Violation of k_T-factorization in Hadronic Collisions

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Outline

- ❑ Collinear QCD factorization is violated for Drell-Yan
- ❑ Single transverse spin asymmetry
- ❑ Sivers function vs twist-3 quark-gluon correlation
- ❑ Di-jet momentum imbalance in hadronic collisions
- ❑ k_T-factorization is violated in a model calculation
- ❑ Summary and outlook

Ji, Qiu, Vogelsang, and Yuan,

PRL97, 082002 (2006), PRD73, 094017 (2006), PLB638, 178 (2006)

Qiu, Vogelsang, and Yuan,

arXiv:0704.1153 [hep-ph] (PLB in press), arXiv:0706.1196 [hep-ph]

Collins and Qiu, arXiv:0705.2141 [hep-ph] (PRD in press),

Perturbative QCD Factorization

- **Cross sections with identified hadrons are infrared sensitive and non-perturbative**

  Typical hadronic scale: $1/R \sim 1\ \text{fm}^{-1} \sim \Lambda_{QCD}$

  Energy exchange in hard collisions: $Q \gg \Lambda_{QCD}$

  $\Longrightarrow$ pQCD works at $\alpha_s(Q)$, but not at $\alpha_s(1/R)$

- **PQCD can be useful iff quantum interference between perturbative and nonperturbative scales can be neglected**

Short-distance

Power corrections

$$\sigma_{\text{phy}}(Q,1/R) \sim \hat{\sigma}(Q) \otimes \varphi(1/R) + O(1/QR)$$

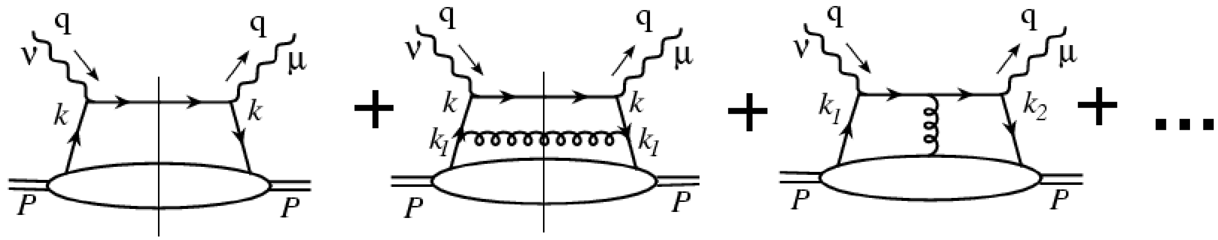
Measured

Long-distance

Factorization $\Longleftrightarrow$ needs a "long-lived" parton state

Long-lived parton states

- **Feynman diagram representation of the inclusive DIS:**

$W^{\mu\nu} \propto$ 

- **Perturbative pinched poles:**

$$\int d^4k \; \mathrm{H}(Q,k) \left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right) \mathrm{T}(k,\frac{1}{r_0}) \;\Rightarrow\; \infty \;\; \text{perturbatively}$$

Dominated by a region where $k^2 \sim 0$ - "long-lived" parton state

- **Perturbative factorization:**

$$k^\mu = xp^\mu + \frac{k^2+k_T^2}{2xp\cdot n} n^\mu + k_T^\mu$$

Nonperturbative matrix element

$$\int \frac{dx}{x} d^2k_T \; \mathrm{H}(Q,k^2=0) \boxed{\int dk^2 \left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right) \mathrm{T}(k,\frac{1}{r_0})}$$

Short-distance

Collinear factorization

❑ **Collinear approximation:**

If all observed physical scales: $Q_i \sim xp \gg k_T,\ \sqrt{k^2}$

$$\int dx\, \mathrm{H}(Q, k = xp) \boxed{\int d^4k\,\delta\left(x - \frac{k^+}{p^+}\right)\left(\frac{1}{k^2 + i\varepsilon}\right)\left(\frac{1}{k^2 - i\varepsilon}\right)\mathrm{T}(k, \frac{1}{r_0})} + \frac{\langle k_T^2 \rangle}{Q^2} + \ldots$$

Parton distribution function: $\varphi(x)$

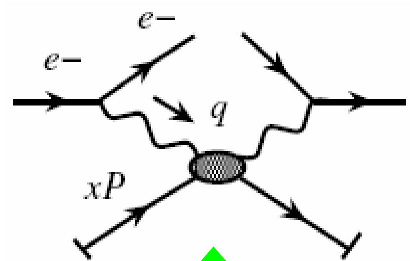
❖ Collinear approximation for *k* entering the hard part

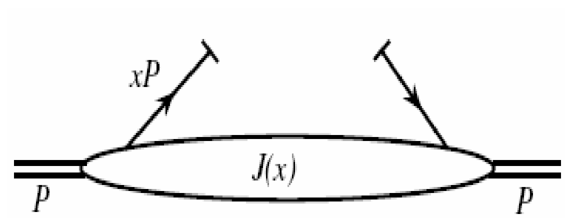
❖ Parton's transverse momentum is integrated into parton distributions, and provides a scale of power corrections

❑ **Parton distributions are process independent – universal**

Factorization: single hard scale

❑ One hadron:

$$\sigma^{\rm DIS}_{\rm tot} \sim$$ 

$$\otimes$$ 

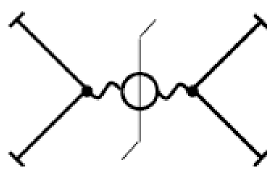
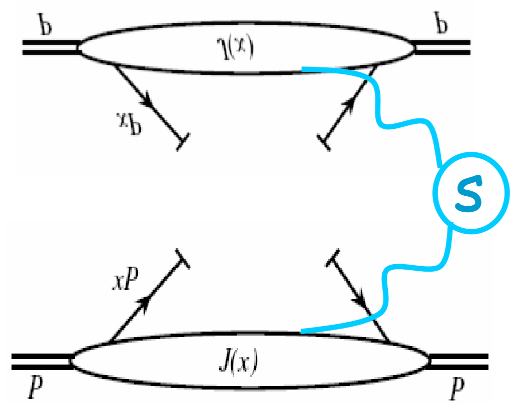
$$+\, O\left(\frac{1}{QR}\right)$$

Now
Hard-part

Past
Parton-distribution

Connection
Power corrections

❑ Two hadrons:

$$\sigma^{\rm DY}_{\rm tot} \sim$$ 
$$\otimes$$ 

$$+\, O\left(\frac{1}{QR}\right)$$

❑ Final-state hadrons:

Future: Fragmentation functions

Necessary condition for factorization

"Any uncanceled long-distance divergence of a partonic scattering cross section has to be process-independent"

On hadron state: $\sigma_{\mathrm{H}}(Q,1/R) \sim \sum_{\mathrm{a}} \hat{\sigma}_{\mathrm{a}}(Q) \otimes \varphi_{\mathrm{a/H}}(1/R) + O(1/QR)$

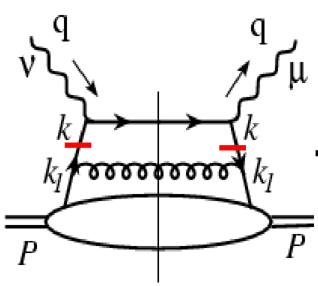
On parton state: $\sigma_{\mathrm{p}}(Q,1/R) \sim \sum_{\mathrm{a}} \hat{\sigma}_{\mathrm{a}}(Q) \otimes \varphi_{\mathrm{a/p}}(1/R) + O(1/QR)$

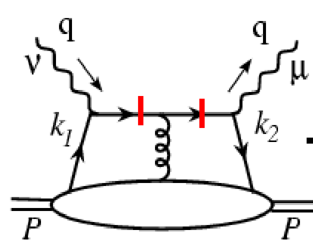
Process dependent partonic cross section (Feynman diagrams)

Process-independent Parton-level pdfs (Feynman diagrams)

Equal long-distance physics

Example:

$W_{\mu\nu}^{(1)} \propto$ 

$+ \ldots$ 

$+ \ldots$

All uncanceled divergences are absorbed into PDFs

Factorization is violated for Drell-Yan

- **Drell-Yan cross section:** $H_A(p_A)+H_B(p_B) \Rightarrow \ell^+\ell^-(Q)+X$

$$\sigma(Q)=\sigma_0(Q)+\sigma_2(Q)\frac{1}{Q^2}+\sigma_4(Q)\frac{1}{Q^4}+\ldots$$

  - Uncanceled partonic divergences of $\sigma_0(Q),\ \sigma_2(Q)$ are all from collinear region, and can be factorized

  - But, $\sigma_4(Q)$ has uncanceled IR divergence, and cannot be factorized into some kind of "PDFs or correlation functions"

- **The violation is small at high** $Q > m_{J/\psi}$

Data is consistent with the factorized first term at large Q

Violation is from soft gluon interactions between incoming partons

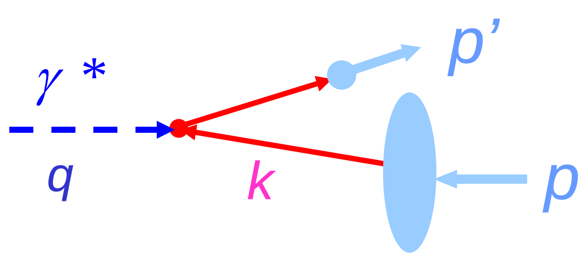
k_T – factorization

- Momentum of the "long-lived" parton is not necessary collinear to the hadron momentum

$$k^2 \approx 0 \;\Rightarrow\; k^\mu \approx x p^\mu + \frac{k_T^2}{2xp\cdot n} n^\mu + k_T^\mu$$

For cross sections with a single hard scale Q,
dk_T leads to power corrections: $\left(k_T^2/Q^2\right)^N$

- Physical processes with two observed scales: Q and q
with a large Q to ensure QCD factorization,
while $q \sim k_T$ probes a parton's transverse momentum



Both p and p' are observed

p' probes the parton's k_T

Effect of k_T is not suppressed by Q

$\varphi(x) \;\Rightarrow\; \varphi\left(x, k_T^2\right)$ = TMD parton distributions

TMD parton distributions

- Transverse momentum dependent (TMD) parton distributions: Belitsky, Ji, Yuan, 2003

$$\mathcal{M}_a = \int \frac{P^+ d\xi^-}{\pi} \frac{d^2\xi_\perp}{(2\pi)^2} e^{-ix\xi^- P^+ + i\xi_\perp\cdot k_\perp} \langle PS|\overline{\psi}_a(\xi)\mathcal{L}_v^\dagger(\infty;\xi)\mathcal{L}_v(\infty;0)\psi_a(0)|PS\rangle$$

$$= \frac{1}{2}\left[\underbrace{f_a^{\rm SIDIS}(x,k_\perp)}_{\text{Spin-averaged}}\gamma_\mu P^\mu + \frac{1}{M_P}\underbrace{q_{Ta}^{\rm SIDIS}(x,k_\perp)}_{\text{Spin-dependent - Sivers function}}\epsilon_{\mu\nu\alpha\beta}\gamma^\mu P^\nu k^\alpha S^\beta + \ldots\right]$$

- Connection to normal parton distributions

$$q_a(x) = \int d^2k_T\, f_a^{\rm SIDIS}(x, k_T) + \mathrm{UVCT}$$

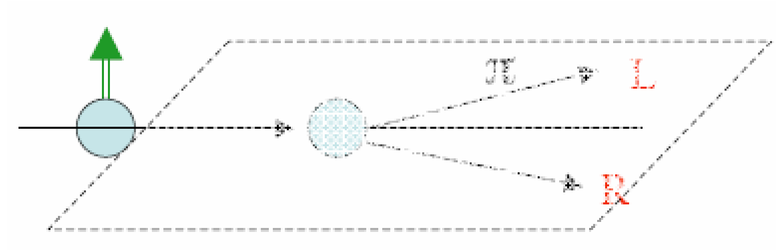
- Spin-dependent TMD parton distributions

→ Parton's orbital motion and
Non-vanishing single transverse-spin asymmetries

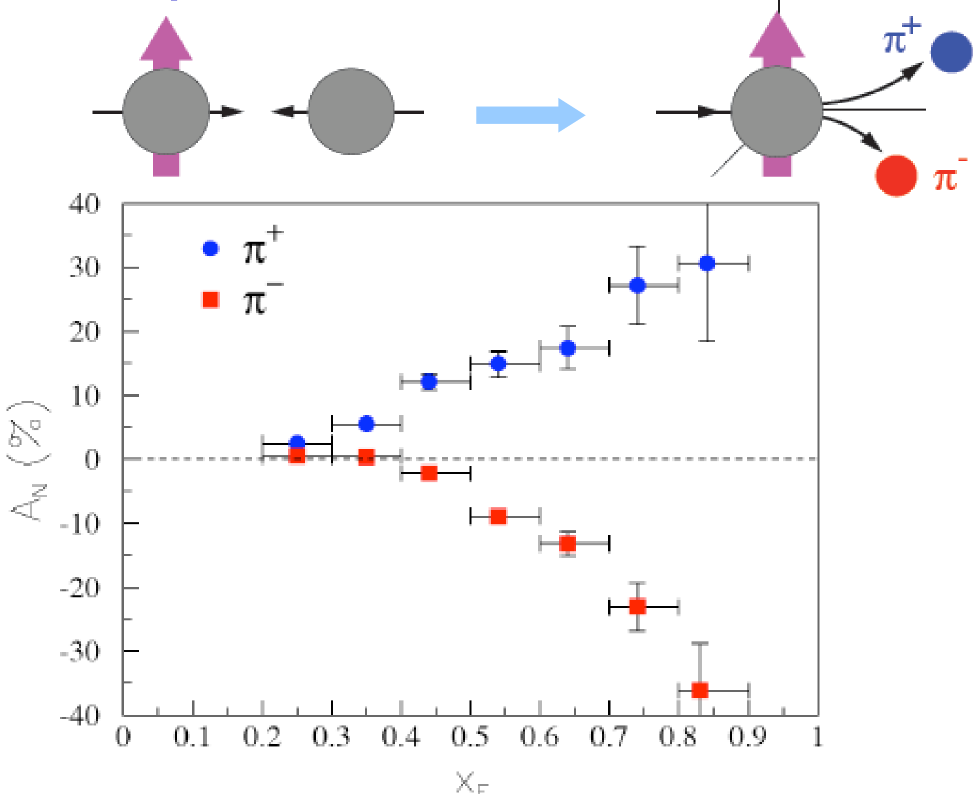
Single transverse spin asymmetry

- **Hadronic** $p\uparrow + p \to \pi(l)X$:

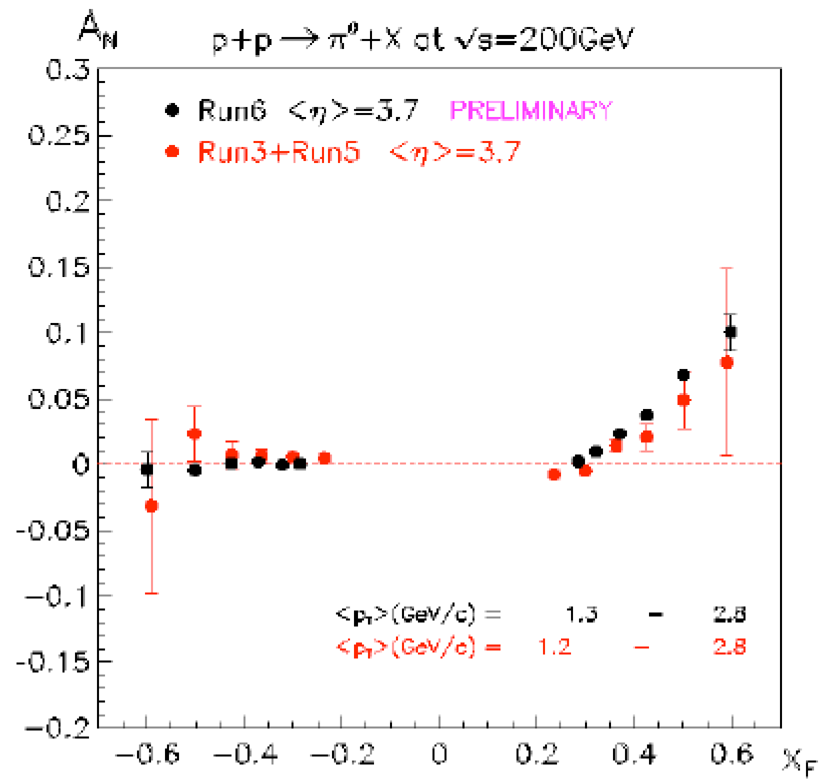
$$A_N = \frac{1}{P_{\rm beam}} \frac{N^{\pi}_{\rm left} - N^{\pi}_{\rm right}}{N^{\pi}_{\rm left} + N^{\pi}_{\rm right}}$$



If partons are collinear, $A_N \propto \alpha_s m_q$, to be small



E704

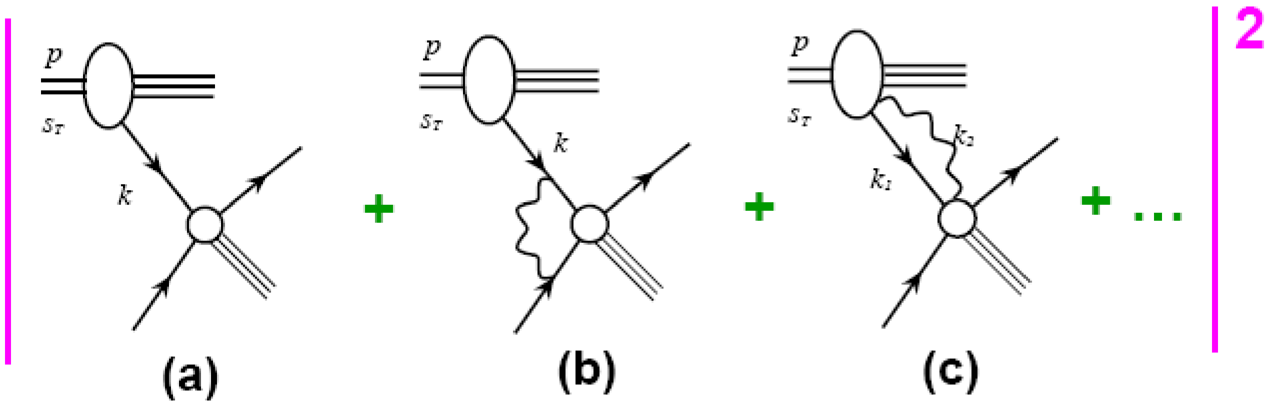


STAR (BRAHMS, too)

Asymmetry in collinear factorization

Efremov, Teryaev, 1982, Qiu, Sterman, 1991

- When there is only one large scale >> Λ_{QCD} collinear factorization should work:

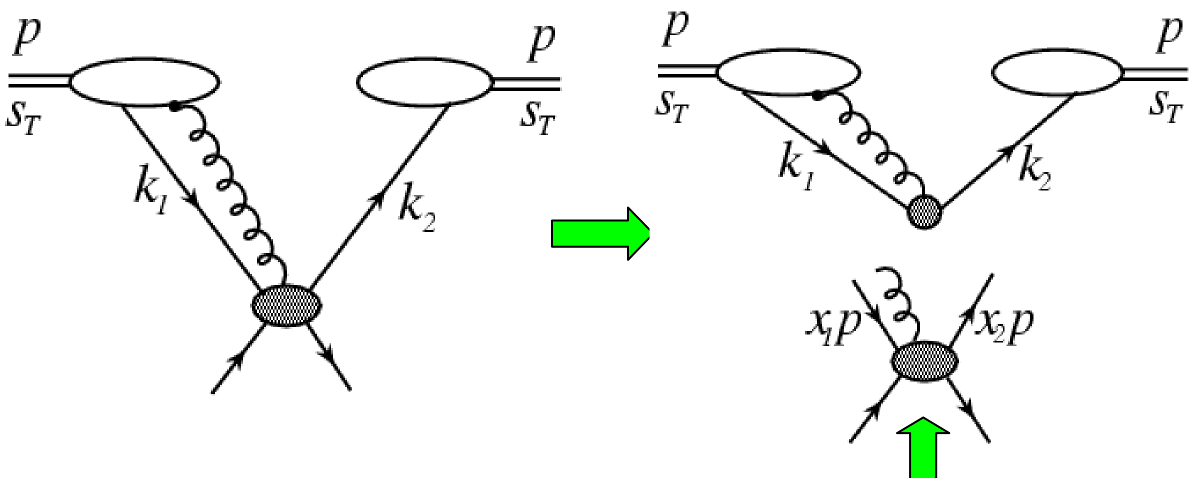
$\sigma(s_T) \sim$ 

- Leading spin dependent part of the cross section
  - ➡ Interference between amplitudes (a) and (b) or (c)
- The hadronic phase – the "i"
  - ➡ $\mathrm{Re}[(a)]$ interferes with $\mathrm{Im}[(b)]$ or $\mathrm{Im}[(c)]$
- $\mathrm{Re}[(a)] \times \mathrm{Im}[(b)] \propto m_Q \;\; \delta q(s_\perp)$

A_N from polarized twist-3 correlations

Qiu, Sterman, 1991, 1999

- **Factorization:**



$$T_F(x_1,x_2) \propto \langle \bar{\psi}\gamma^+ F^{+\perp}\psi \rangle$$

$$T_D(x_1,x_2) \propto \langle \bar{\psi}\gamma^+ D_\perp \psi \rangle$$

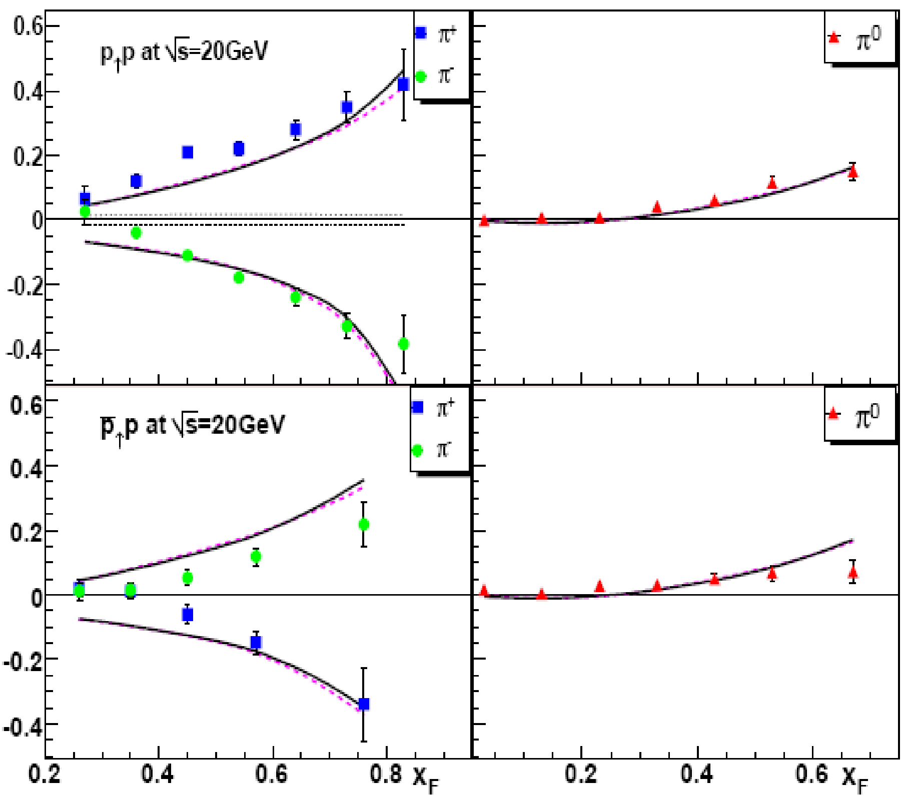
Normal twist-2 distributions

- **Twist-3 correlation functions:**

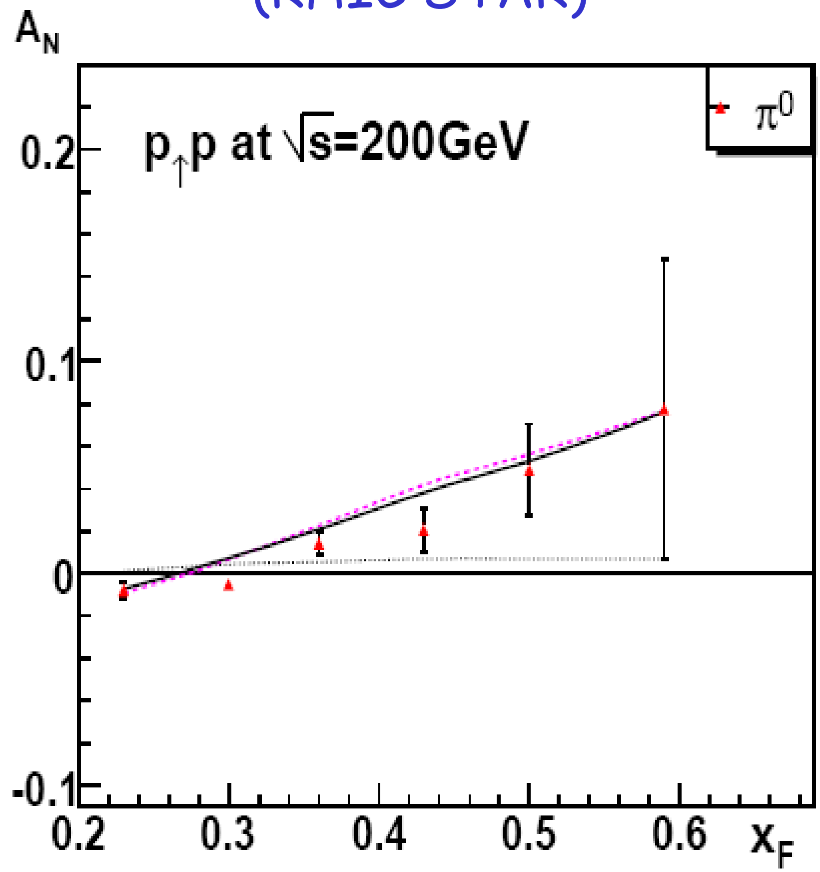
  - $T_F(x_1,x_2)$ and $T_D(x_1,x_2)$ **have different properties under the P and T transformation**
  - $T_D(x_1,x_2)$ **does not contribute to the A_N**
  - $T_F(x_1,x_2)$ **is universal, $x_1=x_2$ for A_N due to the pole**

Asymmetry from the $T_F(x,x)$

(FermiLab E704)



(RHIC STAR)



Kouvaris,Qiu,Vogelsang,Yuan, 2006

Nonvanish twist-3 function ➡ Nonvanish transverse motion

Asymmetry from Sivers function

- Sivers' function: $q_T(x,k_\perp)$

$$\varphi_q(x,k_\perp,\vec{s}_\perp)-\varphi_q(x,k_\perp,-\vec{s}_\perp)=q_T(x,k_\perp)\ \varepsilon_{\mu\nu\rho\sigma}\frac{\gamma^\mu n^\nu k_\perp^\rho s_\perp^\sigma}{M}$$

- Asymmetry in SIDIS:

$$\Delta\sigma(k_\perp,s_\perp)\propto[\varphi_q(x,k_\perp,\vec{s}_\perp)-\varphi_q(x,k_\perp,-\vec{s}_\perp)]\otimes D(z)$$

$$\propto\left(\varepsilon_{\mu\nu\rho\sigma}P^\mu n^\nu k_\perp^\rho s_\perp^\sigma\right)\ q_T(x,k_\perp)\otimes D(z)$$

Single transverse spin asymmetry can be generated by the Sivers function – spin-dependent TMD distribution

- TMD factorization works for SIDIS, as well as Drell-Yan when two physical scales satisfy: $Q \gg q_T\ (> \Lambda_{QCD})$

Ji, Ma, and Yuan

TMD vs Collinear factorization

- Relation between spin-dependent TMD distributions and collinear factorized distributions

$$\frac{1}{M_P}\int d^2\vec{k}_\perp \vec{k}_\perp^2 q_T(x,k_\perp) = T_F(x,x)$$

Compare to the spin-averaged case:

$$\int d^2k_T\, f_a^{\mathrm{SIDIS}}(x,k_T) + \mathrm{UVCT}\left(\mu^2\right) = q_a\left(x,\mu^2\right)$$

- Relation between two mechanisms of SSA:

Connection between both short-distance and long-distance parts

They are valid for different kinematical regions:

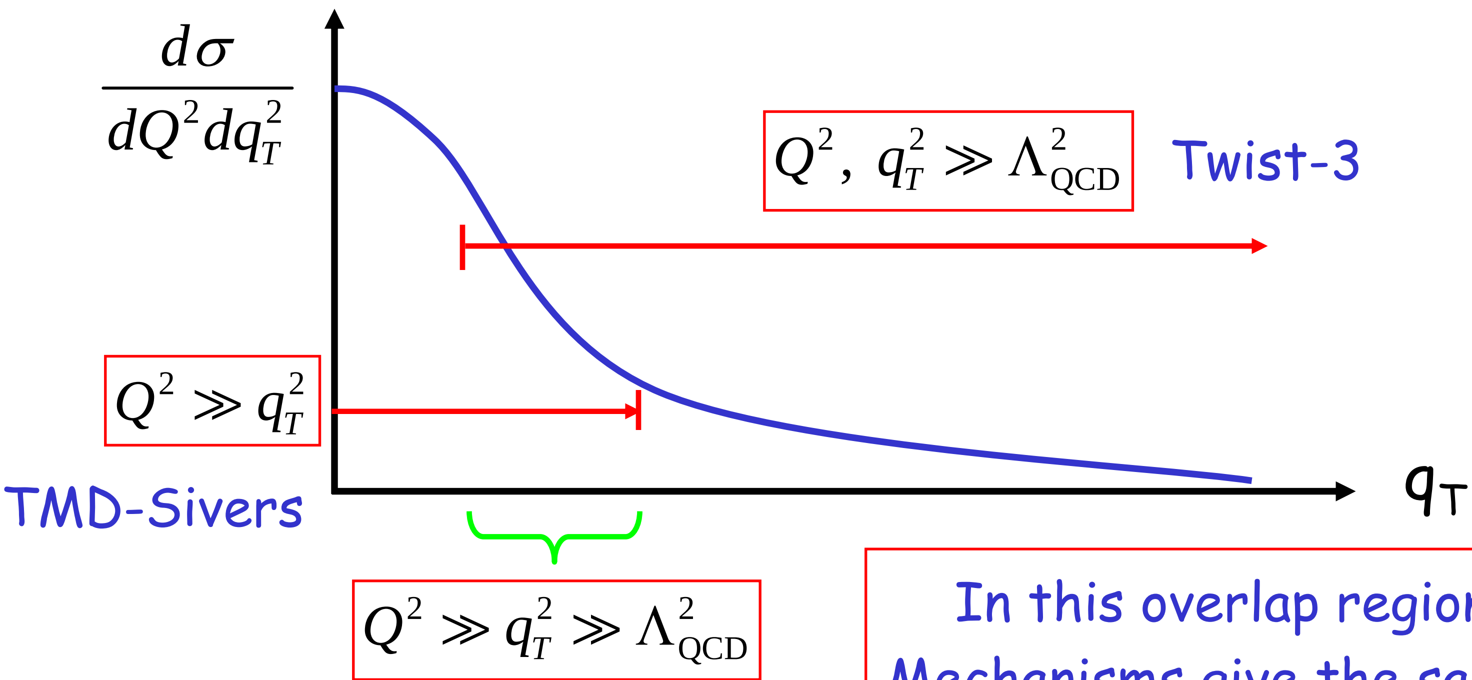
Twist-3 collinear: $Q_1 \ldots Q_n \gg \Lambda_{QCD}$

Sivers TMD: $Q_1 \gg Q_2 > \Lambda_{QCD}$

Unification of two mechanisms

- **Need a process for a case study:**
  - has two observed momentum scales: Q_1 and Q_2
  - both collinear and TMD factorization are valid in their respective kinematical regimes
- **Drell-Yan:**

Ji, Qiu, Vogelsang, and Yuan



In this overlap region, both Mechanisms give the same result

Hadronic di-jet production

$$H_1(p_A)+H_2(p_B)\Rightarrow Jet(p_1)+Jet(p_2)+X$$

- Dominated kinematical region: $\boxed{P_T \gg q_T}$

$$p_1=\frac{P}{2}+q \qquad p_2=\frac{P}{2}-q$$

- Idea: if the TMD factorization is valid in this region, di-jet momentum imbalance is an excellent observable to test the universality of the Sivers function   Boer and Vogelsang

  But, TMD factorization was not proved for this process

- Collinear factorization should be valid for $P_T,\ q_T \gg \Lambda_{\rm QCD}$
  - Calculate di-jet SSA in twist-3 approach for $P_T \gg q_T \gg \Lambda_{\rm QCD}$
  - Calculation itself does not prove TMD factorization, but, it provides the necessary condition for TMD factorization to be valid

SSA of the di-jet imbalance - I

- Need to consider three-particle final-state:
- Asymmetry can be generated by both initial- and final-state interaction
- Very simple representation of qq' -> qq' channel:

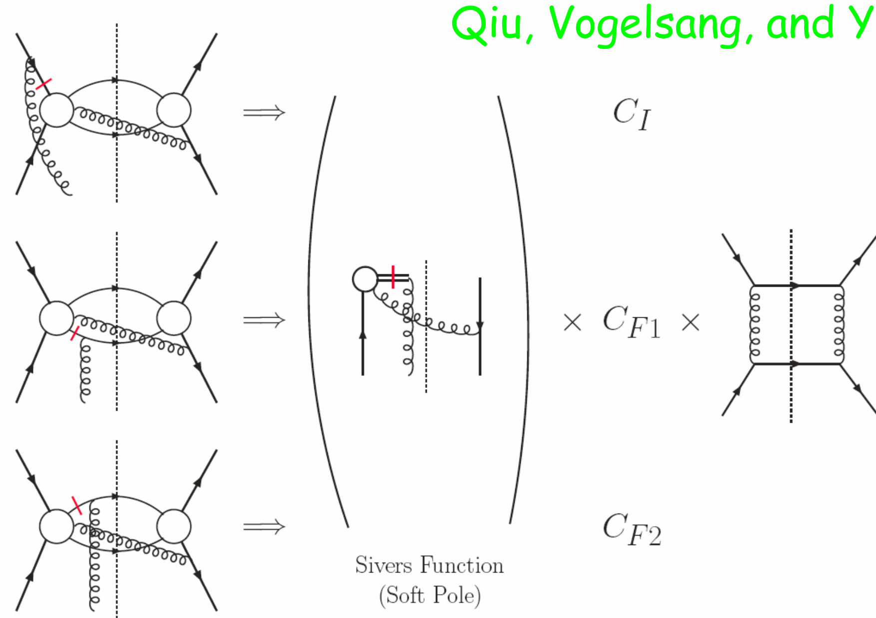
$$\frac{d\Delta\sigma}{dy_1\,dy_2\,dP_\perp^2\,d^2\vec{q}_\perp} \propto q'(x)\,q_T^{\mathrm{SIDIS}}(x,q_\perp)\,\left(C_I + C_{F_1} + C_{F_2}\right)\,\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$$

when k is parallel to the polarized hadron

Perturbatively generated Sivers' function at g^2

Initial- and final-state interactions differ only by a color factor

Qiu, Vogelsang, and Yuan



SSA of the di-jet imbalance - II

- For partonic channels, the hard part depends only P_T

$$H^{\rm Sivers}_{ab\to cd} = \frac{\alpha_s^2\pi}{\hat{s}^2}\sum_i (C_I^i + C_{F1}^i + C_{F2}^i)h^i_{ab\to cd}$$

  - For each diagram "i", there is a sum of color factors from the initial- and final-state interaction
  - For each partonic channel, the hard part is equal to a sum over all partonic diagrams with the proper color factors,

- All q_T dependence at the leading power of q_T/P_T is factorized into the $O(g^2)$ perturbatively generated Sivers function

Is the TMD factorization valid for di-jet momentum imbalance?

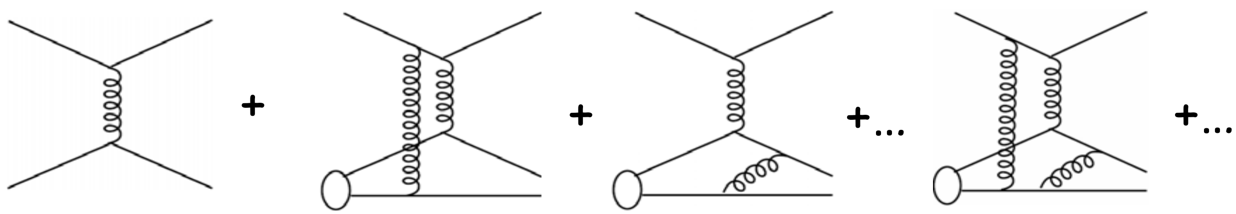
The calculation done in collinear factorization with $P_T \gg q_T \gg \Lambda_{QCD}$ itself does not real tell what happens when $q_T \to 0$

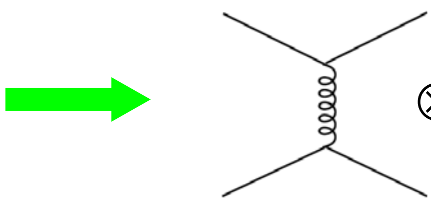
SSA of the di-jet imbalance - III

- Test the TMD factorization by studying long-distance physics of partonic scattering cross section:

  If the factorization is valid, all factorized long-distance information should be process independent

- Consider the poles from collinear gluon attachment to the lowest order partonic diagram in the TMD approach





⊗ TMD distribution with the exponentiated gauge link

Qiu, Vogelsang, and Yuan

Initial-state and final-state have different color flow. If keeps the difference in the hard part, we derive the same leading order hard part

SSA of the di-jet imbalance - IV

Collins and Qiu

- A simple model:
  - similar to the model by Brodsky et al
  - ❖ Hadron is made of a fermion ψ and a scalar φ
  - ❖ There are two hadrons
  - ❖ Gauge field (Abelian) couples g_i to ψ_i and $-g_i$ to φ_i

$$\lambda_i \left(\bar{H}_i \psi_i \, \phi_i^\dagger + \bar{\psi}_i H_i \, \phi_i \right)$$

- Idea:

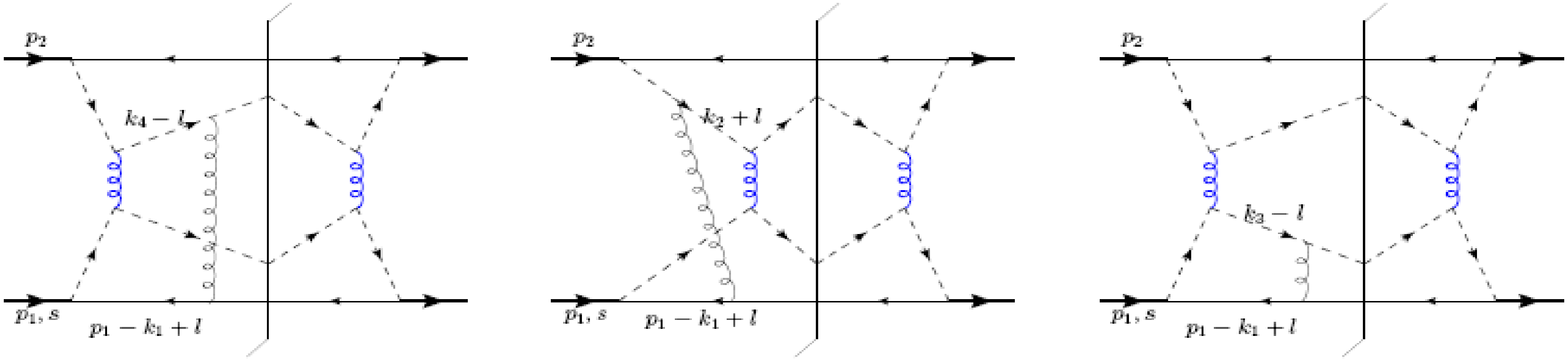
  If the TMD factorization is valid,

  - ❖ Gauge link of hadron H1 should not depend on the property of hadron H2, or any details of the subprocesses
  - ❖ Leading contribution from multiple gluon interaction should be expressed in terms of gauge link times the same lowest order hard part

  Otherwise, the TMD factorization is violated

Results of the model calculation

- Leading contribution to SSA:



Phase from the leading pole: $(i\pi)(g_1 + 2g_2)\delta(\ell^+)$
depends on the g_2!

- Can we keep the g_2 dependence in the hard part?

We find that the $(i\pi)^2$ from two gluon exchange
Also depends on g_2 , which cannot be factored into
The same lowest order hard part with g_2.

Gauge link of hadron 1 depends on the property of hadron 2
– failure of the TMD factorization

Summary

- We calculated di-jet momentum imbalance in hadronic collisions using collinear factorization approach for the $P_T \gg q_T \gg \Lambda_{QCD}$

- Our first non-trivial order result can be factorized into a hard part at a scale P_T, while the $O(g^2)$ Sivers Function keeps all q_T dependence.

- In a simple model calculation, we show that the TMD factorization for hadronic di-jet momentum imbalance is violated

- RHIC measurement on SSA of di-jet production as a function of momentum imbalance probes the transition to the nonfactorized regime