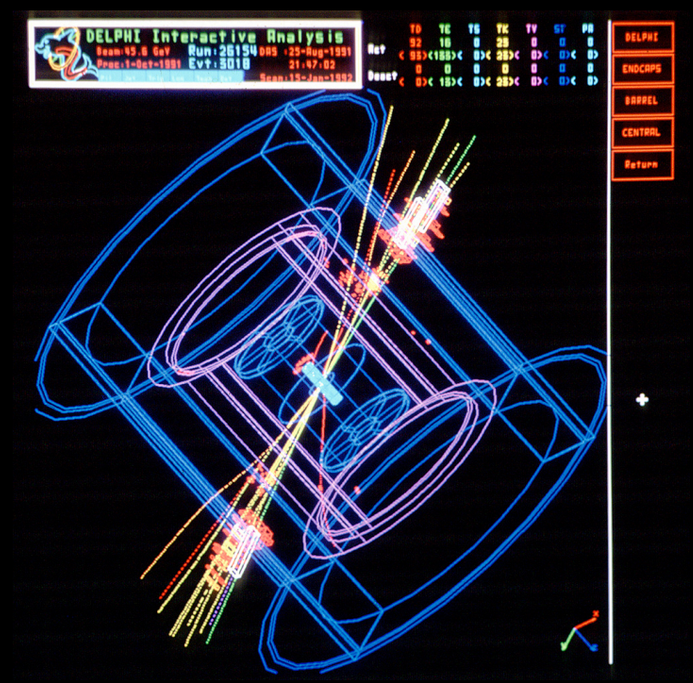
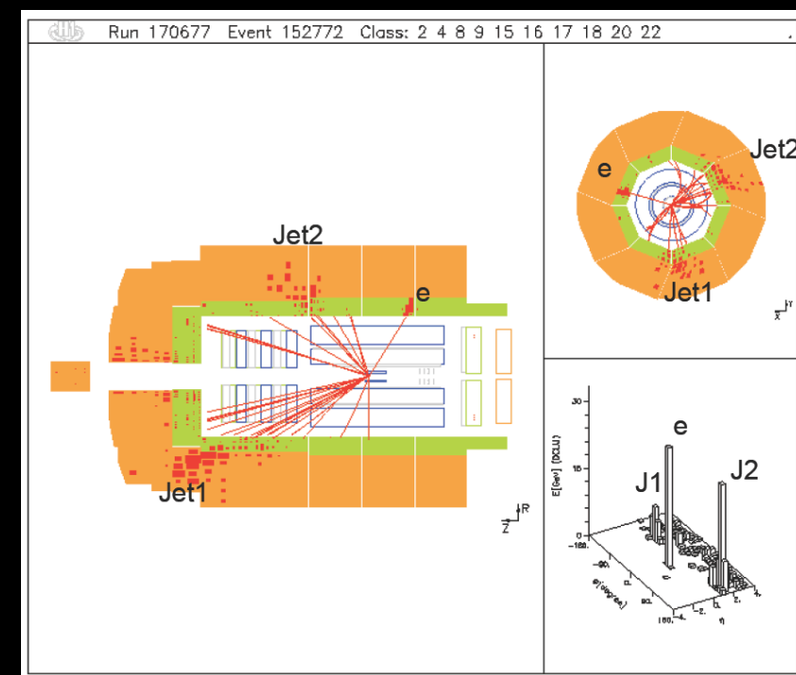
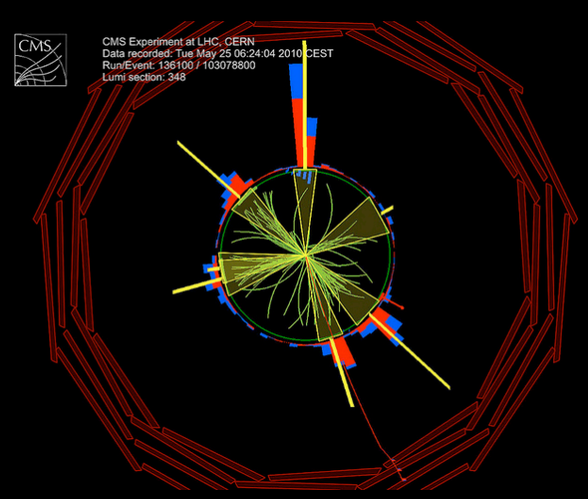
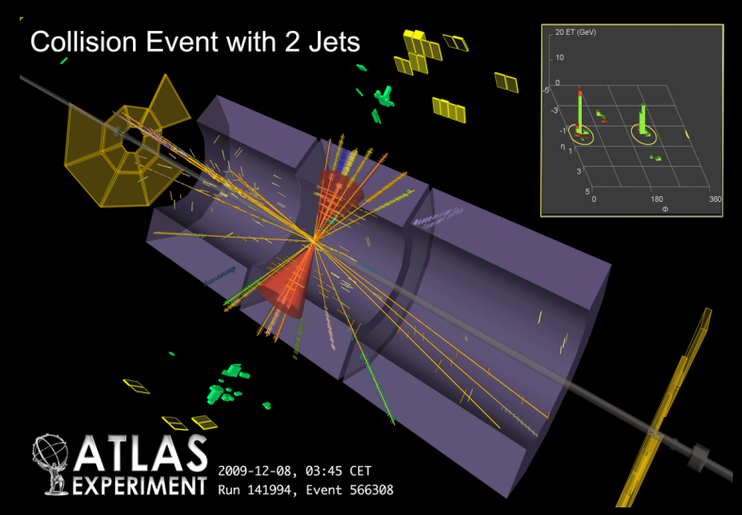
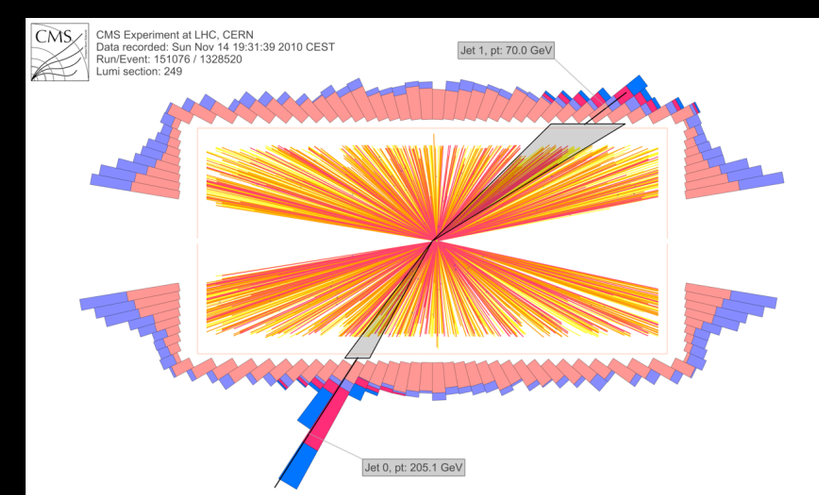
Precision Jet Physics in High-Energy Collisions











Los Alamos NATIONAL LABORATORY

Christopher Lee *T-2*

LANL P-25 Seminar
April 9, 2013

Outline

- QCD and Jets
- Event Shapes
- Theory Toolbox for Precision Jet Physics:
  Factorization, Resummation, and Effective Field Theory
- Predictions for e^+e^- Event Shapes
- Predictions for 1-Jettiness in DIS
- Conclusions

I. QCD and Jets

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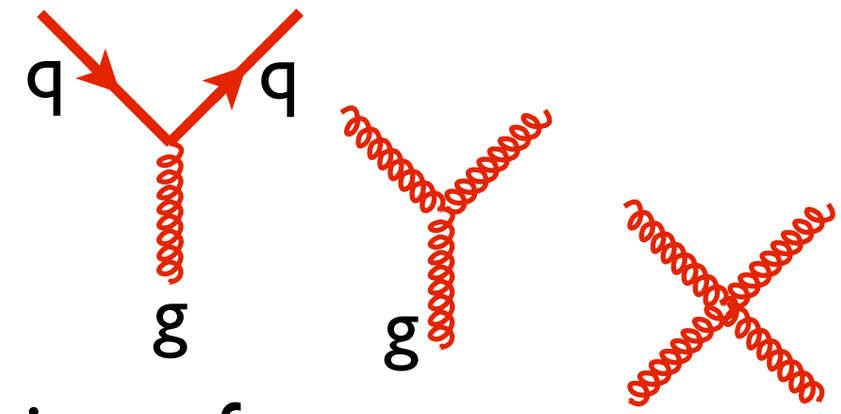
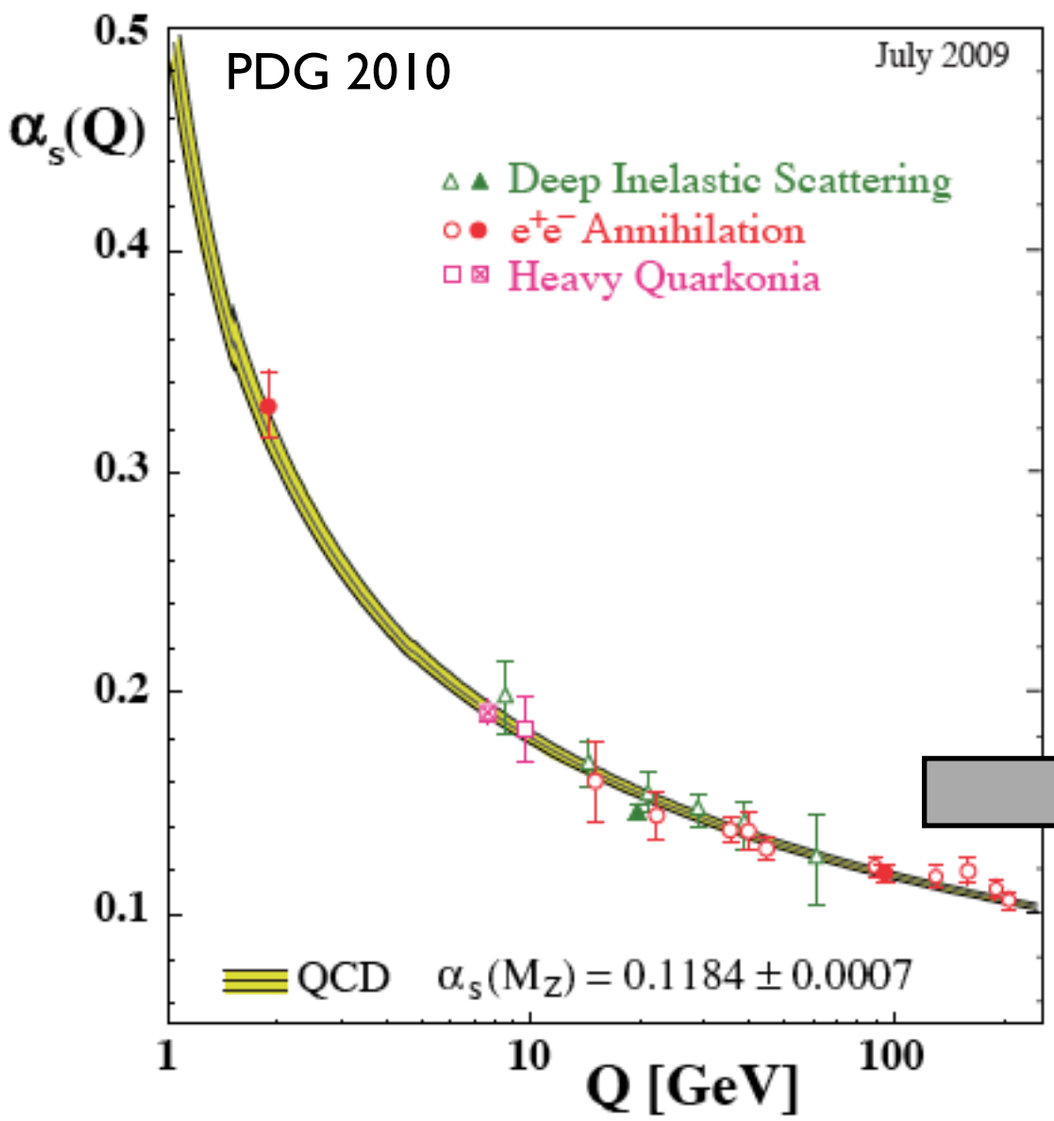
Quantum Chromodynamics

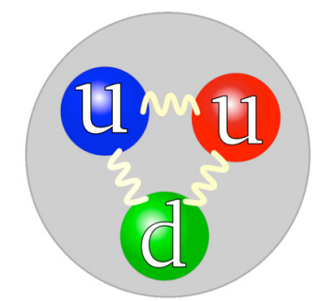
- QCD is theory of "Strong Interactions" amongst quarks and gluons

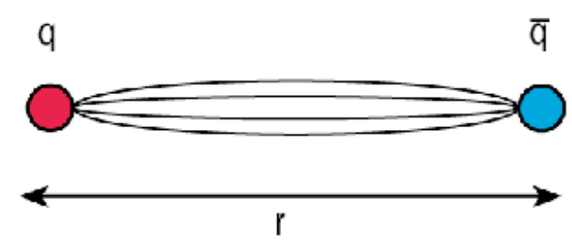


- Observed strength of interaction is function of energy



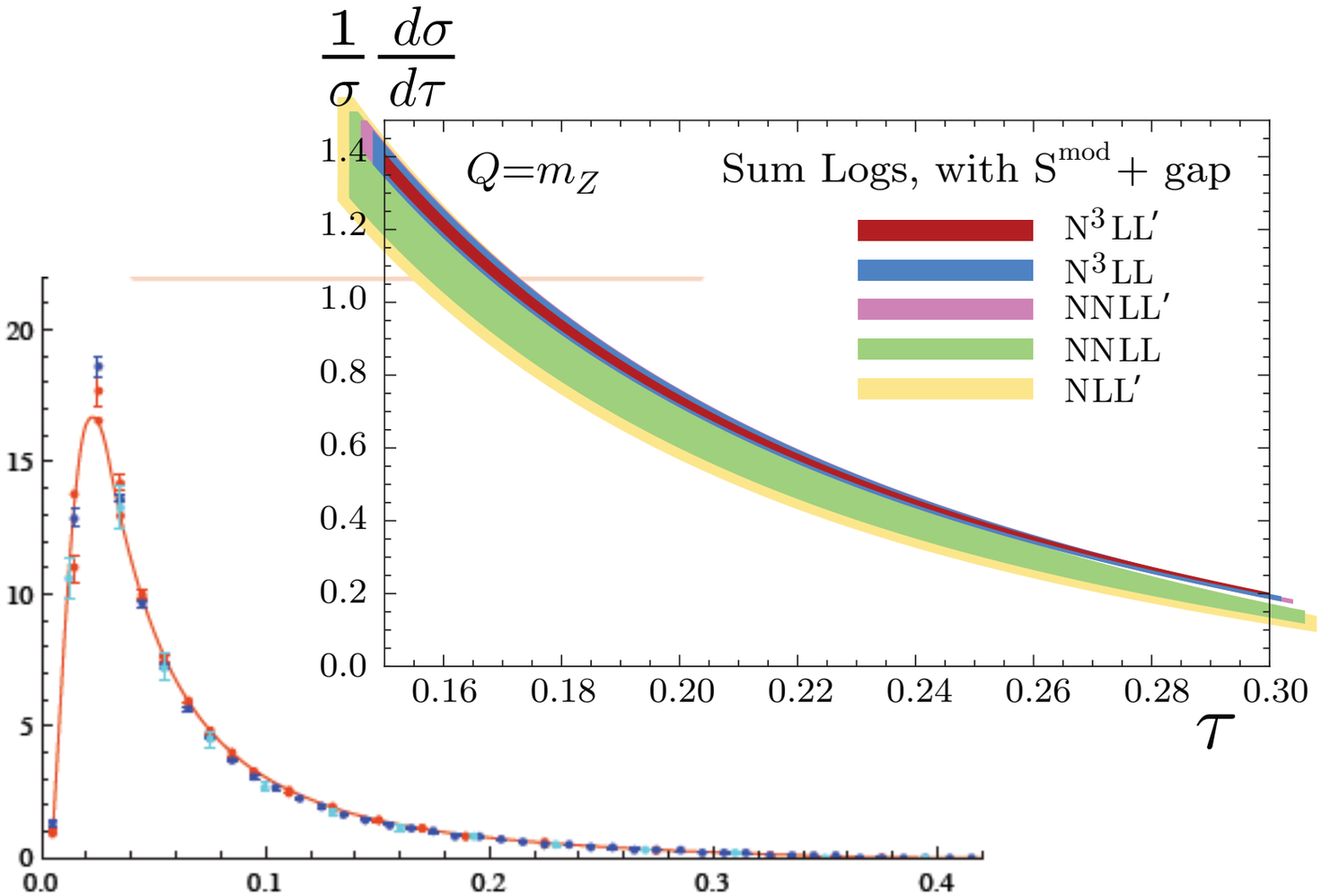
- At ordinary energies, quarks and gluons always "confined" into "colorless" hadrons.

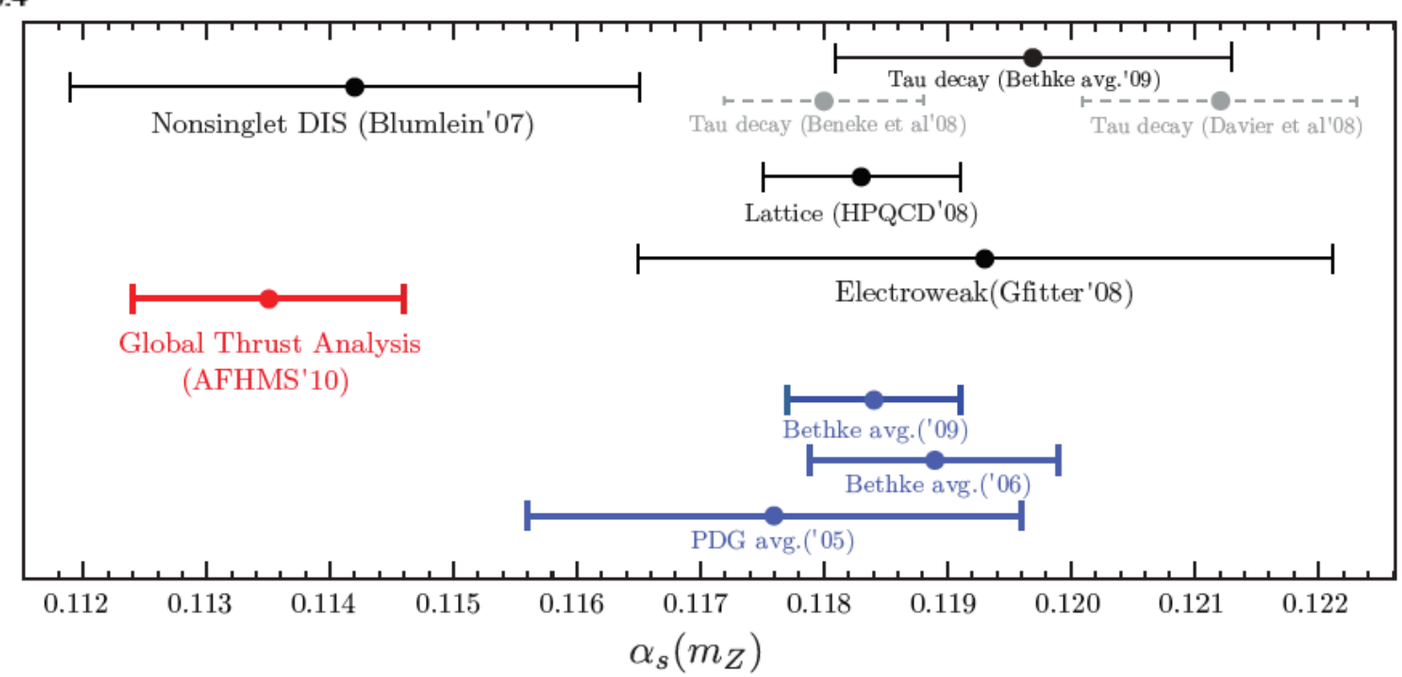




"Asymptotic Freedom"

Precision Extraction from Event Shapes



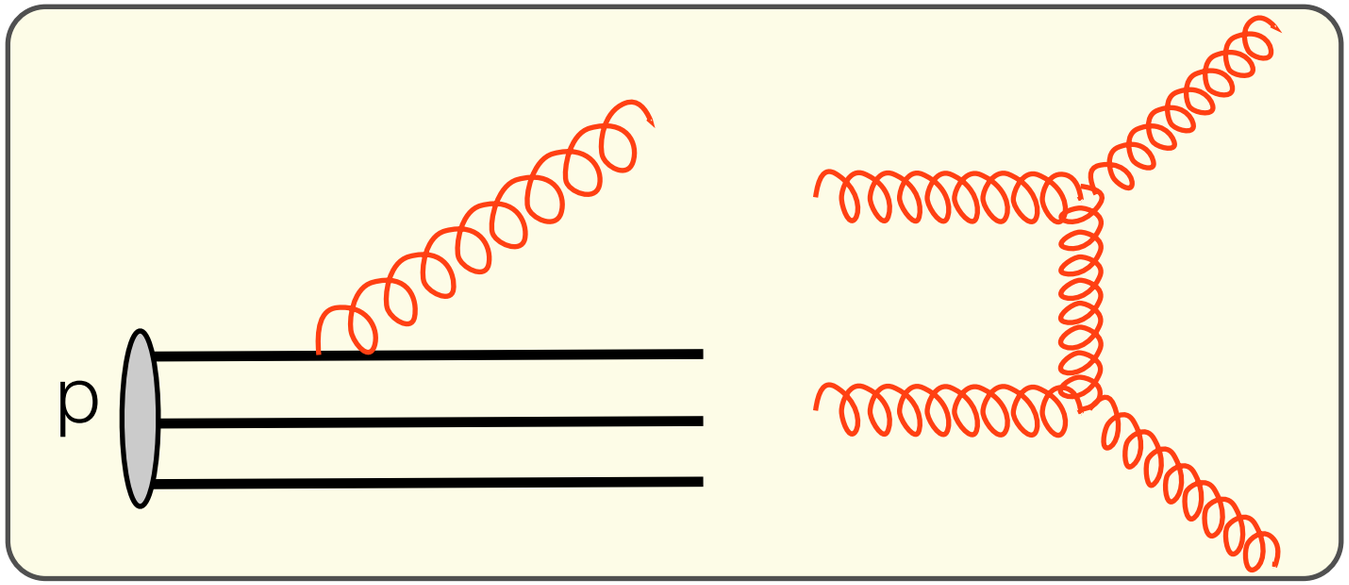


NNNLL perturbative prediction + nonperturbative soft power correction led to most precise extraction of strong coupling from event shapes

Abbate, Fickinger, Hoang, Mateu, Stewart (2010)

$$\alpha_s(m_Z) = 0.1135 \pm .0010$$

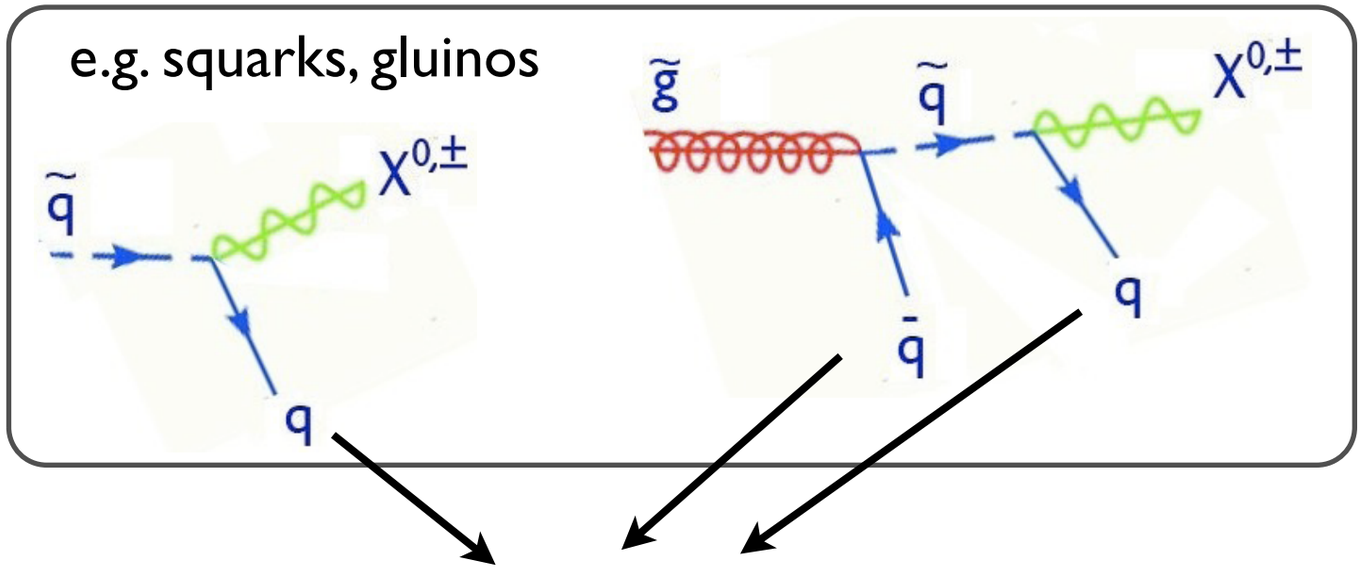
Jets from New Physics



LHC, Tevatron collide quarks and gluons bound in protons

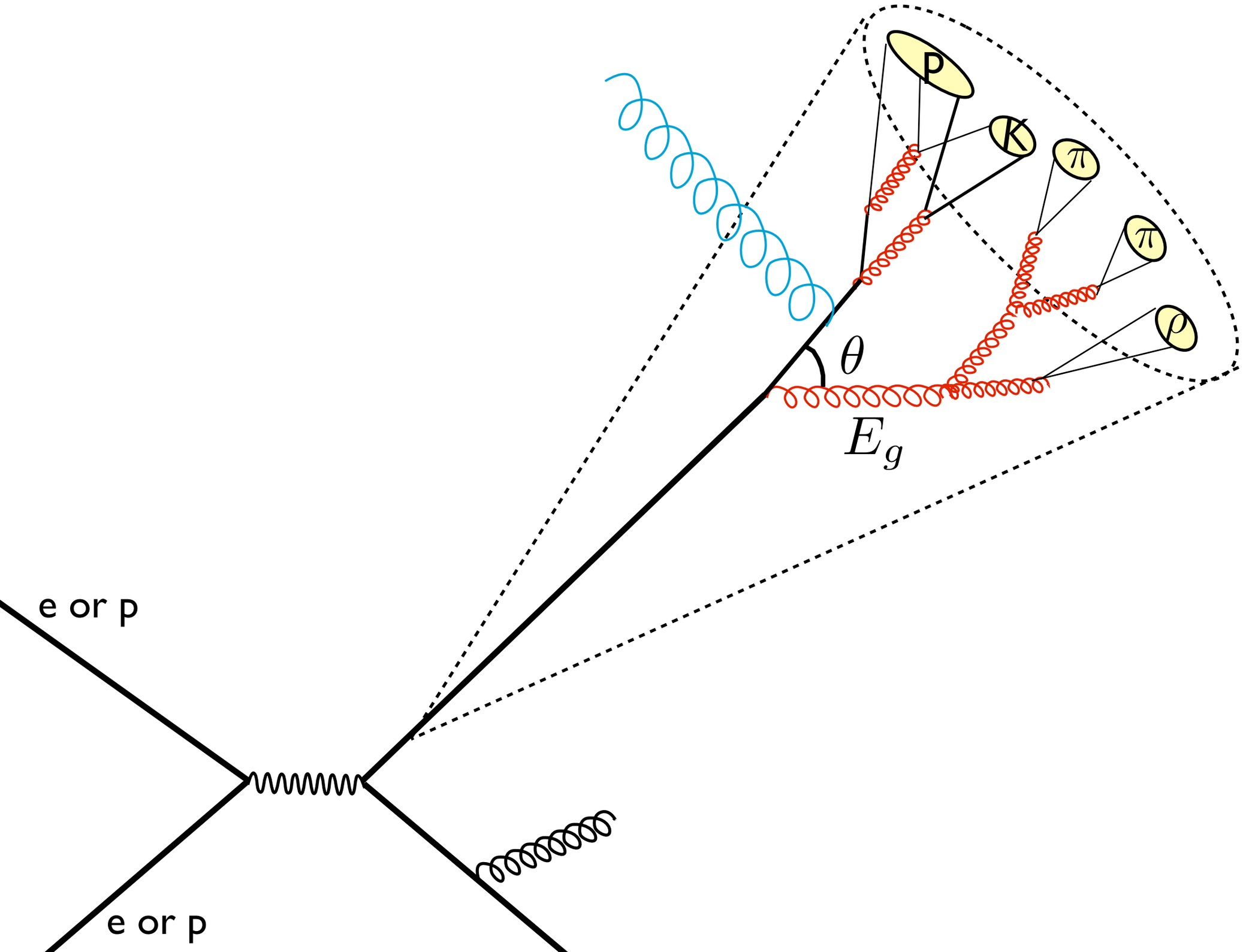
produce jets copiously, and any new particles that interact strongly

Any new particles that feel the strong force will decay to quarks and gluons, producing jets!



produce jets!

Formation of Jets



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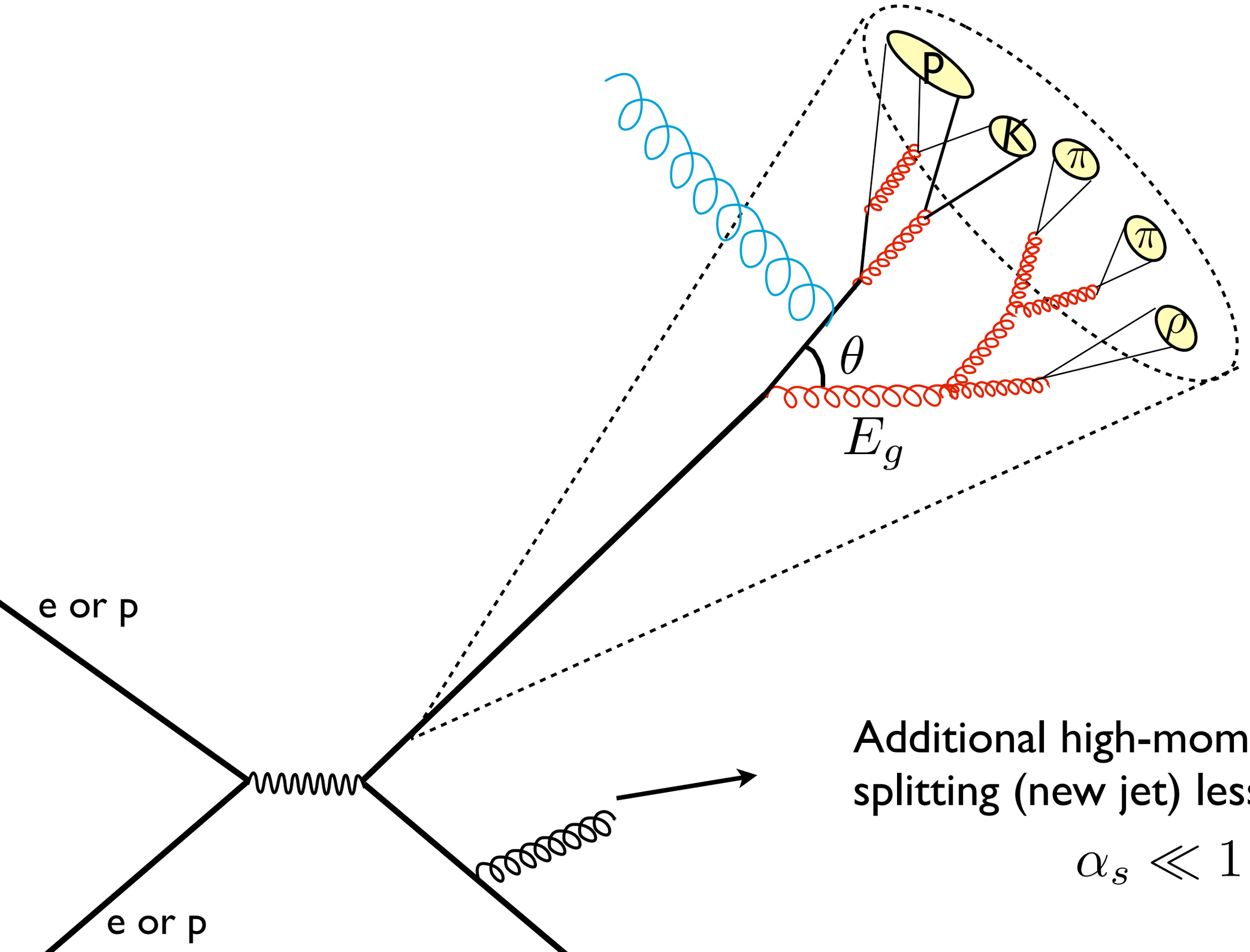
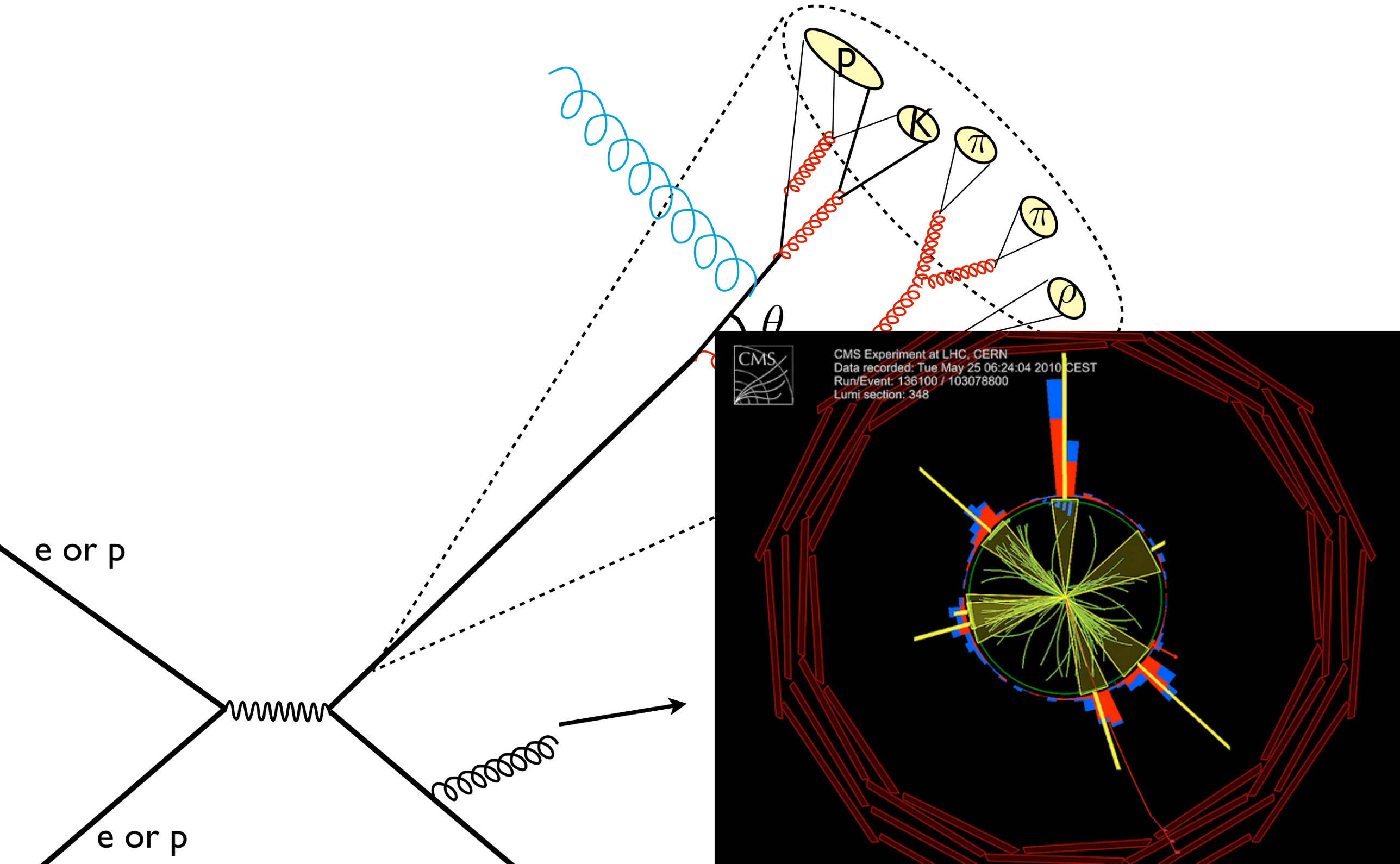


Additional high-momentum splitting (new jet) less likely

$$\alpha_s \ll 1$$

Formation of Jets



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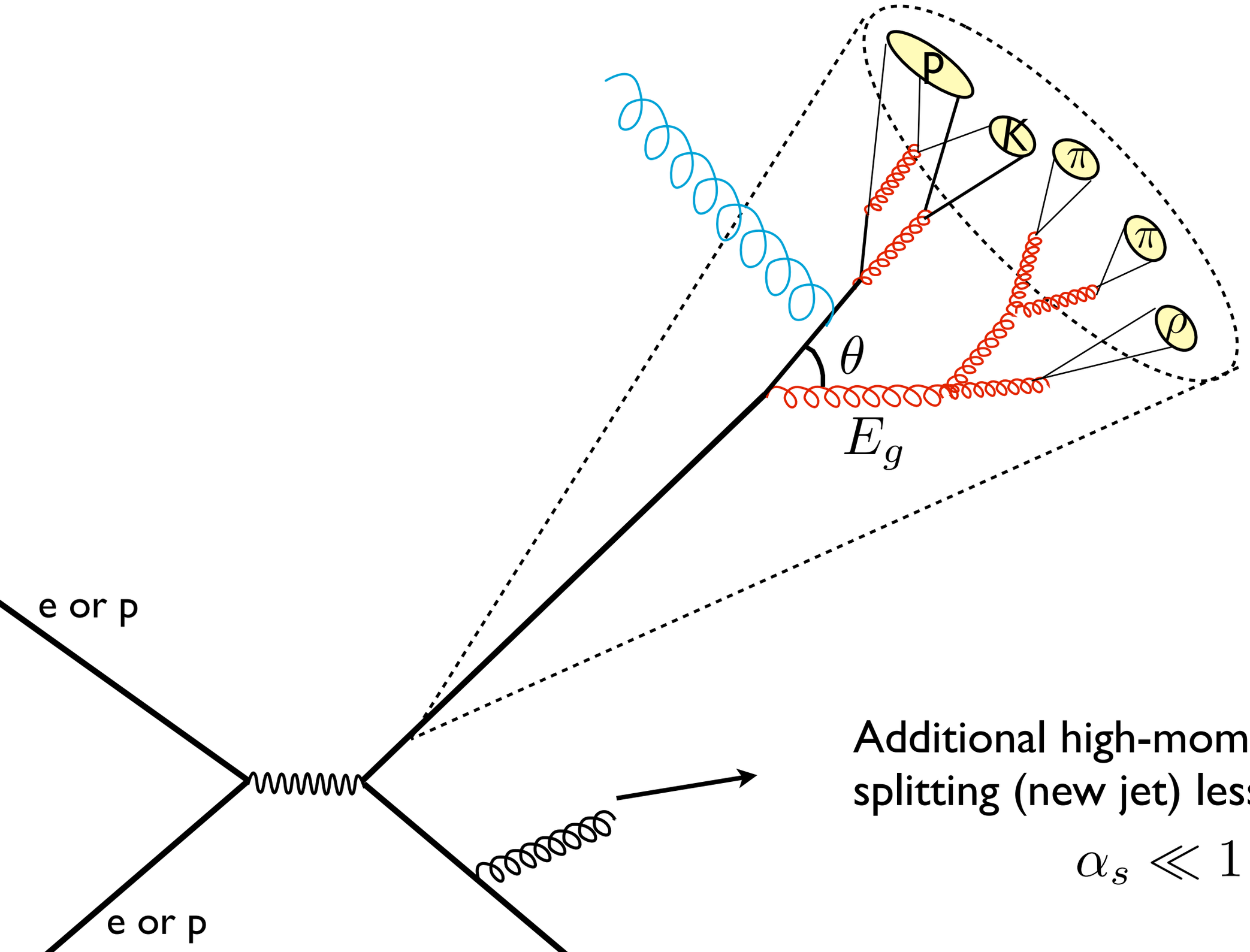

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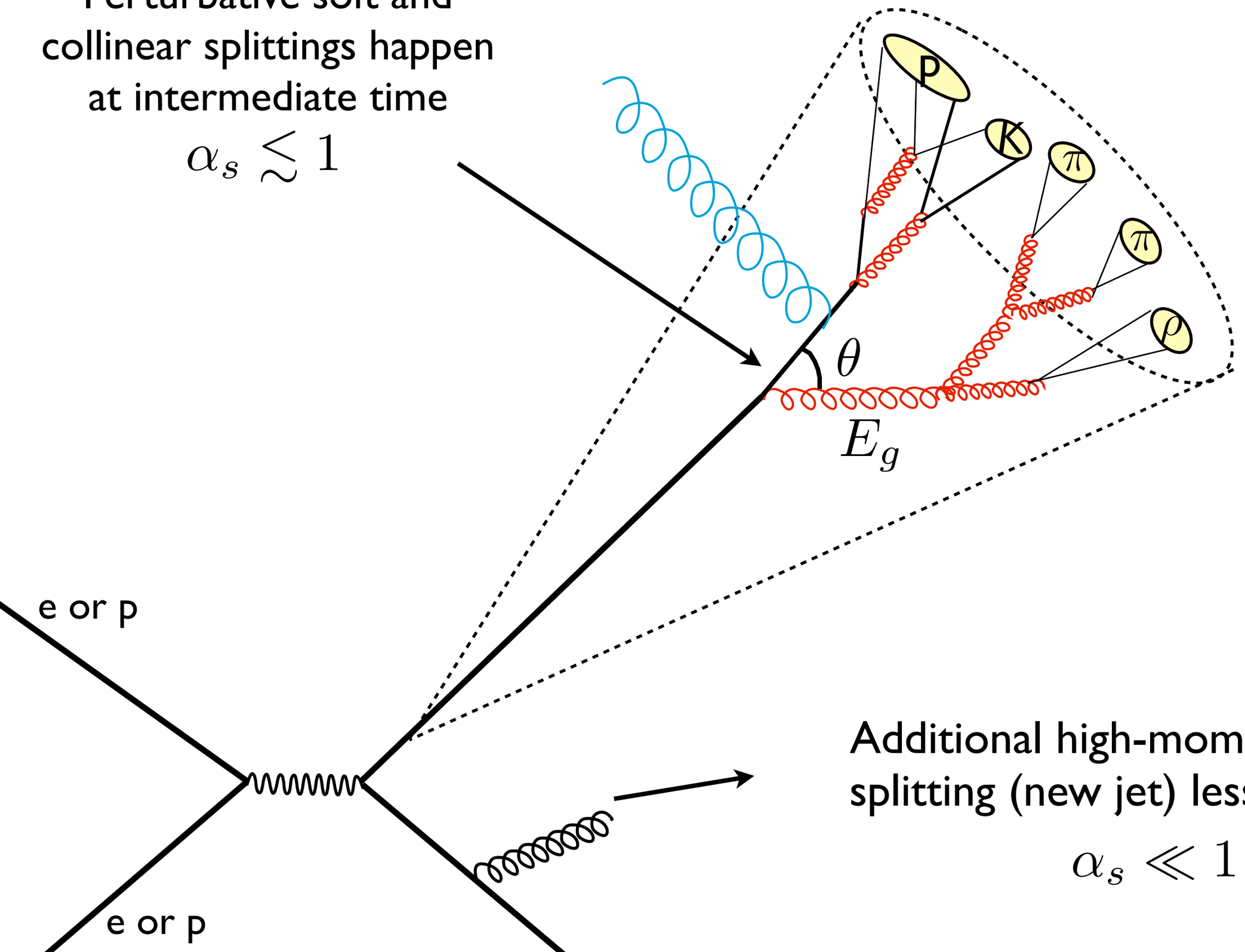
Formation of Jets

Perturbative soft and collinear splittings happen at intermediate time

$\alpha_s \lesssim 1$



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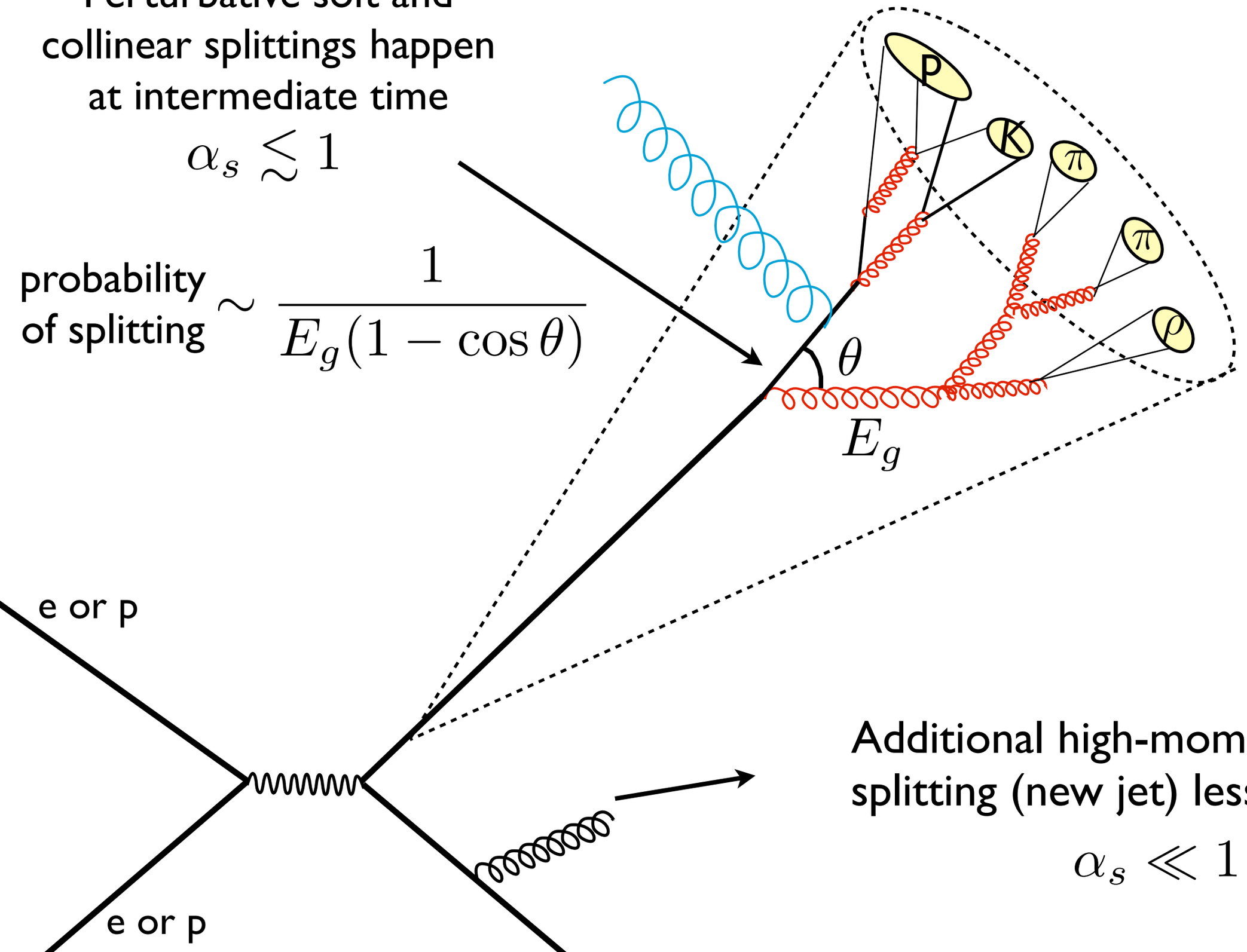
probability of splitting $\sim \dfrac{1}{E_g(1-\cos\theta)}$



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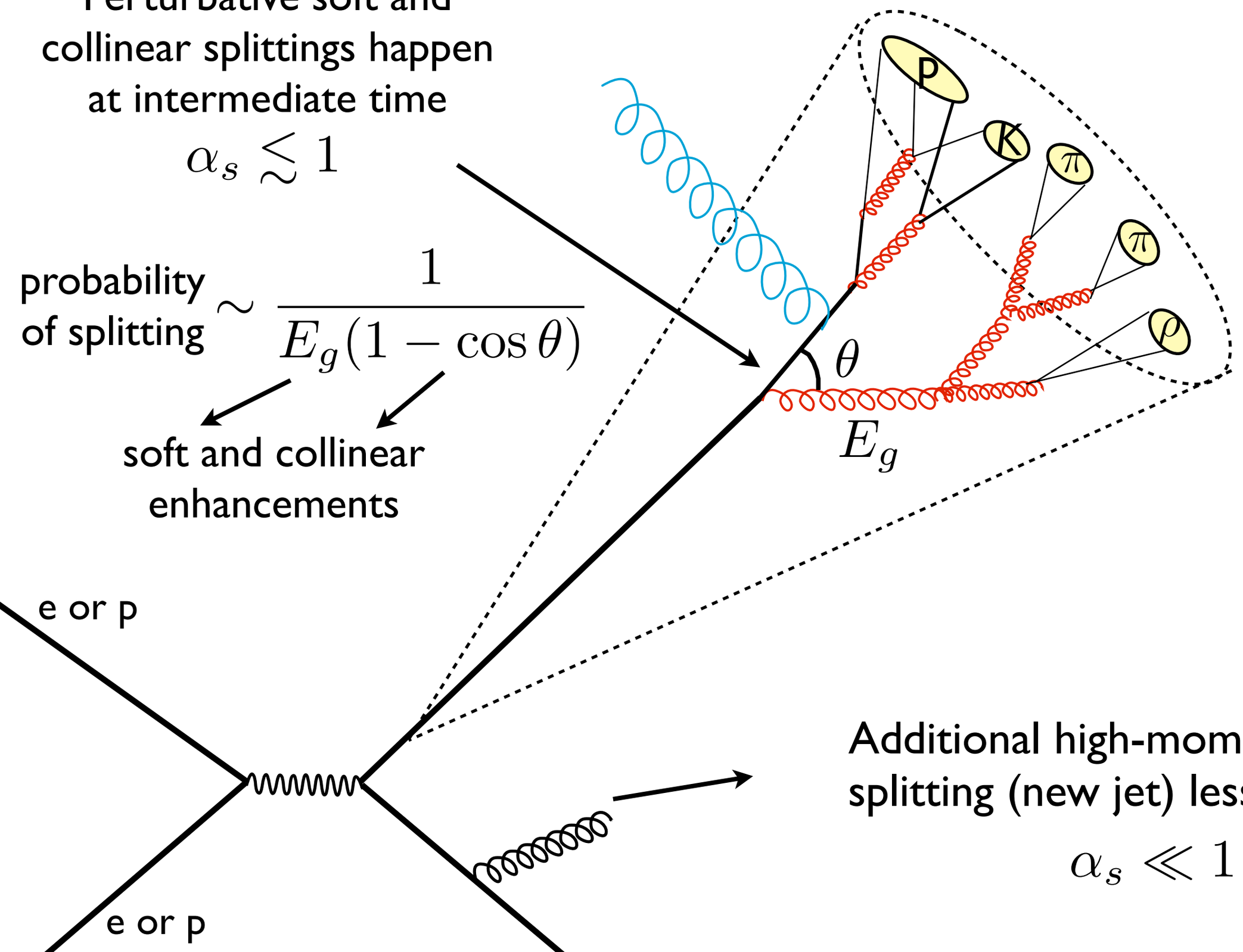
soft and collinear enhancements



Additional high-momentum splitting (new jet) less likely

$$\alpha_s \ll 1$$

Formation of Jets

Perturbative soft and collinear splittings happen at intermediate time

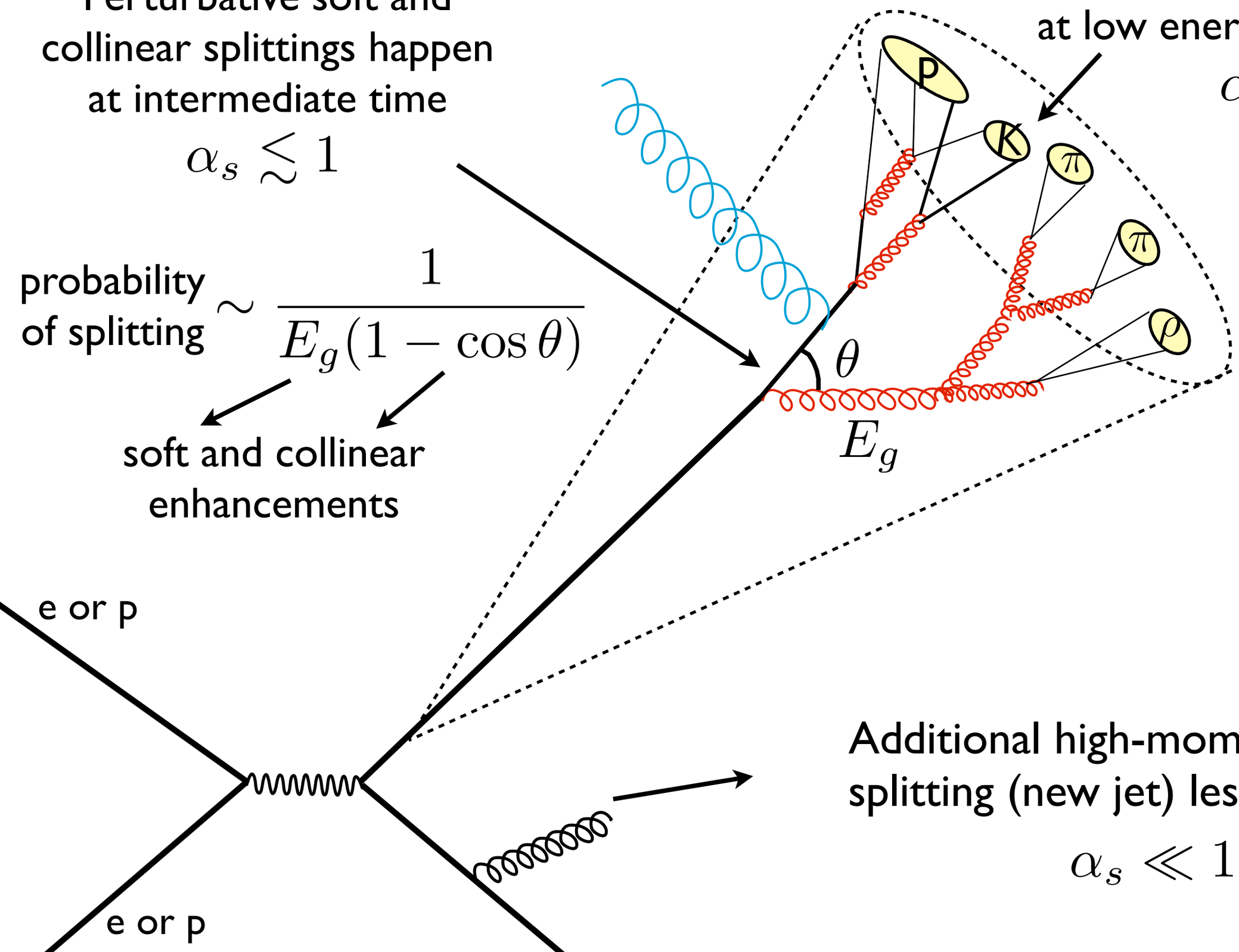
$\alpha_s \lesssim 1$

Hadronization at late time at low energy scale

$\alpha_s \gg 1$

probability of splitting $\sim \dfrac{1}{E_g(1-\cos\theta)}$

soft and collinear enhancements



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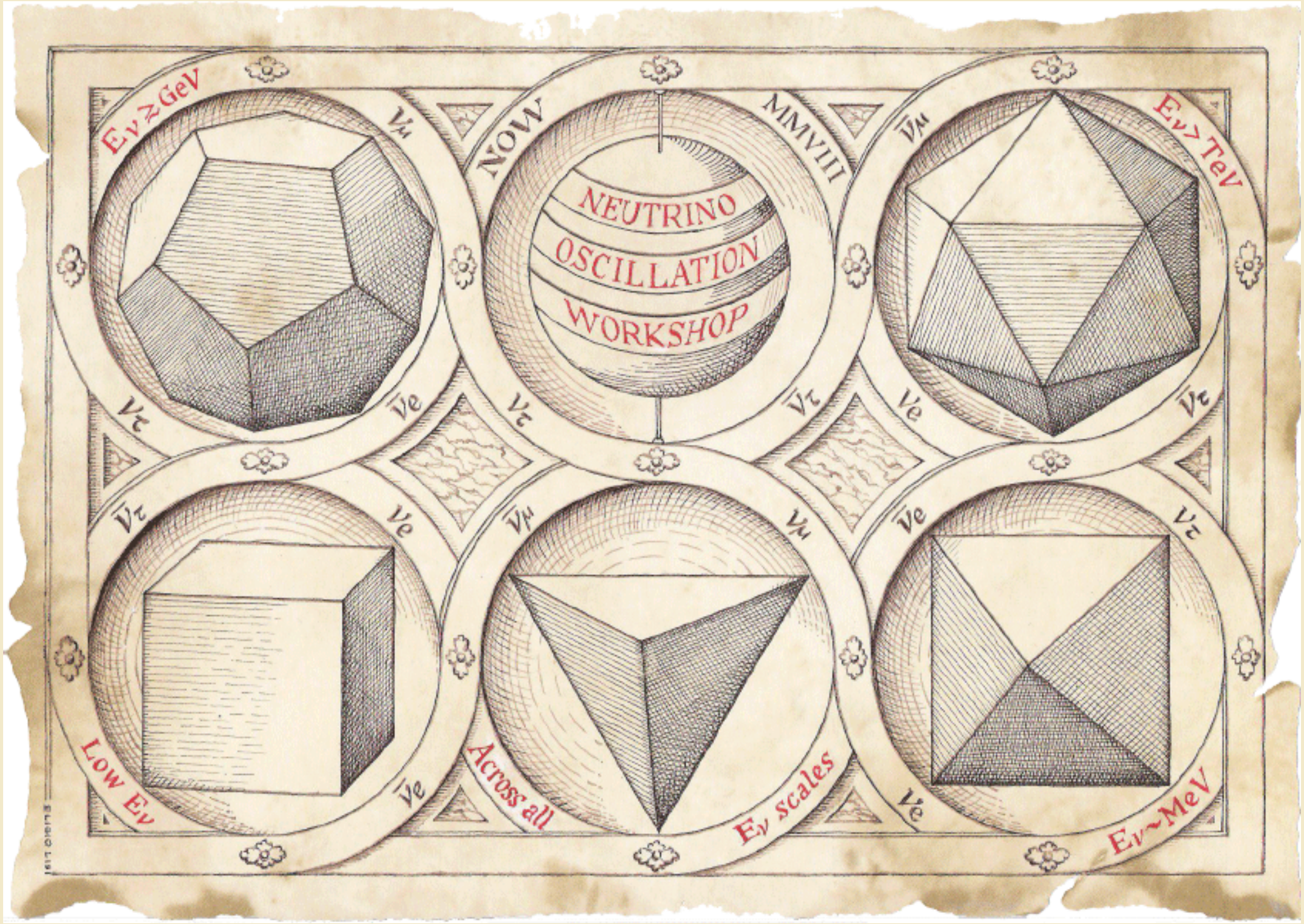
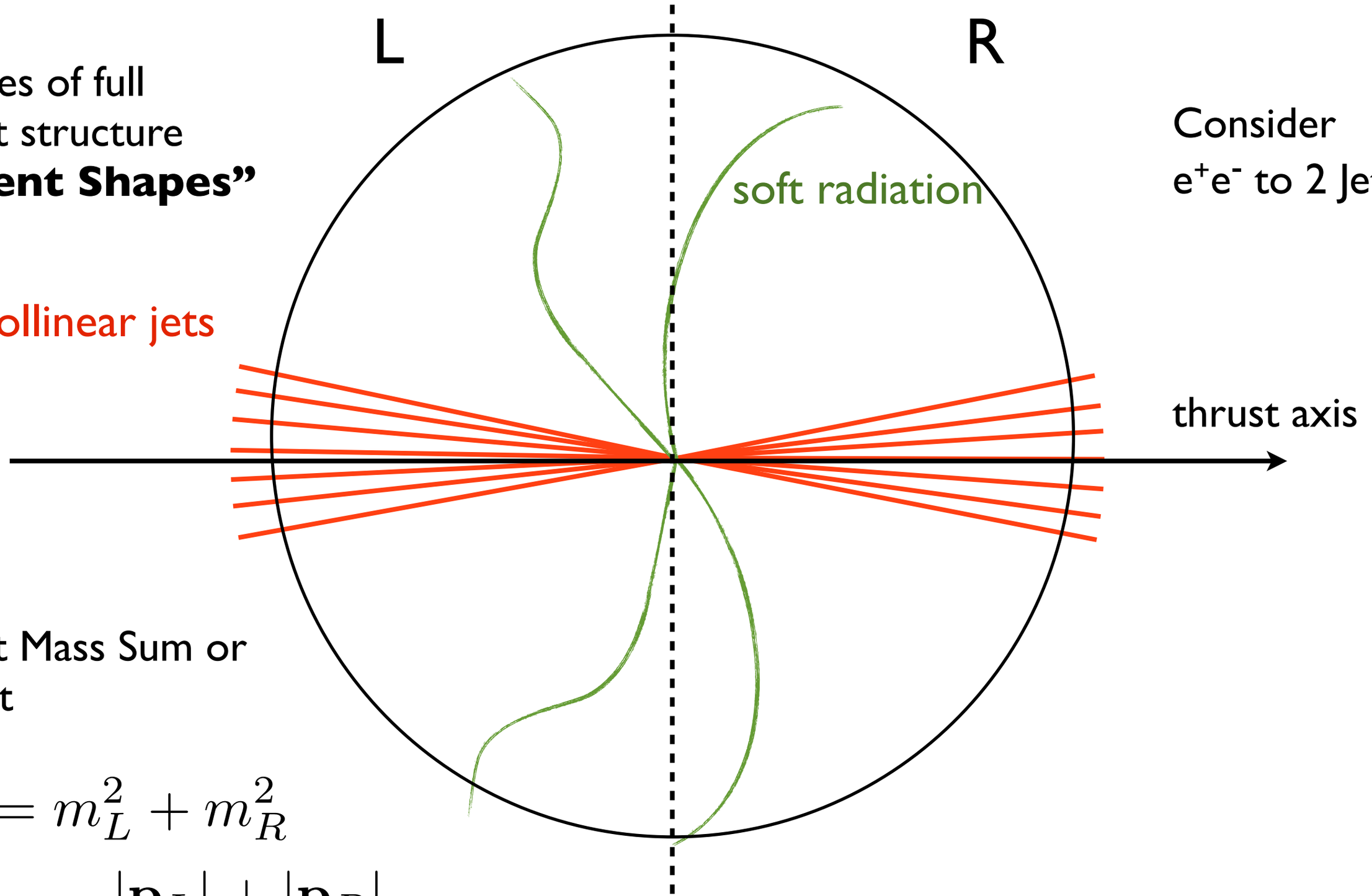
II. Event Shapes



Probing Jets Globally with Event Shapes

Probes of full
event structure
"Event Shapes"

Consider
e^+e^- to 2 Jets



e.g. Jet Mass Sum or
Thrust

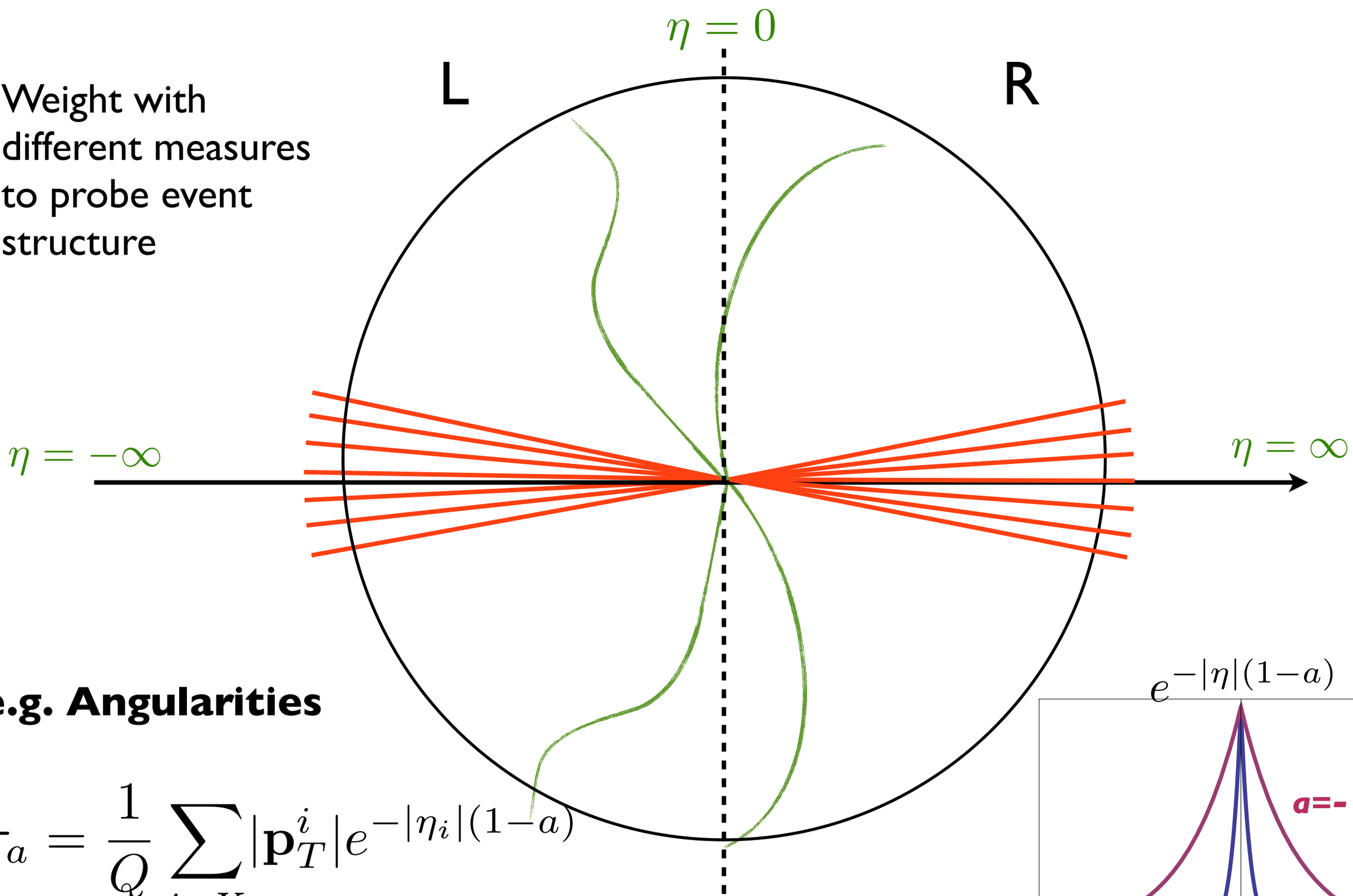
$$m^2 = m_L^2 + m_R^2$$

$$\tau = 1 - \frac{|\mathbf{p}_L| + |\mathbf{p}_R|}{Q}$$

0 in 2-jet limit

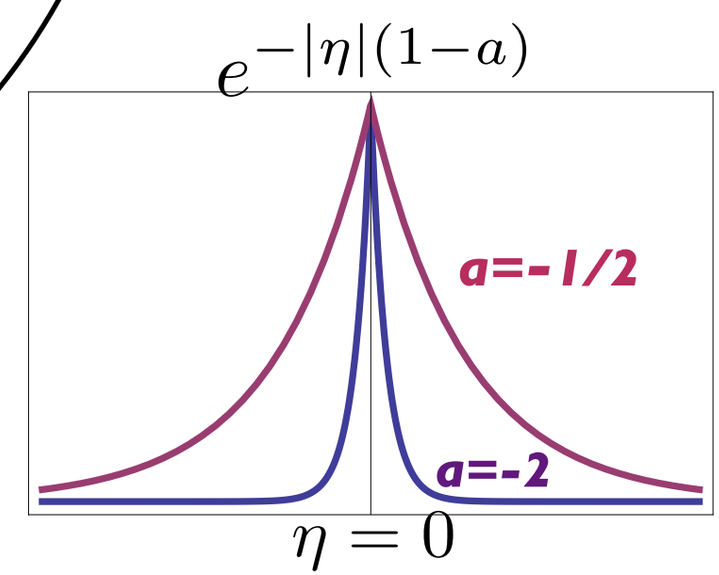
Event Shapes with Variable Weights

Weight with different measures to probe event structure



e.g. Angularities

$$\tau_a = \frac{1}{Q} \sum_{i \in X} |\mathbf{p}_T^i| e^{-|\eta_i|(1-a)}$$



Thrust is $a = 0$

Berger, Kucs, Sterman

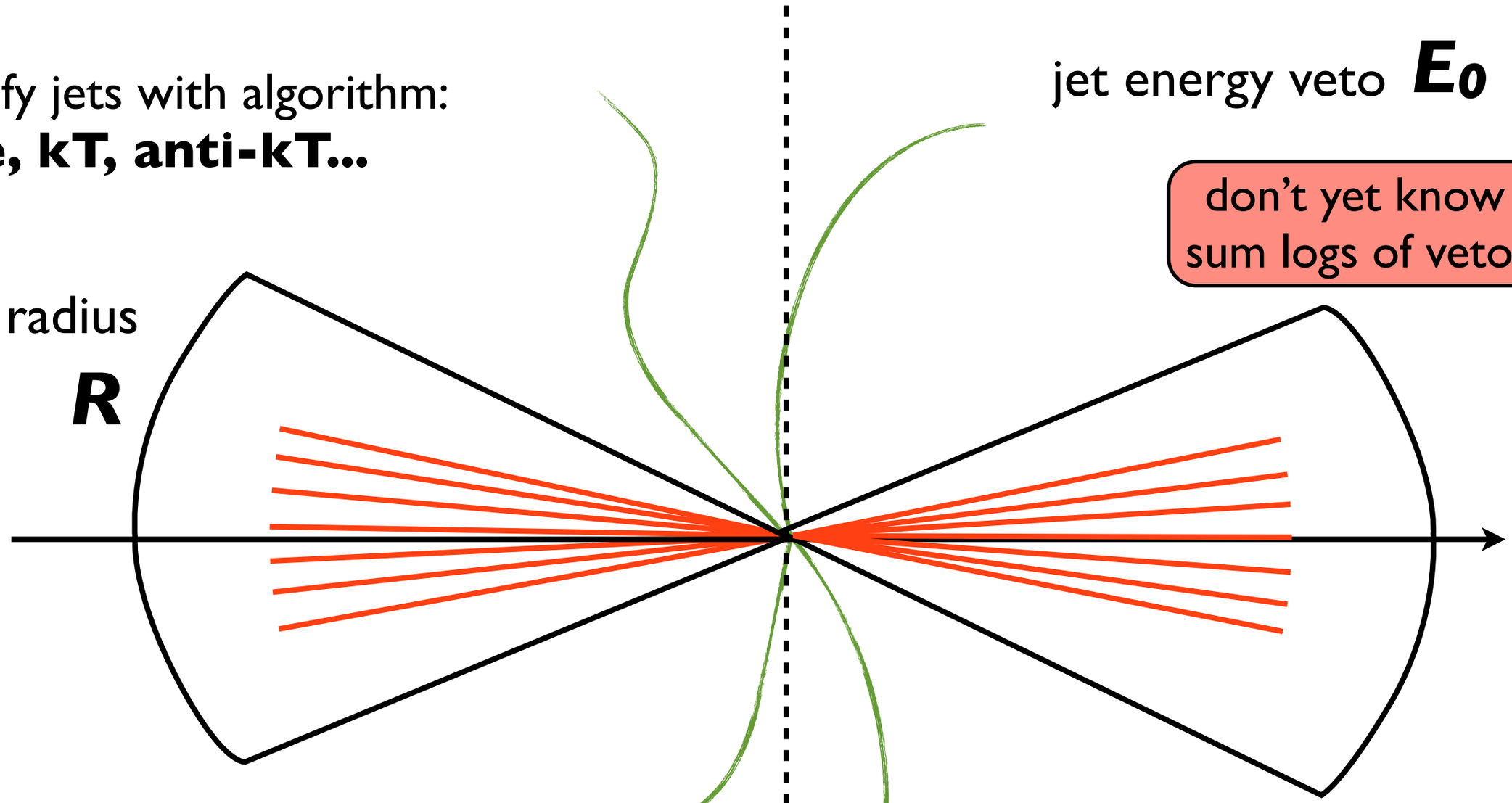
Shapes of Jets with Finite Size

Identify jets with algorithm:
cone, kT, anti-kT...

jet radius
R

jet energy veto ***E_0***

don't yet know how to sum logs of vetoes or *R*'s



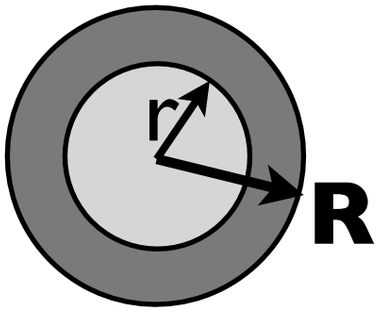
Measure individual jet shapes:

$$\tau_a = \frac{1}{E_J} \sum_{i \in \text{jet}} |\mathbf{p}_T^i| e^{-|\eta_i|(1-a)}$$

Ellis, Hornig, CL, Vermilion, Walsh (2010)

"The" Jet Shape:

$\Psi(r/R)$ = energy fraction inside subcone

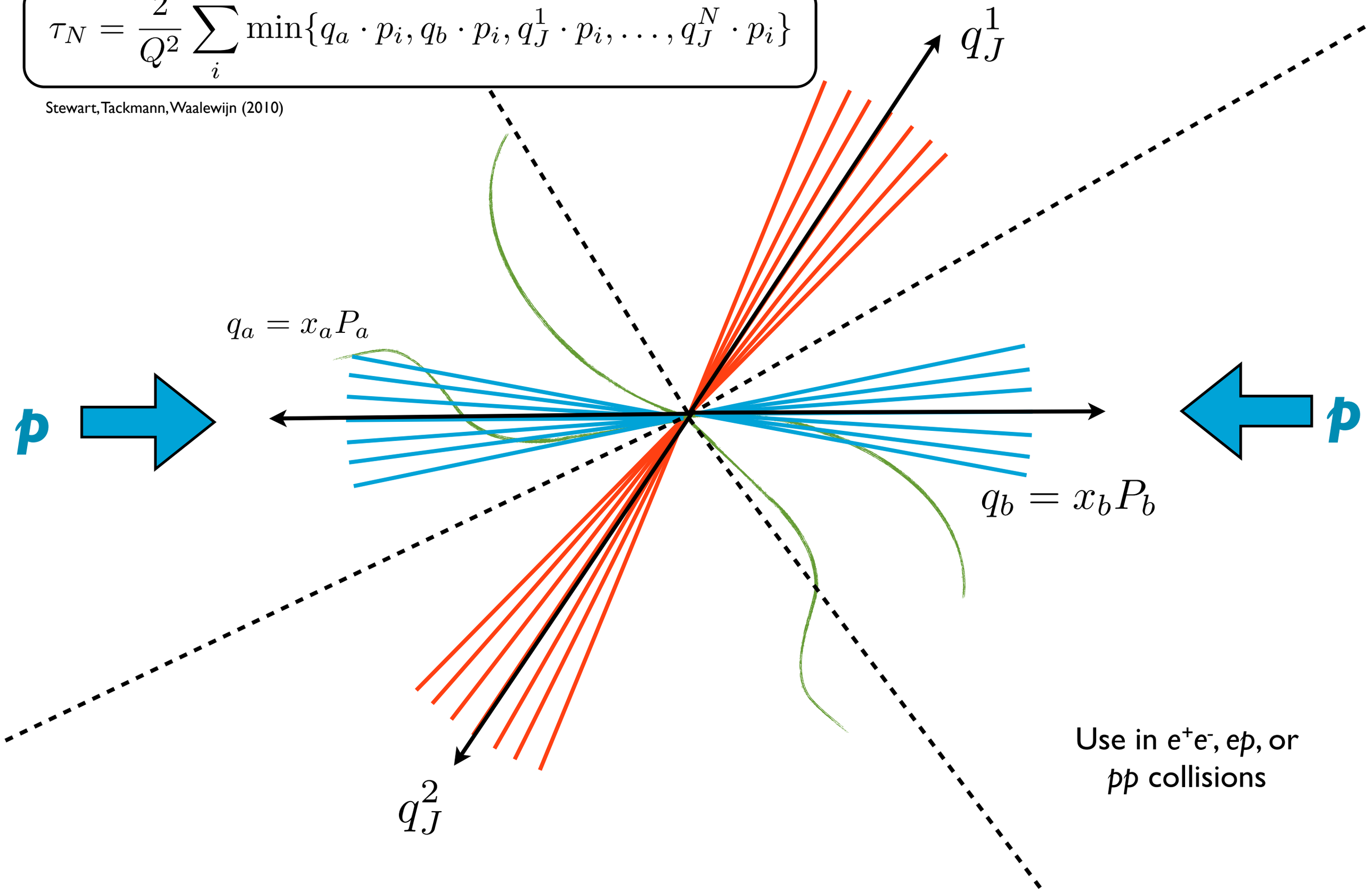


Ellis, Kunzst, Soper;
CDF

A Global Event Shape to Veto Jets: N-Jettiness

$$\tau_N = \frac{2}{Q^2} \sum_i \min\{q_a \cdot p_i, q_b \cdot p_i, q_J^1 \cdot p_i, \ldots, q_J^N \cdot p_i\}$$

Stewart, Tackmann, Waalewijn (2010)



Use in e^+e^-, ep, or pp collisions

III. A Theory Toolbox for Precision Jet Physics

Large Logs in Perturbation Theory

- If we calculate event shape $\mathcal{T}$ cross section in QCD perturbation theory, we will find:

$$\int_0^\tau d\tau \frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau} \sim \Bigg[1 + \frac{\alpha_s}{4\pi}\Big(F_{12}\ln^2\tau - F_{11}\ln\tau + F_{10}\Big)$$
$$+ \left(\frac{\alpha_s}{4\pi}\right)^2\Big(F_{24}\ln^4\tau + F_{23}\ln^3\tau + F_{22}\ln^2\tau + F_{21}\ln\tau + F_{20}\Big) + \ldots\Bigg]$$

- In the narrow-jet limit $\tau \to 0$ the logs grow large and spoil the perturbative expansion. Reorganize the expansion:

$$\begin{aligned}\ln\sigma(\tau) \sim \quad & \alpha_s(\ln^2\tau + \ln\tau) \\ & + \alpha_s^2(\ln^3\tau + \ln^2\tau + \ln\tau) \\ & + \alpha_s^3(\ln^4\tau + \ln^3\tau + \ln^2\tau + \ln\tau) \\ + \quad & \vdots \qquad \vdots \qquad \vdots \qquad \vdots\end{aligned}$$

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Leading Log (LL)

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Leading Log (LL)

Next-to-Leading Log (NLL)

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Leading Log (LL) | Next-to-Leading Log (NLL) | NNLL

Large Logs in Perturbation Theory

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	Leading Log (LL)	Next-to-Leading Log (NLL)	NNLL
New power counting when $\ln\tau \sim \frac{1}{\alpha}$:	$\sim \alpha^{-1}$	~ 1	$\sim \alpha$

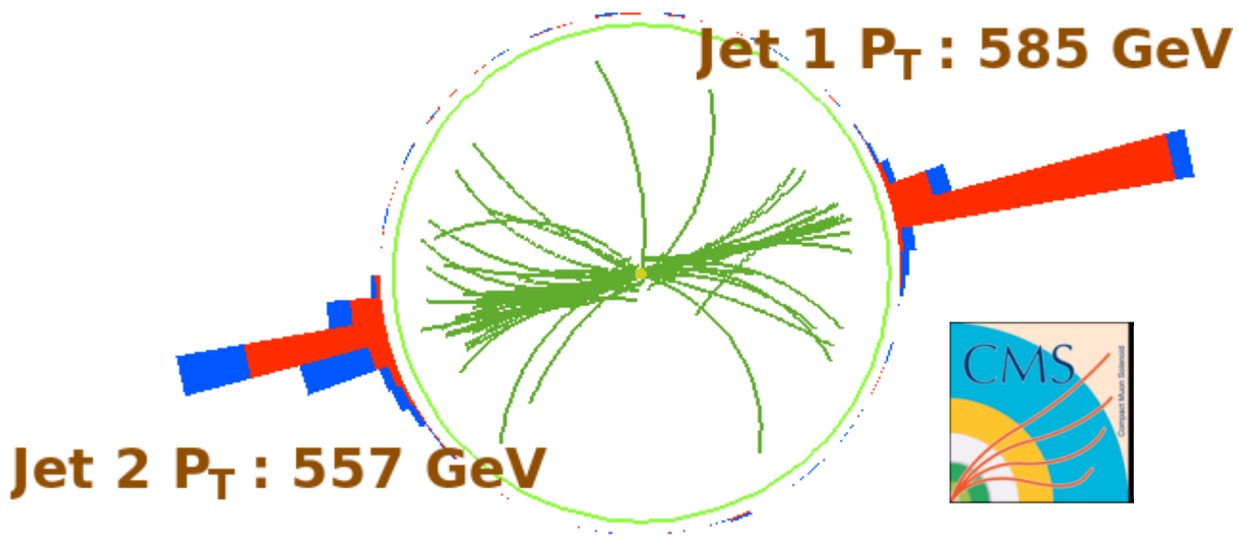
e+e- event shapes so far computed to N^{3}LL
pp and *ep* event shapes to NNLL

Collins, Soper, Sterman
(1980s)

Factorization of Jet Cross Sections

Event shapes in SCET:
Bauer, **CL**, Fleming, Sterman
PRD **78**, 034027 (2008)

Separate into physics at disparate time scales:



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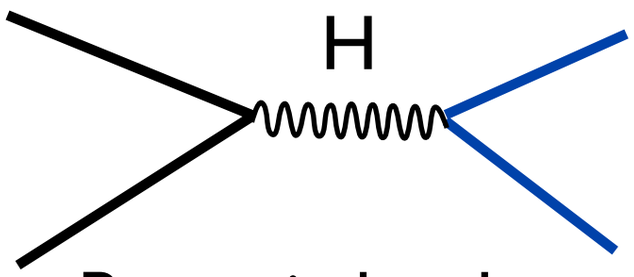
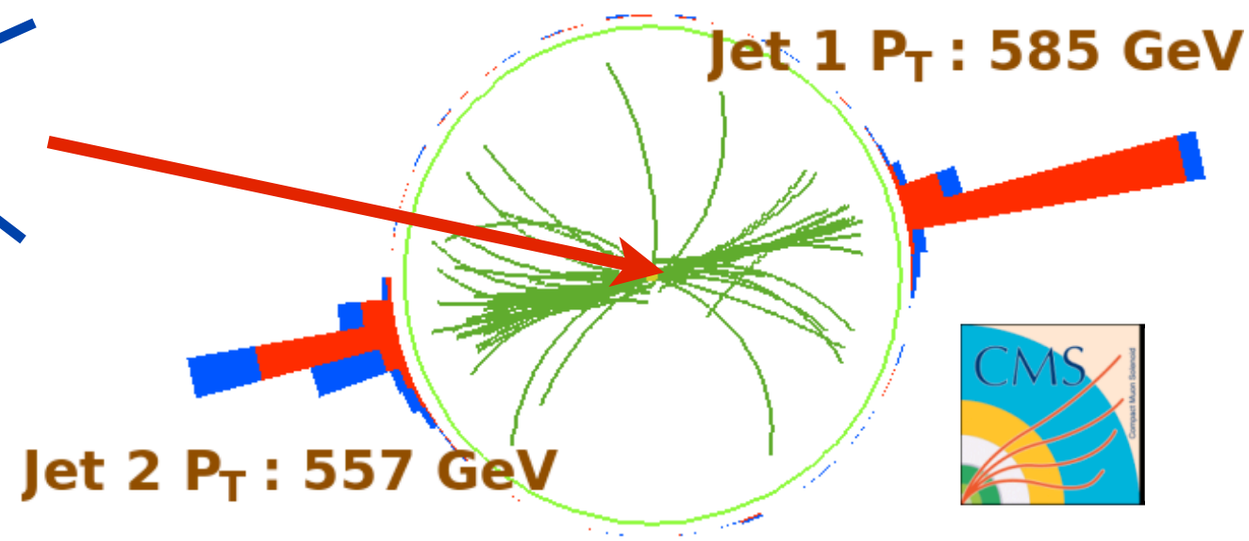


Partonic hard scattering cross section



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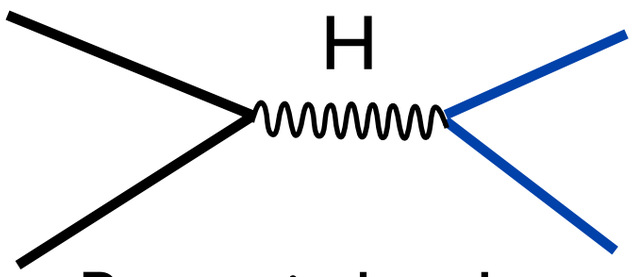

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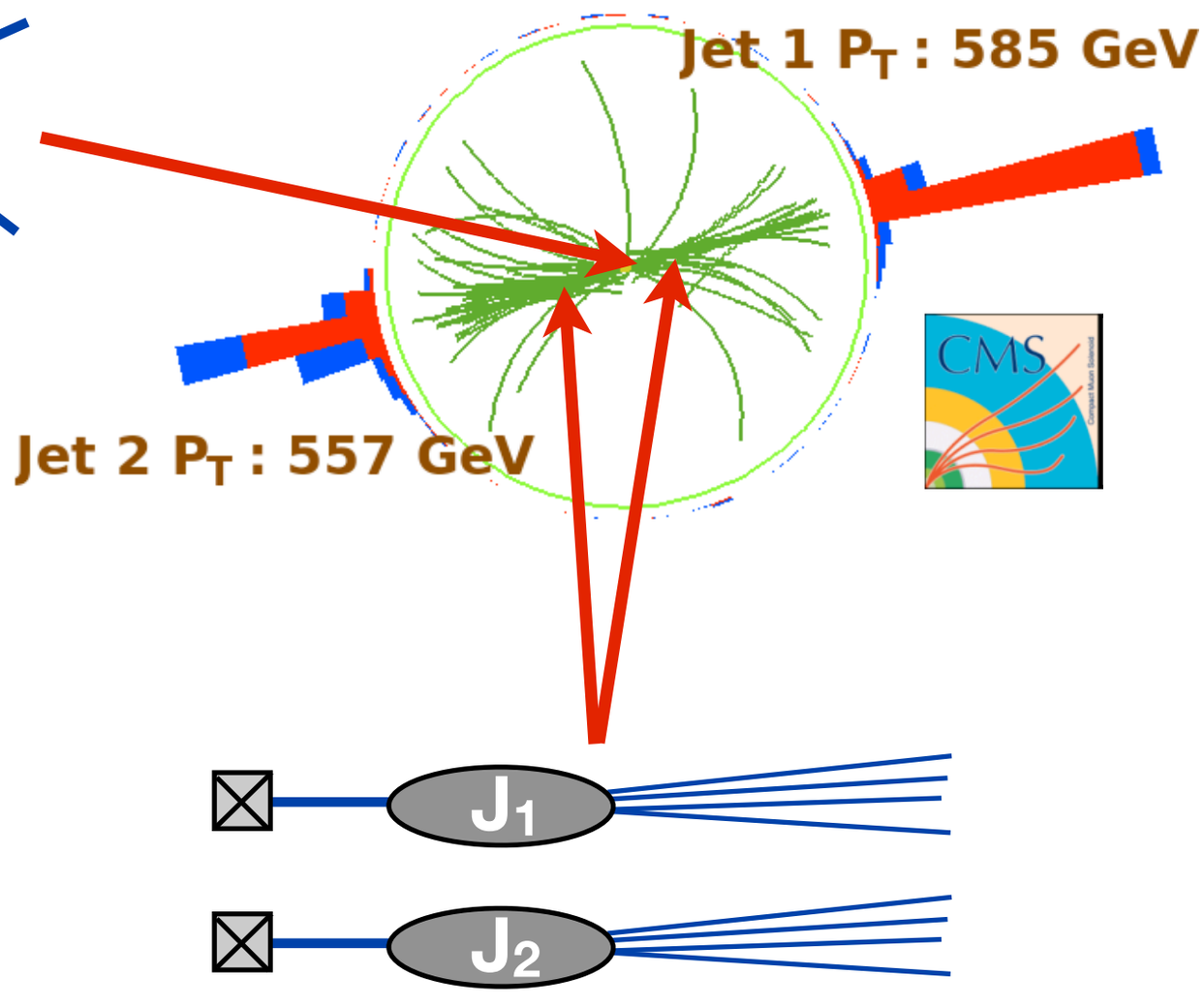


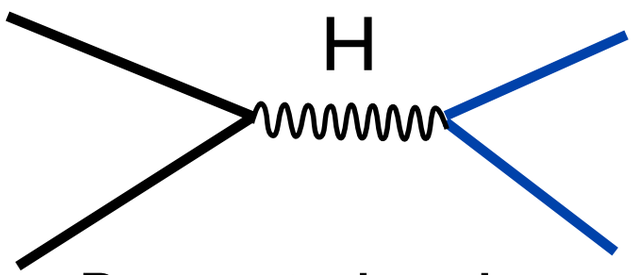
Jet Functions:
Collinear splitting, showering

Collins, Soper, Sterman
(1980s)

Factorization of Jet Cross Sections

Event shapes in SCET:
Bauer, **CL**, Fleming, Sterman
PRD **78**, 034027 (2008)

Separate into physics at disparate time scales:



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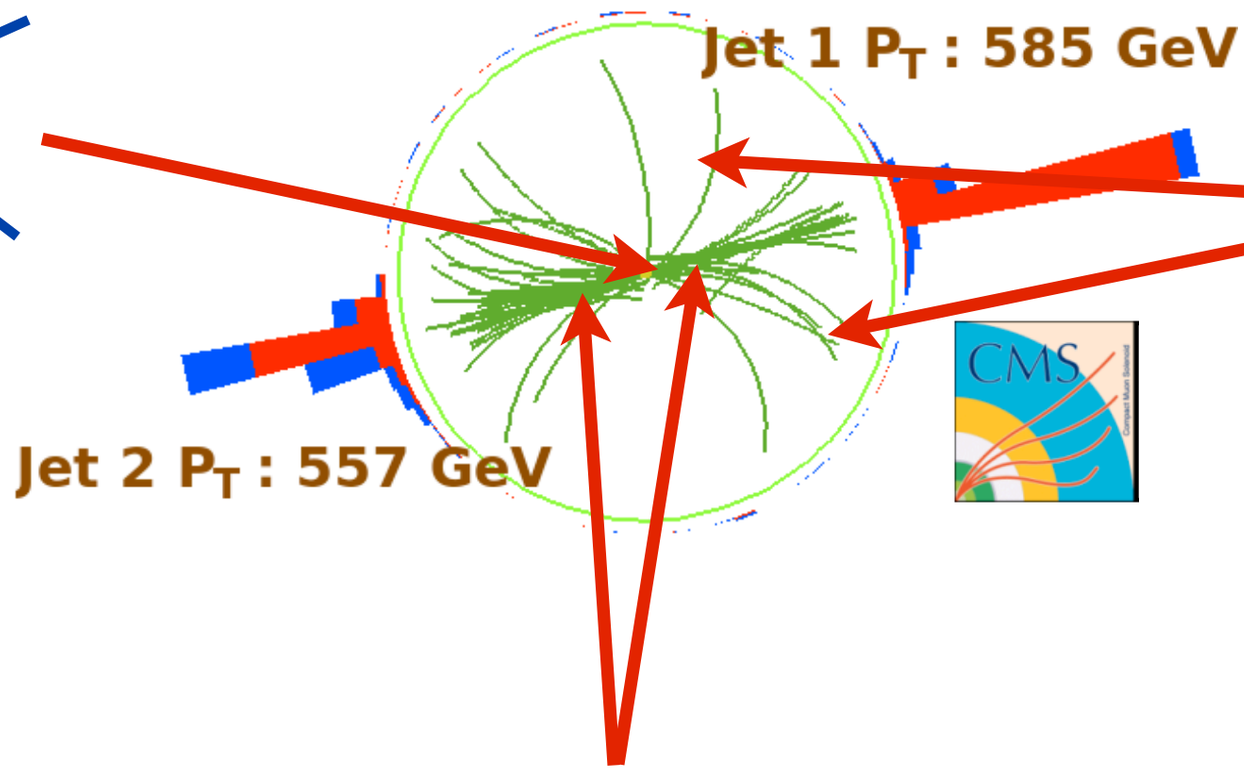
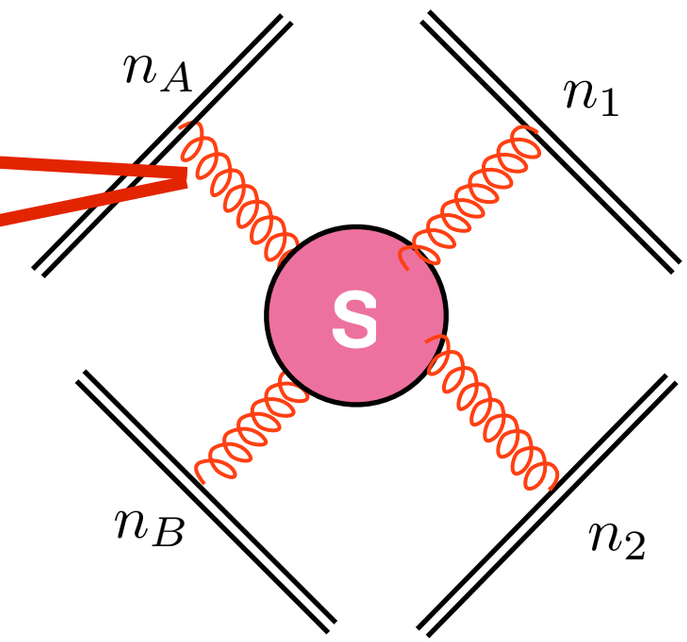
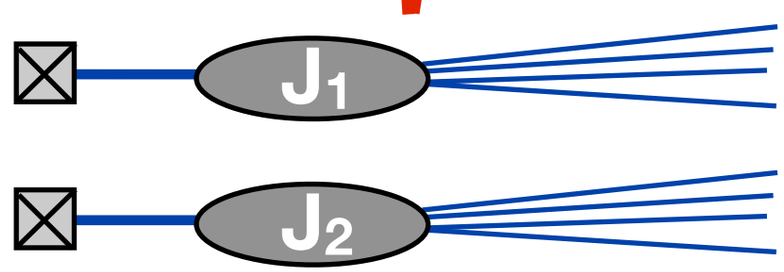




Soft radiation, color exchange (hadronization)



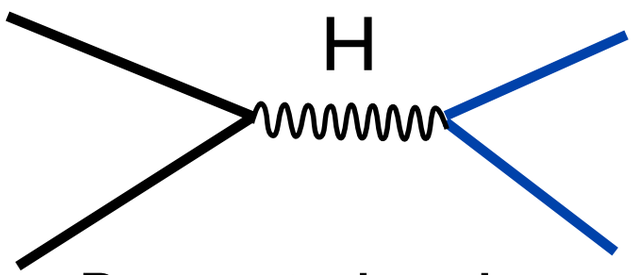
Jet Functions: Collinear splitting, showering

Collins, Soper, Sterman
(1980s)

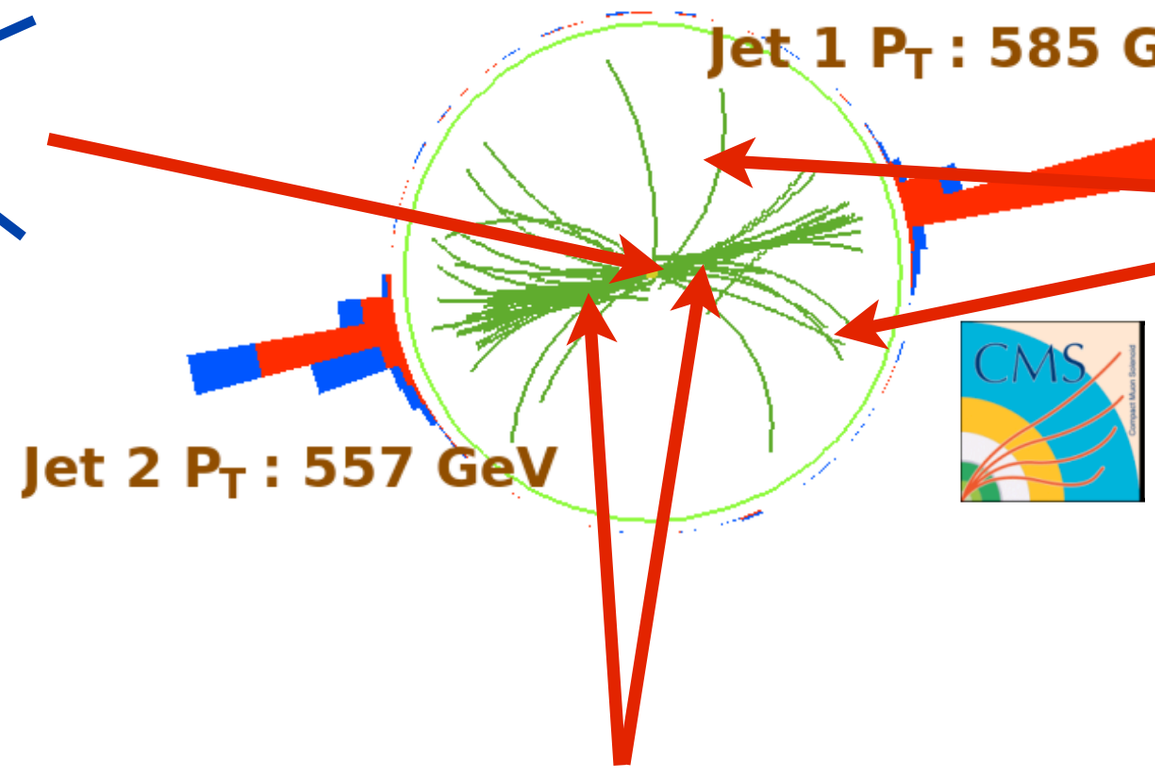
Factorization of Jet Cross Sections

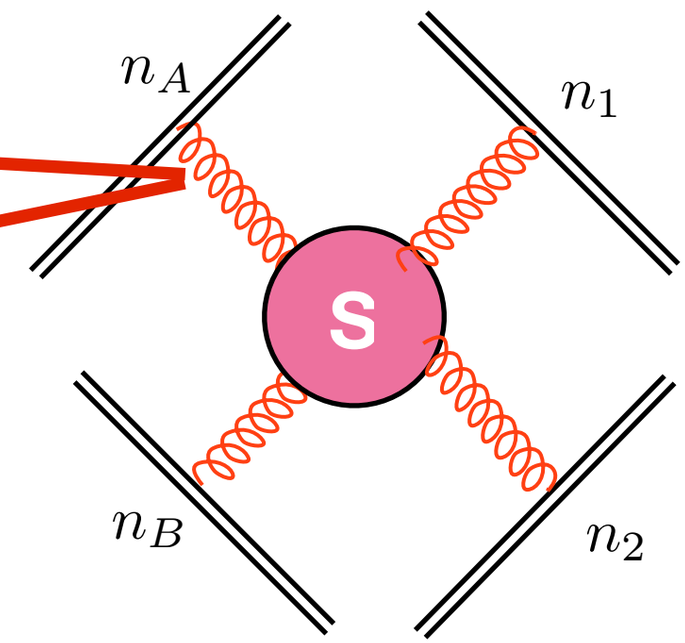
Event shapes in SCET:
Bauer, **CL**, Fleming, Sterman
PRD **78**, 034027 (2008)

Separate into physics at disparate time scales:



Partonic hard scattering cross section





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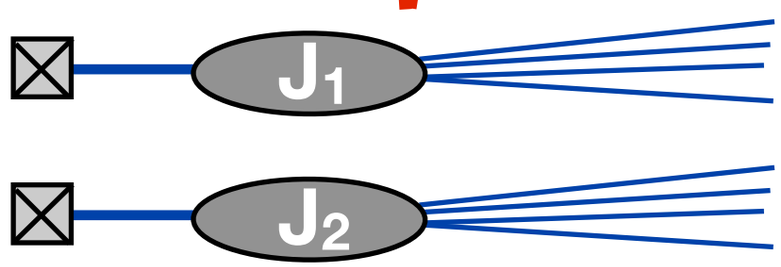
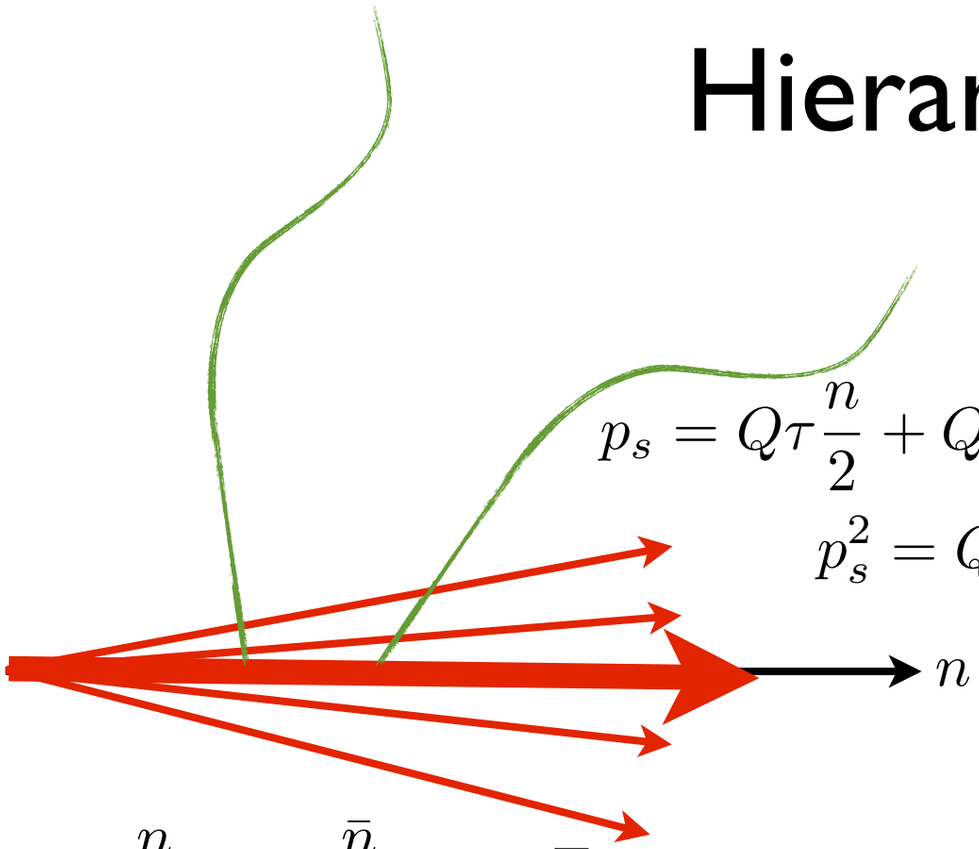


Jet Functions: Collinear splitting, showering

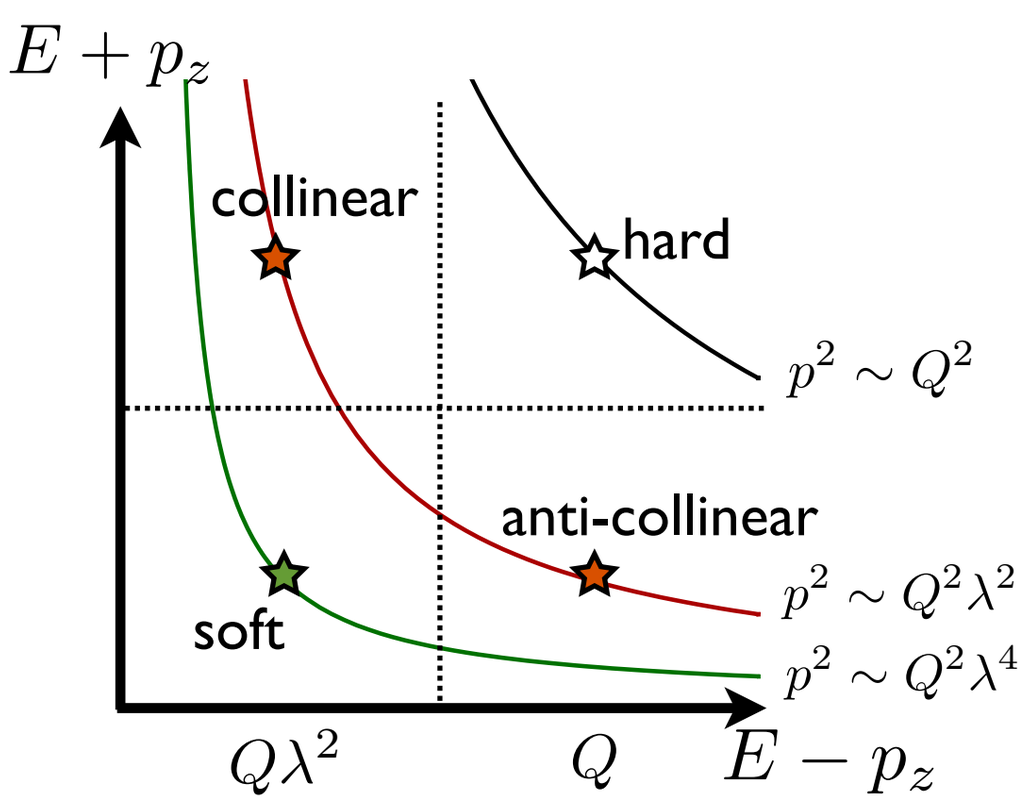
(For pp collisions, also factor into PDFs and "beam functions" for ISR)

Hierarchy of scales



$$p_s = Q\tau\frac{n}{2} + Q\tau\frac{\bar{n}}{2} + Q\tau\hat{n}_\perp$$
$$p_s^2 = Q^2\tau^2$$

$$p_c = Q\frac{n}{2} + Q\tau\frac{\bar{n}}{2} + Q\sqrt{\tau}\hat{n}_\perp$$
$$p_c^2 = Q^2\tau$$



Jet cross sections depend on three separated energy scales:

hard scale	$\mu_H = Q$
jet/beam scale	$\mu_{J,B} = Q\sqrt{\tau}$
soft scale	$\mu_S = Q\tau$

Use **effective field theory:** Integrate out hard degrees of freedom, form theory of collinear and soft DOF

Soft Collinear Effective Theory

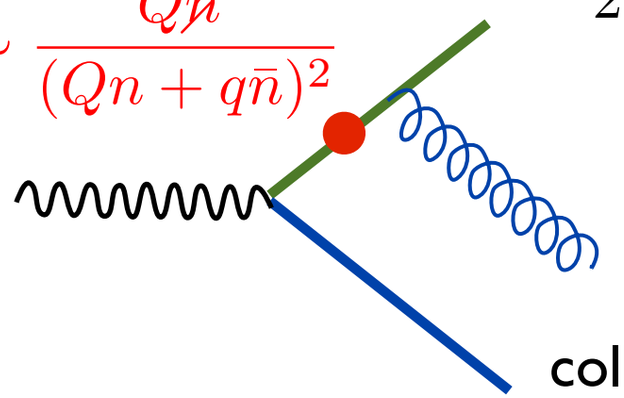
Bauer, Fleming, Luke, Pirjol, Stewart (2000, 2001)

- Integrate out hard modes from QCD, match onto theory of soft and collinear modes in power expansion in $\lambda \sim p_T/Q$

Collinear sector:

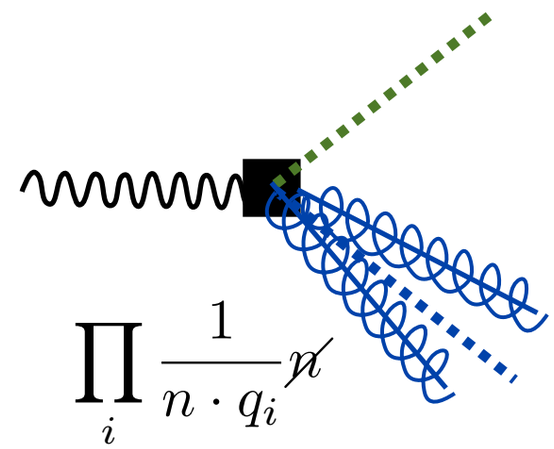
$$\sim \frac{Q\not{n}}{(Qn+q\bar{n})^2}$$

$$p = Q\frac{n}{2}$$ collinear to *n*



$$q\frac{\bar{n}}{2}$$

collinear to $\bar{n}$

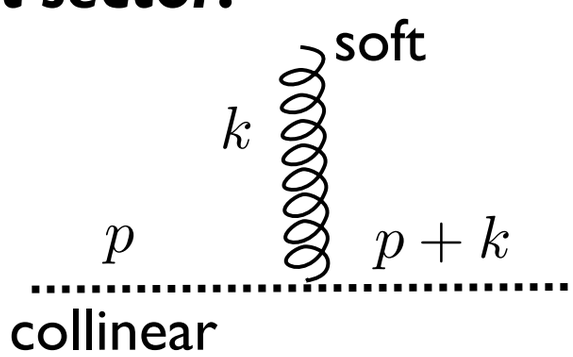


$$\prod_i \frac{1}{n\cdot q_i}\not{n}$$

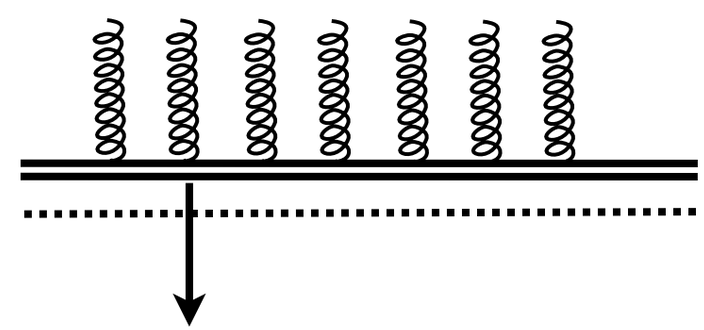
integrate out hard propagators

match onto operators in SCET

Soft sector:



soft gluons "see" only direction and color of jet



$$\sim \not{p}\frac{ig\gamma^\mu t^a}{(p+k)^2}\not{p}$$

keep leading order part of diagram

$$\frac{n^\mu t^a}{n\cdot k}\frac{\not{n}}{2}$$

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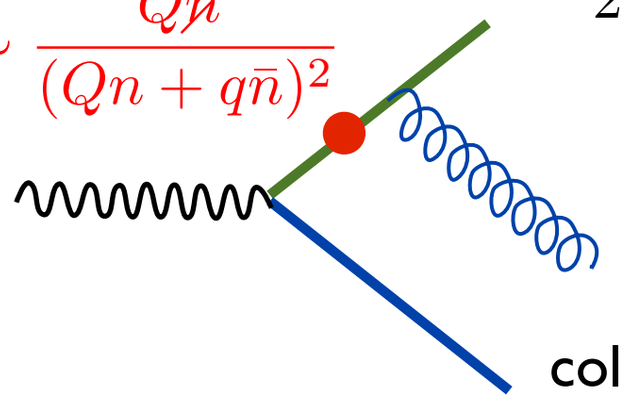

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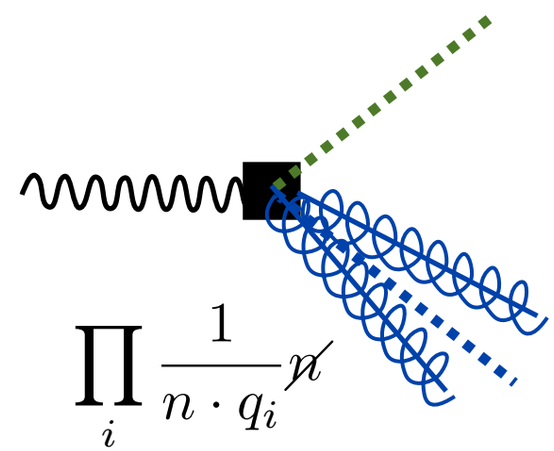
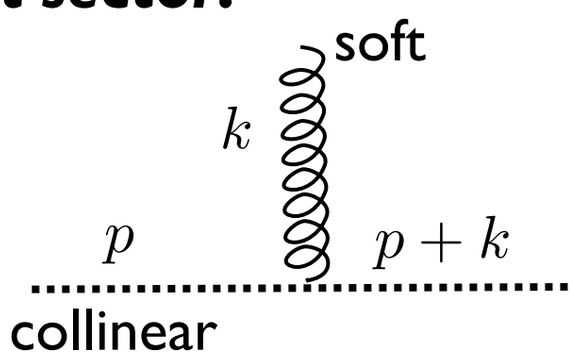


$$\prod_i \frac{1}{n\cdot q_i}\not{n}$$

decouples different collinear directions

match onto operators in SCET

Soft sector:



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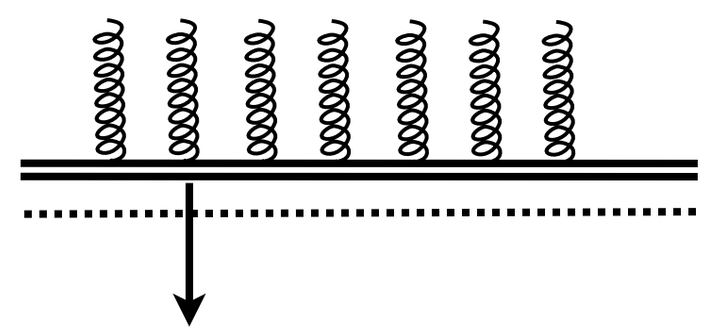

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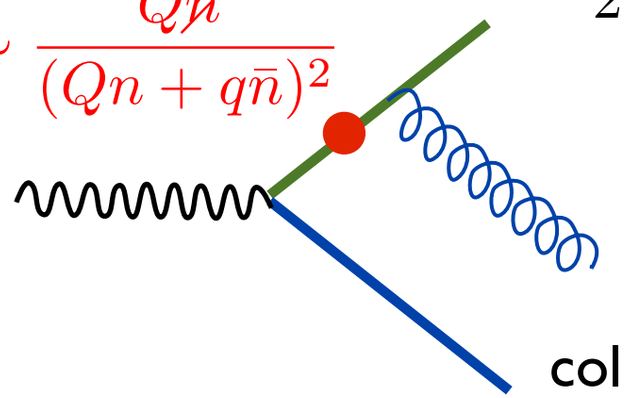

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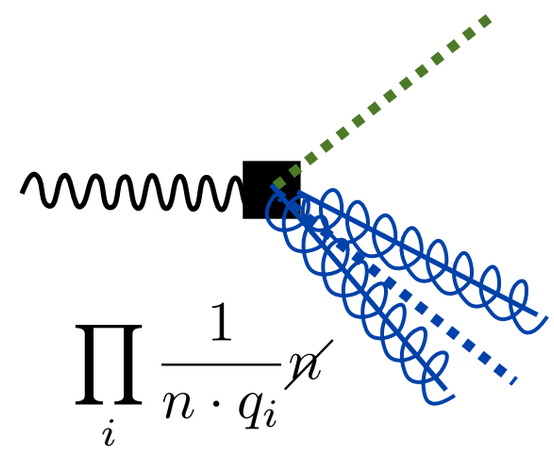

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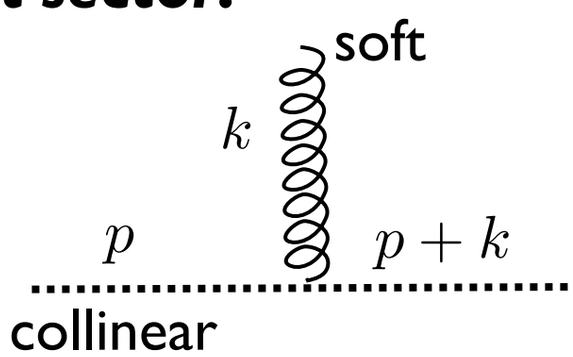

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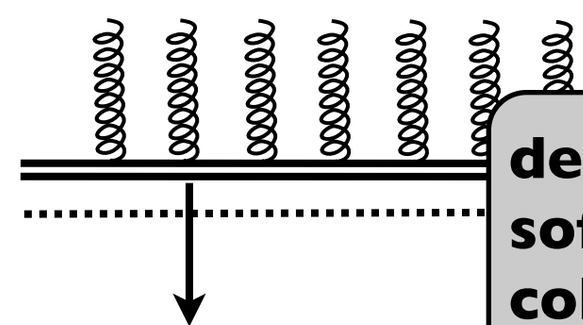

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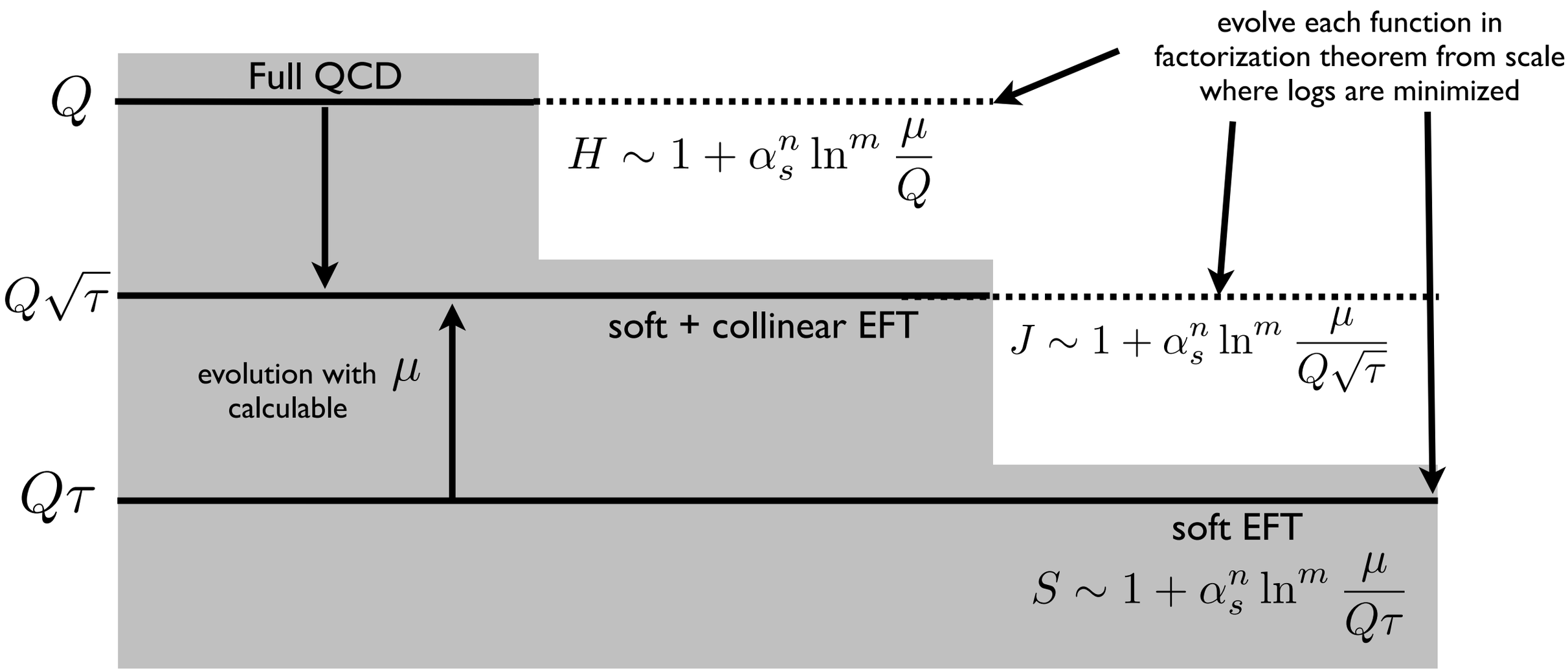
decouples soft from collinear

Resummation from Evolution

- Effective theory gives equations for evolution of hard, jet, and soft functions in factorization theorem with energy scale μ.



- **Solutions of evolution equations contain logs resummed to all orders in α_s**

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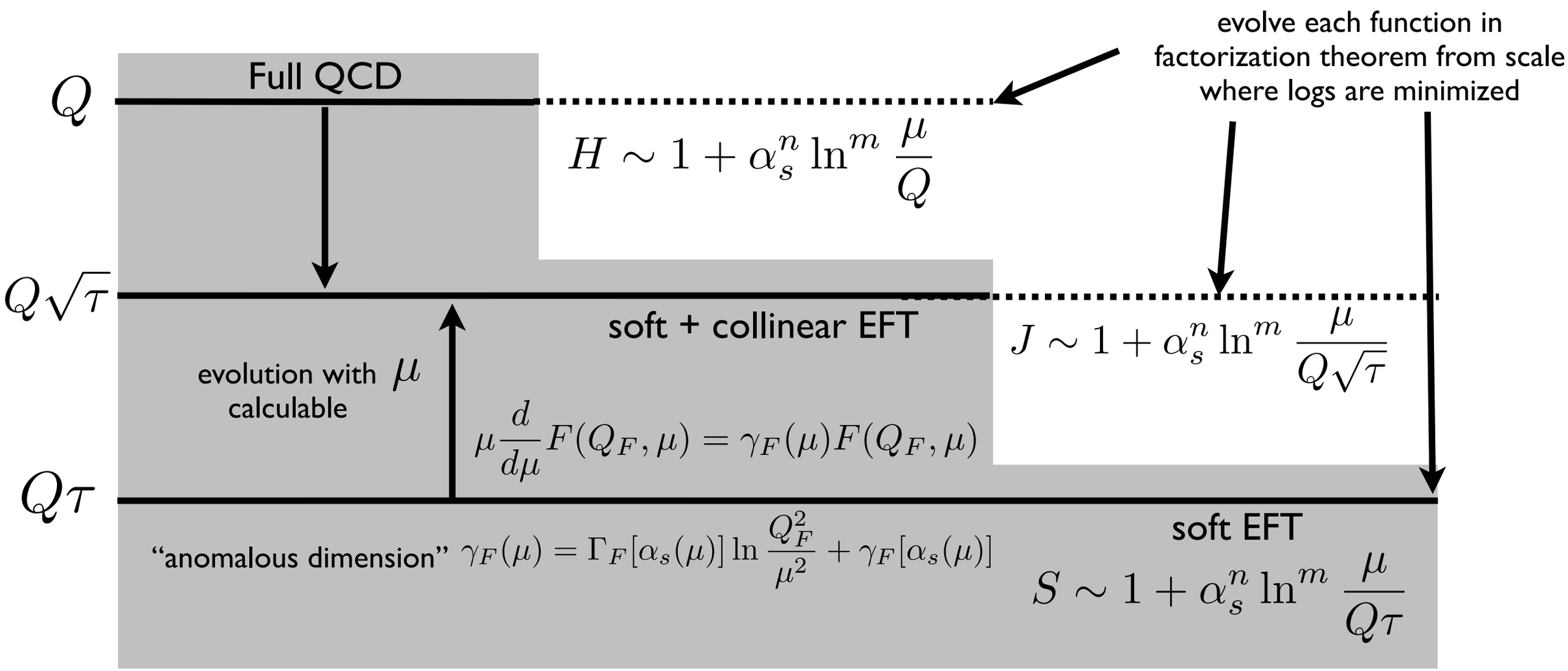

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Resummation of Logs

- Solution of RG Equation automatically resums logs to all orders in α_s
- Order of logarithmic accuracy (LL, NLL, etc.) depends on accuracy to which anomalous dimensions and fixed-order matrix elements are known:

	Γ_F	γ_F	H, J, S	$\beta[\alpha_s]$
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NNLL	α_s^3	α_s^2	α_s	α_s^3
N^3LL	α_s^4	α_s^3	α_s^2	α_s^4

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N³LL	α_s^4	α_s^3	α_s^2	α_s^4

+ 1 order for
"primed" counting

Universal Soft Power Corrections

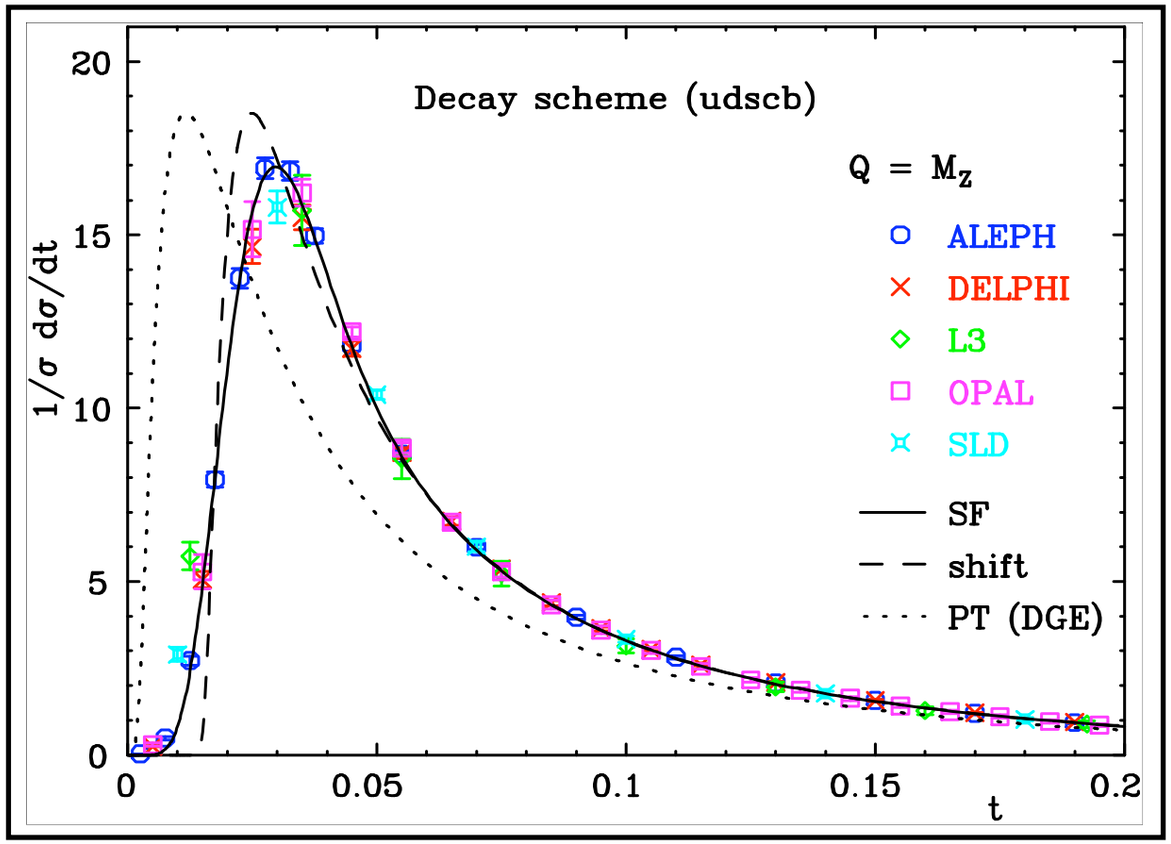
- Soft power corrections shift mean values of event shapes

$$\langle e \rangle = \langle e \rangle_{\mathrm{PT}} + c_e \frac{\Omega_1}{Q}$$

conjecture from single soft gluon emission: Dokshitzer, Webber (1995, 1997)

c_e observable dependent, calculable coefficient

Ω_1 universal nonperturbative parameter



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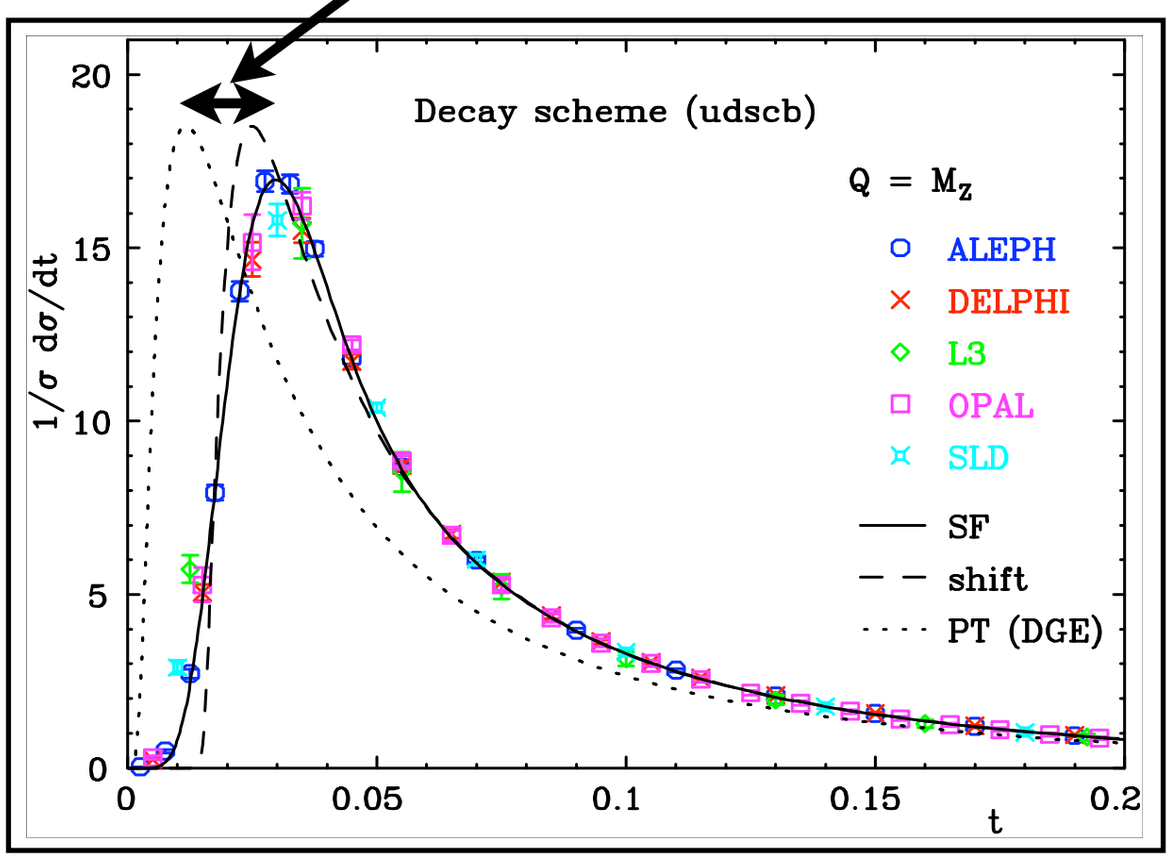

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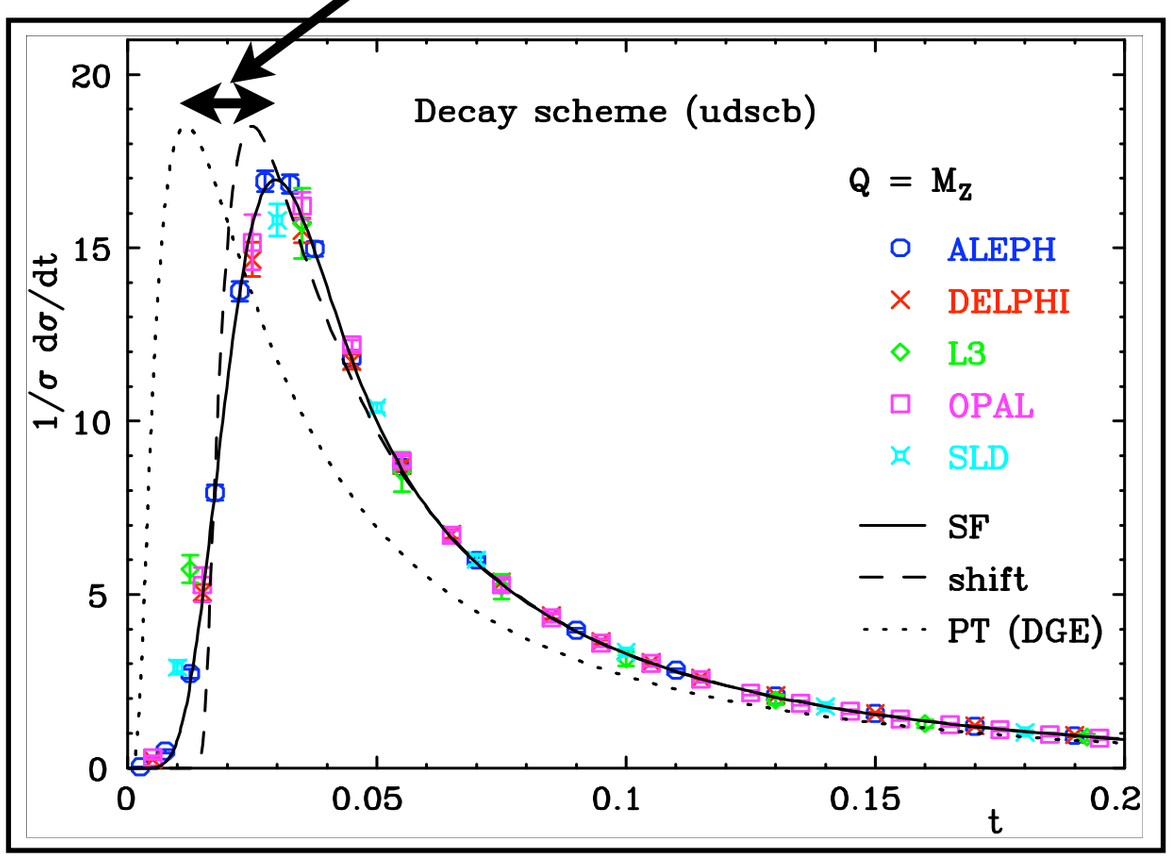
Use factorization to prove true

CL, Sterman PRD **75**, 014022 (2007)



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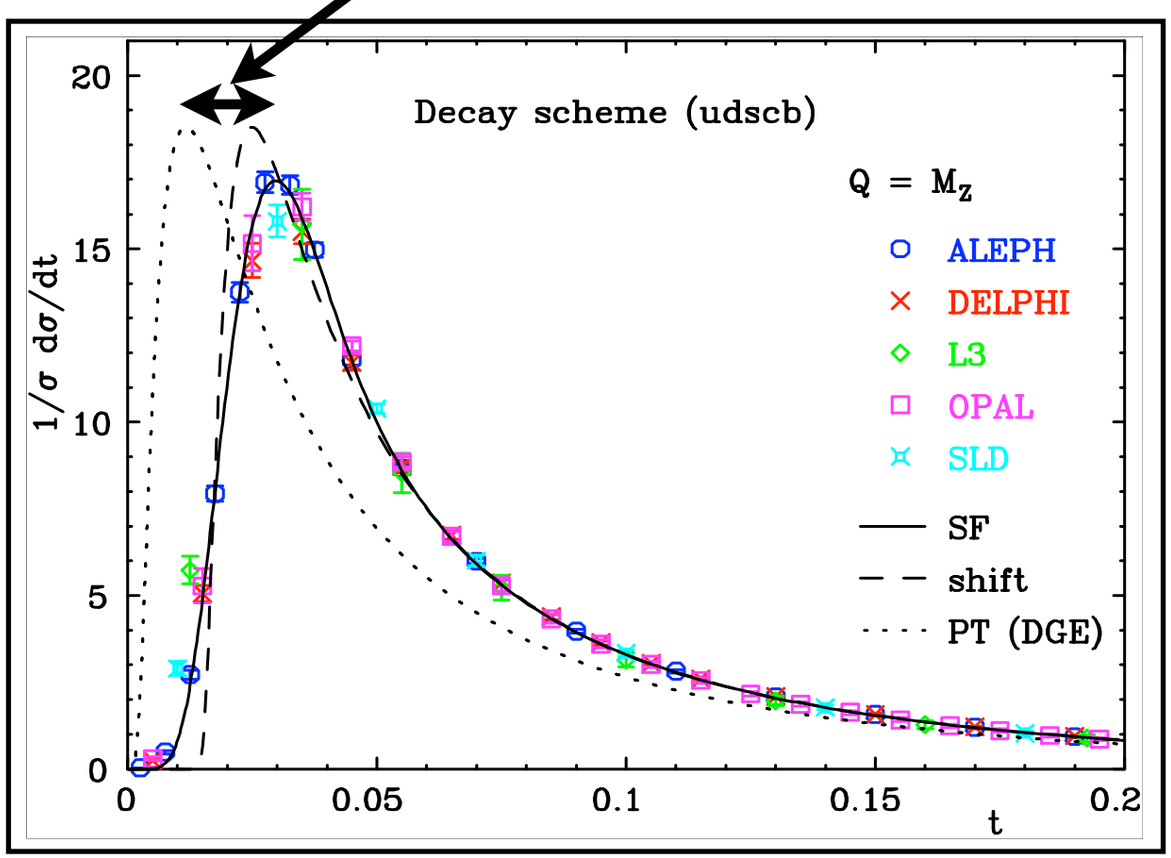

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Crucial to obtain reliable extractions of strong coupling from event shapes

Abbate, Fickinger, Hoang, Mateu, Stewart (2010)

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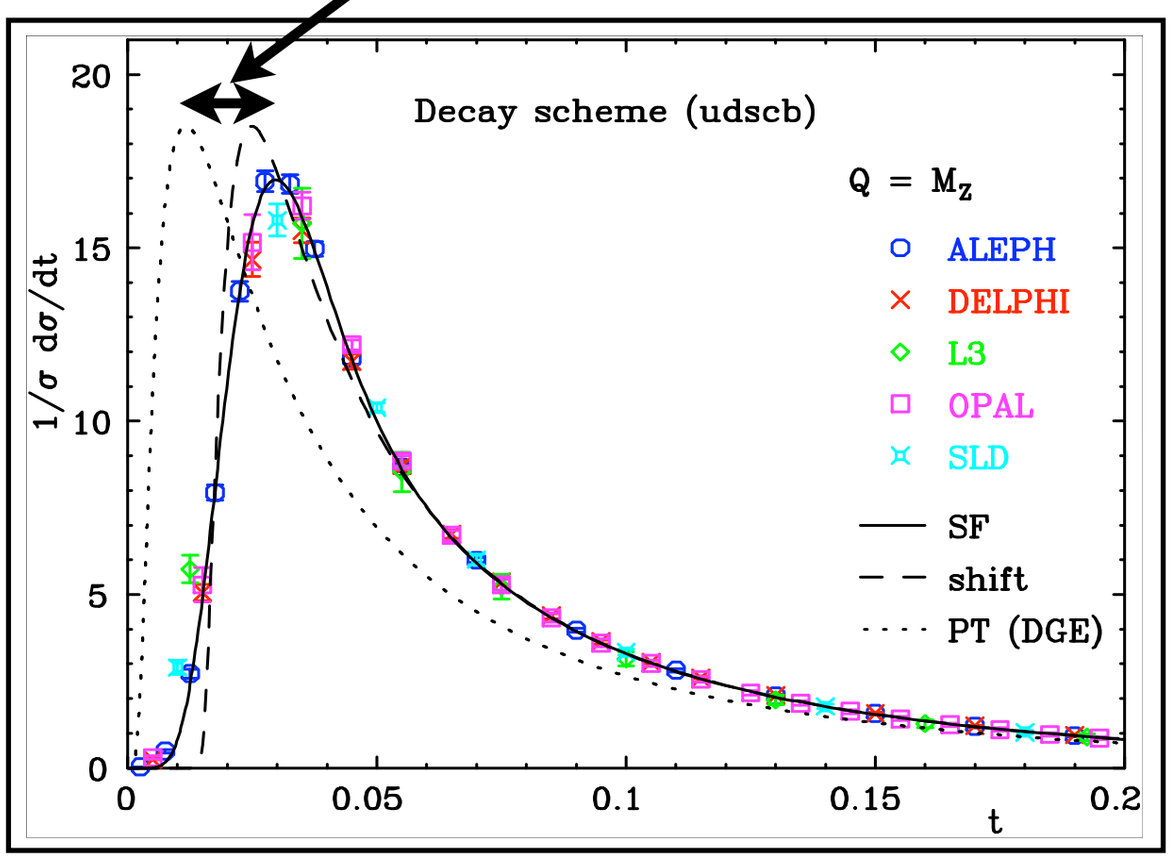

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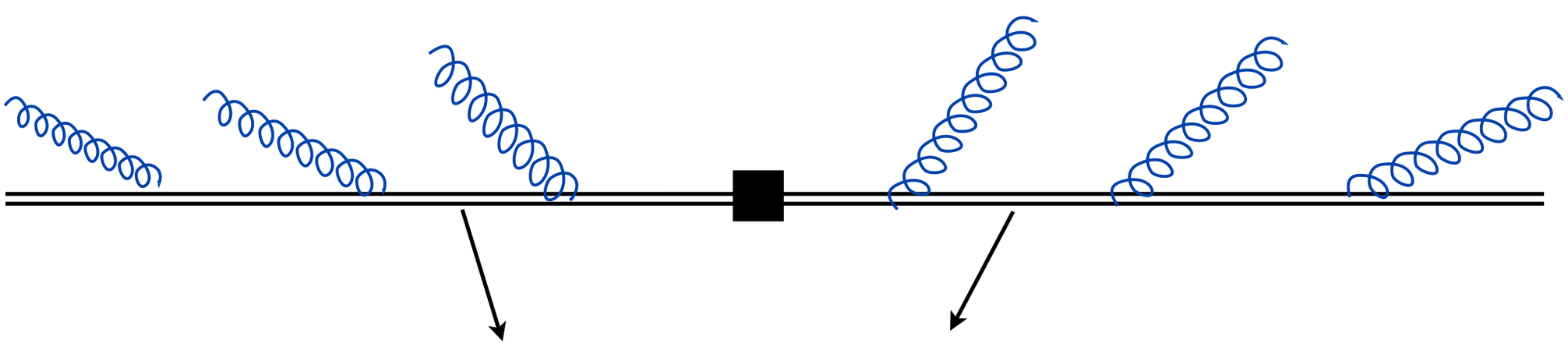
CL, Sterman PRD **75**, 014022 (2007)

- SCET: First rigorous proof (and **field theory** definition of Ω_1) from factorization theorem and boost invariance of soft radiation:

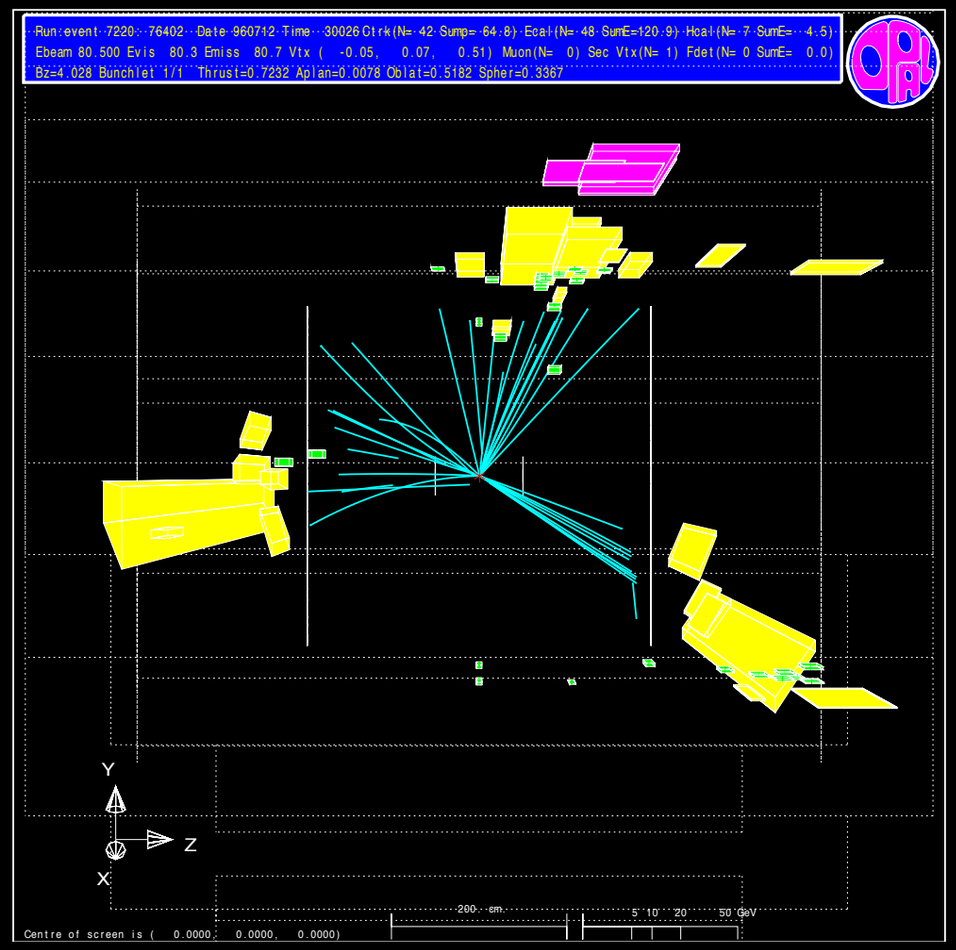
$$c_\tau = 2$$

$$c_{\tau_a} = \frac{2}{1-a}$$

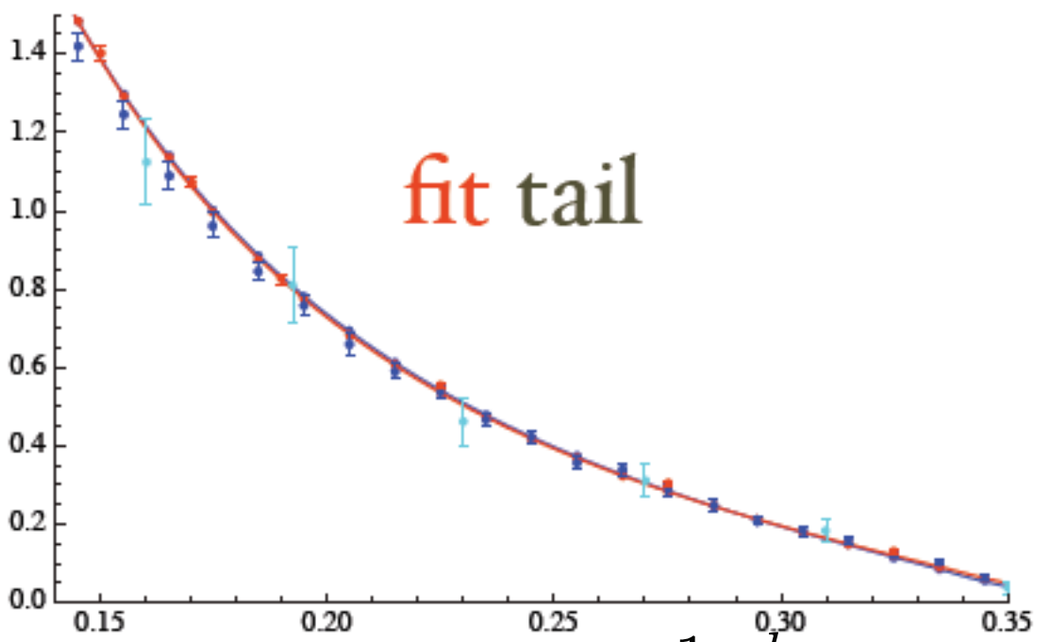


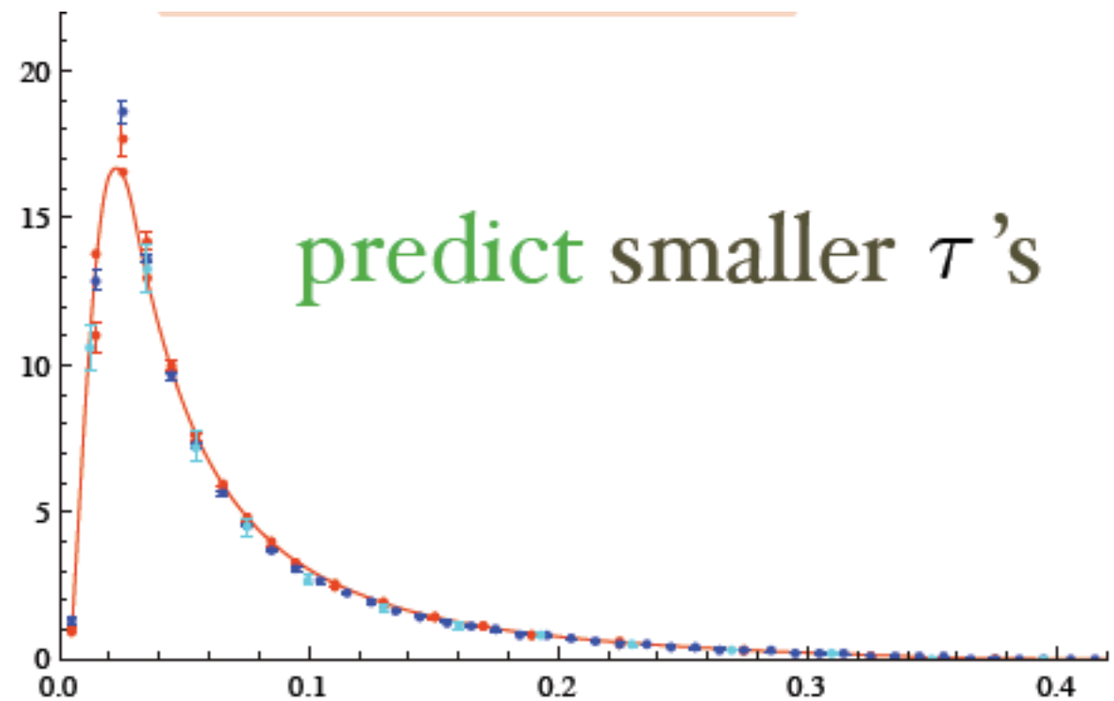
soft radiation sees only direction, not energy, of original collinear partons, invariant to boosts along z

IV. Predictions for e^+e^- event shapes



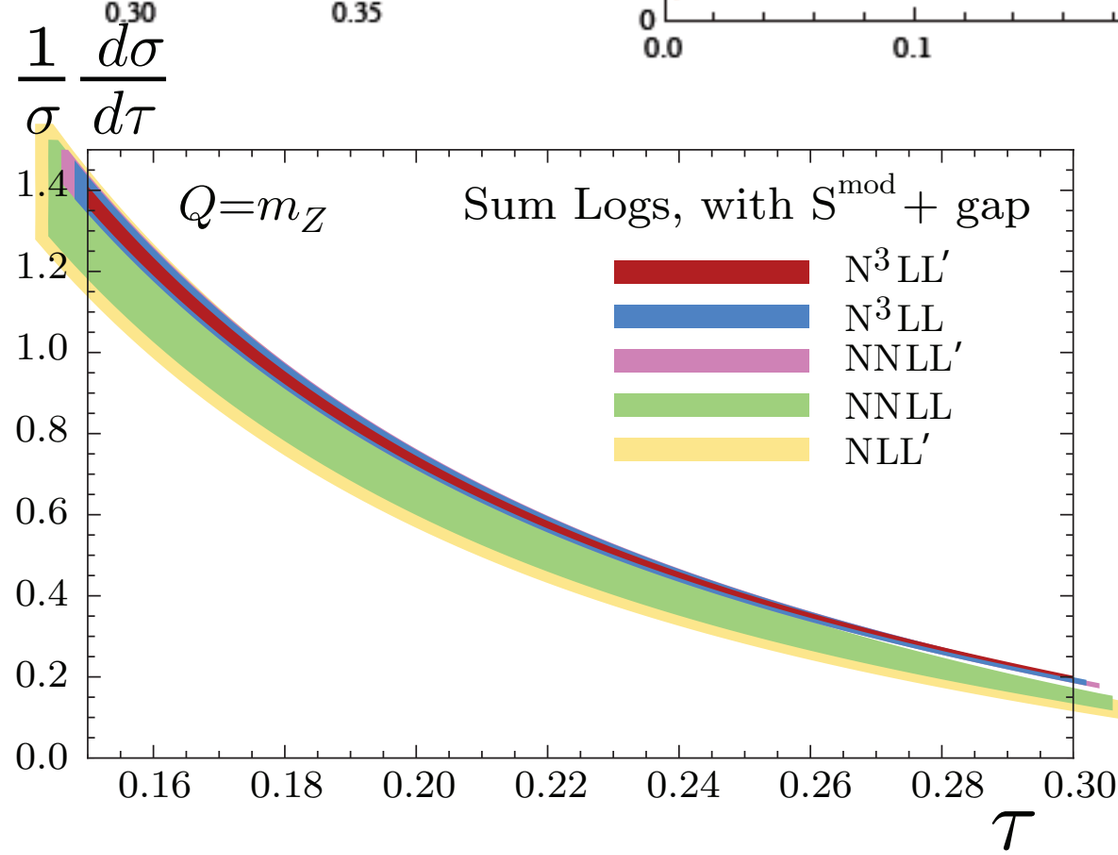
Most precise prediction of thrust comes from SCET





NNNLL perturbative prediction + nonperturbative soft power correction led to most precise extraction of strong coupling from event shapes

Abbate, Fickinger, Hoang, Mateu, Stewart (2010)

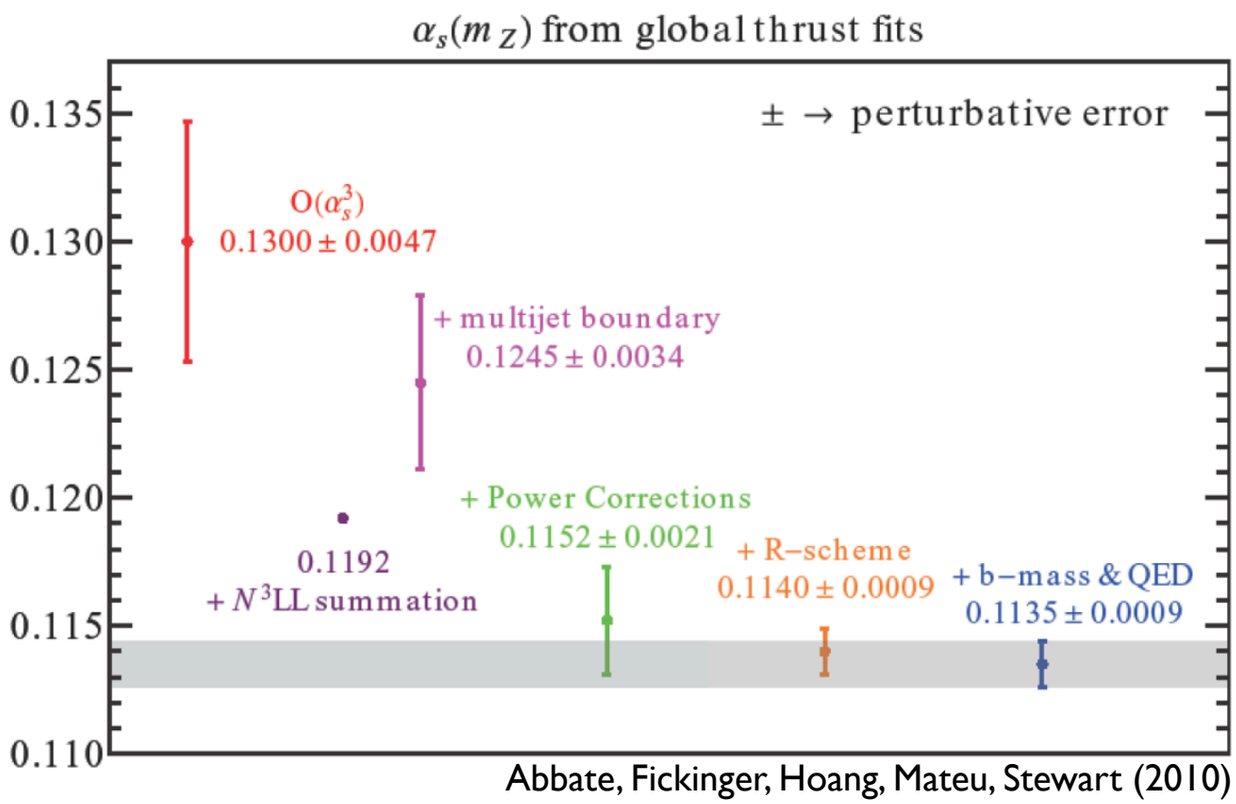


Factorization:
Bauer, **CL**, Fleming, Sterman
PRD **78**, 034027 (2008)

NNNLL resummed perturbative distribution

Becher, Schwartz (2008)

Most precise extractions of strong coupling



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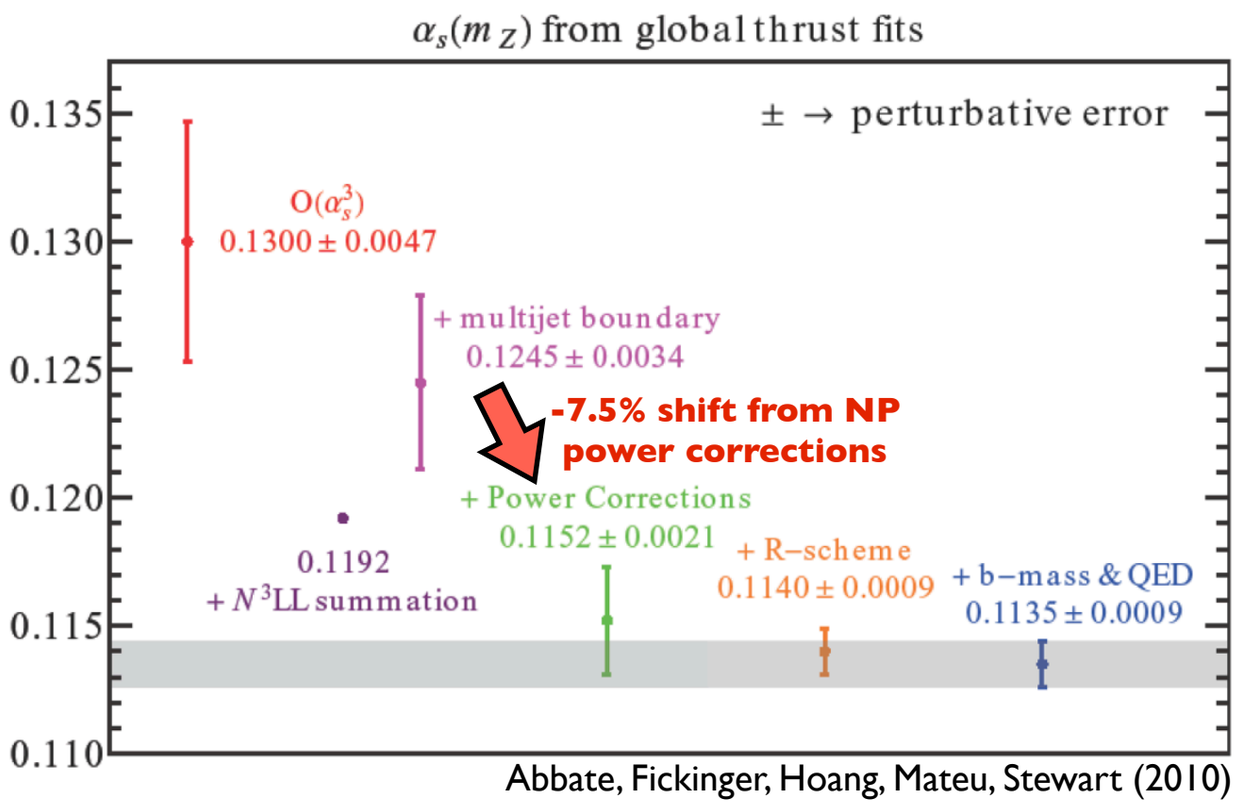

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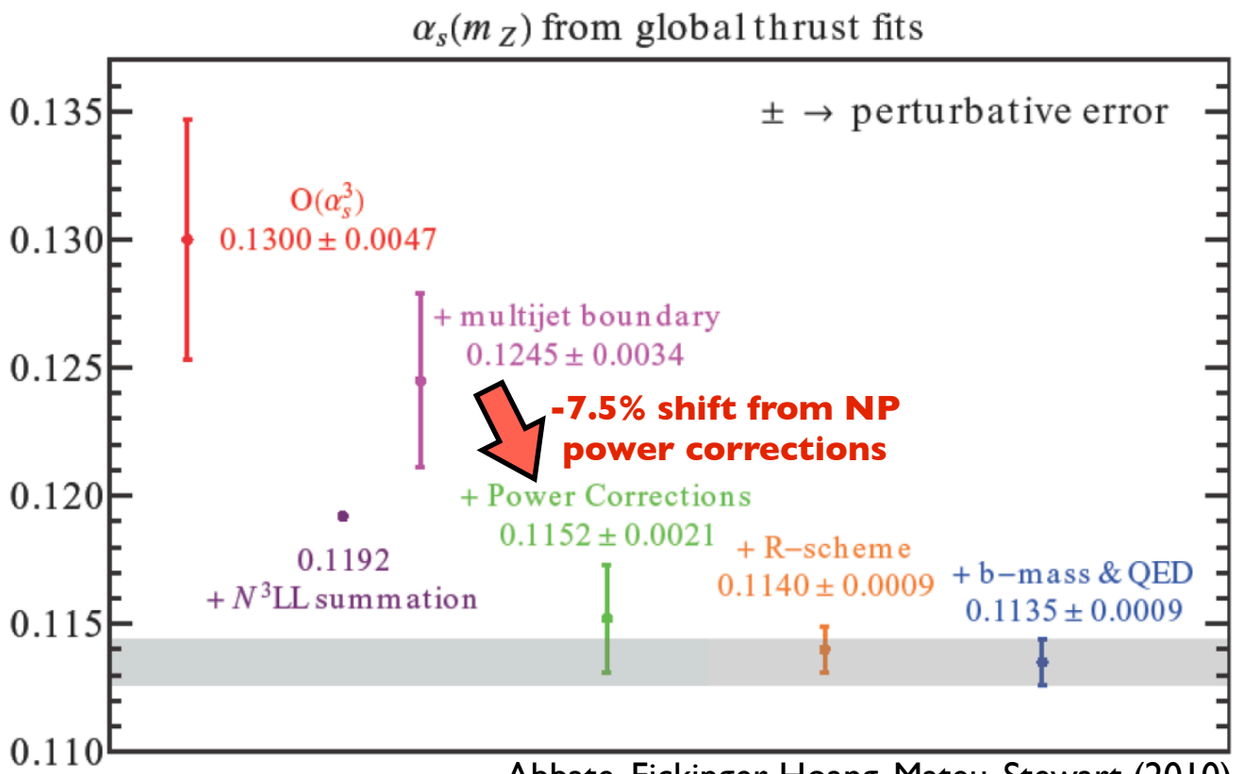

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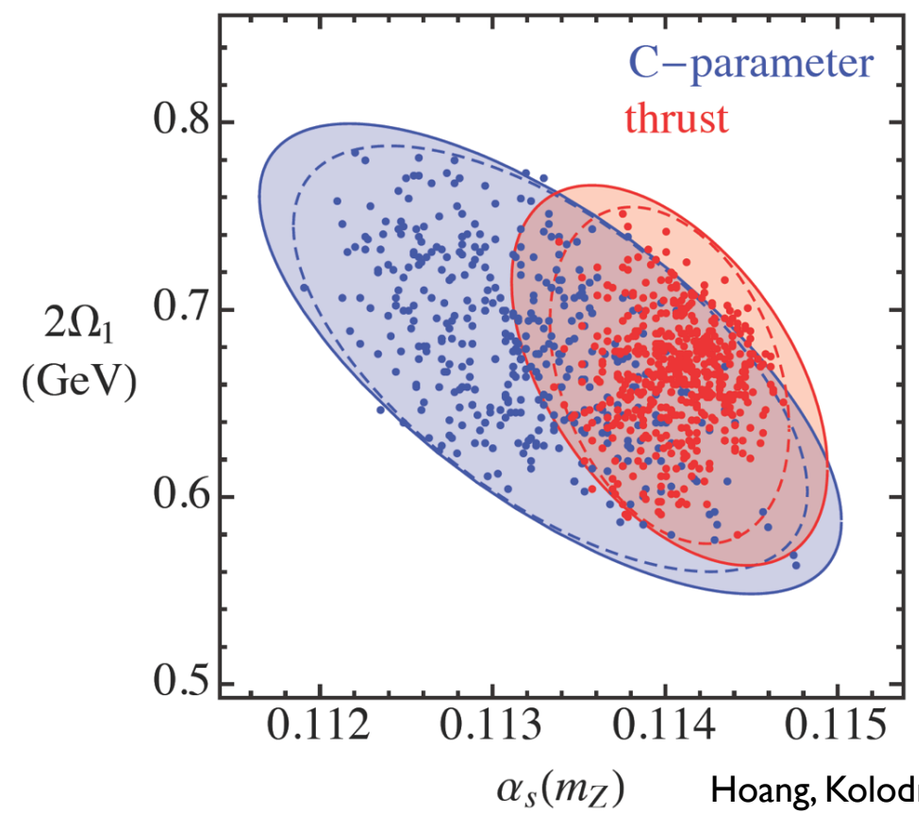

Hoang, Kolodrubetz, Mateu, Stewart (2013)

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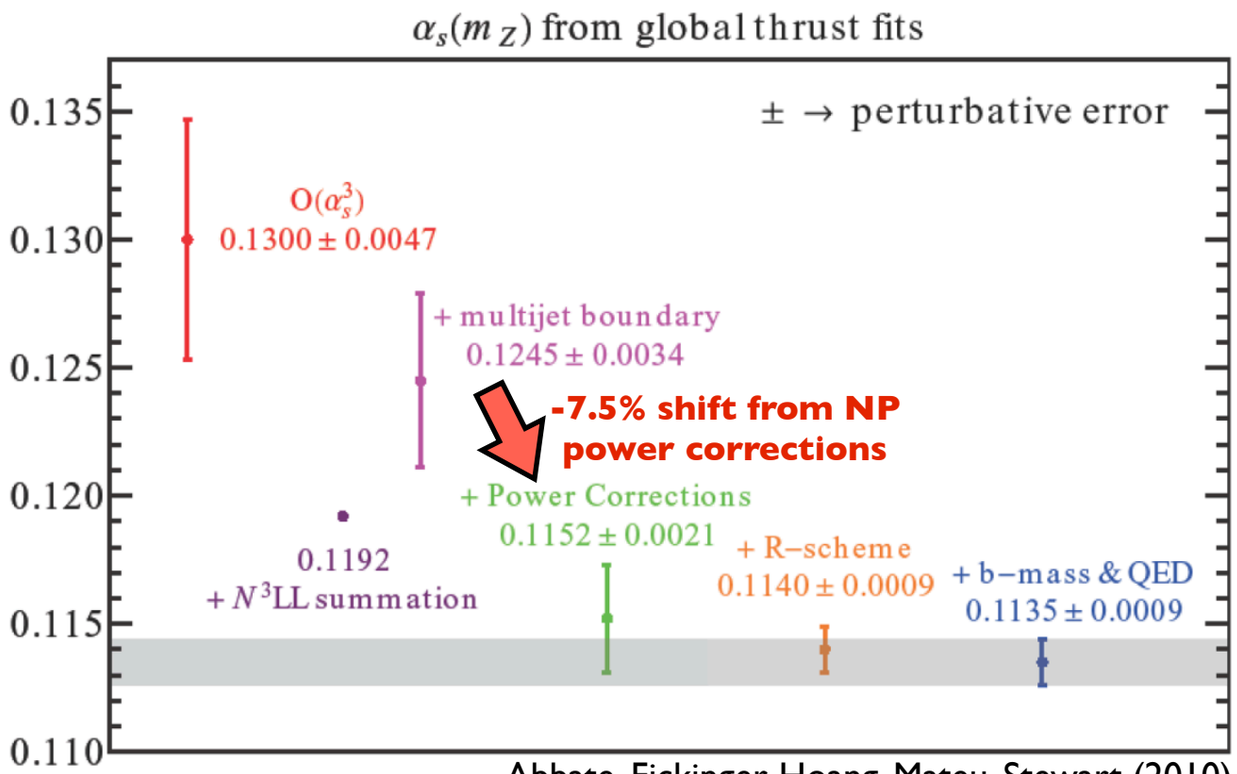

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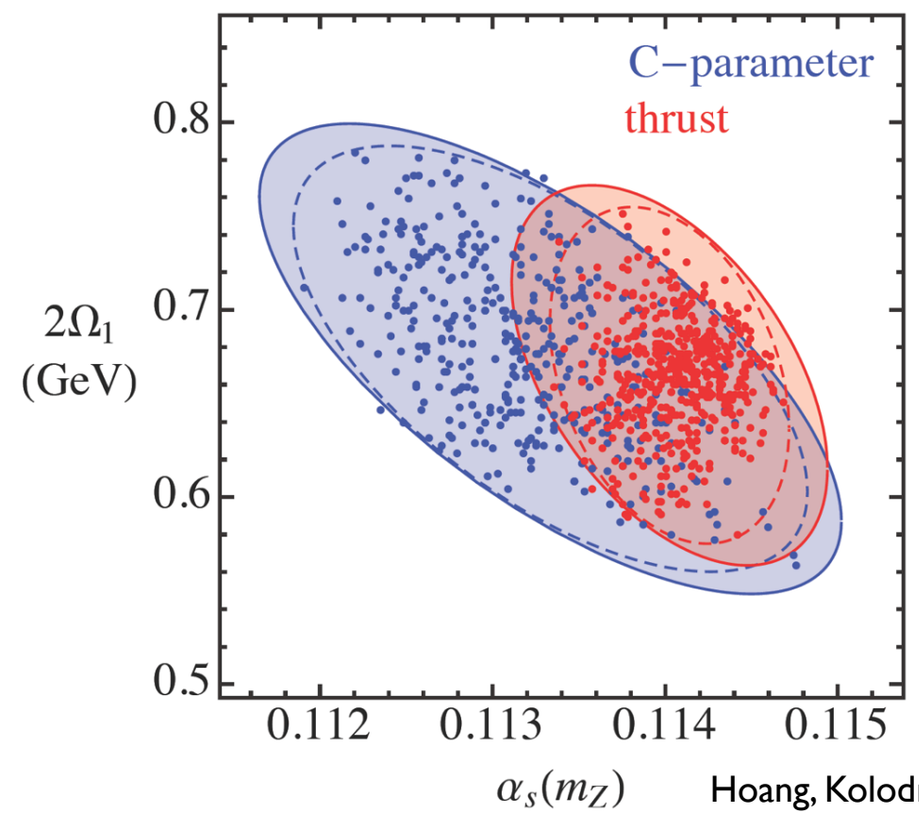

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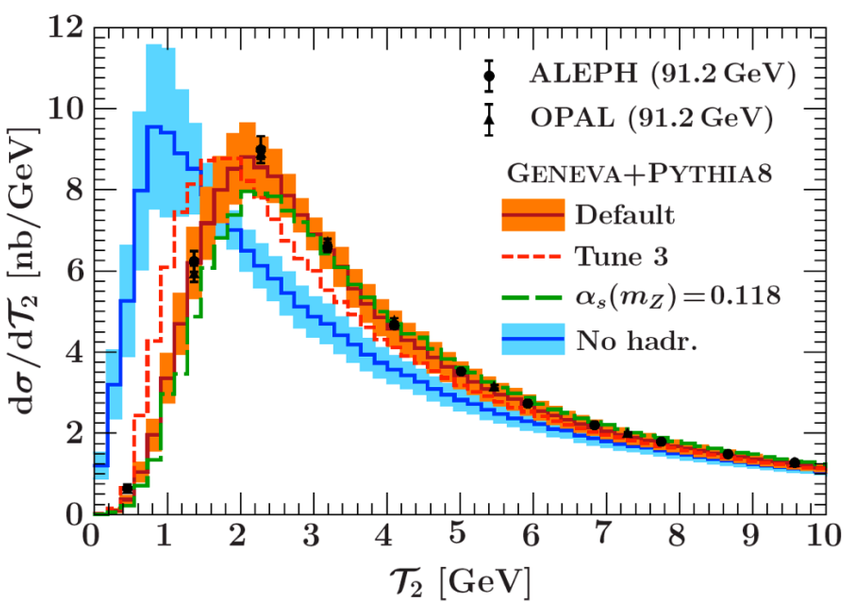
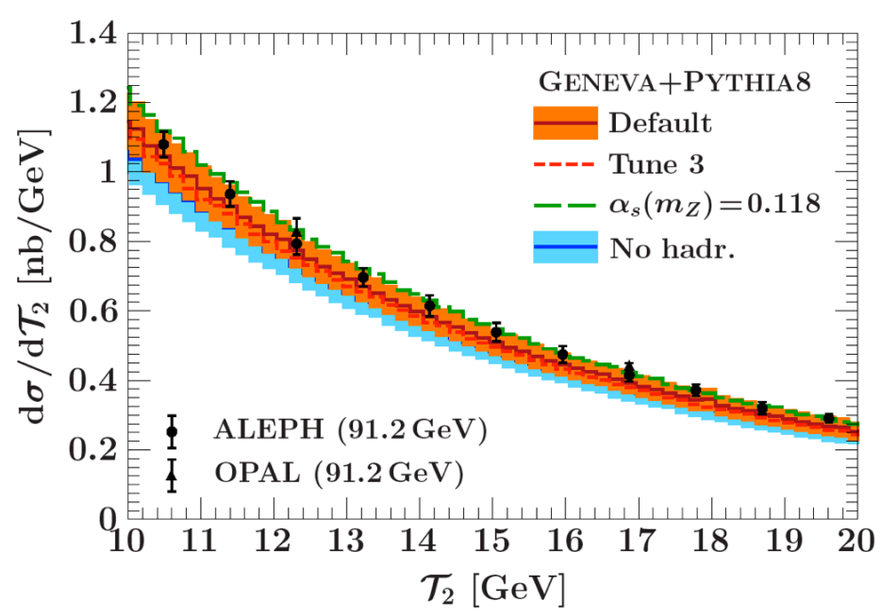


(a) Peak Region



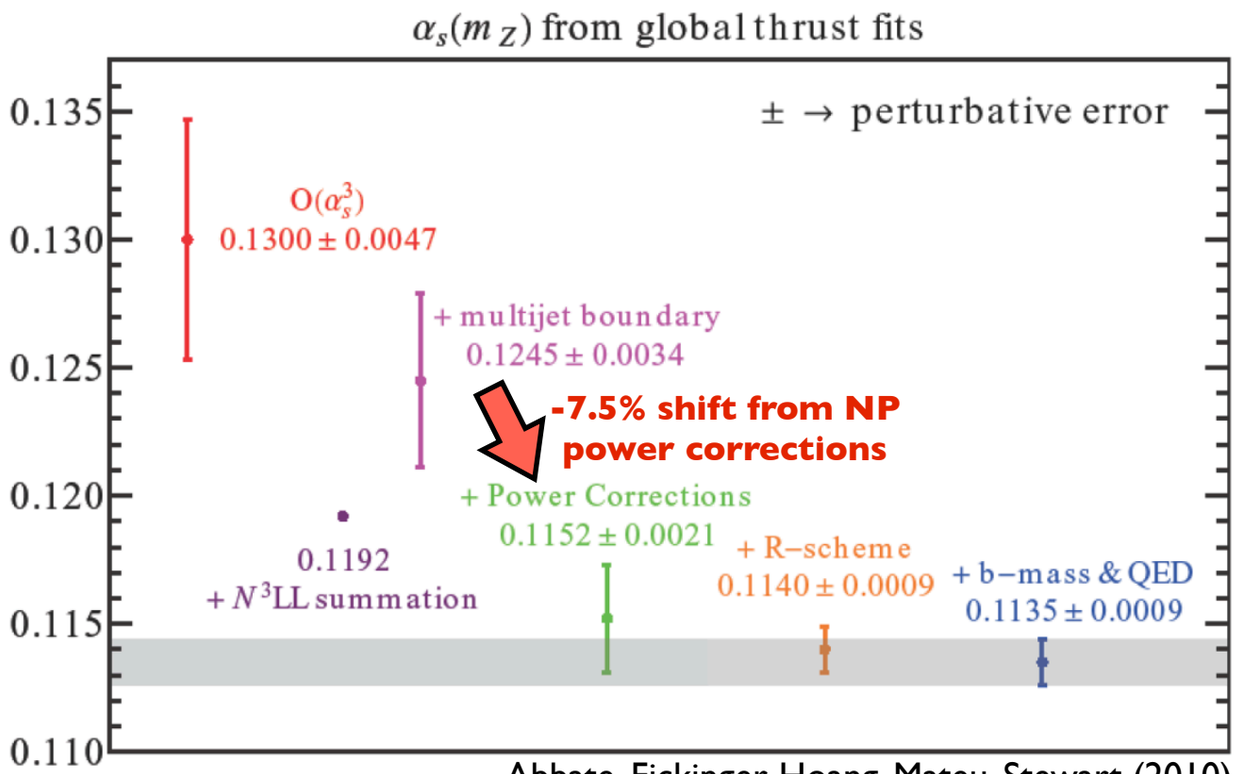
(b) Transition Region

GenEvA Collaboration (2012) (improved Monte Carlo with NLL and higher-order theory input)

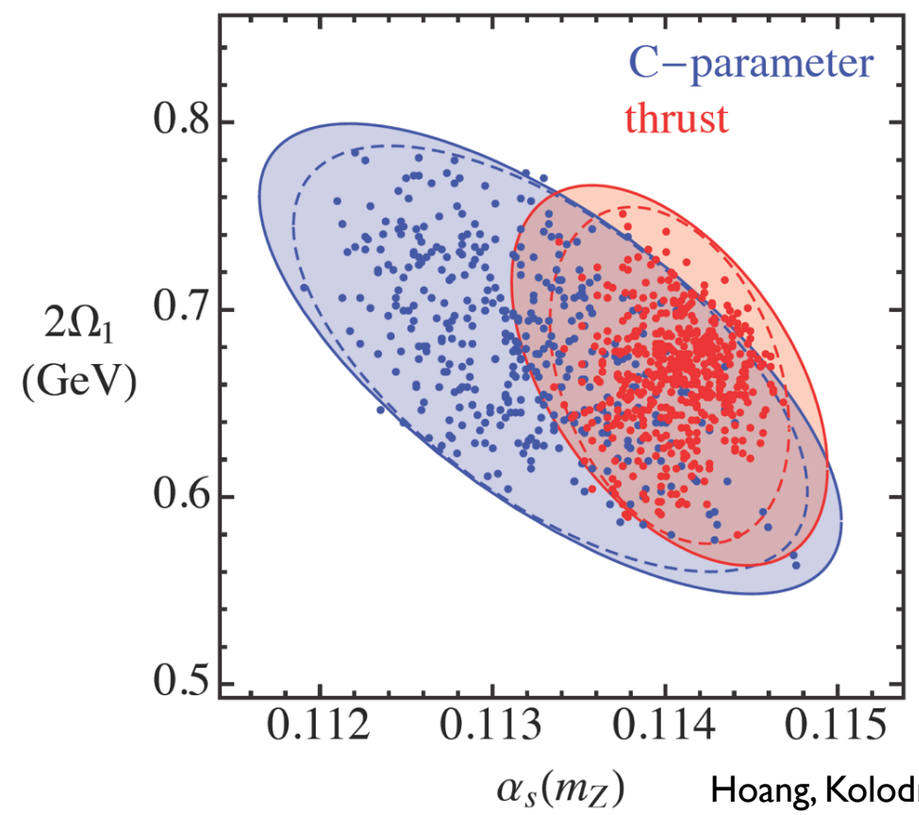
Best fit: $\alpha_s(M_Z) = 0.1135$

with Pythia Tune 1 (similar NP shift to above)

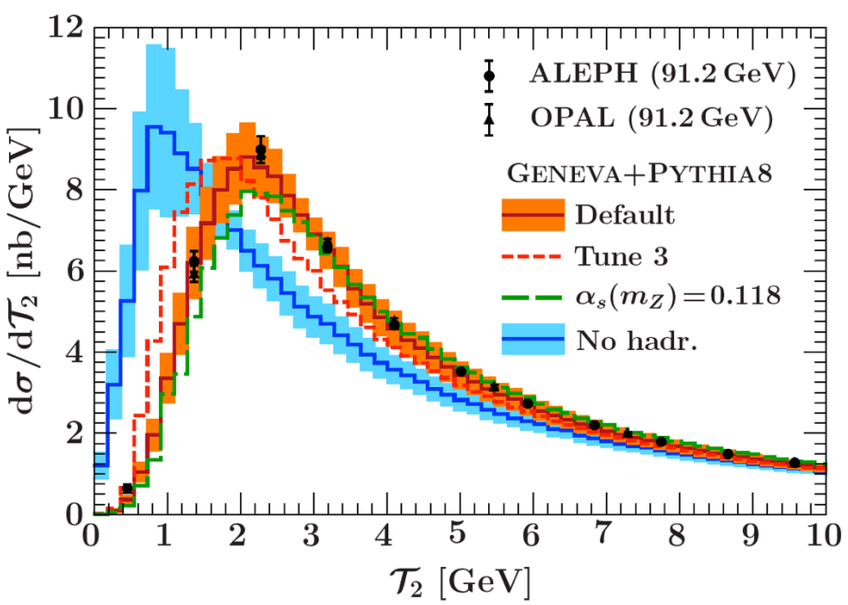
Most precise extractions of strong coupling



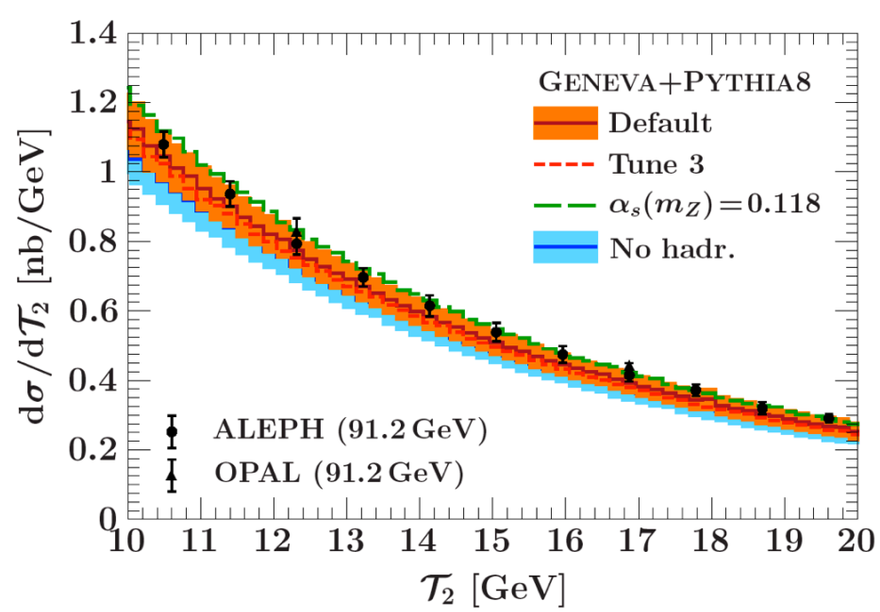
Abbate, Fickinger, Hoang, Mateu, Stewart (2010)



Hoang, Kolodrubetz, Mateu, Stewart (2013)



(a) Peak Region



(b) Transition Region

GenEvA Collaboration (2012) (improved Monte Carlo with NLL and higher-order theory input)

Best fit: $\alpha_s(M_Z) = 0.1135$

with Pythia Tune 1 (similar NP shift to above)

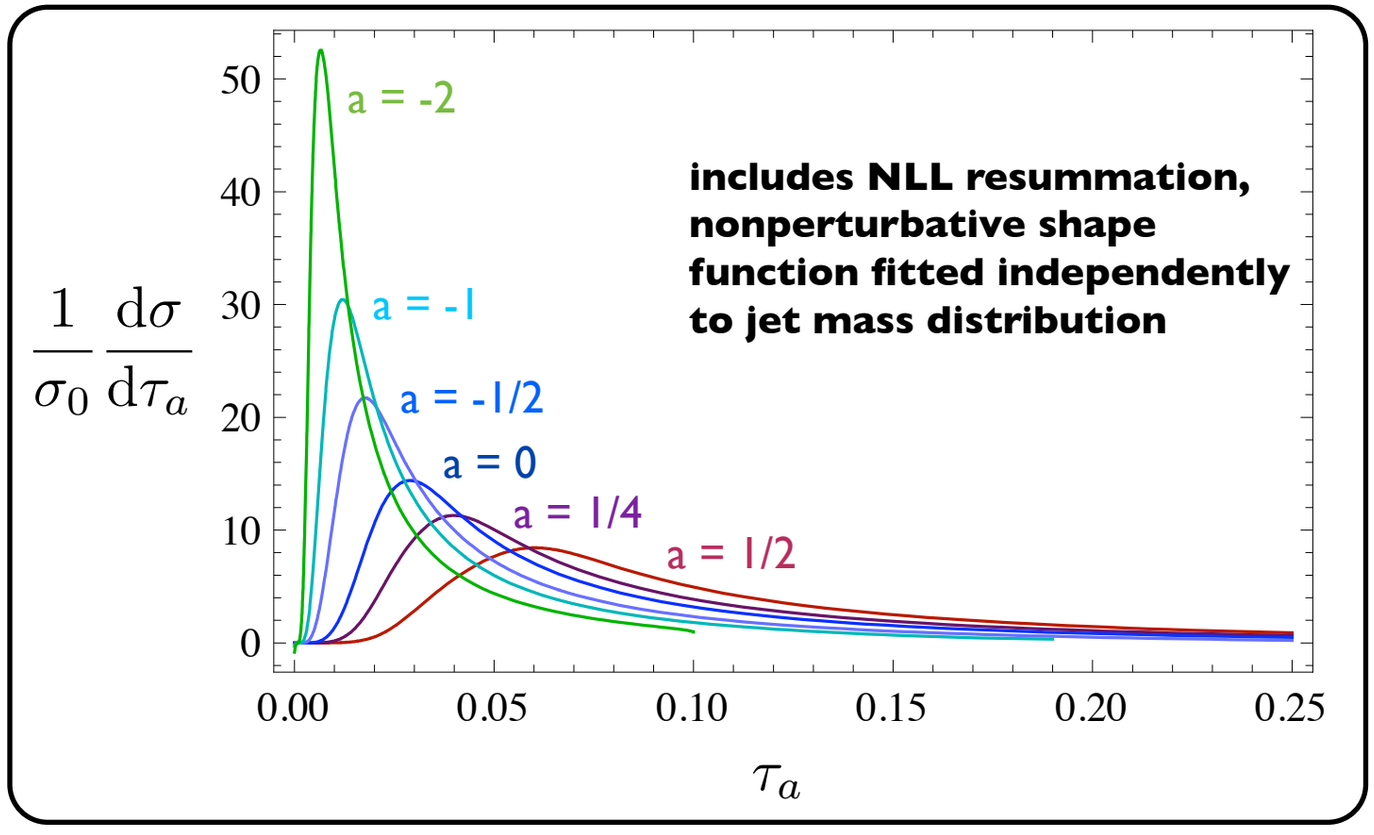
Generically, better perturbative calculations + rigorous treatment of nonperturbative corrections gives smaller α_s

Resummed angularity event shapes

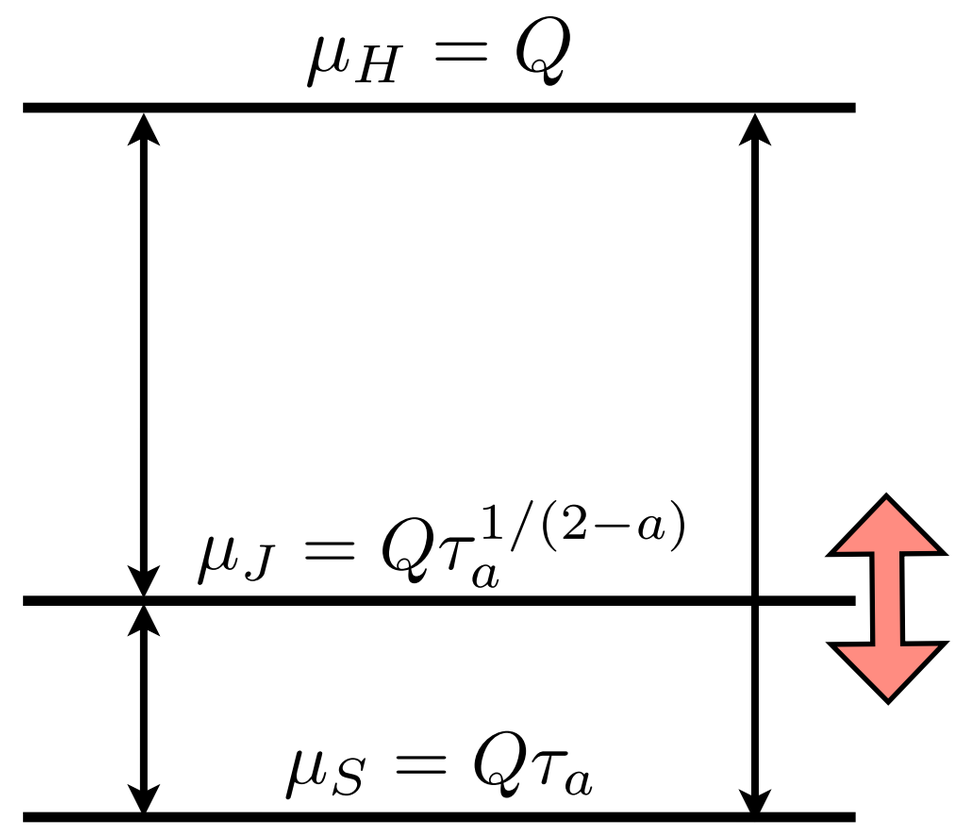
- Tunable class of event shapes:

$$\tau_a = \frac{1}{Q} \sum_{i \in X} |\mathbf{p}_T^i| e^{-|\eta_i|(1-a)}$$

Berger, Kucs, Sterman (2003)



Hornig, **CL**, Ovanesyan
JHEP **05**, 122 (2009)

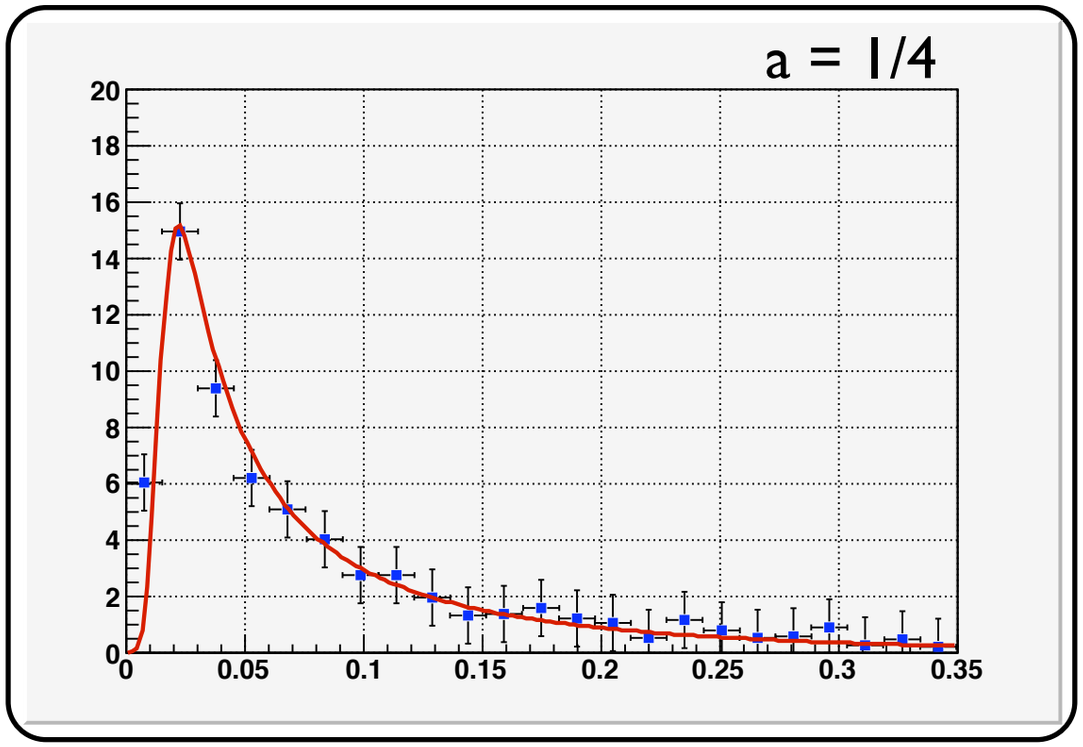


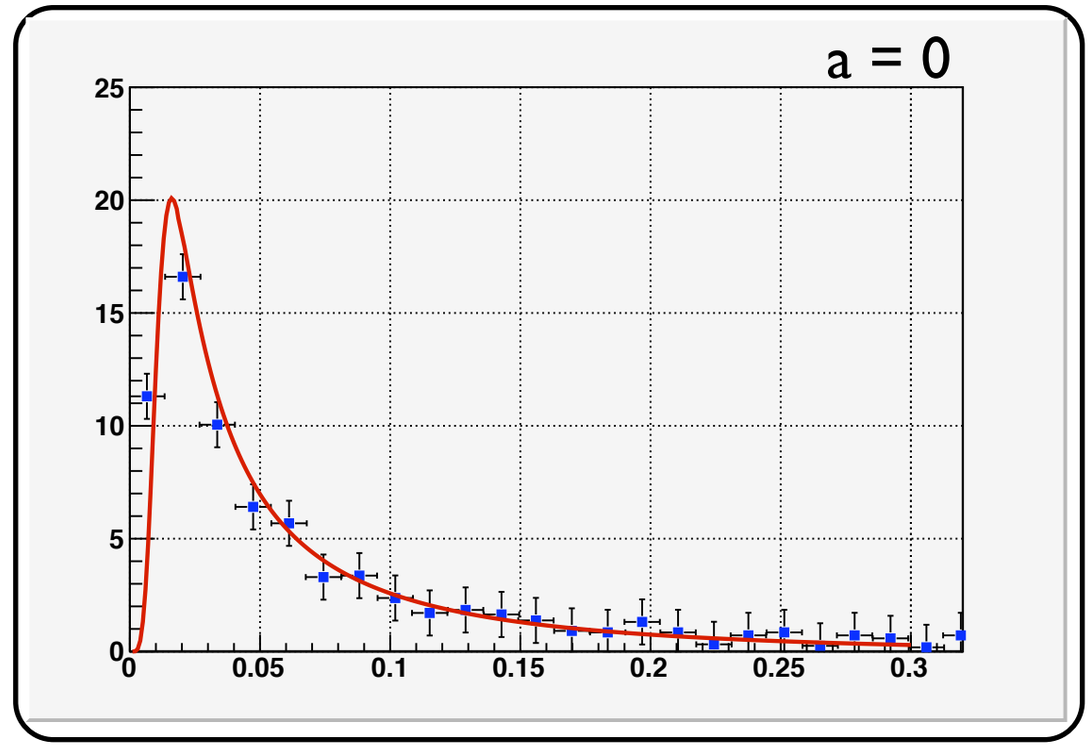
tune collinear scale in SCET

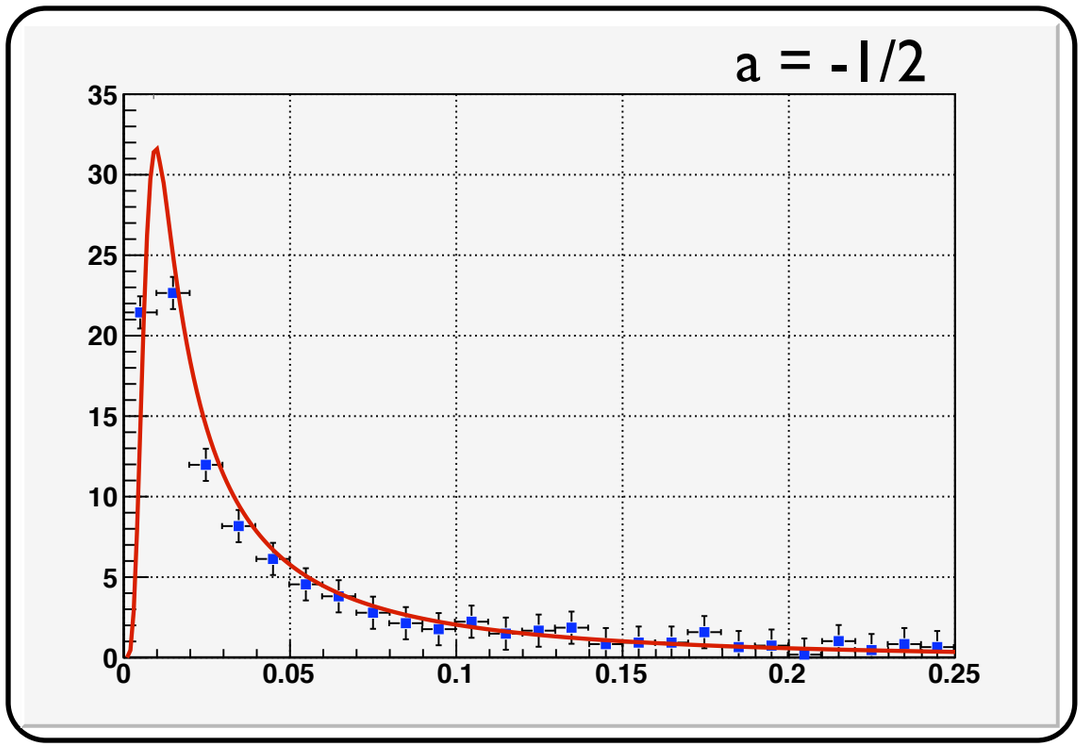
Angularity Distributions at Q = 197 GeV:

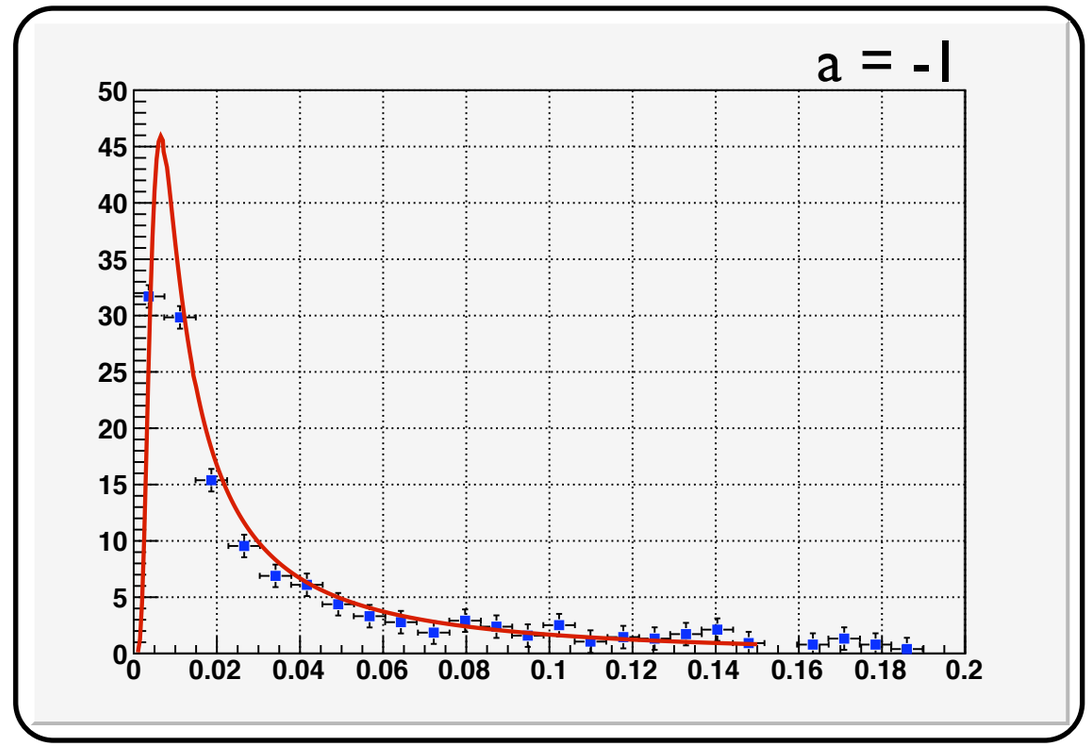
NLL Theory vs. Experiment

Th: Hornig, **CL**, Ovanesyan (2009)
Ex: P. Jindal et al. (LEP) (2011)





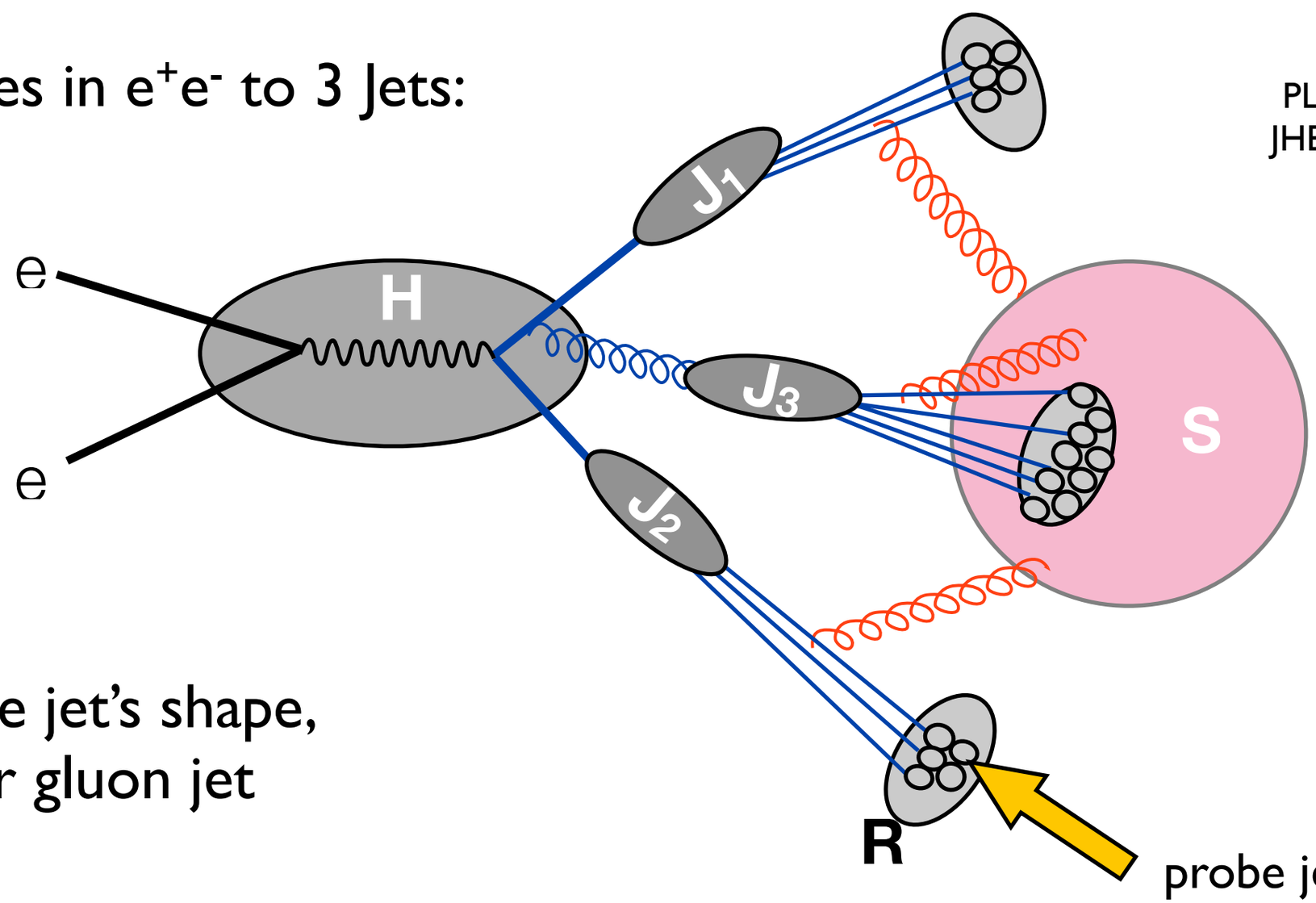




Factorization of Jet Shape Distributions

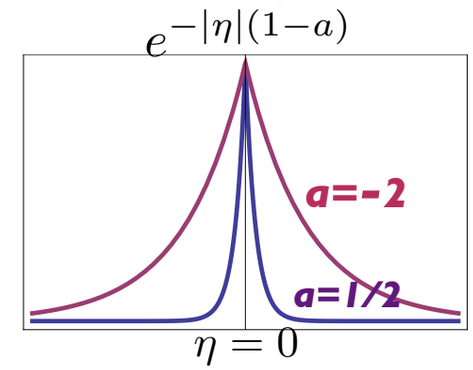
Ellis, Hornig, **CL**, Vermilion, Walsh
PLB **689**, 82 (2010)
JHEP **11**, 101 (2010)

- e.g. jet shapes in e^+e^- to 3 Jets:



- Measure one jet's shape, e.g. quark or gluon jet

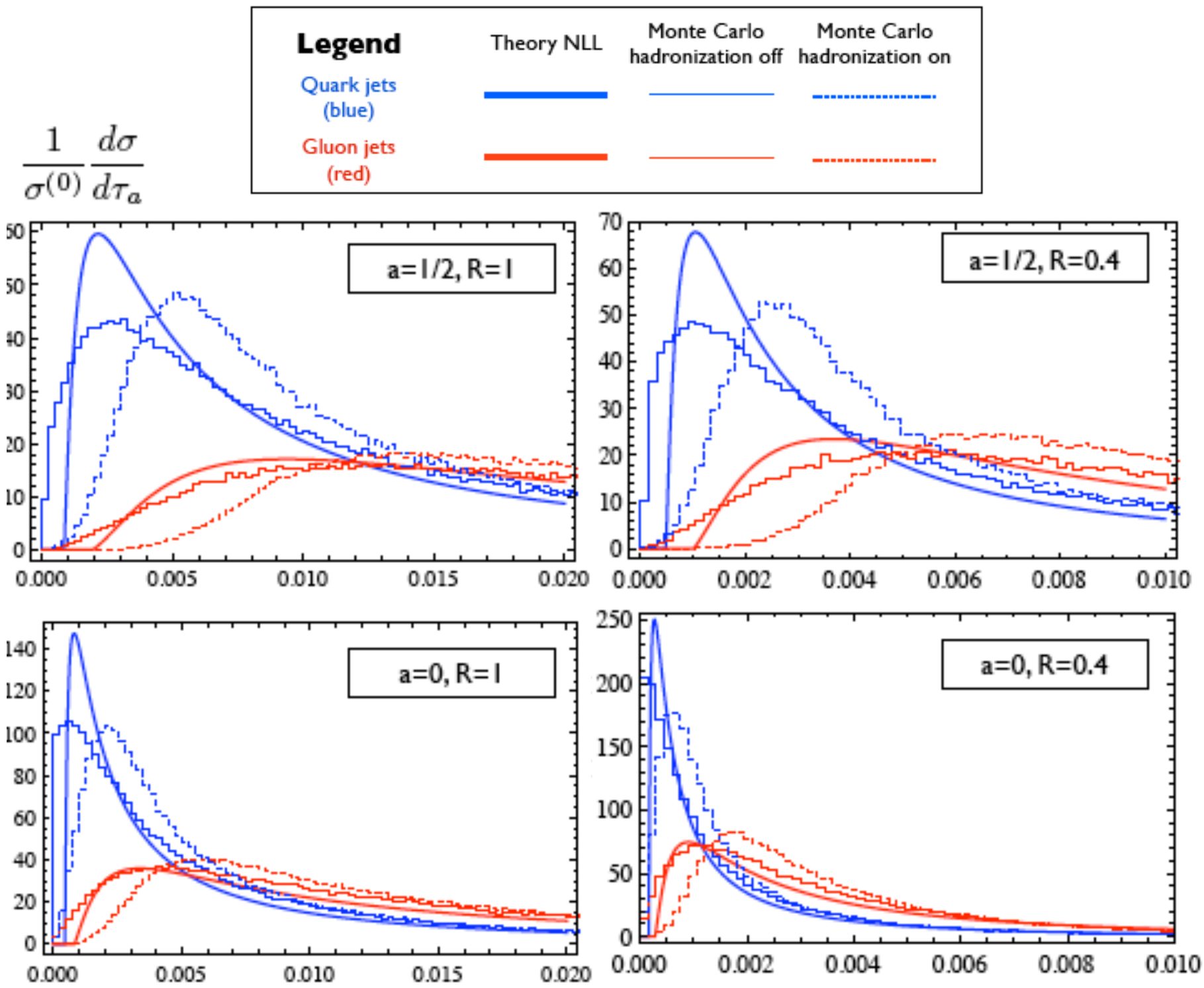
probe jet shape $\hat{\tau}_a$



- We derived new factorization theorem to account for jet algorithm dependence, new jet and soft functions for $a \neq 0$

Resummed Predictions of Jet Shape Distributions vs. Monte Carlo

Ellis, Hornig, **CL**, Vermilion, Walsh (2010)

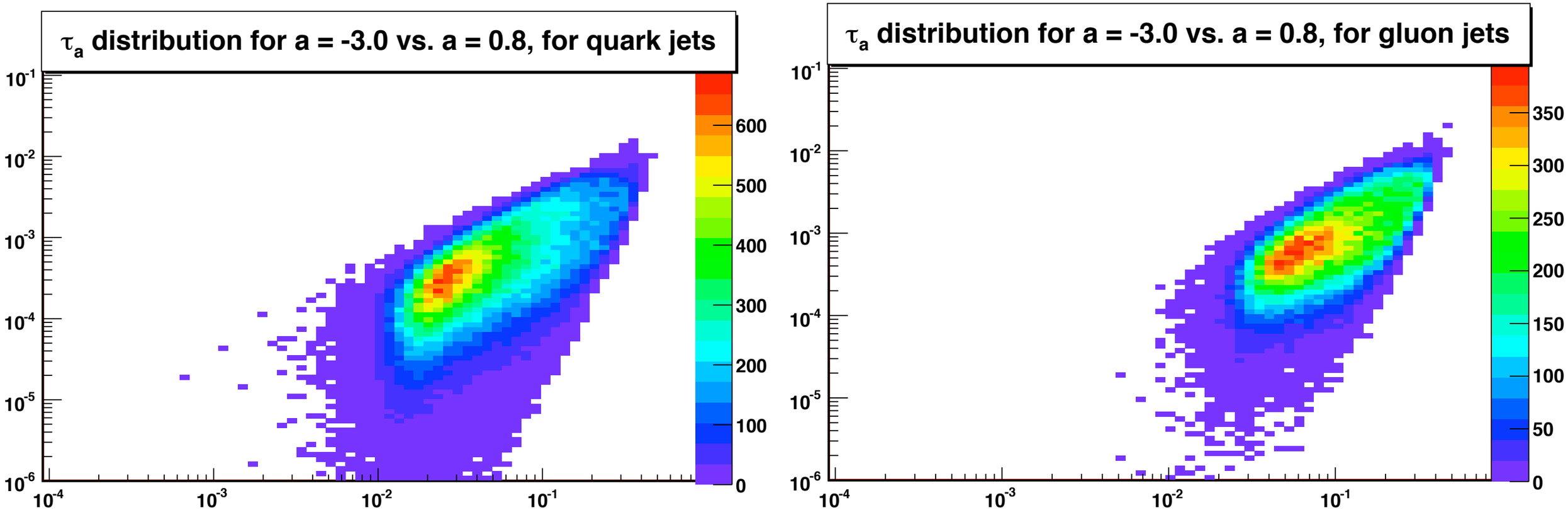


Predictions for e+e- to 3 jets in "Mercedes-Benz" configuration

Q = 500 GeV
E_J = 150 GeV

Gain ability to discriminate between quark and gluon jets!

Using angularities to distinguish quark and gluon jets



Ellis, Hornig, **CL**, Vermilion, Walsh

Measure multiple angularities (different *a*'s) for each jet, use to separate quark and gluon jets statistically.

With two different *a*'s, can find cuts enhancing fraction of quark-to-gluon jets in e^+e^- to 3-jet events by factor of 30-40. Enhance gluon-to-quark jets by factor of 5-10.

Extend to **multivariate analysis** with many *a*'s for greater discrimination.

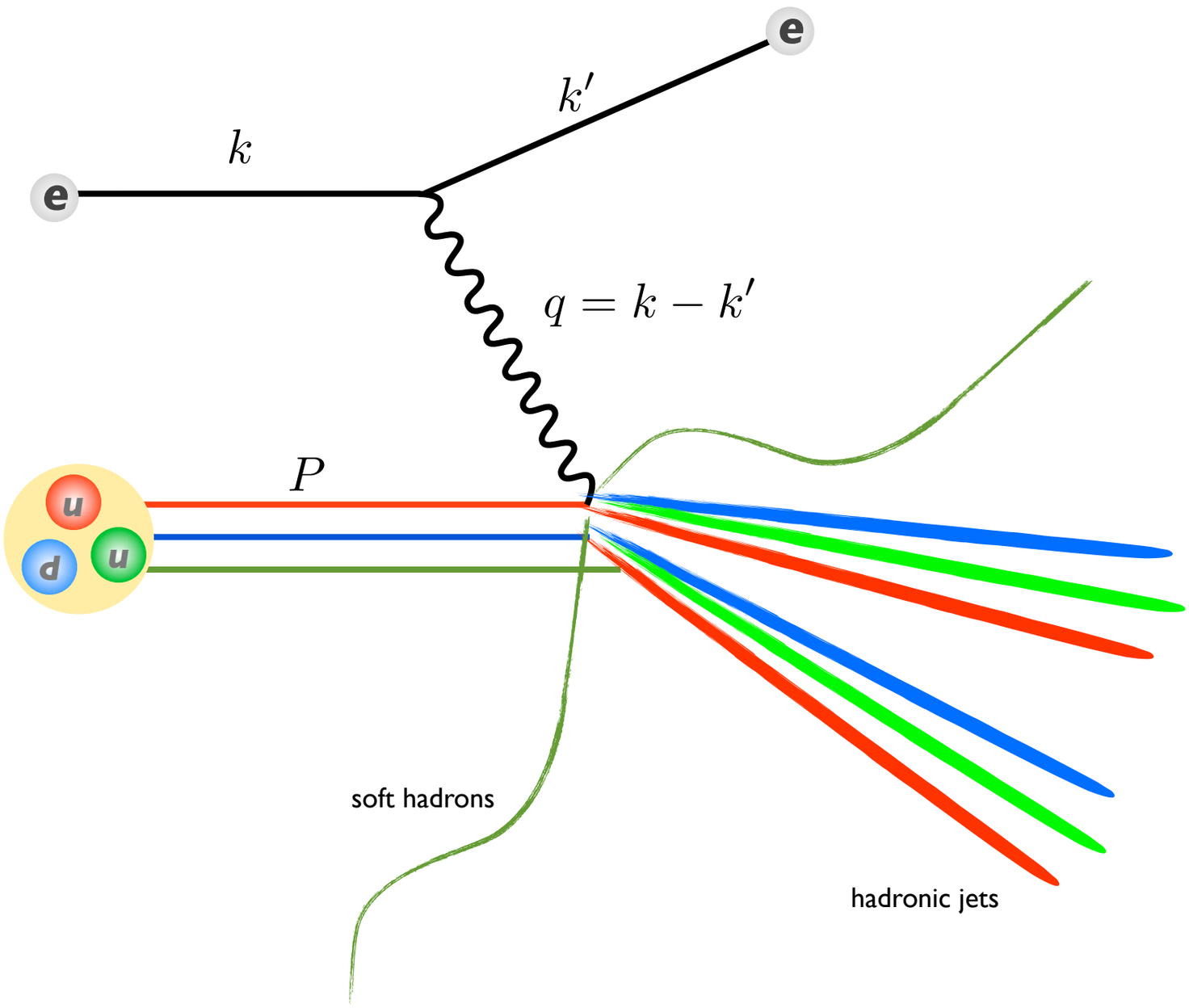
see also recent work by Gallicchio and Schwartz (2012) and Larkoski, Salam, and Thaler (2013)

Using 1-Jettiness to Measure 2 Jets in DIS in 3 Ways

Daekyoung Kang *(MIT)*, **CL, Iain Stewart** *(MIT)*

arXiv: 1303.6952

DIS Kinematics



$$s = (k + P)^2$$ squared center-of-mass energy

$$Q^2 = -q^2$$ momentum transfer

$$x = \frac{Q^2}{2P \cdot q}$$ Björken scaling variable

$$y = \frac{P \cdot q}{P \cdot k}$$ lepton energy loss in proton rest frame

$$Q^2 = xys$$

$$p_X = q + P$$ total momentum of final hadronic state

$$p_X^2 = \frac{1-x}{x} Q^2$$ invariant mass of final hadronic state

Limit $x \longrightarrow 1$ corresponds to single collimated jet in final state

We will look away from *x = 1* at *two*-jet like final states

Strong Coupling

Process	Collab.	Value	Exp.	Th.	Total	(%)
(1) Inc. jets at low Q^2	H1	0.1180	0.0018	$^{+0.0124}_{-0.0093}$	$^{+0.0125}_{-0.0095}$	$^{+10.6}_{-8.1}$
(2) Dijets at low Q^2	H1	0.1155	0.0018	$^{+0.0124}_{-0.0093}$	$^{+0.0125}_{-0.0095}$	$^{+10.8}_{-8.2}$
(3) Trijets at low Q^2	H1	0.1170	0.0017	$^{+0.0091}_{-0.0073}$	$^{+0.0093}_{-0.0075}$	$^{+7.9}_{-6.4}$
(4) Combined low Q^2	H1	0.1160	0.0014	$^{+0.0094}_{-0.0079}$	$^{+0.0095}_{-0.0080}$	$^{+8.2}_{-6.9}$
(5) Trijet/dijet at low Q^2	H1	0.1215	0.0032	$^{+0.0067}_{-0.0059}$	$^{+0.0074}_{-0.0067}$	$^{+6.1}_{-5.5}$
(6) Inc. jets at medium Q^2	H1	0.1195	0.0010	$^{+0.0052}_{-0.0040}$	$^{+0.0053}_{-0.0041}$	$^{+4.4}_{-3.4}$
(7) Dijets at medium Q^2	H1	0.1155	0.0009	$^{+0.0045}_{-0.0035}$	$^{+0.0046}_{-0.0036}$	$^{+4.0}_{-3.1}$
(8) Trijets at medium Q^2	H1	0.1172	0.0013	$^{+0.0053}_{-0.0032}$	$^{+0.0055}_{-0.0035}$	$^{+4.7}_{-3.0}$
(9) Combined medium Q^2	H1	0.1168	0.0007	$^{+0.0049}_{-0.0034}$	$^{+0.0049}_{-0.0035}$	$^{+4.2}_{-3.0}$
(10) Inc. jets at high Q^2 (anti-k_T)	ZEUS	0.1188	$^{+0.0036}_{-0.0035}$	$^{+0.0022}_{-0.0022}$	$^{+0.0042}_{-0.0041}$	$^{+3.5}_{-3.5}$
(11) Inc. jets at high Q^2 (SIScone)	ZEUS	0.1186	$^{+0.0036}_{-0.0035}$	$^{+0.0025}_{-0.0025}$	$^{+0.0044}_{-0.0043}$	$^{+3.7}_{-3.6}$
(12) Inc. jets at high Q^2 (k_T; HERA I)	ZEUS	0.1207	$^{+0.0038}_{-0.0036}$	$^{+0.0022}_{-0.0023}$	$^{+0.0044}_{-0.0043}$	$^{+3.6}_{-3.6}$
(13) Inc. jets at high Q^2 (k_T; HERA II)	ZEUS	0.1208	$^{+0.0037}_{-0.0032}$	$^{+0.0022}_{-0.0022}$	$^{+0.0043}_{-0.0039}$	$^{+3.6}_{-3.2}$
(14) Inc. jets in γp (anti-k_T)	ZEUS	0.1200	$^{+0.0024}_{-0.0023}$	$^{+0.0043}_{-0.0032}$	$^{+0.0049}_{-0.0039}$	$^{+4.1}_{-3.3}$
(15) Inc. jets in γp (SIScone)	ZEUS	0.1199	$^{+0.0022}_{-0.0022}$	$^{+0.0047}_{-0.0042}$	$^{+0.0052}_{-0.0047}$	$^{+4.3}_{-3.9}$
(16) Inc. jets in γp (k_T)	ZEUS	0.1208	$^{+0.0024}_{-0.0023}$	$^{+0.0044}_{-0.0033}$	$^{+0.0050}_{-0.0040}$	$^{+4.1}_{-3.3}$
(17) Jet shape	ZEUS	0.1176	$^{+0.0013}_{-0.0028}$	$^{+0.0091}_{-0.0072}$	$^{+0.0092}_{-0.0077}$	$^{+7.8}_{-6.5}$
(18) Subjet multiplicity	ZEUS	0.1187	$^{+0.0029}_{-0.0019}$	$^{+0.0093}_{-0.0076}$	$^{+0.0097}_{-0.0078}$	$^{+8.2}_{-6.6}$
HERA average 2004		0.1186	± 0.0011	± 0.0050	± 0.0051	± 4.3
HERA average 2007		0.1198	± 0.0019	± 0.0026	± 0.0032	± 2.7

Table 1: Values of $\alpha_s(M_Z)$ extracted from jet observables at HERA together with their uncertainties (rows 1 to 18). The 2004 [10] and 2007 [11] HERA averages are shown in the last two rows.

Extractions from exclusive jet cross sections have order 10-20% uncertainty, dominated by theory

Improve to level of e^+e^-?

C. Glasman, in the Proceedings of the Workshop on Precision Measurements of α_s [1110.0016]

N-jettiness

An *inclusive* event shape over all final state hadrons *excluding* more than *N* jets:

$$\tau_N = \frac{2}{Q^2} \sum_i \min\{q_B \cdot p_i, q_1 \cdot p_i, \ldots, q_N \cdot p_i\}$$

Stewart, Tackmann, Waalewijn (2010)

Vector q_B is aligned with the incoming proton beam and $q_{1,\ldots,N}$ with final state jets.
Final state hadrons *i* are grouped with the axis "closest" to it.

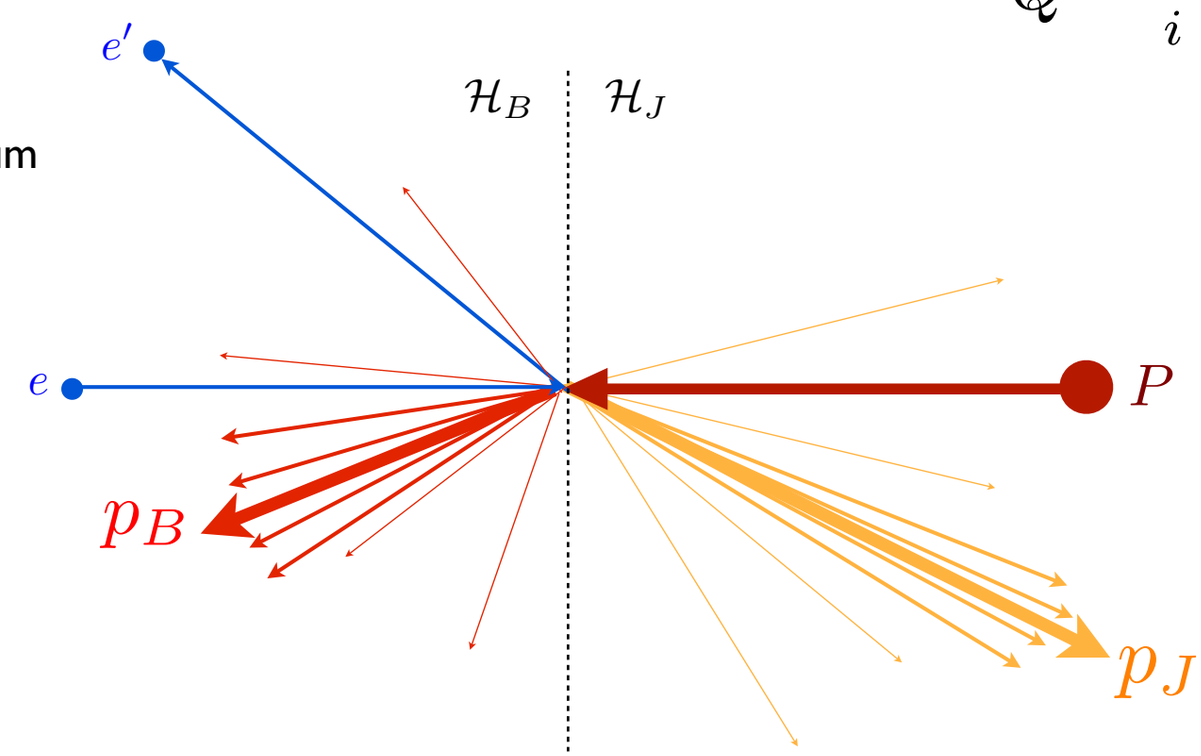
As $\tau_N \to 0$, final state contains exactly *N+1* pencil-like jets (one from beam radiation).

We will look at "1-jettiness" in DIS.

$$\tau_1 = \frac{2}{Q^2} \sum_i \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

Center-of-momentum frame:

$$k = \sqrt{s}\frac{n_z}{2}$$



$$P = \sqrt{s}\frac{\bar{n}_z}{2}$$

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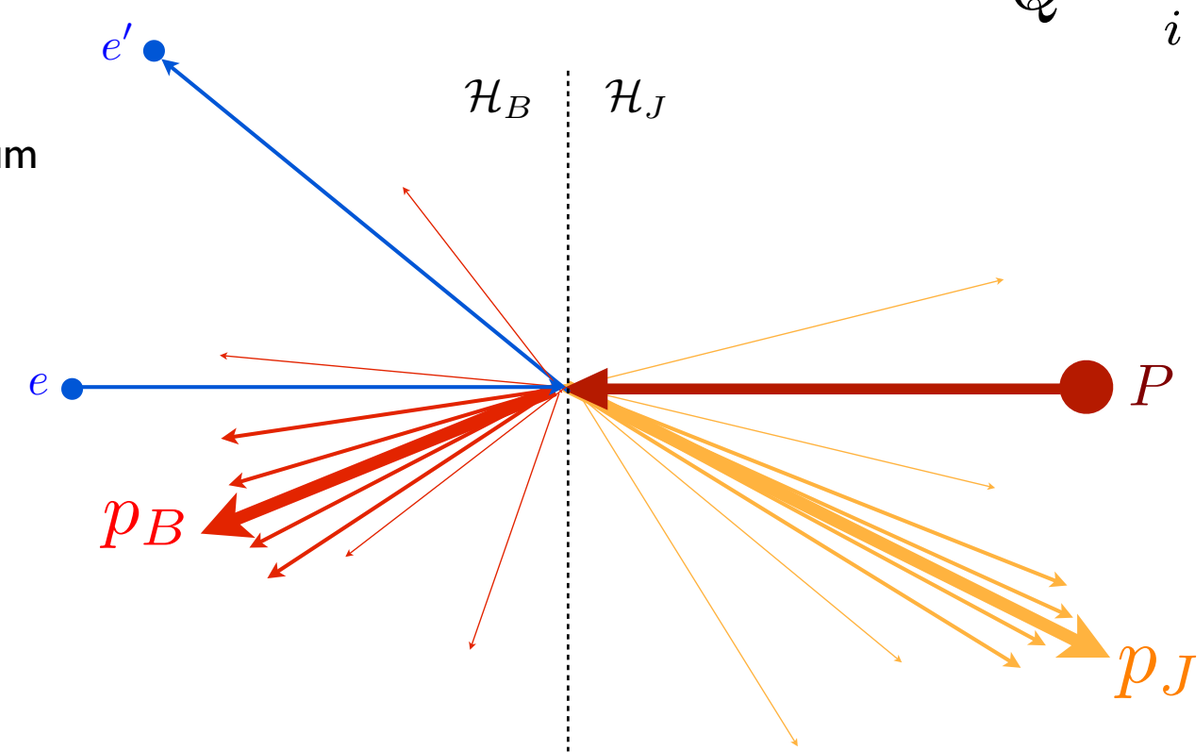
We will look at "1-jettiness" in DIS.

$$\tau_1 = \frac{2}{Q^2} \sum_i \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

How shall we pick $\boldsymbol{q_B}$ and $\boldsymbol{q_J}$?

Center-of-momentum frame:

$$k = \sqrt{s}\frac{n_z}{2}$$

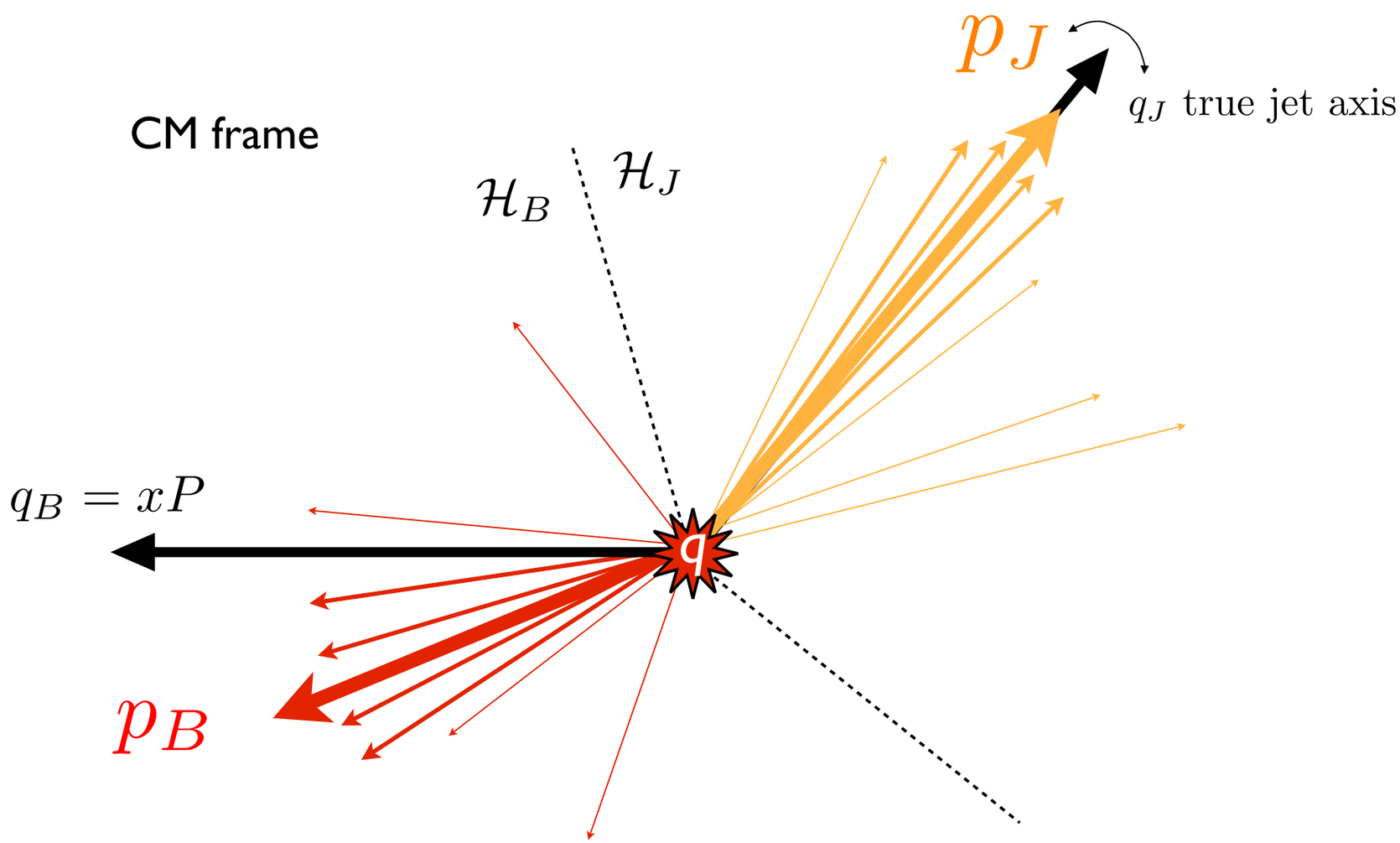


$$P = \sqrt{s}\frac{\bar{n}_z}{2}$$

Three choices for DIS 1-jettiness

τ_1^a

CM frame

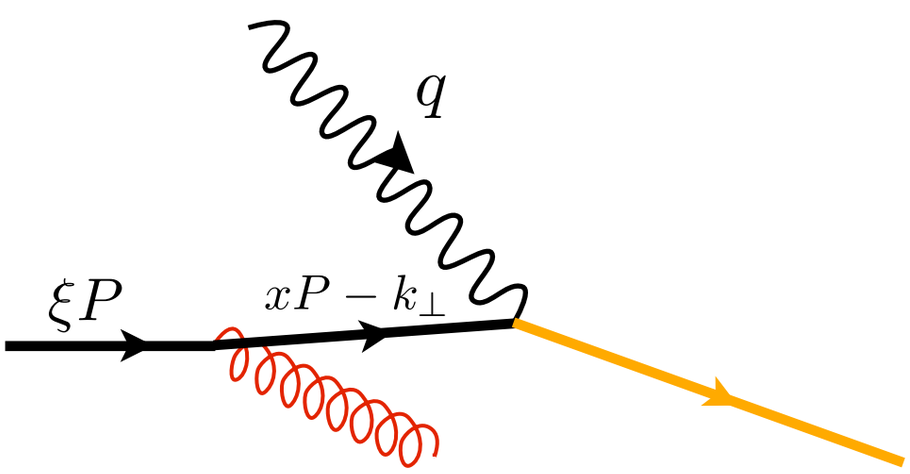


$$q_B = xP$$

$$q_J = \text{true jet axis}$$

Kang, Mantry, Qiu (2012)

q_J is **A**ligned with the jet momentum, with no relative label transverse momentum: find by jet algorithm or minimization



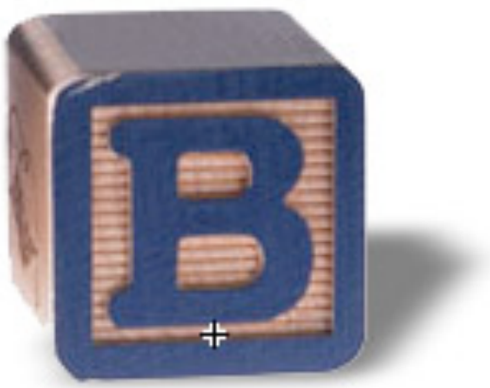
$$p_{\rm ISR} = (\xi - x)P + k_\perp$$

$$q_J = q + xP - k_\perp$$

$$k_\perp \sim Q\lambda$$

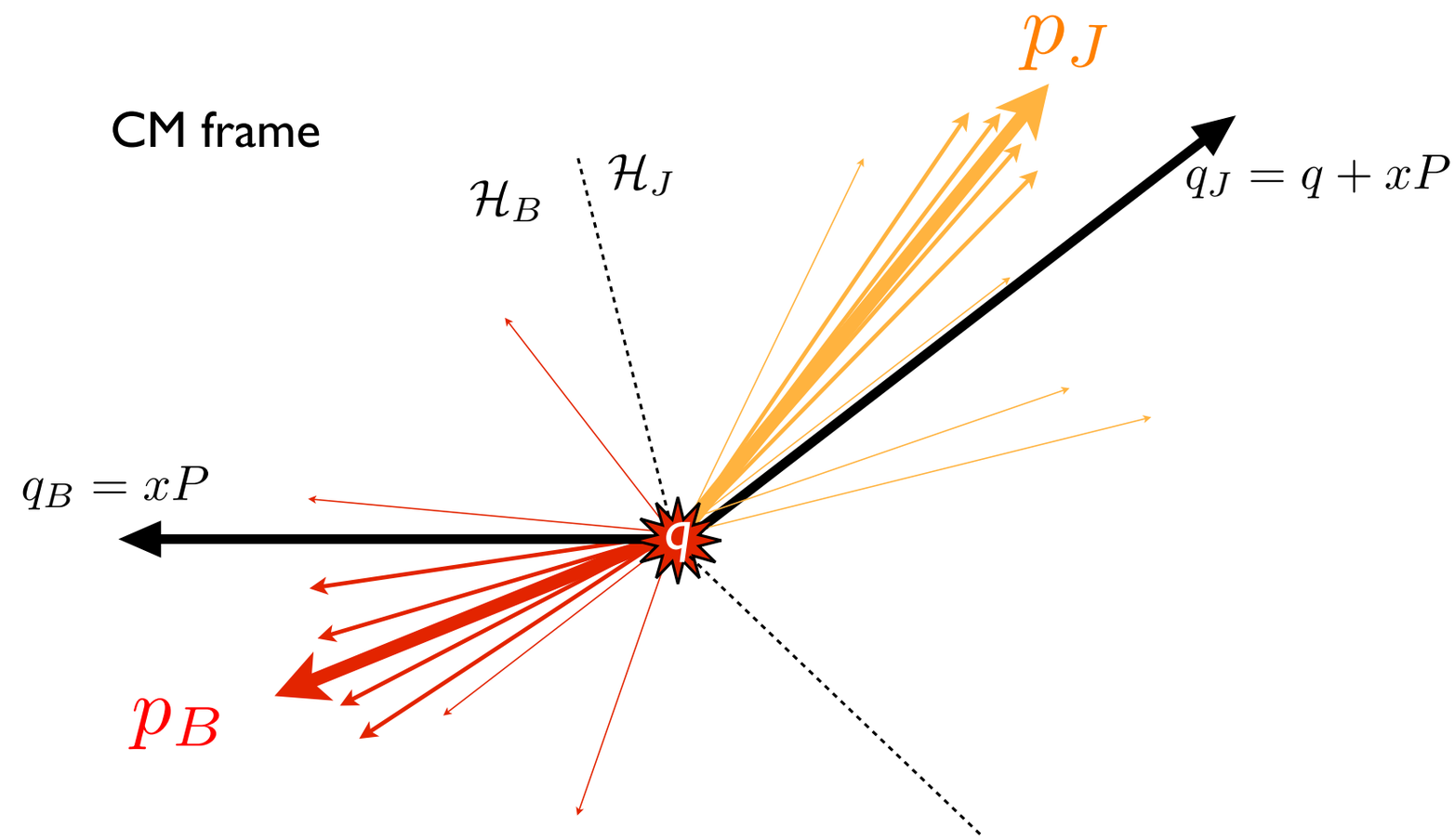
depends on momenta of final-state hadrons

Three choices for DIS 1-jettiness



τ_1^b

CM frame



$$q_B = xP$$

$$q_J = q + xP$$

same as DIS thrust
by Antonelli, Dasgupta, Salam
(1999)

q_J no longer exactly aligned with jet, but simpler in that *q+xP* is given only by lepton and initial-state proton momenta

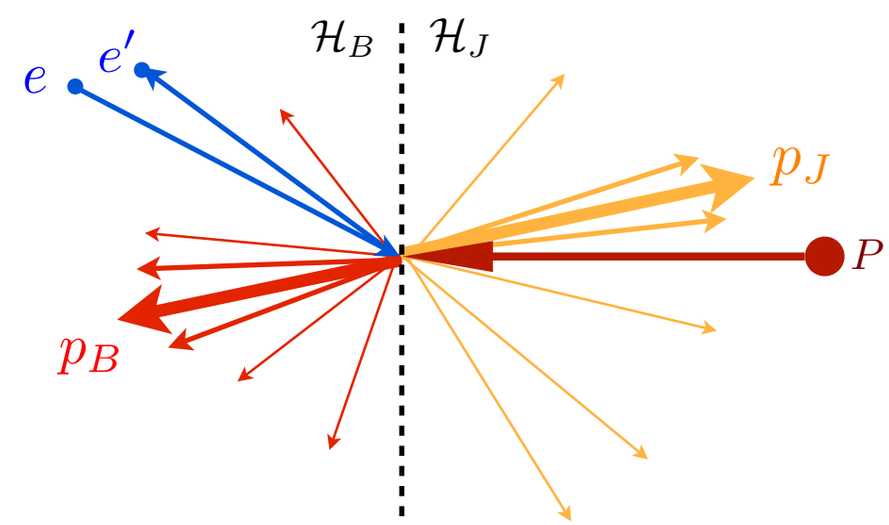
Boost

Breit frame:

$$q = (0, 0, 0, Q)$$

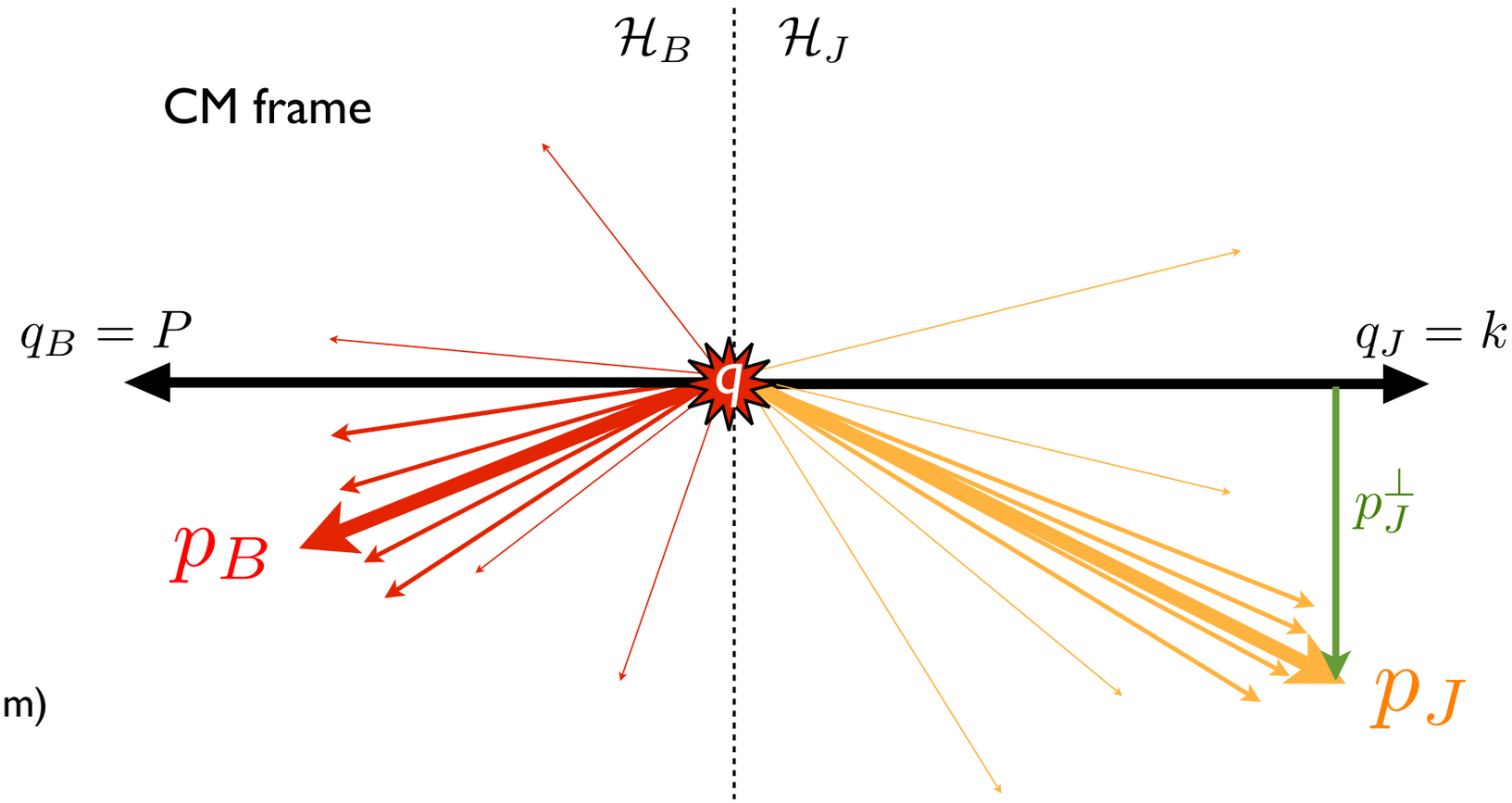
$$q_B = Q\bar{n}_z \quad q_J = Qn_z$$

1-jettiness regions are hemispheres in Breit frame



Three choices for DIS 1-jettiness

τ_1^c



$q_B = P$

$q_J = k$

(electron momentum)

measures thrust in back-to-back hemispheres in **C**enter-of-momentum frame

momentum transfer ***q*** itself has a nonzero transverse component:

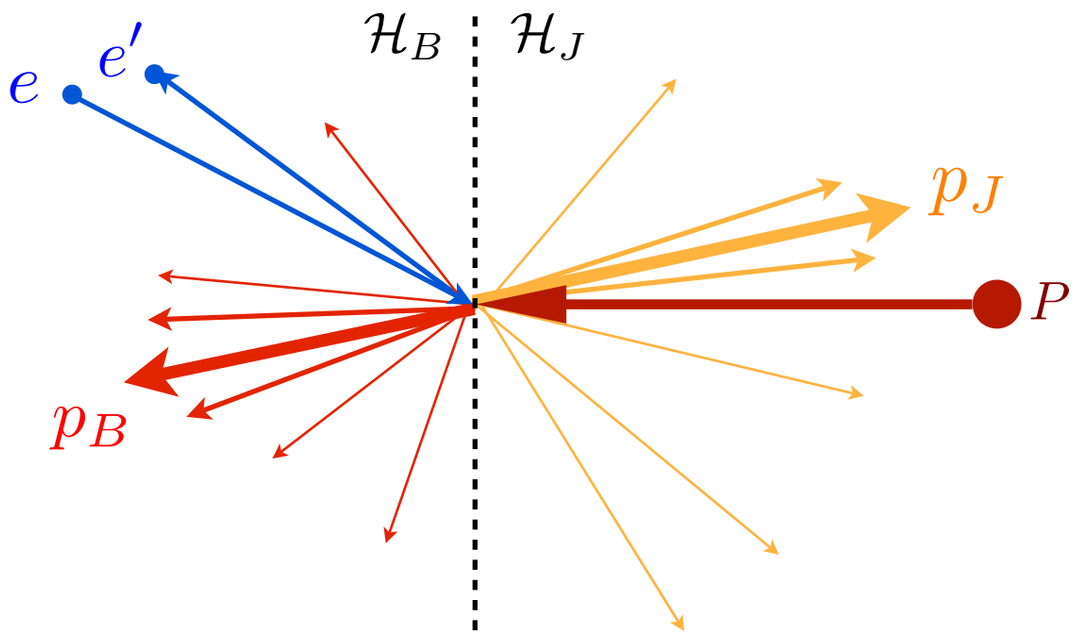
$$q = y\sqrt{s}\frac{n_z}{2} - xy\sqrt{s}\frac{\bar{n}_z}{2} + \sqrt{1-y}\,Q\hat{n}_\perp$$

seemingly simplest definition: *in practice* hardest to calculate!

Restriction: $p_J^\perp$ has to be small for 1-jettiness τ_1^c to be small $\Rightarrow 1 - y \sim \lambda^2$

Do you have to measure the beam region?

- One can use momentum conservation to measure 1-jettiness just from jet region:



e.g. τ_1^b in Breit frame

$$\tau_1^b \overset{\text{Breit}}{=} 1 - \frac{2}{Q} \sum_{i \in \mathcal{H}_J^b} p_z^i$$

Similarly, in CM frame, $\tau_1^c \overset{\text{CM}}{=} \frac{1}{x}\left(1 - \frac{2}{y\sqrt{s}} \sum_{i \in \mathcal{H}_J^c} p_z^i\right)$

And, in the 2-jet limit, $\tau_1^a \overset{\tau_1 \ll 1}{=} \tau_1^b + \frac{2}{Q^2} \sum_{i \in \mathcal{H}_J^b} (q_J^a - q_J^b) \cdot p_i$

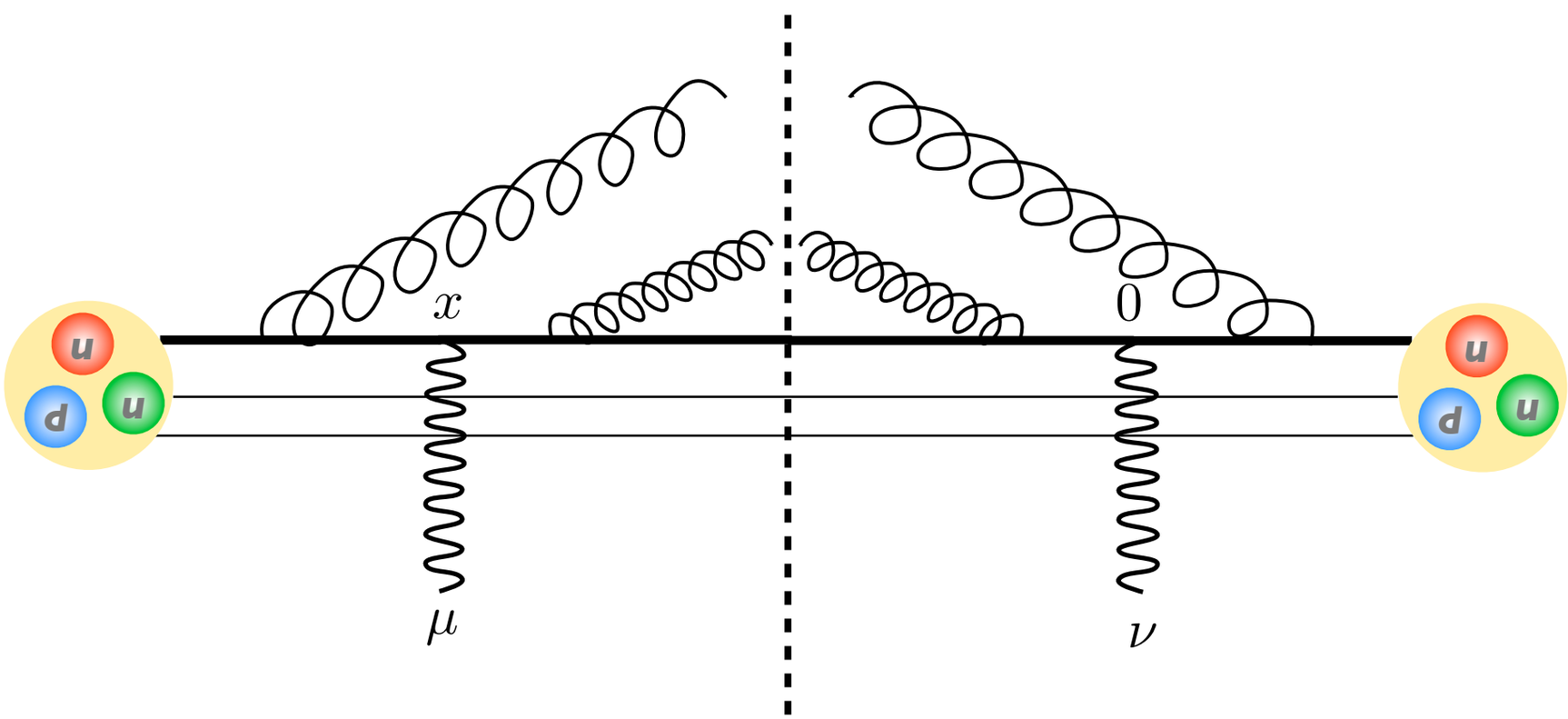
Factorization Theorem for 1-Jettiness

$$\frac{d\sigma(x,Q^2)}{d\tau_1} = \underbrace{L_{\mu\nu}(x,Q^2)}_{\text{leptonic tensor}}\underbrace{W^{\mu\nu}(x,Q^2,\tau_1)}_{\text{hadronic tensor}}$$

Start in QCD:

$$W^{\mu\nu}(x,Q^2,\tau_1) = \int d^4x\, e^{iq\cdot x}\langle P|\bar{q}\gamma^\mu q(x)\delta(\tau_1-\hat{\tau}_1)\bar{q}\gamma^\nu q(0)|P\rangle$$

$$\hat{\tau}_1|X\rangle = \tau_1(X)|X\rangle$$



Measure τ_1 of particles crossing the cut

Factorization Theorem for 1-Jettiness

Factor collinear and soft matrix elements:

hard function

$$W_{\mu\nu}(x,Q^2,\tau_1) = \int d^2\tilde{p}_\perp \int d\tau_J d\tau_B d\tau_S \, C^*(Q^2,\mu)C(Q^2,\mu)\,\delta\Big(\tau_1 - \frac{t_J}{s_J} - \frac{t_B}{s_B} - \frac{k_S}{Q_R}\Big)$$

soft function

$$\times \langle 0|[Y^\dagger_{n'_J} Y^\dagger_{n'_B}](0)\delta(k_S - n'_J\cdot\hat{p}_{J'} - n'_B\cdot\hat{p}_{B'})[Y_{n'_B} Y_{n'_J}](0)|0\rangle$$

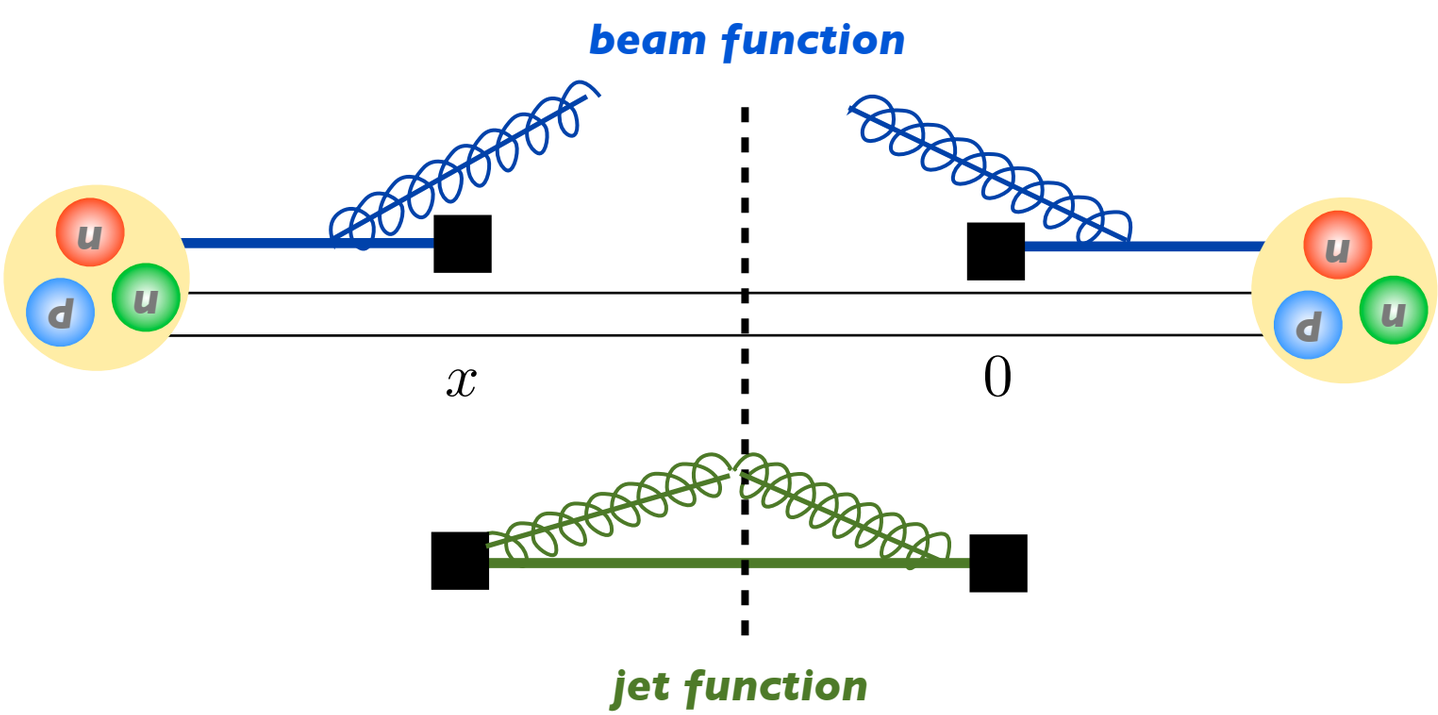
beam function

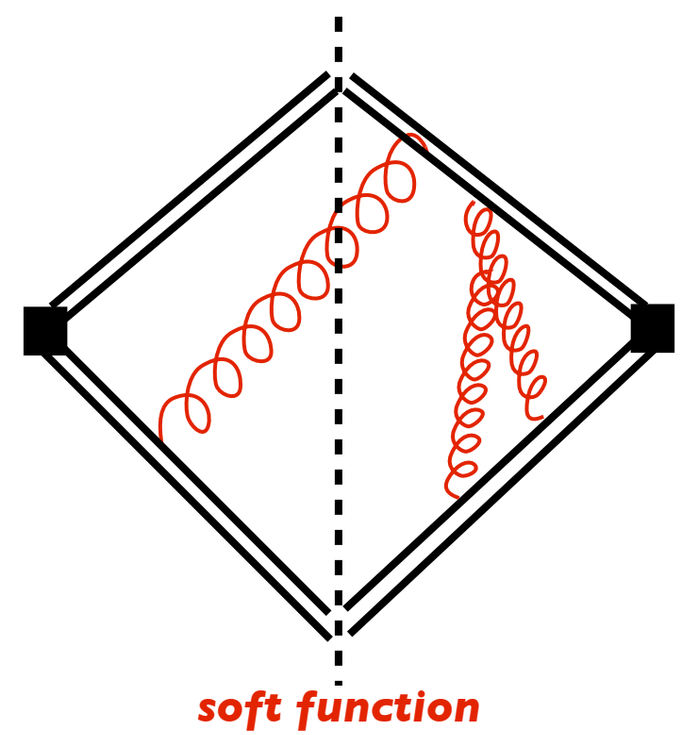
$$\times \langle P_{n_B}|\bar{\chi}_{n_B}(0)\delta(Q_B\tau_B - n_B\cdot\hat{p}^{n_B})[\delta(\bar{n}_B\cdot q + \bar{n}_B\cdot\mathcal{P})\delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)\chi_{n_B}](0)|P_{n_B}\rangle$$

jet function

$$\times \langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\delta^2(q_\perp + \tilde{p}_\perp + \mathcal{P}_\perp)\bar{\chi}_{n_J}(0)|0\rangle$$

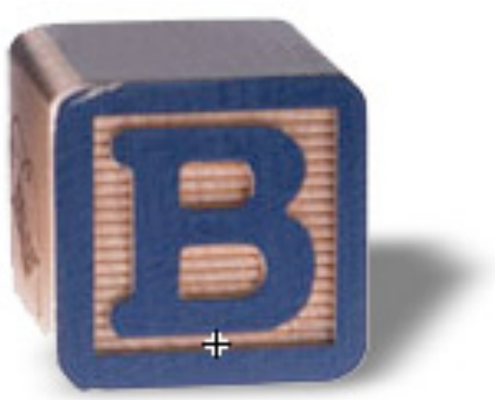
(+ permutations)





Factorization Theorems for 1-Jettiness

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^a} = H(Q^2,\mu)\int dt_J dt_B dk_S \delta\left(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right)$$
$$\times J_q(t_J,\mu)B_q(t_B,x,\mu)S(k_S,\mu)$$



$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\left(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right)$$
$$\times J_q(t_J - \mathbf{p}_\perp^2,\mu)\mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu)S(k_S,\mu)$$

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Factorization Theorems for 1-Jettiness

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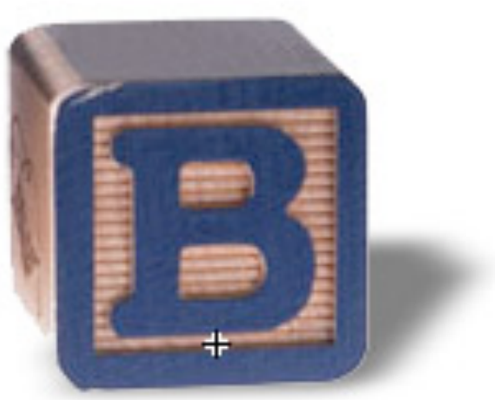

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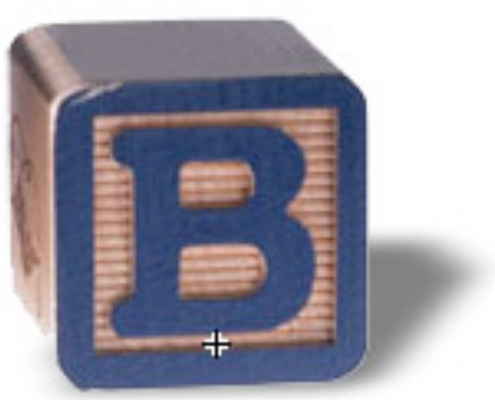

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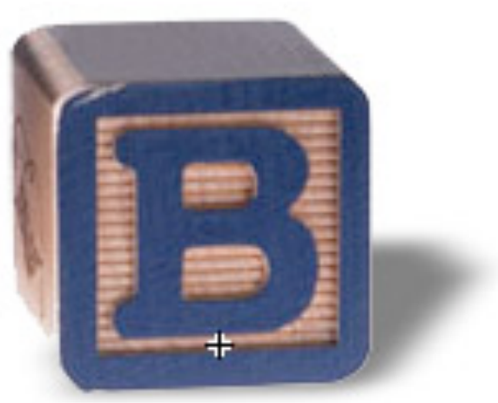

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Theoretical Uncertainty

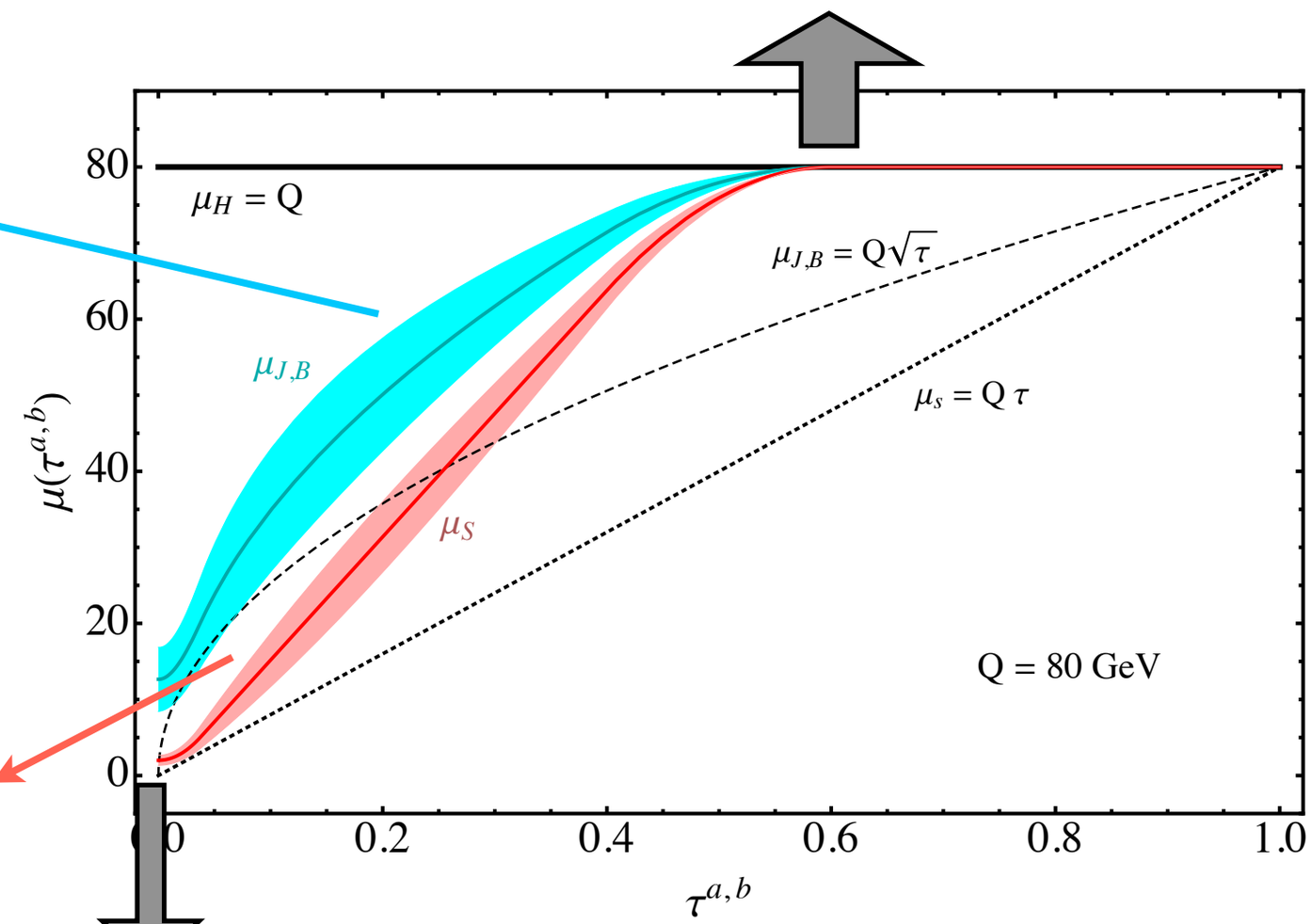
Uncertainty from missing terms of higher order in perturbation theory estimated by varying the scales in three ways:

Stewart, Tackmann, Waalewijn (2010)

need to turn off resummation for matching to fixed order at large τ

2. jet/beam scale within cyan band

1. overall scale between Q/2 and 2Q



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nonzero minimum values of scales at small τ

variation of multiple scales allows more realistic estimate of uncertainty than fixed-order or traditional pQCD predictions which only allow one (overall) scale variation

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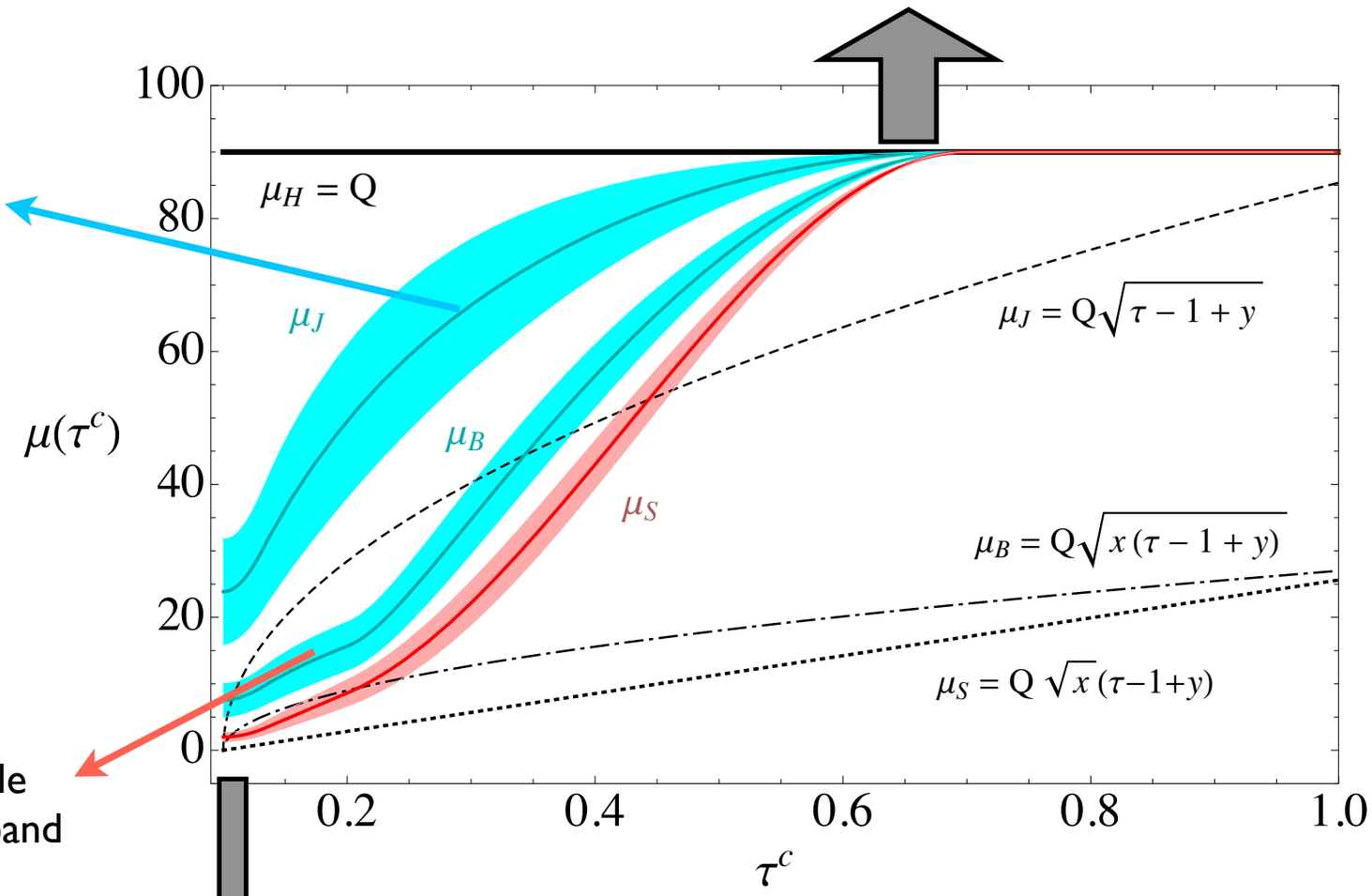

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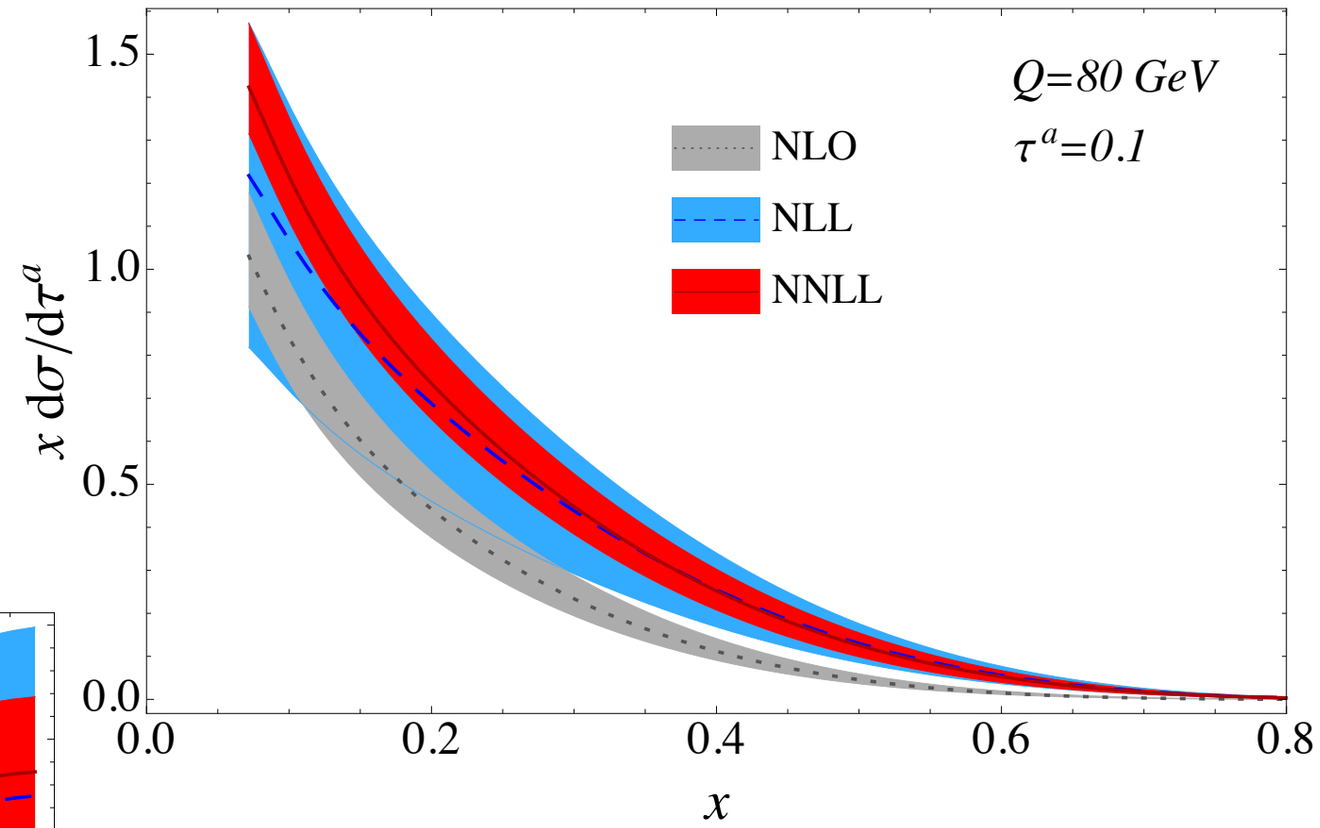
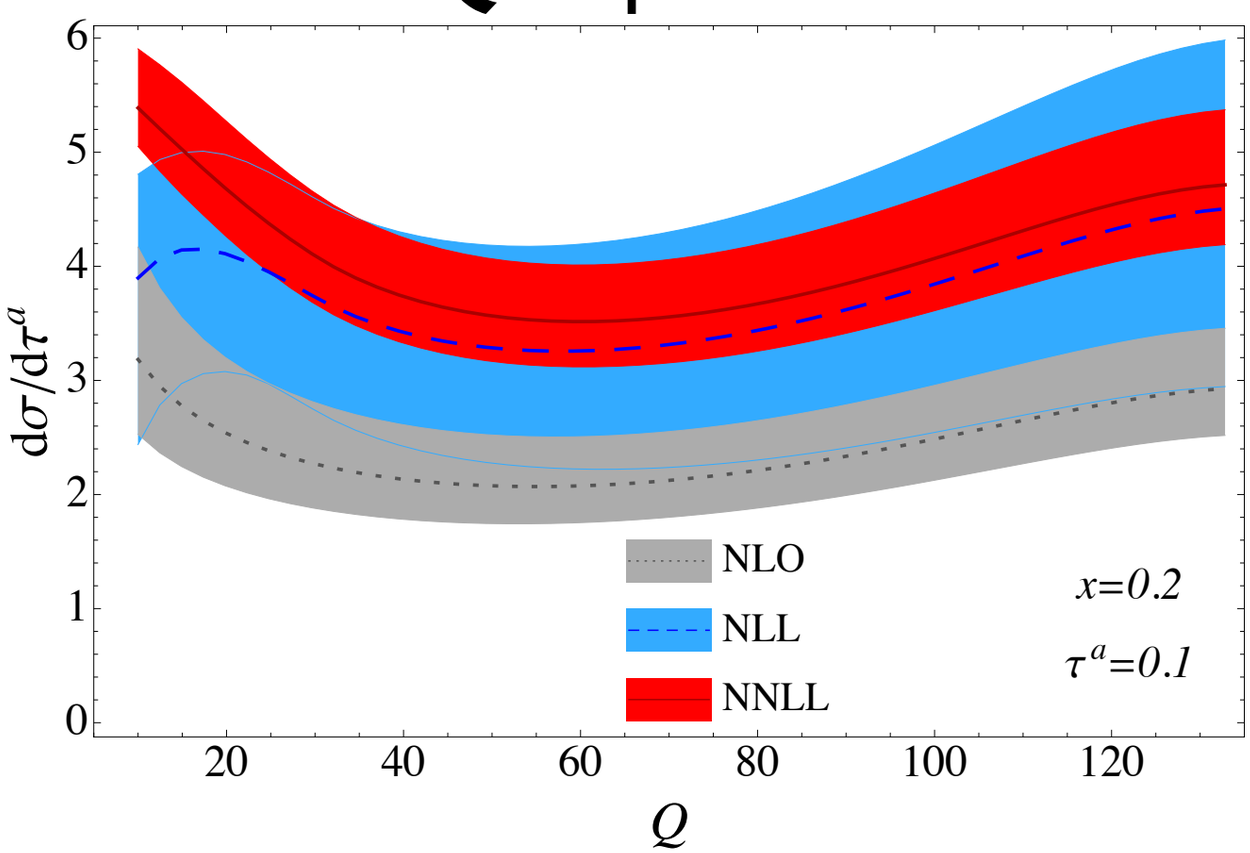
Predictions for DIS 1-jettiness

x dependence:

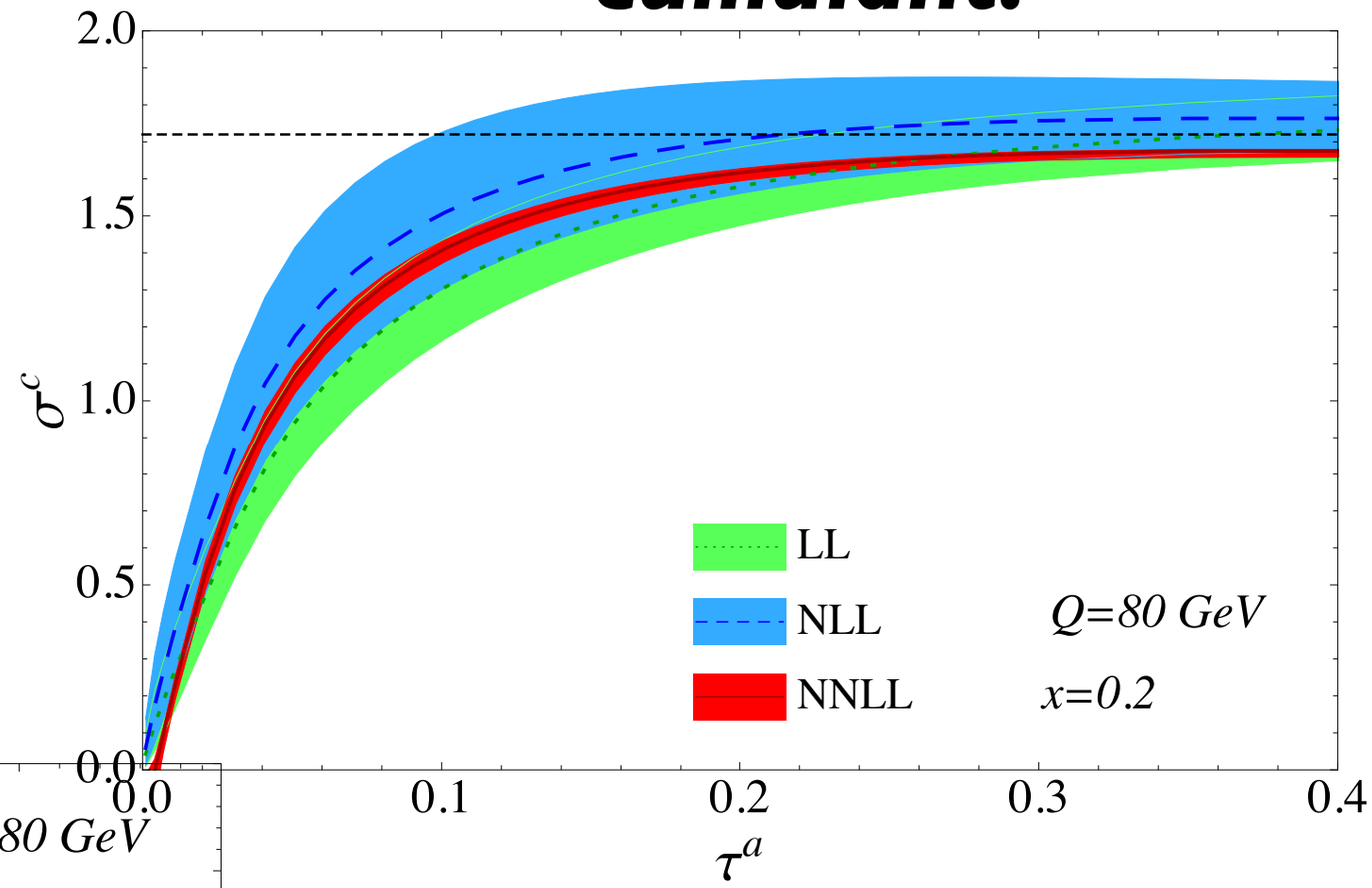


Q dependence:

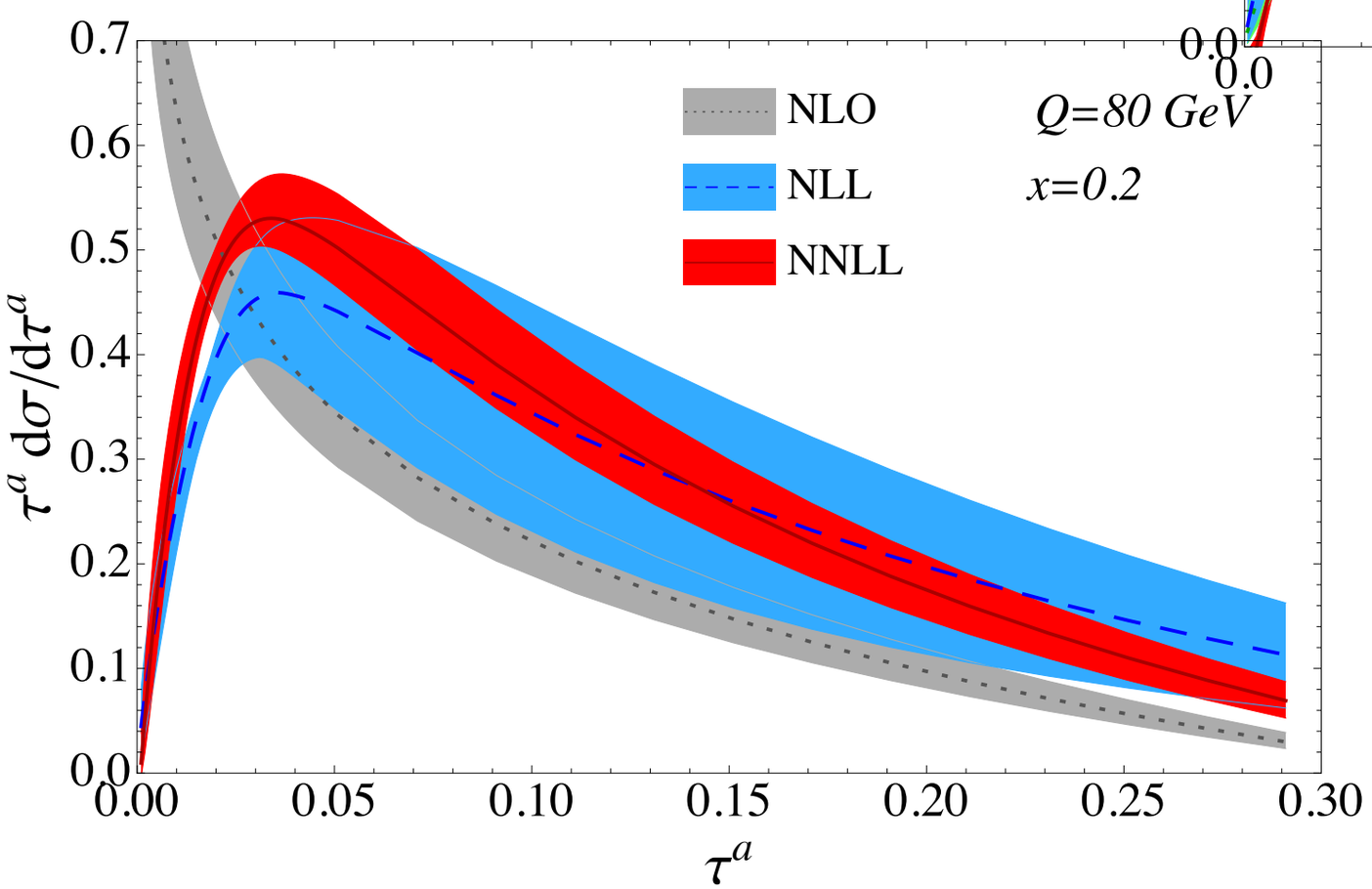


Predictions for DIS 1-jettiness

cumulant:



differential distribution:



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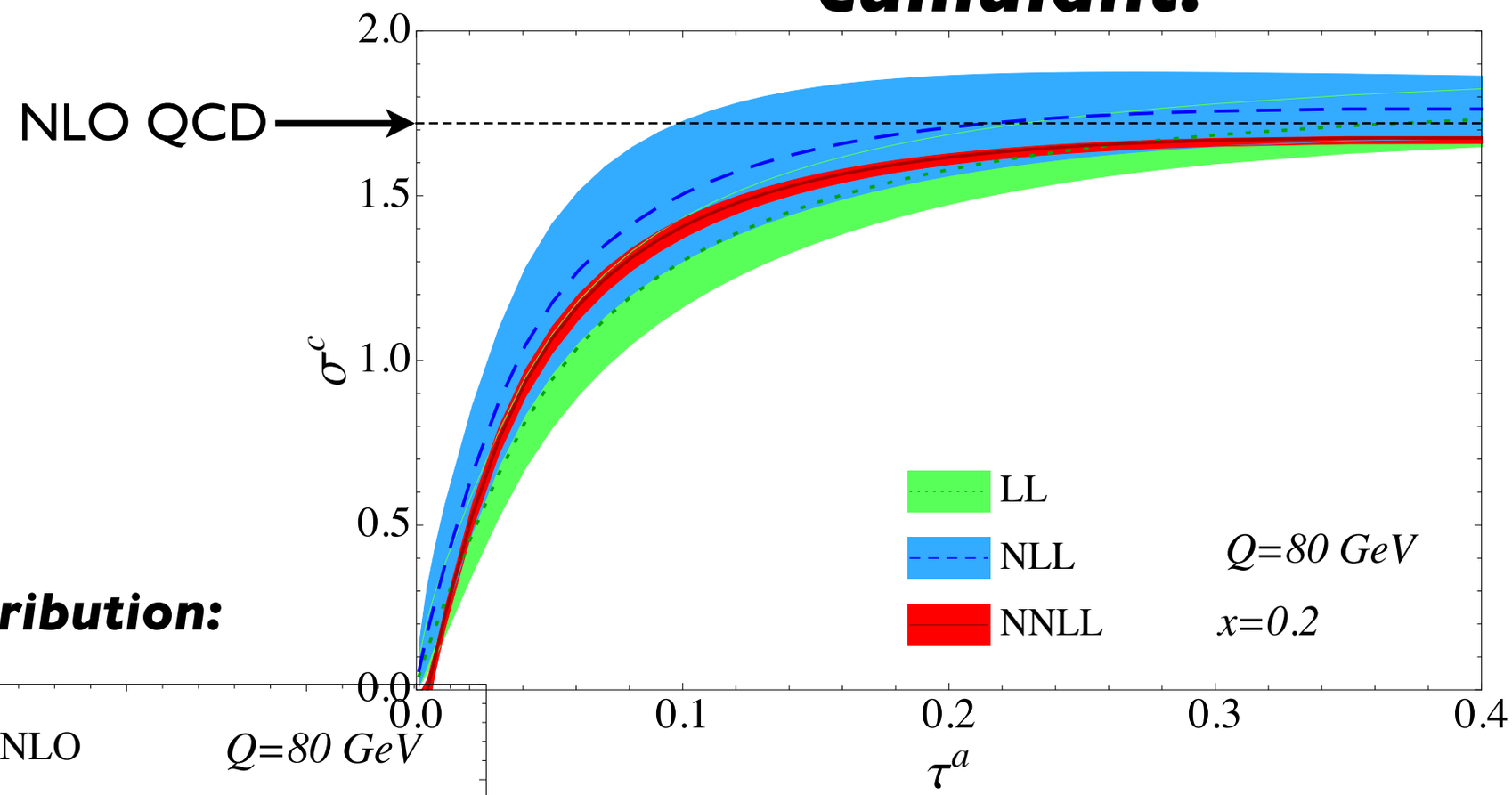

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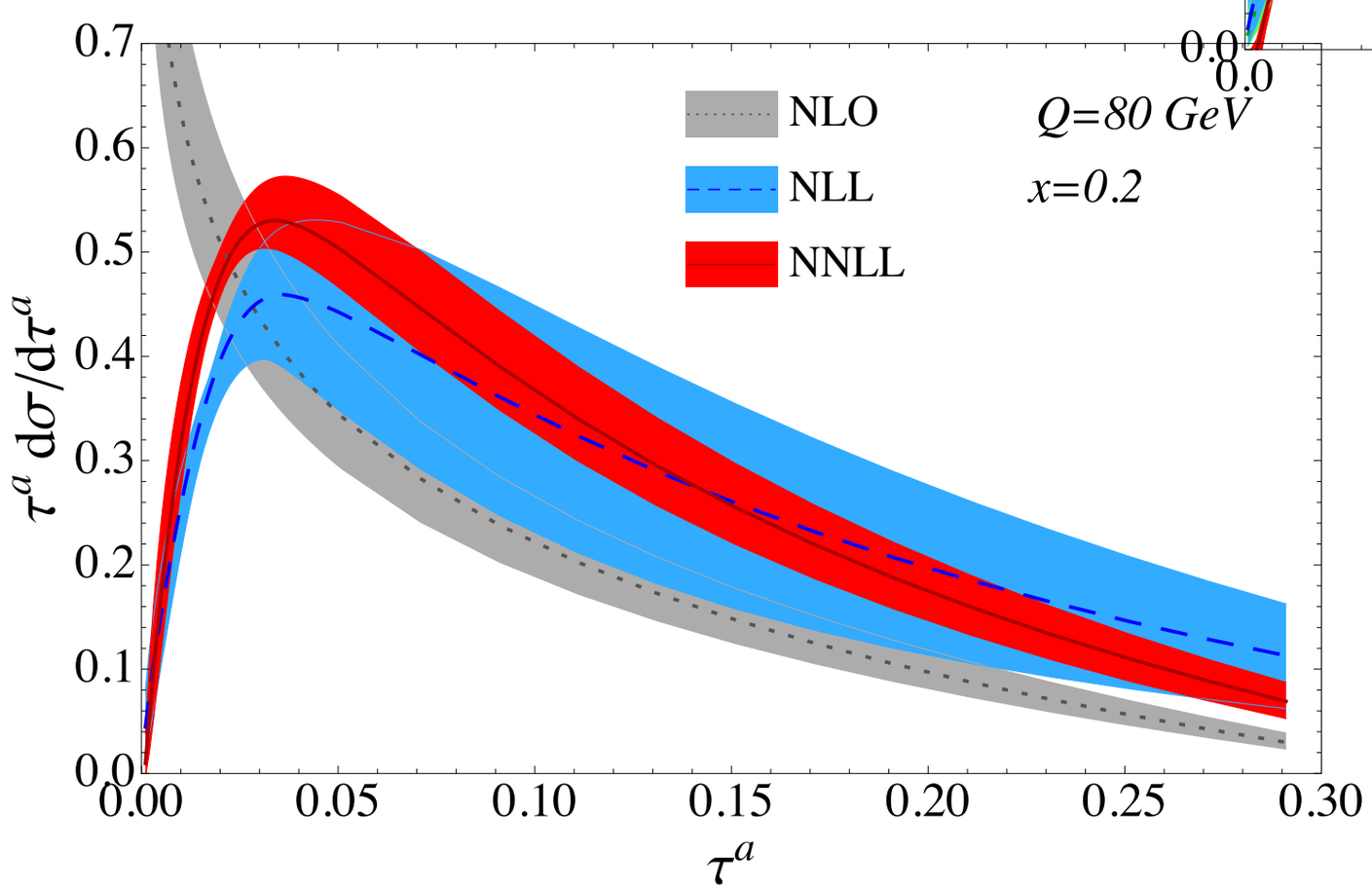

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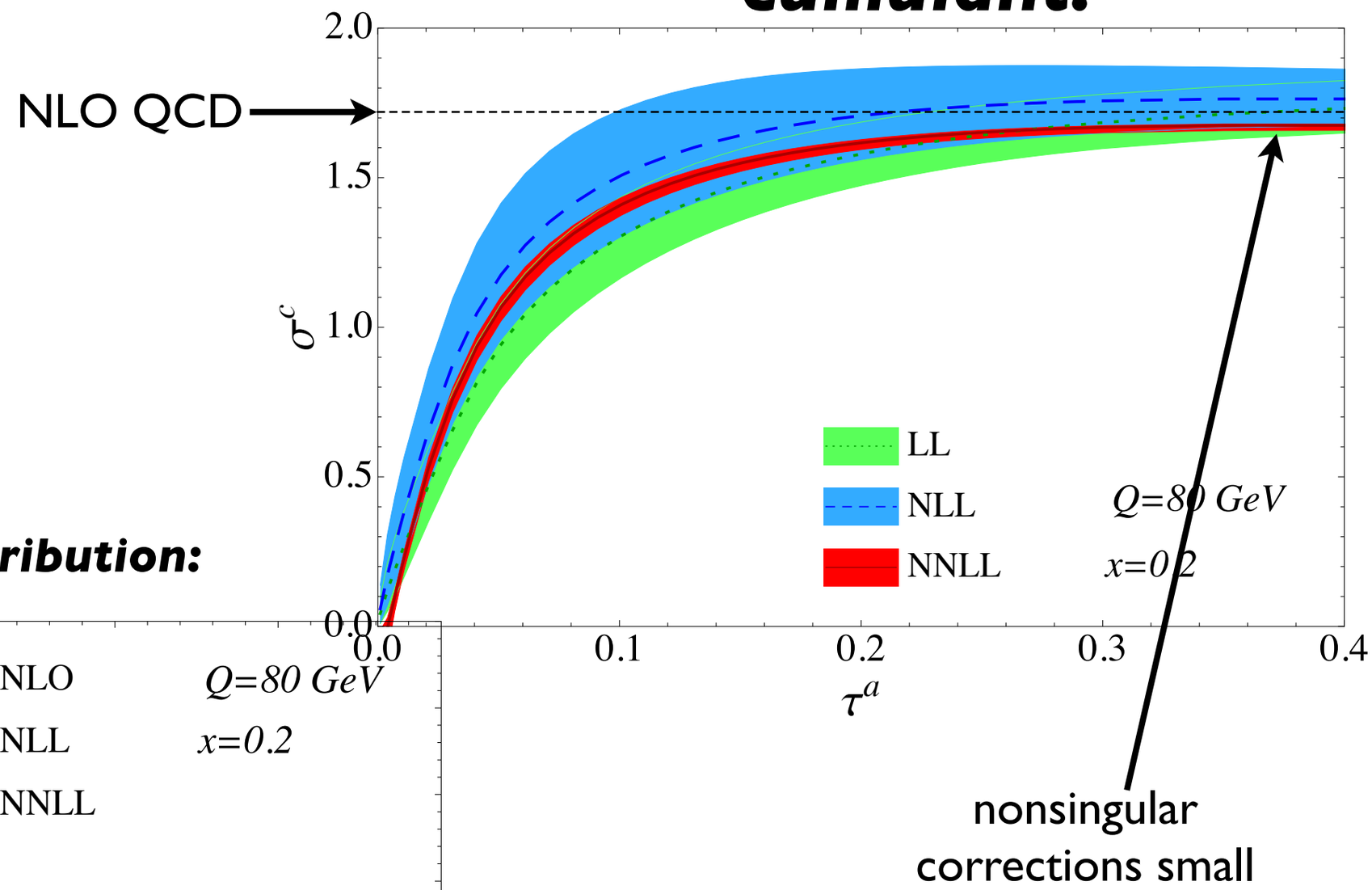

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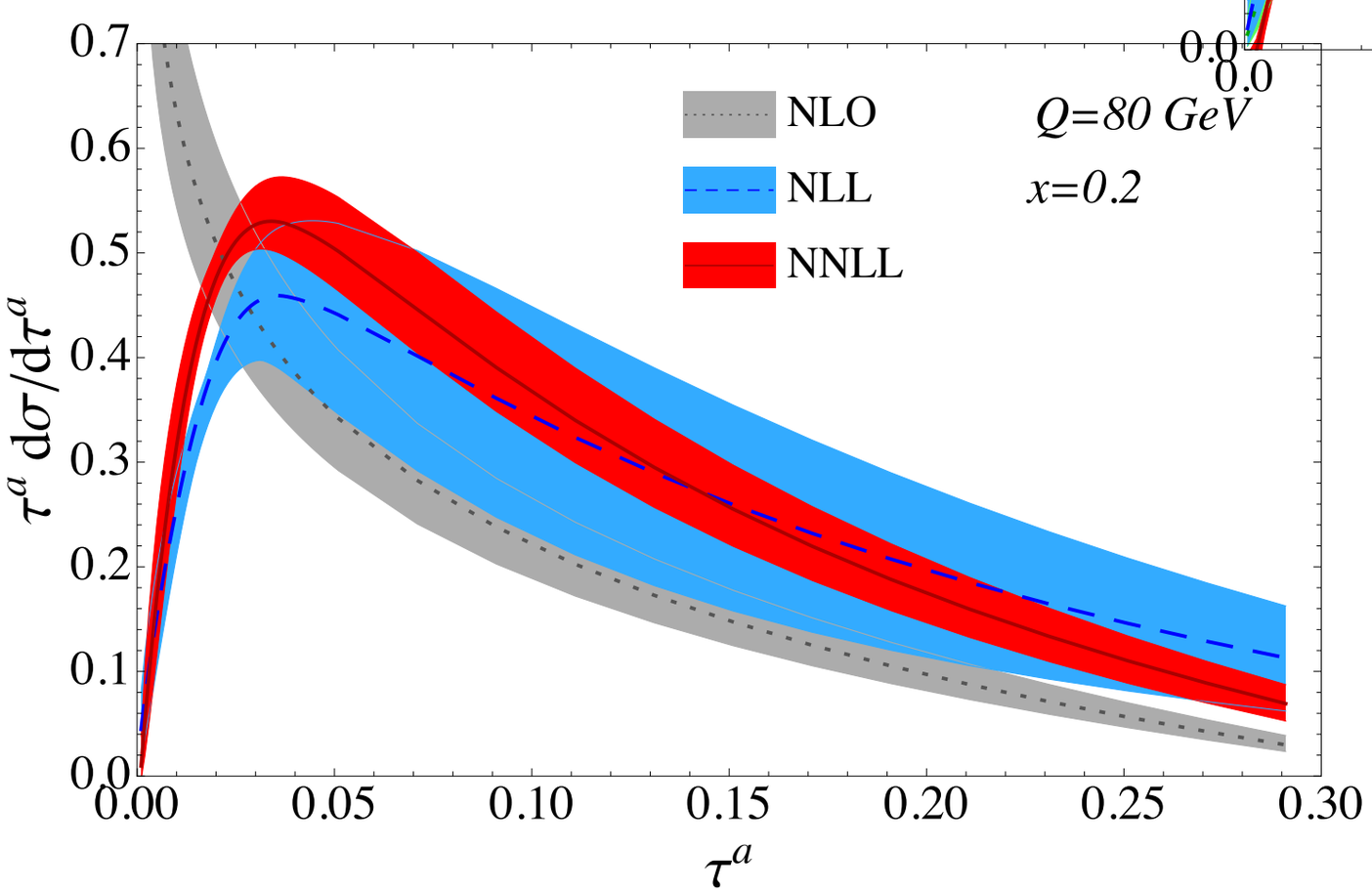
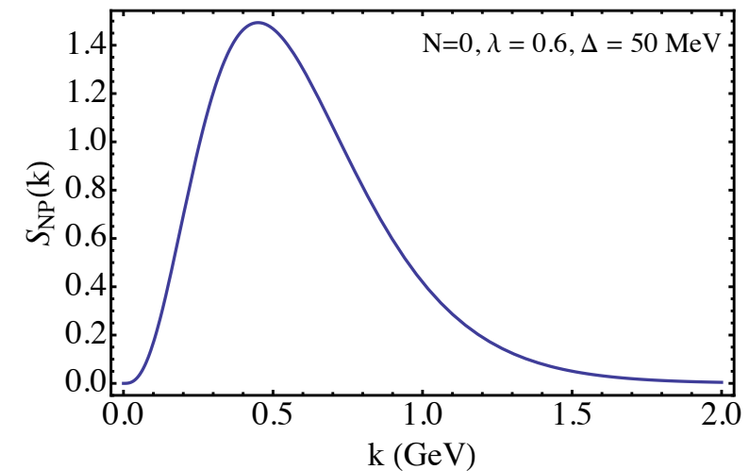


Predictions for DIS 1-jettiness

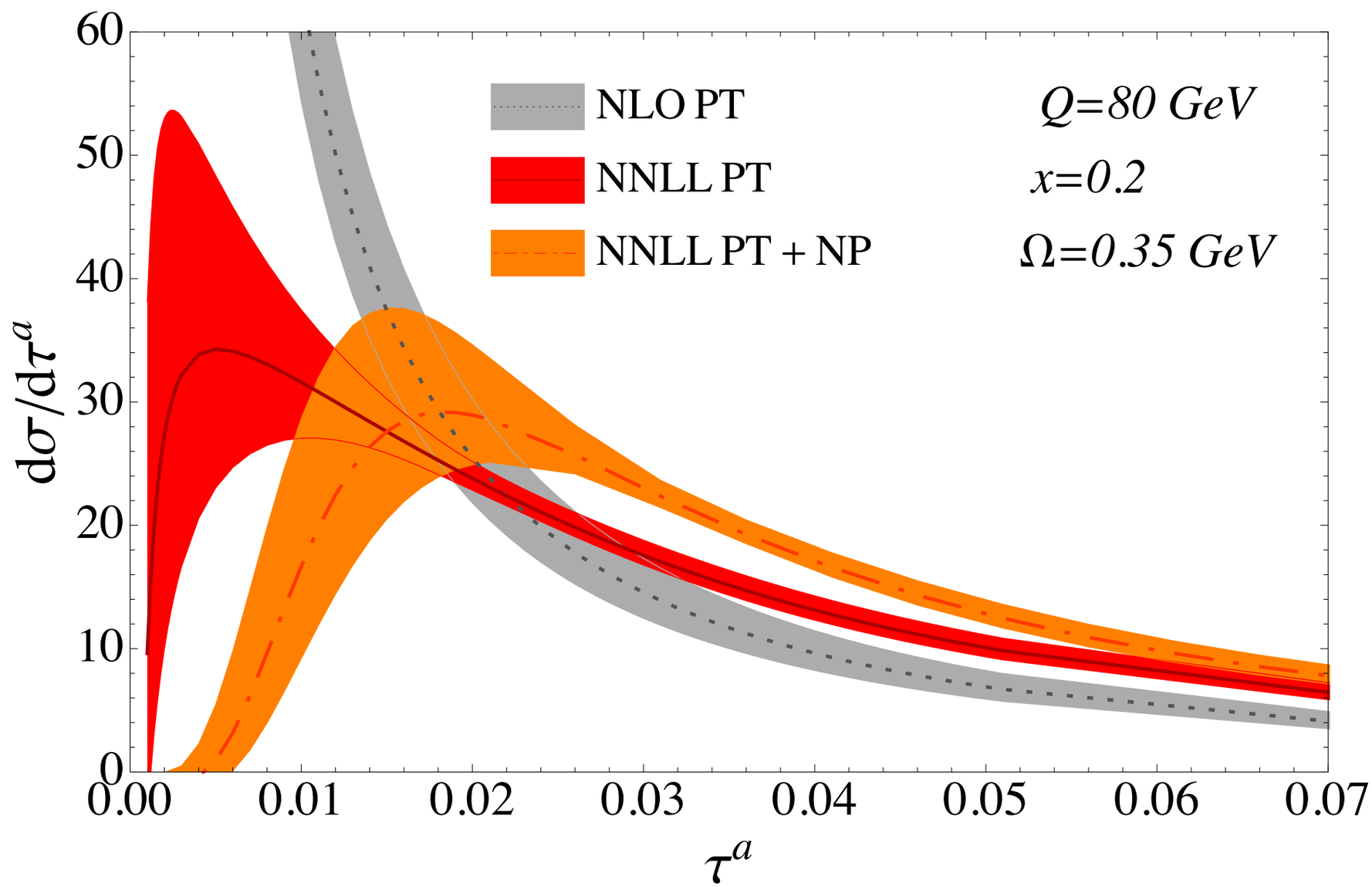
$$S_{NP}(l) = f(l - \Delta)$$

$$f(l) = \frac{1}{\lambda} \sum_{n=0}^{N} c_n f_n\left(\frac{l}{\lambda}\right)$$

Ligeti, Stewart, Tackmann (2008)



convolution with NP shape function:



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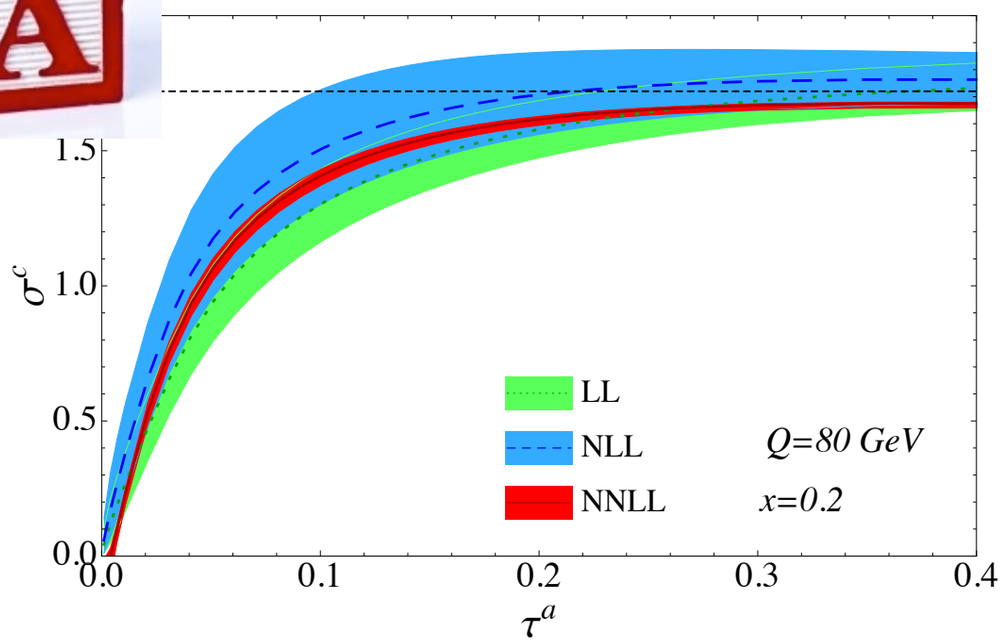

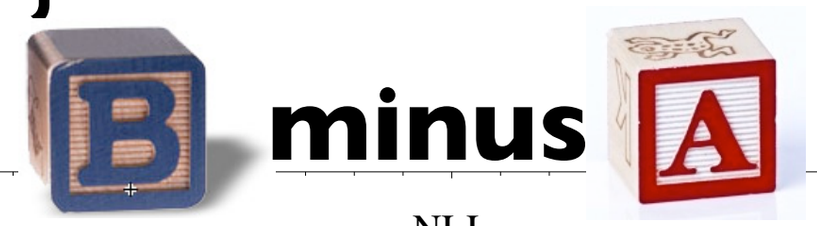

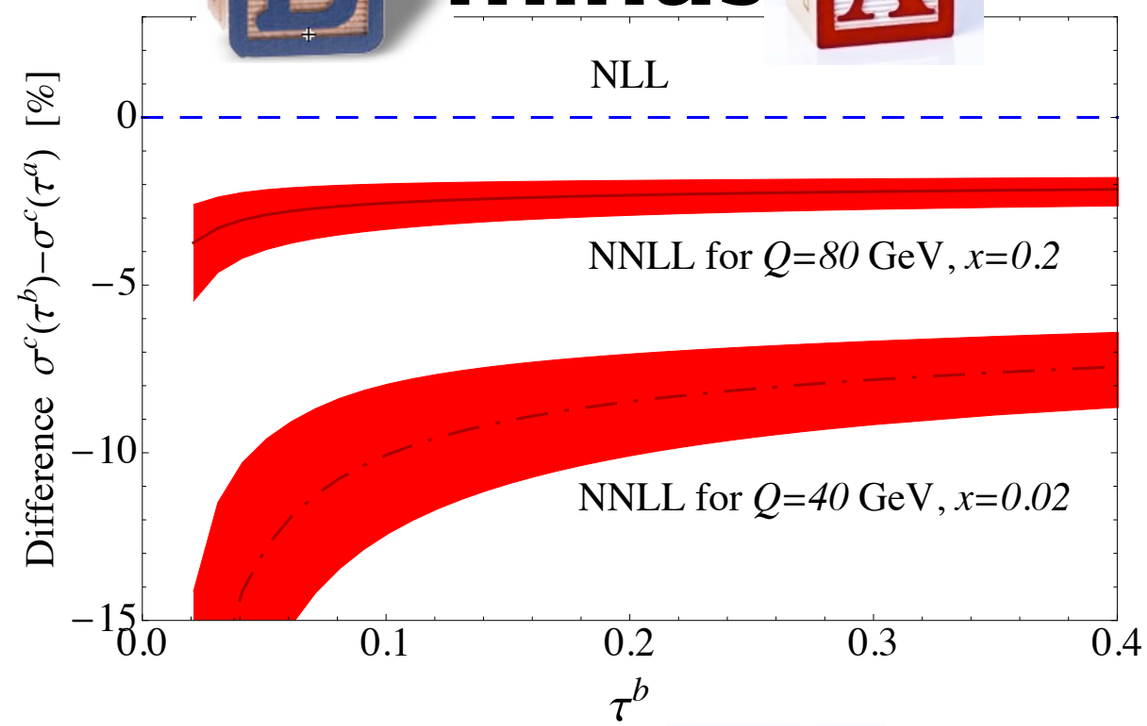

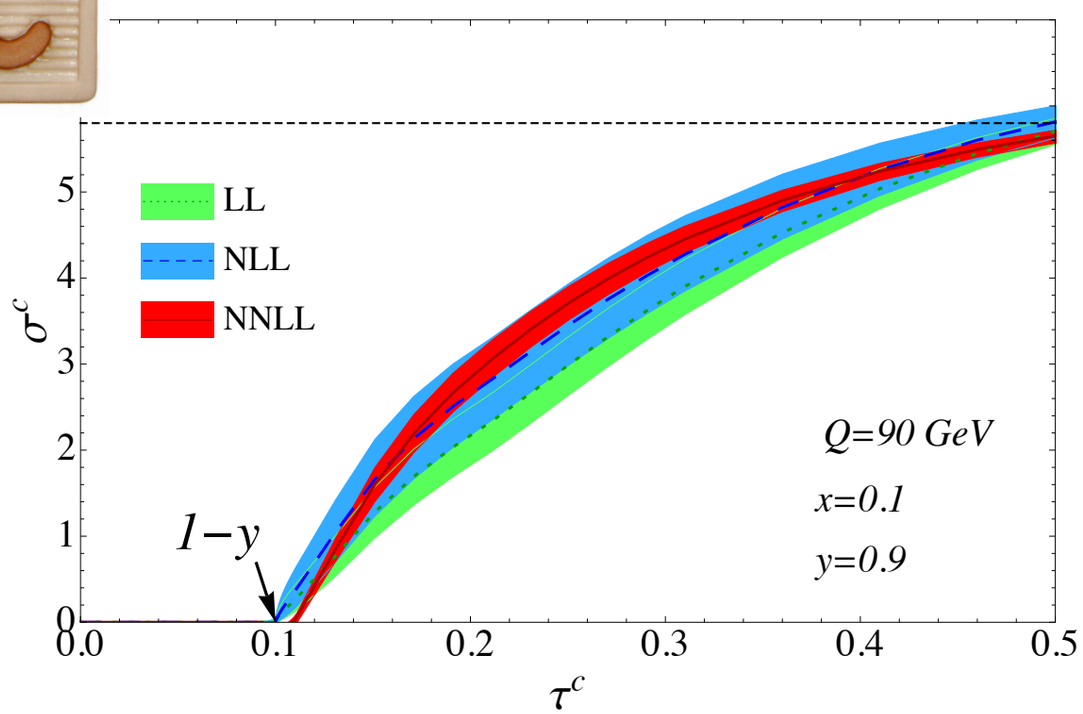

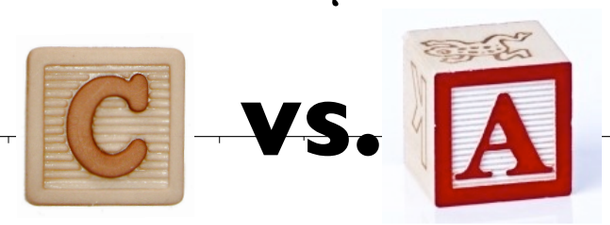

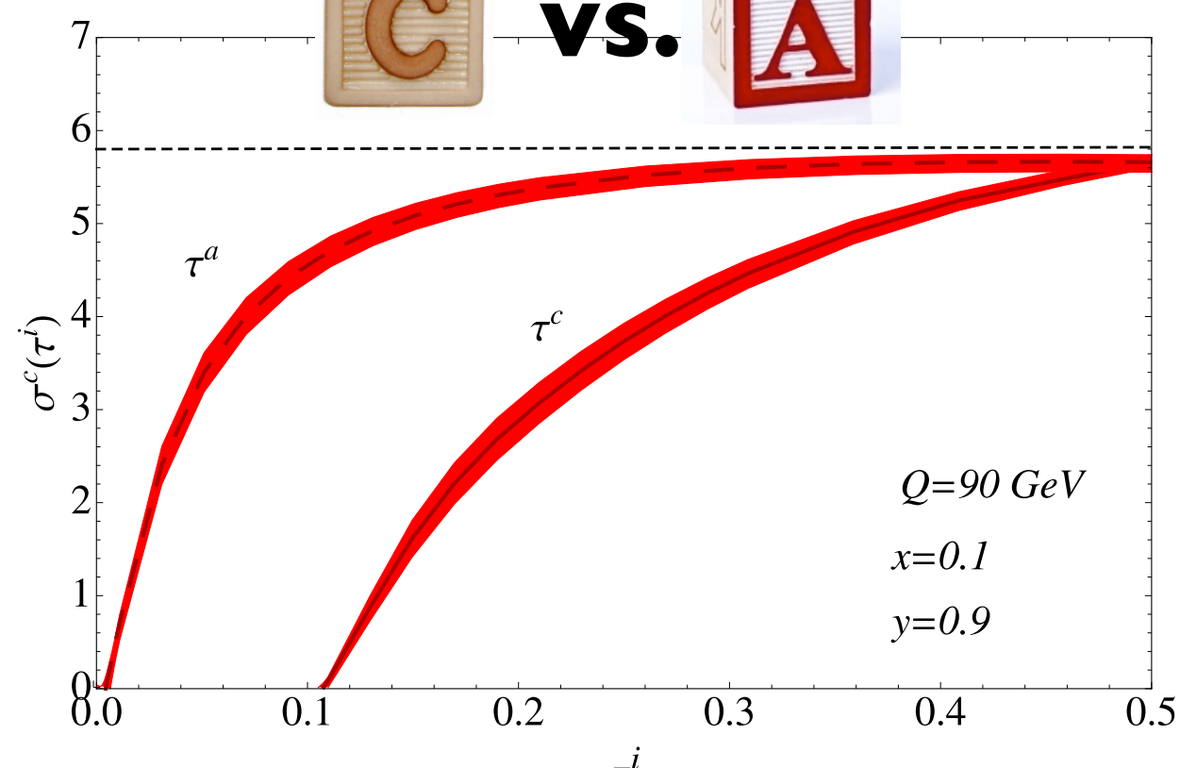

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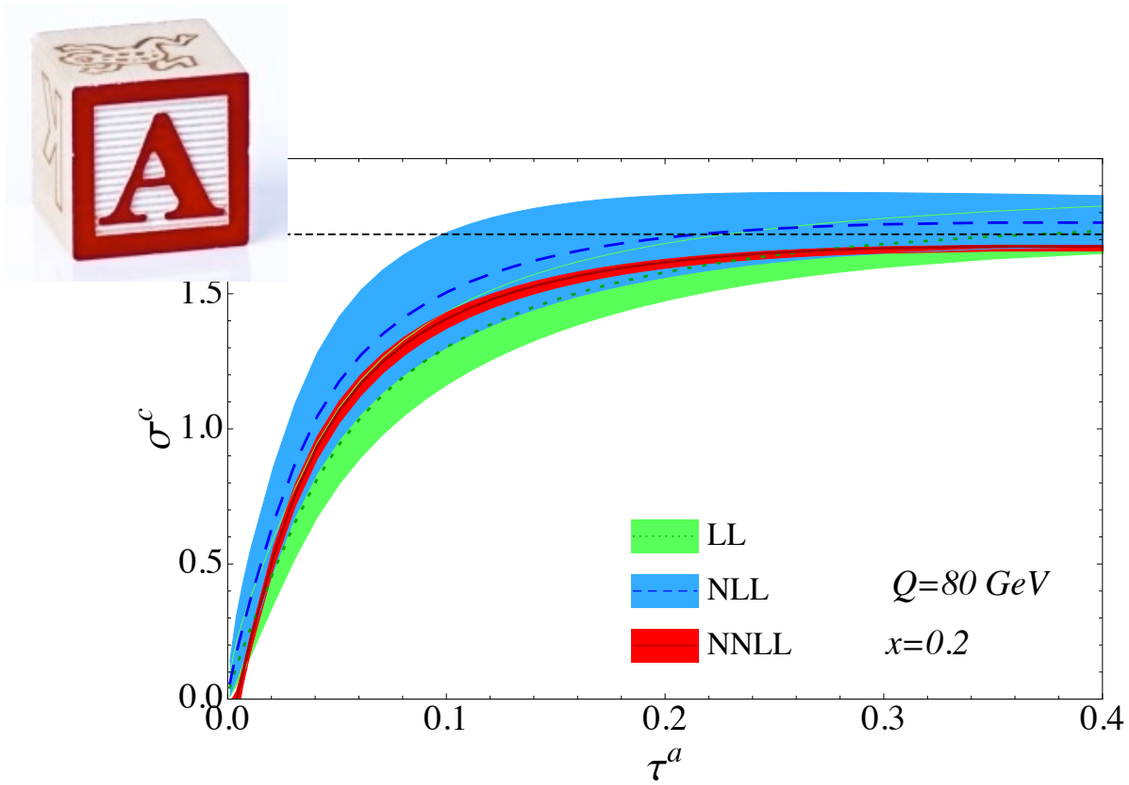
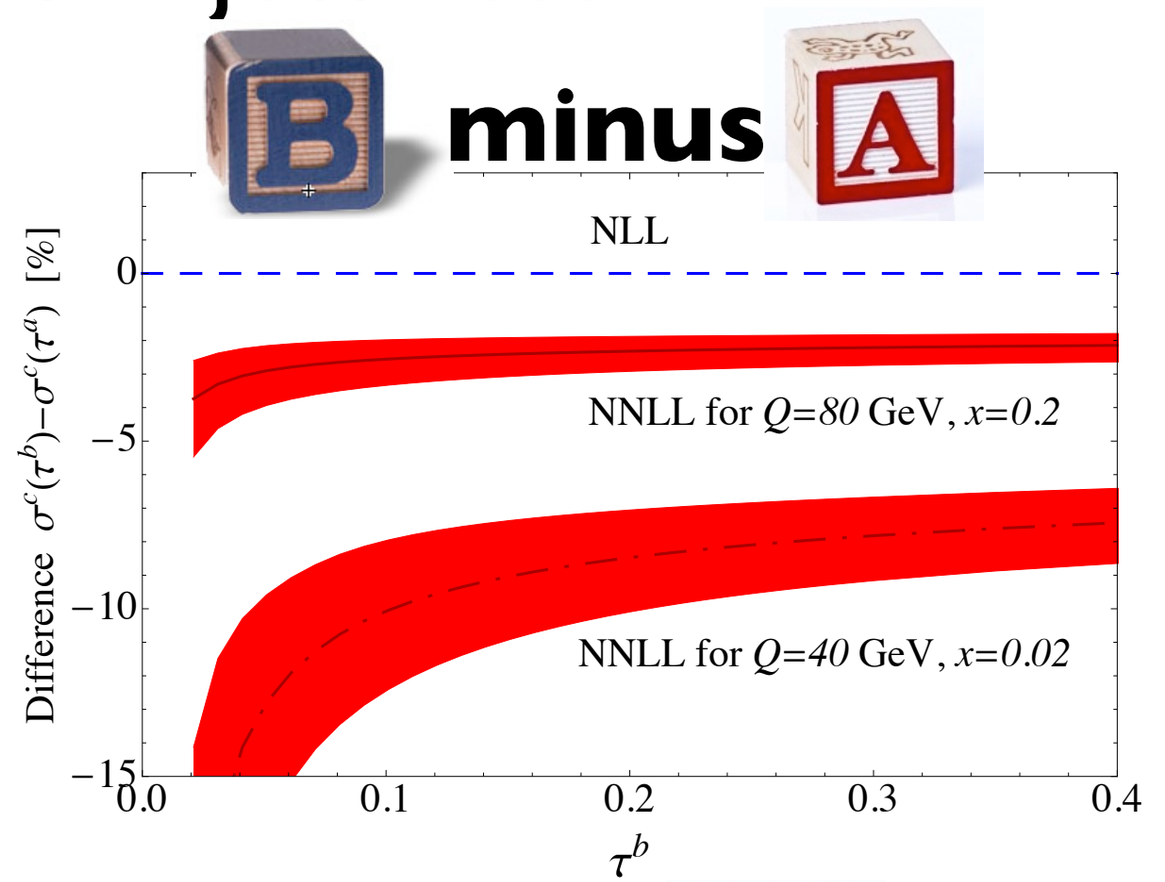
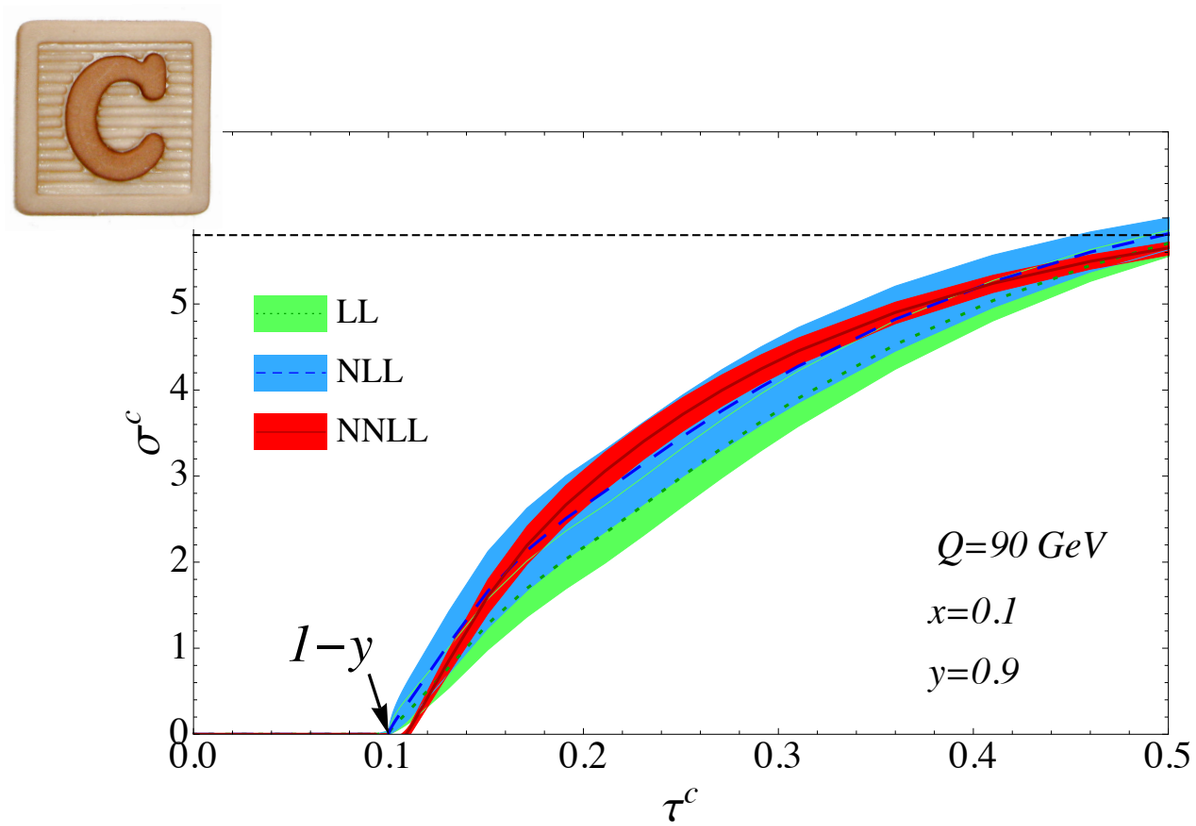
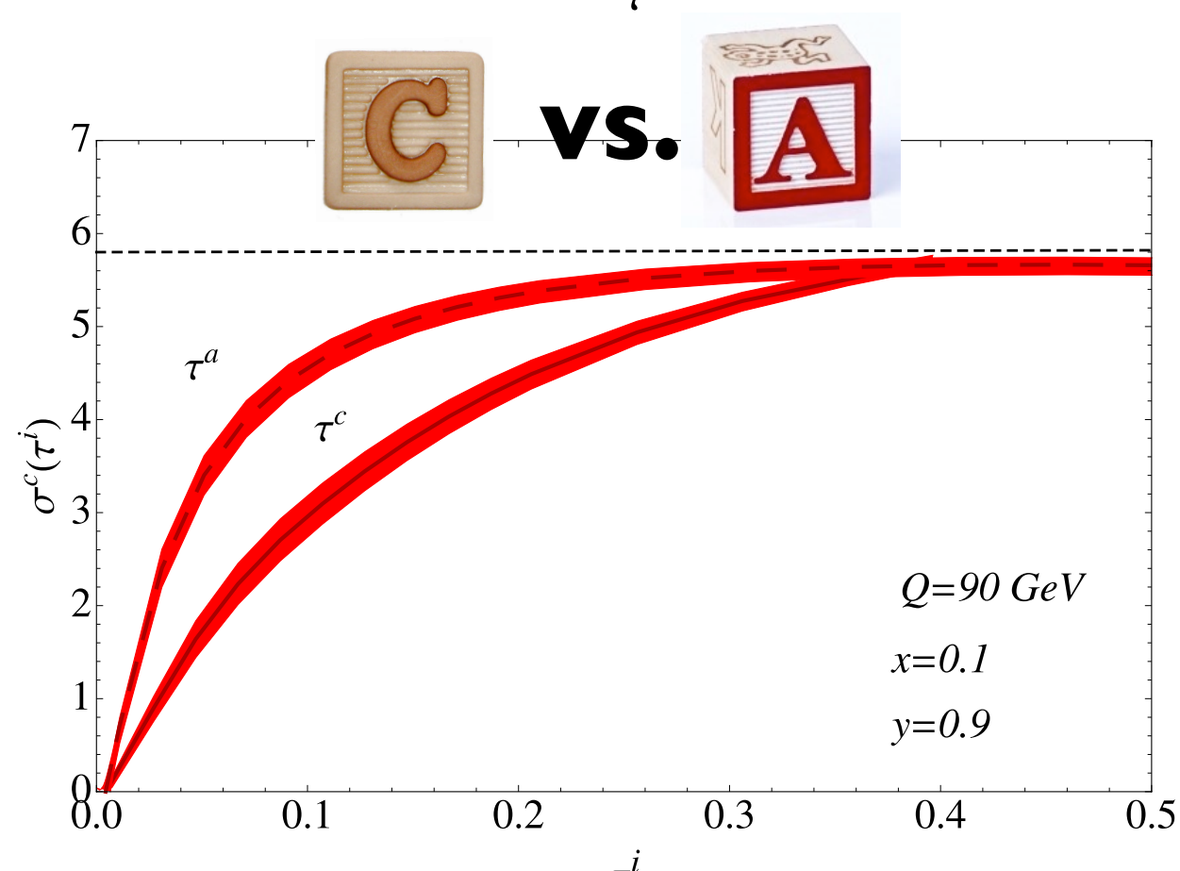


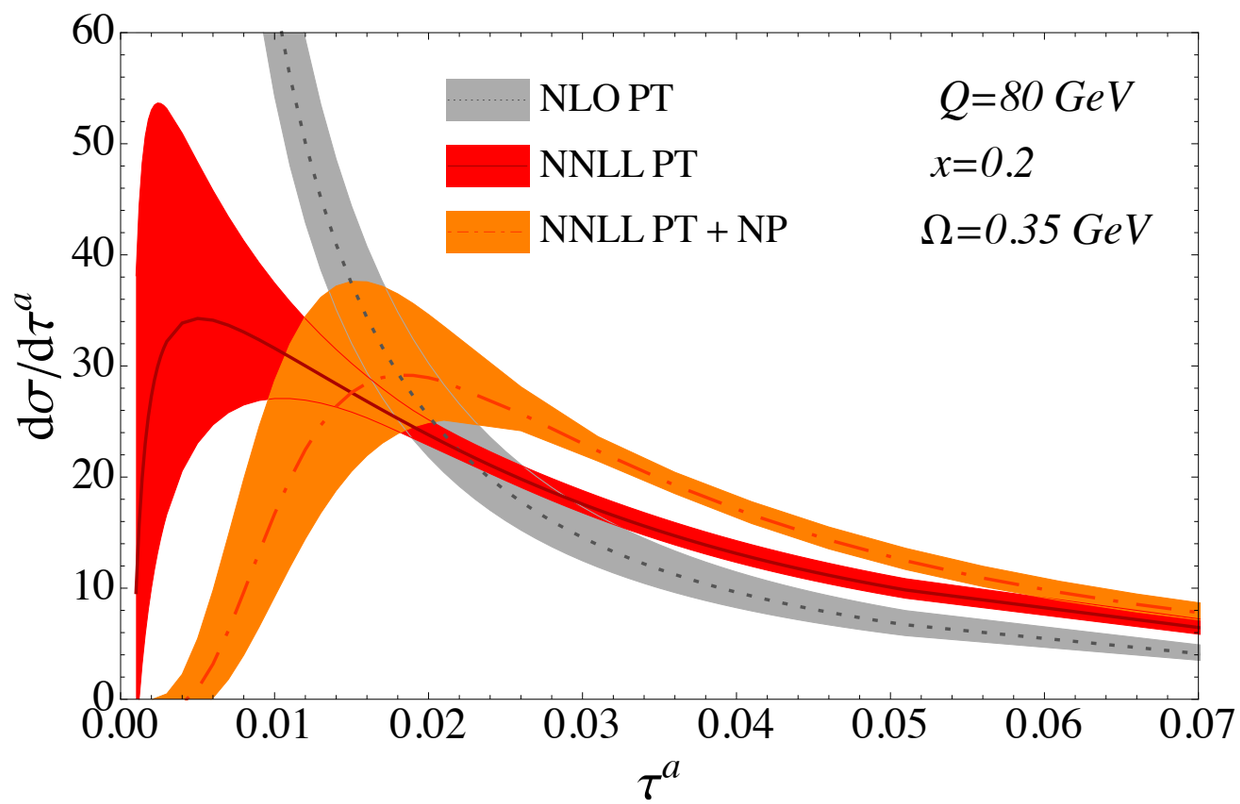






Universal Nonperturbative Corrections

- In the tail region, the leading nonperturbative correction is again just a shift:



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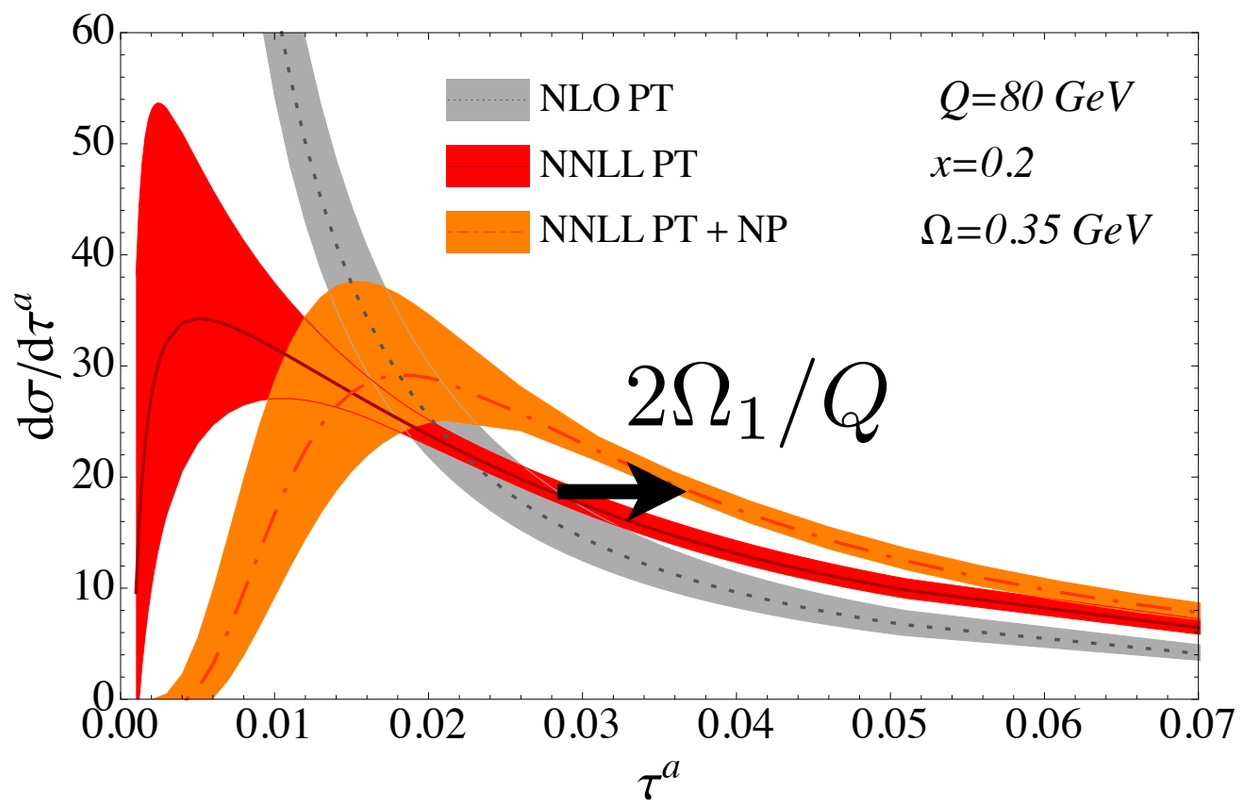

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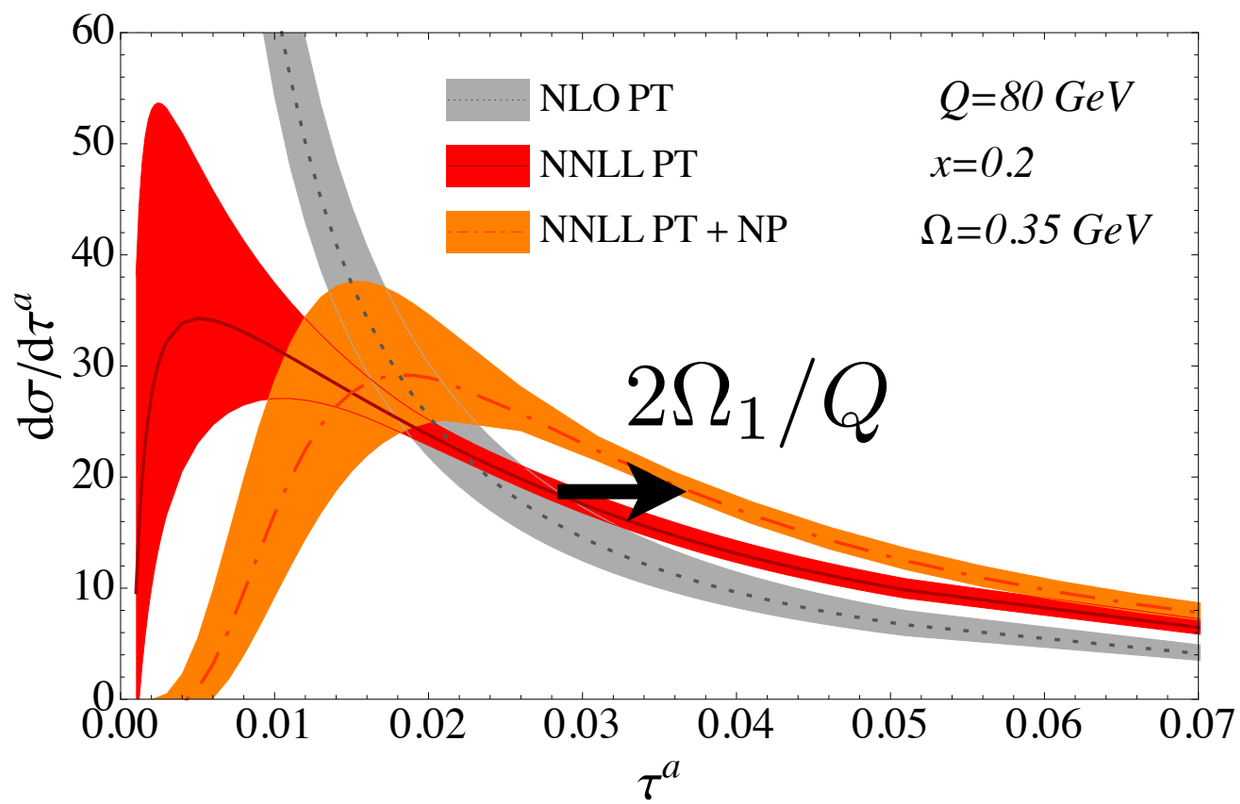

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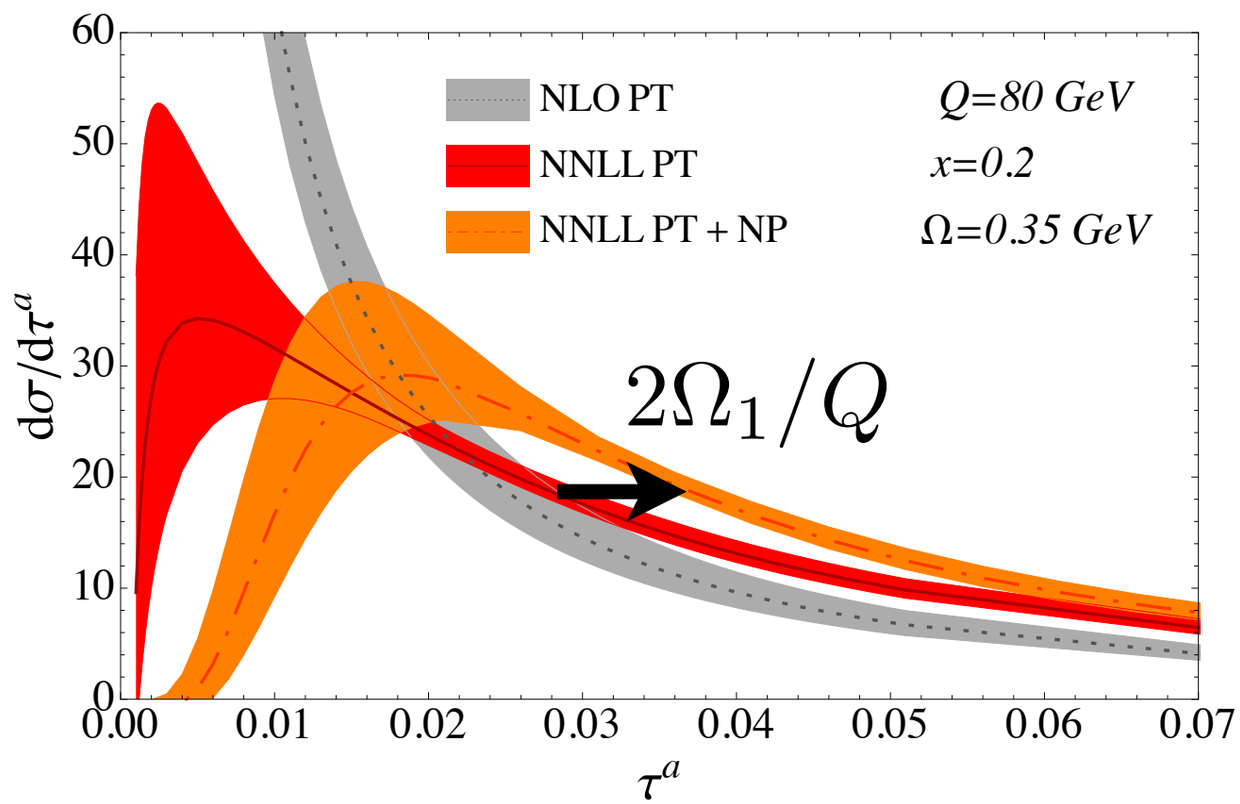

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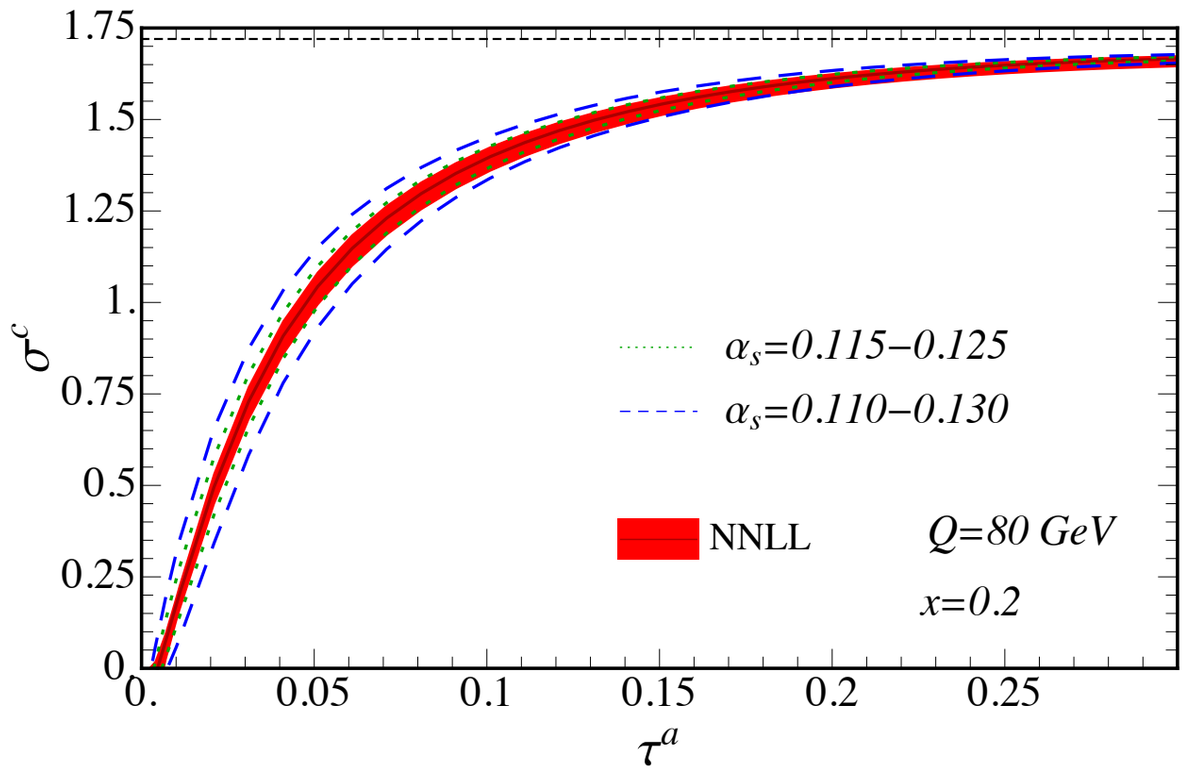
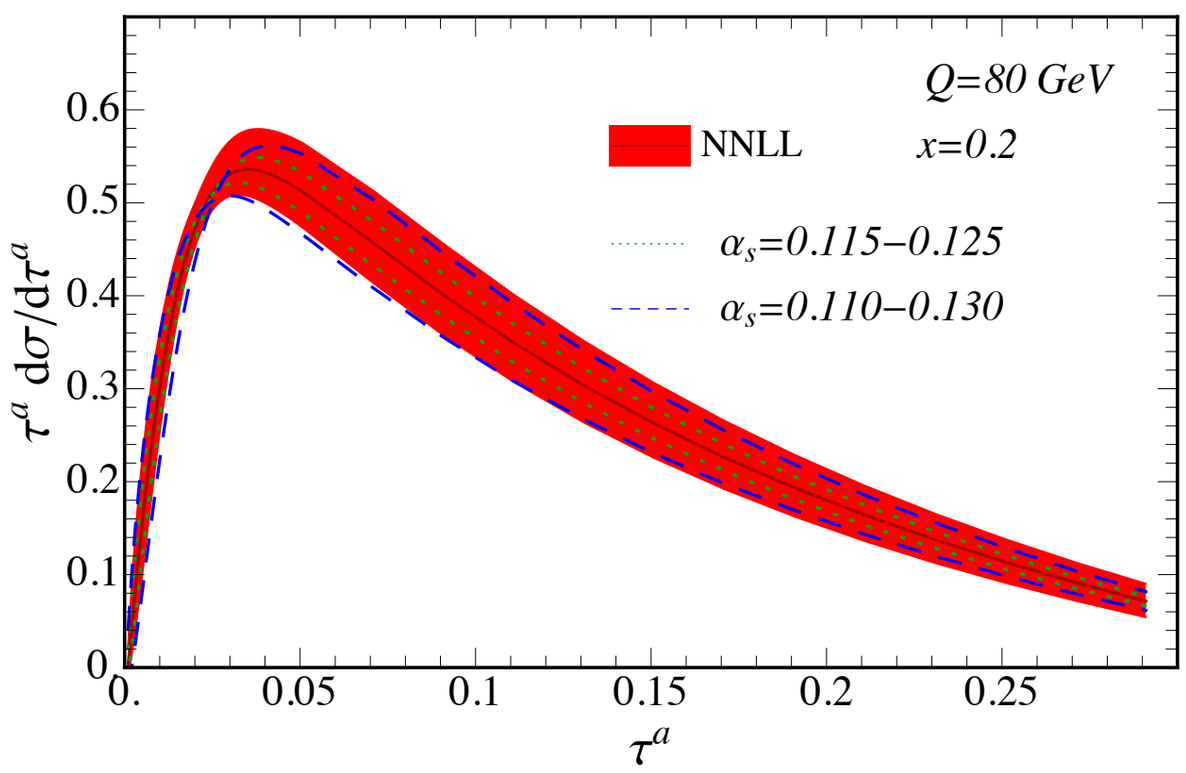
powerful reduction in uncertainty from nonperturbative hadronization corrections

more reliable extractions of strong coupling and PDFs

Sensitivity to Strong Coupling





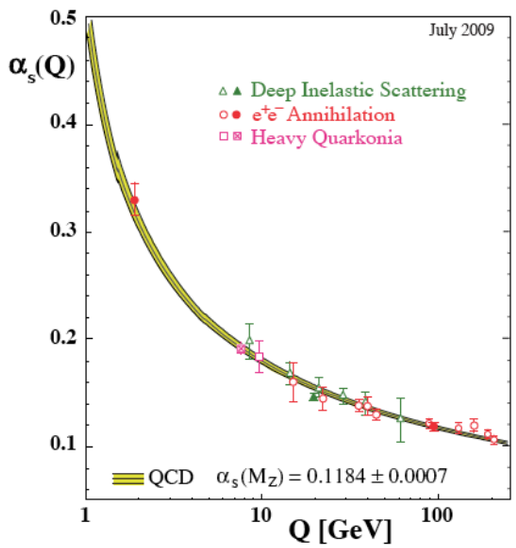
Perturbative uncertainty in NNLL cumulant similar to the effect of a $\pm 5\%$ variation in $\alpha_s(M_Z)$

This sensitivity will be improved by:

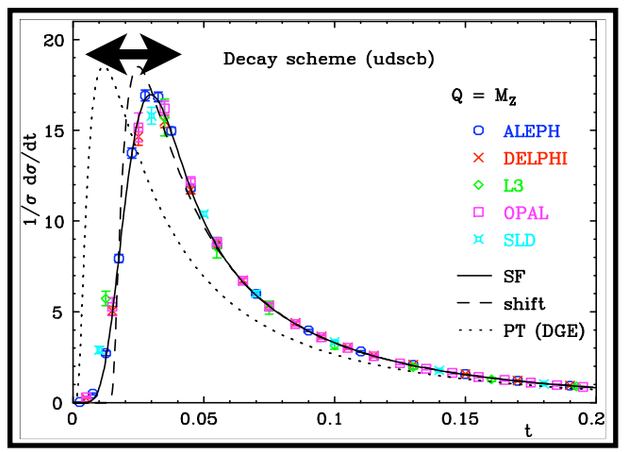
- Combining information from different τ's
- Combining info from different x, Q^2
- Improvement of theoretical calculation to NNLL' or N^3LL accuracy

Summary and Outlook

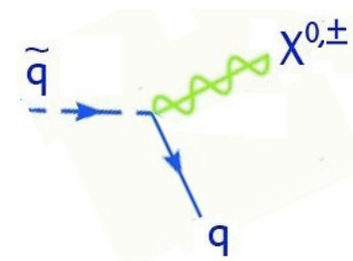
- **Jets probe predictions and parameters of QCD**

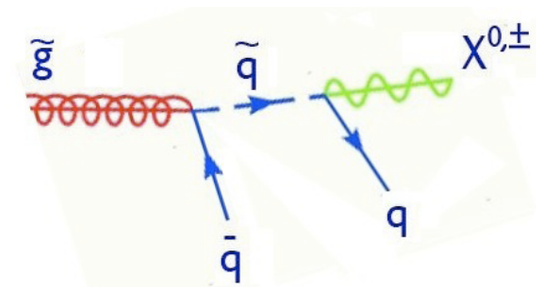


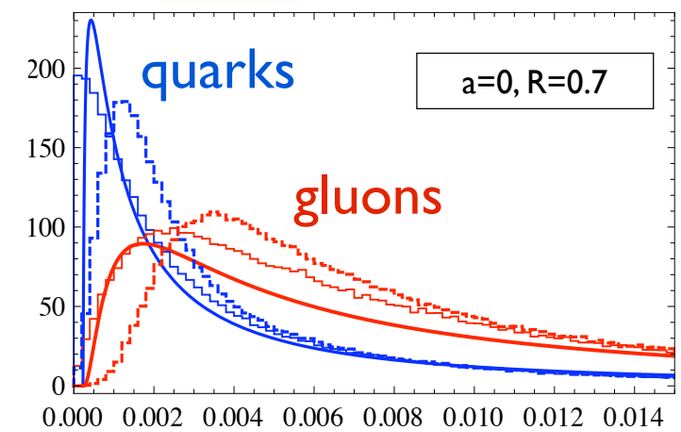
$\sim \Omega_1/Q$



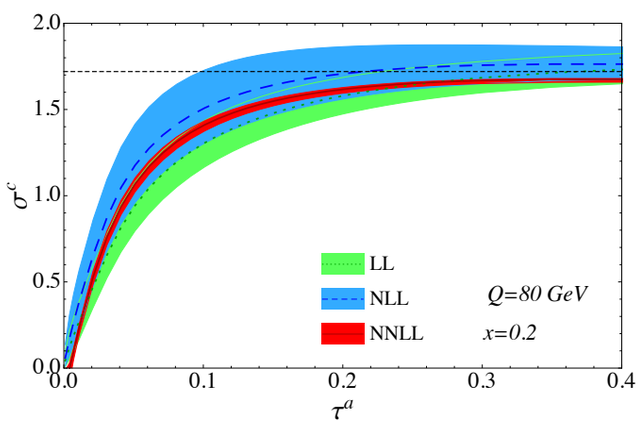
- **Jet Shapes can distinguish jet origins**

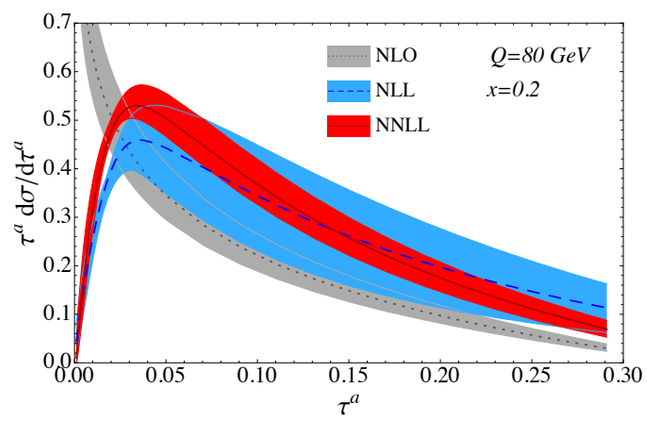




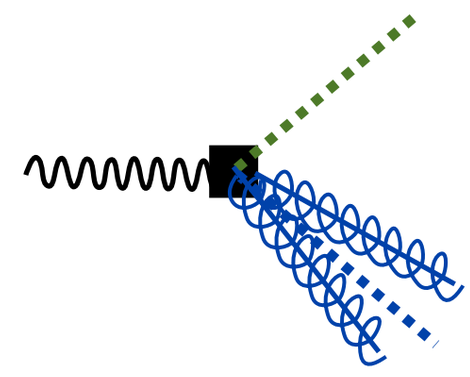


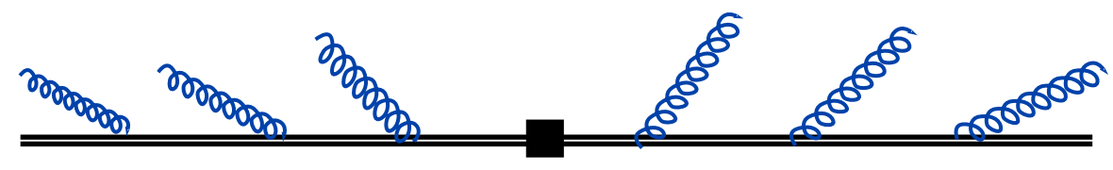
- **New predictions of DIS event shapes can improve precision of strong coupling, PDF extractions**





- **Tools of Effective Field Theory allow improved precision in all applications of jet physics**



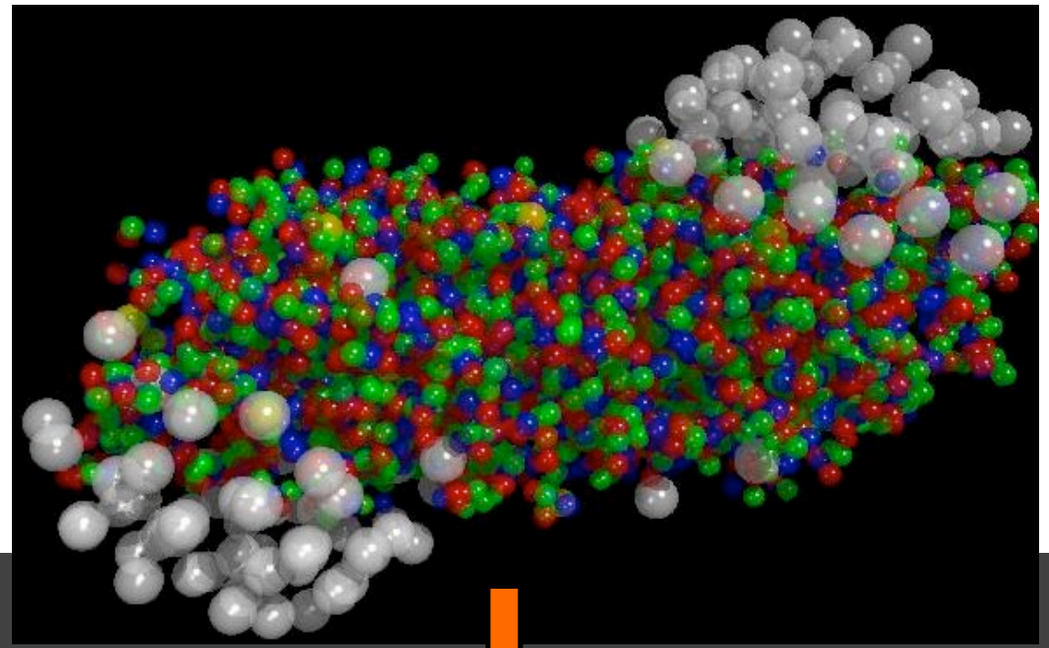


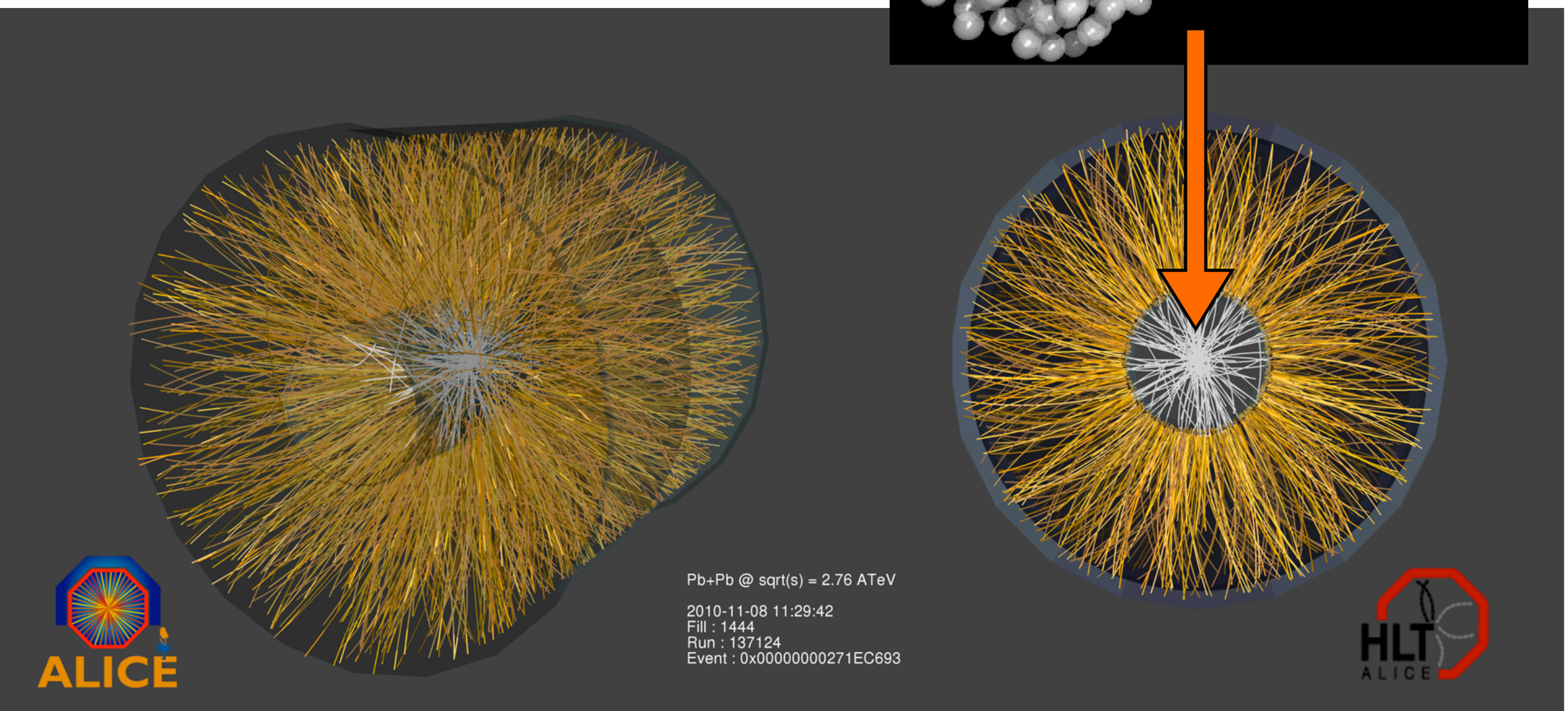
How can we probe new states of matter with jets?

The Quark Gluon Plasma

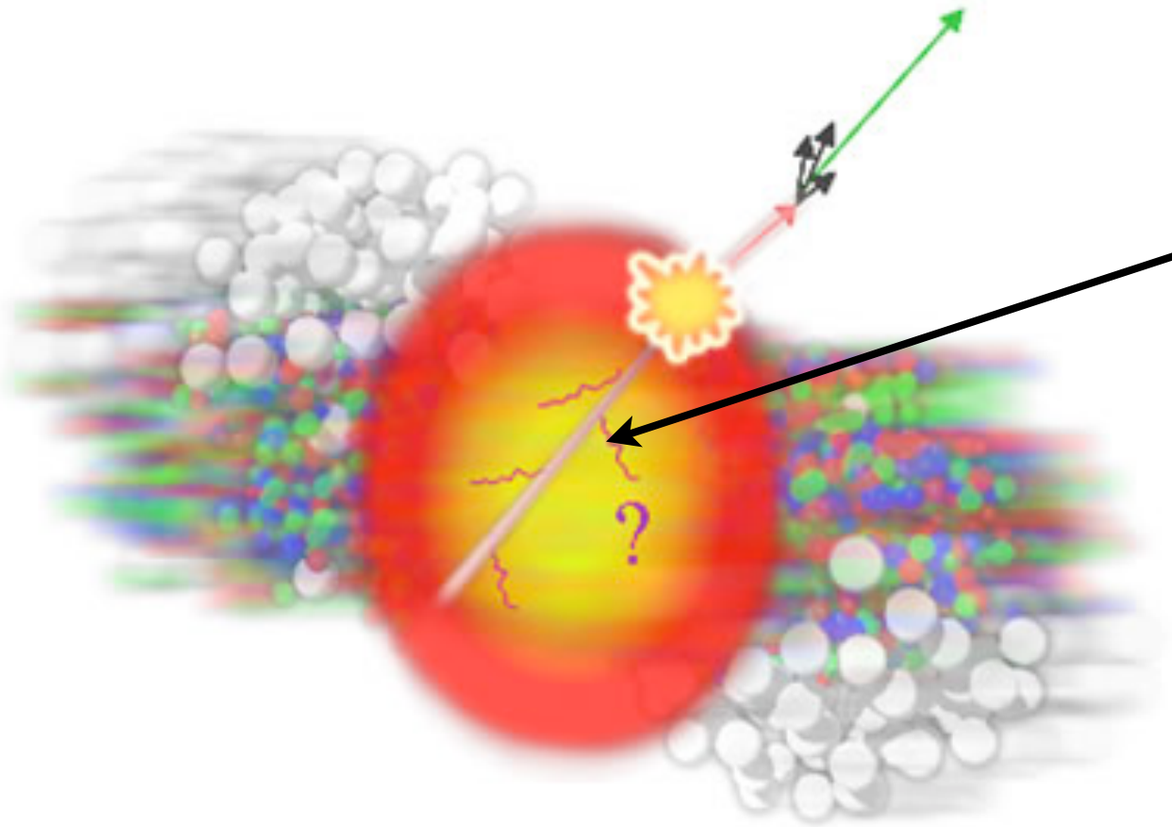
Is a new hot, dense state of matter of liberated quarks and gluons created in Heavy Ion Collisions?

What are its properties?



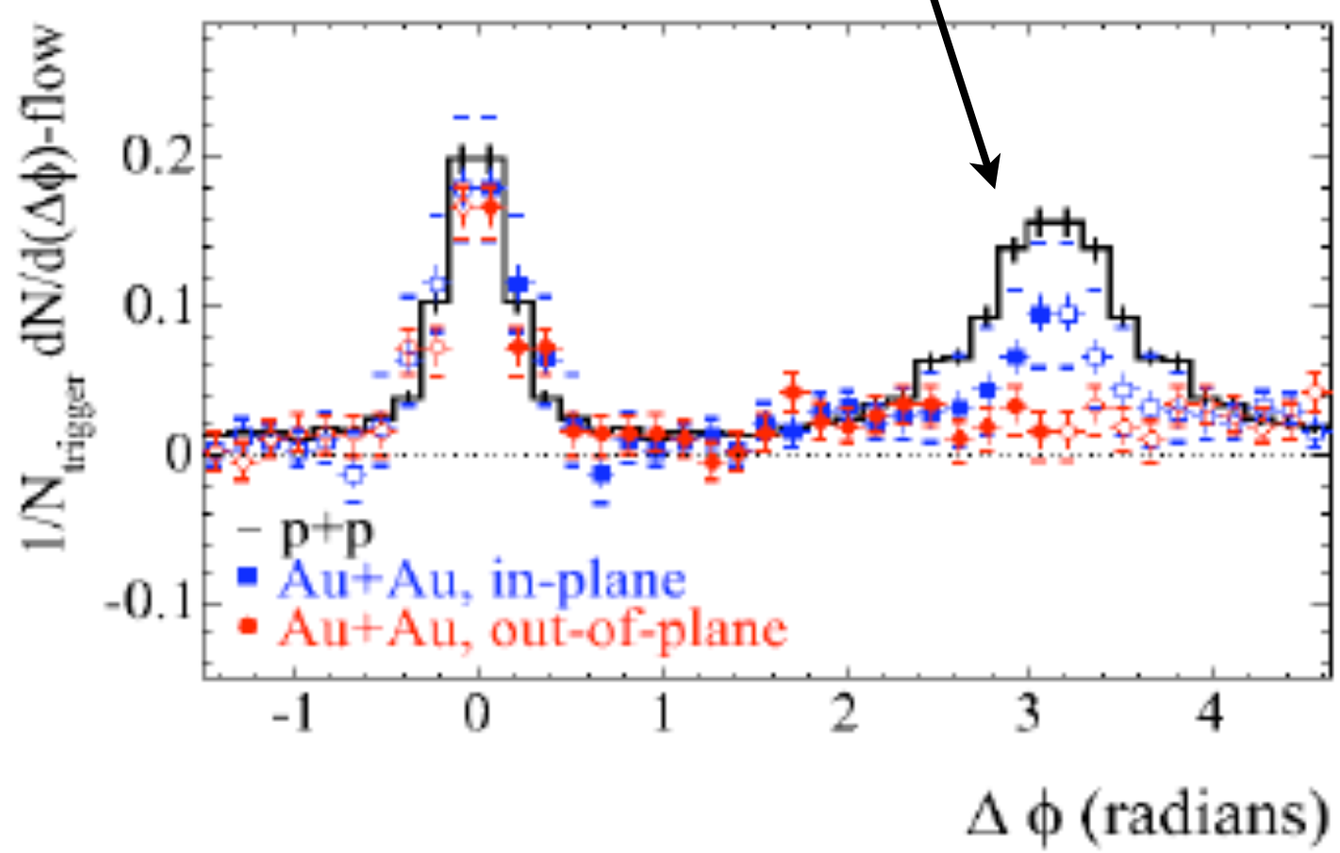


Probe the Medium with Jets!



When back-to-back jets are produced in the medium, one may not escape

"jet quenching"



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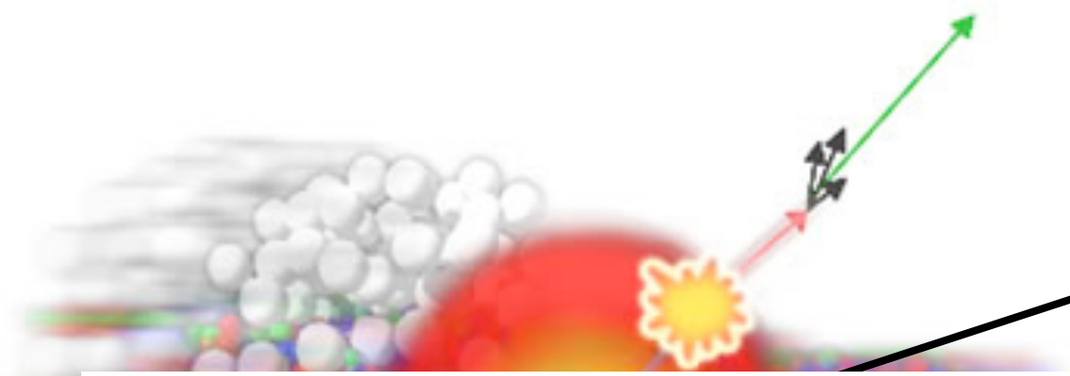

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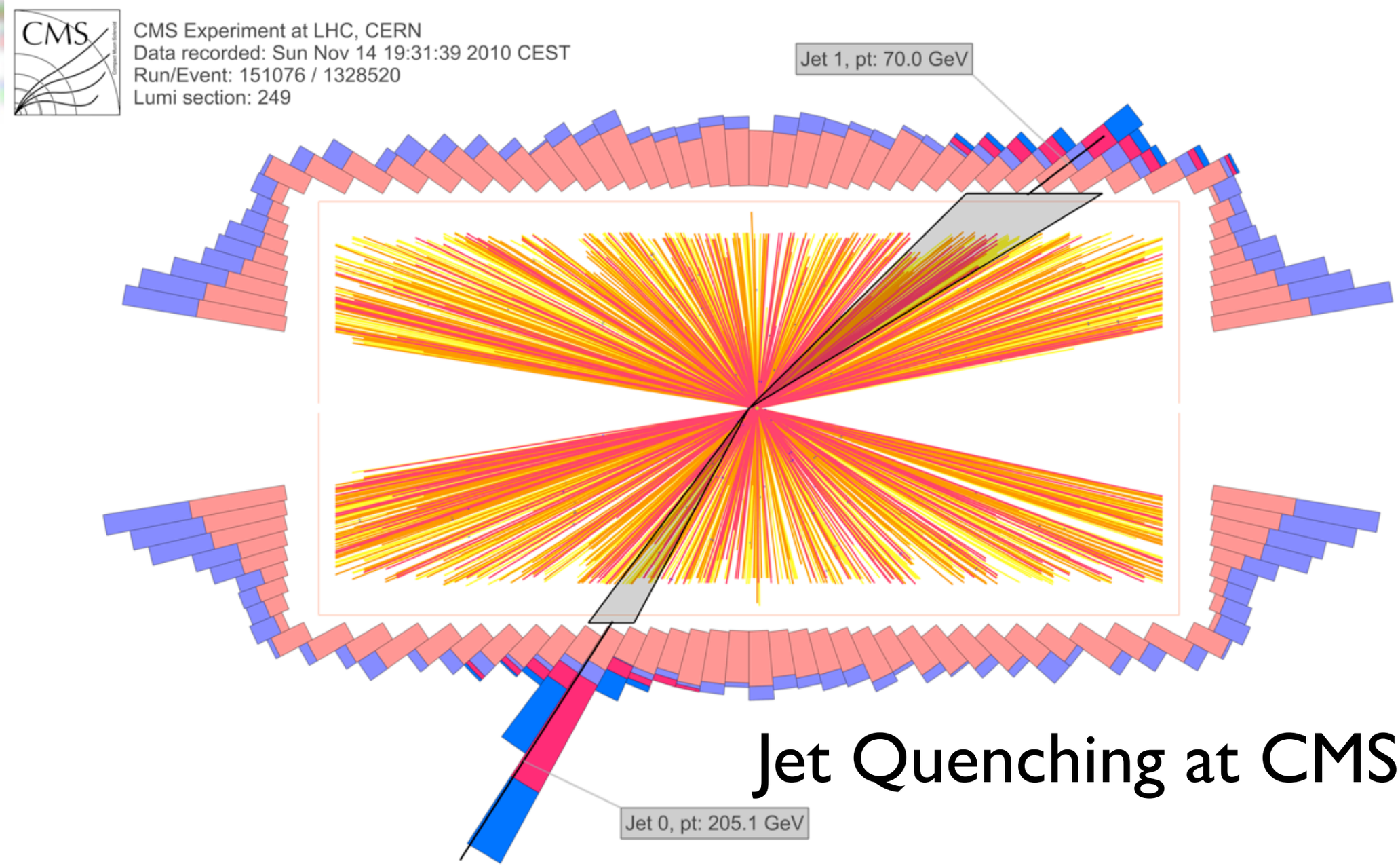
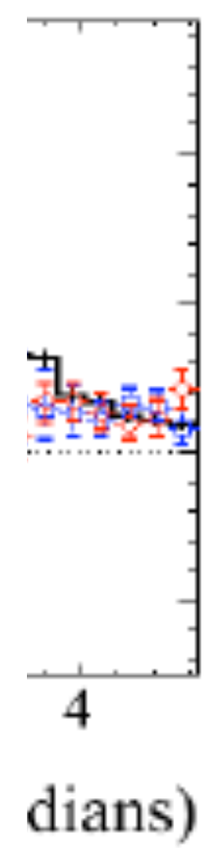
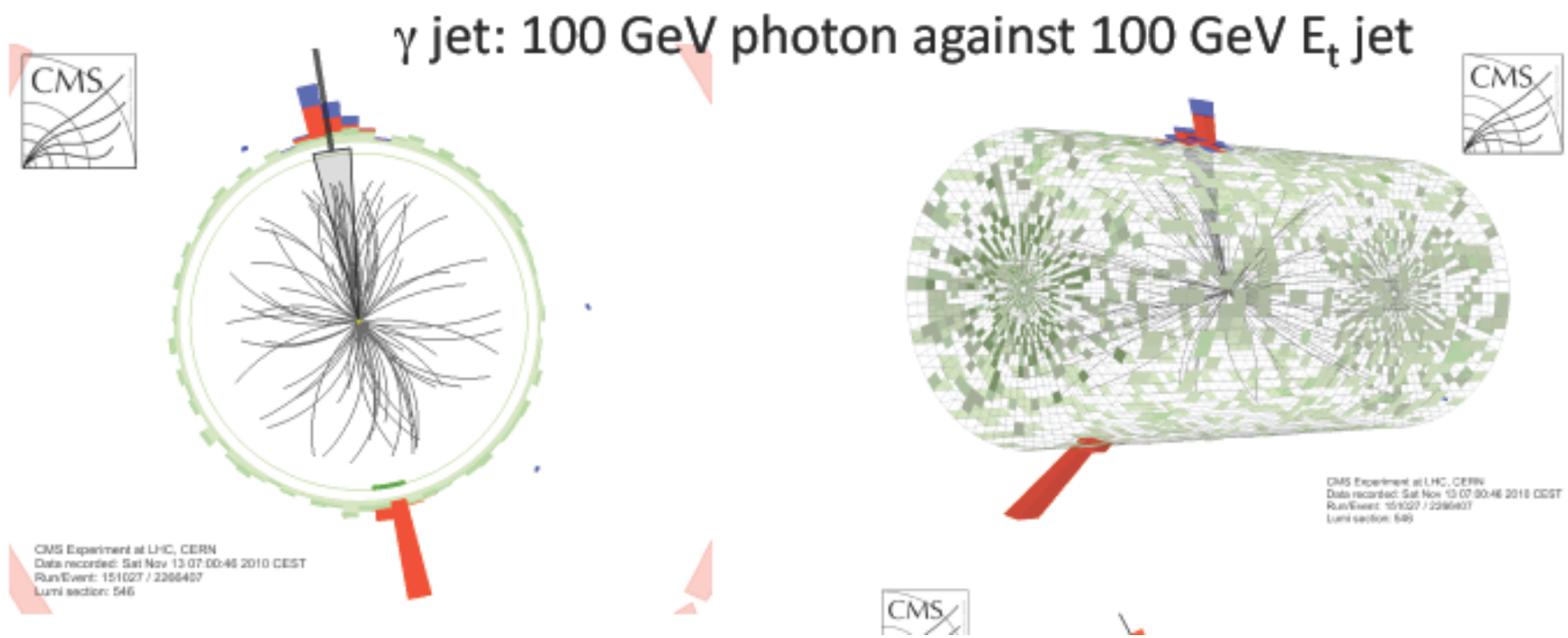


Jet Quenching at CMS!

ching”



Jet Modification in the Medium



- Heavy Ion Physicists use Jet Energy Loss, Jet Broadening in the medium to probe its properties
- **Proposal:** Use jet shapes to gain more detailed information about these medium properties

Vitev, Wicks, Zhang

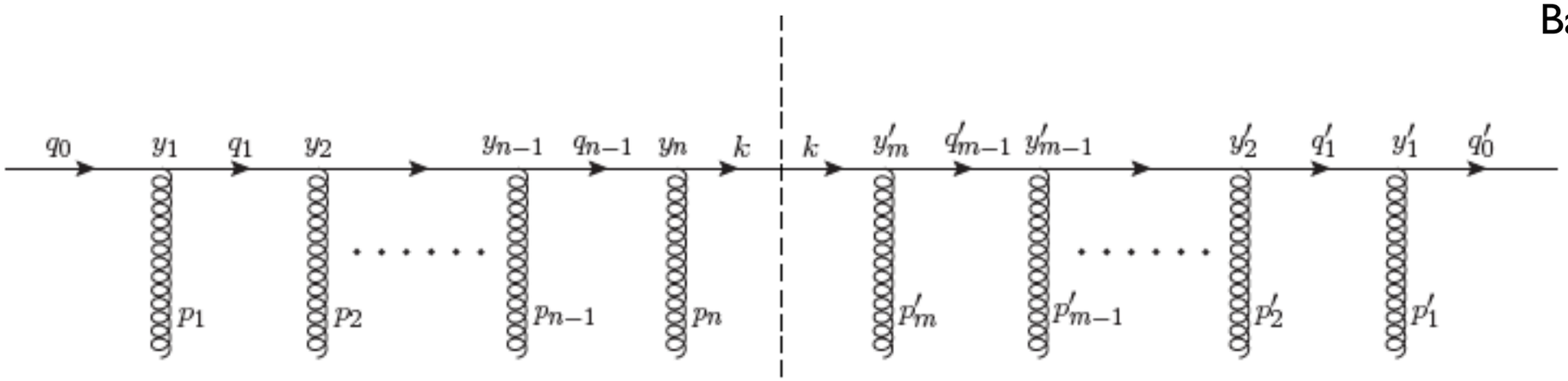
work in progress by
D'Eramo, Lekaveckas, **CL**, Liu, Rajagopal

- **New tools:** Apply power of SCET to prove factorization, calculate jet shapes precisely, resum logarithms!

SCET for Jets in Dense Media

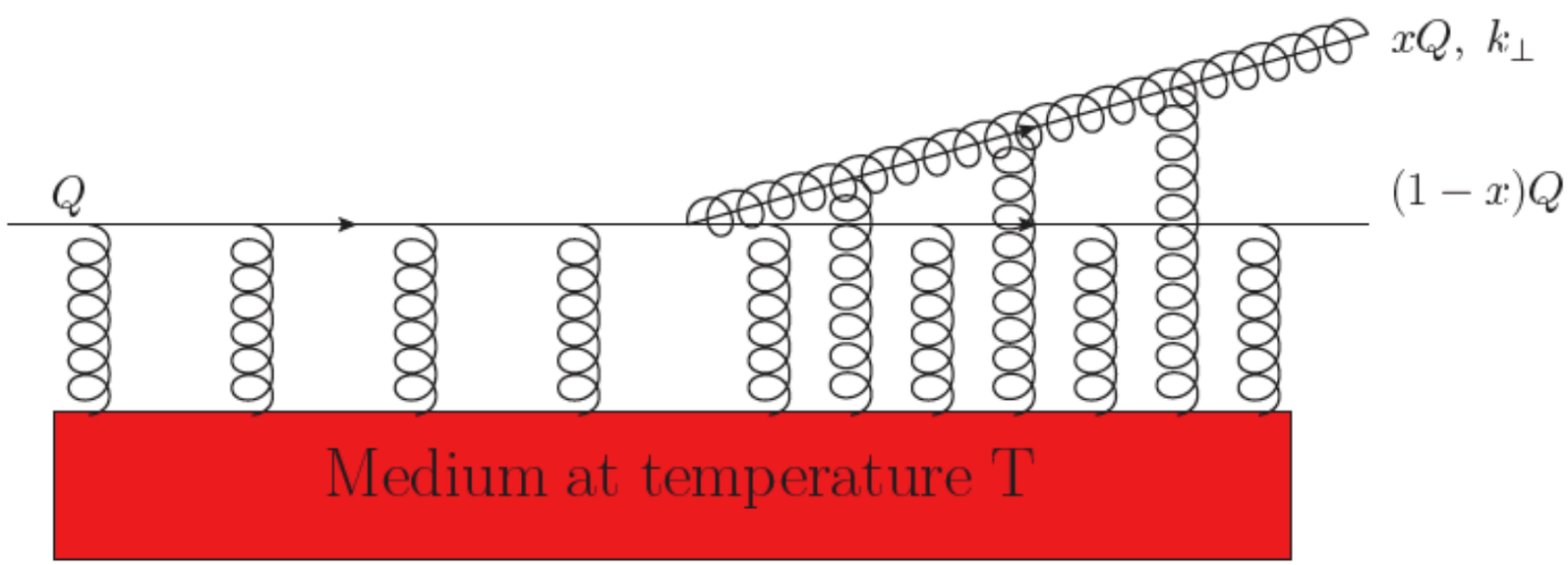
- So far, added to SCET gluons in medium which change (broaden) jet's transverse momenta but do not contribute to their energy loss

Idilbi, Majumder;
Bauer, Lange, Ovanesyan;
D'Eramo, Liu, Rajagopal



- In progress: inclusion of jet energy loss in the medium and soft & collinear radiation from jets

in progress by D'Eramo, Lekaveckas, **CL**, Liu, Rajagopal; Ovanesyan, Vitev



First observations of angularity jet shapes

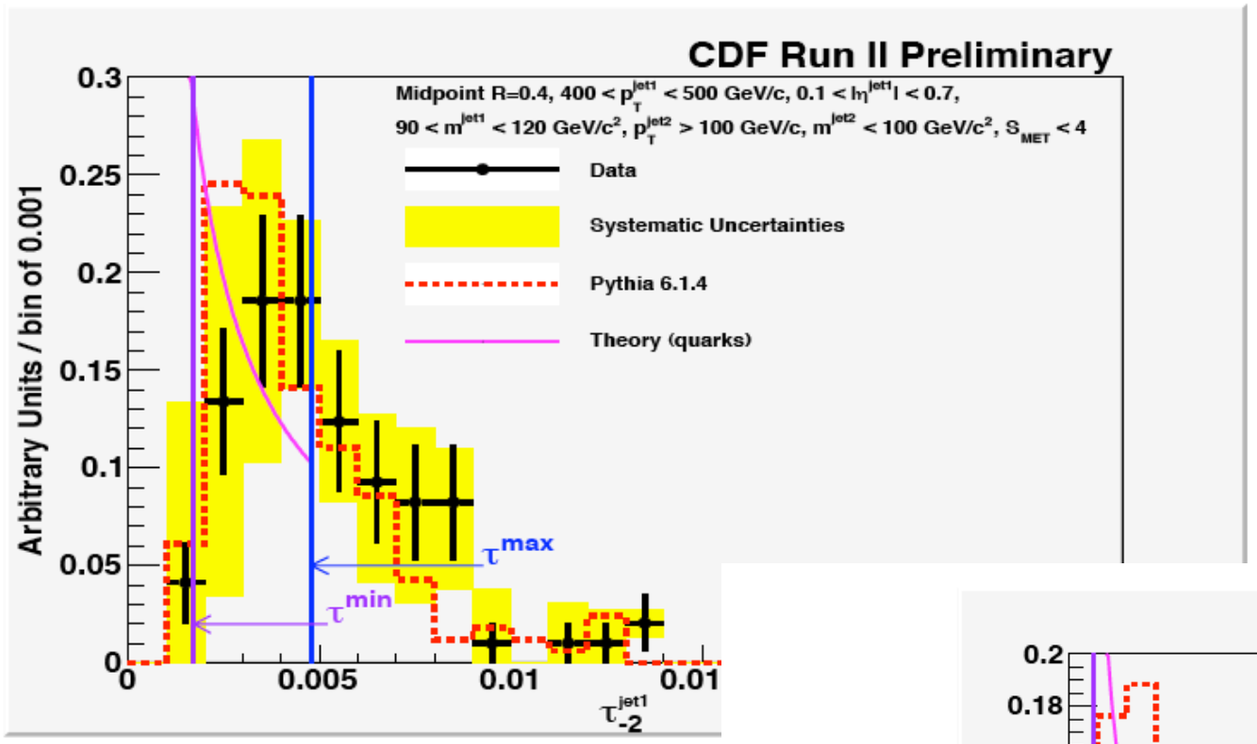


Figure 16: The angularity distribution for jets with $p_T \in (40$
$R = 0.4$ Midpoint cone algorithm. We have rejected the $t\bar{t}$ ev
We also show the results from the PYTHIA calculation, as

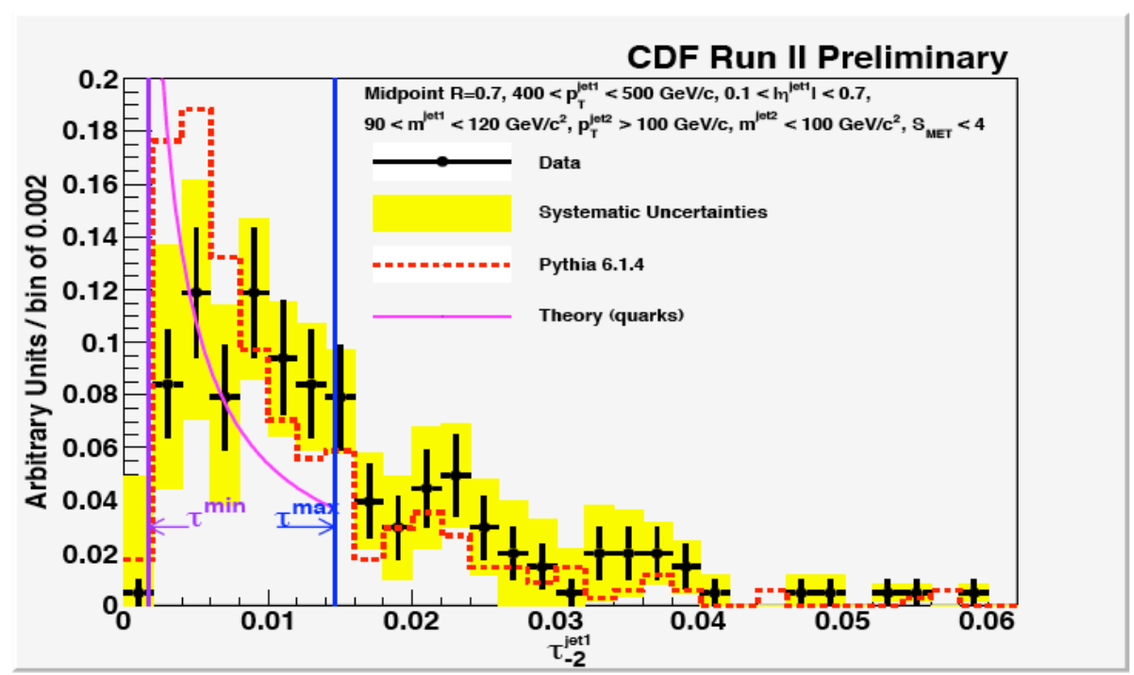


Figure 17: The angularity distribution for jets with $p_T \in (400, 500)$ and $|\eta| \in (0.1, 0.7)$ reconstructed with an $R = 0.7$ Midpoint cone algorithm. We have rejected the $t\bar{t}$ events and required that $m^{jet1} \in (90, 120)$ GeV/c^2. We also show the results from the PYTHIA calculation (red dashed line), as well as the analytic QCD prediction (dotted).

from CDF Public Note,
July 19, 2010

First observations of angularity jet shapes

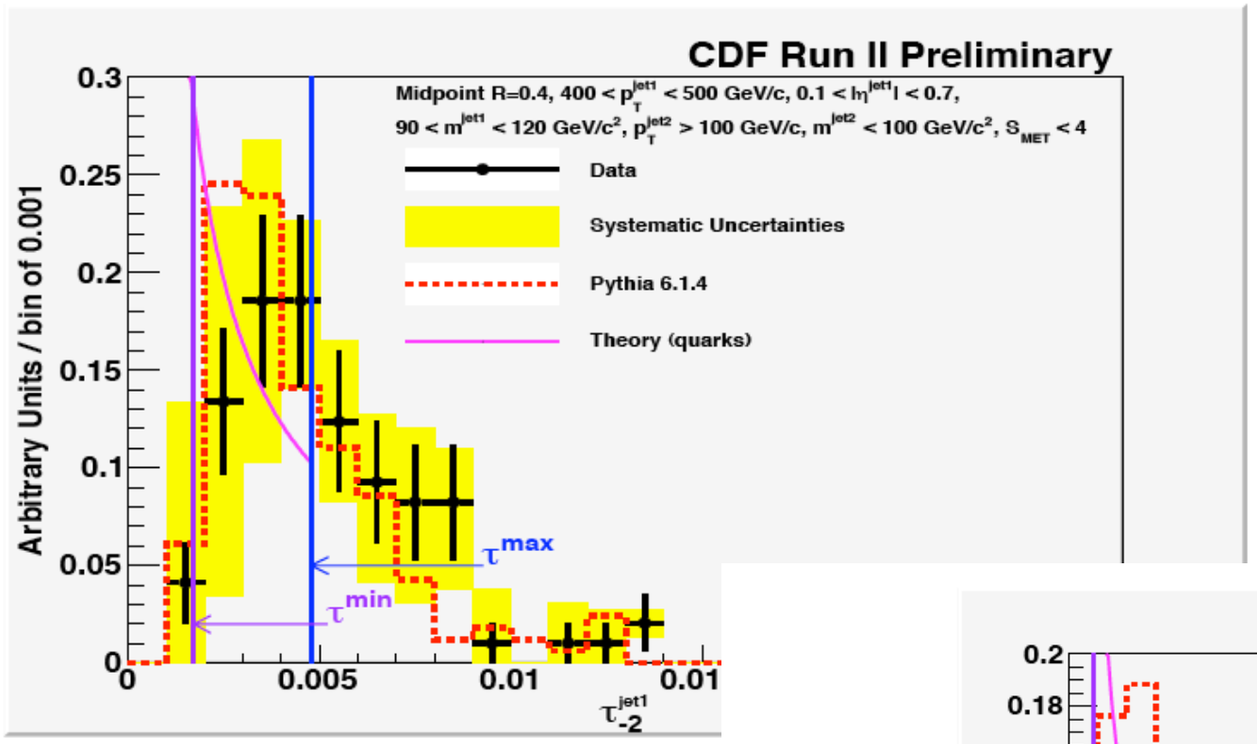


Figure 16: The angularity distribution for jets with $p_T \in (40$
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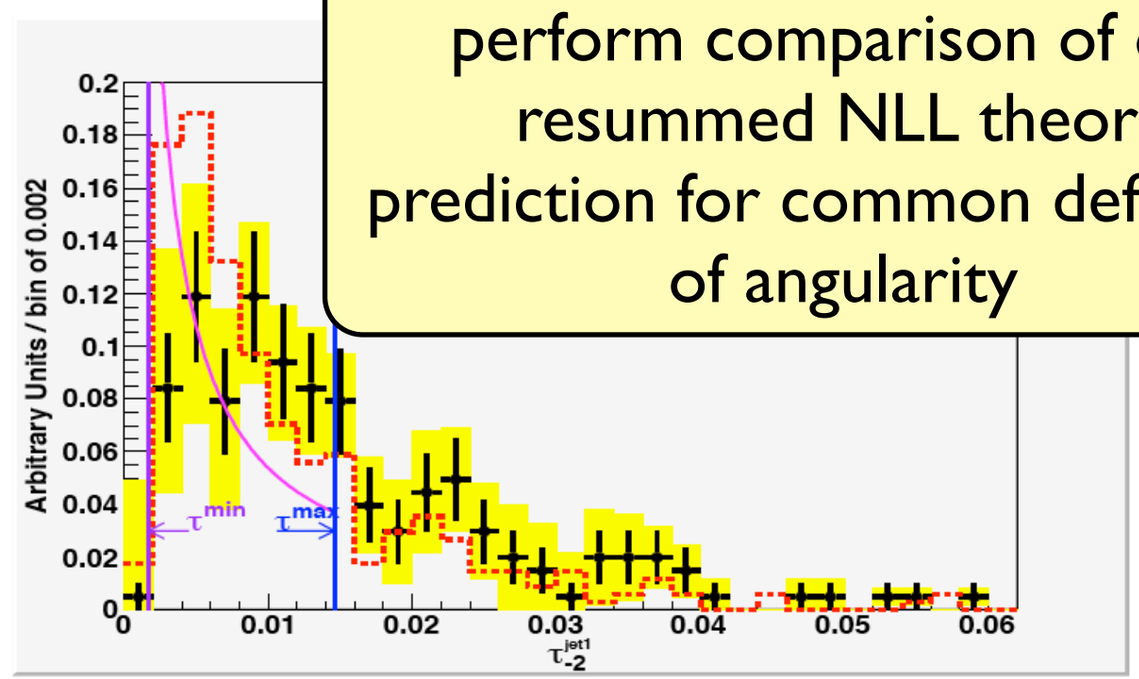
We also show the results from the PYTHIA calculation, as

Caveat: different definition of angularity

CDF only plots leading-order, leading-log theory prediction

We are now in conversations to perform comparison of our resummed NLL theory prediction for common definition of angularity



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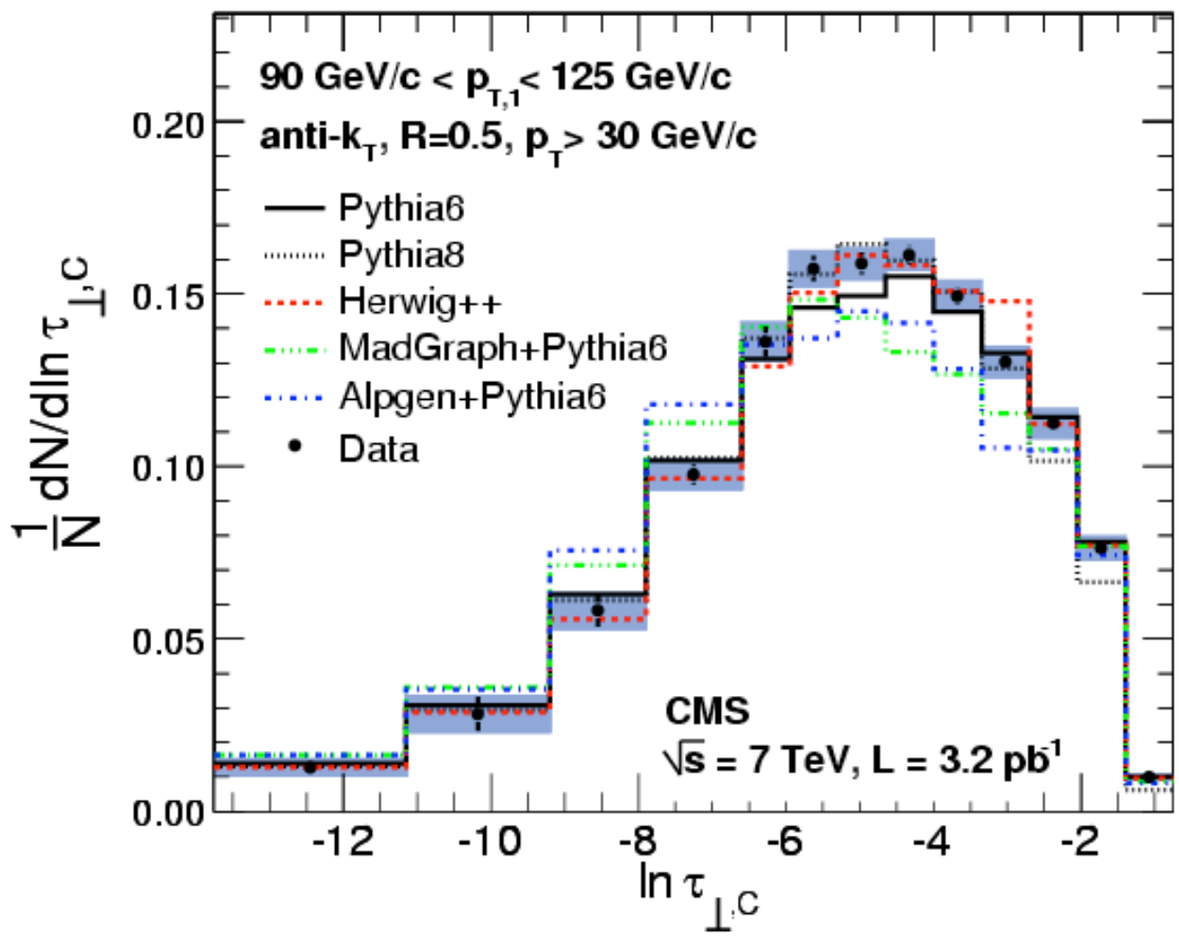
from CDF Public Note, July 19, 2010

Event shapes at LHC vs Monte Carlo

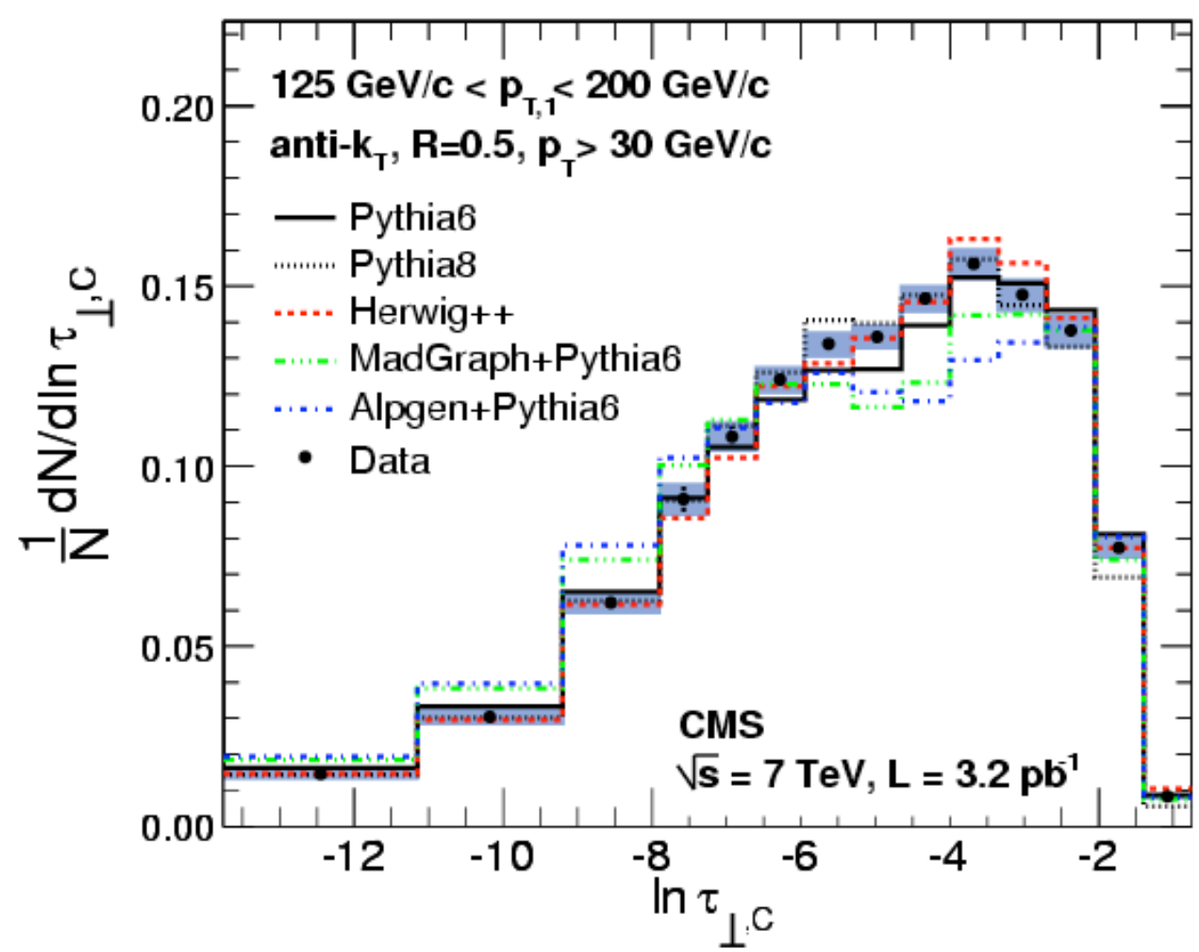
CMS 1102.0068



"Central tranverse thrust"

$$\tau_\perp = 1 - \max_{\hat{n}_T} \frac{\sum_i |\mathbf{p}_\perp^i \cdot \hat{n}_T|}{\sum_i |\mathbf{p}_\perp^i|}$$

Banfi, Salam, Zanderighi



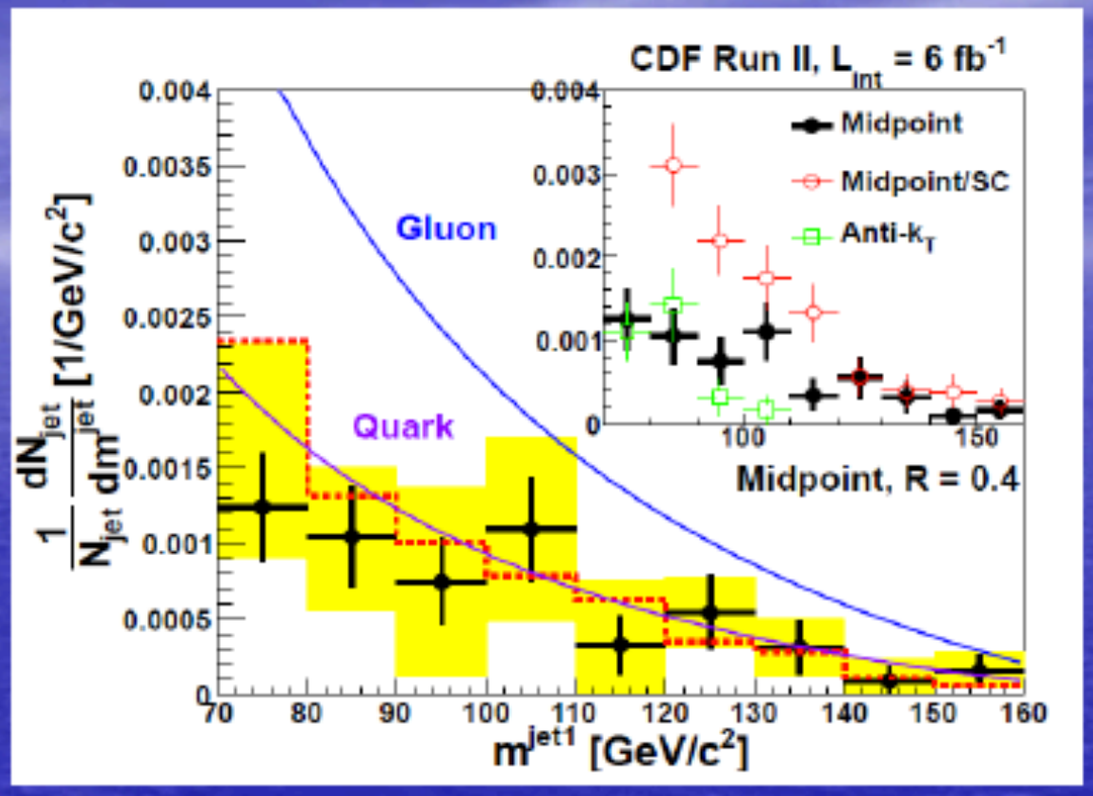
- Fair agreement but noticeable discrepancies between data and MC
- How to assign discrepancy into perturbative and nonperturbative contributions?

Jet mass distributions at CDF

CDF Note 10199

Substructure of High E_T Jets

- Jet shape variables for high p_T (> 400 GeV) QCD jets
  Jet mass, Angularity, Planar Flow
- Study of massive jets:
  -- Test of pQCD predictions
  -- Tuning MC generators
  -- massive boosted jets comprise important background for high p_T top, Higgs and various BSM searches

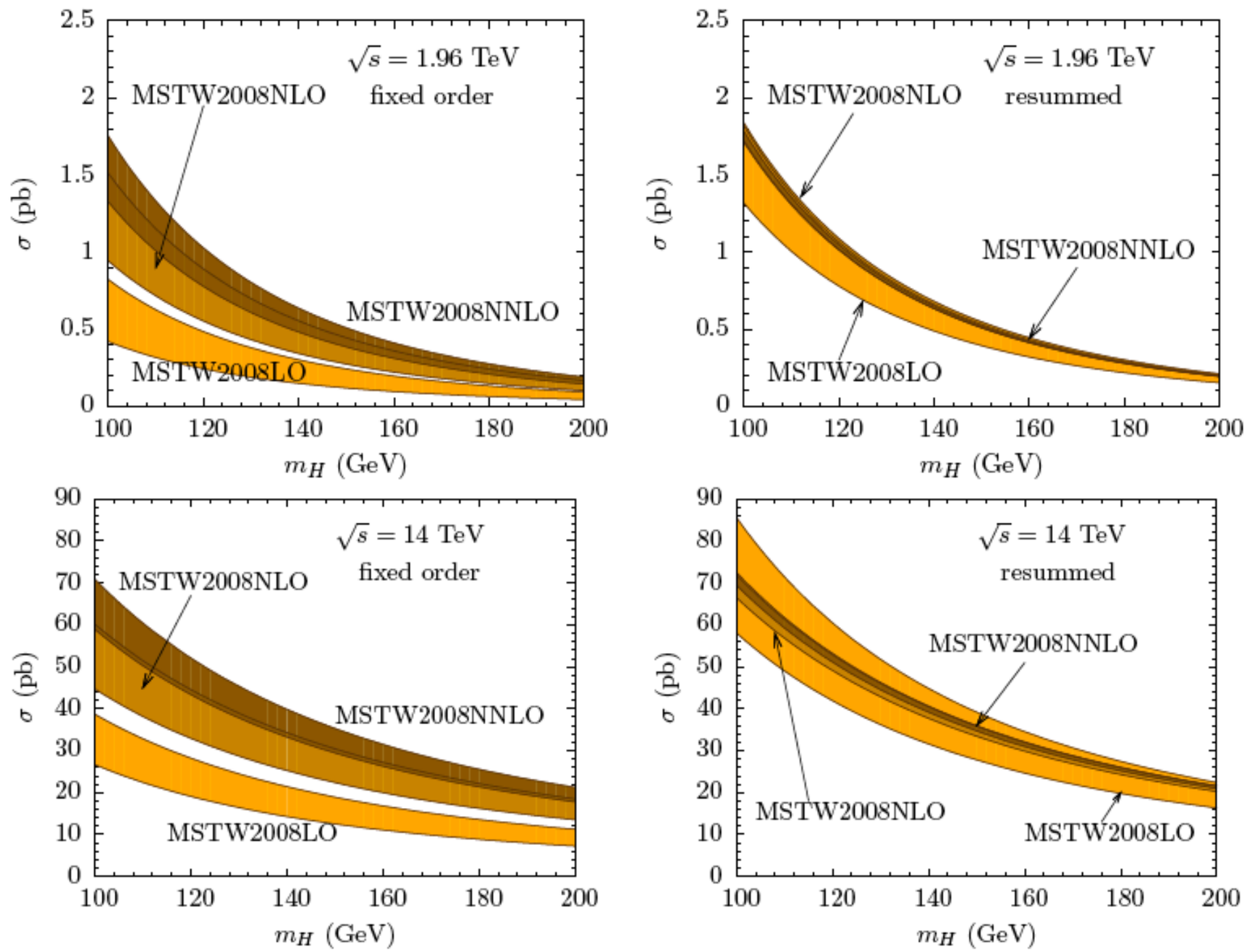


- Jet Mass : Standard E-scheme for mass calculation : vector sum over (E,px,py,pz) of towers in jets.

- Data in agreement with PYTHIA prediction
- Data between quark and gluon prediction (consistent with the expectation that over 80% of jets would arise from quark showering)

Resummed Higgs production cross section

Ahrens, Becher, Neubert, Yang(2008)



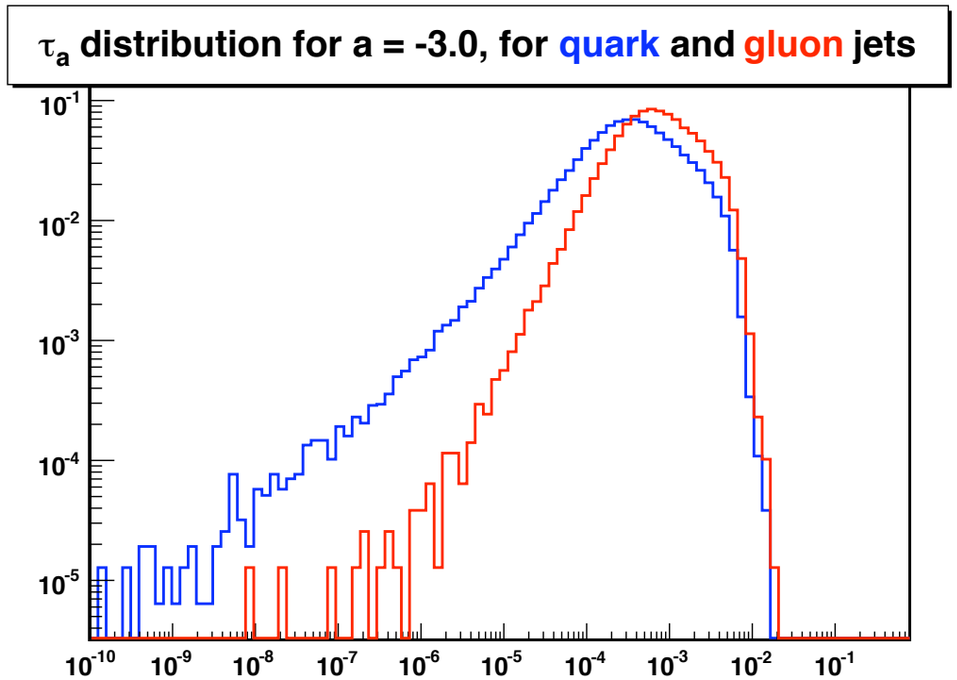
Fixed order

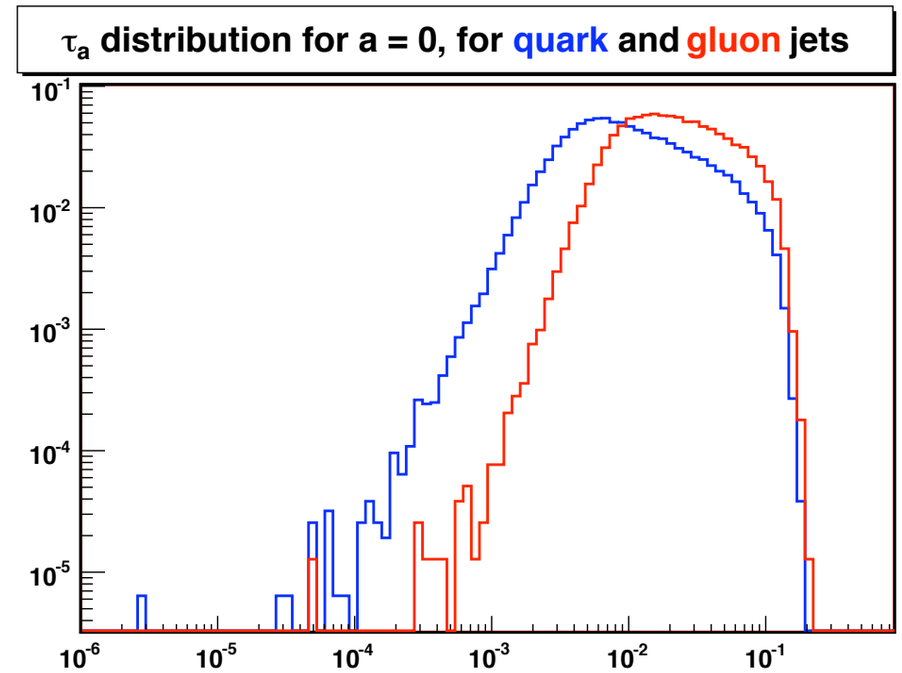
Resummed

Using angularities to distinguish quark and gluon jets

PRELIMINARY

Ellis, Hornig, CL, Vermilion, Walsh



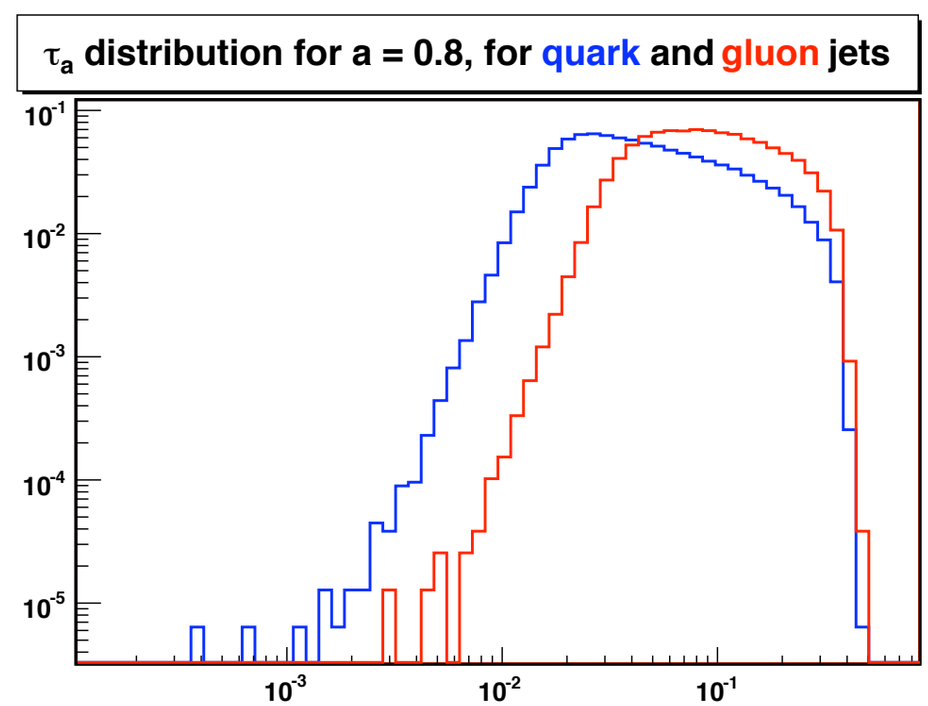


Studied quark v. gluon jets 3 well-separated jets in e^+e^-

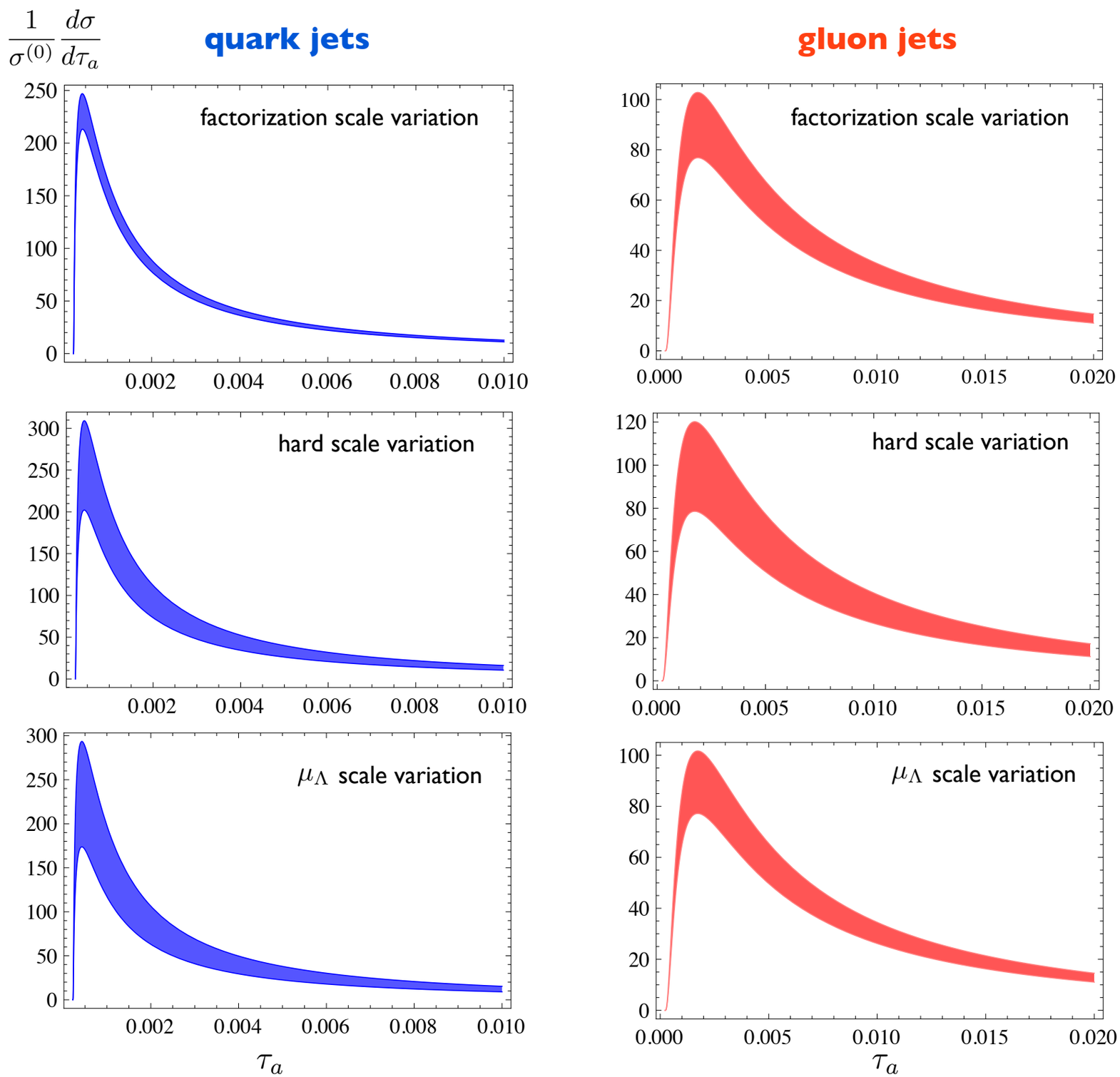
cuts exist keeping ~2% of gluon jets and ~20% of quark jets (**10:1** enhancement)

or ~15% of gluons and 8% quarks (**2:1** enhancement).

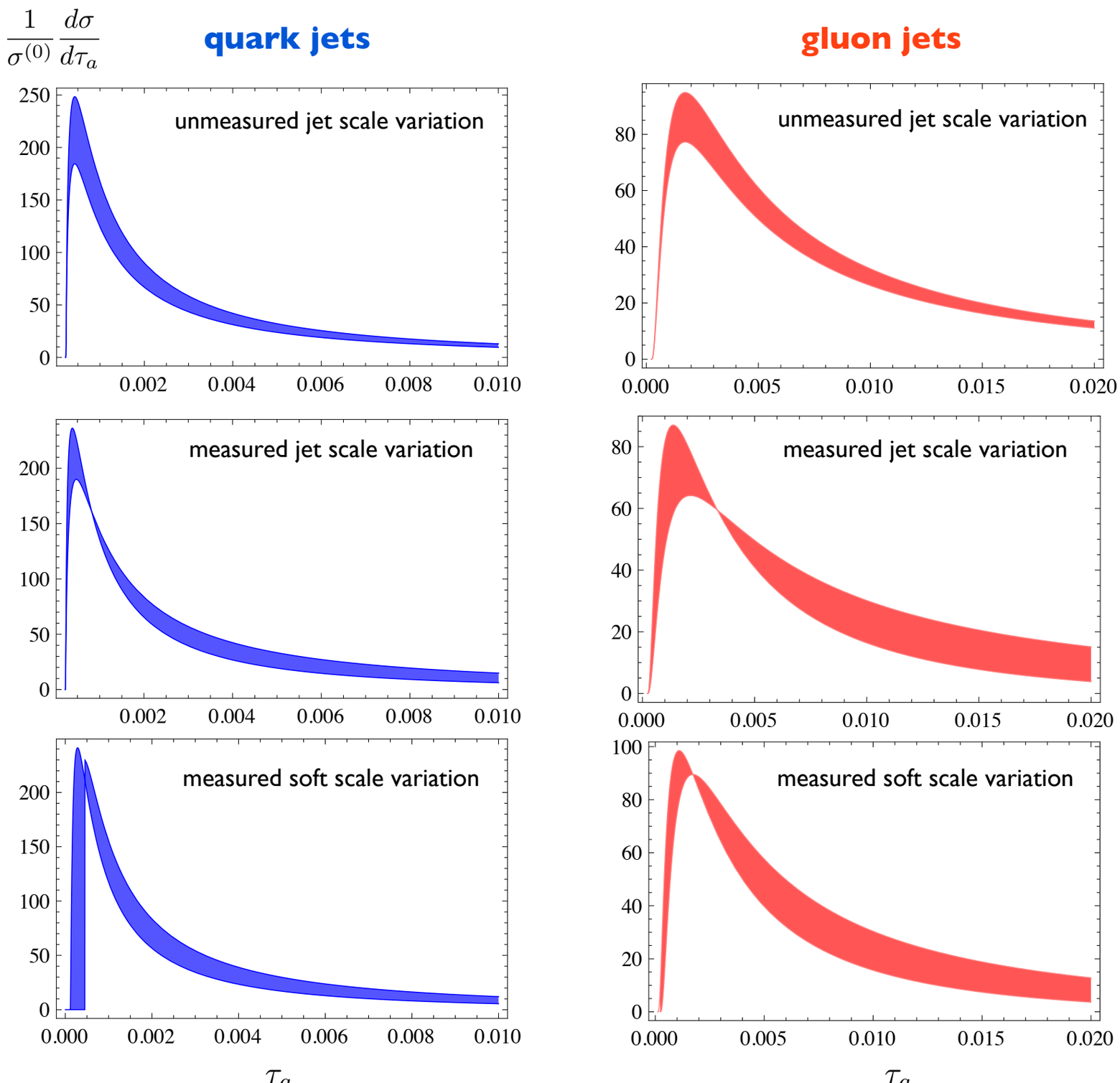
Expect greater discriminating power in correlated distributions for multiple values of *a*



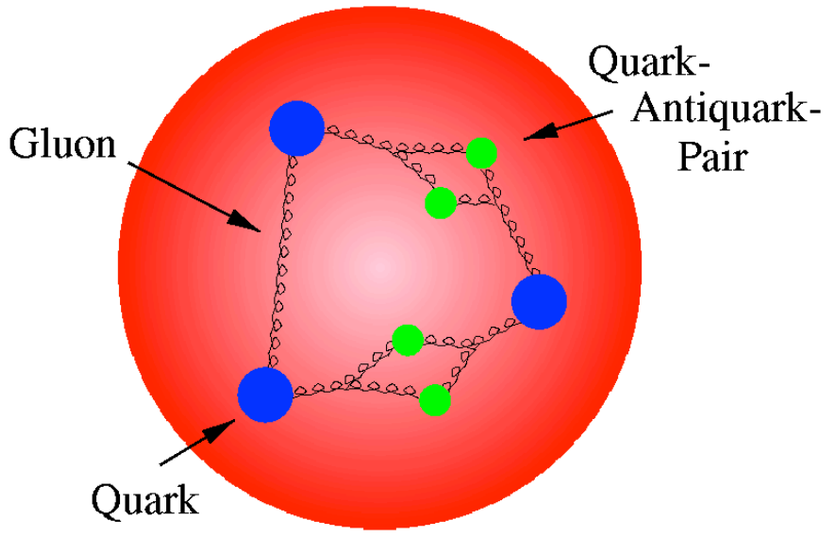
Estimate of Theoretical Uncertainty

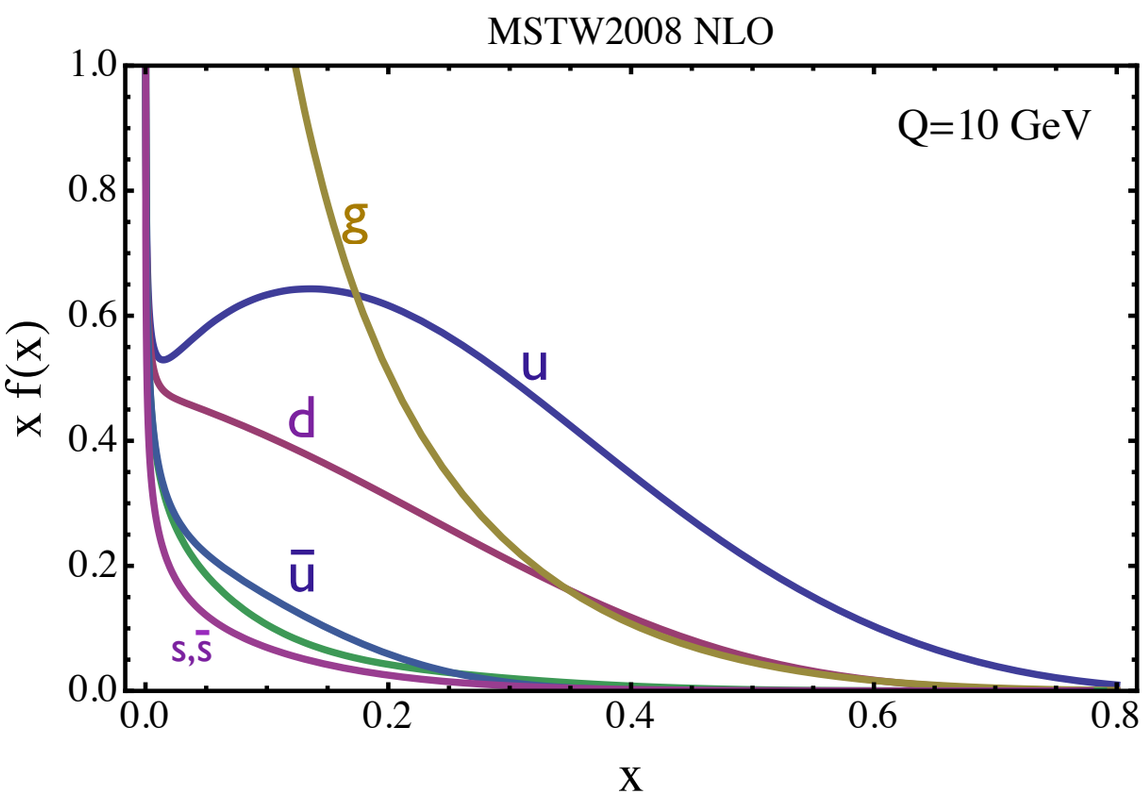


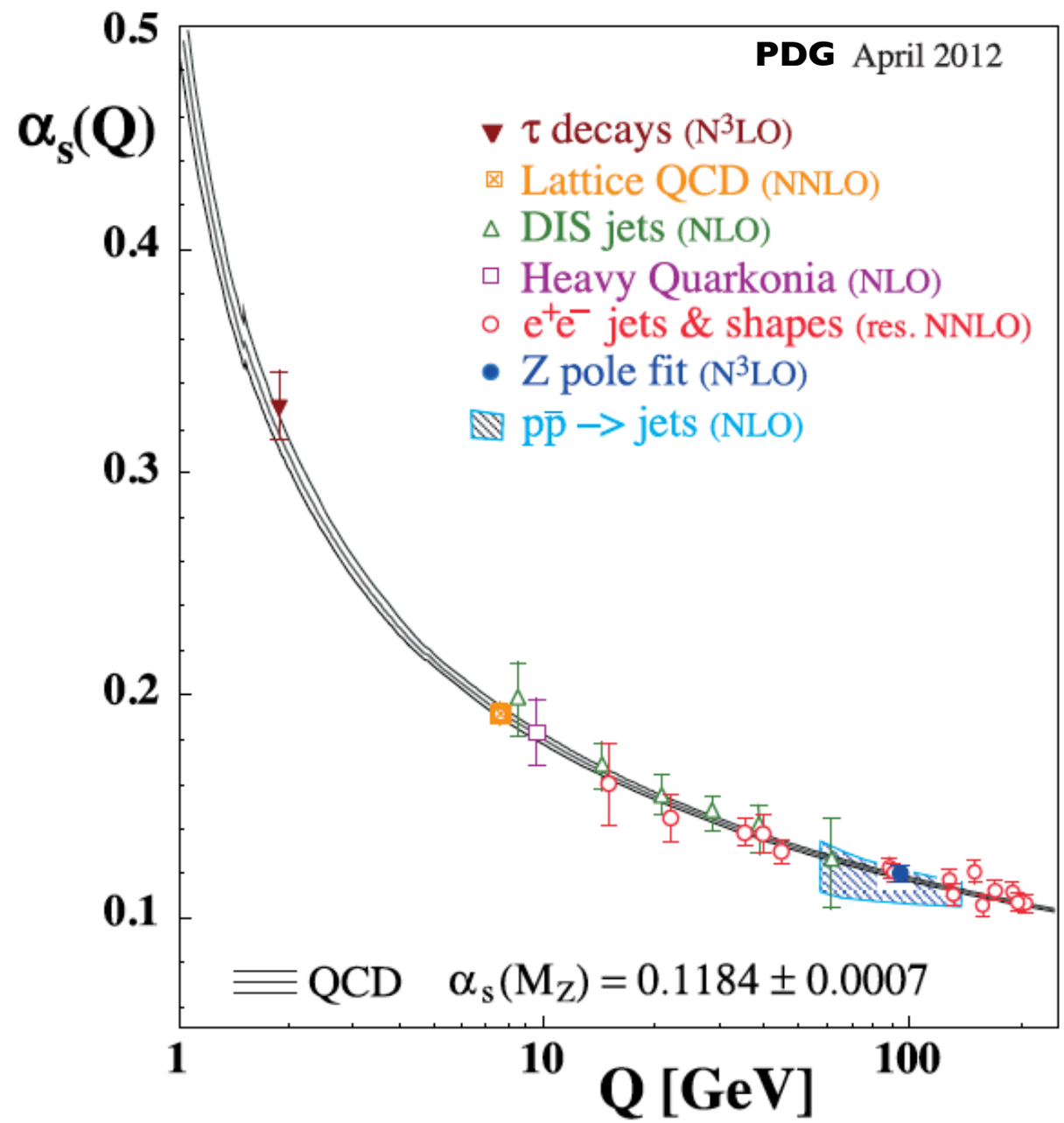
Estimate of Theoretical Uncertainty



QCD, PDFs, and the Strong Coupling

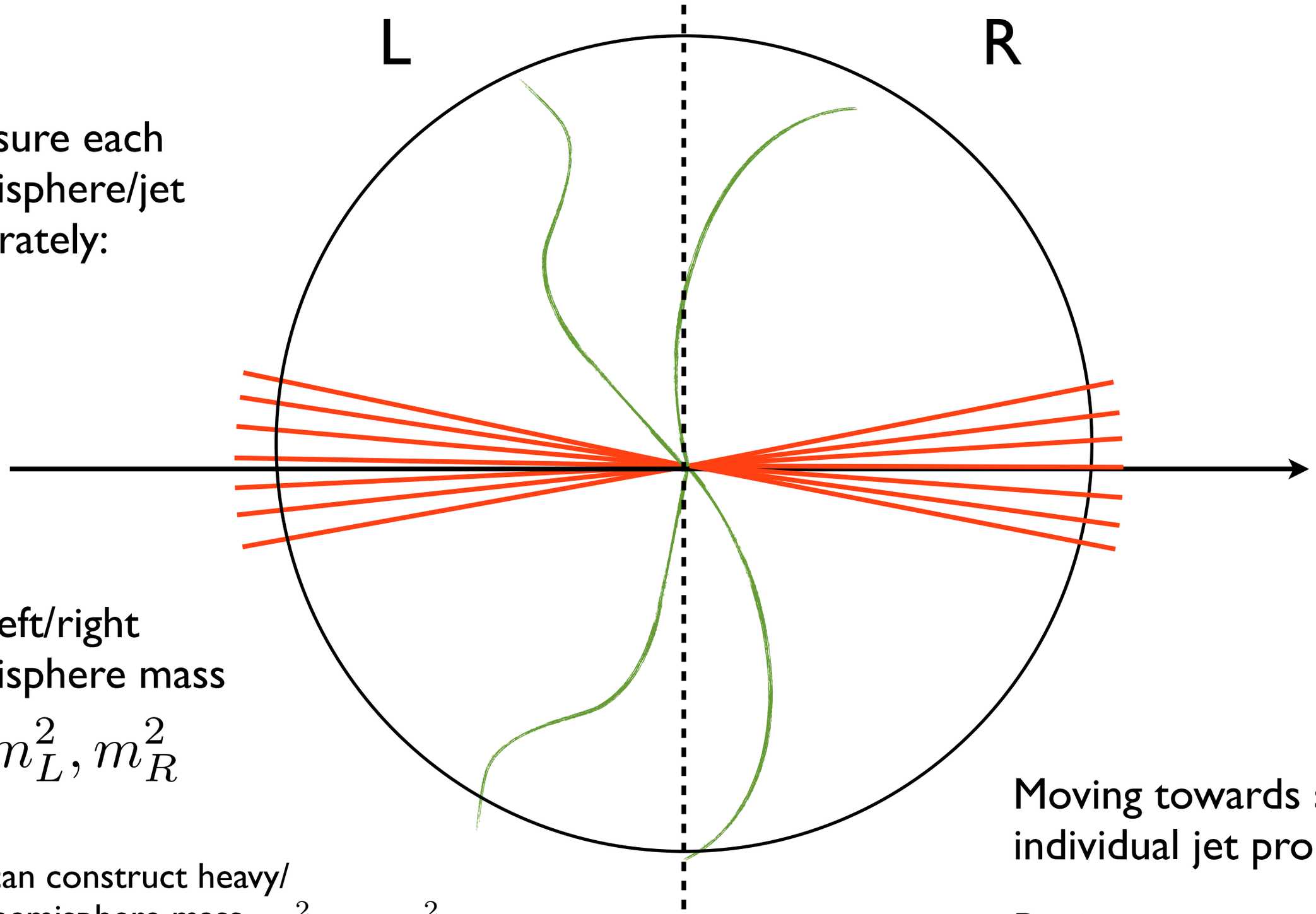






Probing Jets At Ever Finer Scales

Measure each hemisphere/jet separately:



e.g. left/right hemisphere mass

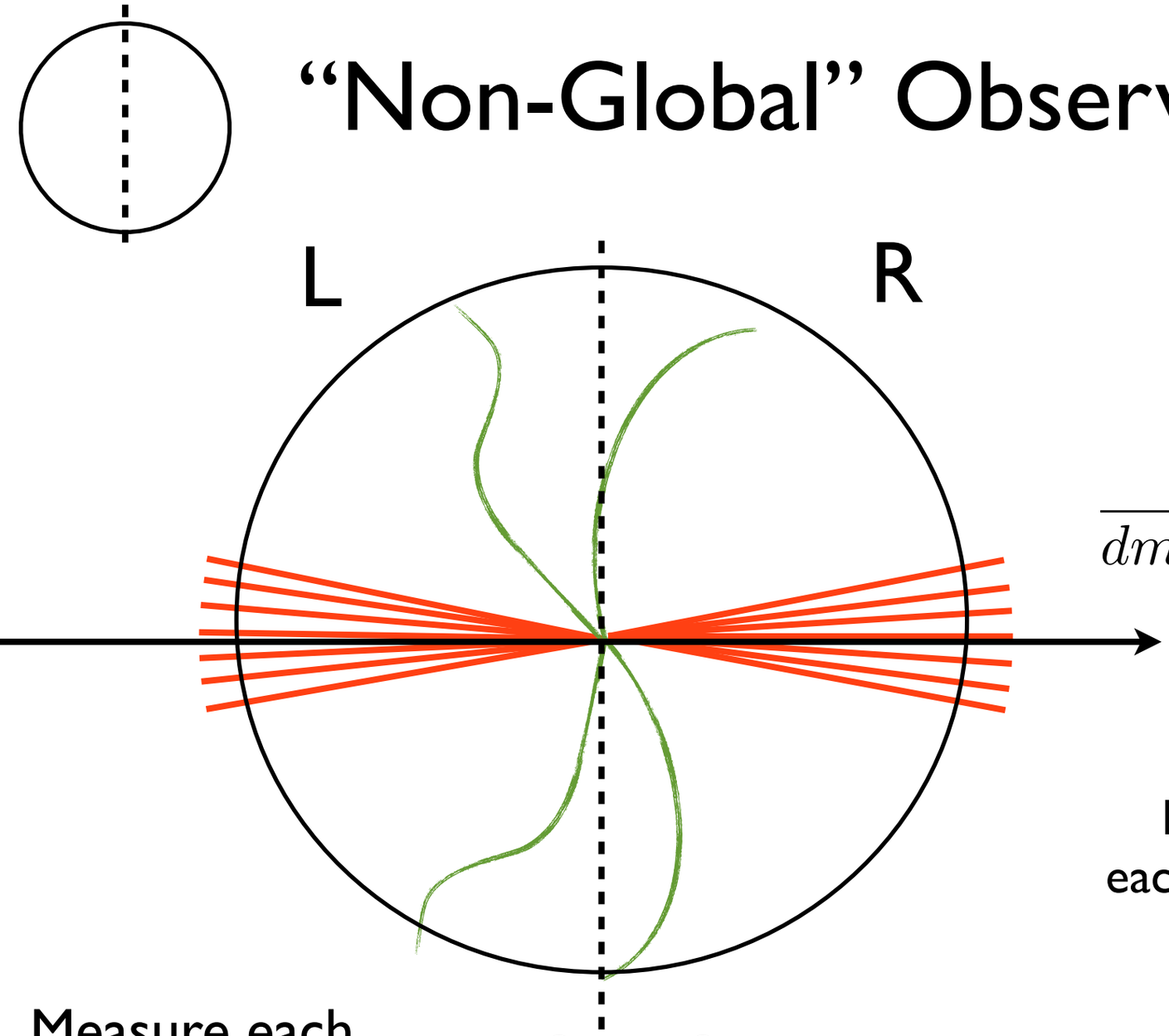
$$m_L^2, m_R^2$$

also can construct heavy/light hemisphere mass $m^2_{\text{heavy}}, m^2_{\text{light}}$

Moving towards studying individual jet properties...

Begin to encounter so-called "non-global" logs

"Non-Global" Observables and Logs



$$\frac{d\sigma}{dm_L^2 dm_R^2} \sim \sigma(m_L)\sigma(m_R)S_{ng}(m_L, m_R)$$

logs of m_L/Q or m_R/Q each
each summable using "ordinary" SCET

Measure each hemisphere/jet separately: m_L^2, m_R^2

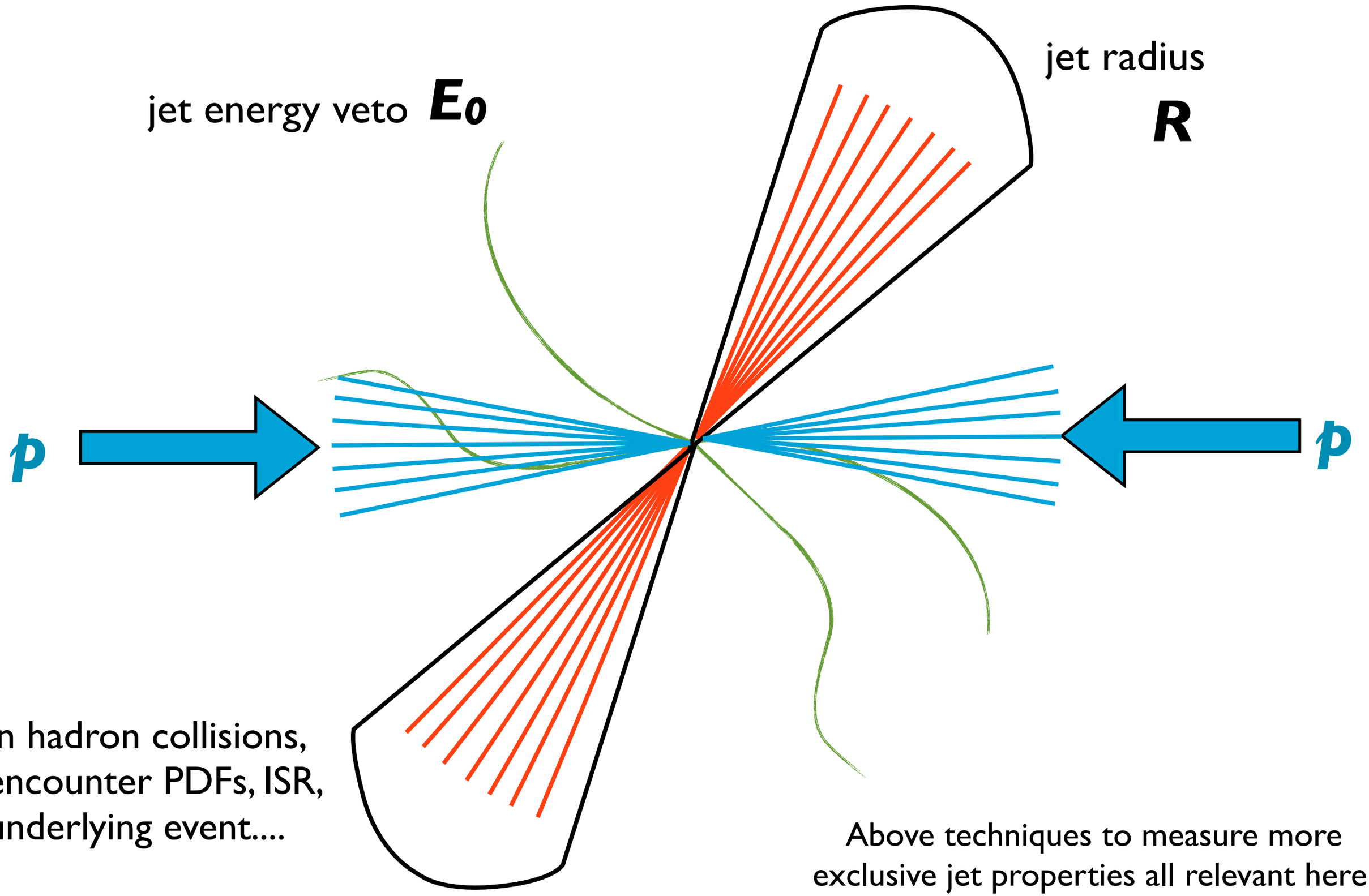
unknown how to resum "non-global" logs of m_L/m_R

$$S_{ng}(m_L, m_R) = 1 + \left(\frac{\alpha_s}{2\pi}\right)^2 C_F C_A \frac{\pi^2}{3} \ln^2 \frac{m_L}{m_R} + \cdots$$

Soft radiation is sensitive/ probed at more than one scale

Dasgupta, Salam
Banfi, Dasgupta, Khelifa-Kerfa, Marzani
...

Probing Jets At Ever Finer Scales



In hadron collisions, encounter PDFs, ISR, underlying event....

Above techniques to measure more exclusive jet properties all relevant here.

Jet Substructure

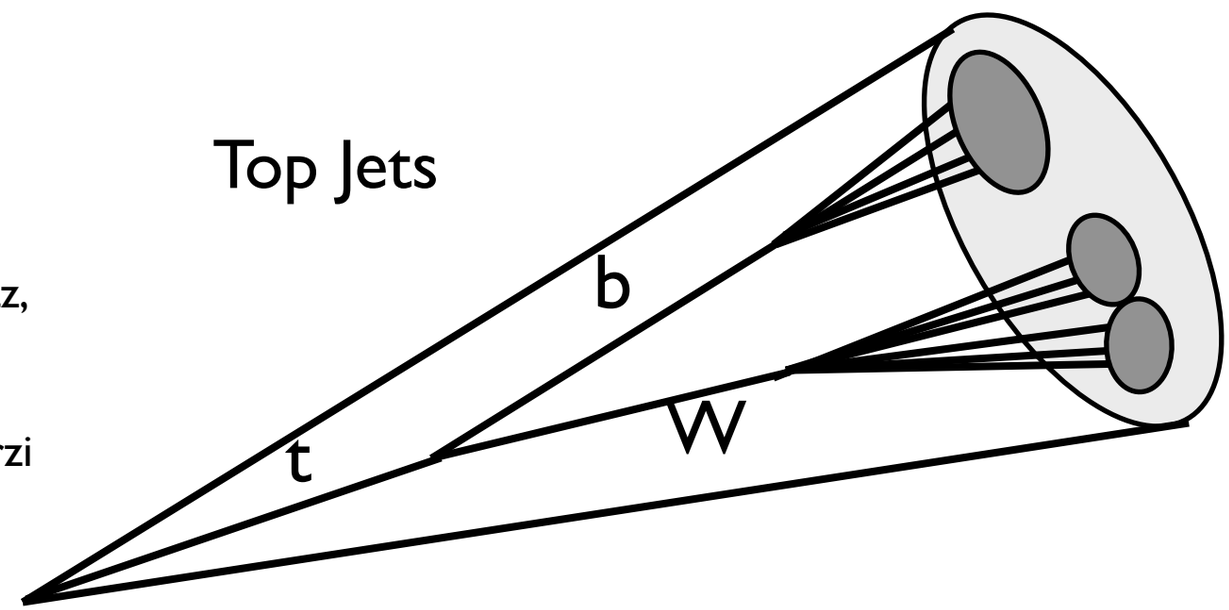
Probe **substructure** of jets to look for evidence of highly-boosted heavy particle

Thaler, Wang

Kaplan, Rehermann, Schwartz, Tweedie

Almeida, Lee, Perez, Sung, Virzi

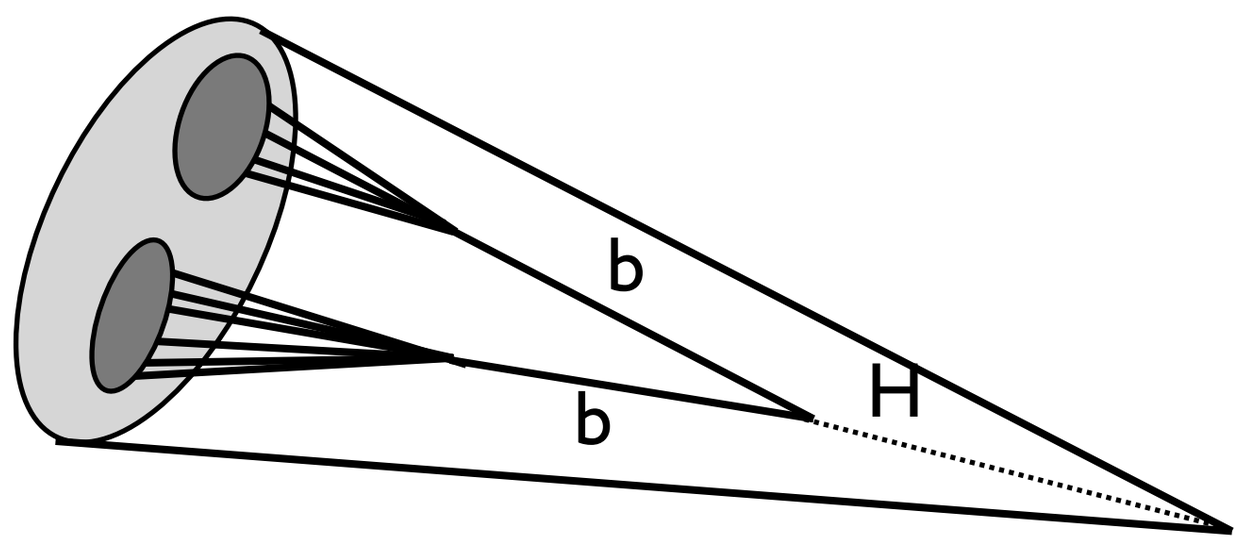
Top Jets



Butterworth, Davison, Rubin, Salam

Plehn, Salam, Spannowsky

Kribs, Martin, Roy, Spannowsky



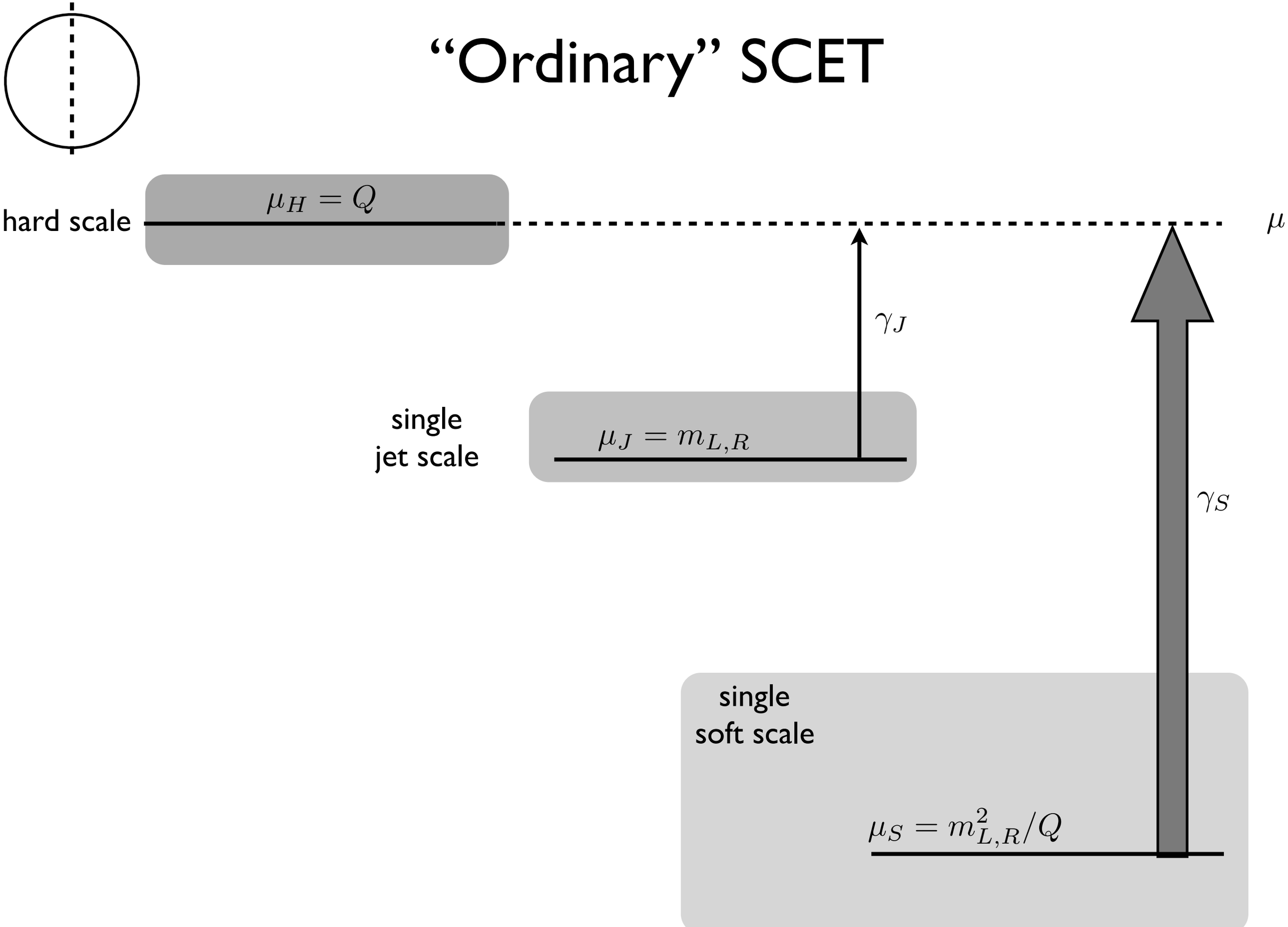
Higgs Jets

...BSM Jets

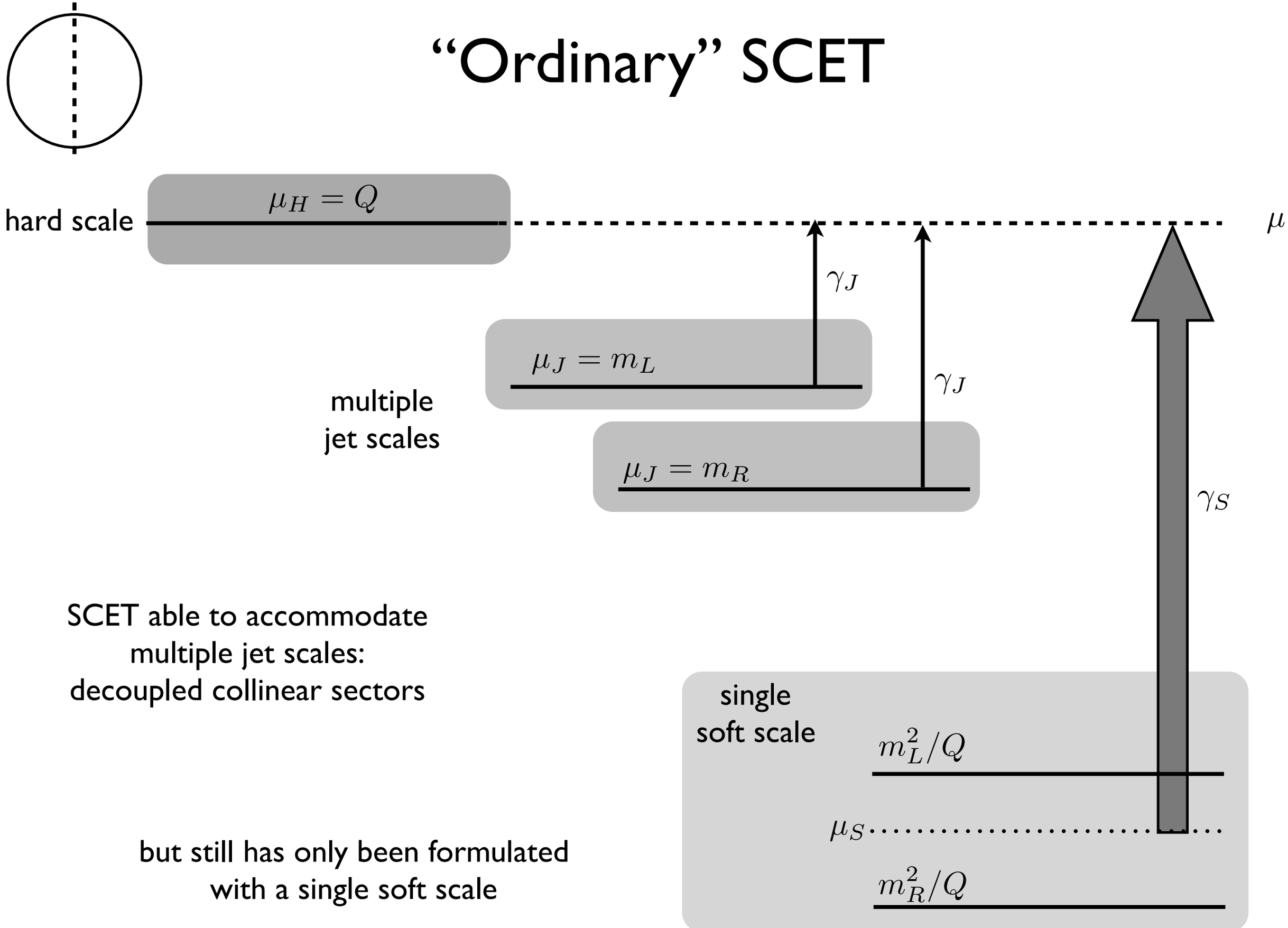
e.g. Squark Jets

Fan, Krohn, Mosteiro, Thalapillil, Wang

"Ordinary" SCET

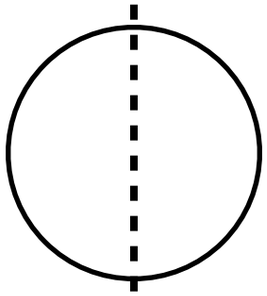


"Ordinary" SCET



SCET able to accommodate
multiple jet scales:
decoupled collinear sectors

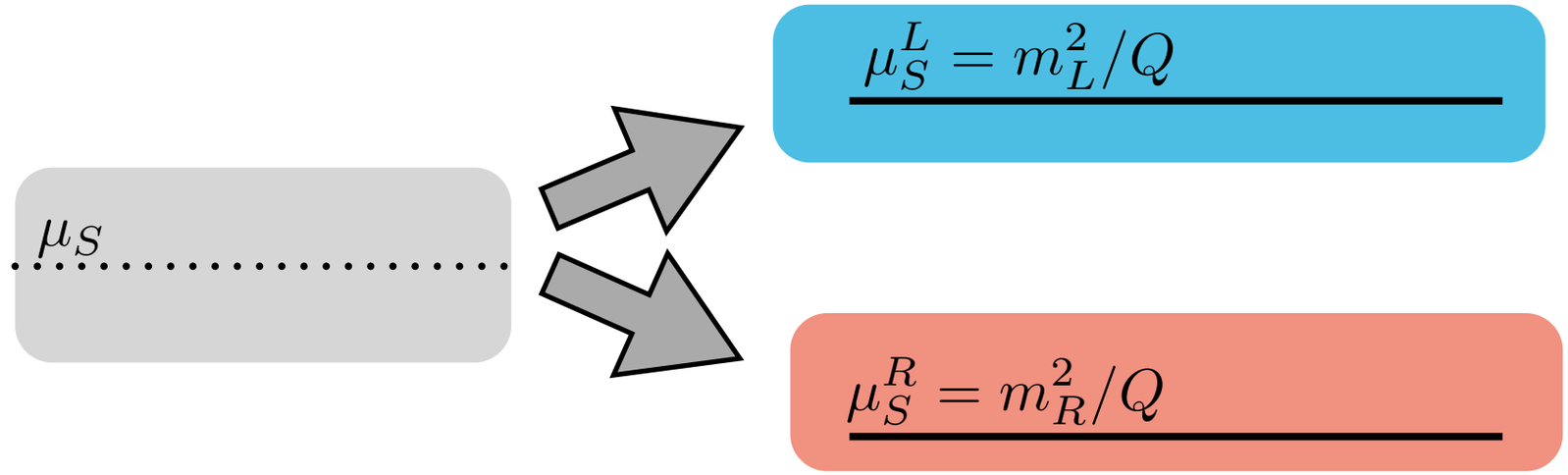
but still has only been formulated
with a single soft scale



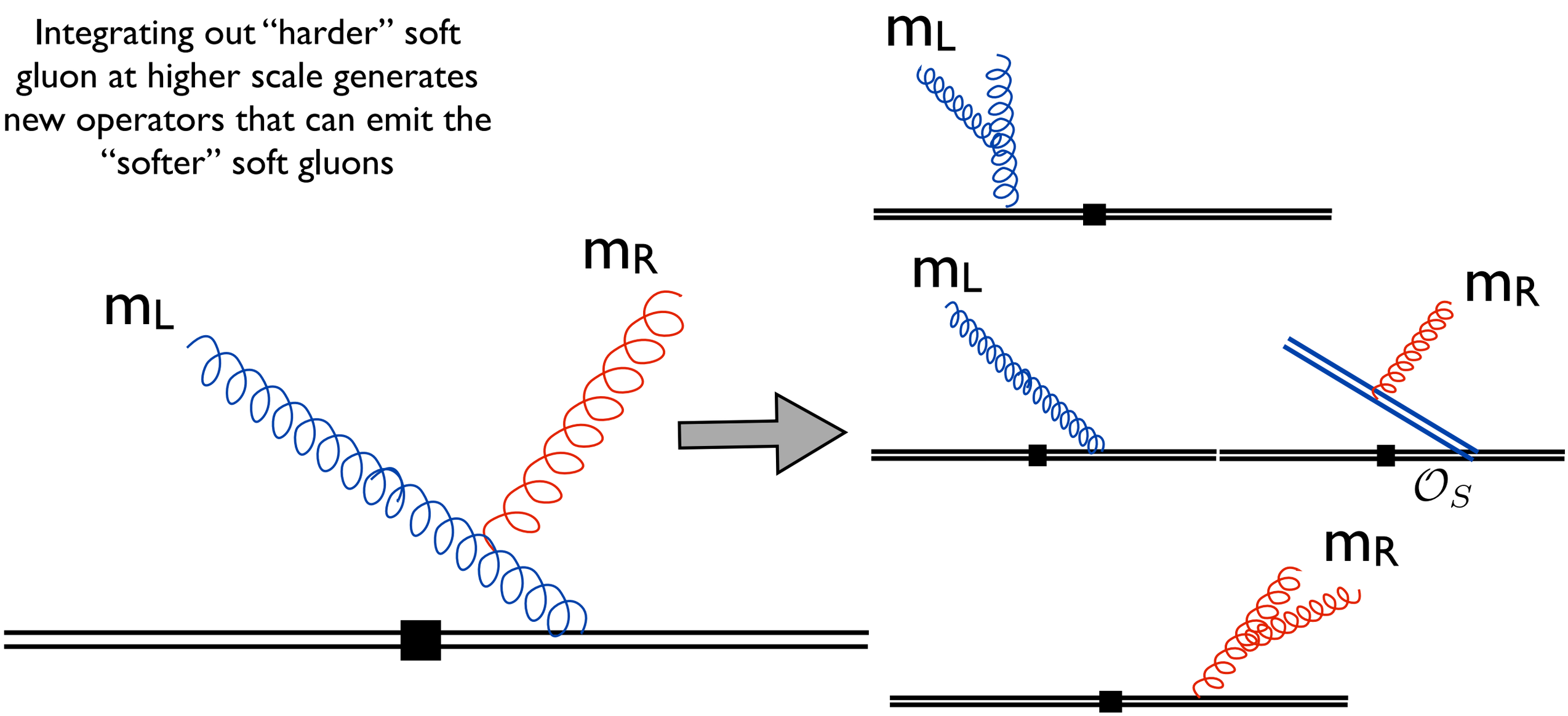
Soft "Refactorization"

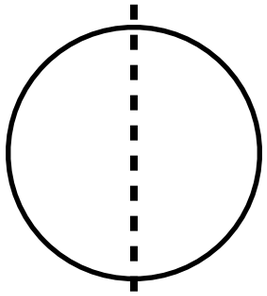
Hornig, **CL**, Stewart,
Walsh, Zuberi
(work in progress)

Factorize soft sector into two separate soft modes:



Integrating out "harder" soft gluon at higher scale generates new operators that can emit the "softer" soft gluons

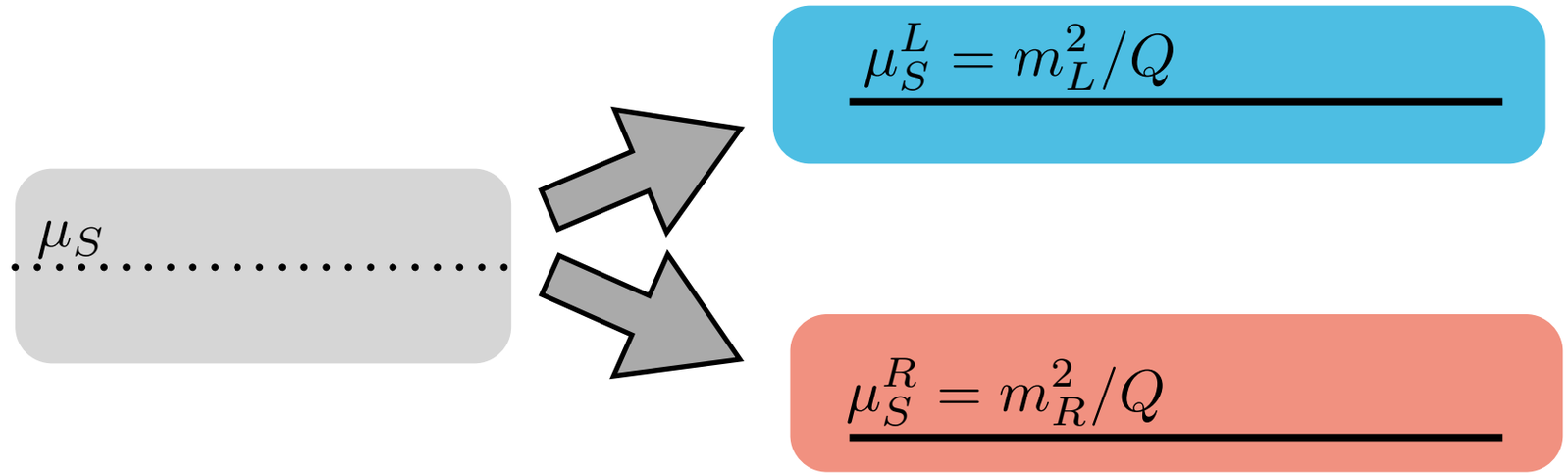




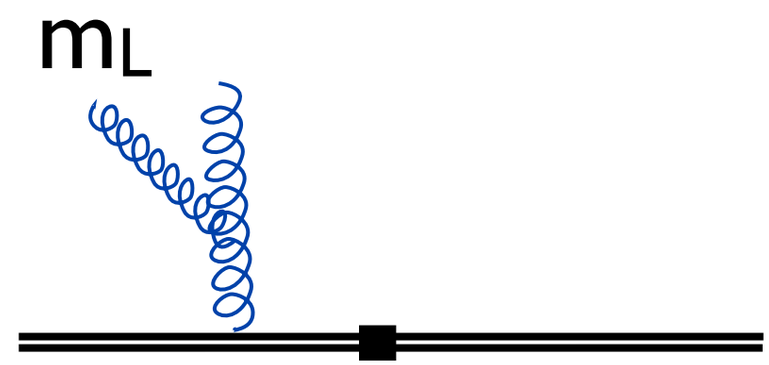
Soft "Refactorization"

Hornig, **CL**, Stewart, Walsh, Zuberi
(work in progress)

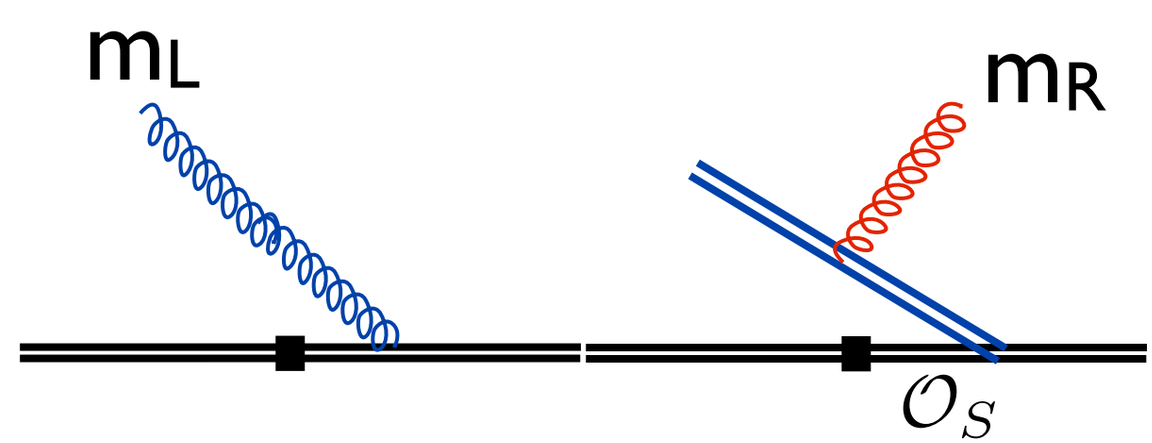
Original "non-global" log factorized into separate functions which can be evolved using EFT renormalization group:



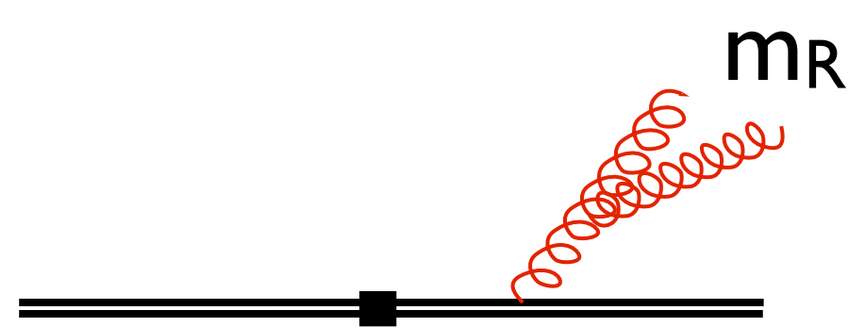
$$S_L(m_L) \sim \left(\frac{\alpha_s}{2\pi}\right)^2 C_F C_A \frac{\pi^2}{3} \ln^2 \frac{\mu}{m_L}$$



$$C_S(m_L)\langle \mathcal{O}_S(m_R)\rangle \sim \frac{\alpha_s C_F}{2\pi} \ln \frac{\mu}{m_L} \times \frac{\alpha_s C_A}{2\pi} \frac{\pi^2}{3} \ln \frac{\mu}{m_R}$$

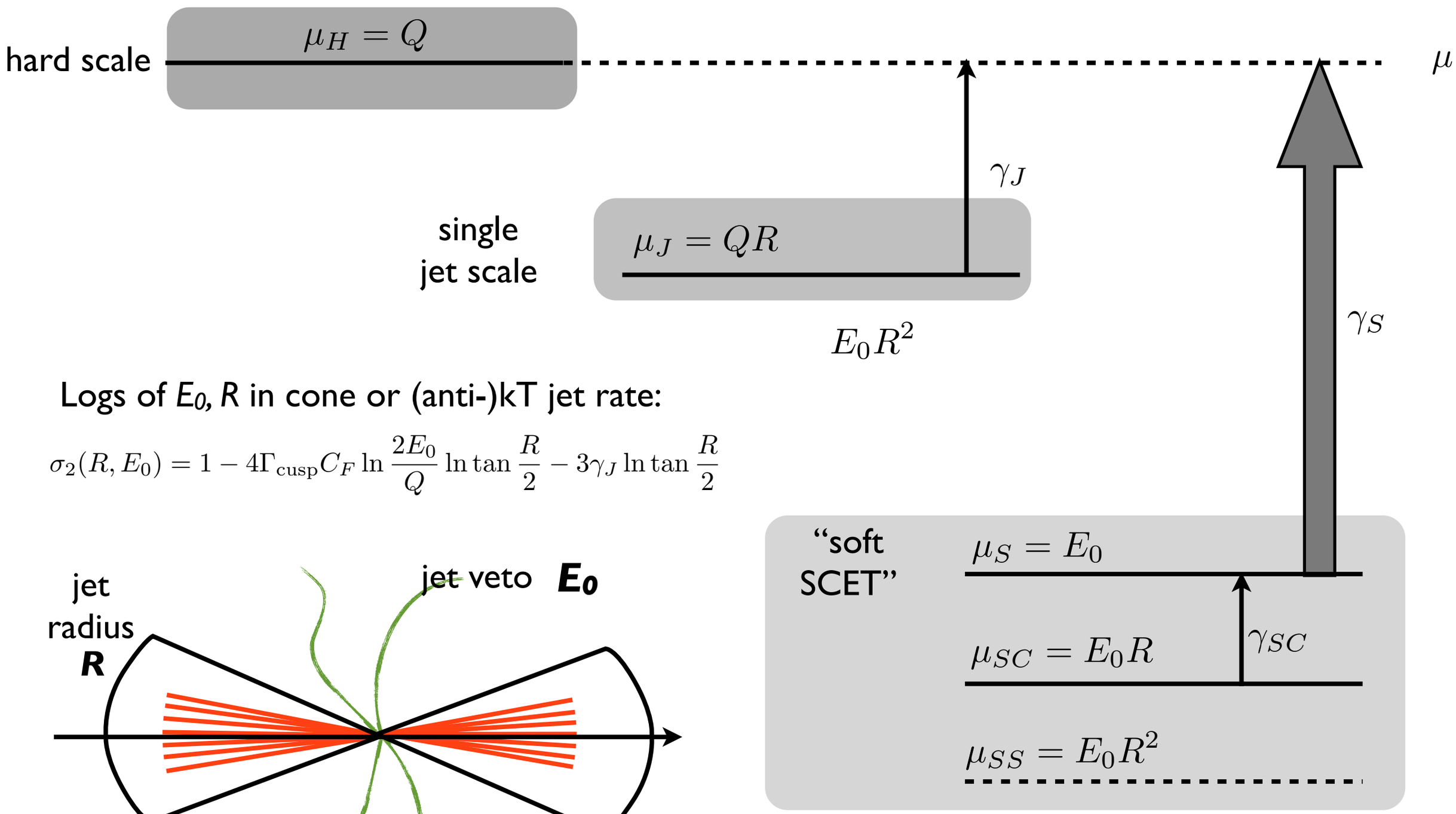


$$S_R(m_R) \sim \left(\frac{\alpha_s}{2\pi}\right)^2 C_F C_A \frac{\pi^2}{3} \ln^2 \frac{\mu}{m_R}$$



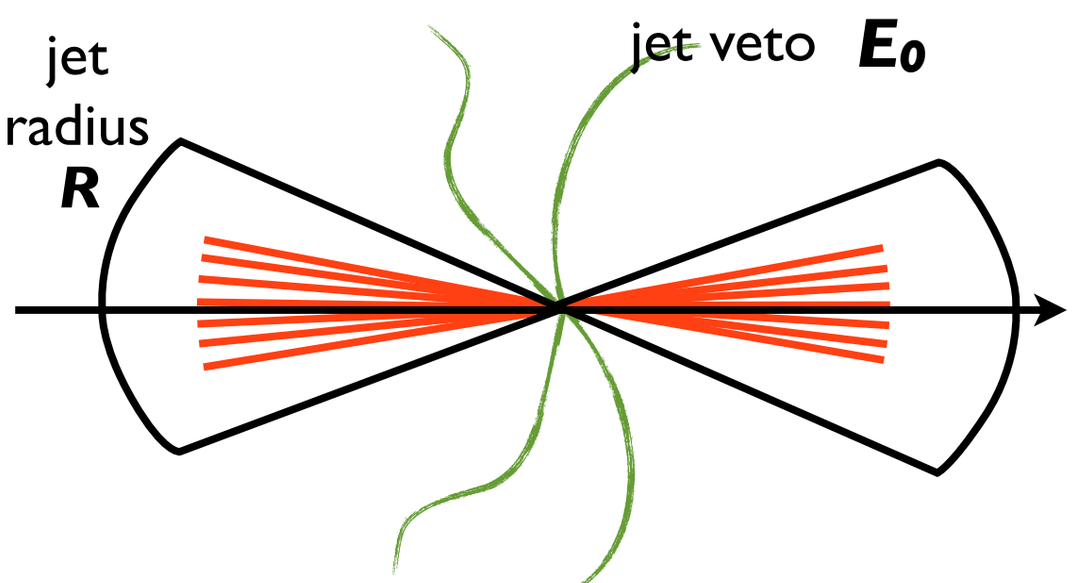
CL, Walsh, Zuberi
(work in progress)

Summing Jet Algorithm Parameters in SCET



Logs of E_0, R in cone or (anti-)kT jet rate:

$$\sigma_2(R, E_0) = 1 - 4\Gamma_{\rm cusp} C_F \ln \frac{2E_0}{Q} \ln \tan \frac{R}{2} - 3\gamma_J \ln \tan \frac{R}{2}$$



Needs addition of "soft-collinear" mode and matching onto new operators at $\mathcal{O}(\alpha_s^2)$.

Recap: Jet Observables at Ever Finer Scales

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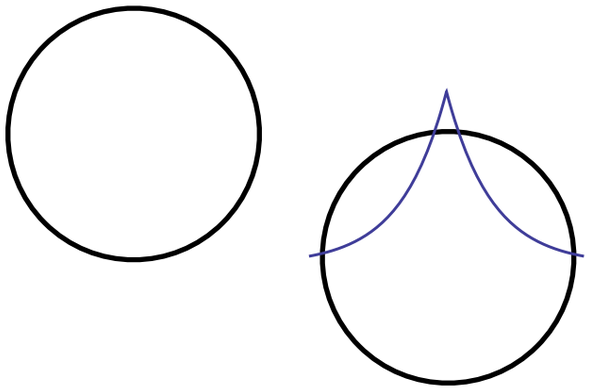
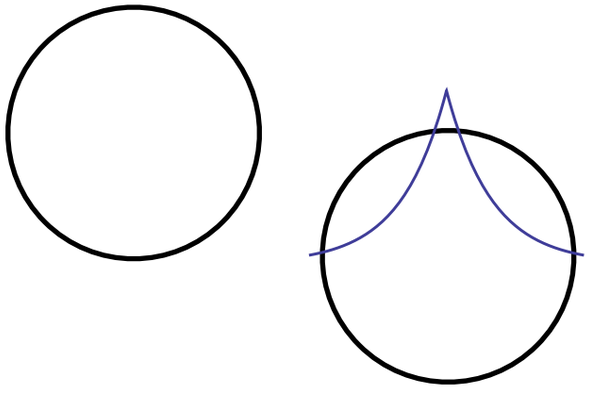


- **SCET has resummed global jet measures to high precision**

Universal nonperturbative shift: **CL**, Sterman (2007)
Factorization: Bauer, **CL**, Fleming, Sterman (2008)
Angularity event shapes: Hornig, **CL**, Ovanesyan (2009)

Recap: Jet Observables at Ever Finer Scales



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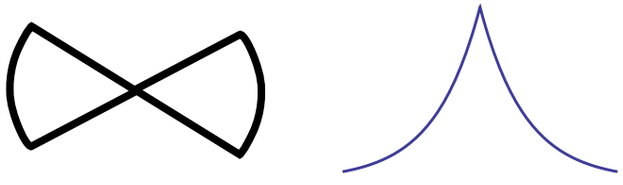

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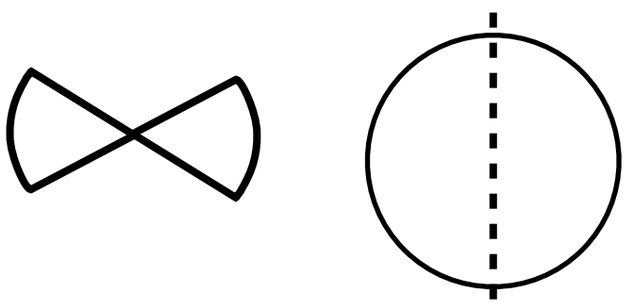
- **More exclusive jet observables sensitive to more scales, require additional resummations of logs**



In Progress:

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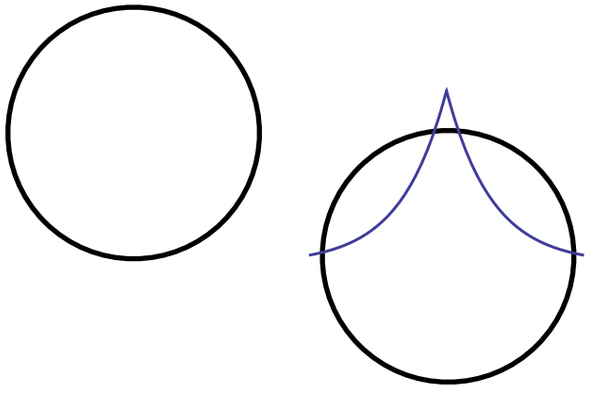
Recap: Jet Observables at Ever Finer Scales



- **SCET has resummed global jet measures to high precision**

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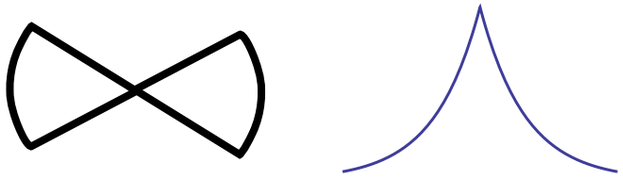

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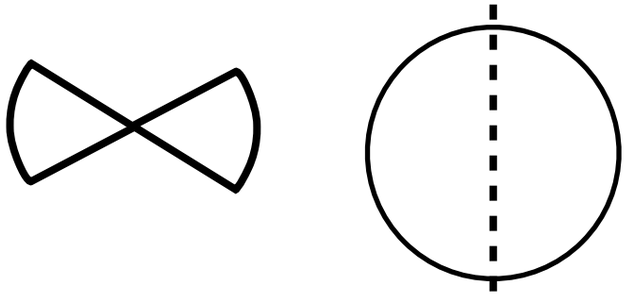

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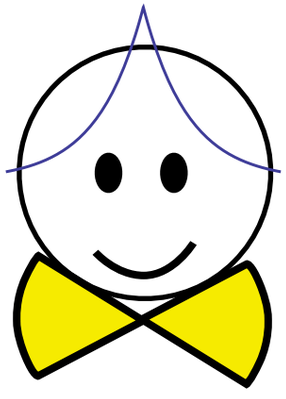

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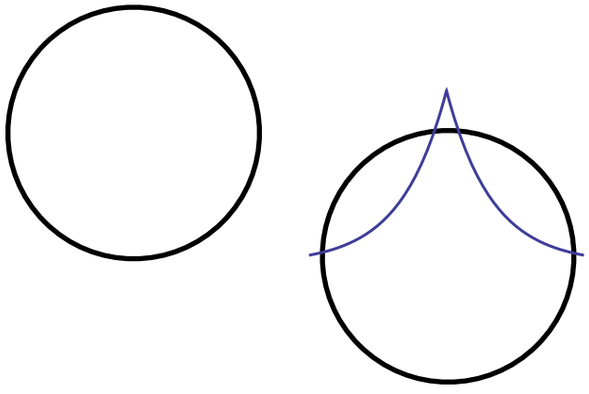

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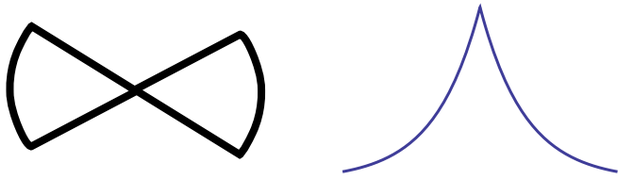

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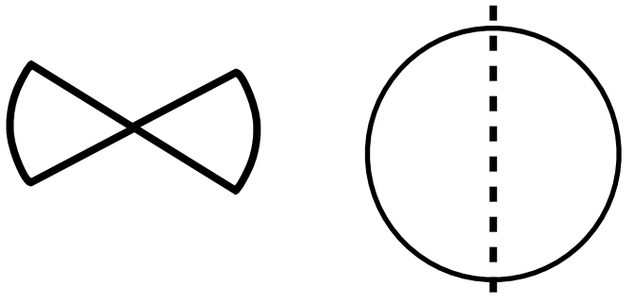

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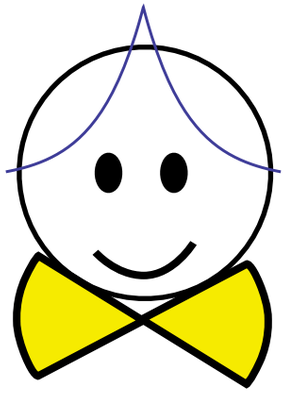


Future Projects:

quark vs. gluon discrimination

measures of subjet masses/shapes

light QCD vs. top, Higgs, BSM jets

accurate dependence on jet vetoes

Brief History and Motivation

- In SCET, DIS near the endpoint $x = 1$ has been studied extensively.

  Manohar; Becher, Neubert, Pecjak; Chay, Kim; Chen, Idilbi, Ji, Yuan; Fleming, Zhang

- Away from $x = 1$, the final state can have more than one jet.

- DIS event shapes:
  - In QCD, thrust to NLO, resummed to NLL. Antonelli, Dasgupta, Salam (1999)
  - In SCET, (one version of) 1-jettiness at NLL [now NNLL] Kang, Mantry, Qiu (2012) [Kang, Liu, Mantry, Qiu (yesterday)]

- e^+e^- event shapes have been resummed in SCET to N^3LL accuracy matched to N^3LO, leading to precise extraction of α_s Becher, Schwartz; Chien, Schwartz; Abbate, Fickinger, Hoang, Mateu, Stewart

- Extractions of α_s from DIS jet cross sections not yet at same precision.

- Theoretical tools now available for significant improvement of DIS event shape predictions and α_s extraction

Problems with Jet Cross Sections

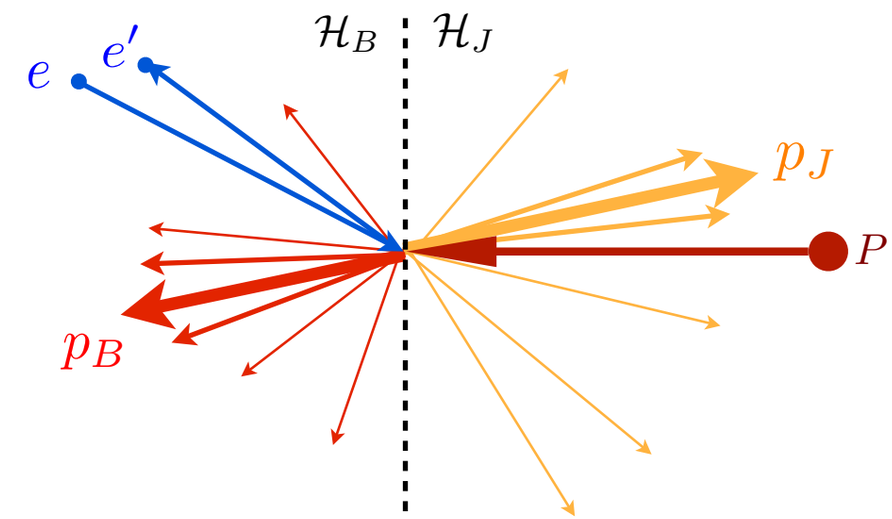
- Exclusive jet cross sections (fixed number of jets) typically depend on
  - choice of jet algorithm
  - jet sizes
  - jet vetoes
- These parameters generate a number of logarithms (NGLs, clustering logs, log R) in perturbation theory which we do not yet know how to resum
- ***N*-Jettiness**: an *inclusive* observable picking out *N*-jet final states by measurement of a single parameter, logs of which *can* be resummed in perturbation theory

DIS event shapes

Breit frame: $q^\mu = (0, 0, 0, Q)$

DIS thrust (review by Dasgupta & Salam '03)

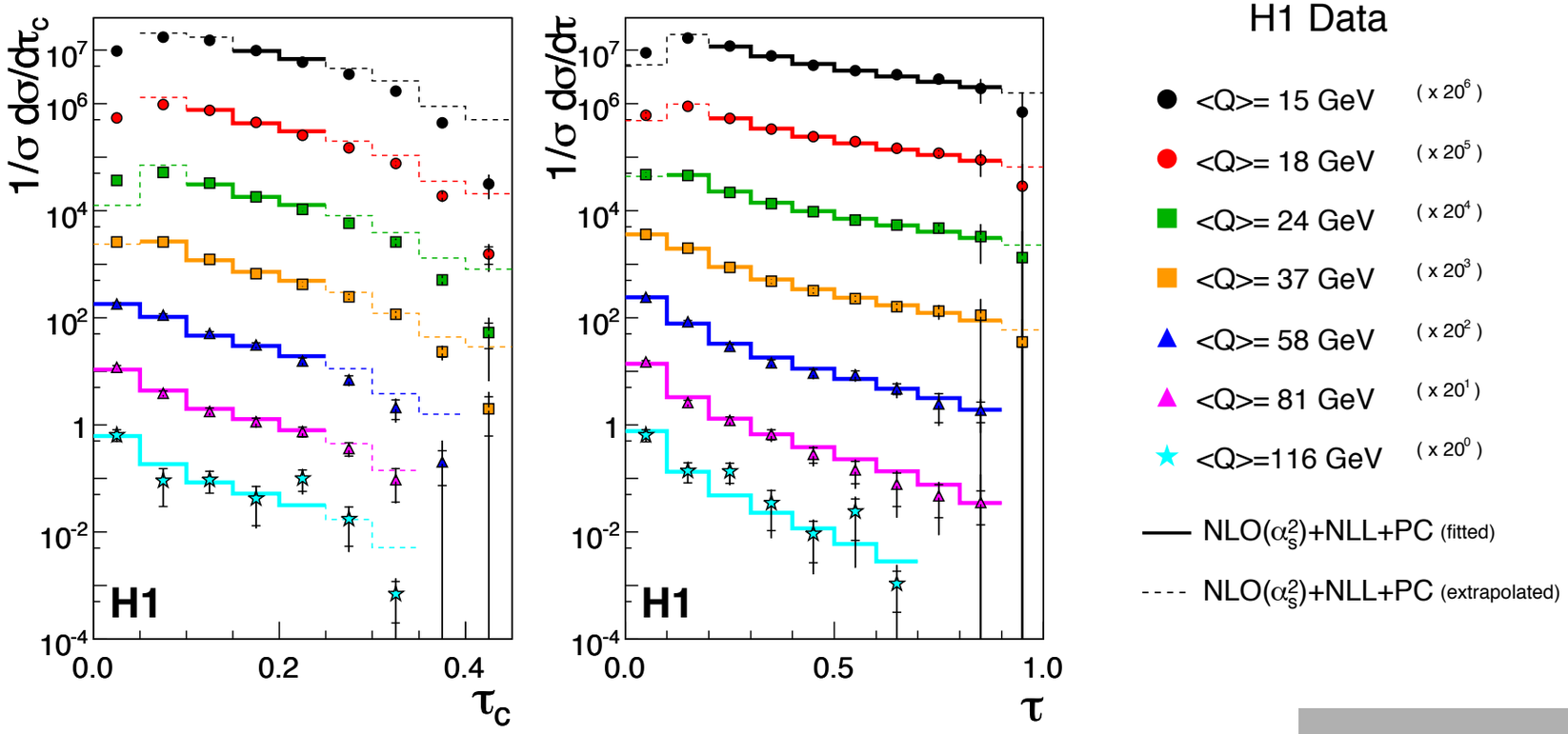
$$\tau_{nQ} = 1 - \frac{2}{Q} \sum_{i \in \mathcal{H}_J} |\vec{p}_i \cdot \vec{n}|$$

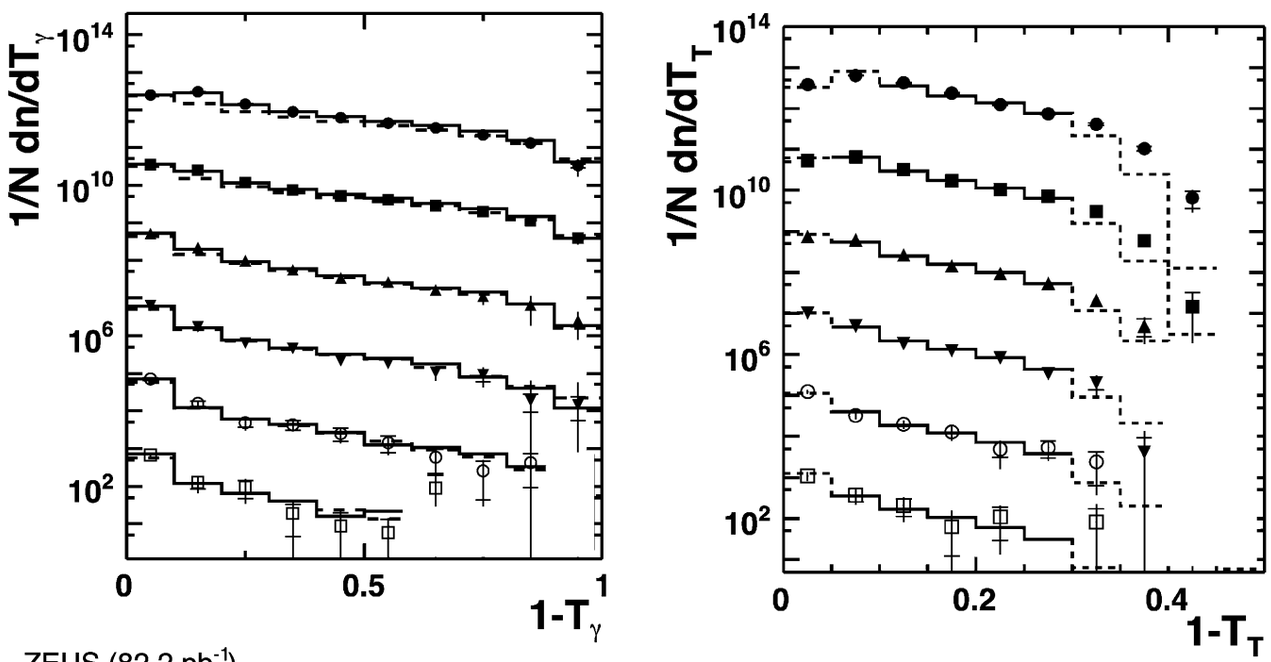


different versions:

- $\frac{Q}{2} \to \sum_{i \in \mathcal{H}_J} E_i$
- $\vec{n} = \hat{z}$ fixed to photon/weak boson's axis τ
- vary $\vec{n}$ to minimize τ_{nQ} τ_C

calculations to NLO & NLL Dasgupta, Salam, ...

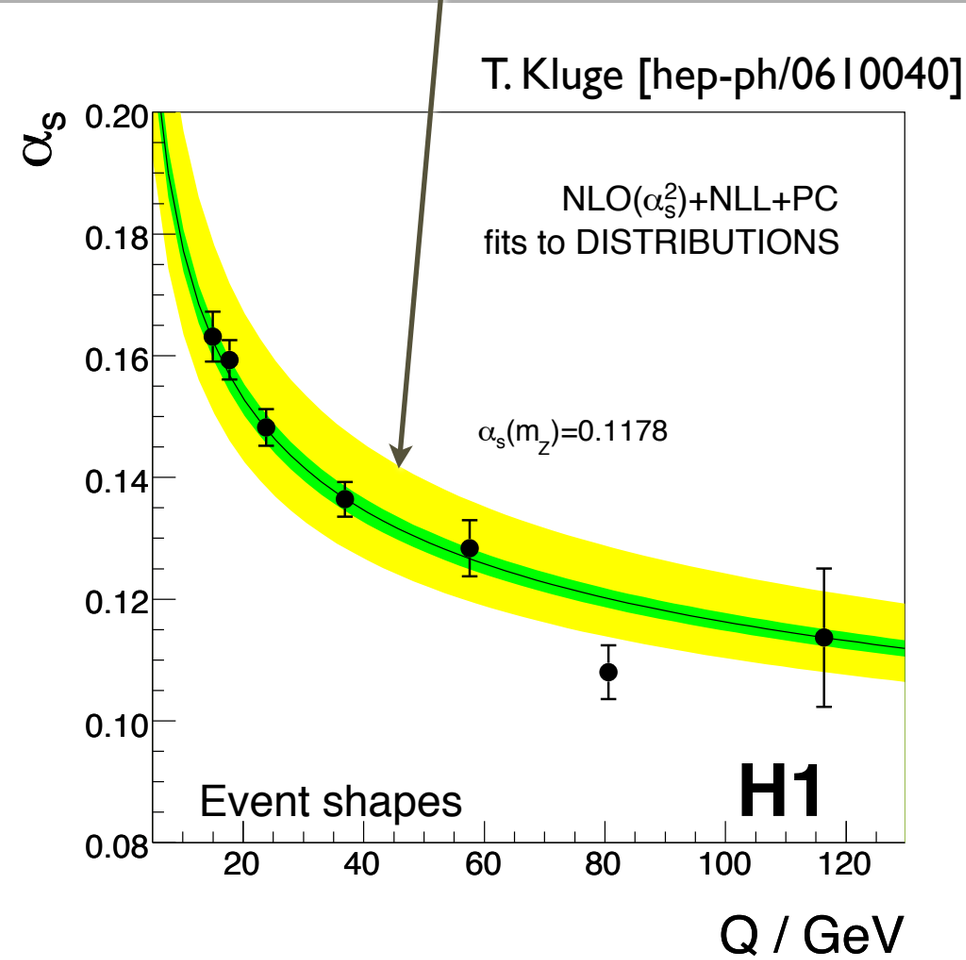




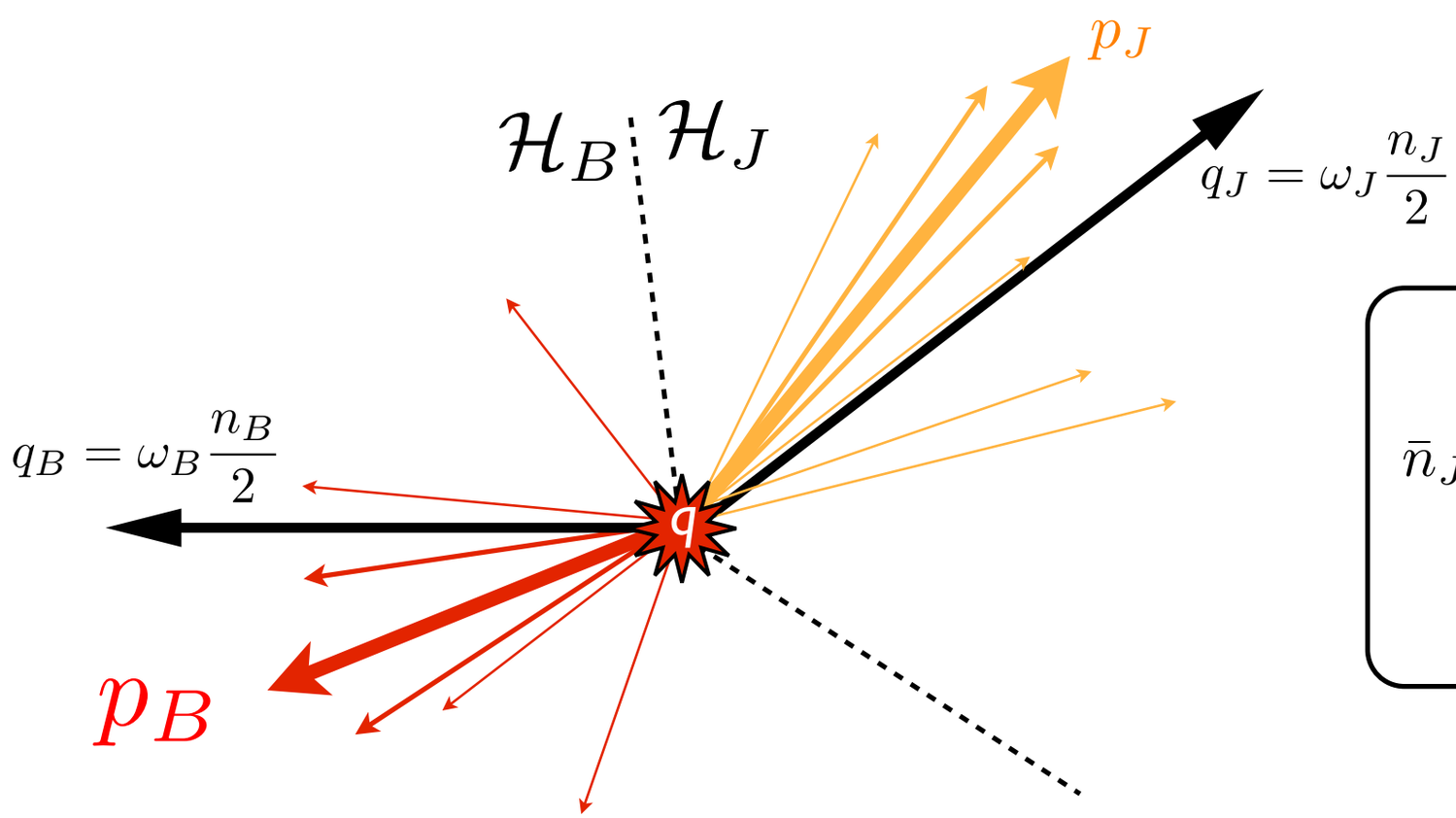
ZEUS (82.2 pb^{-1})
- <Q> = 21 GeV
- <Q> = 29 GeV
- <Q> = 42 GeV
- <Q> = 59 GeV
- <Q> = 82 GeV
- <Q> = 113 GeV
- ARIADNE Hadrons
- ARIADNE Partons

Higher precision possible?

perturbative uncertainty



Light-Cone Directions



Choose conjugate directions:

$$\bar{n}_J = \frac{2}{n_J \cdot n_B} n_B \qquad \bar{n}_B = \frac{2}{n_J \cdot n_B} n_J$$

$$\Rightarrow n_J \cdot \bar{n}_J = n_B \cdot \bar{n}_B = 2$$

Sizes of beam and jet regions:

$$\frac{n_J \cdot p}{\bar{n}_J \cdot p} < \frac{\omega_B}{\omega_J} \frac{n_J \cdot n_B}{2} \equiv R_J^2$$

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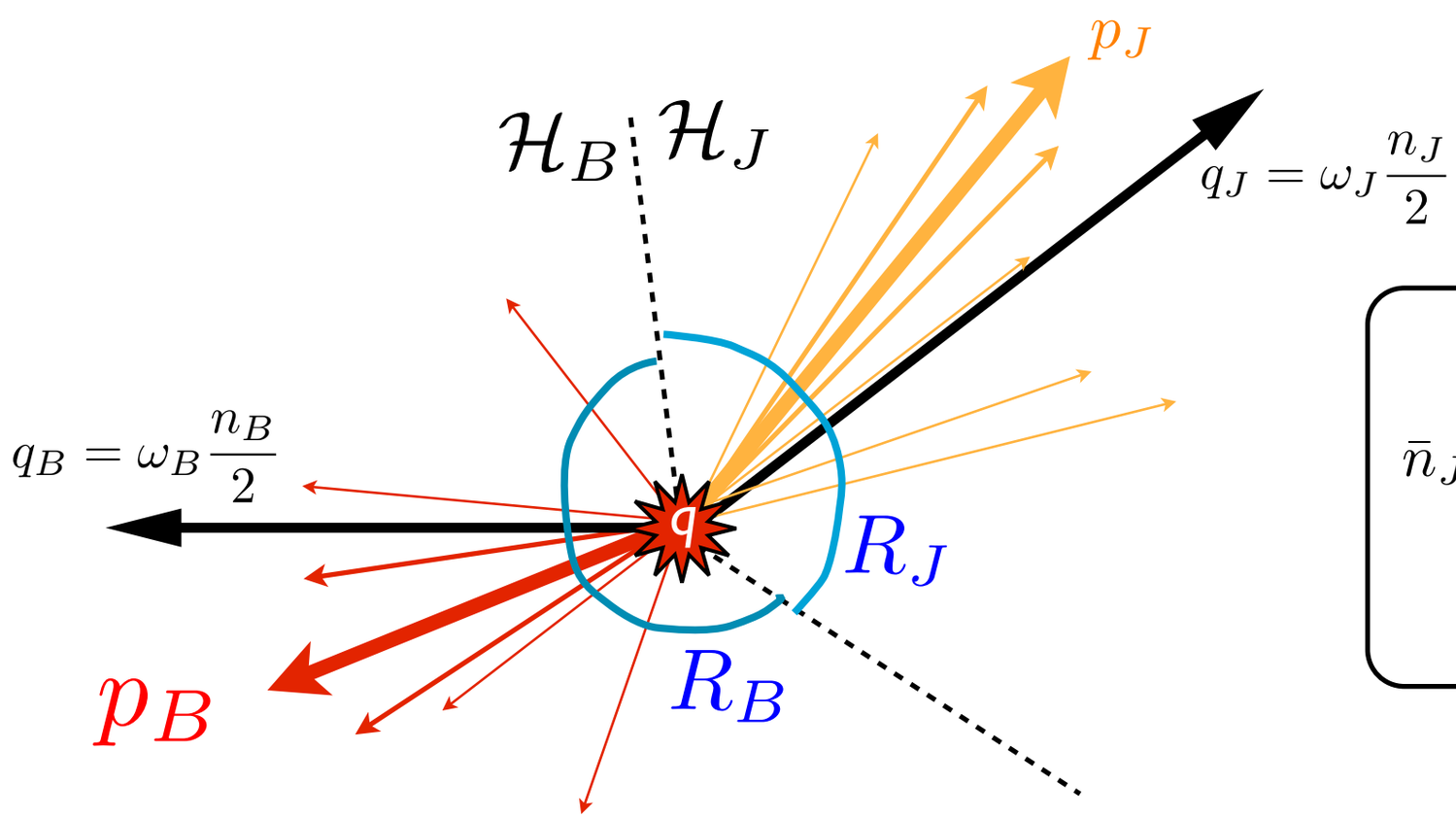

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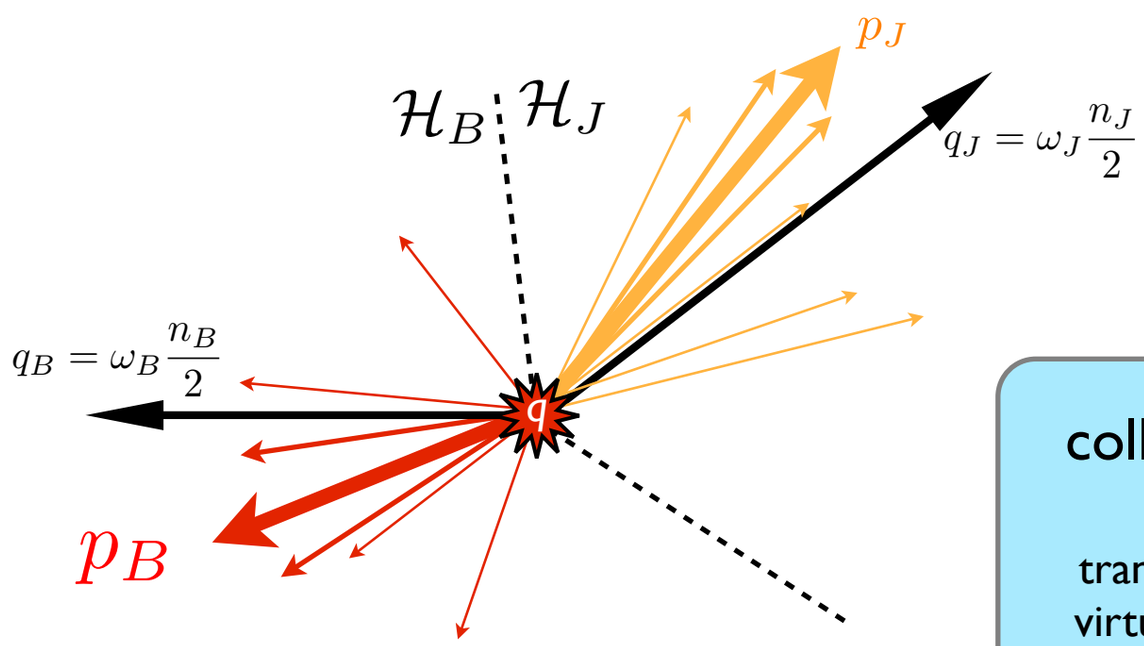
Beam, Jet, and Soft Contributions



In each case of 1-jettiness, $\tau_1 = \frac{n_J \cdot p_J}{Q_J} + \frac{n_B \cdot p_B}{Q_B}$

contributions from different modes: $\tau_1 = \tau_J^{n_J} + \tau_B^{n_B} + \tau_S$

collinear contributions: $\tau_J^{n_J} = \frac{t_J}{s_J}\,,\ \tau_B^{n_B} = \frac{t_B}{s_B}$

transverse virtualities: $t_J = \bar{n}_J \cdot p_J\, n_J \cdot p_J^{n_J}$ $\quad t_B = \bar{n}_B \cdot p_B\, n_B \cdot p_B^{n_B}$

soft contribution: $\tau_S = \frac{n_J \cdot k_J}{Q_J} + \frac{n_B \cdot k_B}{Q_B} \longrightarrow \frac{n'_J \cdot k_J + n'_B \cdot k_B}{Q_R}$

soft boost invariance: $n_{J,B} \longrightarrow n'_{J,B} = \frac{n_{J,B}}{R_{J,B}}$ $\quad Q_R \equiv \frac{Q_J}{R_J} = \frac{Q_B}{R_B}$

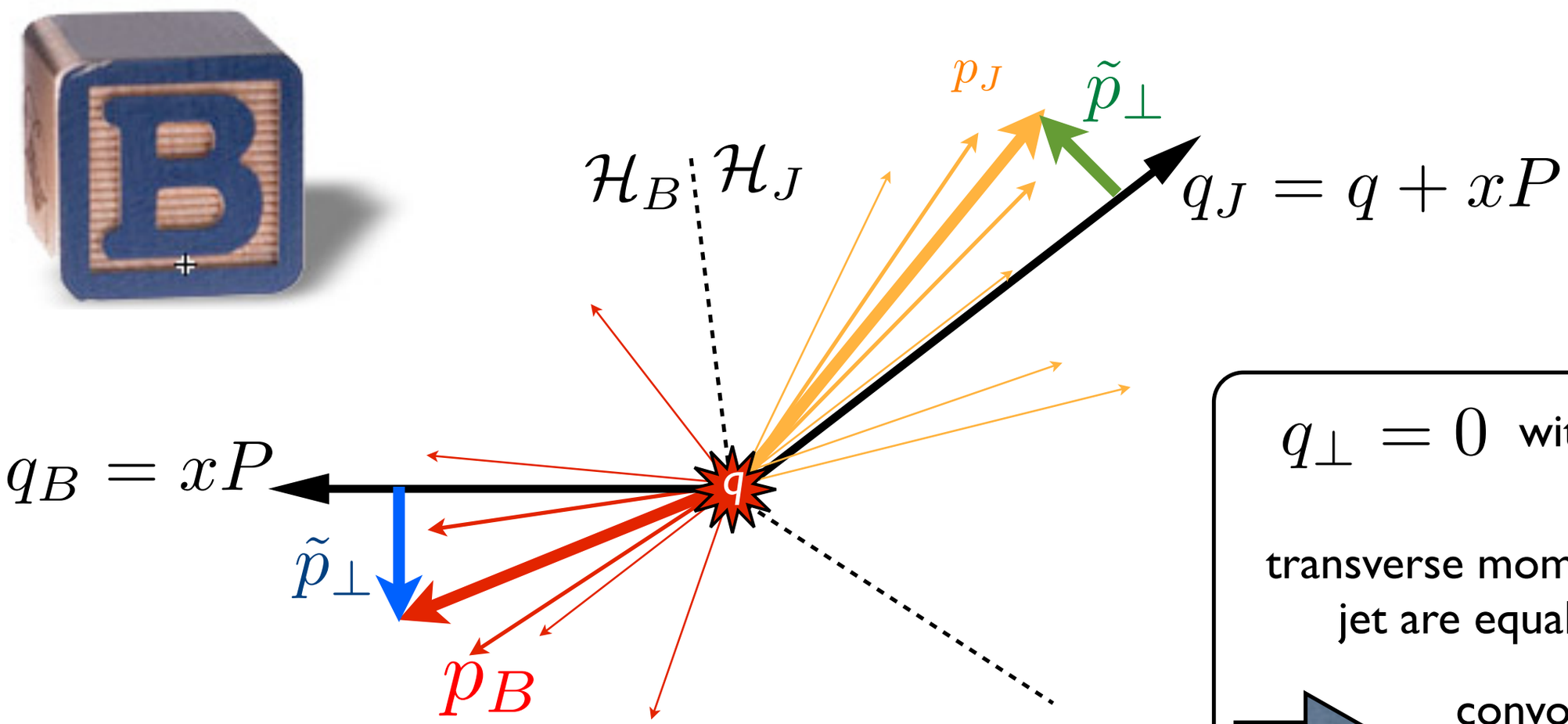
Lorentz invariant constants:

	s_J	s_B	Q_R
τ_1^a	Q^2	Q^2	Q
τ_1^b	Q^2	Q^2	Q
τ_1^c	Q^2	xQ^2	$\sqrt{x}\,Q$

Transverse jet and beam momenta

Convolution between jet and beam transverse momenta:

$$W \supset \int d^2\tilde{p}_\perp \langle P_{n_B}|\bar{\chi}_{n_B}(0)\delta(Q_B\tau_B - n_B\cdot\hat{p}^{n_B})[\delta(\bar{n}_B\cdot q + \bar{n}_B\cdot\mathcal{P})\delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)\chi_{n_B}](0)|P_{n_B}\rangle$$
$$\times \langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\delta^2(q_\perp + \tilde{p}_\perp + \mathcal{P}_\perp)\bar{\chi}_{n_J}(0)|0\rangle$$



$q_\perp = 0$ with respect to this $\boldsymbol{q_J}$ and $\boldsymbol{q_B}$

transverse momenta of beam and jet are equal and balanced

→ convolution over $\tilde{p}_\perp$ remains, depends on kT-dependent beam function (perturbative kT)

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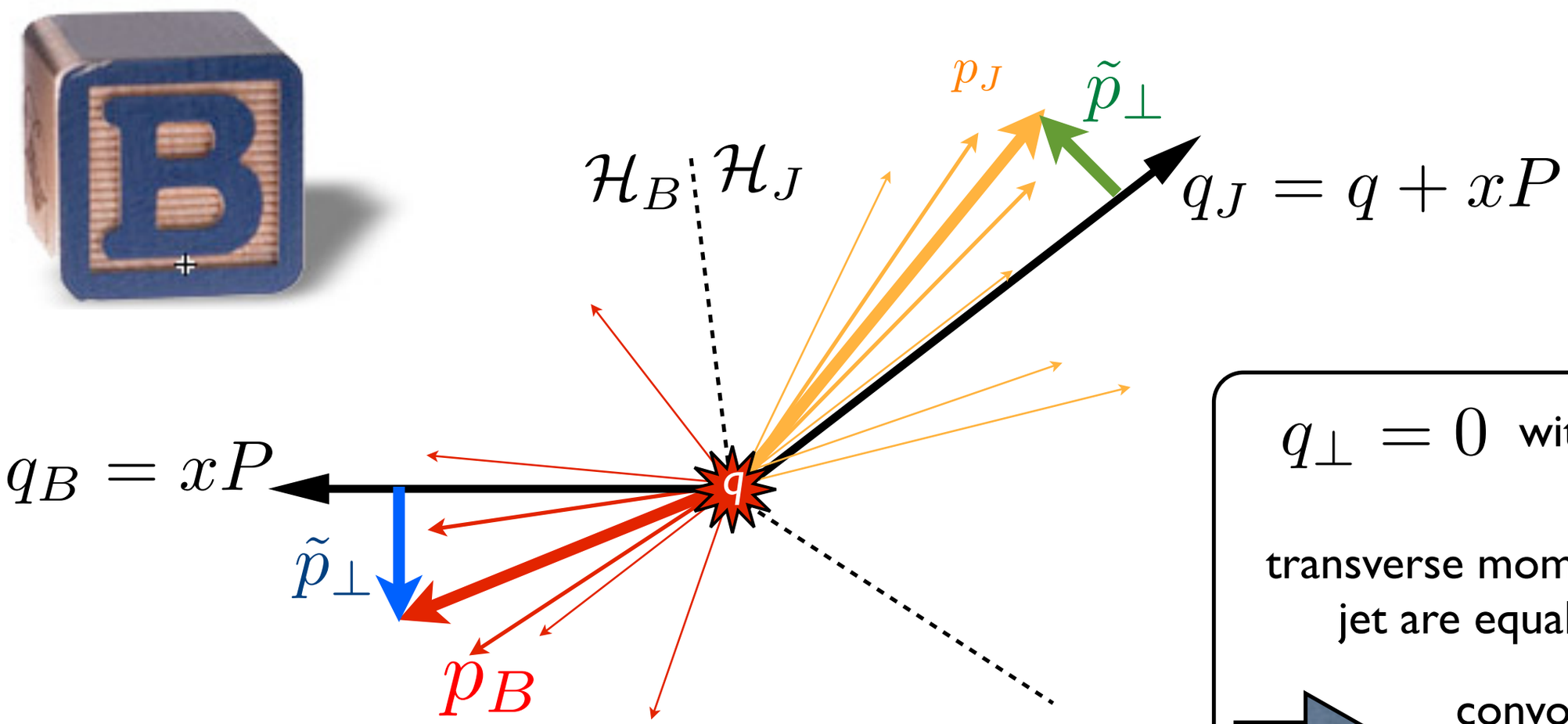

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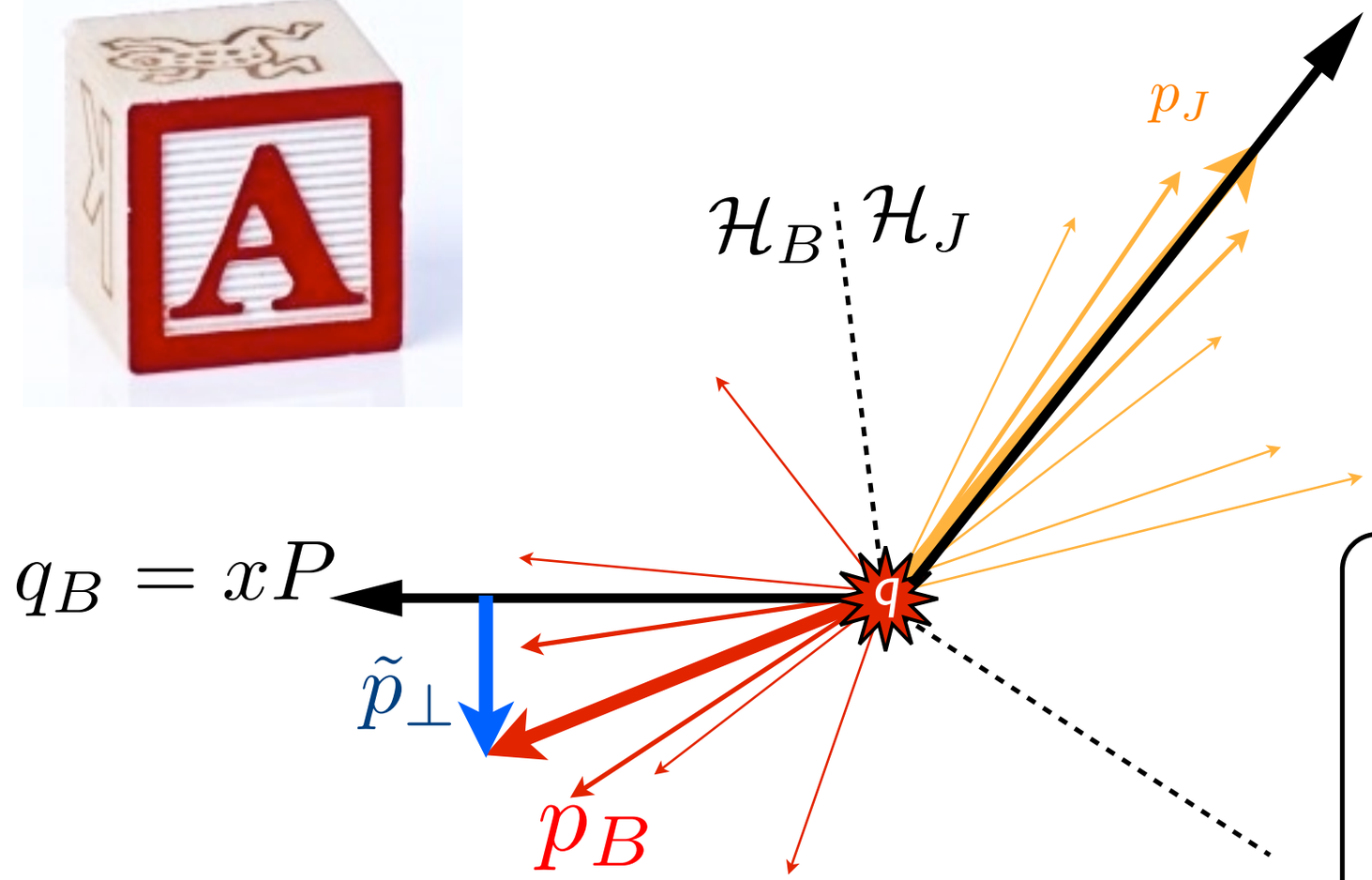

$\Rightarrow n_J$ aligned so that transverse label momentum on χ_{n_J} is zero

jet function independent of beam $\tilde{p}_\perp$

integral averages over beam $\tilde{p}_\perp$

turns into ordinary beam function

difference between q_J axes for case A and B is a leading-order effect on the argument of beam and jet functions

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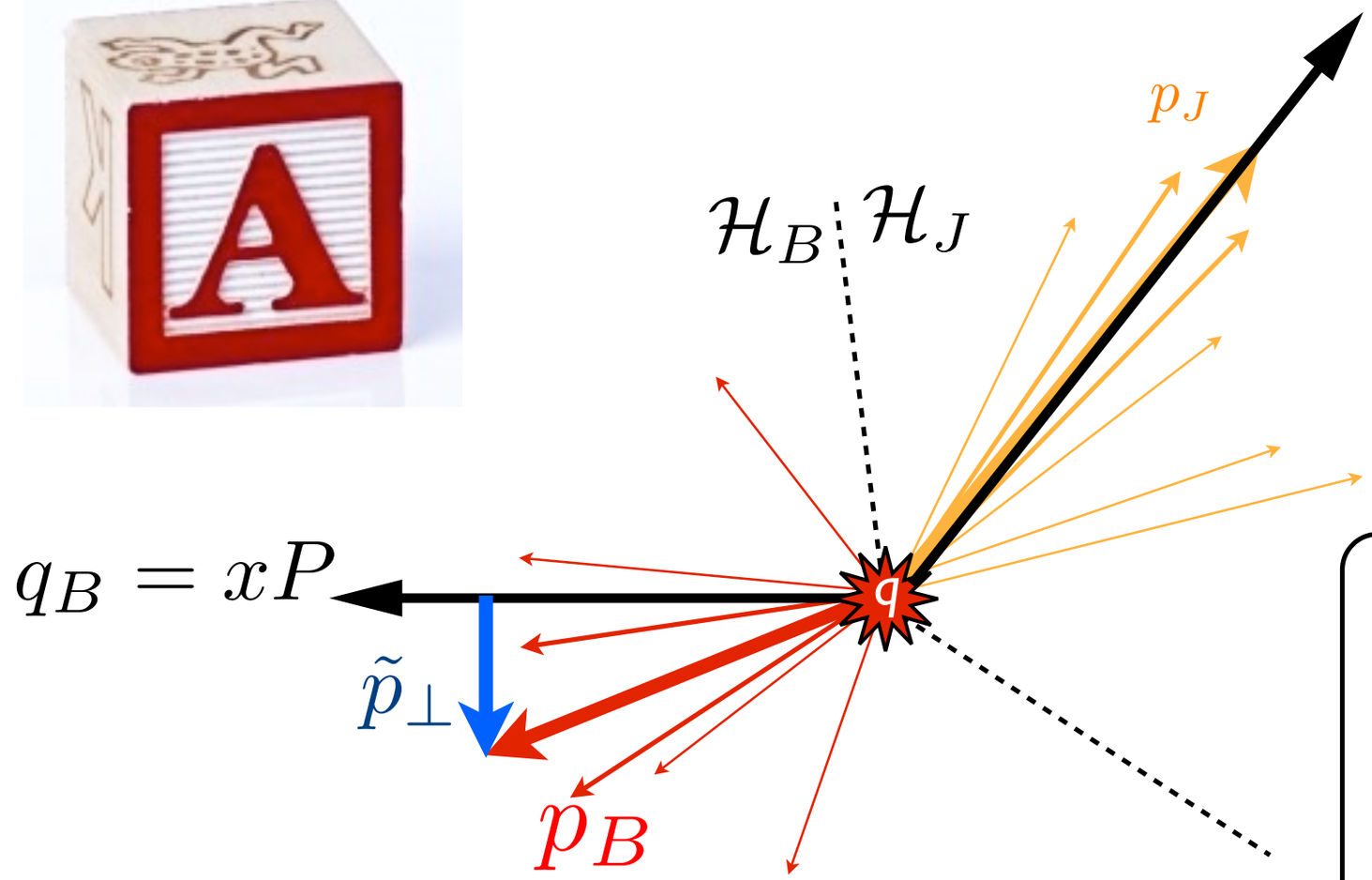

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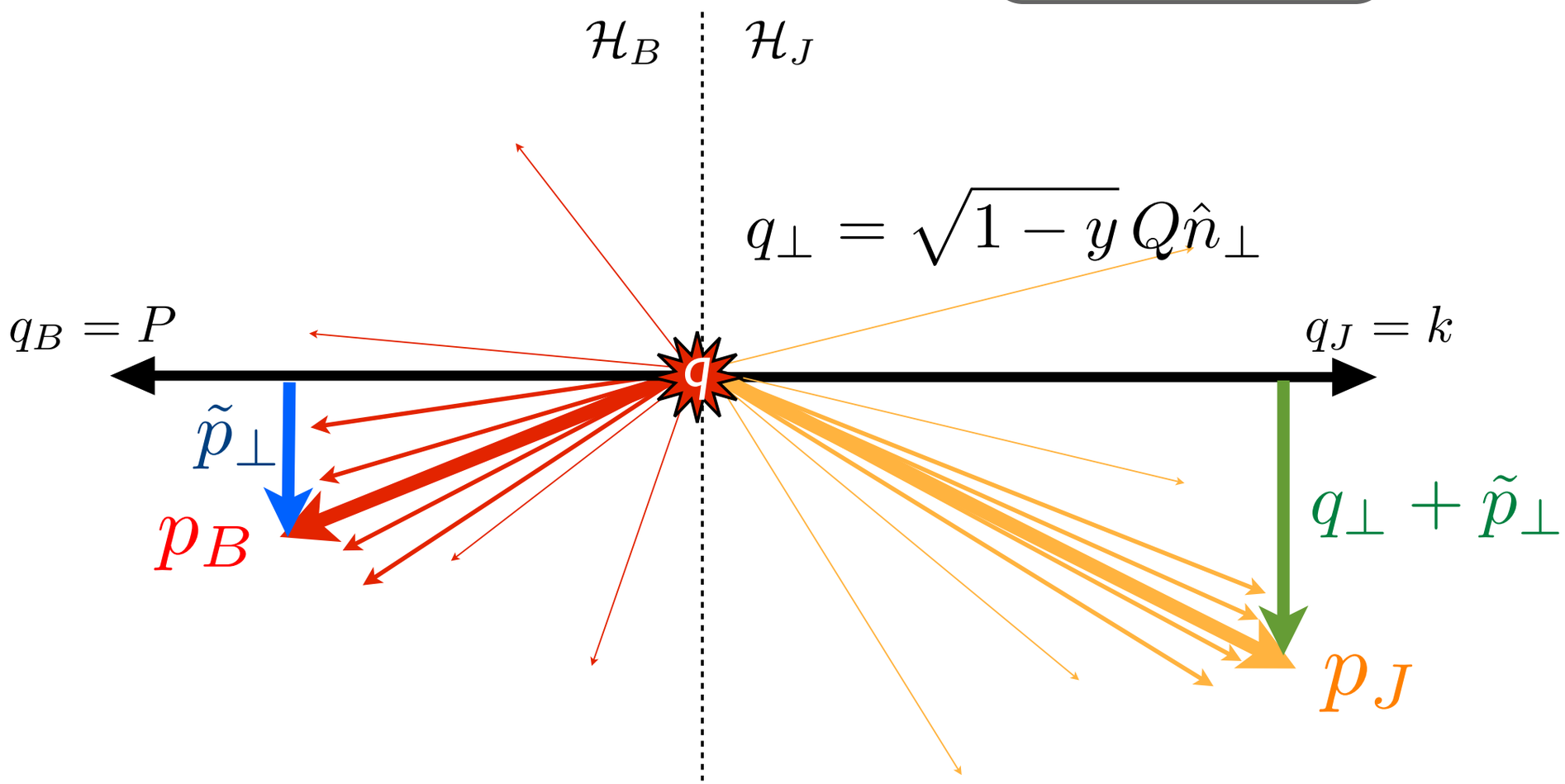


momentum transfer ***q*** itself has nonzero transverse component relative to ***P, k*** → nontrivial convolution between jet function and pT-dependent beam function

Factorization Theorem for 1-Jettiness

$$\frac{d\sigma(x,Q^2)}{d\tau_1} = L_{\mu\nu}(x,Q^2)W^{\mu\nu}(x,Q^2,\tau_1)$$

Match onto 2-jet operators in SCET:

$$W_{\mu\nu}(x,Q^2,\tau_1) = \int d^4x\, e^{iq\cdot x} \sum_{n_1,n_2} \int d^3\tilde{p}_1 d^3\tilde{p}_2 e^{i(\tilde{p}_2-\tilde{p}_1)\cdot x} C^*_\mu(\tilde{p}_1,\tilde{p}_2) C_\mu(\tilde{p}_1,\tilde{p}_2)$$
$$\times \langle P_{n_B}|\bar{\chi}_{n_2,\tilde{p}_2}(x)\overline{T}[Y^\dagger_{n_2}(x)Y_{n_1}(x)]\chi_{n_1,\tilde{p}_1}(x)$$
$$\times\, \delta(\tau_1 - \hat{\tau}_1^{n_1} - \hat{\tau}_1^{n_2} - \tau_1^s)$$
$$\times\, \bar{\chi}_{n_1,\tilde{p}_1}(0)T[Y^\dagger_{n_1}(0)Y_{n_2}(0)]\chi_{n_2,\tilde{p}_2}(0)|P_{n_B}\rangle$$

collinear jet operators in SCET

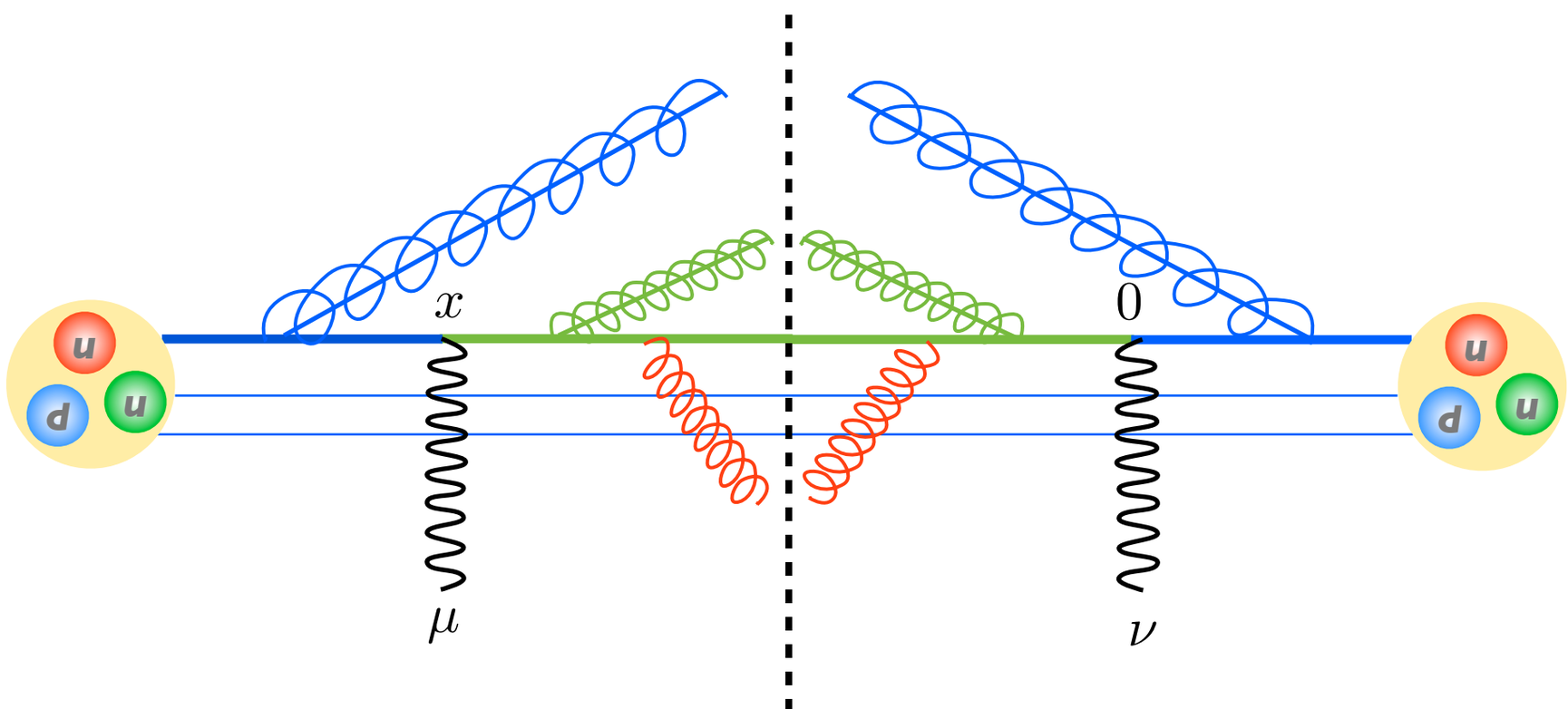
$$\chi_n = [W_n \xi_n]$$

collinear Wilson line

collinear quark field

soft gluon Wilson lines

$$Y_{n_{1,2}}$$



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$$\times\, \bar{\chi}_{n_1,\tilde{p}_1}(0)T[Y^\dagger_{n_1}(0)Y_{n_2}(0)]\chi_{n_2,\tilde{p}_2}(0)|P_{n_B}\rangle$$

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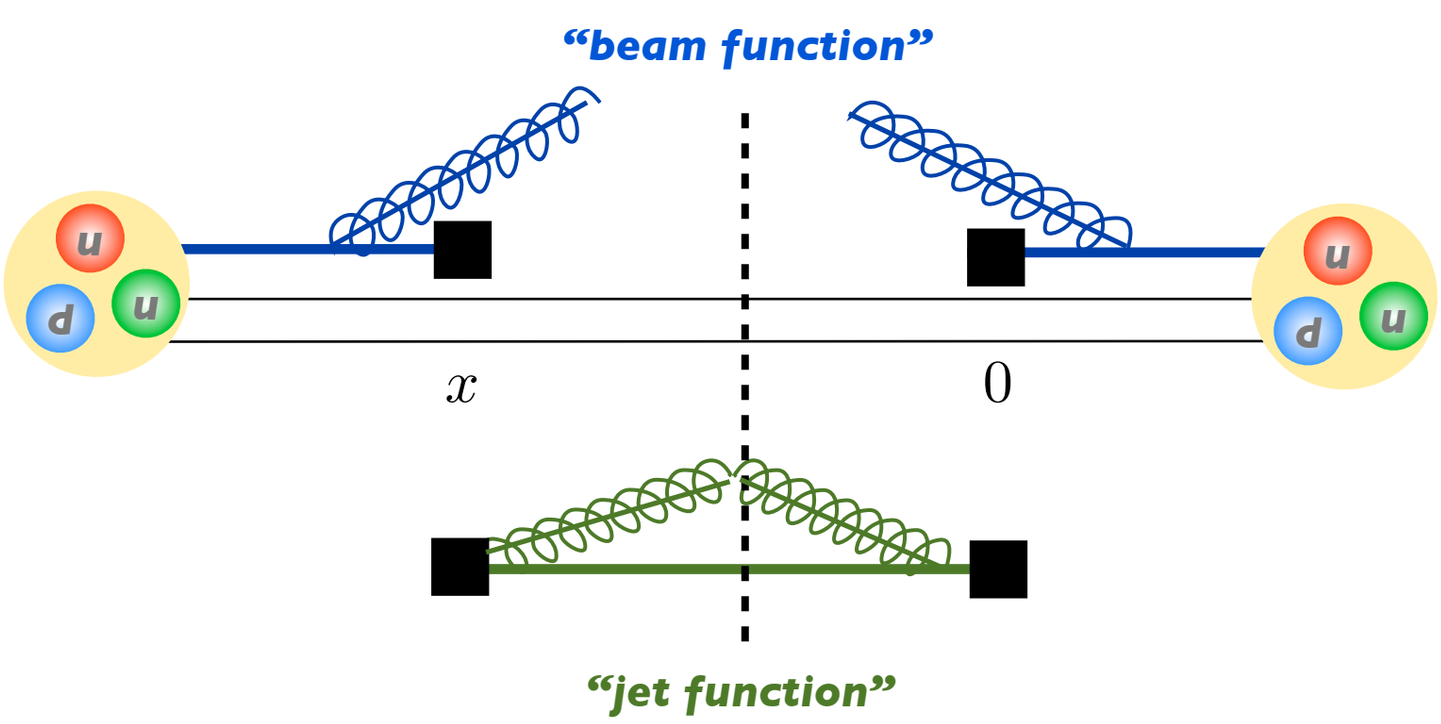
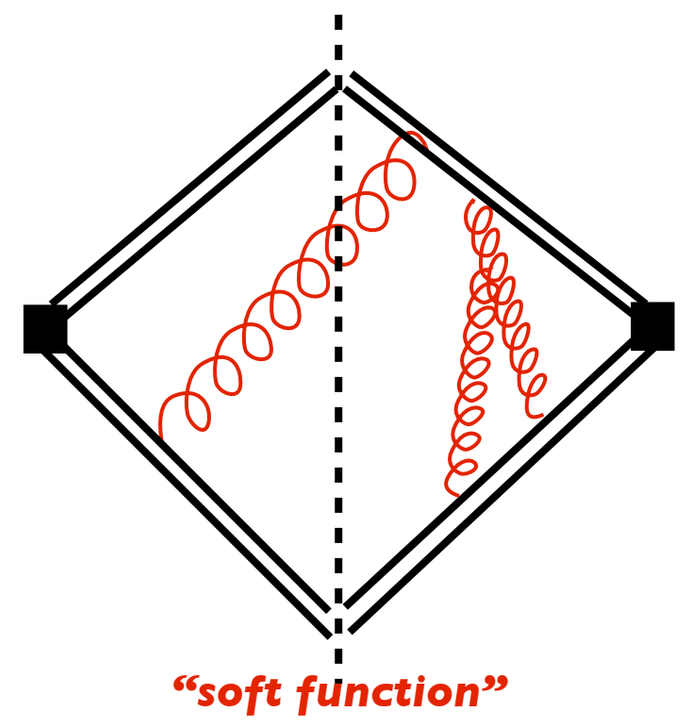
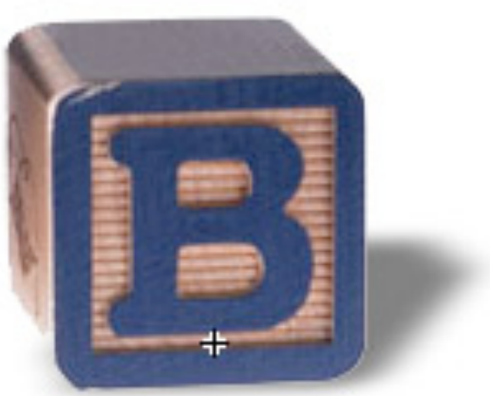




Differences between versions A and B

$$n_J^A = n_J^B + \mathcal{O}(\lambda)$$

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^a} = H(Q^2,\mu)\int dt_J dt_B dk_S \delta\left(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right)$$
$$\times\, J_q(t_J,\mu) B_q(t_B,x,\mu) S(k_S,\mu)$$



$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\left(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right)$$
$$\times\, J_q(t_J - \mathbf{p}_\perp^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

Differences between versions A and B

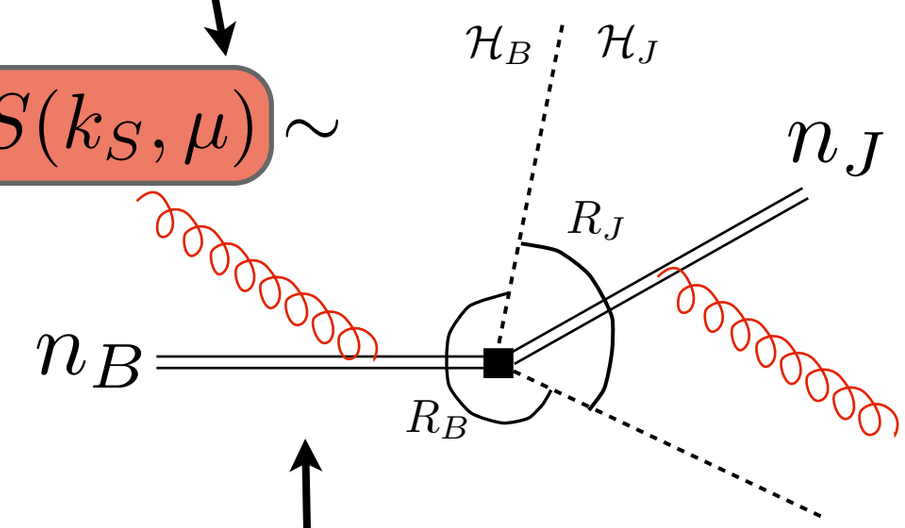
$$n_J^A = n_J^B + \mathcal{O}(\lambda)$$

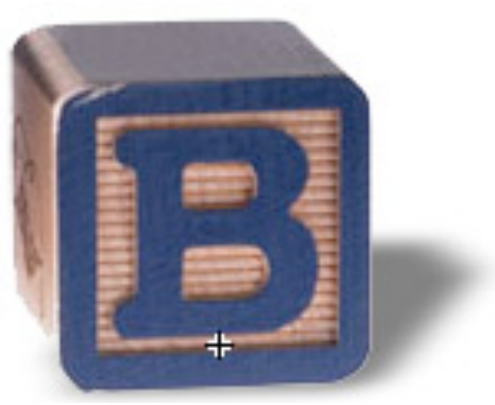
$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^a} = H(Q^2,\mu)\int dt_J dt_B dk_S \delta\left(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right) \times J_q(t_J,\mu) B_q(t_B,x,\mu) S(k_S,\mu)$$

Differences **power suppressed:**

$$H(Q^2,\mu) = |C(Q^2,\mu)|^2 \,\mathrm{Tr}\left(\Gamma \frac{\not{n}_J}{4} \Gamma' \frac{\not{n}_B}{4}\right)$$

$$S(k_S,\mu) \sim$$





$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\left(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right) \times J_q(t_J - \mathbf{p}_\perp^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

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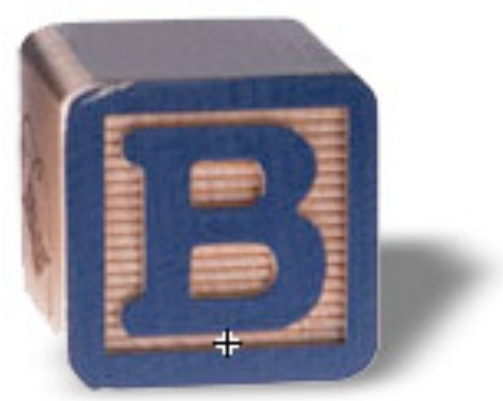
$$\times J_q(t_J,\mu) B_q(t_B,x,\mu) S(k_S,\mu)$$

Differences
leading order:

argument of jet function

type of beam function



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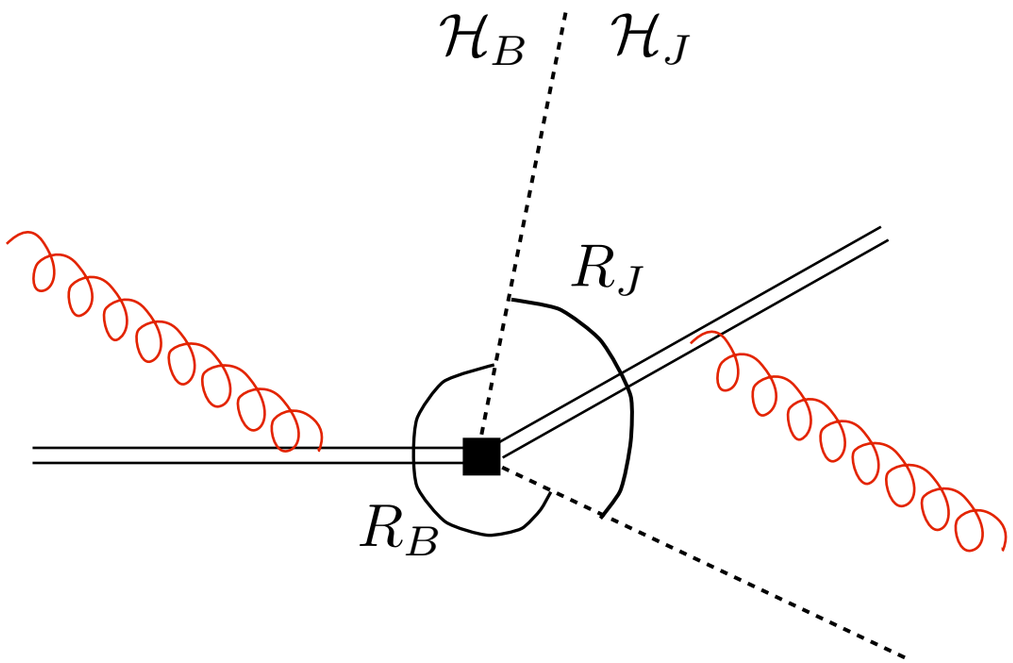
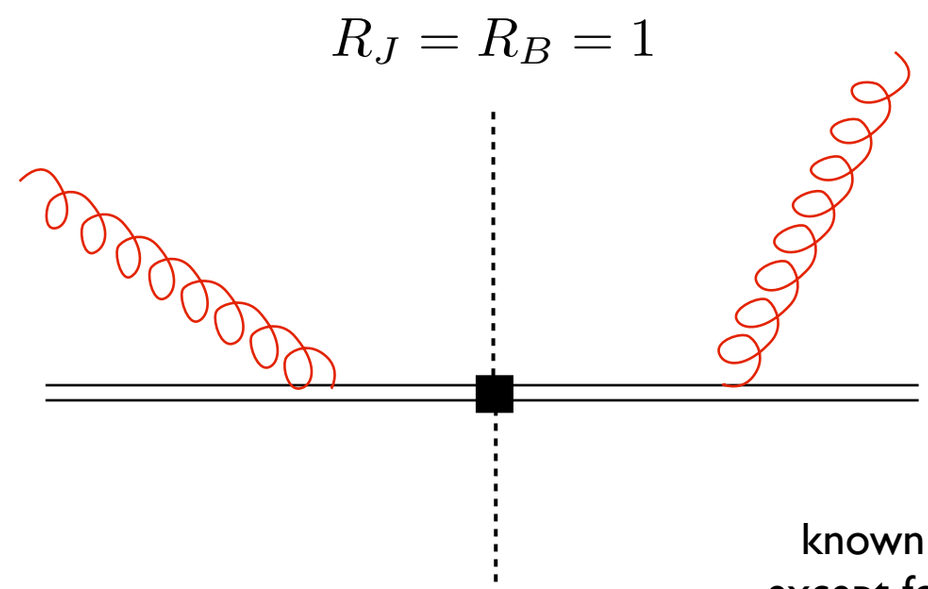
$$\times J_q(t_J - \mathbf{p}_\perp^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

Boost to Hemisphere Soft Function



1-jettiness soft function



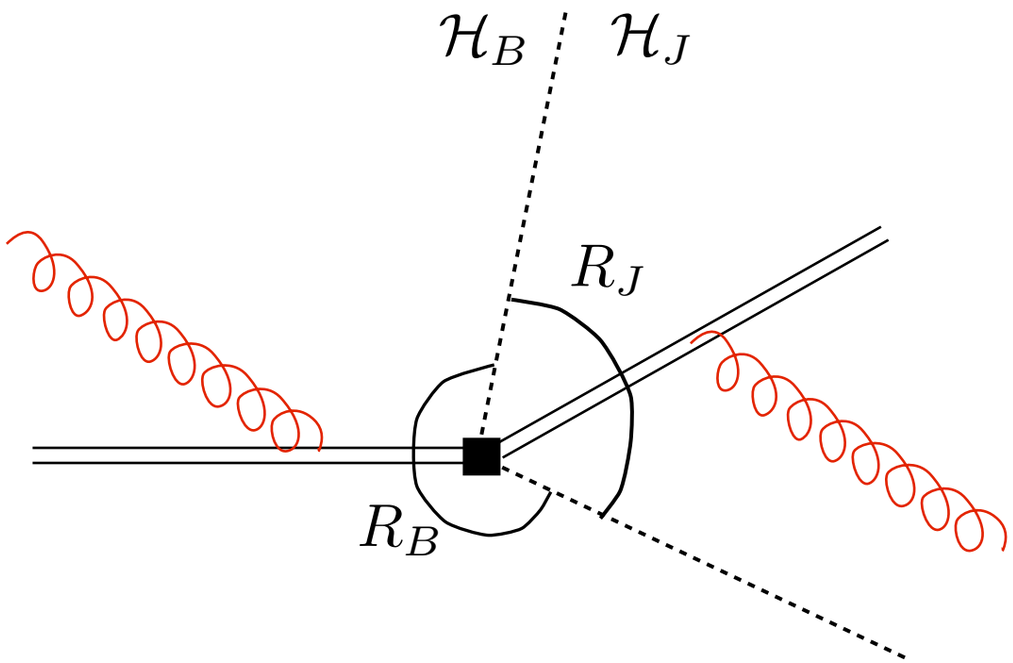
hemisphere soft function

known to 2 loops except for constant

anomalous dimension known to 3 loops

$$S(k_J, k_B, q_J, q_B, \mu) = \frac{1}{N_C} \operatorname{tr} \sum_{X_s} \left| \langle X_s | \, T[Y^\dagger_{n_B} Y_{n_J}](0) \, |0\rangle \right|^2 \delta\Big(k_J - \sum_{i \in X_s} \theta(q_B \cdot k_i - q_J \cdot k_i) n_J \cdot k_i\Big)$$
$$\times \, \delta\Big(k_B - \sum_{i \in X_s} \theta(q_J \cdot k_i - q_B \cdot k_i) n_B \cdot k_i\Big)$$

Boost to Hemisphere Soft Function



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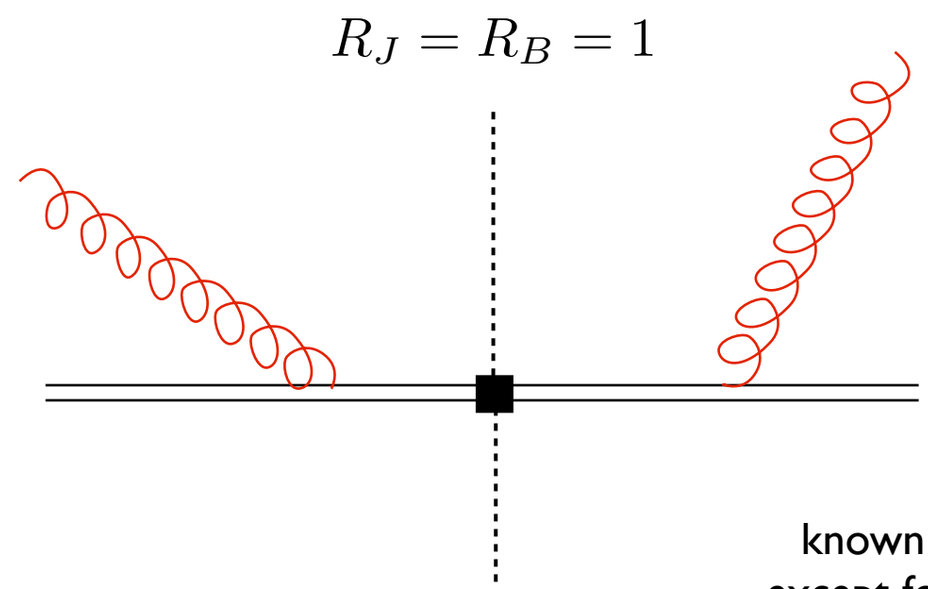

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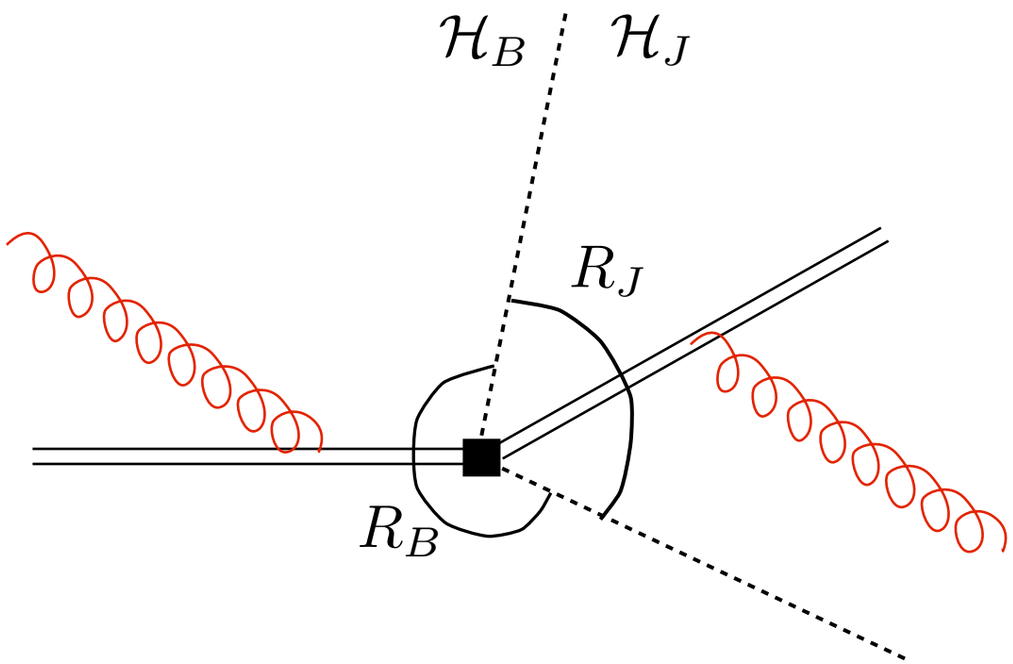
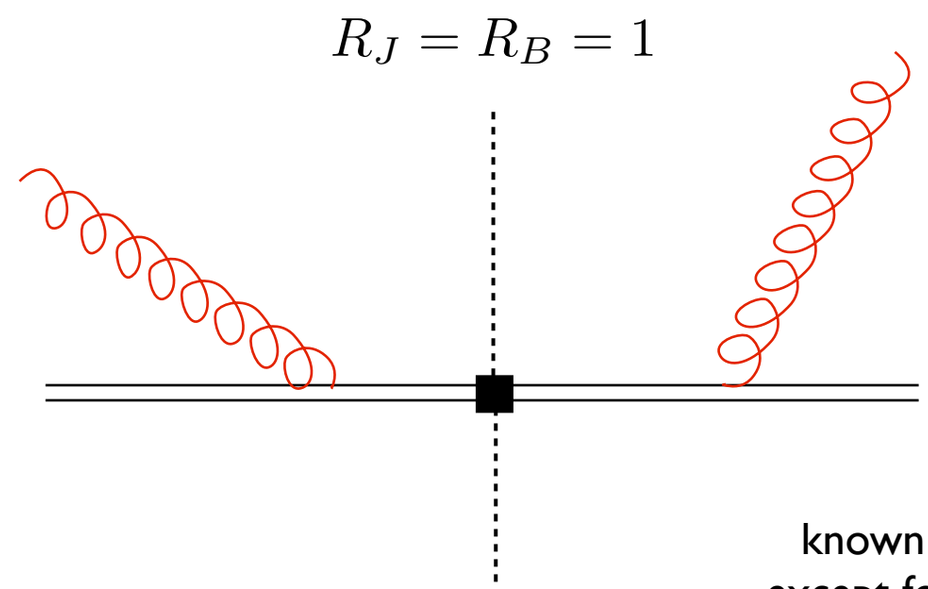
$$\times\, \delta\Big(k_B - \sum_{i \in X_s} \theta(q_J \cdot k_i - q_B \cdot k_i) n_B \cdot k_i\Big)$$

$$n_J \to n'_J = \frac{n_J}{R_J} \qquad R_J = \sqrt{\frac{\omega_B n_J \cdot n_B}{2\omega_J}}$$
$$n_B \to n'_B = \frac{n_B}{R_B} \qquad R_B = \sqrt{\frac{\omega_J n_J \cdot n_B}{2\omega_B}}$$

Boost to Hemisphere Soft Function



1-jettiness soft function



hemisphere soft function

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$$S(k_J, k_B, q_J, q_B, \mu) = \frac{1}{N_C R_J R_B} \operatorname{tr} \sum_{X_s} \left| \langle X_s | \, T[Y^\dagger_{n'_B} Y_{n'_J}](0) \, |0\rangle \right|^2 \delta\Big(\frac{k_J}{R_J} - \sum_{i \in X_s} \theta(n'_B \cdot k_i - n'_J \cdot k_i) n'_J \cdot k_i \Big)$$
$$\times \, \delta\Big(\frac{k_B}{R_B} - \sum_{i \in X_s} \theta(n'_J \cdot k_i - n'_B \cdot k_i) n'_B \cdot k_i \Big) = \frac{1}{R_J R_B} S_{\rm hemi}\Big(\frac{k_J}{R_J}, \frac{k_B}{R_B}, \mu \Big) \, .$$

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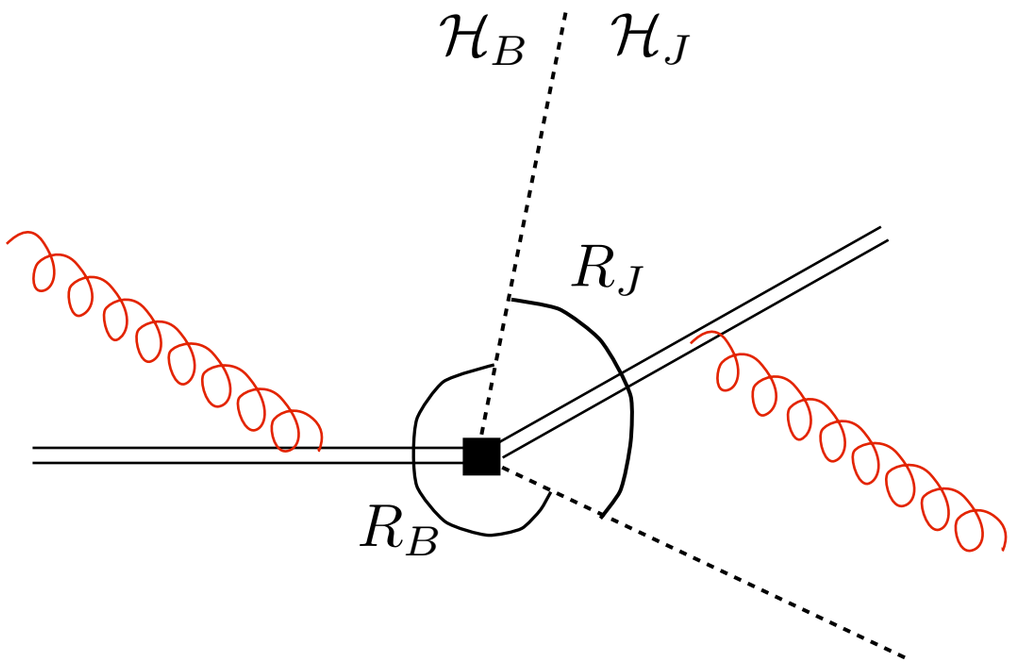
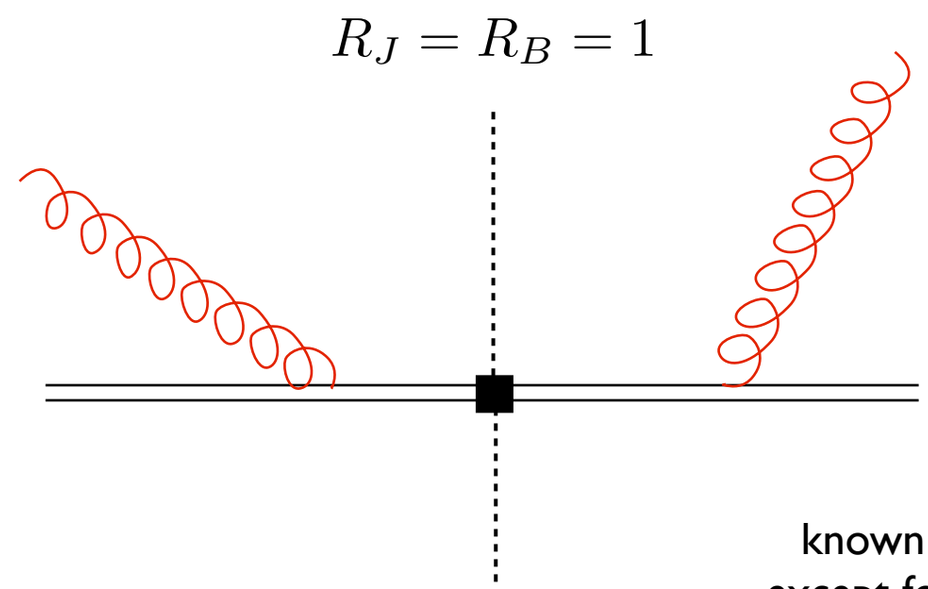
Boost to Hemisphere Soft Function



1-jettiness soft function



hemisphere soft function

known to 2 loops except for constant

anomalous dimension known to 3 loops

$$S(k_J, k_B, q_J, q_B, \mu) = \frac{1}{N_C R_J R_B} \operatorname{tr} \sum_{X_s} \left| \langle X_s | T[Y^\dagger_{n'_B} Y_{n'_J}](0) | 0 \rangle \right|^2 \delta\Big(\frac{k_J}{R_J} - \sum_{i \in X_s} \theta(n'_B \cdot k_i - n'_J \cdot k_i) n'_J \cdot k_i \Big)$$
$$\times \delta\Big(\frac{k_B}{R_B} - \sum_{i \in X_s} \theta(n'_J \cdot k_i - n'_B \cdot k_i) n'_B \cdot k_i \Big) = \frac{1}{R_J R_B} S_{\rm hemi}\Big(\frac{k_J}{R_J}, \frac{k_B}{R_B}, \mu \Big) .$$

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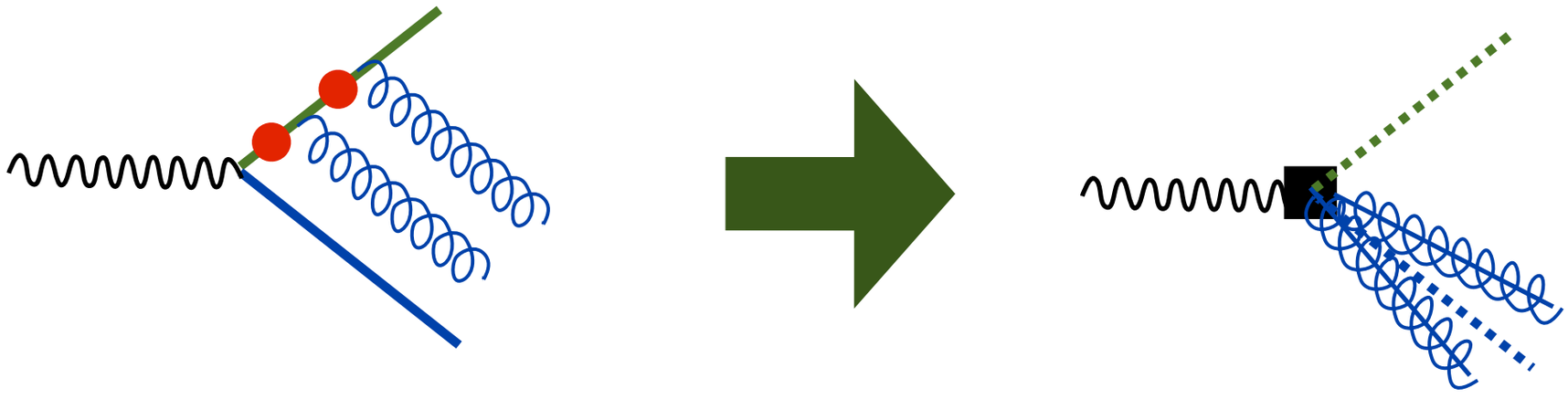
$$n_B \to n'_B = \frac{n_B}{R_B} \qquad R_B = \sqrt{\frac{\omega_J n_J \cdot n_B}{2\omega_B}}$$

$$S(k_J, k_B, R_J, R_B, \mu) = \frac{1}{R_J R_B} S_{\rm hemi}\Big(\frac{k_J}{R_J}, \frac{k_B}{R_B}, \mu \Big)$$

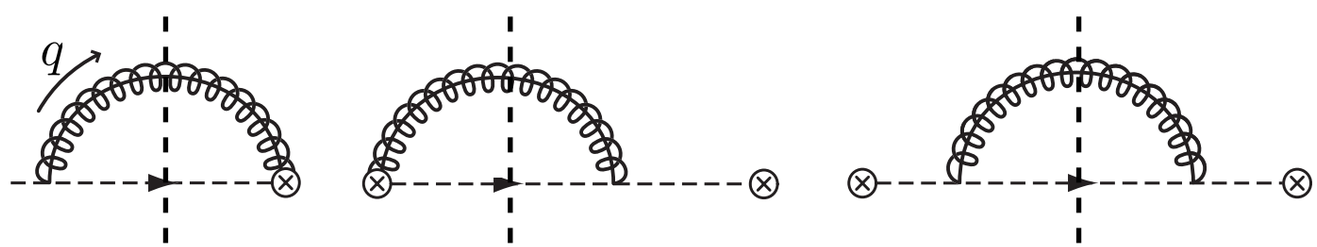
cf. Feige, Schwartz, Stewart, Thaler (2012)

Hard and Jet Functions



$$H(Q^2,\mu) = 1 + \frac{\alpha_s(\mu)C_F}{2\pi}\left(-\ln^2\frac{\mu^2}{Q^2} - 3\ln\frac{\mu^2}{Q^2} - 8 + \frac{\pi^2}{6}\right) + \ldots$$

known to 3 loops



$$J(t,\mu) = \delta(t) + \frac{\alpha_s(\mu)C_F}{4\pi}\left\{(7-\pi^2)\delta(t) - \frac{3}{\mu^2}\left[\frac{\mu^2\theta(t)}{t}\right]_+ + \frac{4}{\mu^2}\left[\frac{\theta(t)\ln(t/\mu^2)}{t/\mu^2}\right]_+\right\} + \ldots$$

known to 2 loops

anomalous dimension known to 3 loops

Beam Function and PDFs

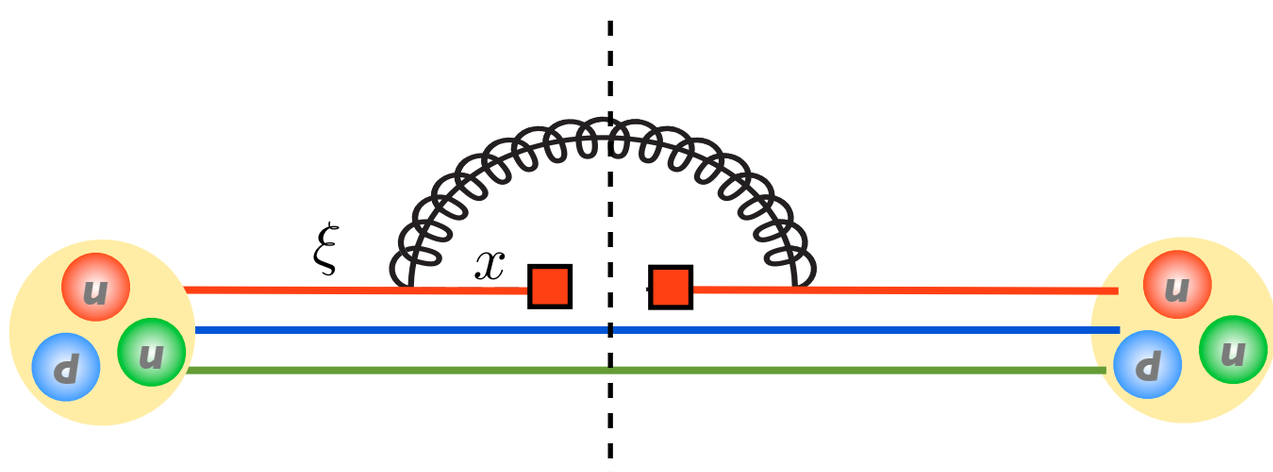
transverse momentum dependent beam function:

$$B(\omega k^+, x, k_\perp^2, \mu) = \frac{\theta(\omega)}{\omega} \int \frac{dy^-}{4\pi} e^{ik^+ y^-/2} \langle P_n(P^-)|\bar{\chi}_n\Big(y^- \frac{n}{2}\Big)\delta(xP^- - \bar{n}\cdot\mathcal{P})\delta(k_\perp^2 - \mathcal{P}_\perp^2)\chi_n(0)|P_n(P^-)\rangle$$

match onto PDF

$$f(x,\mu) = \theta(\omega)\langle P_n(P^-)|\bar{\chi}_n(0)\delta(xP^- - \bar{n}\cdot\mathcal{P})\chi_n(0)|P_n(P^-)\rangle$$

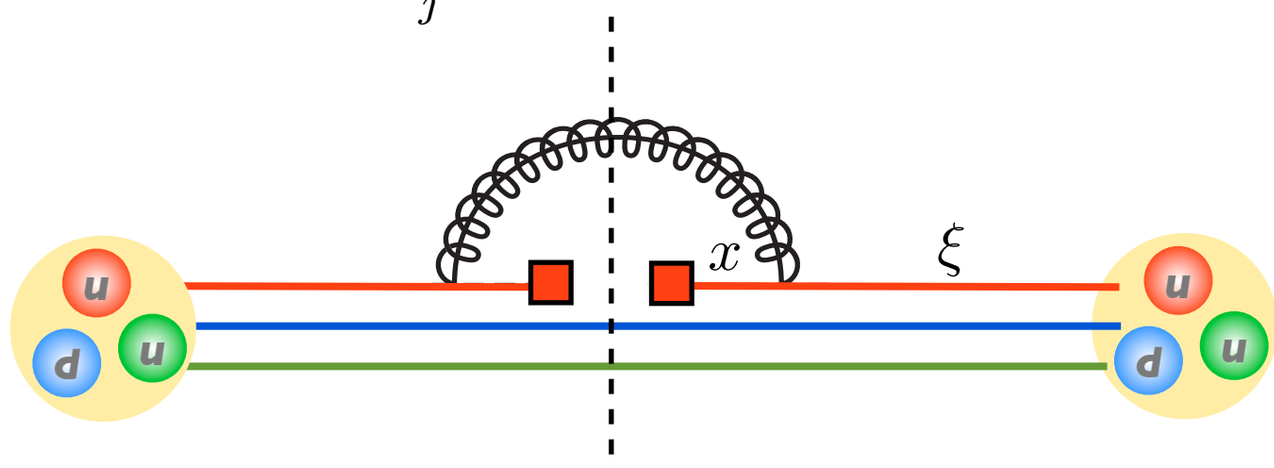
$$\mathcal{B}_q(t, x, \mathbf{k}_\perp^2, \mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ij}\Big(t, \frac{x}{\xi}, \mathbf{k}_\perp^2, \mu\Big) f_j(\xi, \mu)$$



Measure small light-cone momentum $k^+ = t/P^-$
and transverse momentum $\mathbf{k}_\perp$
of initial state radiation

Generalized Beam Function at 1-loop

$$\mathcal{B}_q(t,x,\mathbf{k}_\perp^2,\mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ij}\left(t,\frac{x}{\xi},\mathbf{k}_\perp^2,\mu\right) f_j(x,\mu)$$



$$\begin{aligned}\mathcal{I}_{qq}(t,z,\mathbf{k}_\perp^2,\mu) = \frac{1}{\pi}\delta(t)\delta(1-z)\delta(\mathbf{k}_\perp^2) + \frac{\alpha_s(\mu)C_F}{2\pi^2}\theta(z)\Bigg\{ &\frac{2}{\mu^2}\left[\frac{\theta(t)\ln(t/\mu^2)}{t/\mu^2}\right]_+ \delta(1-z)\delta(\mathbf{k}_\perp^2) \\ &+ \frac{1}{\mu^2}\left[\frac{\theta(t)}{t/\mu^2}\right]_+ \left[P_{qq}(z) - \frac{3}{2}\delta(1-z)\right]\delta\Big(\mathbf{k}_\perp^2 - \frac{(1-z)t}{z}\Big) \\ &+ \delta(t)\delta(\mathbf{k}_\perp^2)\left[\left[\frac{\theta(1-z)\ln(1-z)}{1-z}\right]_+ (1+z^2) - \frac{\pi^2}{6}\delta(1-z) + \theta(1-z)\Big(1-z-\frac{1+z^2}{1-z}\ln z\Big)\right]\Bigg\}\end{aligned} \tag{162a}$$

Jain, Procura, Waalewijn (2009)

anomalous dimension known to 3 loops

$$\mathcal{I}_{qg}(t,z,\mathbf{k}_\perp^2,\mu) = \frac{\alpha_s(\mu)T_F}{2\pi^2}\theta(z)\Bigg\{\frac{1}{\mu^2}\left[\frac{\theta(t)}{t/\mu^2}\right]_+ P_{qg}(z)\delta\Big(\mathbf{k}_\perp^2 - \frac{(1-z)t}{z}\Big) + \delta(t)\delta(\mathbf{k}_\perp^2)\left[P_{qg}(z)\ln\frac{1-z}{z} + 2\theta(1-z)z(1-z)\right]\Bigg\}, \tag{162b}$$

Tells us that PDFs should be evaluated at the beam radiation scale *t*

ordinary beam function: $B(t,x,\mu) = \int d^2k_\perp \mathcal{B}(t,x,\mathbf{k}_\perp^2,\mu)$ Stewart, Tackmann, Waalewijn (2009)

Nonperturbative Soft Model Function

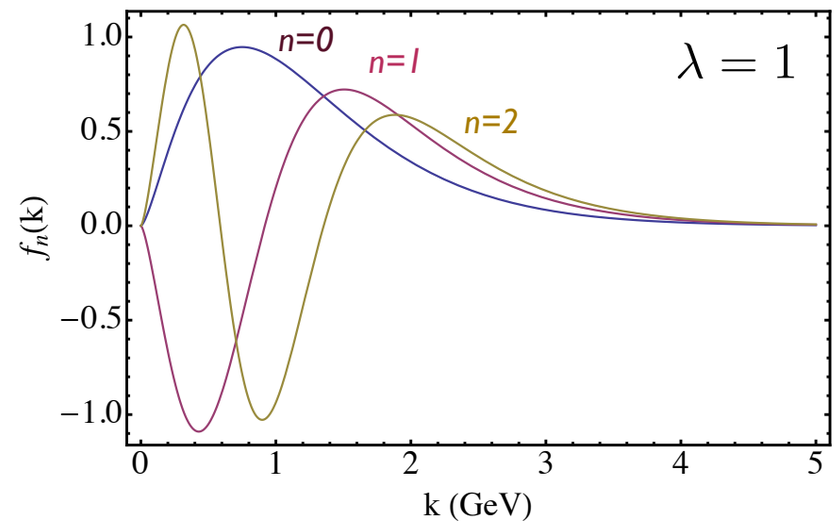
Convolution of perturbative soft function (soft radiation) with nonperturbative model function (hadronization):

$$S(k_S,\mu) = \int dl\, S_{\rm PT}(k_S - l,\mu) S_{\rm NP}(l)$$

$$S_{NP}(l) = f(l-\Delta)$$

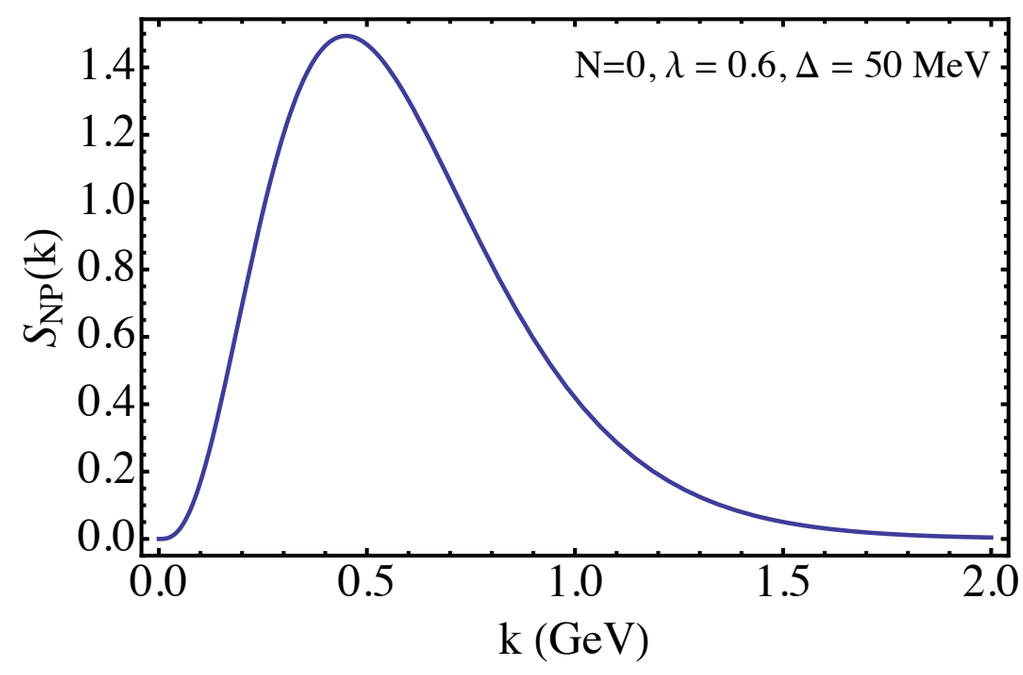
$$f(l) = \frac{1}{\lambda}\sum_{n=0}^{N} c_n f_n\left(\frac{l}{\lambda}\right)$$

Ligeti, Stewart, Tackmann (2008)



Basis coefficients, width and gap should be fit to data for one event shape and value of *Q*. Universality allows predictions for other event shapes and values of *Q*.

In following results, the following model function will be used:



III. Predictions for DIS 1-Jettiness

at Next-to- Next-to- Leading-Log

Resummation of Logs

- Solution of RG Equation automatically resums logs to all orders in α_s
- Order of logarithmic accuracy (LL, NLL, etc.) depends on accuracy to which anomalous dimensions and fixed-order matrix elements are known:

	Γ_F	γ_F	c_F	$\beta[\alpha_s]$
LL	α_s	1	1	α_s
NLL	α_s^2	α_s	1	α_s^2
NNLL	α_s^3	α_s^2	α_s	α_s^3

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+ 1 order for primed counting

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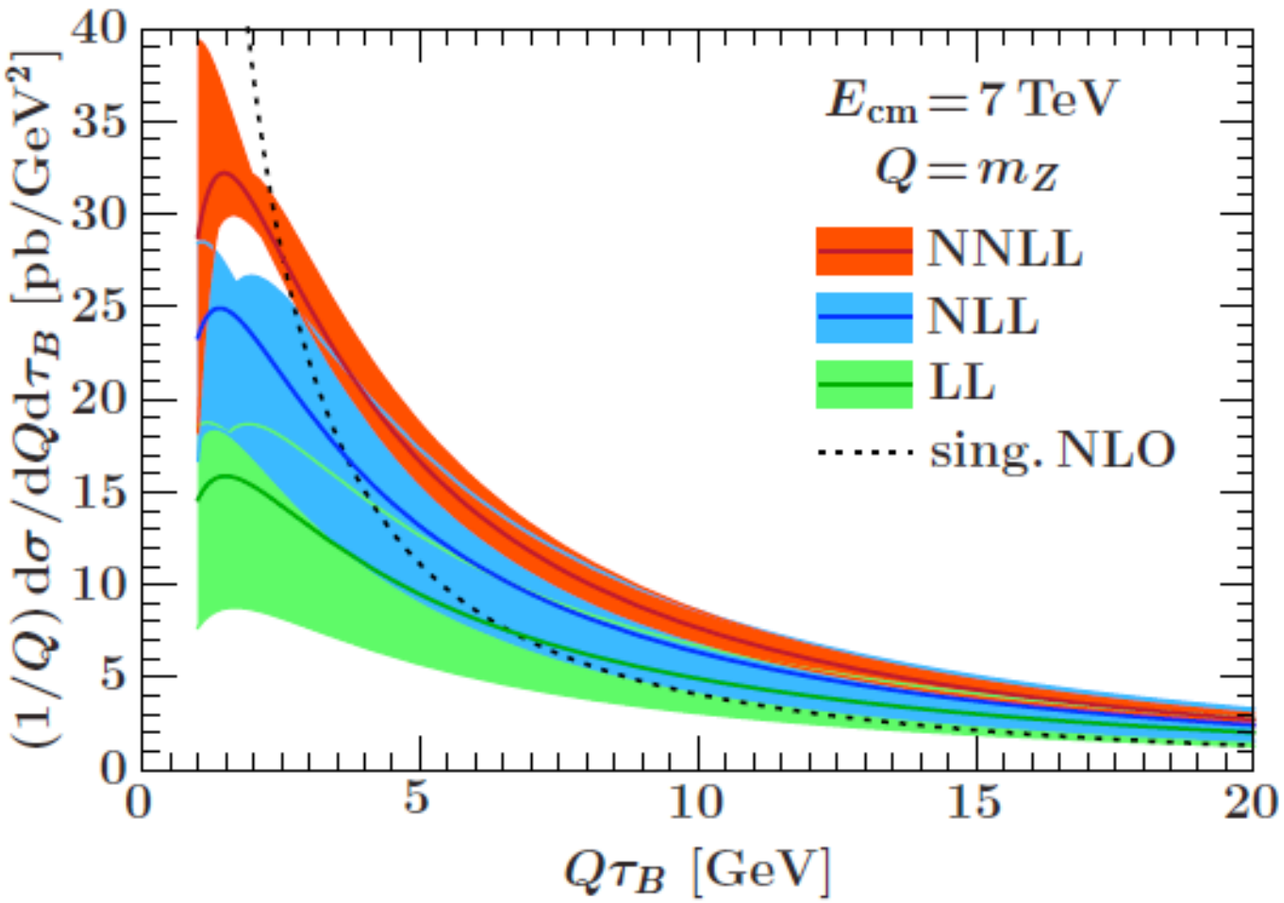
All pieces known for DIS 1-jettiness

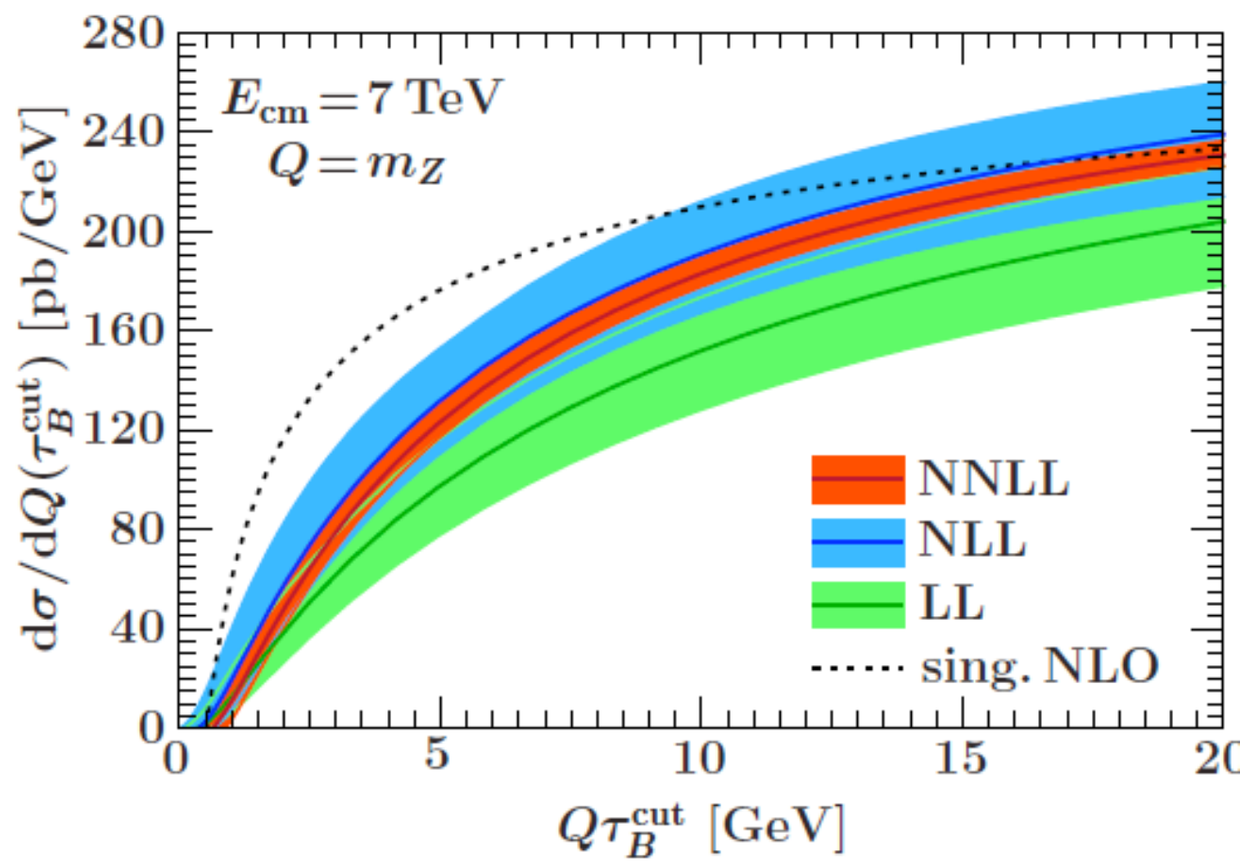
+ 1 order for primed counting

The Beam Thrust Cross Section for Drell-Yan at NNLL Order

Iain W. Stewart, Frank J. Tackmann, and Wouter J. Waalewijn

Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA





The Future

- Calculate rapidity gap distribution
- Two-loop beam function and constant in two-loop soft funciton can be used to achieve N^3LL accuracy
  - Heard about a 2-loop quark beam function here [Lübbert, Gehrmann, Yang]
- Calculate fixed order 1-loop 1-jettiness cross sections in QCD to accurately predict far tail (large τ_1) region
- Comparison to data for improved extraction of strong coupling, nonperturbative soft function, and PDFs

Conclusions

- We have computed 1-jettiness cross sections probing 2 jets in DIS in 3 different ways

- We used SCET to factorize and resum the cross sections to NNLL accuracy, the highest achieved to date for DIS jet cross sections

- SCET provides the tools:

  - to vastly improve the perturbative accuracy of large classes of cross sections in medium and high-energy nuclear and particle physics

  - to improve the extraction of universal nonperturbative functions and the strong coupling