HBT Tutorial

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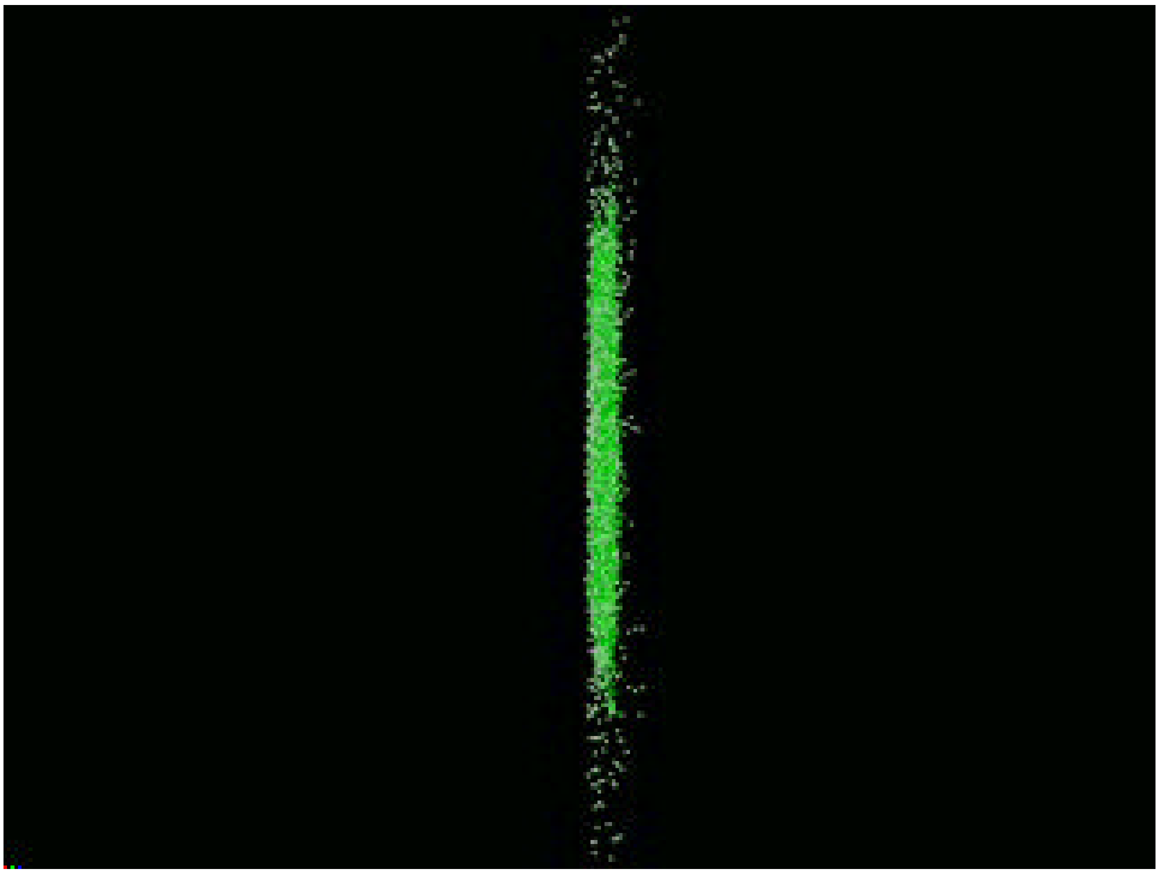
A Motivation

General Observations

- **Heavy Ion Collisions requires understanding particle distributions in coordinate *and* momentum space**



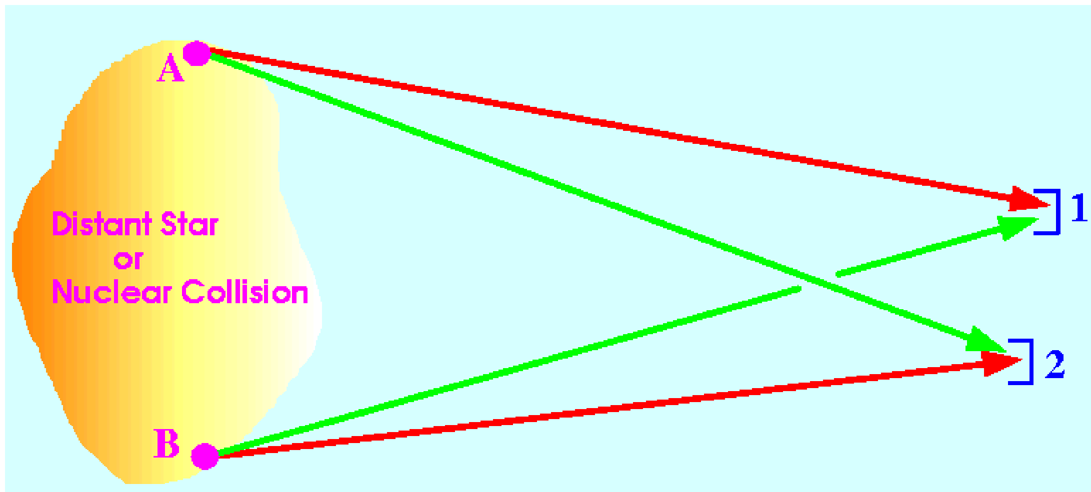
- **The complexity of these collisions forces us to employ all tools at our disposal:**
  - **Coalescence**
  - **Reaction rates ~ $n_1\ n_2\ s_{12}\ v_{12}$**
  - **Like-particle correlations**
  - **Unlike particle correlations**
- **~All such techniques measure the distribution of the final strong interaction of the various species**

Some Abbreviations

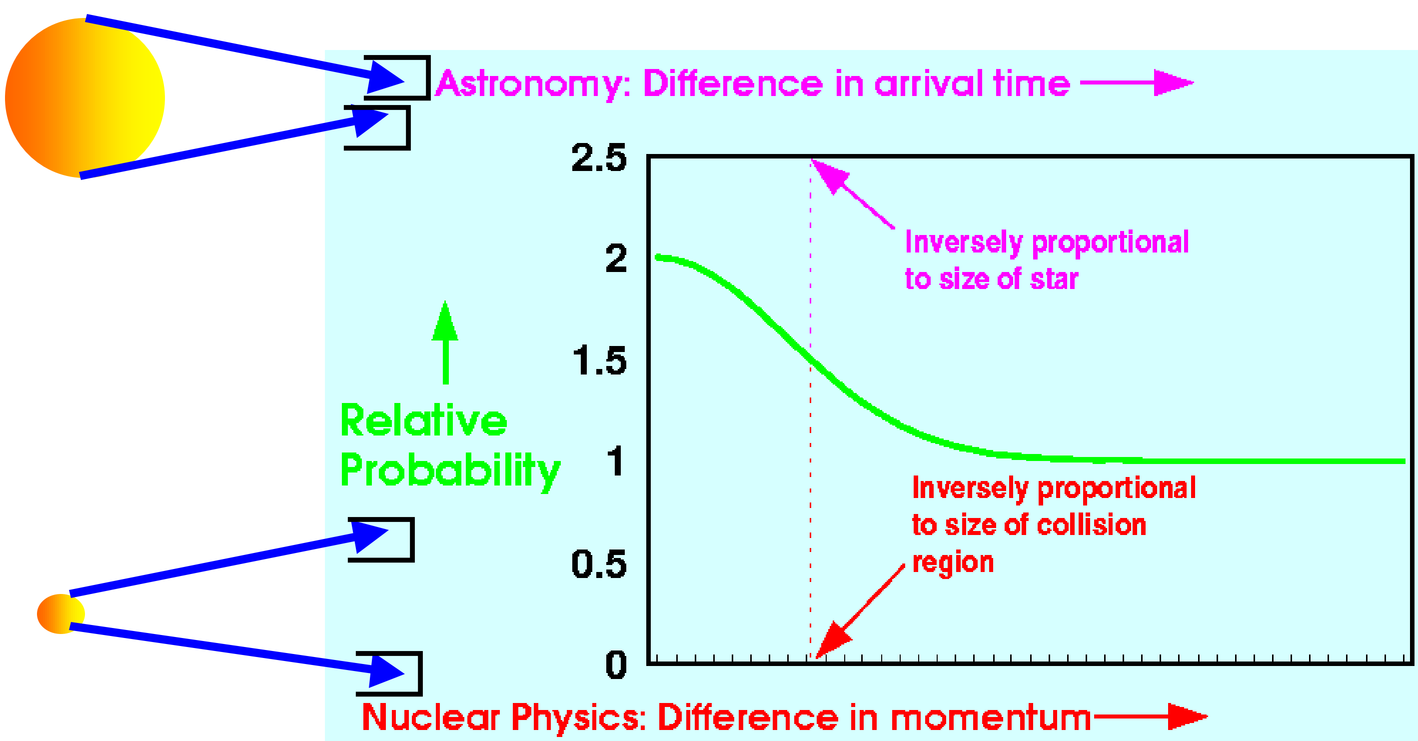
- **GGLP**
  - Goldhaber, Goldhaber, Lee and Pais (1960)
  - First application of “intensity interferometry” in particle physics
- **HBT**
  - Hanbury-Brown and Twiss (1950’s)
  - Revolutionary development of intensity interferometry in astronomy
- **B-E**
  - Bose-Einstein (1920’s)
  - Role of Bose statistics in fluctuations
- **G**
  - General
  - Rating of this talk
    - That is, a broad overview for non-specialists
    - No attempt at total coverage of the field

An Invocation

- There is a connection (not even an analogy) between stellar HBT measurements and “HBT” in heavy ion collisions:

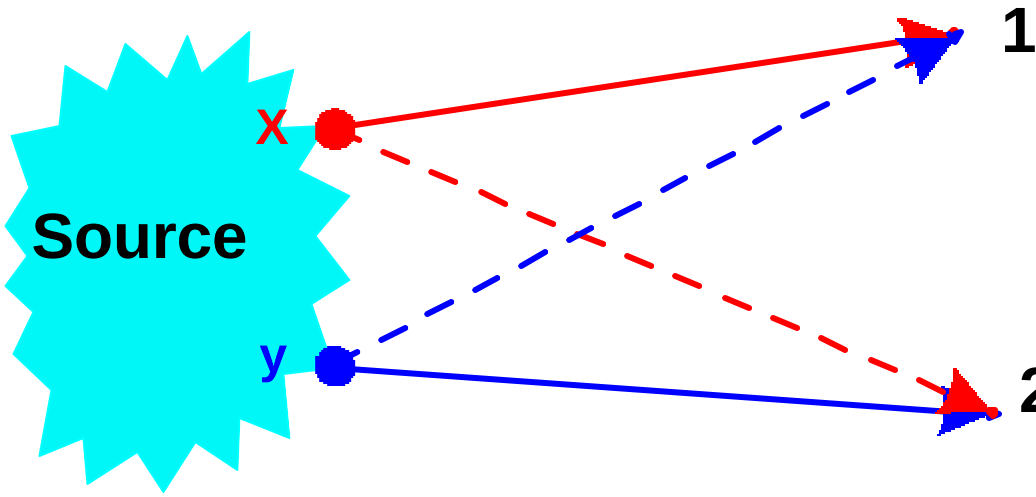


- In particular, the geometry of a collision is not a scale model of imaging a star



An Inspiration

- The usual "derivation" is really more inspiration:



$$A_{12} = \frac{1}{\sqrt{2}}\left[e^{ip_1\cdot(r_1-x)}e^{ip_2\cdot(r_2-y)} + e^{ip_1\cdot(r_1-y)}e^{ip_2\cdot(r_2-x)}\right]$$

so that

$$\mathcal{P}_{12} = \int d^4x\, d^4y\, |A_{12}|^2 \rho(x)\rho(y) = 1 + |\tilde{\rho}(q)|^2 \equiv C_2(q)$$

- Neglects
  - Momentum dependence of source
  - Quantum mechanics *up to* x and y
  - Final State Interactions *after* x and y
- Nonetheless
  - C2(q) contains shape information
  - True component-by-component in q

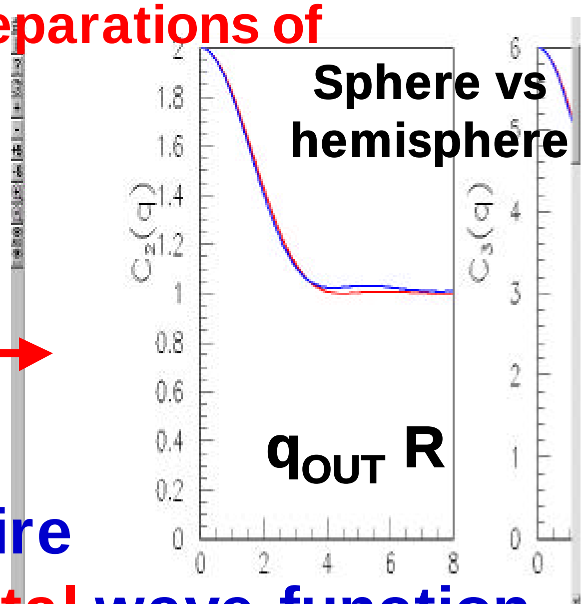
Two Clarifications

- Note the following property of F.T.'s:
  for $A_{12} = \frac{1}{\sqrt{2}}[e^{ip_1\cdot(r_1-x)}e^{ip_2\cdot(r_2-y)} + e^{ip_1\cdot(r_1-y)}e^{ip_2\cdot(r_2-x)}]$

  so that

  $$\mathcal{P}_{12} = \int d^4x\, d^4y\, |A_{12}|^2 \rho(x)\rho(y) = 1 + |\tilde{\rho}(q)|^2 \equiv C_2(q)$$

  we have $|A_{12}|^2 = 1 + \cos(p_1-p_2)\,(x-y)$
  - That is, *squared* Fourier Transforms measure (only) *relative* separations of source coordinates
  - Very different sources $\rho(x)$ can give very similar distributions in relative separation →



- Fermions of course require anti-symmetrization of total wave-function

$$|\Psi|^2 = \frac{1}{4}|\varphi_S(1,2) + \varphi_S(2,1)|^2 + \frac{3}{4}|\varphi_T(1,2) - \varphi_T(2,1)|^2$$

(Singlet) (Triplet)

- But note spatial wave-functions must be strong + (sometimes) Coulomb waves

A Qualification

- **Naively:**

$$C_2(q) = 1 + |\rho(q)|^2 \,,\quad \rho(q) \equiv \int \rho(x) e^{iqx}\, d^4x$$

$$q \equiv p_1 - p_2 = (E_1 - E_2, \vec{p}_1 - \vec{p}_2) \equiv (q_0, \vec{q})$$

$$\Rightarrow q \cdot x = (q_0 t - \vec{q} \cdot \vec{r})$$

Fourier Transform provides one time and three space extensions of source

BUT:

$$q_0 \equiv E_1 - E_2 = \frac{E_1^{\,2} - E_2^{\,2}}{E_1 + E_2} = \frac{\vec{p}_1^{\,2} - \vec{p}_2^{\,2}}{E_1 + E_2}$$

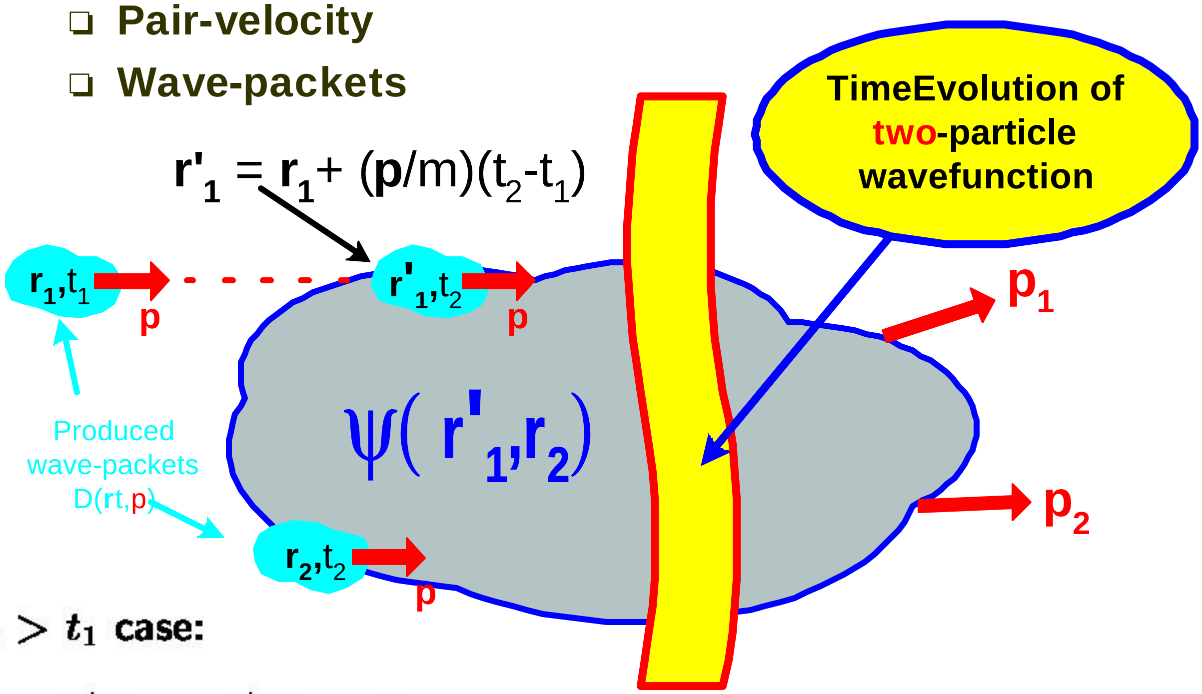
$$= (\vec{p}_1 - \vec{p}_2) \cdot \frac{\vec{p}_1 + \vec{p}_2}{E_1 + E_2} \equiv \vec{q} \cdot \vec{V}_{PAIR}$$

- **That is, q_0 is not an independent quantity in the F.T.**
  (e.g., note that $q_0 < |\vec{q}|$)

  Fourier Transform has support in only half of the q_0-q plane (??)

A Revelation

- **Inspired ansatz by Koonin for treatment of pp correlations** (Phys. Lett. 70B, 43, 1977)
  - Correctly anticipates role of
    - Pair-velocity
    - Wave-packets



For $t_2 > t_1$ case:

$$\frac{dn_{(1>2)}}{d\vec{p}_1 d\vec{p}_2} = \int_{t_1}^{+\infty} dt_2 \int_{-\infty}^{+\infty} dt_1 \int d\vec{r}_1 d\vec{r}_2 D(\vec{r}_1 t_1, \vec{p}) D(\vec{r}_2 t_2, \vec{p}) |\Psi_{\vec{p}_1\vec{p}_2}(\vec{r}_{1'}, \vec{r}_2)$$

where

$$\vec{r}_{1'} \equiv \vec{r}_1 + \frac{\vec{p}}{m}(t_2 - t_1)$$

and

$D(\vec{r}t, \vec{p})$ is space-time distribution of final scatterings that produces a particle of momentum $\vec{p}$.

and $\Psi_{\vec{p}_1\vec{p}_2}(\vec{r}_1, \vec{r}_2)$ is the two-particle wave-function.

 (Considering $t_2 < t_1$ case simply extends time integrals to all times.)

A re-formulation

- **Return to "naïve" formulation in terms of Fourier transform:**

$$C_2(q) = 1 + |\rho(q)|^2 \,,\; \rho(q) \equiv \int \rho(x) e^{iqx}\, d^4x$$

$$q_0 = \vec{q} \cdot \vec{V}_{PAIR}$$

$$\Rightarrow\; q \cdot x = (\vec{q} \cdot \vec{V}_{PAIR} t - \vec{q} \cdot \vec{r}) = -\vec{q} \cdot (\vec{r} - \vec{V}_{PAIR} t)$$

$$\Rightarrow\; \rho(q) = \rho(\vec{q}; \vec{V}_{PAIR}) = \int \rho(x) e^{-i\vec{q} \cdot (\vec{r} - \vec{V}_{PAIR} t)}\, d^4x$$

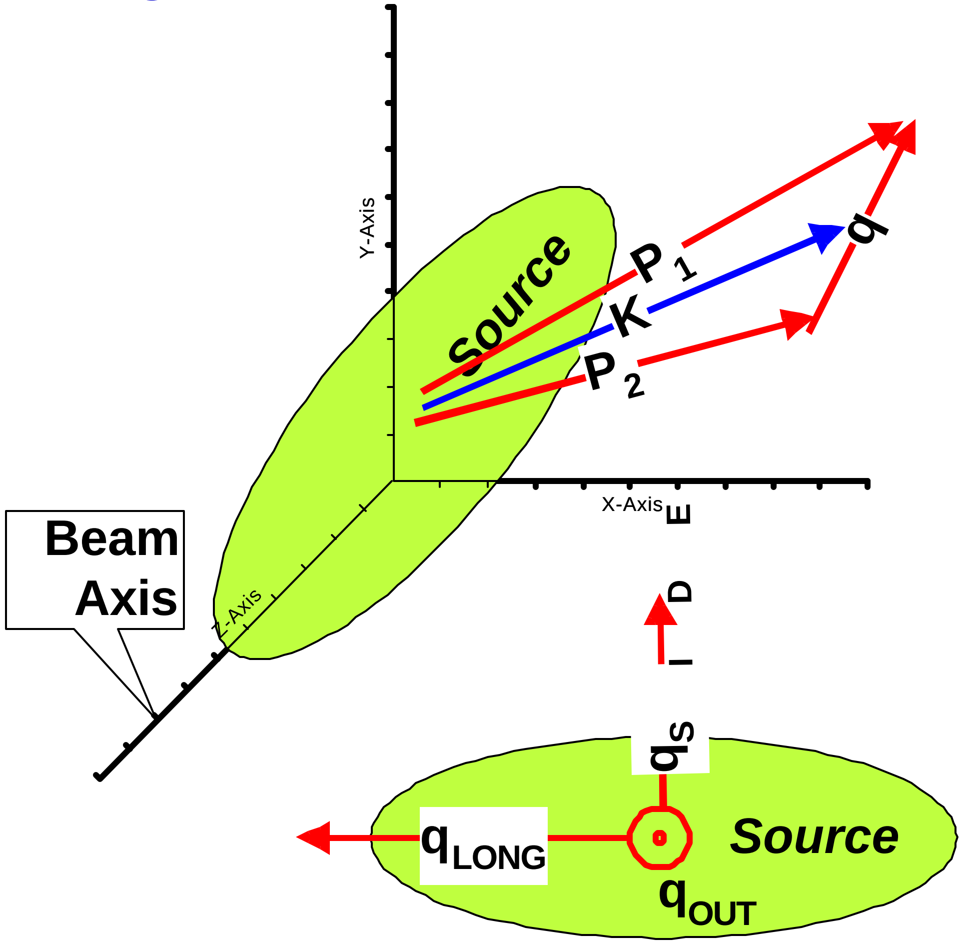
- **So Fourier transform (+ two-particle kinematics) already provides conclusion:**

$$C_2(q) = 1 + |\rho(q)|^2 = C_2(\vec{q}\,; \vec{V}_{PAIR}) \;\Leftrightarrow\; < (\vec{r} - \vec{V}_{PAIR} t)^2 >$$

- **Additional dynamic effects (expansion, thermal source, …) will also lead to systematic dependence of extracted "radii" on V_{PAIR} (or m_T or k_T or …)**

An Education

- Quick guide to conventional Q variables



- There are other decompositions...
- Note vector components applicable only to plane waves, e.g.,
  - pp
  - KK
- pp Dominated by s-wave final state interaction
  - Function of |q| only

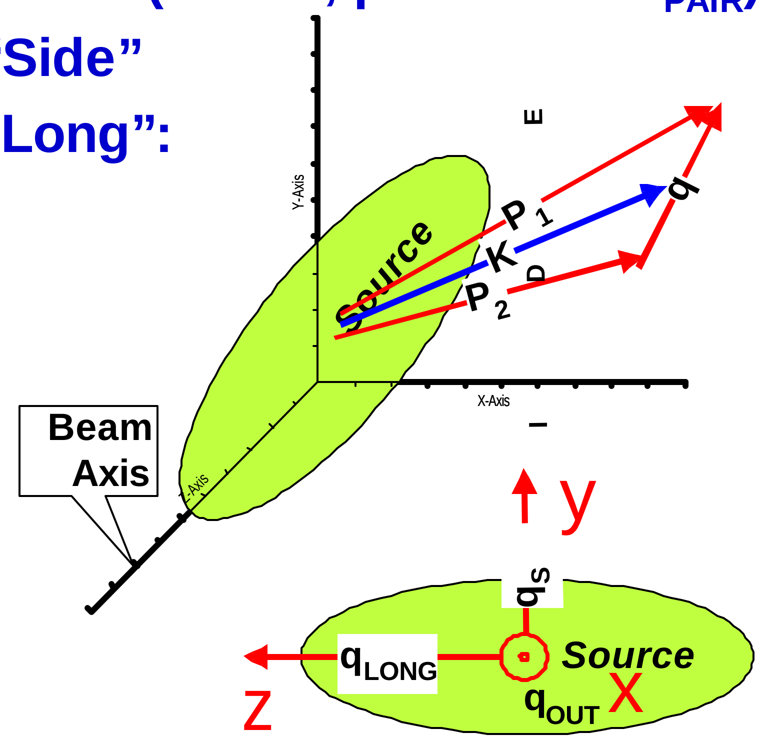
An Orientation

- Simplest possible (yet still very useful) case:

$$\rho(\vec{r},t) \sim \exp[-\frac{1}{2}(\frac{x^2}{R_x^2}+\frac{y^2}{R_y^2}+\frac{z^2}{R_z^2} \quad +\frac{t^2}{\tau^2})]$$

$$\rho(\vec{q};\vec{V}_{PAIR}) = \exp[-\frac{1}{2}(q_x^2 R_x^2 + q_y^2 R_y^2 + q_z^2 R_z^2 + (\vec{q}\cdot\vec{V}_{PAIR})^2\tau^2)]_{)]}$$

- Choose frame so that
  - x = "Out" (that is, parallel to V_{PAIR})
  - y = "Side"
  - z = "Long":



$$(q_x^2 R_x^2 + q_y^2 R_y^2 + q_z^2 R_z^2 + (\vec{q}\cdot\vec{V}_{PAIR})^2\tau^2)$$

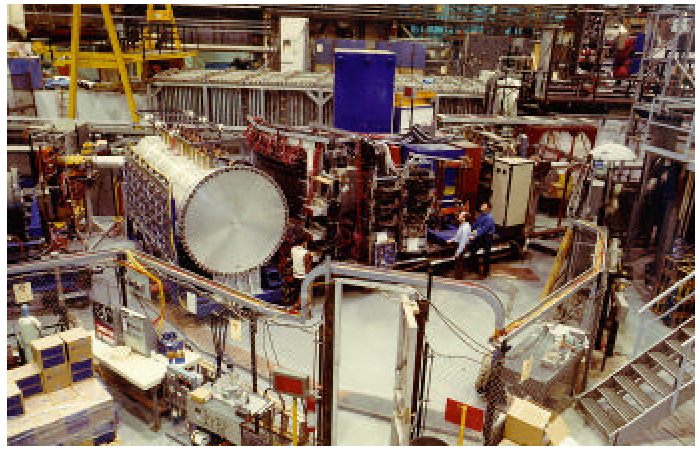
$$= q_{SIDE}^2 R_{SIDE}^2 + q_{LONG}^2 R_{LONG}^2 + q_{OUT}^2 R_{OUT}^2 \quad \textit{with} \quad R_{OUT}^2 = R_x^2 + V_{PAIR}^2\tau^2$$

$$\textit{Symmetry} \quad "\Rightarrow" \quad R_y \approx R_x \Rightarrow R_{OUT}^2 = R_x^2 + V_{PAIR}^2\tau^2 > R_{SIDE}^2 = R_y^2$$

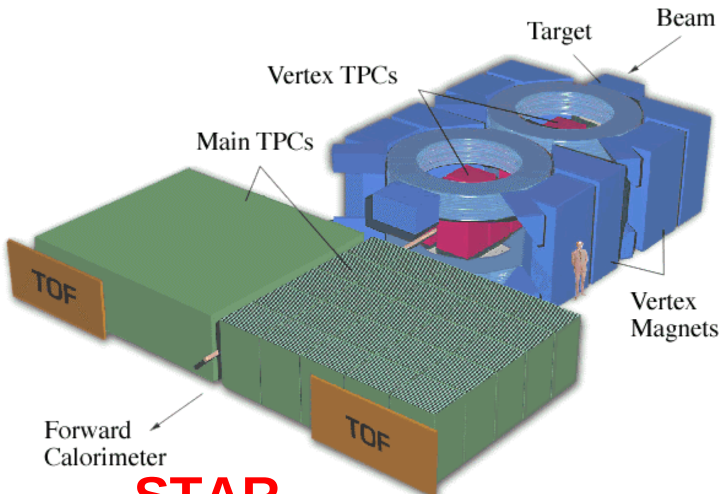
The Investigations

Most experiments with charged tracking and particle identification have HBT results:

- **BNL AGS:**
  - E859, E866
  - E877
  - E895



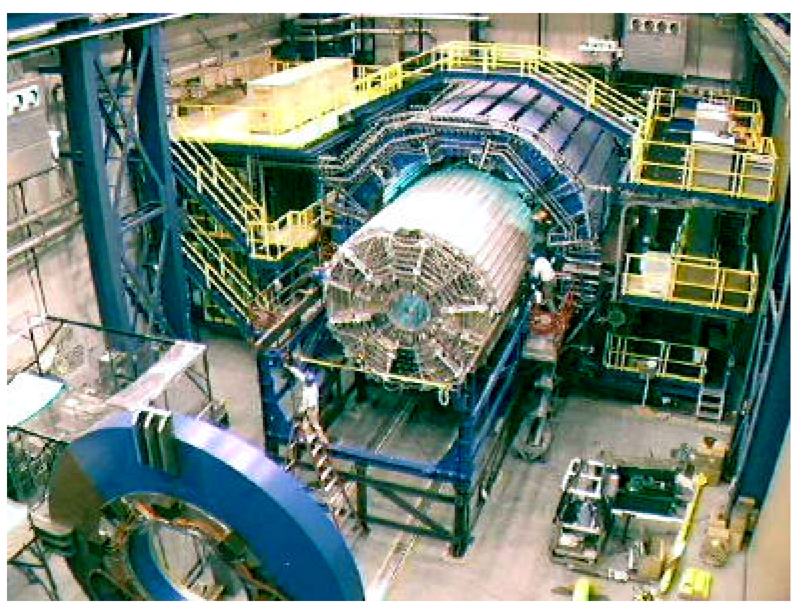
- **CERN SPS:**
  - NA44
  - NA49
  - WA98



- **RHIC**

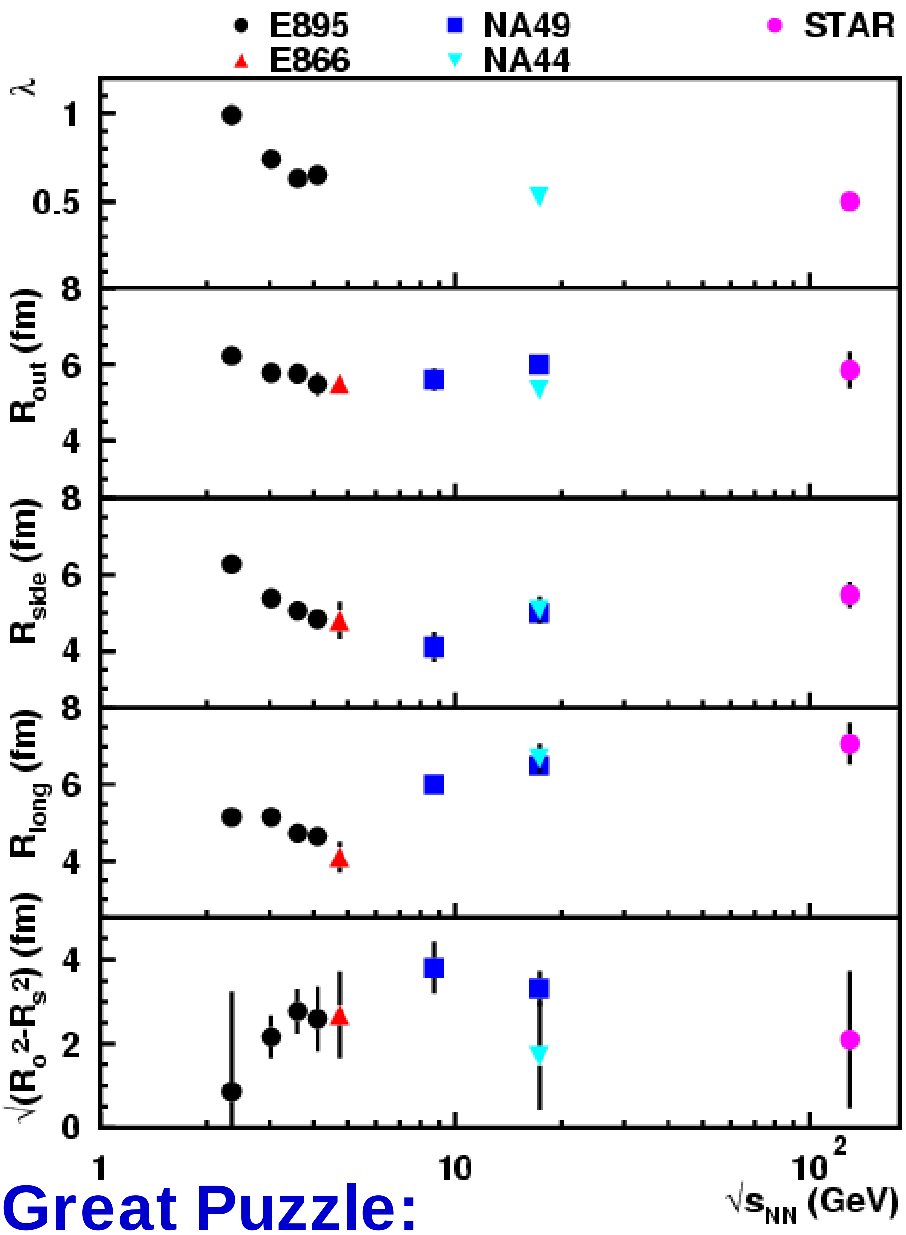
PHENIX

STAR



The Instigation

- **This (Side,Long,Out) decomposition has been applied to nucleus-nucleus collisions from E_{CM} ~1 to 200 GeV**



- **The Great Puzzle:**
- ***Why so little variation with* E_{CM} ?**
- ***Why are lifetimes* (τ) ~ 0 ??**
  (especially *at RHIC*)

Provocation

- **Solution to the Great Puzzle**
  - **The data are wrong**
  - **The theories are wrong**

- **Better said:**
  - **A better understanding of systematic errors in experimental data is required:**
    - Definition of C_2
    - Coulomb corrections
    - Role of I, dependence of R on same
    - Contributions from resonances

  (especially when comparing between different experiments)

  - **A better understanding of theoretical assumptions and uncertainties is required:**
    - Quantum corrections
    - Resonance contributions
    - Lorentz effects
    - Modeling of expansion, velocity profiles

A Complication

- **Our favorite (charged) bosons never have pure plane-wave states**
  - **Even our ideal-case Fourier transform becomes a "Coulomb transform"**
  - **This particular case is easy to treat analytically via "Gamow" or "Coulomb" corrections:**

- Exact solution to two-body Coulomb is

$$\Psi_k(\vec{r}) = e^{ikz}{}_1F_1(-i\eta; 1; ik(r-z))$$

where

- $\eta = \frac{\mu e^2}{\hbar^2 k}$ (principal quantum number)
- ${}_1F_1(a; c; x)$ is the confluent hypergeometric function

- Effect on two-particle wave-function given in terms of the usual Gamow function $G_0(k)$ is given by

$$G_0(k) \equiv \frac{|\Psi_k(\vec{r}=0)|^2}{|\Psi_k(\vec{r}\to\infty)|^2} = \frac{2\pi\eta}{e^{2\pi\eta}-1}$$

- The effect of an extended source distribution $\rho(\vec{r})$ is given by

$$G_\rho(k) \equiv \frac{\int |\Psi_k(\vec{r})|^2 \rho(\vec{r})\, d\vec{r}}{|\Psi_k(\vec{r}\to\infty)|^2} \sim G_0(k)\cdot\{1 + 2\frac{\langle r\rangle}{a_0} + \cdots\}$$

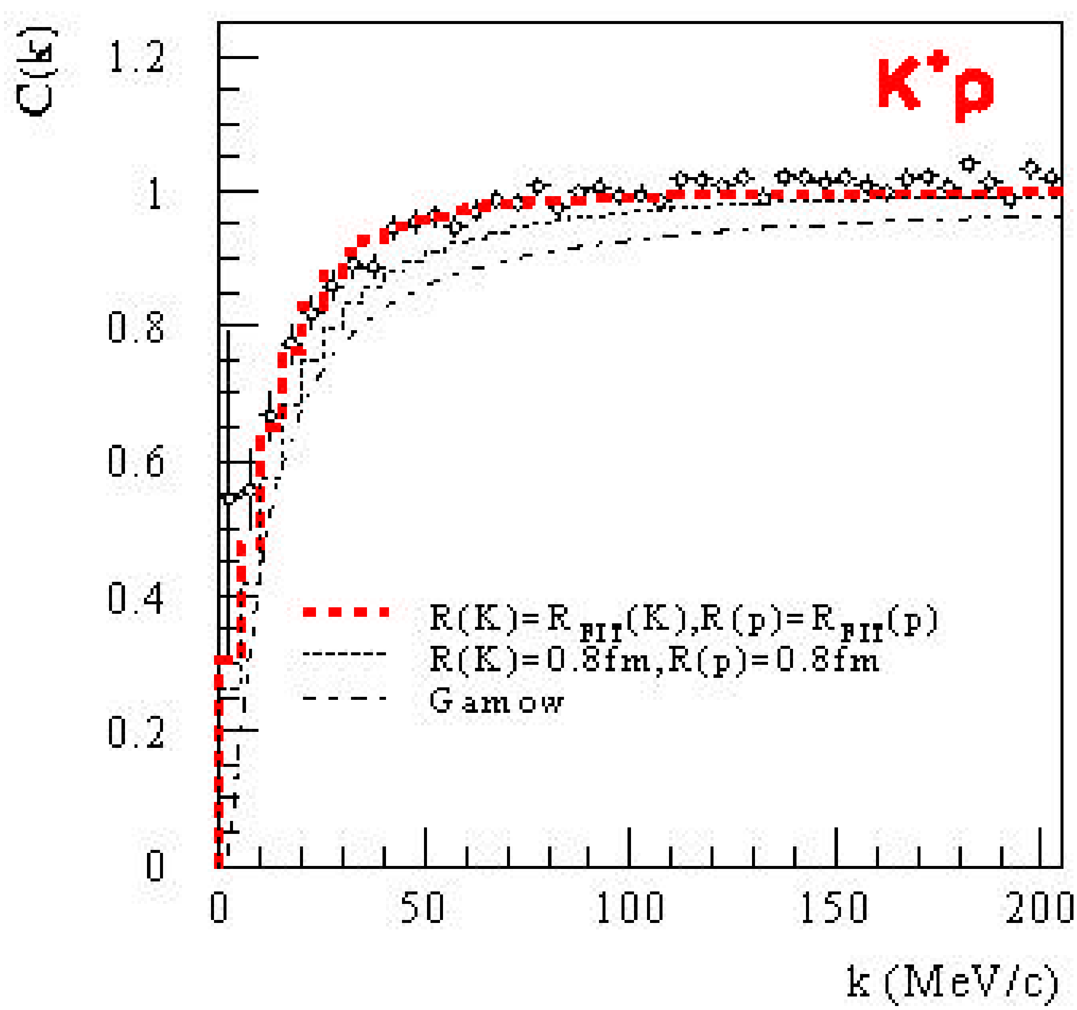
where $a_0 \equiv \frac{\hbar^2}{\mu e^2}$ is the Bohr radius for the system.

$\Rightarrow$ We can pick the Bohr radius of choice:

System	a_o (fm)
$\pi\pi$	387
πK	249
πp	224
KK	108
Kp	83

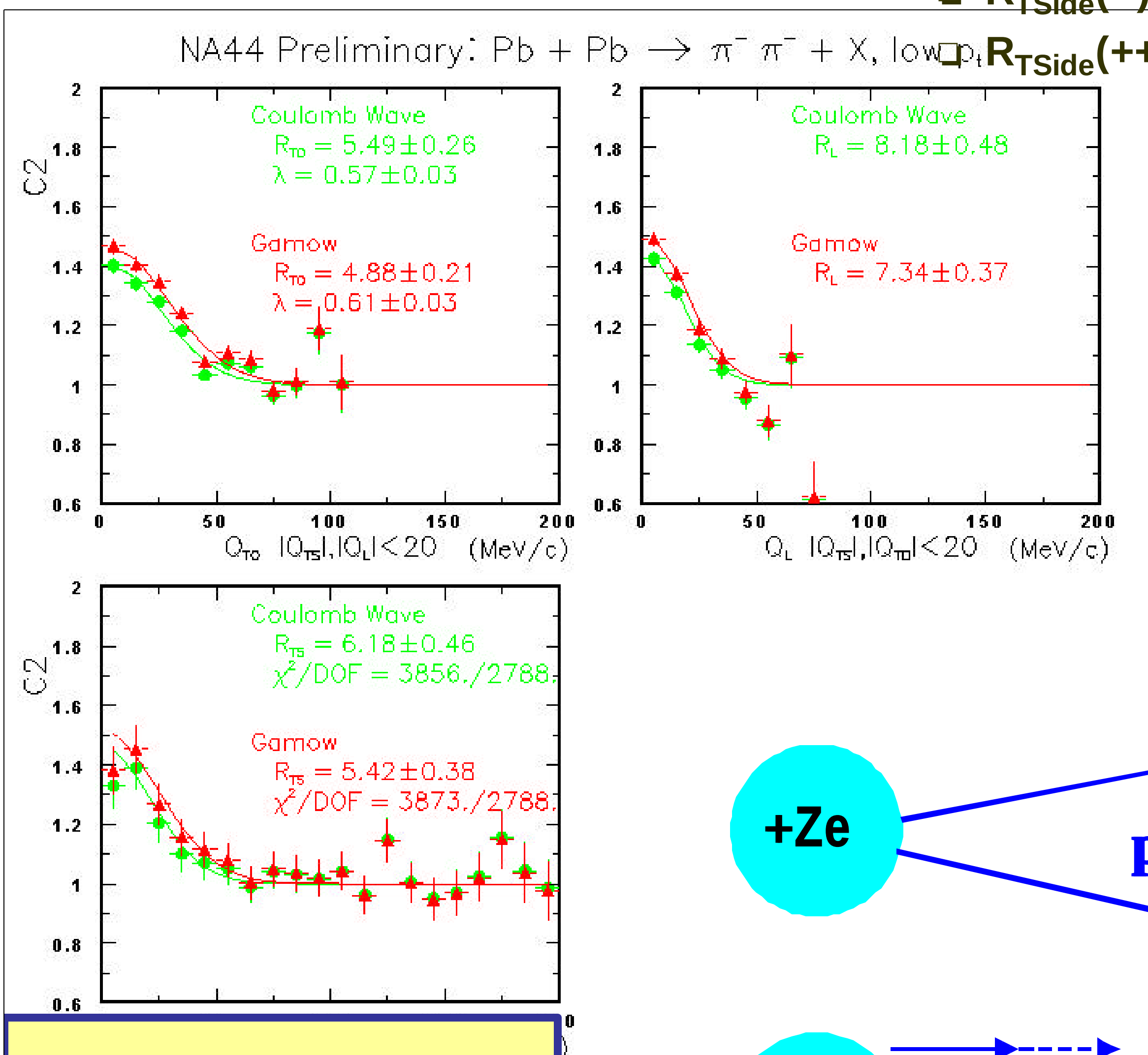
Sensation to Calibration

- **Circa 1991 considerable emphasis placed on getting K-K correlations**
- **In 1997, use K+'s from the same data set to calibrate the finite-size Coulomb corrections:**
  - **Note data extend well inside classical turning point**
    - **Futility of semi-classical approximations to Coulomb solutions**

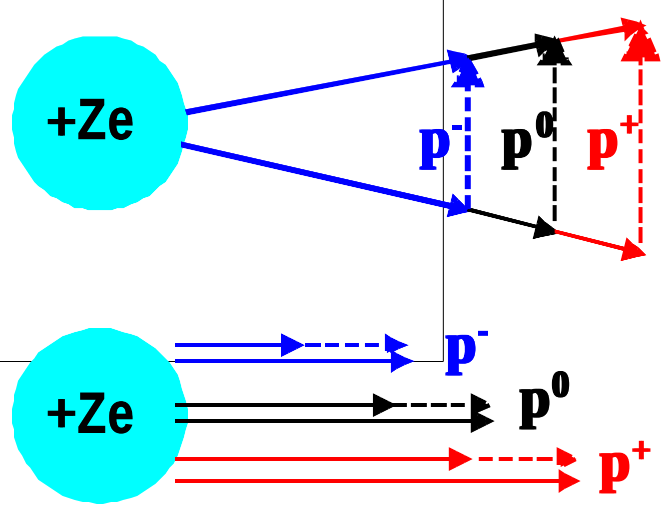


An Indication

- **What about all the other (positive) charge in the system?**
- **Some indication from both AGS and SPS that $R_{Side}(--) > R_{Side}(++)$**
  - **E877 (AGS): $R_{Side}(--) \sim (1.5 +/- 0.4)*R_{Side}(++)$**
  - **NA44 (SPS): Same pattern, and also see**
    - **$R_{TSide}(--) > R_{TOut}(--)$**
    - **$R_{TSide}(++) < R_{TOut}(++)$**

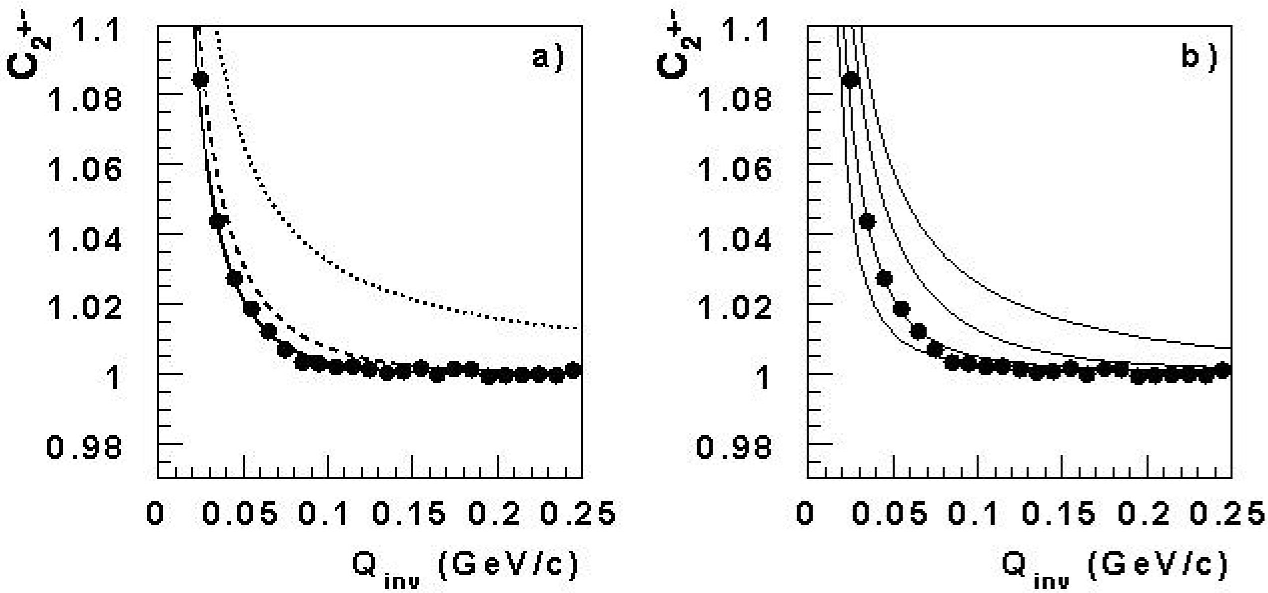


Note external Coulomb
- *Systematic in R_{SIDE}*
- *Cancels in R_{OUT}*



An Evaluation

- **NA49 has used nice $\pi^+\pi^-$ data to derive screening correction:**



Q. Is Coulomb correction screened by other charges in the system?

A. Perhaps, but …

- **Estimates of static screening lengths are 7-20 fm**
  - ➡ **Not a dominant effect (further reduced by expansion)**
- **Entangles (well-known) two-particle correction with poorly-known external Coulomb correction**
- **Using vintage result I guess-timate external Coulomb shifts in single-particle momenta of**
  - ❑ **6% at $p_T \sim 400$ MeV/c**
  - ❑ **15% at $p_T \sim 100$ MeV/c**

  (M. Gyulassy and S.K. Kaufmann, Nucl. Phys. A362, 503 (1981))

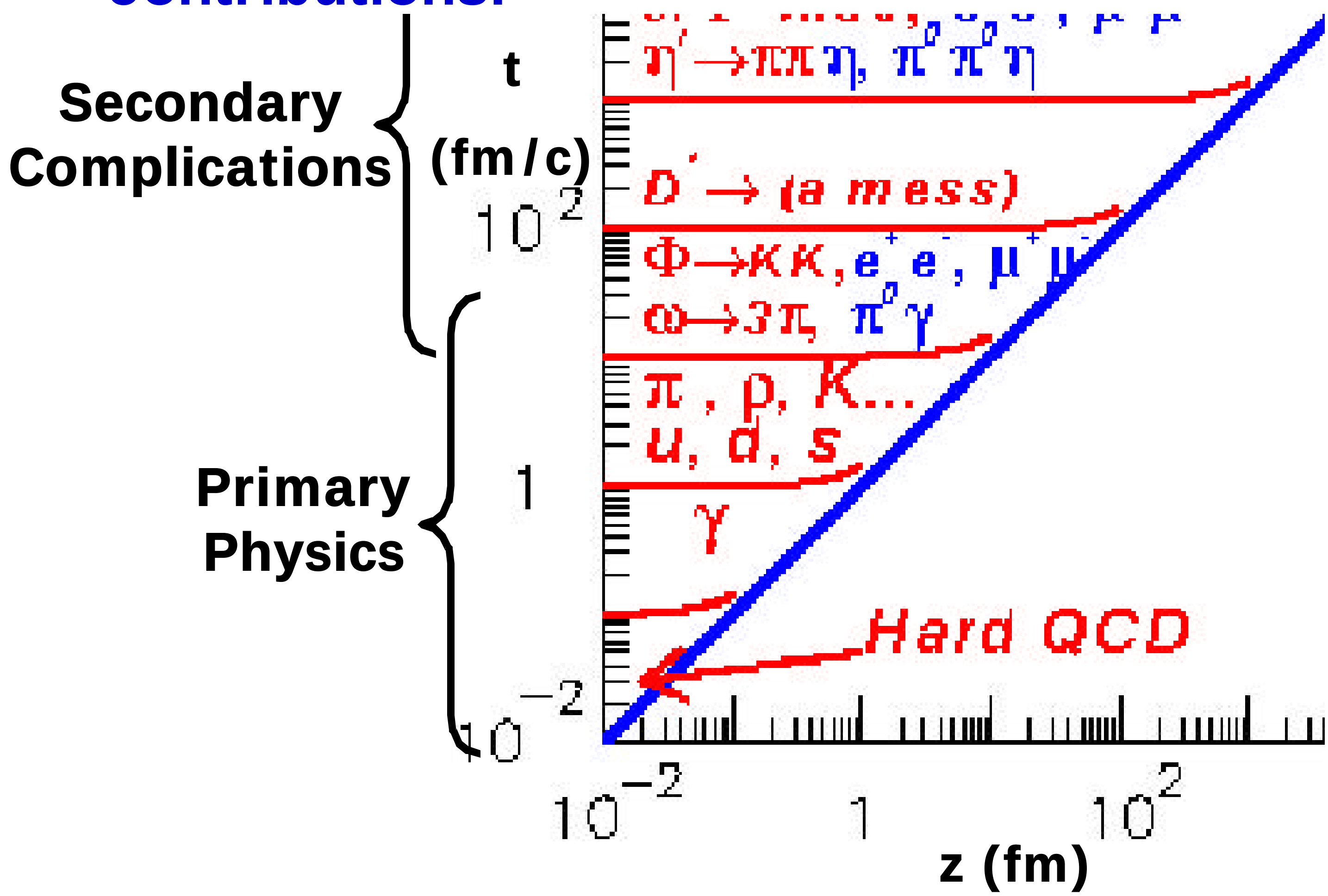
A Recantation

- Previous statement that $C_2(q) = 1 + |\rho(q)|^2$ a little bit untrue…

  $\rho(q) \equiv \int \rho(x) e^{iqx}\, d^4x \Rightarrow \rho(q=0) = 1$

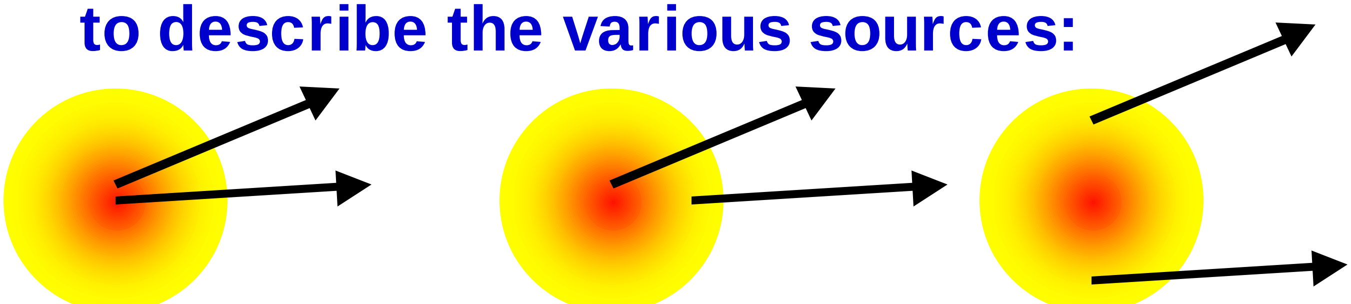
  $\Rightarrow C_2(q=0) = 2$

  versus observed value of 1.1-1.5
- This is parameterized away via $C_2(q) = 1 + \lambda\, |\rho(q)|^2$, λ ~0.1-0.5
- “Understood” via assumed resonance contributions:



A Degradation

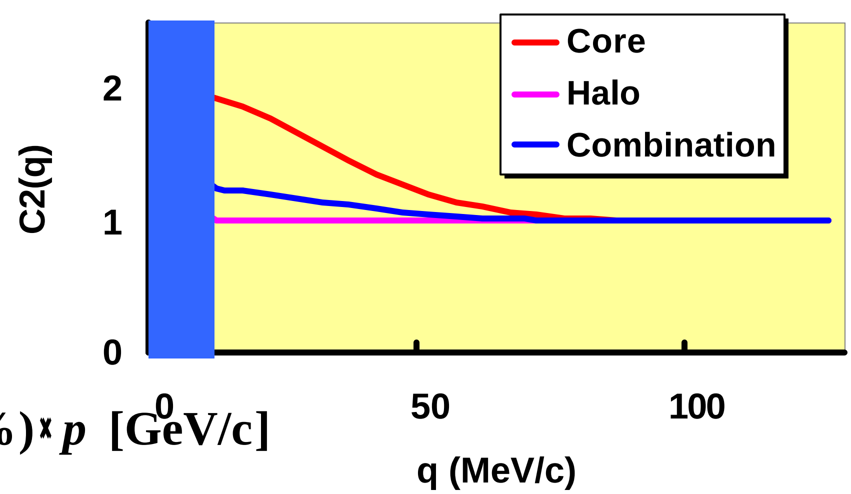
- How do resonances affect λ ?
- By creating a “halo” around the source “core”:
- Some culprits:
  - r pp (G = 150 MeV S ~ 1.3 fm)
  - K* Kp (G = 50 MeV S ~ 4 fm)
  - w ppp (G = 8.4 MeV S ~ 20 fm)
  - h’ hpp (G = 0.2 MeV S ~ 200 fm)
  - (and many more…)
- Effect: C_2 develops structure near q=0 to describe the various sources:



$f_{CORE} f_{CORE}$ $2(1-f_{CORE})f_{CORE}$ $(1-f_{CORE})(1-f_{CORE})$

Momentum resolution

$$\frac{dp}{p} = (\sim 1\%) \oplus (\sim 1\%) \cdot p \ [\text{GeV/c}]$$



A Reformulation

Quick review of Wigner functions:

- Motivate via density matrix $\rho \equiv \sum_i P(i)\,|\,i><i\,|$

  where *P(i)* are probabilities for i-th something.
- Probability *P* (K) to have momentum K is given by *P* (K) = <K| ρ|K>

Q. Is there a function <R|ρ|K> ≡ ρ(R,K)
with the (classical-looking) property

$$P(K) = \int <R\,|\,\rho\,|\,K> dR \quad ?$$

A. Yes:

$$\int <R\,|\,\rho\,|\,K> dR \overset{?}{=} P(K) = <K\,|\,\rho\,|\,K>$$

$$= \int\int <K\,|\,x><x\,|\,\rho\,|\,y><y\,|\,K> dx\,dy$$

Transform to relative coordinates

$$= \int\int <K\,|\,R+\tfrac{1}{2}r><R+\tfrac{1}{2}r\,|\,\rho\,|\,R-\tfrac{1}{2}r><R-\tfrac{1}{2}r\,|\,K> dr\,dR$$

Plane wave $e^{iK(R-r/2)}$

$$= \int \left\{ \int <R+\tfrac{1}{2}r\,|\,\rho\,|\,R-\tfrac{1}{2}r> e^{iKr} dr \right\} dR$$

$$\equiv \int <R\,|\,\rho\,|\,K> dR = \int \rho_W(R,K)\,dR$$

- Show: $P(K) = \int \rho_W(R,K)\,dR$

$$P(R) = \int \rho_W(R,K)\frac{dK}{2\pi\hbar}$$

$$<F> = \int F(R)\,\rho_W(R,K)\,dR\frac{dK}{2\pi\hbar}$$

A Derivation

- **Make QM look ~ "classical":**

Write the single-particle inclusive in terms of

- Evolution operator $U(t)$
- Sum over discrete probabilities $\mathcal{P}(i)$:

$$\begin{aligned}
\frac{dn}{d\vec{p}} &= \sum_i \mathcal{P}(i)\,|\int d^4x \langle p|U(t\to\infty|x\rangle\langle x|i\rangle|^2 \\
&= \int d^4x\, d^4y \rho(x,y)\Pi^*(p;x,y) \quad \text{where} \\
\rho(x,y) &\equiv \sum_i \mathcal{P}(i)\langle x|i\rangle\langle i|y\rangle \equiv \langle x|\rho|y\rangle \\
\Pi^*(p;x;y) &\equiv \langle y|U^\dagger|p\rangle\langle p|U|x\rangle = e^{ip(x-y)} \quad \text{(plane waves)}
\end{aligned}$$

In terms of the Wigner transforms of ρ and Π :

$$\frac{dn}{d\vec{p}} = \int d^4r\, \frac{d^4k}{(2\pi)^4}\, \rho_W(r,k)\Pi^*_W(p;r,k) \quad .$$

Repeat for two-particles:

- Remember to symmetrize $\Pi^*(p;x,x';yy')$
- Assume $\rho(xy;x'y') \approx \rho(x,x')\cdot\rho(y,y') \Rightarrow$

$$\frac{dn}{d\vec{p}_1 d\vec{p}_2} = \int d^4x\, d^4y\, [\rho_W(x,p_1)\rho_W(y,p_2) + \rho_W(x,K)\rho_W(y,K)\cos q(x-y)]$$

where the four-vectors q and K are defined by

- $q \equiv p_2 - p_1$
- $K = \frac{1}{2}(\,E_1 + E_2,\ \vec{p}_1 + \vec{p}_2\,)$

$\Rightarrow$ Wigner functions are off mass-shell(!)

Use ~this in cascade codes

$$C_2(p_1,p_2) \equiv \frac{P_2(p_1,p_2)}{P_1(p_1)\cdot P_1(p_2)} = 1 + \frac{|\int e^{iqx}\rho_W(x,K)\,d^4x|^2}{[\int \rho_W(x,p_1)\,d^4x]\cdot[\int \rho_W(x,p_2)\,d^4x]}$$

A Justification

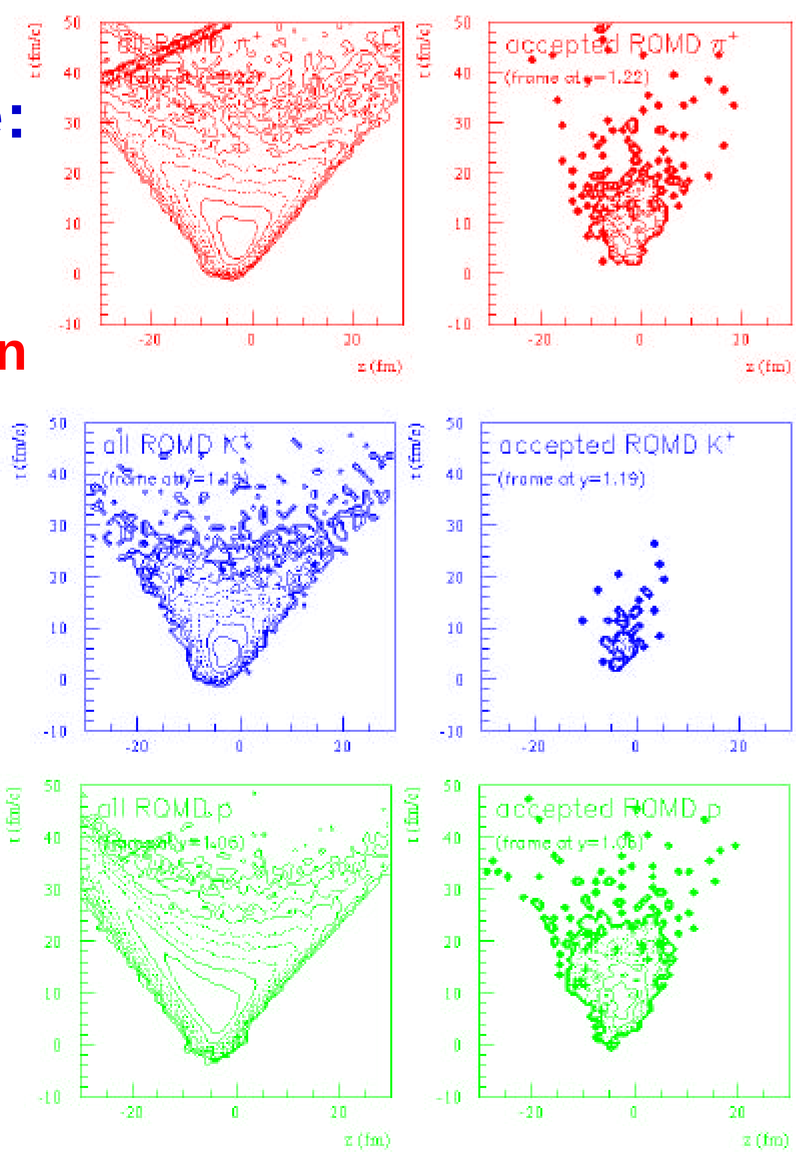
Q: Why compare to cascades?

- They're complicated
- They have ~infinite number of parameters

A: The sources are

- complicated
- dynamic (i.e., x and p are not independent)

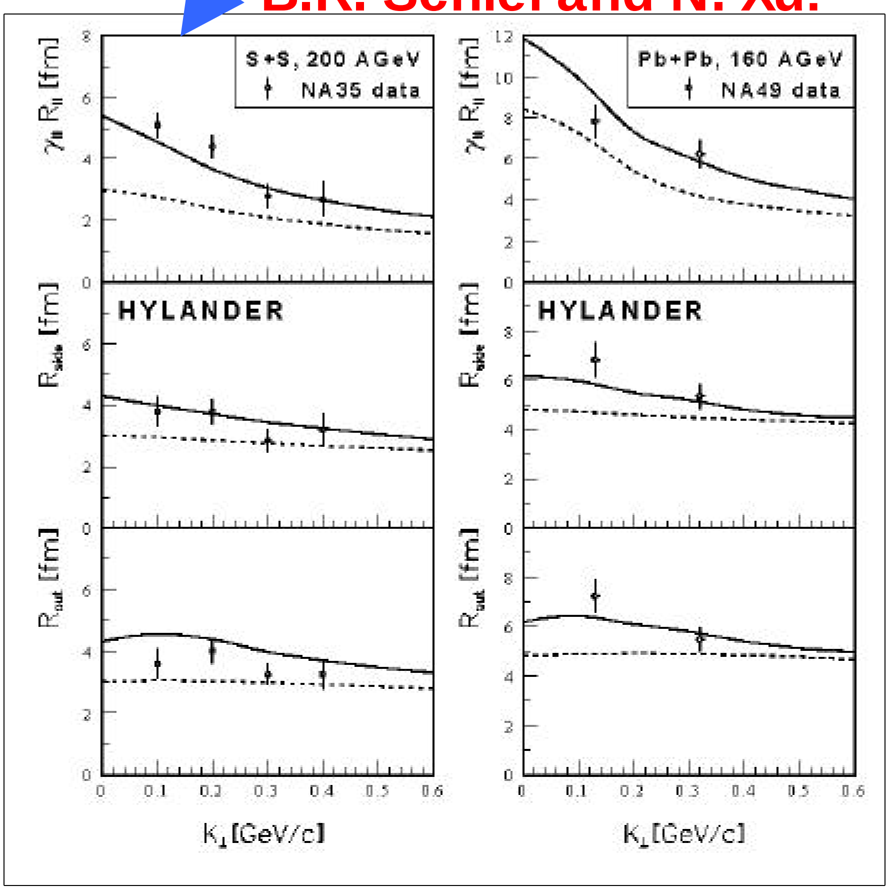
- **RQMD example:**
  - Si+Au
  - 14.6 A*GeV
  - Look at location of last strong interaction for pions, kaons and protons in (z,t) plane
- **From O.Vossnack Ph.D Thesis (Columbia)**

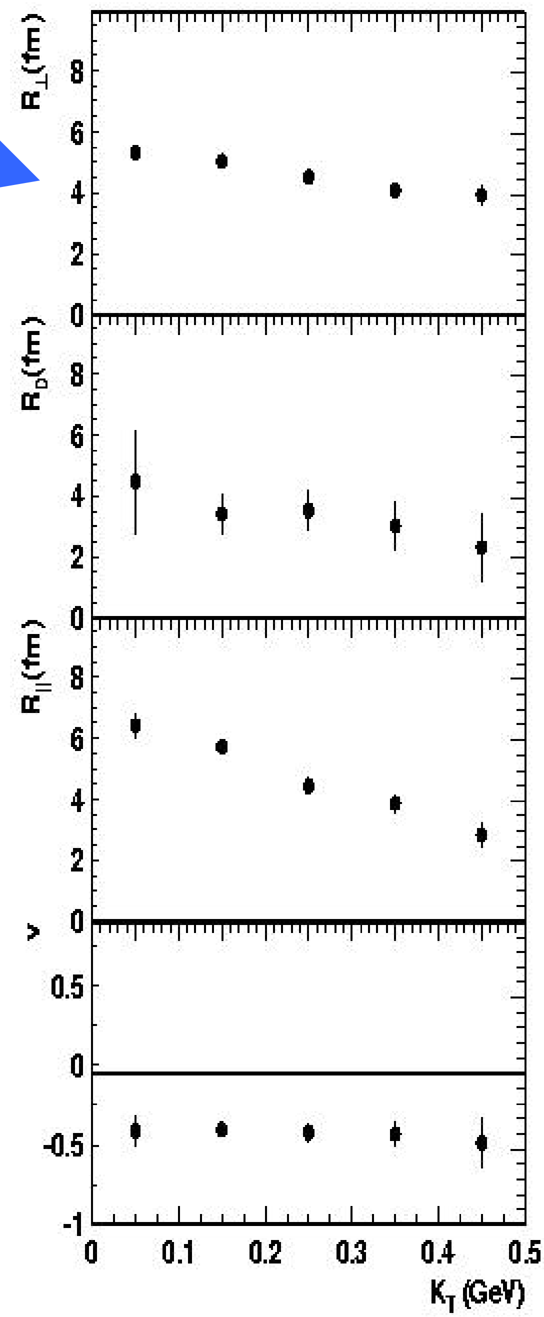


An Application

(of transport codes)

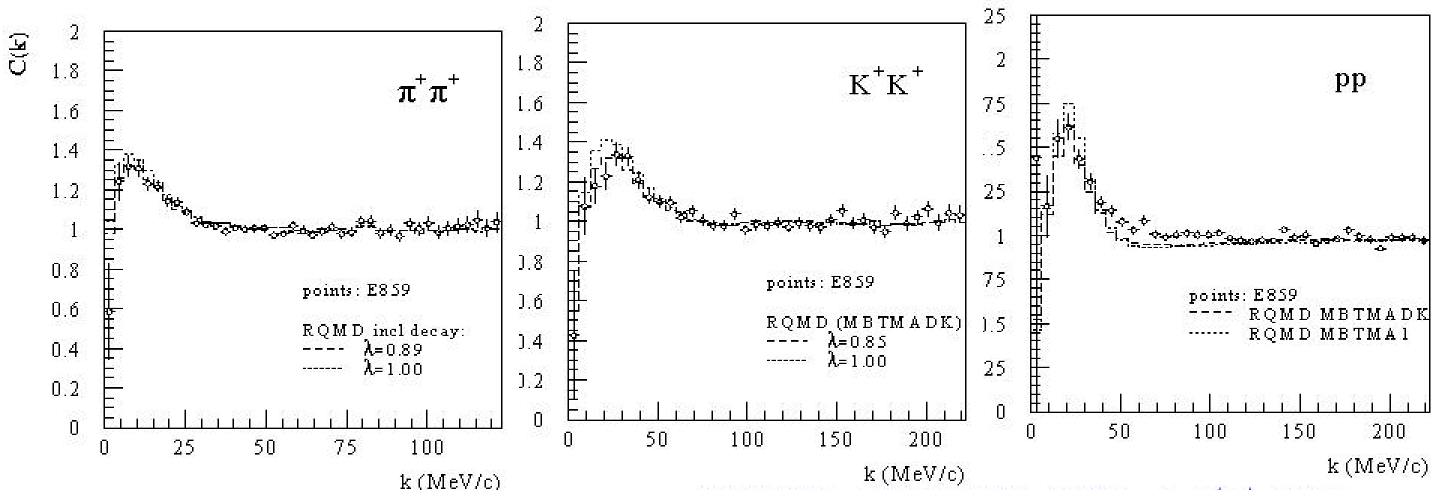
- **Expect C_2 to depend on q *<u>and</u>* K**
  - No easy analytic demonstration (?)
  - Results from $q_0 = q \cdot K / E(K)$
  - Usually produces decreasing R with increasing K
  - E.g., NA49 data
  - Explained in terms of resonance decays by B.R. Schlei and N. Xu:



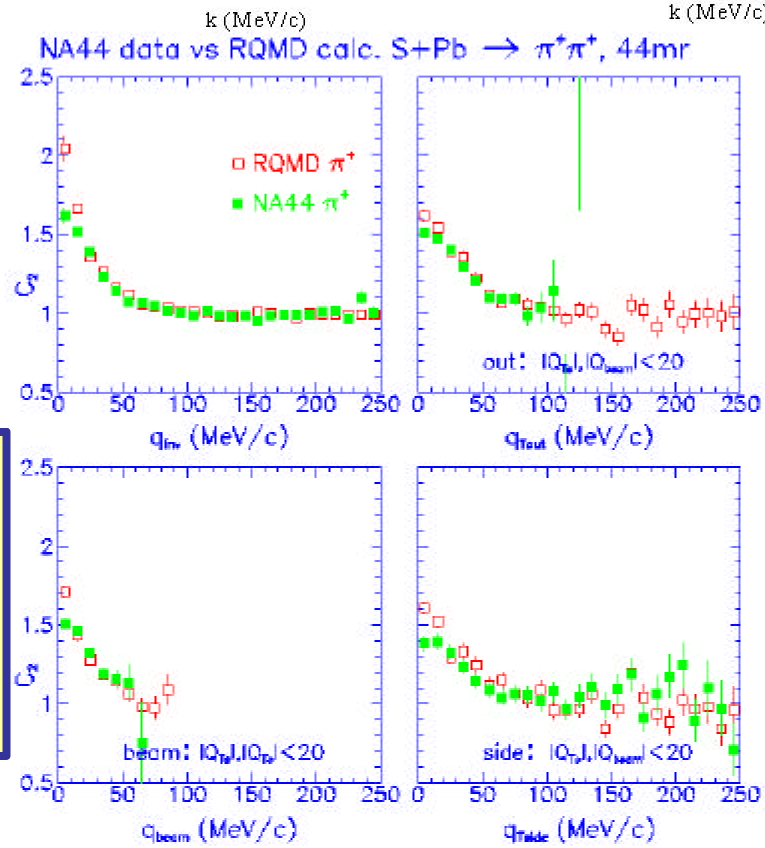


An Observation

- **HBT predictions from cascade codes describe data better than they have any right to:**
- **E859**
  - **AGS Si+Au: (note *uncorrected* data)**



- **NA44**
  - **SPS S+Pb:**
  - **Note 3 projections**
- **Both cases:**



Final state phase space points x_i are weighted by

$$1 + \cos[(p_a-p_b)(x_a-x_b)]$$

Some Trepidation

- **This approach is**
  - **Plausible**
  - ***WRONG***

Final state phase space points x_i are weighted by

$1 + \cos[(p_a-p_b)(x_a-x_b)]$

- **Corresponds to replacing this correct, positive-definite result**

$$C_2(p_1,p_2) \equiv \frac{P_2(p_1,p_2)}{P_1(p_1)\cdot P_1(p_2)} = 1 + \frac{\quad}{[\int \rho_W(x,p_1)\,d^4x]\cdot[\int \rho_W(x,p_2)\,d^4x]}$$

with

$$\left|\int e^{iqx}\rho_W(x,p_a)\,d^4x \cdot \int e^{iqx}\rho_W(x,p_b)\,d^4x\right|$$

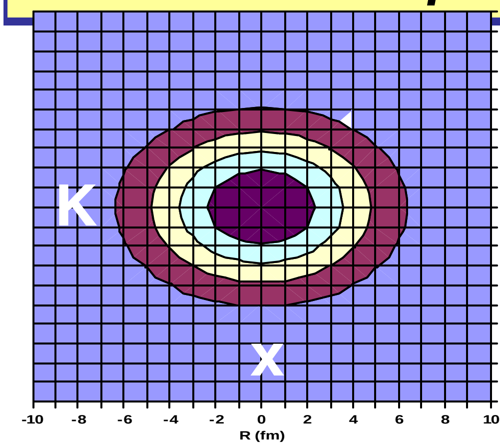
- **Advantage:**
  - **Uses precisely what codes produce**
- **Disadvantage**
  - **Can lead to non-physical oscillations in C2**
    - **First noted empirically by M. Martin *et al*.**
    - **Theoretical analysis by Q.H. Zhang *et al*.**
  - **Quantitatively, is it significant for heavy ion collisions?**
    - **Answer 1: No (Empirical)**
    - **Answer 2: Not yet? (see next slide)**

A Demonstration

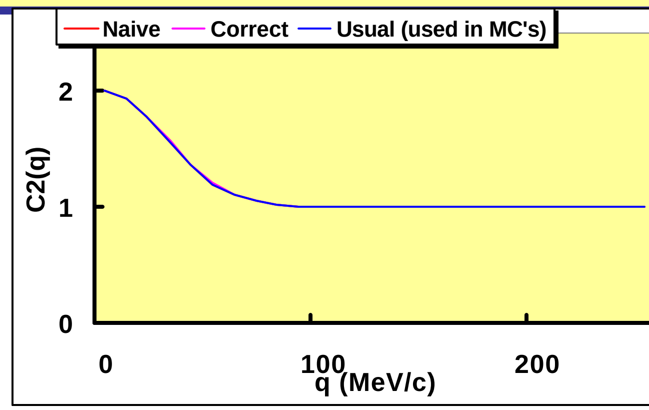
- **Study these effects with a source that explicitly parameterizes x-p correlations:**

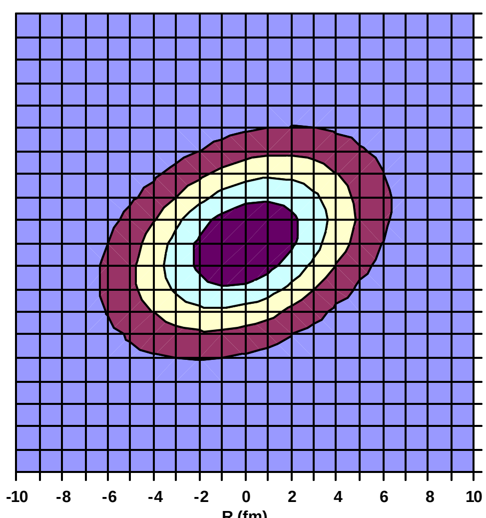
$$\rho_W(x,K) \sim \exp\left\{-\frac{1}{1-s^2}\left[\frac{x^2}{R^2} - 2s\frac{Kx}{PR} + \frac{K^2}{P^2}\right]\right\}$$

Examples for R=5fm, P=200 MeV/c

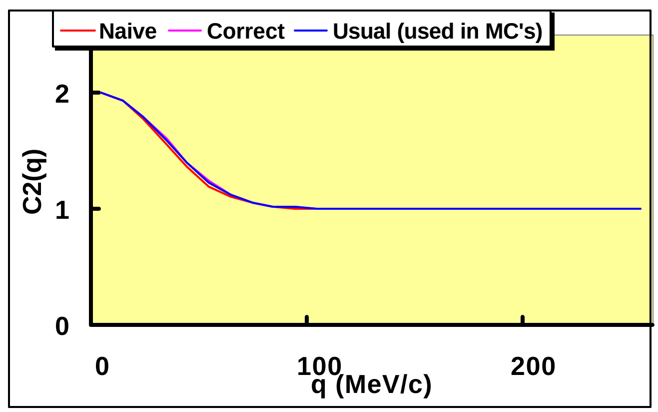


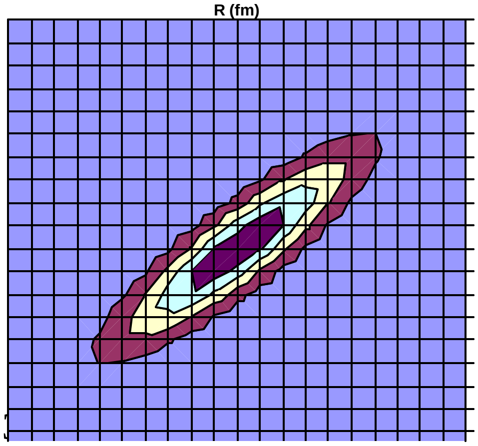
S=0



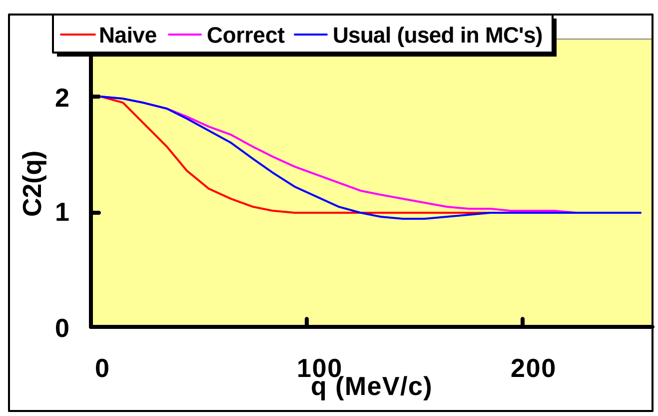


S=0.33





S=0.9



An Amelioration

Q. Can one remove these pathologies from ~classical predictions?

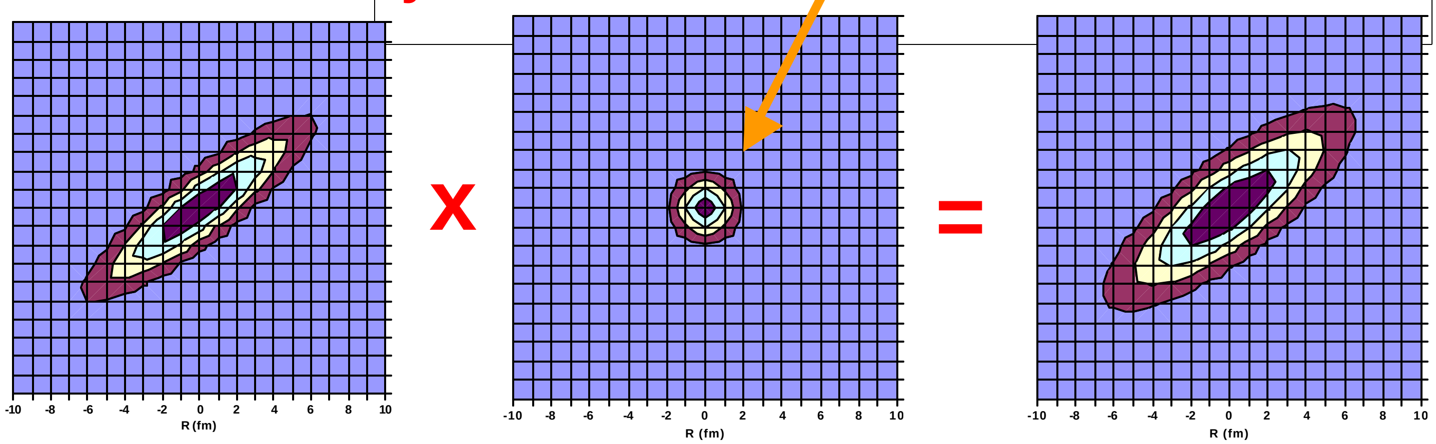
A. Yes (Q.H. Zhang *et al.*):

- **Smear the phase space points (x,p) with minimum uncertainty wave-packets:**

$$g(x,p) \sim \exp\left\{-\left[\frac{x^2}{2\sigma^2} + 2p^2\sigma^2\right]\right\}$$

  with s ~ 1 fm

- **Pictorially:**



- **This can be done *analytically* with parameterization on previous slide:**
  - **Get back same(!) functional form, with**

$$R^2 \rightarrow R^2 + \sigma^2$$

$$P^2 \rightarrow P^2 + \left(\frac{1}{2\sigma}\right)^2$$

$$s \rightarrow s\,\frac{RP}{R_{smeared}P_{smeared}}$$

  - **C_2 is demonstrably well-behaved**

A Transformation

- **What are the Lorentz properties of $C_2(q)$?**

$$C_2(p_1,p_2) \equiv \frac{\dfrac{d^6n}{dp_1{}^3 dp_2{}^3}}{\dfrac{d^3n}{dp_1{}^3}\cdot\dfrac{d^3n}{dp_1{}^3}} = \frac{E_1 E_2 \dfrac{d^6n}{dp_1{}^3 dp_2{}^3}}{E_1 \dfrac{d^3n}{dp_1{}^3}\cdot E_2 \dfrac{d^3n}{dp_1{}^3}} = \text{Lorentz Invariant}$$

- **How to write our favorite practice Gaussian in an explicitly Lorentz invariant way?**
- **Answered in**

F.B. Yano and S.E. Koonin, *Phys. Lett*. B78, 556 (1978).

$$\rho(\vec{r},t) \sim \exp[-\frac{1}{2}(\frac{r^2}{R^2} + \frac{t^2}{\tau^2})]$$

$$\Rightarrow \rho(\vec{q};\vec{V}_{PAIR}) = \exp\{-\frac{1}{2}[\,|\vec{q}|^2 R^2 + (\vec{q}\cdot\vec{V}_{PAIR})^2 \tau^2)\,]\}$$

so

$$C_2(p_1,p_2) = 1 + |\rho(\vec{q};\vec{V}_{PAIR})|^2 = 1 + \exp\{-|\vec{q}|^2 R^2 - (\vec{q}\cdot\vec{V}_{PAIR})^2 \tau^2\}$$

i.e., not explicitly Lorentz covariant

Fix by introducing source four-velocity $u = \gamma_s(1,\vec{v}_s)$

$q\cdot q = q_0{}^2 - |\vec{q}|^2$,

$q\cdot u = \gamma_s(q_0 - \vec{q}\cdot\vec{v}_s) \rightarrow q_0$ (in source rest frame)

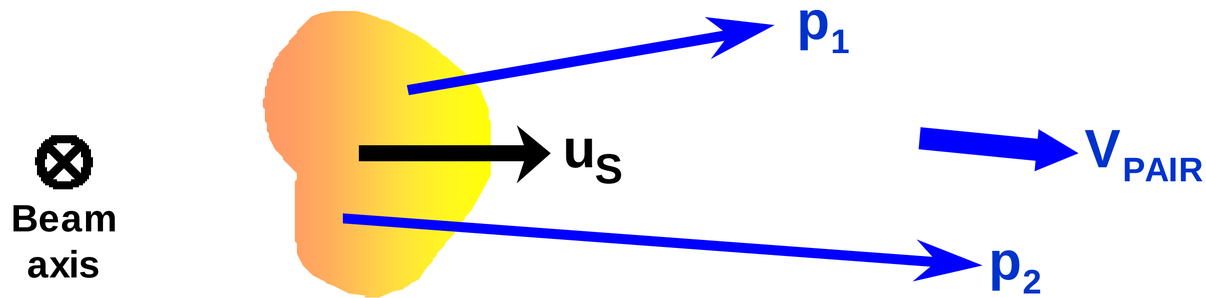
Then

$$C_2(p_1,p_2) = 1 + \exp\{-(q\cdot q)^2 R^2 + (q\cdot u)^2[R^2 + \tau^2]\}$$

i.e., explicitly Lorentz covariant

A Demonstration

- **Can Lorentz effects "distort" information?**
- **Apply Yano-Koonin-Podgoretzsky result to another toy model:**



- **Apply this to ~1-d motion in "Out" direction:**

Study argument of exponential

$\mathrm{ARG} \equiv -(q \cdot q)^2 R^2 + (q \cdot u)^2 [R^2 + \tau^2]$ **for various cases:**

• **Source at rest** $\vec{v}_S = 0$:

$$\Rightarrow ARG = |\vec{q}|^2 [R^2 + V_{PAIR}{}^2 \tau^2]$$

$$\Rightarrow R_{OUT}{}^2 = R^2 + V_{PAIR}{}^2 \tau^2$$

• **Pair at rest** $\vec{V}_{PAIR} = 0$:

$$\Rightarrow ARG = |\vec{q}|^2 \gamma_s{}^{*2} [R^2 + v_S{}^{*2} \tau^2]$$

$$\Rightarrow R_{OUT}{}^2 = \gamma_s{}^{*2} [R^2 + v_S{}^{*2} \tau^2]$$

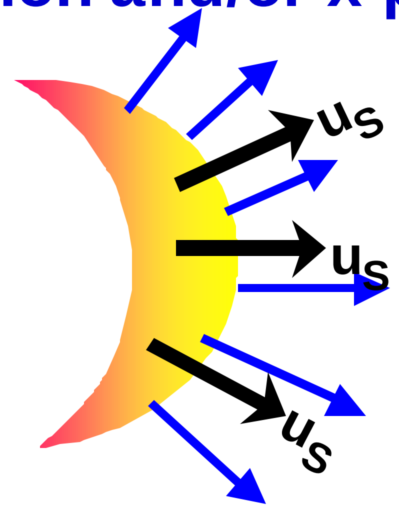
• **Numerical example:** $|\vec{V}_{PAIR}| \approx 1$, $|\vec{v}_S| = 0.7$

$$\Rightarrow R_{OUT}{}^2 \approx (0.4)^2 [R^2 + \tau^2]$$

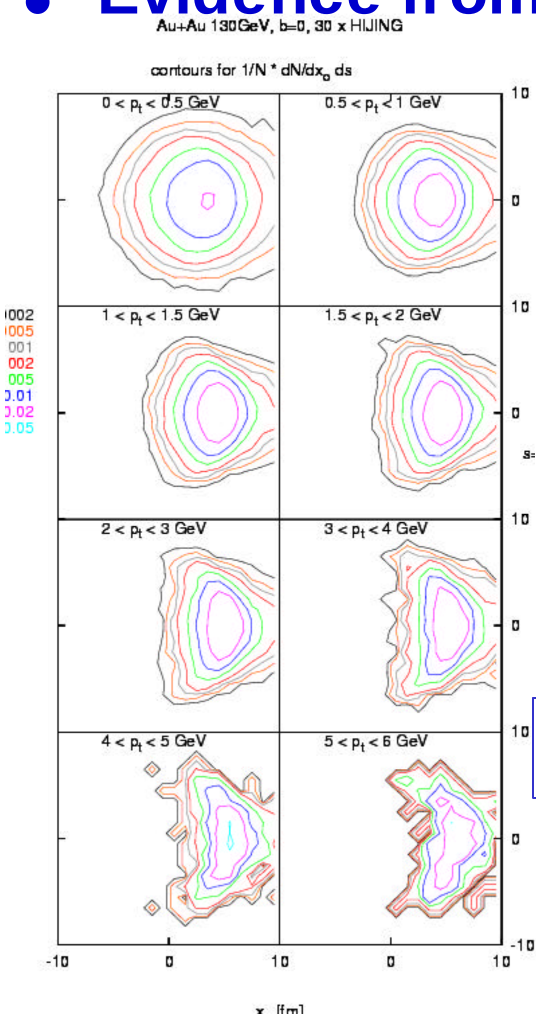
- **Nota Bene: This last case allows $R_{OUT} < R_{SIDE}$ even for $\tau \sim R_{SIDE}$**

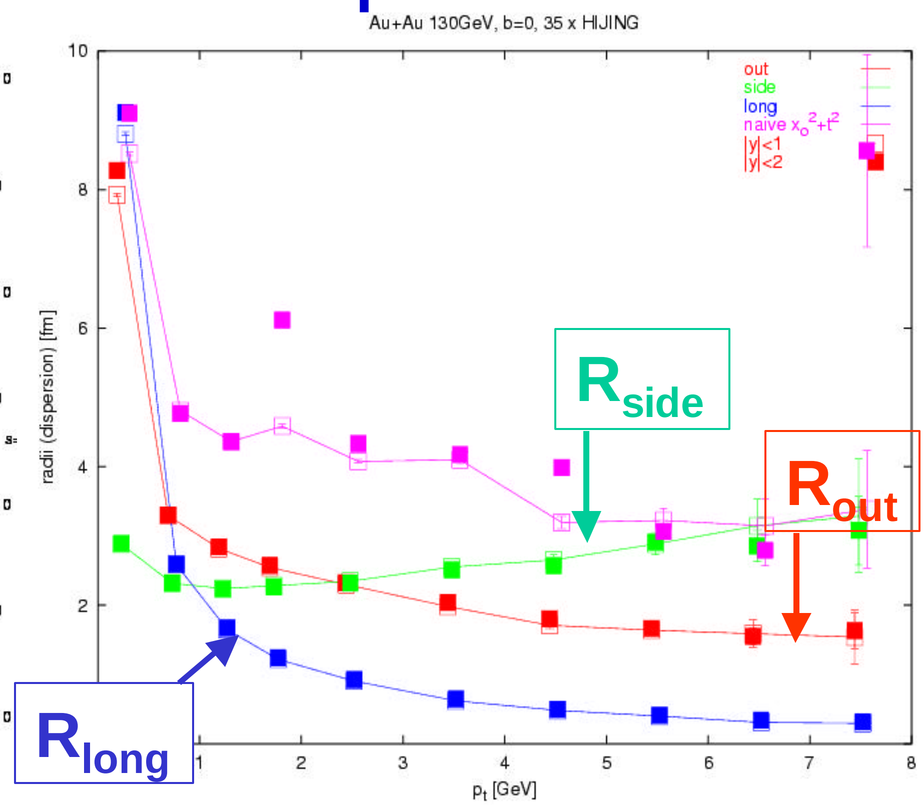
A Prognostication

- My guess: Our "usual" prejudices about HBT "radii" are being distorted by strong expansion and/or x-p correlations:



- Evidence from D. Molnar parton cascade:





(Courtesy of M. Gyulassy)

A Gradation

- **The chief lesson from last 10-15 years of (theoretical) HBT work:**
- **The "radii" are a (potentially) complicated mixture of**
  - Space-time distribution
  - Spatial flow gradients ~ dv / dr
  - Temperature gradients ~ dT / dr
  - This average $\rightarrow \langle (\vec{r} - \vec{V}_{PAIR} t)^2 \rangle$ is over space, time, flow profile, temperature, ...
- **More correct terminology:**
  "radii" "lengths of homogeneity"
- **In the presence of source dynamics, "radii" depend on mean pair (energy, momentum, transverse mass, ...)**

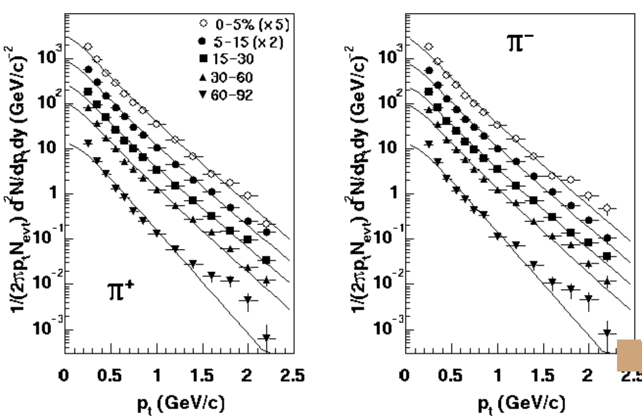
$$q \equiv p_1 - p_2 \equiv (q_0, \vec{q}) \quad , \quad \vec{K} \equiv \frac{1}{2}(\vec{p}_1 + \vec{p}_2)$$

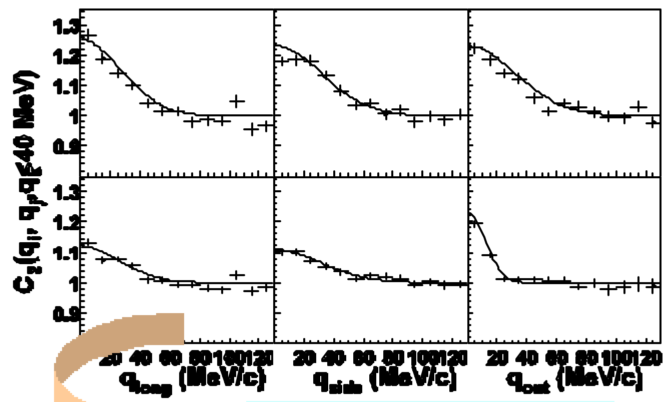
$$C_2(\vec{q}, \vec{V}_{PAIR}) \quad \leftrightarrow \quad C_2(\vec{q}, \vec{K})$$ (Pratt, 1984)

$$\text{Lengths} \sim \frac{dr}{dv}\Delta v \quad and/or \quad \frac{dr}{dT}\Delta T \quad \sim \frac{R_0}{f(K)}$$

A Frustration

- **In principle, combining**
  - **Single-particle (momentum distribution) data**
  - **Two-particle ("HBT") data**
- **can determine freeze-out temperatures, flow velocity, source lengths, …**
- **In practice, this is *complicated:***



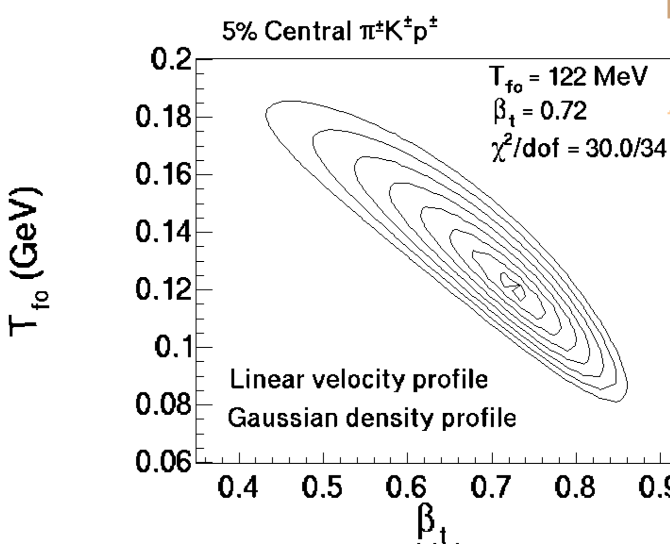


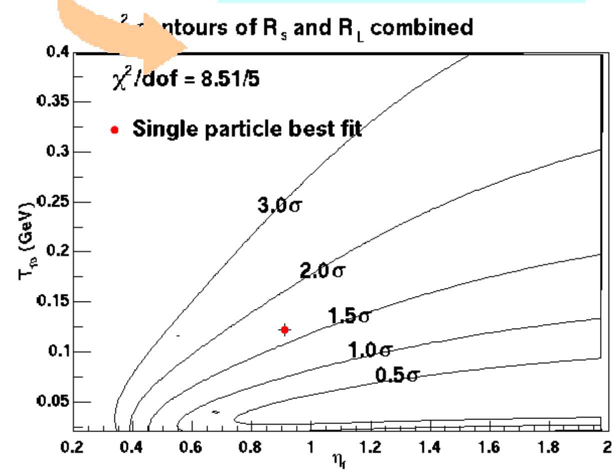
$E\, d^3N/dp^3 = \int d^4x\, S(x,p)$.

$1/m_t\, dN/dm_t$
$= A\, m_T \int \xi \exp(-\xi^2)\, d\xi\, K_1(m_T/T_{fo} \cosh\rho)\, I_0(p_T/T_{fo} \sinh\rho)$
with $\rho = \tanh^{-1}(\beta_t(\xi) = \beta_t \xi)$ and $\xi = r/R$.

$$R_s^2 = \langle y^2 \rangle \approx \frac{\sum_{n=0}^{p} c_n F_n R_s^2(n)}{\sum_{n=0}^{p} c_n F_n}$$

$$R_l^2 = \langle x^2 \rangle \approx \frac{\sum_{n=0}^{p} \tilde{c}_n F_n R_s^2(n)}{\sum_{n=0}^{p} c_n F_n}$$





A Combination

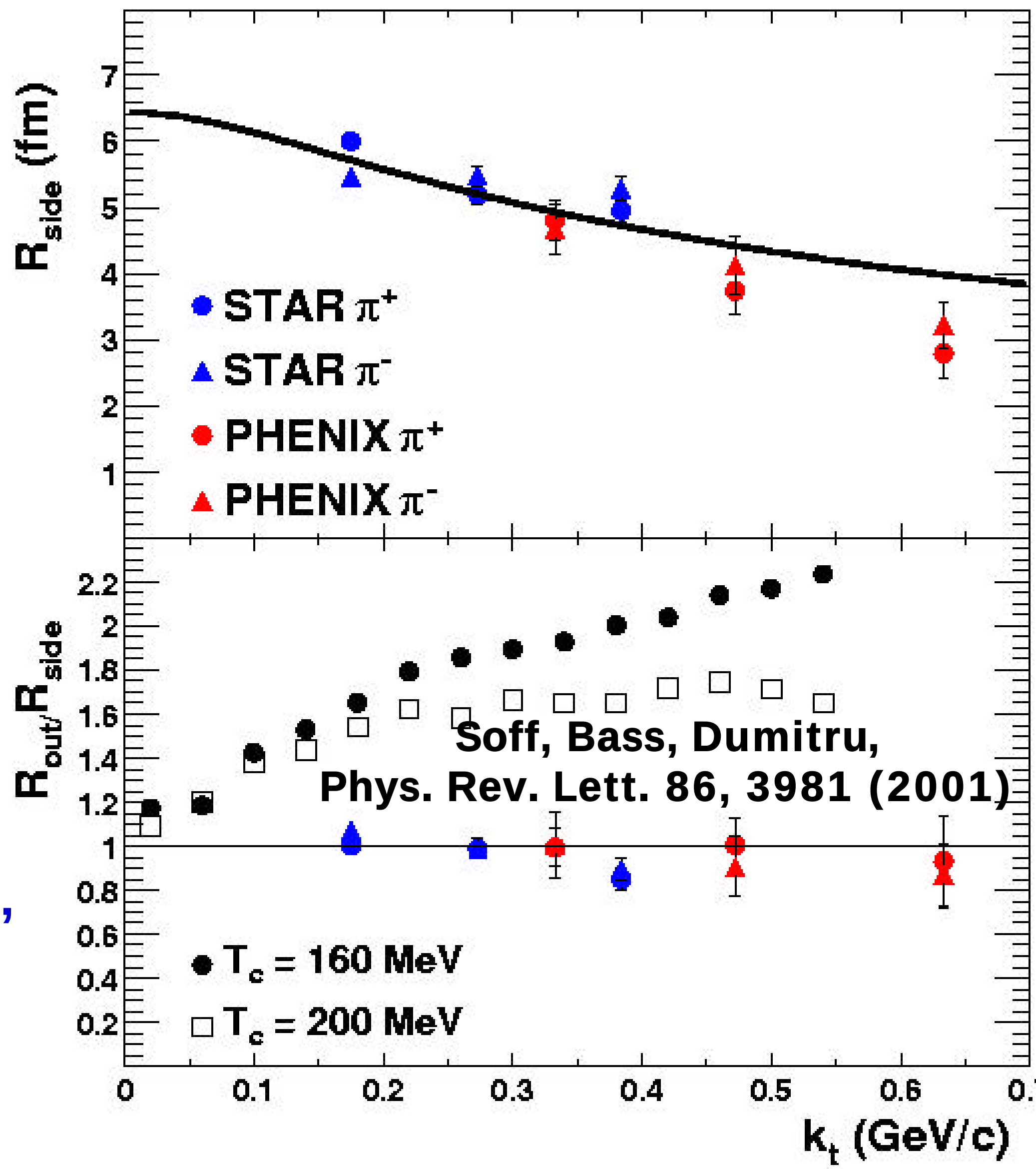
- Combining STAR and PHENIX HBT "radii" :
- Fit

$$R^2_{SIDE} = \frac{R^2_{GEOM}}{1 + m_T \frac{h_f^{\,2}}{T}}$$

Wiedemann, Scotto, Heinz, Phys. Rev. C53, 918 (1996)

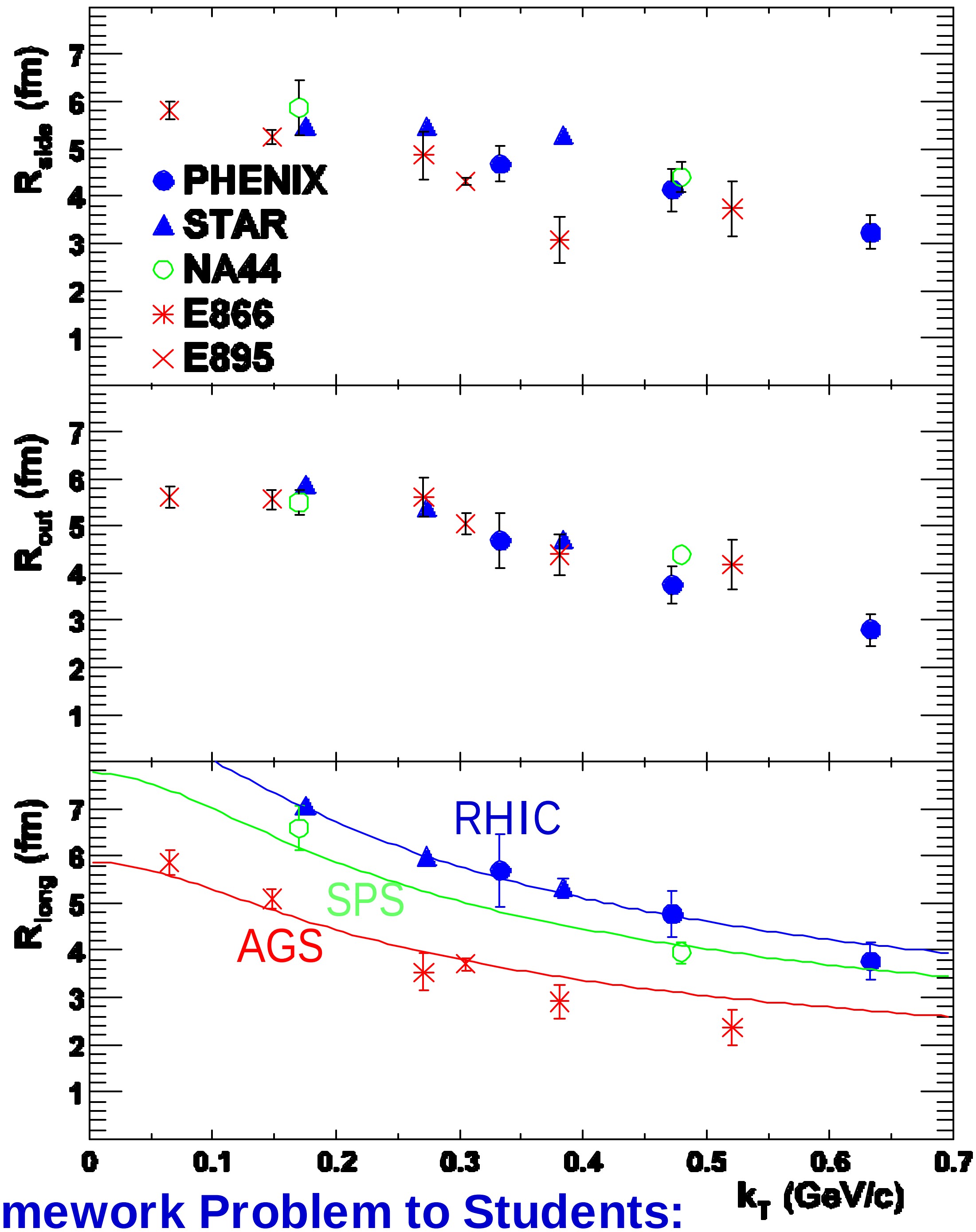
$$\Rightarrow R_{GEOM} = 8.6 \pm 0.8 \; fm \quad , \quad \frac{h_f^{\,2}}{T} = 5.6 \pm 1.7 \, GeV^{-1}$$

- **But :**
  R_{OUT}/R_{SIDE} trend completely eliminates one "hydro" calculation



A Unification

- Plotted versus K_T, there is an amazing consistency between data from AGS, SPS, and RHIC, spanning a factor of 100 in CM energy(!):



- Homework Problem to Students:

☛ ***Explain This !!***

Pontification

- There is now an extensive experimental and theoretical literature built on 20+ years of "HBT" studies
- We're now ready to start doing things right:
  - Experimentally
    - Measure resonance contributions
    - Measure like and unlike particle effects
    - Stop applying ersatz Coulomb corrections

      Or
    - Stop applying any Coulomb corrections
    - Test frame assumptions (do we need LCMS?)
    - Understand systematic errors
  - Theoretically
    - Cascade codes for RHIC
    - Improve understanding of
      - Lorentz transformations (do we need LCMS?)
      - Shape sensitivity
      - Quantum corrections to classical densities
      - Coulomb systematics
    - Understand systematic errors
- Together:

  Let's compare caipirinhas to caipirinhas !