Imaging the Final-State Phase Space in Heavy Ion Collisions

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The status of Bose-Einstein correlations in relativistic heavy ion collisions is discussed. Due to advances in experimental technique which have greatly extended the available data sets, it is now possible to obtain quite detailed informatiom regarding the final-state phase distribution of the produced hadrons. The role of systematic effects in the theoretical modeling of these effects is also presented.

1. INTRODUCTION

1.1. Motivation

The major goals of heavy ion collisions at relativistic energies are the creation and detection of new forms of matter. Generally, one is interested in the bulk properties of such matter (i.e., as opposed to the production of new particles). For instance, while the energy density is thought to be a fundamental parameter controlling the evolution of dense nuclear matter, it is clearly better to have access to both the energy and the volume rather than merely their quotient. Therefore, in contrast to particle production in high energy physics, one is interested in measuring the distributions in both coordinate space as well as momentum space.

The complexity of these collisions forces physicists to employ all tools at their disposal to extract space-time information. It has long been known that correlations between identical particles provide a measure of the source size[1,2]. More generally, virtually any (interacting) pairs have this property, since their coalescence/interactions/reactions are determined by the interplay between their "interaction radius" and the volume over which they are distributed. Present-day relativisitic heavy ion experiments avail themselves of all such measurements, often with very good statistical precision. This precision in turn means that the limitations of the analyses are more often systematic than statistical. This brief discussion reviews some of the issues that arise in such measurements.

1.2. Abbreviations and Invocations

Properly speaking, the use of identical particle measurements in heavy ion physics should be labeled the "GGLP" technique, following the work of Goldhaber, Goldhaber, Lee and Pais[1] in determining the reaction volume in proton-anitproton annihilations. However, it is far more commom to refer to "HBT" measurements, in reference to the

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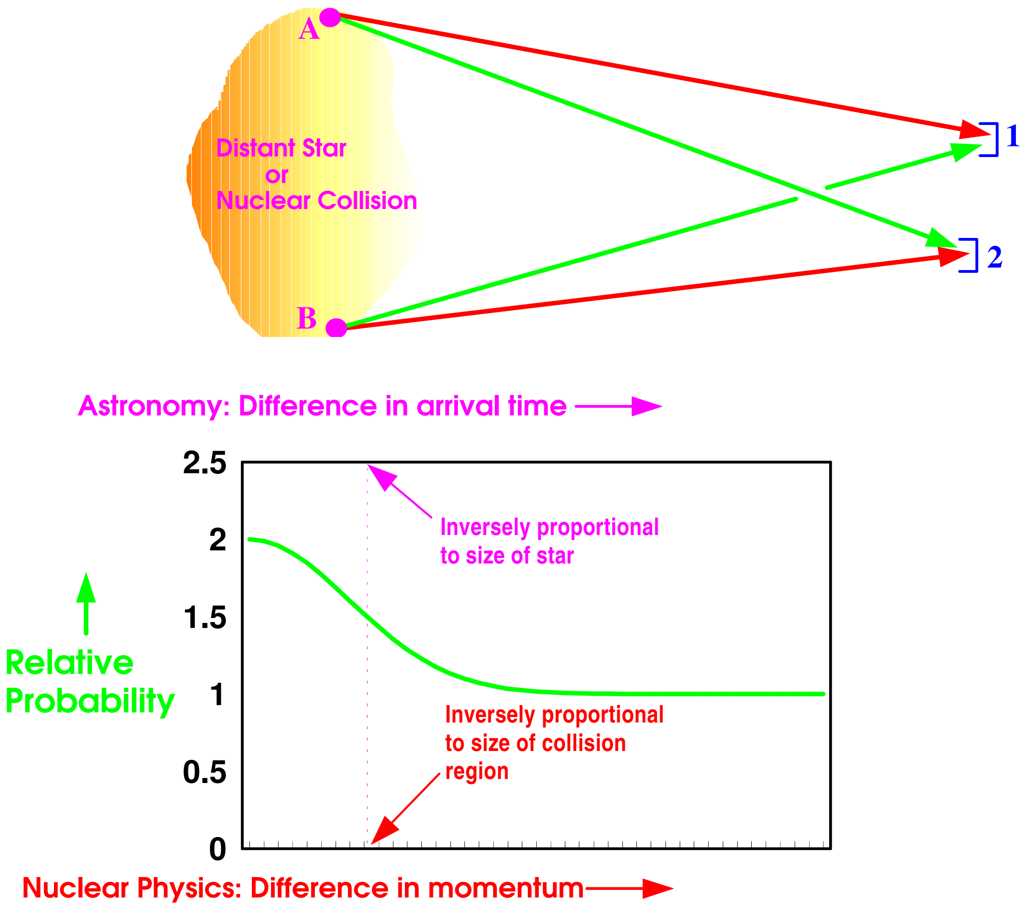

Figure 1. The analogy between stellar and nuclear HBT. See text for important caveats.

work of Hanbury-Brown and Twiss in developing intensity interferometry as a means of measuring stellar diameters[3] (a convention I will follow here).

1.3. Invocation

There is tremendous appeal in drawing an analogy between the measurement of distant stellar diameters a la HBT and "remote" nuclear collisions via like-particle correlations[4]. This is at best an analogy: in both cases, it illustrates the key role of indistinguishability of the two amplitudes in the problem. However, it is *not* an identity, in the sense that the geometry of a nuclear "HBT" measurement is *not* a scale model of a stellar HBT measurement. I have tried to illustrate this in the lower half of Figure 1. In words:

- Astronomy: Source separation $>>$ detector separation $\Rightarrow$ measure in coordinate space (difference of arrival times).

- Nuclear Physics: Source separation $<<$ detector separation $\Rightarrow$ measure in momentum space (difference of momenta).

1.4. Inspiration

How then does one demonstrate the HBT effect for nuclear collisions? The usual "derivation" sketched below of adding amplitudes for the alternative possibilities for prop-

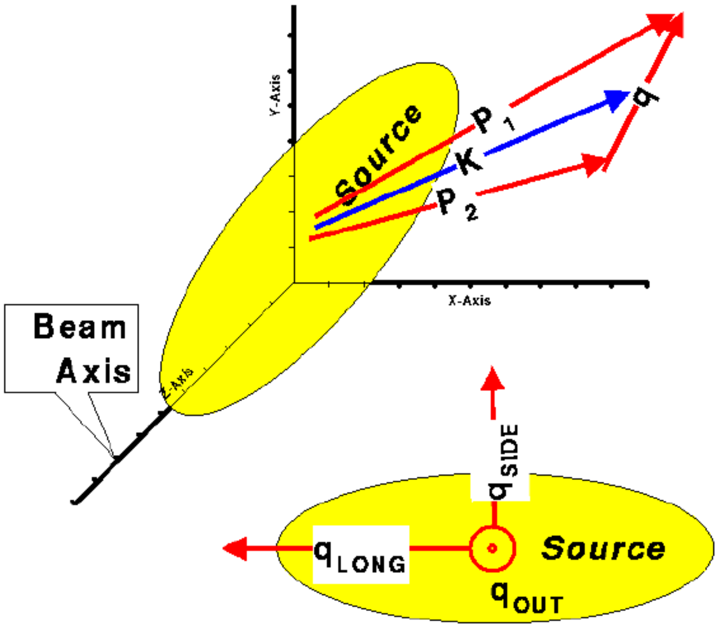

Figure 2. Projection of the relative three-momentum $\vec{q}$ onto directions parallel to the beam axis and to the two-particle average momentum $\vec{K}$.

agation from (x, y) to (r_1, r_2)

$$A_{12} = \frac{1}{\sqrt{2}}\left[e^{ip_1\cdot(r_1-x)}e^{ip_2\cdot(r_2-y)} + e^{ip_1\cdot(r_1-y)}e^{ip_2\cdot(r_2-x)}\right] \tag{1}$$

so that

$$\mathcal{P}_{12} = \int d^4x\, d^4y\, |A_{12}|^2\rho(x)\rho(y) = 1 + |\tilde{\rho}(q)|^2 \equiv C_2(q) \tag{2}$$

correctly accounts for the crucial role of symmetrizing amplitudes over an extended source $\rho(x)$, but ignores the momentum dependence and the quantum mechanics of the production mechanism, and also the final-state interactions "after" the production. Nonetheless, it contains the key message that the two-particle probability, and/or the correlation function $C_2(q)$ contains shape information, which is true component-by-component in q. A choice of components common in and appropriate to heavy ion physics is shown in Fig. 2. It is important to note that this decomposition assumes that the interactions between the two particles are in some sense small. In the non-interacting limit, the decomposition obviously obtains for plane-waves. In the limit where the two-particle final-state interaction is large, then the dependence of the correlation function on $\vec{q}$ is given by the appropriate partial waves. For example, in the case of pp correlations, the interaction is dominated by an s-wave term, and to a very good approximation the correlation function depends on $|\vec{q}|$ only. Physically, this implies that pp correlations provide size (volume) but not shape information.

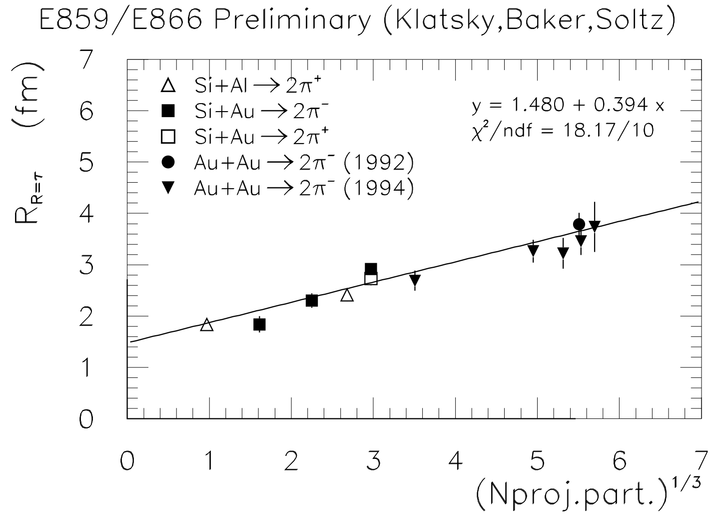

Figure 3. Extracted radii at AGS energies plotted versus the cube root of the number of projectile participants. See text for additional details.

2. A Calibration

While theoretical analyses argue that HBT correlations can in principle measure geometric sizes, there are sufficient complications that one would like some experimental confirmation that this is indeed the case. Experimenters have two approaches to varying the size of the nuclear collision region: Change the nuclei themselves, or "vary" the "impact parameter" for a fixed target-projectile combination[5]. The first such systematic study was presented in the Ph.D thesis of R. Soltz[6]. These data shown in Figure 3 along with additional results from M. Baker[7] and L. Klatsky[8] leave little doubt that the extracted radius R varies as $(Projectile\ Participants)^{1/3}$ (as determined by subtraction of the measured residual energy from the beam energy). Additional details extending this confirmation of geometry to a calibration thereof are available in Soltz's thesis, where it is shown that while these radii scale in some sense as $A^{\frac{1}{3}}$, their absolute values are larger than the values one would expect from unexpanded nuclear geometry.

Similar data on the variation of $R_{Transverse}$ with charged particle density $dN_{ch}/d\eta$ at CERN energies are now available from NA44[9]. Here it remains an open question, with significant implications for RHIC and LHC detectors, what the functional form of R_T vs $dN_{ch}/d\eta$ is. The data in Figure 4 are compatible with scaling laws ranging from $(dN_{ch}/d\eta)^{\frac{1}{3}}$ to $(dN_{ch}/d\eta)^1$, with corresponding predictions for RHIC multiplicities of R_T from 9 to 20 fm. This is more than just an academic exercise; radii in excess of 10 fm are extremely challenging to measure.

3. A Derivation and a Demonstration

A rigorous derivation of the joint two-particle probability distribution[11] shows that the correlation function is given by

$$C_2(p_1, p_2) \equiv \frac{P_2(p_1, p_2)}{P_1(p_1) \cdot P_1(p_2)} = 1 + \frac{|\int e^{iqx} \rho_W(x, K)\, d^4x|^2}{[\int \rho_W(x, p_1)\, d^4x] \cdot [\int \rho_W(x, p_2)\, d^4x]} \quad , \tag{3}$$

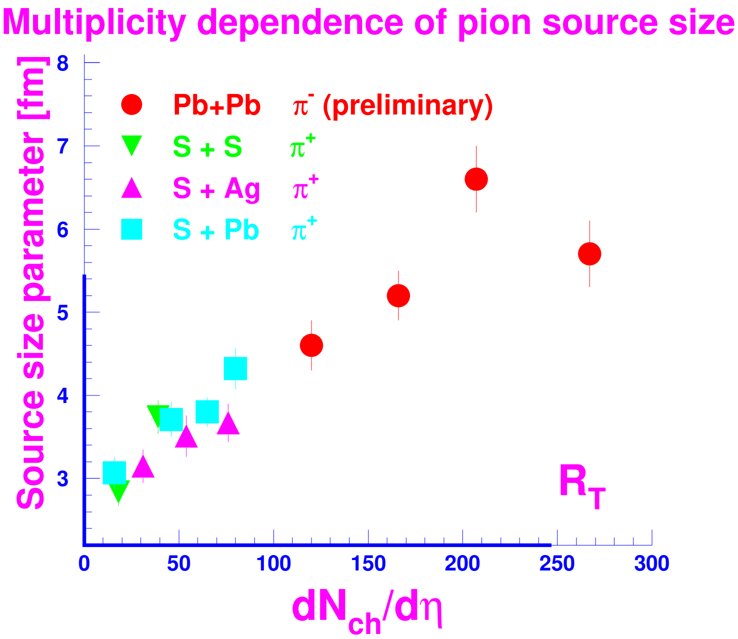

Figure 4. The multiplicity dependence of the transverse radius R_T as observed by NA44[9].

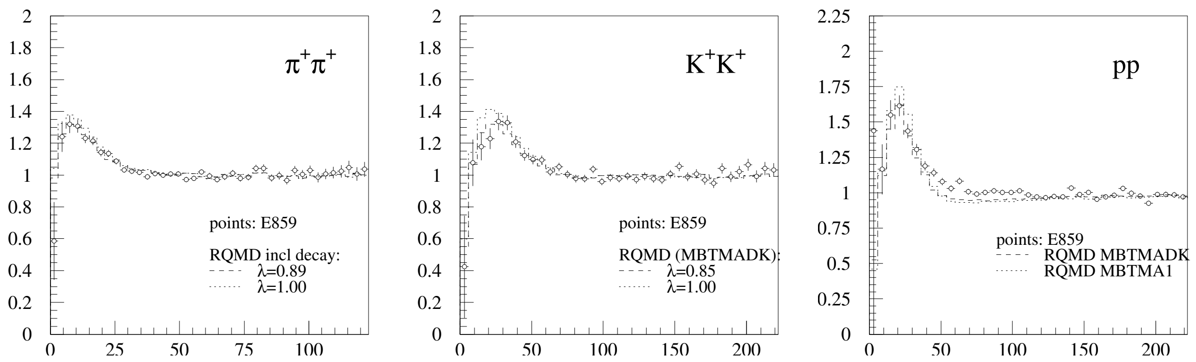

Figure 5. The uncorrected correlation function for $\pi^+\pi^+$, $\mathrm{K}^+\mathrm{K}^+$ and pp pairs measured by E859[10], as compared to RQMD calculations for the same quantity. See text for further details.

where ρ_W is the Wigner transform of the single-particle density matrix $\rho(x, y)$, $q \equiv p_2 - p_1$ and $K \equiv \frac{1}{2}(\,E_1 + E_2,\ \vec{p}_1 + \vec{p}_2\,)$. This result is often cited as a basis for incorporating Bose-Einstein symmetrization into cascade codes, since the Wigner transform of the density matrix produces a quantity $\rho_W(x, K)$ that is similar to a classical phase-space distribution. Indeed, weighting phase-space points produced by essentially classical transport codes produces impressive agreement with the experimental data, as shown in Figures 5 and 6. However, the theoretical basis for this agreement is at best shaky, since the actual weighting of pairs from the cascade code is performed not with the correct result of Equation 3, but rather with an approximation, where (in effect) one replaces

$$|\int e^{iqx}\rho_W(x,K)\,d^4x|^2 \quad \rightarrow \quad |\int e^{iqx}\rho_W(x,p_a)\,d^4x \int e^{iqx}\rho_W(x,p_b)\,d^4x| \quad , \tag{4}$$

where p_a and p_b are the momenta of the two particles under consideration. (What is actually done in the codes is to weight a pair produced at phase-space points (x_a, p_a)

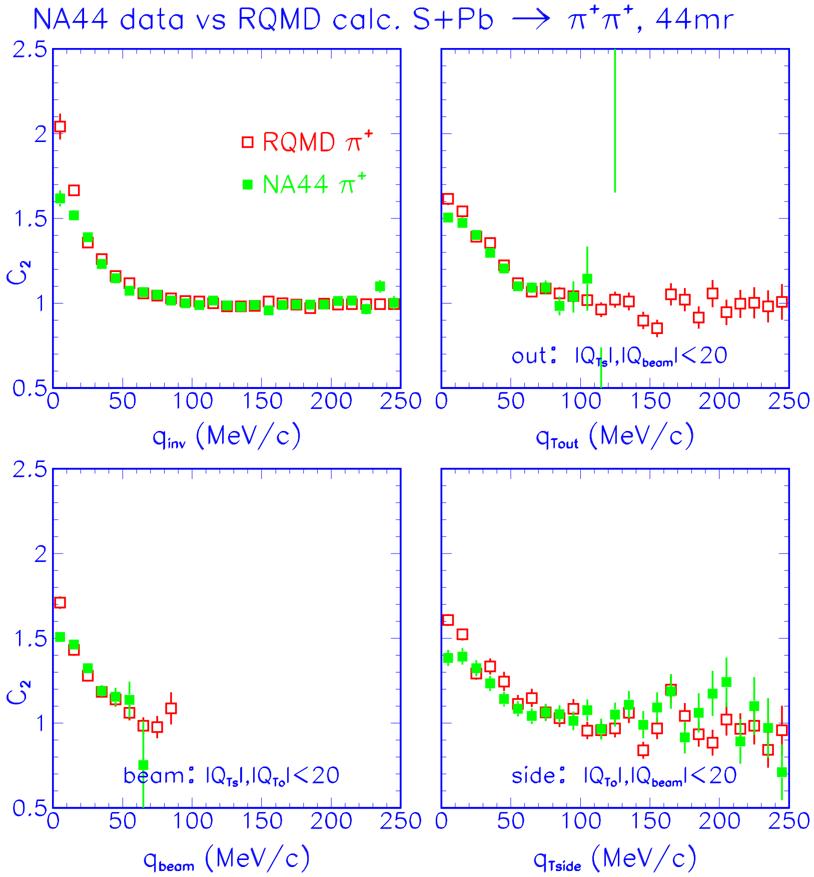

Figure 6. Four different projections of the correlation function from NA44 for S+Pb collisions compared to RQMD calculations.

and (x_b, p_b) with $1 + \cos[(p_a - p_b)(x_a - x_b)]$.) This has the advantage of making direct contact with the produced single-particle distributions, since it does not require knowledge of off-shell production implied by the variable "K", but with this comes a substantial disadvantage: the resulting form for C_2 is no longer positive definite. This was first noted empirically by Martin *et al.*[12], and described analytically by Zhang *et al.*[13], who also showed that a wave-packet smearing prescription can (and should) be used to insure that the uncertainty principle is not violated when simultaneously specifying the positions and momenta of the source distribution.

A particularly convenient form for expressing these results is given by the bi-variate Gaussian form

$$\rho_W(x, K) \sim \exp\left\{-\frac{1}{1-s^2}\left[\frac{x^2}{R^2} - 2s\frac{Kx}{PR} + \frac{K^2}{P^2}\right]\right\} \quad . \tag{5}$$

As noted by the author some time ago[14], this form describes a source with correlated emission between location x and momentum p with correlation coefficient strength s, while preserving $\langle x^2 \rangle = R^2$ and $\langle p^2 \rangle = P^2$. In particular, the correlation function constructed from such a source demonstrates analytically the pathological behavior when the weighting procedure of Equation 4 is used, namely,

$$C_2^{Approx}(q) = 1 + e^{-q^2R^2(1-s^2)} \cos(s\frac{R}{P}q^2) \tag{6}$$

versus the exact result

$$C_2^{Exact}(q) = 1 + e^{-q^2R^2[(1-s^2)-\frac{1}{(2PR)^2}]} \quad . \tag{7}$$

(Note that this last result nicely separates the effects due to the x-p correlation from the localization due to size of the region governed by R and P.) An additional benefit is that the source in Equation 5 is *form-invariant*, in that applying the smearing presription proposed in Ref. [13], by convoluting this source with a minimum uncertainty Gaussian wave-packe

$$g(x,p) \sim \exp\left\{-\left[\frac{x^2}{2\sigma^2} + 2p^2\sigma^2\right]\right\} \tag{8}$$

produces a source of the same functional form, with modified parameters given by

$$R^2 \rightarrow R^2 + \sigma^2 \tag{9}$$

$$P^2 \rightarrow P^2 + (\frac{1}{2\sigma})^2 \tag{10}$$

$$s \rightarrow s\ \frac{RP}{R_{smeared}P_{smeared}} \quad . \tag{11}$$

It is straightforward to check by direct substitution into Equation 7 that these modified parameters also result in a well-behaved correlation function.

Quantitatively, how important are these effects? The answer, for "usual" heavy ion collisions, is "Not very". With typical values[15] $P \sim 200$ MeV/c, $R \sim 5$ fm, $s \sim 0.33$, the difference between the exact and approximate correlation function is less than 1% everywhere. Only for highly correlated sources with $s \approx 0.8$ or greater (e.g., from longitudinal growth), or for smaller sources, does one find a significant deviation from the approximate result. However, substantial care should be taken when studying systems which are simultaneously small ($PR \sim 1$) and in which dynamical correlations are large $s \sim 1$, such as p-p and perhaps p-A collisions.

4. AN APPLICATION

As indicated by Equation 3, the correlation function can in principle depend not only on relative momentum $\vec{q}$ but also the average momentum of the pair $\vec{K}$. As noted by Pratt in a prescient work[18], expansion of the source leads to an explicit dependence of the source "radius" on the component of $\vec{K}$ along the expansion direction. Since then, an active literature has developed in calculating and parameterizing this dependence and using it to determine the expansion parameters[19]. As shown in Figure 7, the NA49 data[16] show that all of the various radii decrease with increasing mean transverse momentum K_T, which could be indicative of an expanding source. This dependence can be used

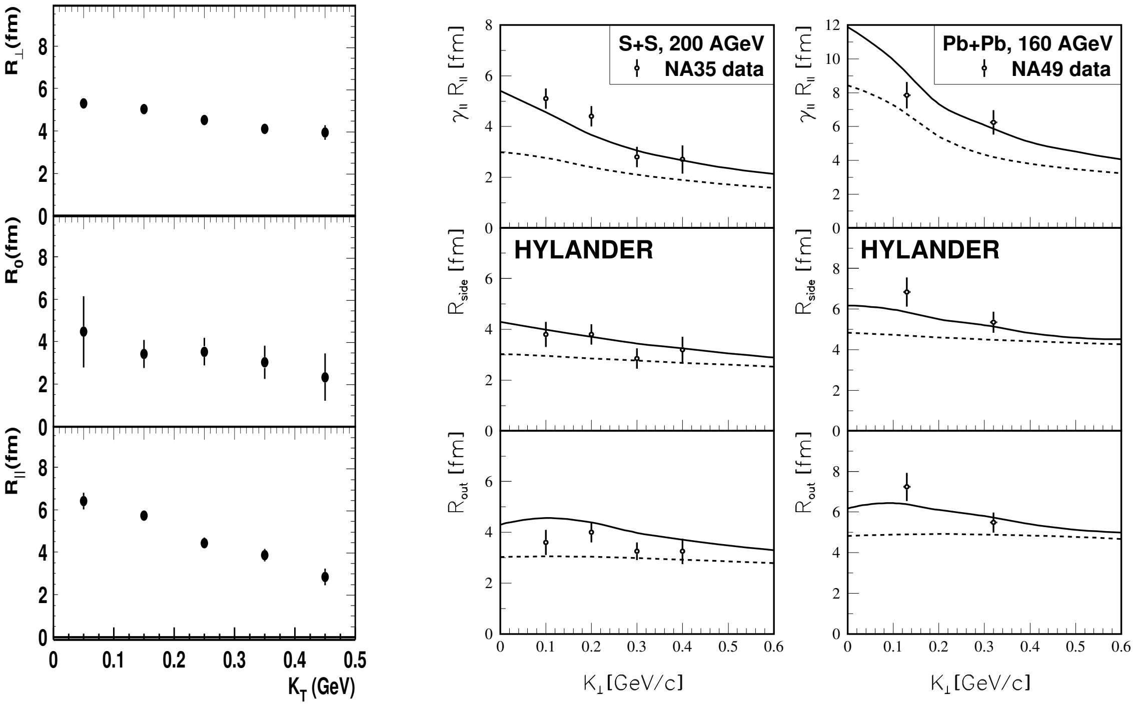

Figure 7. In the left column are NA49 data for the dependence of radius parameters on K_T in Pb+Pb collisions[16]. The right two columns show a partial accounting for this effect due to contributions from resonances[17]. Note that there is a difference of $\sqrt{2}$ between the parameterizations used for the radii on the right versus those on the left.

to remove the ambiguity between temperature and expansion velocity that results in fits to the single-particle spectra alone[20]. However, as shown in the right hand column of Figure 7, there is a complication. Work by Schlei and Xu[17] has shown that the expansion dynamics is complicated due to the contributions from resonances. Specifically, the transverse momentum spectrum of pions from the decays of ω's and η's is significantly softer than that for directly produced pions. These decays occur at distances larger (very much larger in the case of the η's) than the direct source size. Thus, a cut on low K_T has the effect of enhancing the resonance contribution to the spectrum, leading to larger radii. Clearly the source model must account for the long-lived resonances before the role of expansion can be isolated and quantified.

5. A COMPLICATION

It should not have escaped the reader's attention that the correlation functions presented in Figure 5 are quite different from those typically shown (e.g., in Figure 6). That is due to the Coulomb interaction: In Figure 6 it has been corrected for, while in Figure 5 the theoretical curves has been distorted to account for it. It has been known since the earliest days of HBT measurements in heavy ion collisions that the two-particle Coulomb interaction would present difficulties in analysis of sufficiently large source sizes. Early

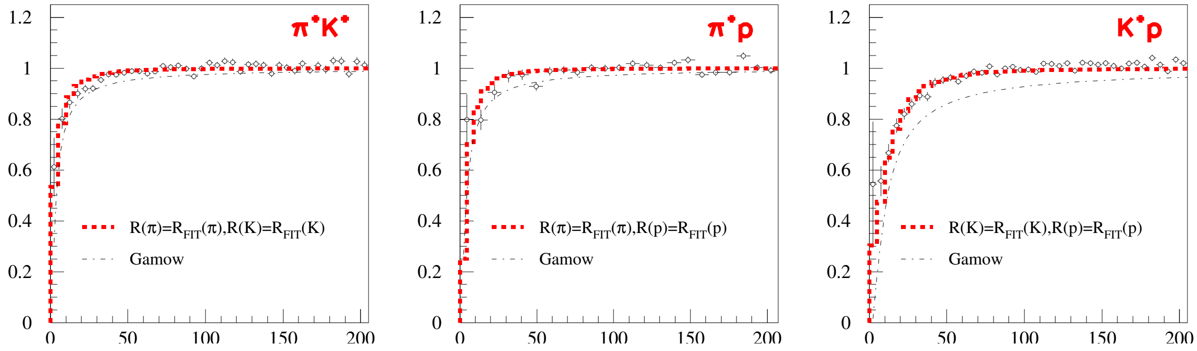

Figure 8. The uncorrected correlation function for π^+-p, π^+K$^+$ and pK$^+$ pairs measured by E859[10], as compared to calculations for the same quantity using exact Coulomb waves integrated over the source distributions extracted from the like-particle analysis. See text for further details.

theoretical[22] and experimental[23,24] work concentrated on removing the effect via the "Gamow correction:"

$$G_0(k) \equiv \frac{|\Psi_k(\vec{r}=0)|^2}{|\Psi_k(\vec{r}\to\infty)|^2} = \frac{2\pi\eta}{e^{2\pi\eta}-1}, \quad \eta = \frac{\mu e^2}{\hbar^2 k} \quad , \tag{12}$$

although in both cases the correction due to the extended nature of the source was explicitly noted and estimated as

$$G_\rho(k) \equiv \frac{\int |\Psi_k(\vec{r})|^2 \rho(\vec{r})\; d\vec{r}}{|\Psi_k(\vec{r}\to\infty)|^2} \sim G_0(k)\cdot\{1+2\frac{\langle r\rangle}{a_0}+\cdots\} \tag{13}$$

where $a_0 \equiv \frac{\hbar^2}{\mu e^2}$ is the Bohr radius for the two particle system and k is the momentum of *one* of the particles in the two-particle center-of-mass system (for equal mass particles, k is one-half the "usual" invariant relative momentum $Q=\sqrt{-(p_1-p_2)^2}$). Since then, some effort has gone into extending these approximations. For example, Baym and Braun-Munzinger have calculated the modification at large k to $G_0(k)$ via a semi-classical calculation[25]. (The approximation in Equation 13 is for the limit $k\to 0$.) NA49 has attempted to estimate finite-size and screening effects by using the $\pi^+\pi^-$ correlations functions (shown in Figure 9) to parameterize an empirical modification to the Gamow function[16]:

$$F(k) = 1 + (G_0^{\pm}(k)-1)e^{-k/k_{eff}} \quad . \tag{14}$$

Neither of these approaches is entirely satisfactory: The semi-classical approach fails entirely for relative momenta within the classical turning radius, and the experimental data shown in Figure 8 are of sufficient quality to require this. The empirical prescription based on $\pi^+\pi^-$ data is confused by the opposite-sign corrections for the momentum shifts of π^+'s relative to π^-'s from the residual positive charge in the system; a rough estimate using the Gyulassy-Kaufmann formalism[22] and the $dn(+)/dy$ distribution from NA49 indicates that the momentum shifts are of order 6% for $p_T \sim 400\ MeV/c$ pions and increase to 15% for $p_T \sim 100$ MeV/c. There is also weak but consistent evidence

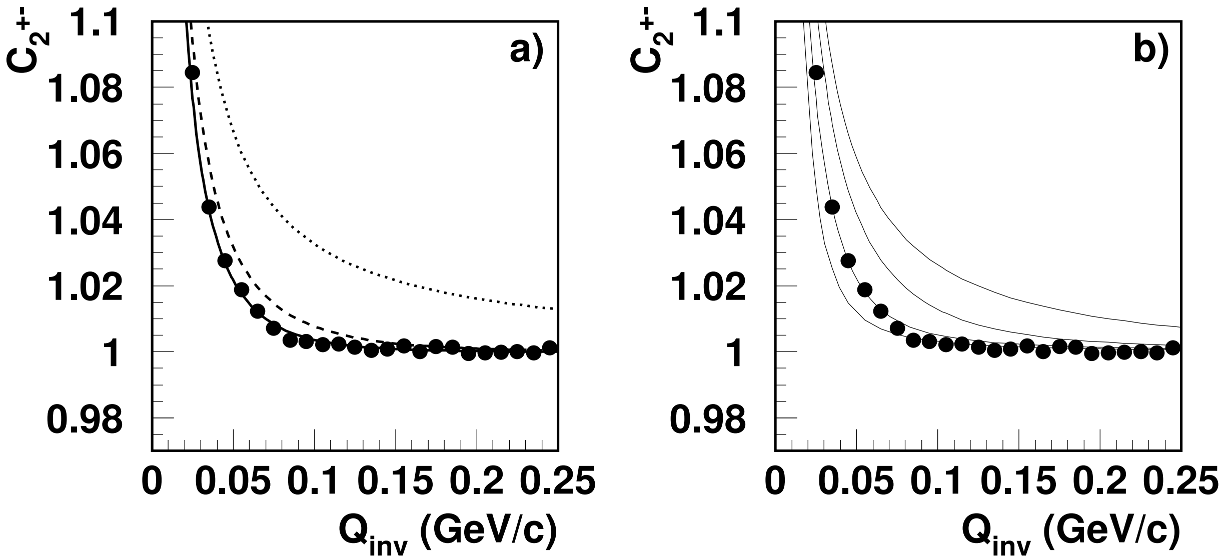

Figure 9. a.) The $\pi^+\pi^-$ correlation function measured by NA49[16], along with fits to Equation 14 with $Q_{eff} = 2k_{eff} = 46$ MeV (full curve) and $Q_{eff} = 75$ MeV (dashed curve). The dotted curve is the pure Gamow result given by Equation 12 b.) The same data as a.), but now compared to exact Coulomb wave functions for a source radii of 8, 5, 2 and 0.55 fm, (from lowest to highest curves).

from E877[26], who find that $R_{SIDE}(--) \sim (1.5 \pm 0.4) R_{SIDE}(++)$ and from NA44[21], who see $R_{TSide}(--) > R_{TOut}(--)$ while $R_{TSide}(++) < R_{TOut}(++)$ (shown in Figure 10). Both of these are indicative of a increase(decrease) in the single-particle momentum for positive(negative) pions due to the residual nuclear charge (note that this affects R_{TSide} but cancels in R_{TOut}), demonstrating that these Coulomb effects on the *single*-particle distributions play a significant role in sufficiently precise HBT studies even at SPS energies.

A better approach is to treat that part of the problem which is amenable to exact calculation directly via numerical integration of the two-particle Coulomb wave-function. A particularly notable example of this is the work of Vossnack[10], who has performed a unified treatment of $\pi^+\pi^+$, $\mathrm{K}^+\mathrm{K}^+$, pp, $\pi^+\mathrm{K}^+$, π^+p and pK^+pairs. The source parameters extracted from the like-particle measurements are used as input to a calculation of the two-particle Coulomb wave-functions for the unlike-particle combinations. The results shown in Figure 8 demonstrate the self-consistency of this approach: the data exhibit a clear preference for the exact Coulomb treatment over the simple Gamow correction.

6. A SUMMATION

It is not possible to do justice to the breadth and quality of current HBT measurements in such a condensed format. Instead, I hope that I have been able to demonstrate that over the past decade this technique has developed from a qualitative method to a quantitative tool fully as precise and important as single-particle data in determining the dynamics of the source.

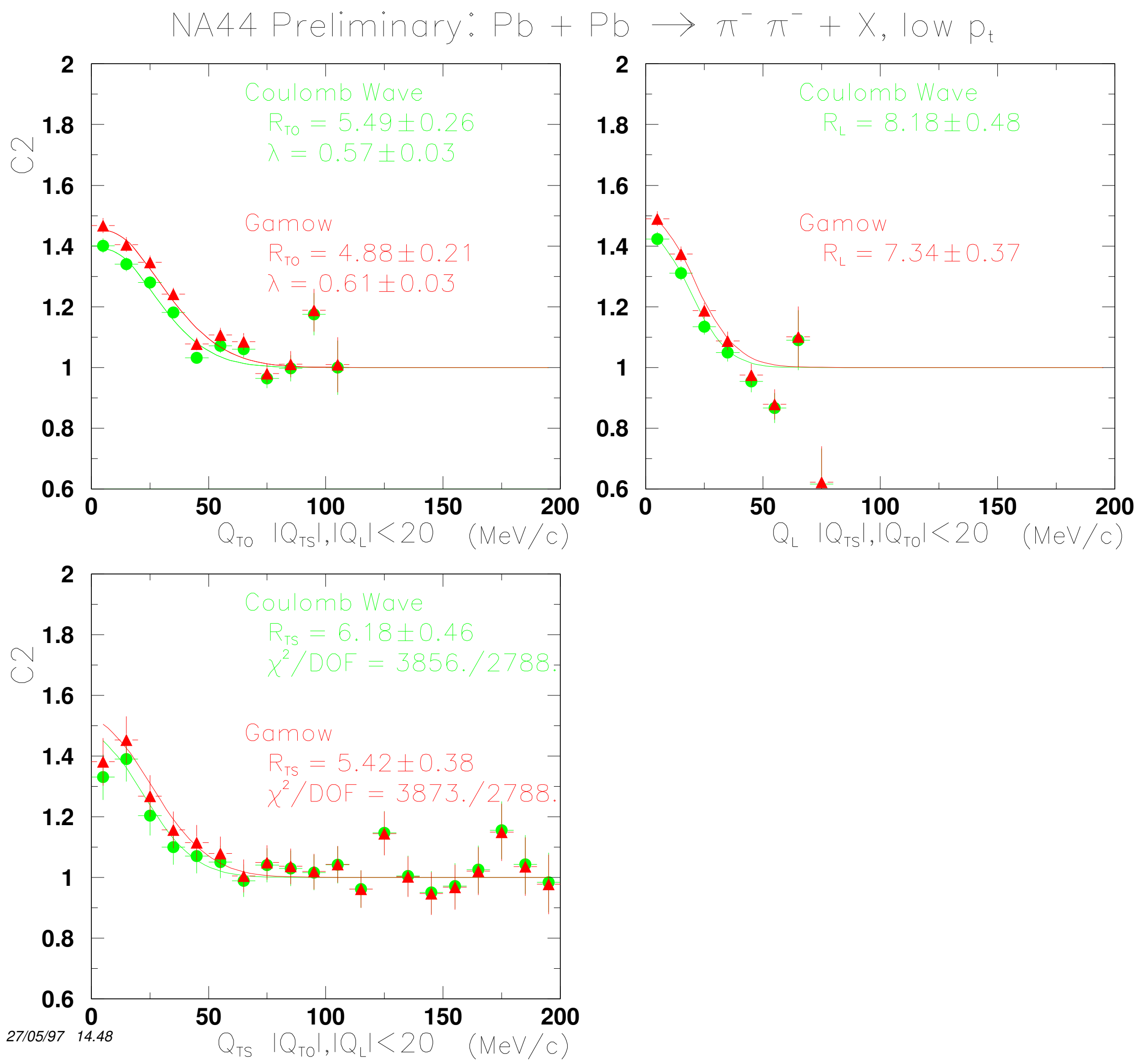

Figure 10. NA44 data[21] for $\pi^-\pi^-$ pairs demonstrating $R_{TSide} > R_{TOut}$.

7. ACKNOWLEDGEMENTS

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4. For instance, in the February, 1996 NSAC Report *Nuclear Science: A Long-Range Plan* there is a sidebar (for which I am partially responsible) that makes this analogy, but without the key elucidation of Fig 1b (the omission of which I cannot take credit

for).
5. The variation of course is performed by nature, and the selection is made not on (an unobserved) impact parameter but some quantity related to it such as forward energy or produced multiplicity.
6. *Two-Pion Correlation Measurements for 14.6* $\mathrm{A}\cdot\mathrm{GeV}/c$ ^{28}Si *+X and 11.6* $\mathrm{A}\cdot\mathrm{GeV}/c$ ^{197}Au *+Au,* R. Soltz, MIT Ph.D Thesis, 1994.
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