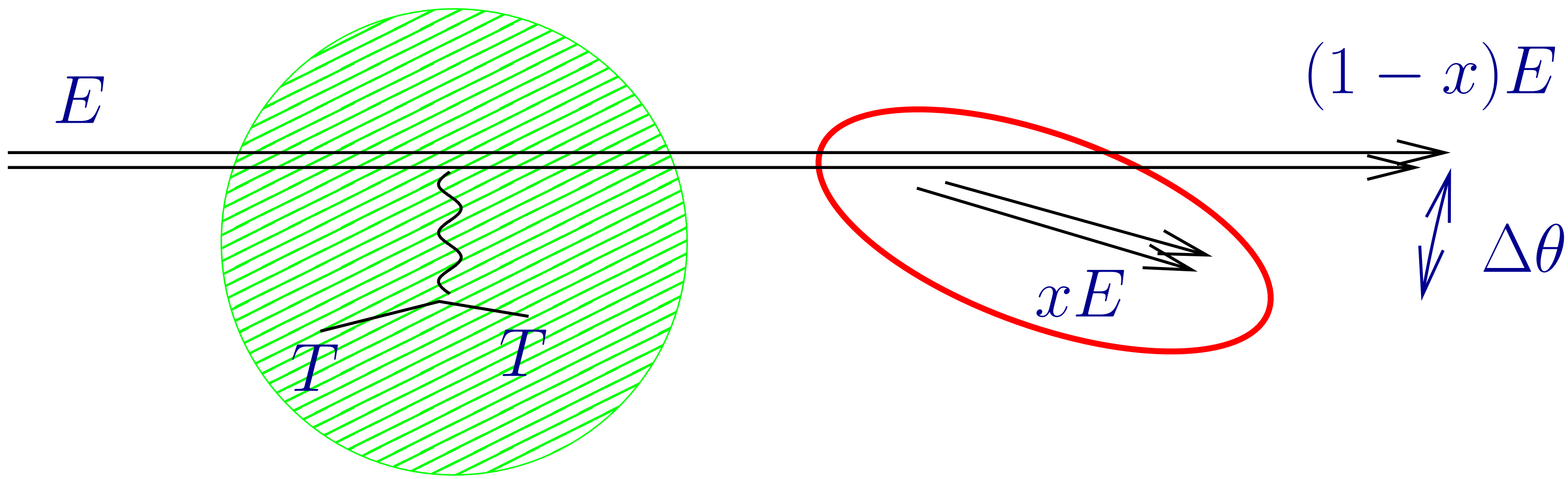
My real motivations:

1. Energy loss.

2. The shear viscosity.

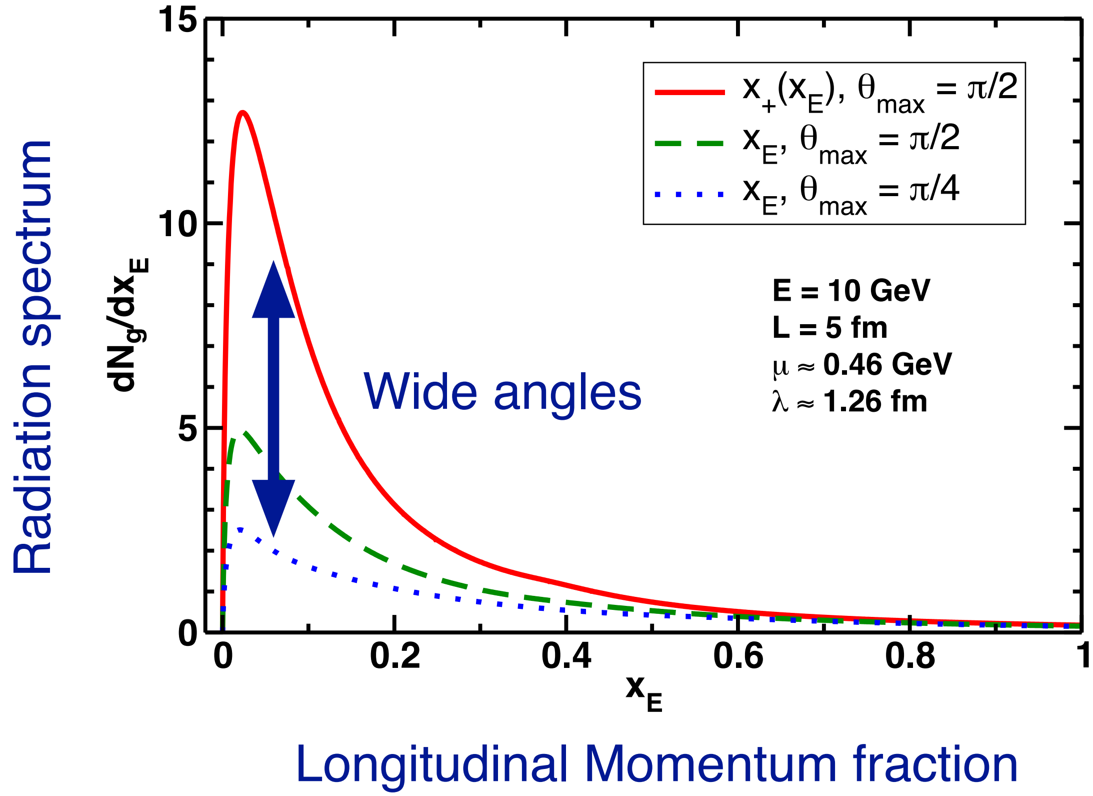
My real motivation. Energy loss at sub-asymptotic energies is important:

1. Kinematic constraints limit the agreement between energy loss formalisms
   - Higher Twist versus AMY versus W-DGLV
   - See the report of the Jet Collaboration: arXiv:1106.1106
2. Finite energy leads to large angle emission outside of radiative loss formalism

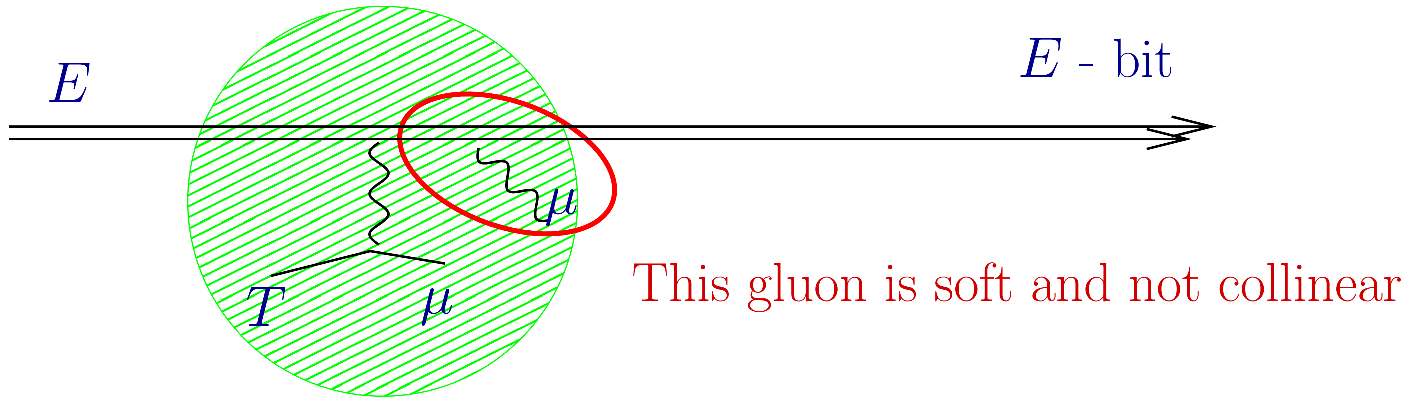


As the bremmed energy gets lower and lower, the angle $\Delta\theta$ gets larger and larger, limiting the agreement

A sample plot from DGLV in the Jet Collaboration Report:

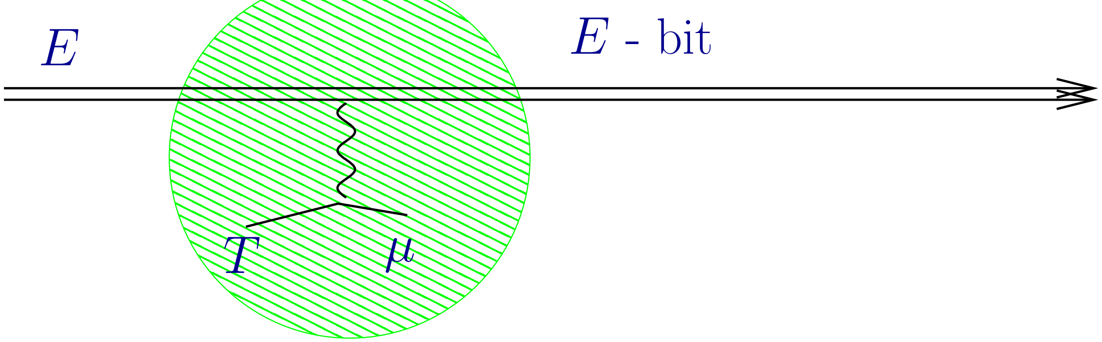


This is what is going on:



Radiative and Collisional Loss:

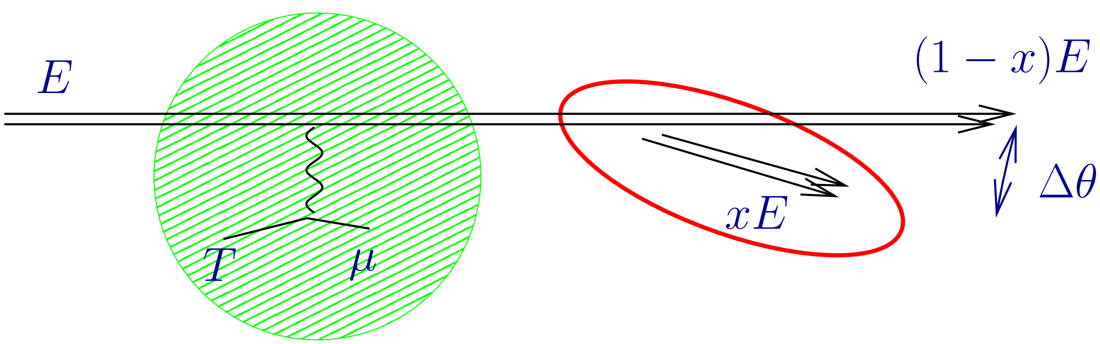
Collisional Energy Loss: $\frac{dp^{LO}_{\rm coll}}{dt}(\mu)$



Features:

1. Plasma is excited: $T \ll \mu \ll E$
2. Hard particle in hard particle out

Radiative Loss: $\frac{dp_{\rm rad}}{dt}(\mu)$



Features:

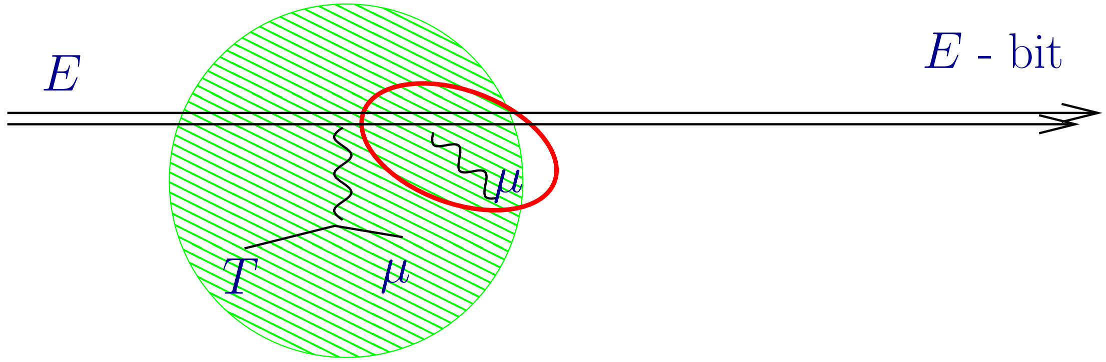
1. Plasma is excited: $T \ll \mu \ll E$
2. Hard particle in, two hard part. out

- We require $xE \gg \mu$

As the bremmed energy gets lower and lower, the angle $\Delta\theta$ gets larger and larger

Radiative and Collisional Loss

Soft Radiative Loss: $\frac{dp^{NLO}_{\rm coll}}{dt}(\mu)$

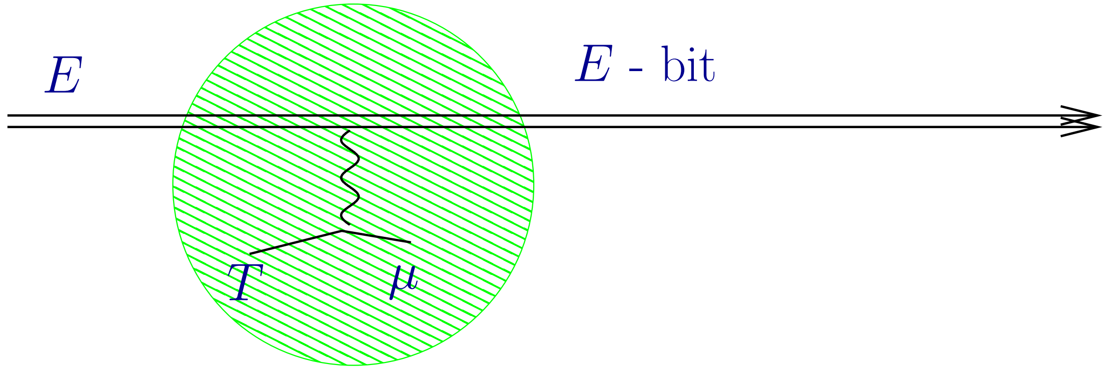


Features:

1. Plasma is excited: $T \ll \mu \ll E$
2. Hard particle in, one hard particle out

This is higher order correction to the collisional E-loss rate

Collisional Energy Loss: $\frac{dp^{LO}_{\rm coll}}{dt}(\mu)$



Final result is independent of μ:

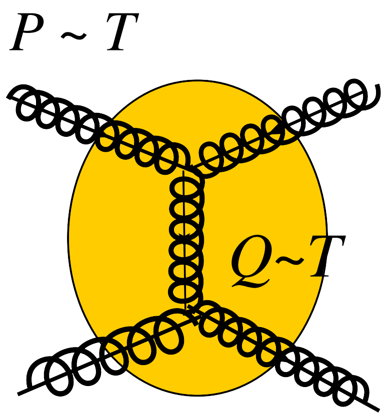
$$\underbrace{\frac{dp^{LO}_{\rm coll}}{dt} + \frac{dp^{NLO}_{\rm coll}}{dt}}_{\text{Phenomenological Coll E-loss}} + \underbrace{\frac{dp_{\rm rad}}{dt}}_{\text{Radiative Loss}}$$

My real motivations:

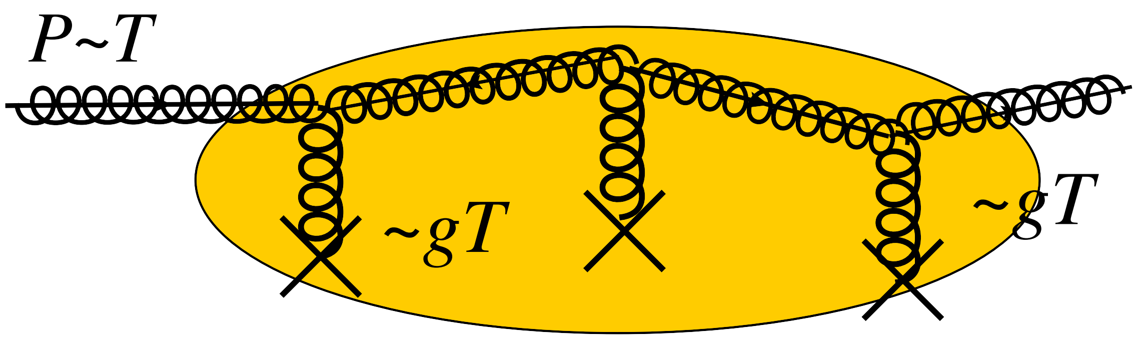
- ✓ Energy loss
2. The shear viscosity

My real motivation. Shear viscosity and the kinetics of weakly coupled QGP

1. Hard Collisions: $2 \leftrightarrow 2$



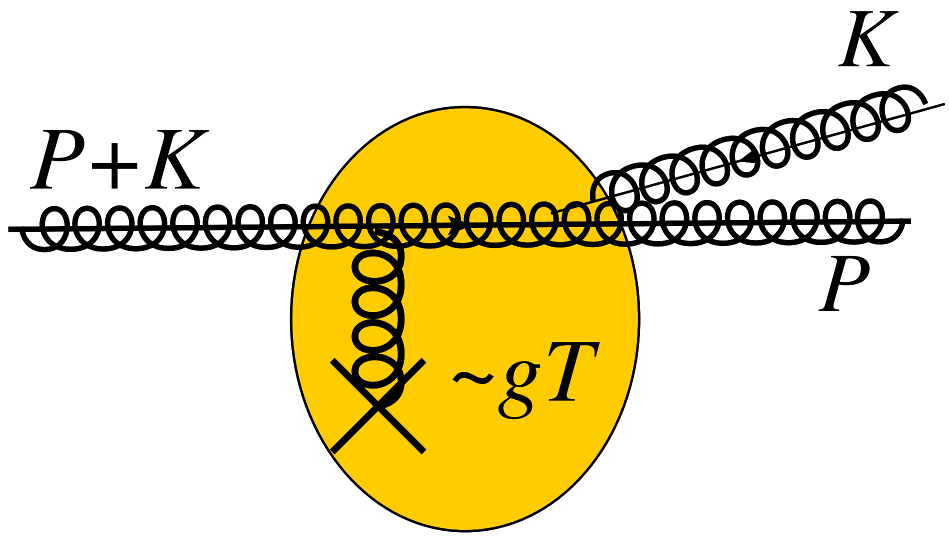
2. Diffusion: collisions with soft random classical field



$$C_{GLV}[q_\perp] = \frac{T g^2 m_D^2}{(q_\perp^2 + m_D^2)^2} \rightarrow$$ Probability of a tranverse kick $q_\perp$

3. Brem: $1 \leftrightarrow 2$

- random walk induces collinear bremsstrhalung



NLO involves corrections to these processes and the relation between them

But shear viscosity is too hard . . .

My real motivations:

- ✓ Energy loss
- ✓ The shear viscosity

Photon production at NLO is a good warm-up calculation.

Lets do it!

$$2k(2\pi)^3\frac{\mathrm{d}\Gamma}{\mathrm{d}^3k} = \text{Photon emission rate per phase-space}$$

The photon emission rate at weak coupling:

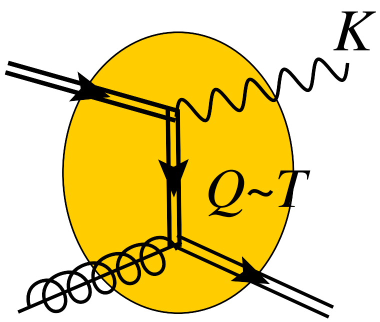
- The rate is function of the coupling coupling constant and k/T:

$$2k(2\pi)^3\frac{\mathrm{d}\Gamma}{\mathrm{d}^3k} \propto e^2T^2\Big[\underbrace{O(g^2\log) + O(g^2)}_{\text{LO AMY}} +$$

$$\underbrace{O(g^3\log) + O(g^3)}_{\text{From soft } gT \text{ gluons, } n_B \simeq \frac{T}{\omega} \simeq \frac{1}{g}} + \ldots$$

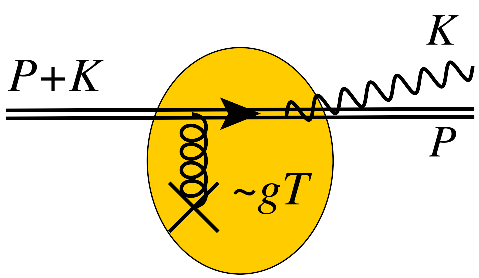
Three rates for photon production at Leading Order — Baier,Kapusta, AMY

1. Hard Collisions – a $2 \leftrightarrow 2$ processes



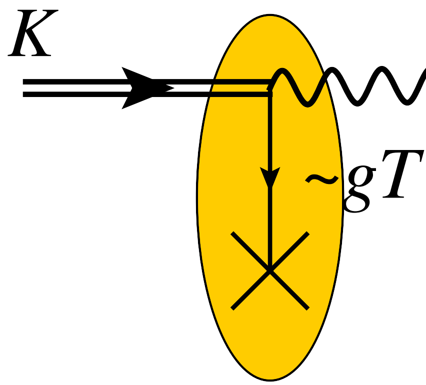
$$\sim e^2 \underbrace{m_\infty^2}_{g^2 C_F T^2/4} \times \underbrace{n_F(k)}_{\text{fermi dist.}} \times \left[\log(T/\mu) + C_{2\text{to}2}(k) \right]$$

2. Collinear Bremsstrhalung – a $1 \leftrightarrow 2$ processes

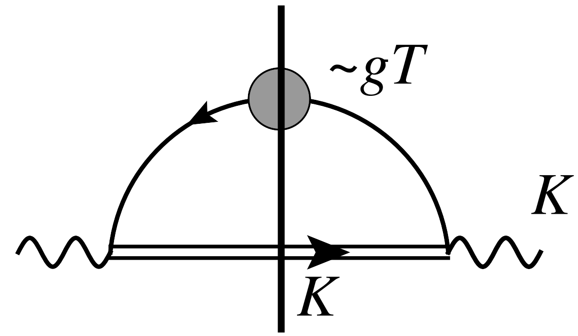


$$\sim e^2 m_\infty^2 n_F \left[\underbrace{C_{\text{brem}}(k)}_{\text{LPM + AMY and all that stuff!}} \right]$$

3. Quark Conversions – $1 \leftrightarrow 1$ processes (analogous to drag)



or



$$= \sim e^2 m_\infty^2 n_F \left[\log(\mu_\perp/m_\infty) + C_{\rm cnvrt}\right]$$

Full LO Rate is independent of scale $\mu_\perp$:

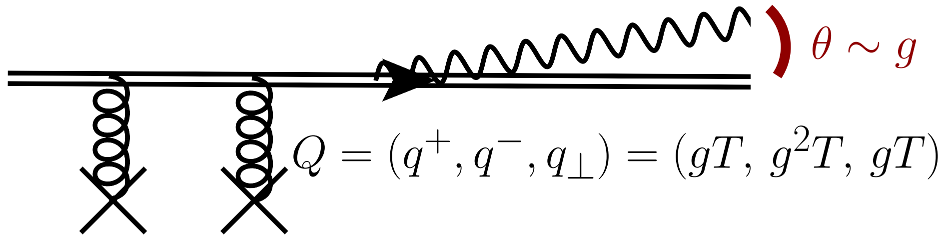
$$2k\frac{d\Gamma}{d^3k} \propto e^2 m_\infty^2 n_F \left[\log\left(T/m_\infty\right) + \underbrace{C_{\rm 2to2}(k) + C_{\rm brem}(k) + C_{\rm cnvrt}(k)}_{\equiv C_{LO}(k)} \right]$$

$O(g)$ Corrections to Hard Collisions, Brem, Conversions:

1. No corrections to Hard Collisions:

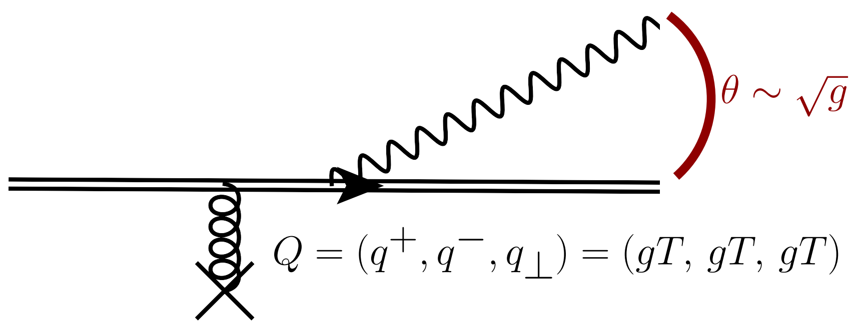
2. Corrections to Brem:

   (a) Small angle brem. Corrections to AMY coll. kernel. (Caron-Huot)

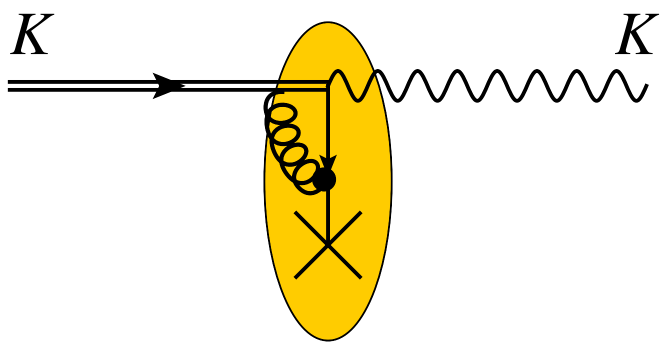


$$C_{LO}[q_\perp] = \frac{Tg^2m_D^2}{q_\perp^2(q_\perp^2 + m_D^2)} \rightarrow \text{A complicated but analytic formula}$$

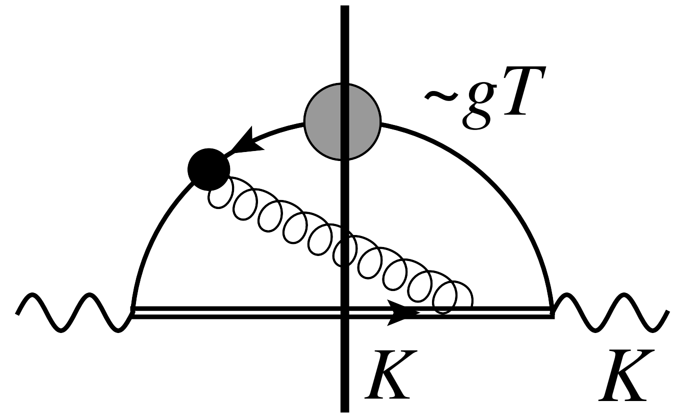
   (b) Larger angle brem. Include collisions with energy exchange, $q^- \sim gT$.



3. Corrections to Conversions:



or



- Doable because of HTL sum rules (light cone causality)

Simon Caron-Huot

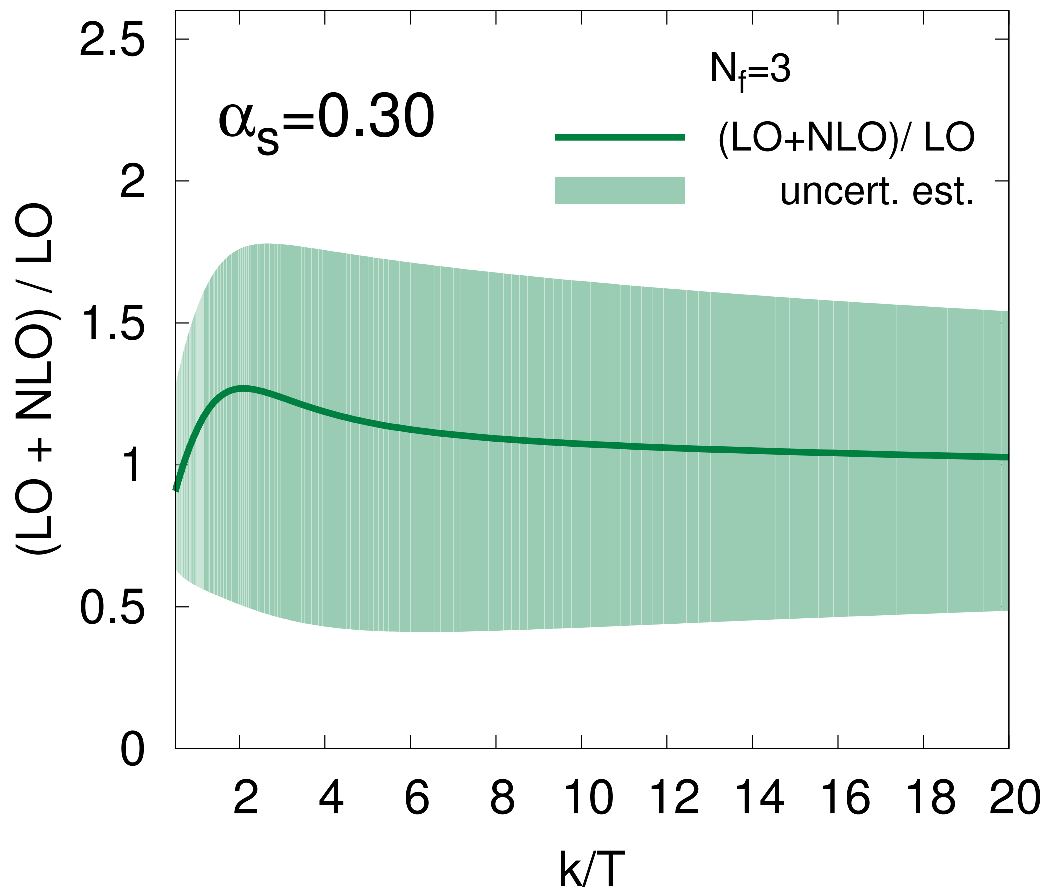
- Gives a numerically small and momentum indep. contribution to the NLO rate

Full results depend on all these corrections.

These rates smoothly match onto each other as the kinematics change.

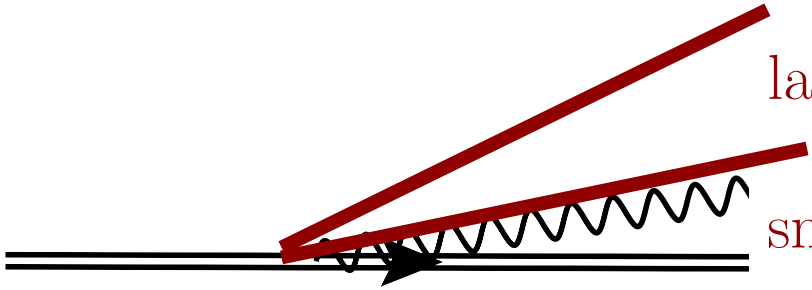
NLO Results: $\Gamma_{LO+NLO} \sim \mathrm{LO} + g^3 \log(1/g) + g^3$

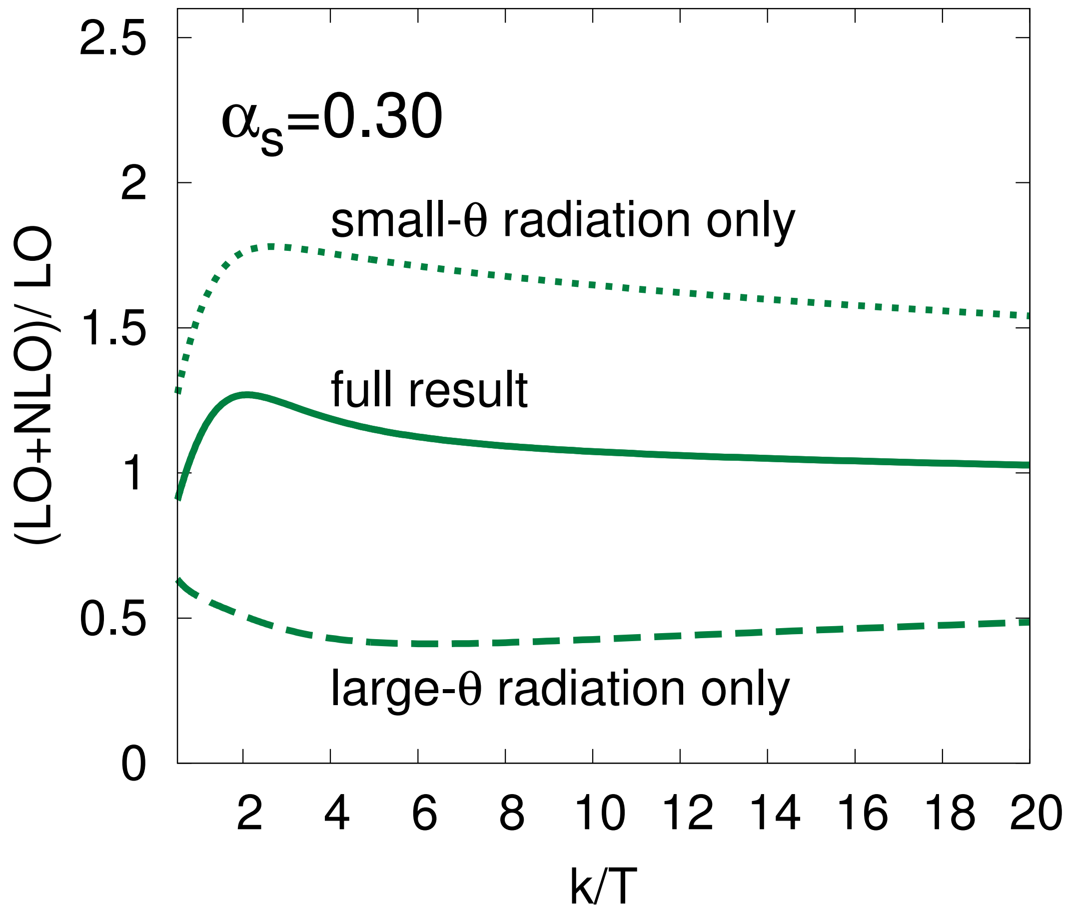
$$2k\frac{\mathrm{d}\Delta\Gamma_{NLO}}{\mathrm{d}^3k} \propto e^2 m_\infty^2 n_F(k)\Big[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\left(\frac{\sqrt{2Tm_D}}{m_\infty}\right)}_{\text{conversions}} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2} C_{\mathrm{large}-\theta}(k)}_{\text{large-}\theta\text{-brem}} + \underbrace{\frac{g^2 C_A T}{m_D} C_{\mathrm{small}-\theta}(k)}_{\text{small-}\theta\text{-brem}}\Big]$$



Corrections are small and k independent

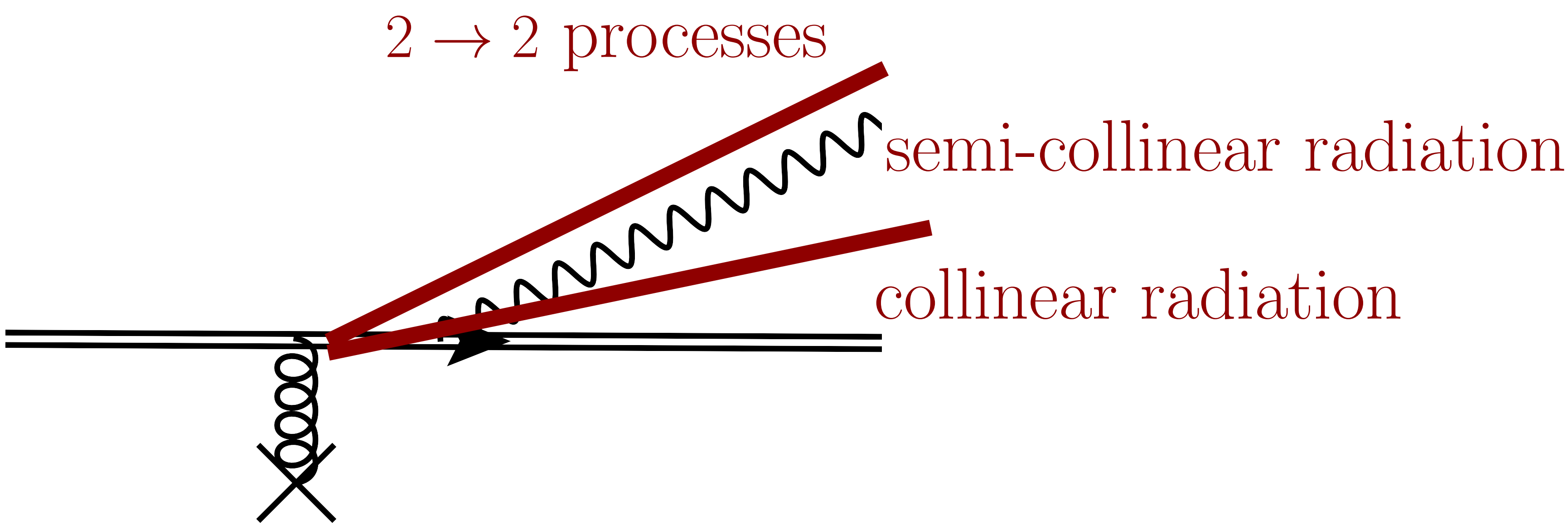
The different contributions at NLO (conversions are not numerically important)





The calculation

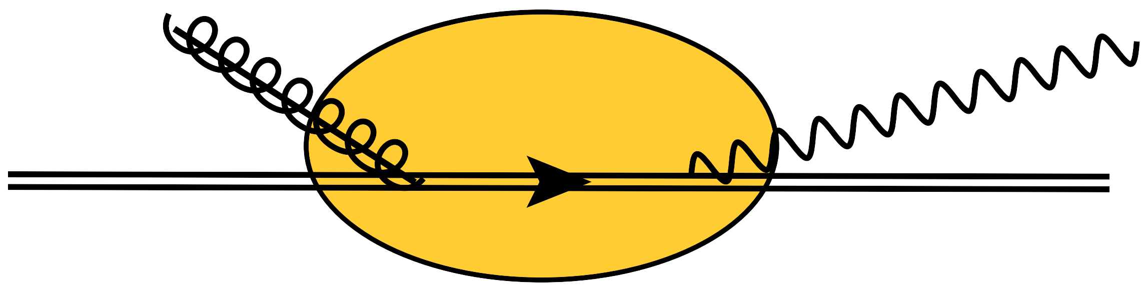
Semi-collinear radiation – a new kinematic window



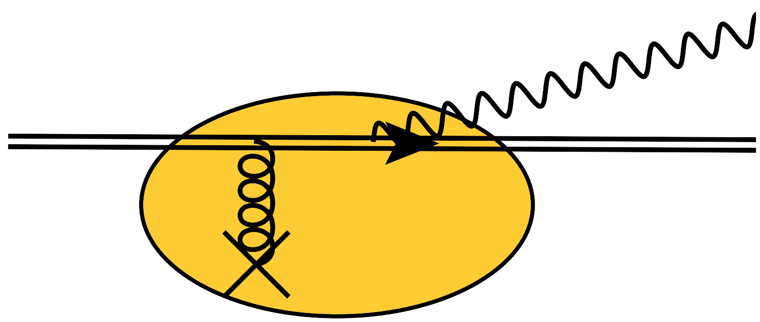
The semi-collinear regime interpolates between brem and collisions

Matching collisions to brem

- When the gluon is hard the $2 \leftrightarrow 2$ collision:

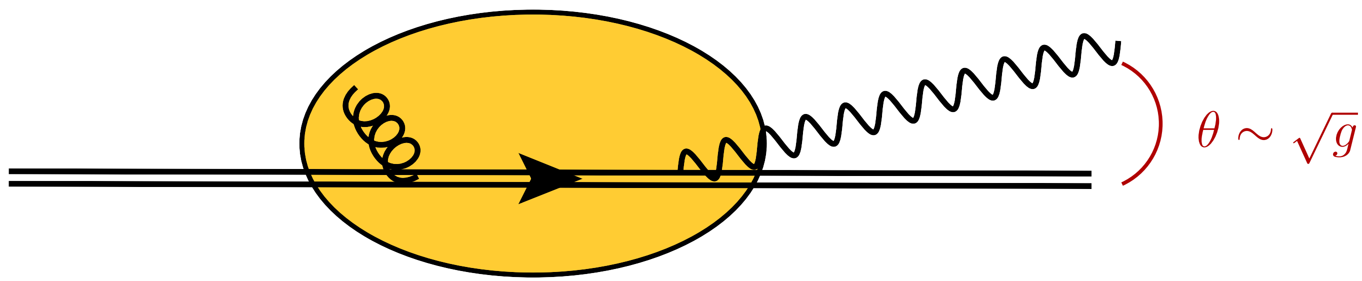


is physically distinct from the wide angle brem

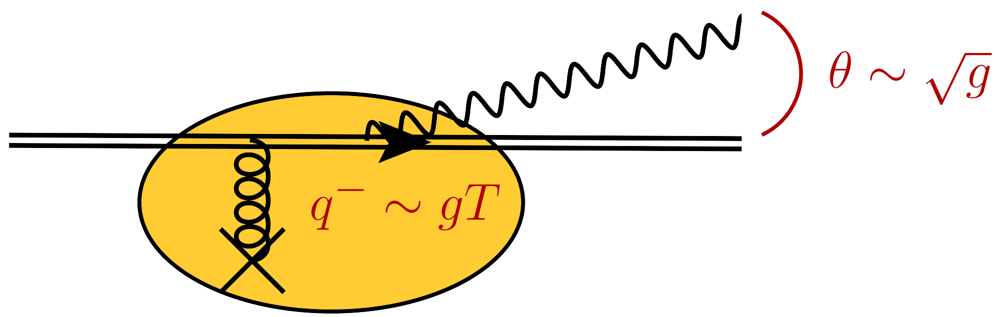


Matching collisions to brem

- When the gluon becomes soft (a plasmon), the $2 \leftrightarrow 2$ collision:



is not physically distinct from the wide angle brem

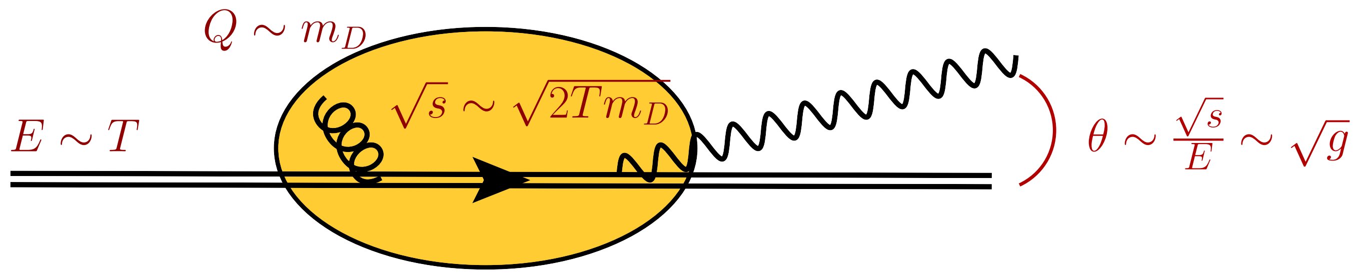


Need both processes

- For harder gluons, $q^- \to T$, this becomes a normal $2 \to 2$ process.
- For softer gluons, $q^- \to g^2 T$, this smoothly matches onto AMY.

Matching collisions to brem

- When the gluon becomes soft (a plasmon), the $2 \leftrightarrow 2$ collision:



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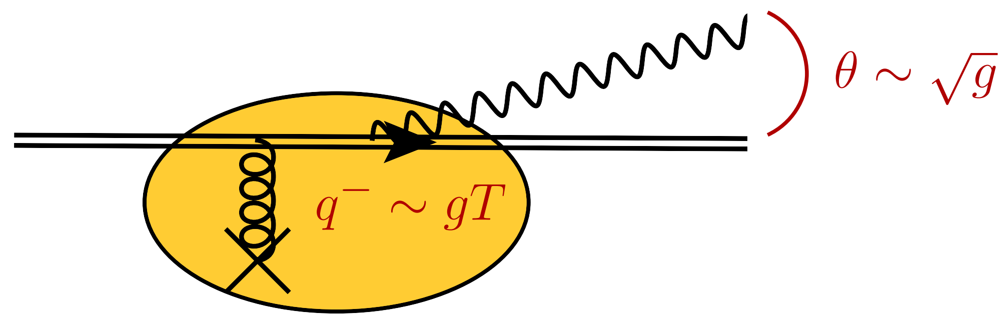

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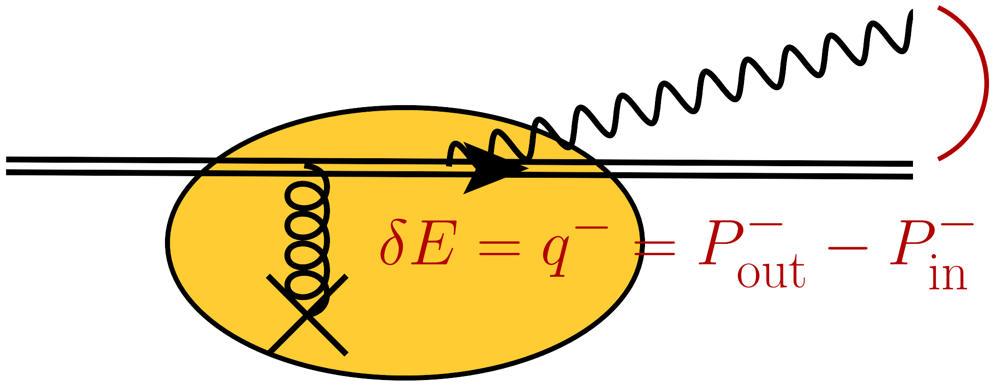


- The AMY collision kernel $C[q_\perp]$ involves

Aurenche, Gelis, Zakarat

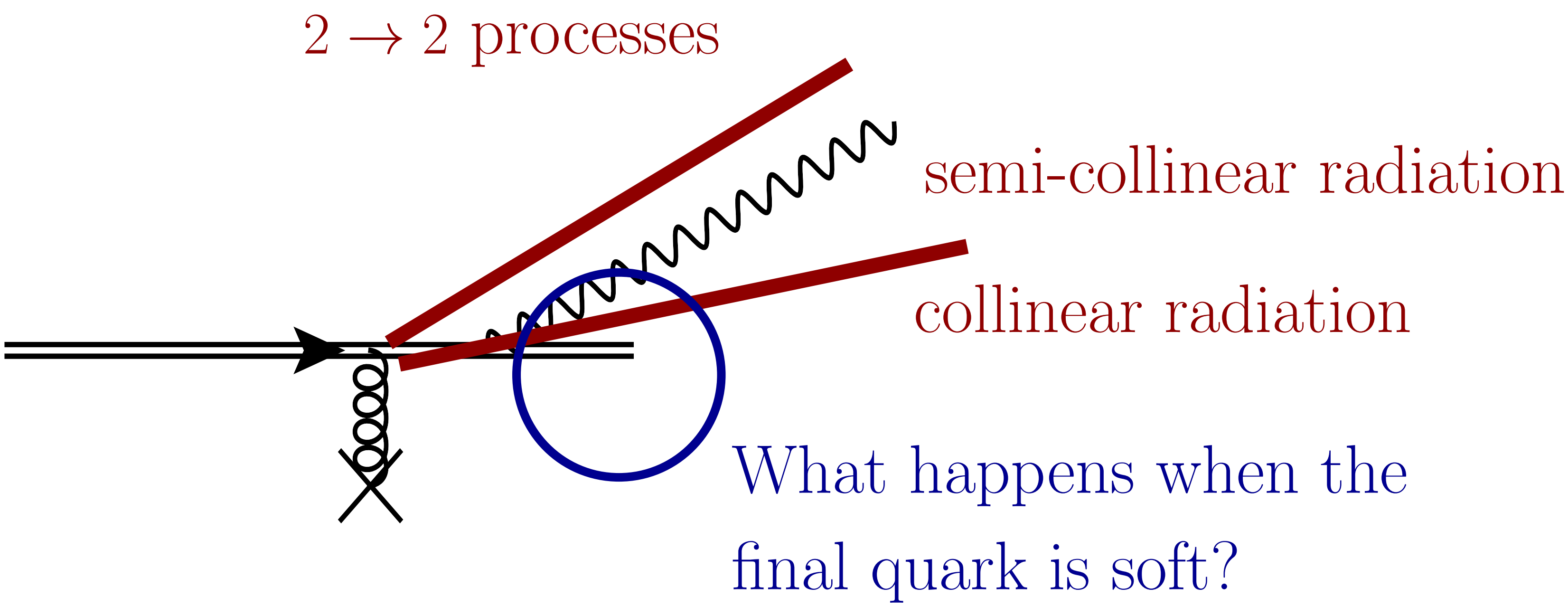
$$\underbrace{q_\perp^2 C[q_\perp] = \int_{-\infty}^{\infty} \frac{\mathrm{d}q^+}{2\pi} \left\langle F_{i+} F_{i+}(Q) \right\rangle\Big|_{q^-=0} = \frac{T m_D^2}{q_T^2 + m_D^2}}_{\text{Probability of a transverse kick } q_\perp}$$

- We need a finite $q^- = \delta E$ generalization:

$$\underbrace{\int_{-\infty}^{\infty} \frac{\mathrm{d}q^+}{2\pi} \left\langle F_{i+} F_{i+}(Q) \right\rangle\Big|_{q^-=\delta E} = T\left[\frac{2(\delta E)^2(\delta E^2 + q_\perp^2 + m_D^2) + m_D^2 q_\perp^2}{(\delta E^2 + q_\perp^2 + m_D^2)(\delta E^2 + q_\perp^2)}\right]}_{\text{Probability of a transverse kick } q_\perp \text{ and energy transfer } \delta E}$$

Wider angle emissions can be included by a “simple” modified collision kernel

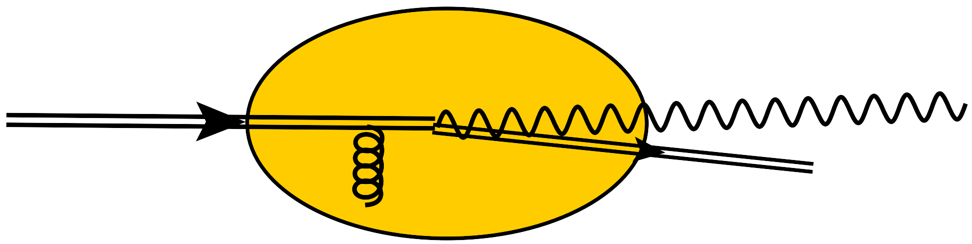
Matching between brem and conversions



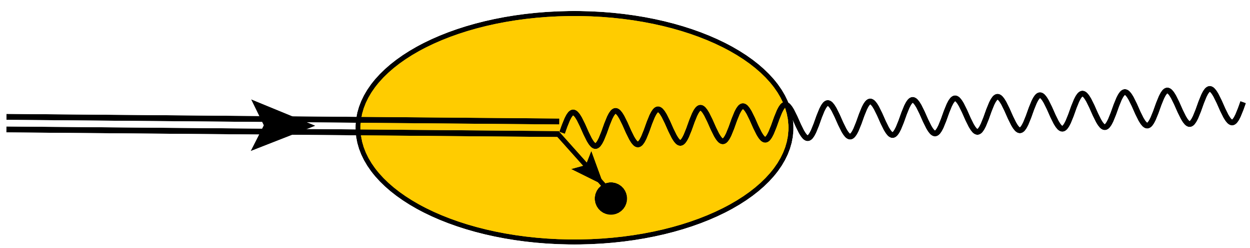
When the quark becomes soft need to worry about conversions.

Matching between brem and conversions

- When the final quark line is hard, the brem process :



is physically distinct from the conversion process:



Matching between brem and conversions

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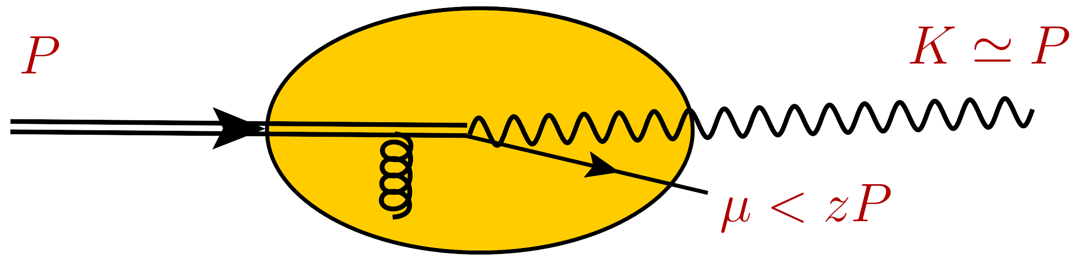
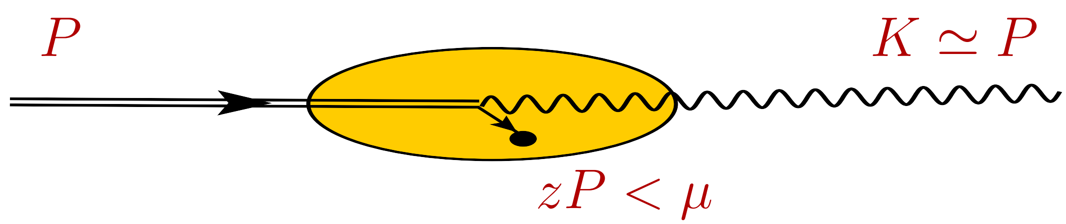


is not physically distinct from the conversion process



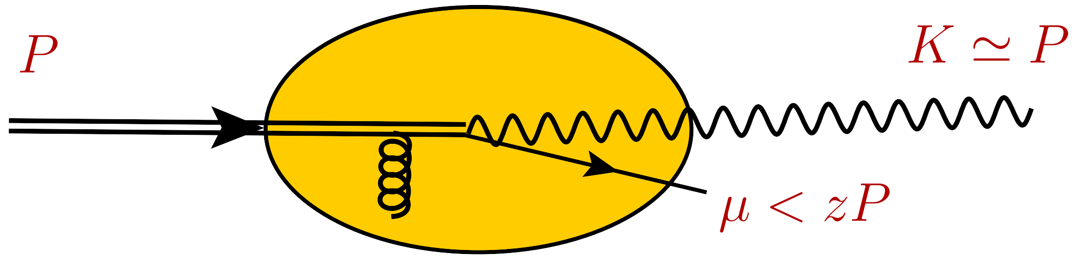
Separately both processes depend on the separation scale, $\mu \sim gT$, but ...

the μ dep. cancels when both rates are included

- The LO small-θ and large-θ brem rates depend linearly and logarithmically on an infrared separation scale, μ.

The NLO conversion rate will depend on a UV cutoff μ and cancels this dependence

Brem rates with a soft quark



- Small angle brem

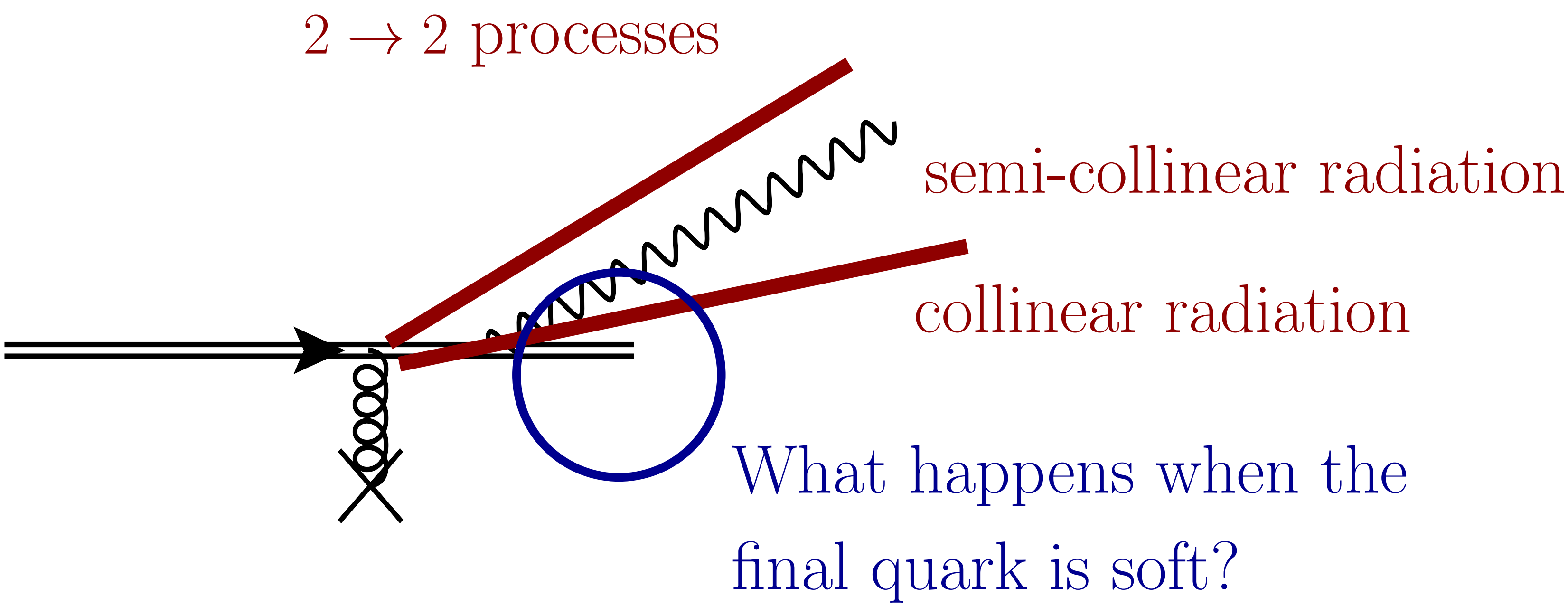
$$2k \frac{\mathrm{d}\Gamma}{d^3k}\bigg|_{zP>\mu} = \text{Finite } - \text{ linear IR dependence } \mu$$

- Wide angle brem

$$2k \frac{\mathrm{d}\Gamma}{d^3k}\bigg|_{zP>\mu} \propto \text{Log IR dependence on } \mu \; + \; \text{Finite}$$

The conversion rate should cancel this dependence on μ

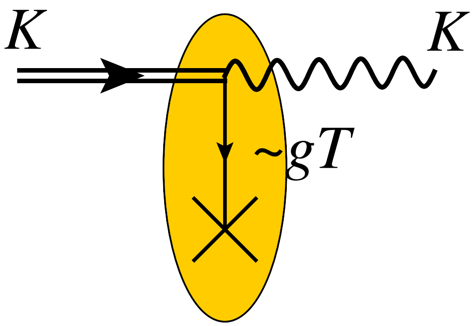
Matching between brem and conversions



When the quark becomes soft need to worry about conversions.

Computing the conversion rate with sum-rules (LO):

(see also Bodeker)



$$2k(2\pi)^3 \frac{\mathrm{d}\Gamma_{\mathrm{cnvrt}}}{\mathrm{d}^3 k} \propto e^2 n_F(k)\; \hat{q}_{\mathrm{cnvrt}}(\mu)$$

- $\hat{q}_{\mathrm{cnvrt}}$ is the quark version of $\hat{q}$

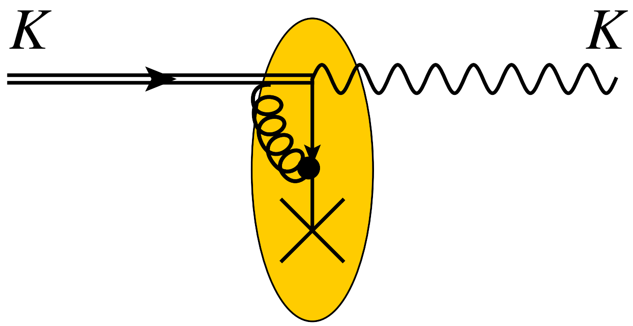
$$\hat{q}_{\mathrm{cnvrt}}(\mu_\perp) = \int^{\sim\mu} \frac{\mathrm{d}^2\mathbf{p}_T}{(2\pi)^2} \underbrace{\int_{-\mu}^{\mu} \frac{\mathrm{d}p^z}{2\pi} \,\mathrm{Tr}\left[\gamma_+ S^<(\omega,\mathbf{p})\right]_{\omega=p^z}}_{\text{unintegrated soft quark parton-dist fcn of QGP}}$$

$$= \int^{\mu} \frac{d^2\mathbf{p}_T}{(2\pi)^2} \frac{m_\infty^2}{p_T^2 + m_\infty^2}$$

where

$$S_R(X) = \left\langle \psi(X) e^{ig\int_0^X dx^\mu A_\mu} \bar{\psi}(0) \right\rangle$$

Computing the conversion rate at NLO with sum-rules:



$$2k(2\pi)^3 \frac{\mathrm{d}\Gamma_{\mathrm{cnvrt}}}{\mathrm{d}^3 k} \propto e^2 \, n_F(k) \, \hat{q}_{\mathrm{cnvrt}}(\mu)$$

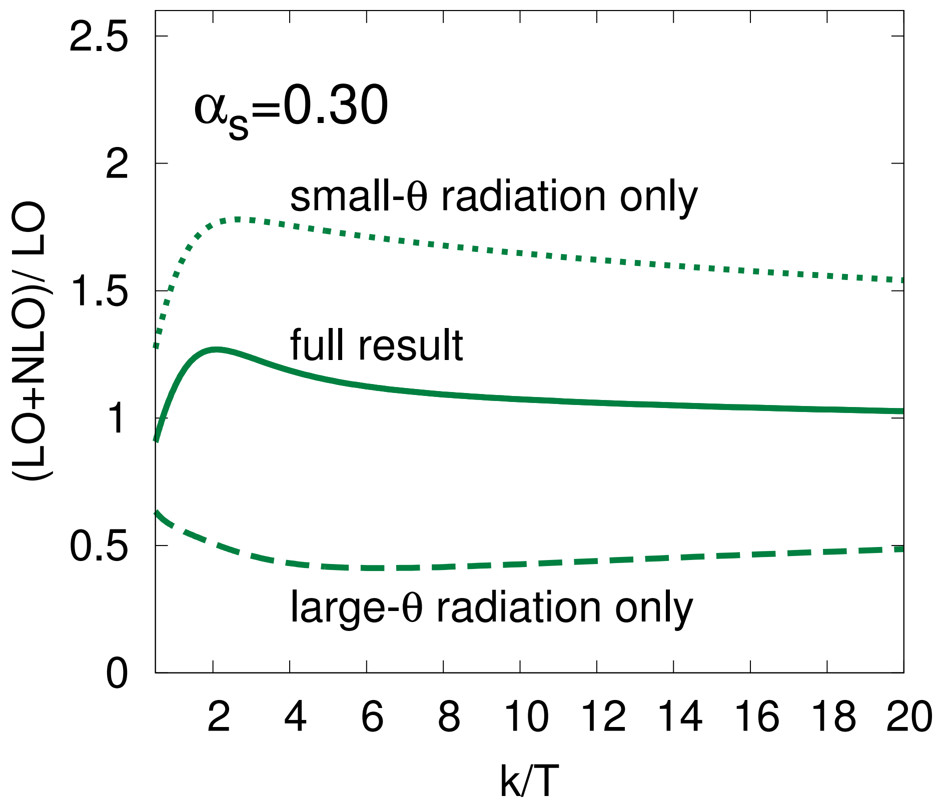
- At NLO we have only to replace $m_\infty^2 \to m_\infty^2 + \delta m_\infty^2$

$$\hat{q}_{\mathrm{cnvrt}} = \underbrace{\int^{\mu} \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \frac{m_\infty^2 + \delta m_\infty^2}{p_T^2 + m_\infty^2 + \delta m_\infty^2}}_{\text{finite + UV logarithmic divergence in } \mu} + \underbrace{\# g^2 \mu}_{\text{linear UV divergence in } \mu}$$

The UV divergences of conversion rate match with the IR divergences of large and small angle brem giving a finite answer

Conclusion

- The result again



- All of the soft sector buried into a few coefficients, $C[q_\perp, \delta E]$ and $\hat{q}_{\rm cnvrt}$
  - Can we compute these non-perturbatively ?
  - Can constrain experimentally with medium-energy jets $E \simeq 30 \div 50\,\mathrm{Gev}.$

Many things can be computed next (e.g. shear viscosity and e-loss)