Heavy Flavour Productions &
Hot/Dense Quark Matter
BNL 14/12/2005

(Theoretical) review of heavy quark production

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LPTHE - Paris 6

Outline

- What can be calculated
- How accurately we can/do calculate it? What tools are available?
- Does it work? Comparisons with data
- Implications for RHIC

A heavy quark production process

$$pp \xrightarrow{pQCD} Q \xrightarrow{NP\,fragm.} H_Q \xrightarrow{decay} e$$

+ quenching here if in dense matter

A generic final state observable

This part is QCD.
How accurately can we predict it?
What ingredients do we need?

Factorization "theorem" for heavy quark hadroproduction

Collins, Soper, Sterman, Nucl. Phys. B263 (1986) 37

$$\sigma_Q(S,m^2) = \sum_{i,j\in L}\int dx_1 dx_2 \hat\sigma_{ij\to QX}(x_1x_2S,m^2;\alpha_s(\mu_R^2),\mu_R^2,\mu_F^2)F_{i/A}(x_1,\mu_F)F_{j/B}(x_2,\mu_F) + O\left(\frac{\Lambda}{m}\right)^p$$

Light flavours only

contribute most of the total cross section. The hard scattering function is perturbatively calculable in an expansion in powers of $\alpha_s(M)$: potential singularities in H have been factorized into the parton distribution functions. Corrections to this formula are suppressed by powers of (hadron mass scale/M).

We have by no means proved this result in this paper, but we believe that the analysis given here should make the result plausible. We are arguing that heavy

NLO implementation of factorization theorem

Hadroproduction

Nason, Dawson, Ellis, NP B327 (1989) 49, NP B303 (1988) 607
Beenakker, van Neerven, Meng, Schuler, Smith, NP B351 (1991) 507

Photoproduction

Nason, Ellis, NP B312 (1989) 551
Smith, van Neerven, NP B374 (1992) 36

This is still the state of the art for fixed order perturbative calculations, and should be the building block of all phenomenological predictions:

- it incorporates in a rigorous manner production "channels" like flavour excitation and gluon splitting which Monte Carlo calculations have to include by hand (beware MC tunes and recipes!!)
- it allows a **rough estimate of the theoretical uncertainty**

NNLO is perhaps in the works, but it's not for tomorrow

The rule of thumb on uncertainties

- A LO calculation gives you a **rough estimate** of the cross section

- A NLO calculation gives you a **good estimate** of the cross section and a **rough estimate** of the uncertainty

- A NNLO calculation gives you a **good estimate** of the uncertainty

Believe it or not, this is one of the motivations for embarking in years-long NNLO calculations (I'm serious.....)

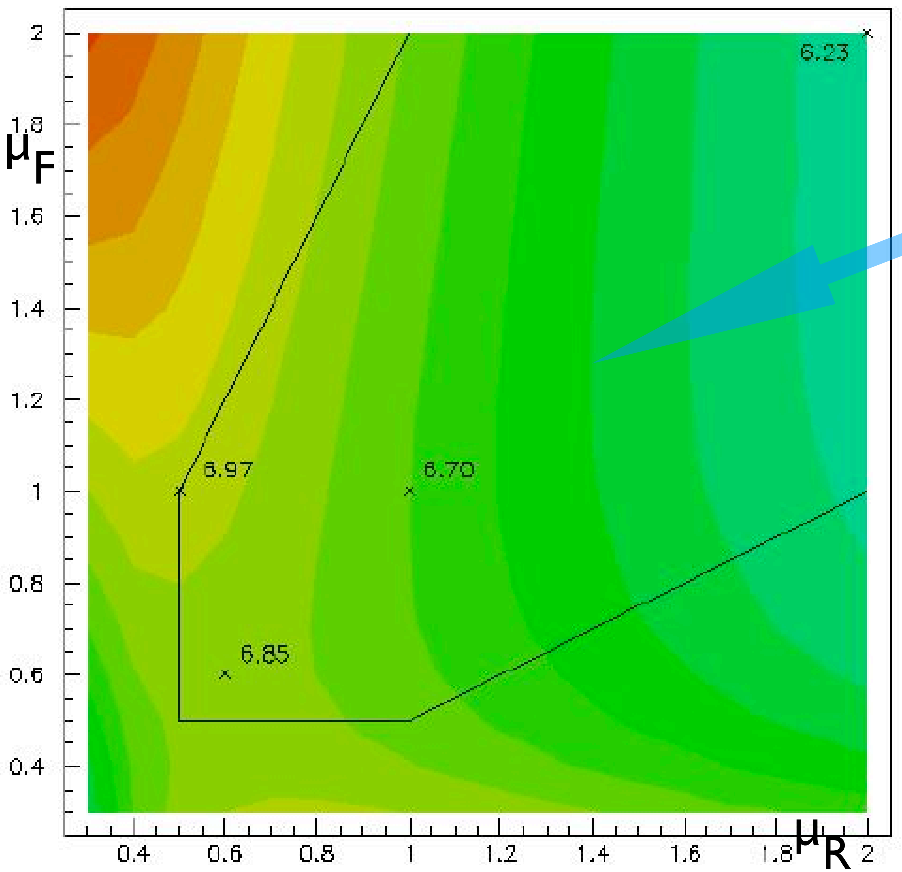
Of course, when the NLO calculation still suffers from large uncertainties an NNLO one can also bring real improvements in the determination of the cross section

Uncertainties estimate: top @ Tevatron

Standard procedure: vary renormalization and factorization scales

σ: 6.82 > 6.70 > 6.23 pb $\quad 0.5 < \mu_{R,F}/m < 2$

σ: 6.97 > 6.70 > 6.23 pb $\quad 0.5 < \mu_{R,F}/m < 2$ && $0.5 < \mu_R/\mu_F < 2$



"Fiducial" region

Order ±5% uncertainty along the diagonal, a little more when considering independent scale variations

NB. The PDF uncertainty (±10-15%) is the dominant one here

Uncertainties estimate: bottom @ Tevatron Run II

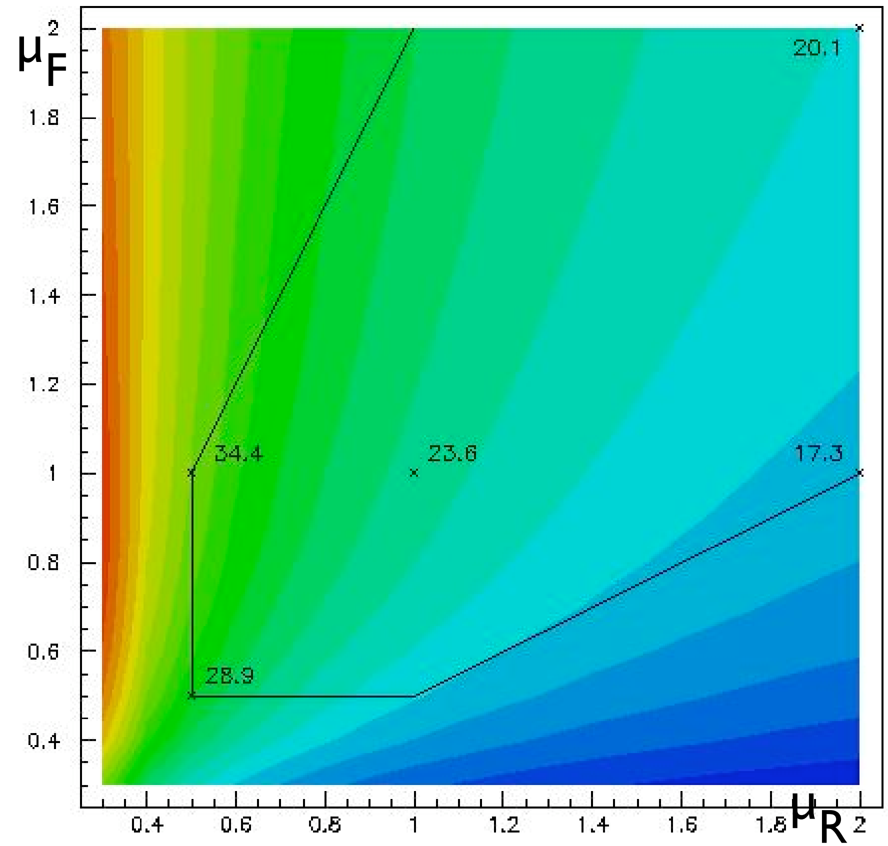
Scale uncertainties:

σ: 28.9 > 23.6 > 20.1 μb

$0.5 < \mu_{R,F}/\mu_0 < 2$

σ: 34.4 > 23.6 > 17.3 μb

$0.5 < \mu_{R,F}/\mu_0 < 2$ && $0.5 < \mu_R/\mu_F < 2$



1) the scale uncertainties increase from ±18% to ±35% when going off-diagonal

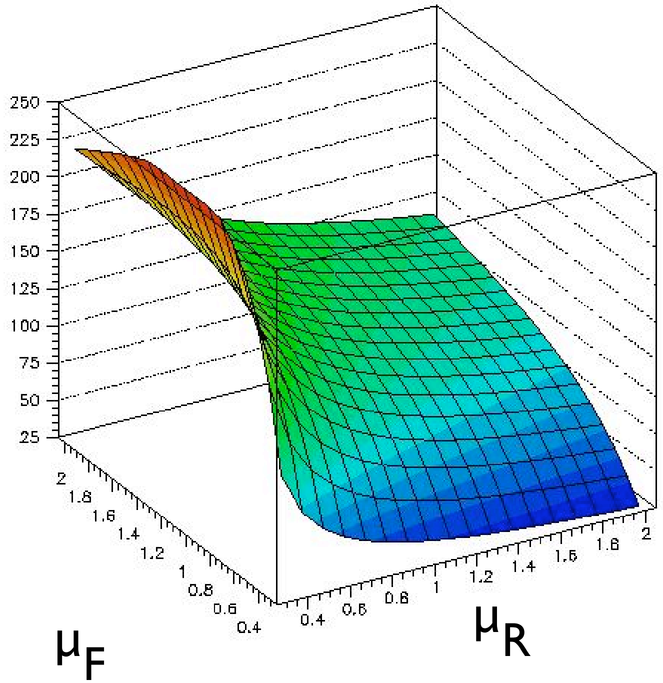
2) PDF uncertainties are here less important than perturbative ones. At large p_T they become instead similar

Uncertainties estimate: bottom @ LHC

σ: 122 > 120 > 115 μb

$0.5 < \mu_{R,F}/\mu_0 < 2$

Only a ±4% uncertainty when varying the scales together.......



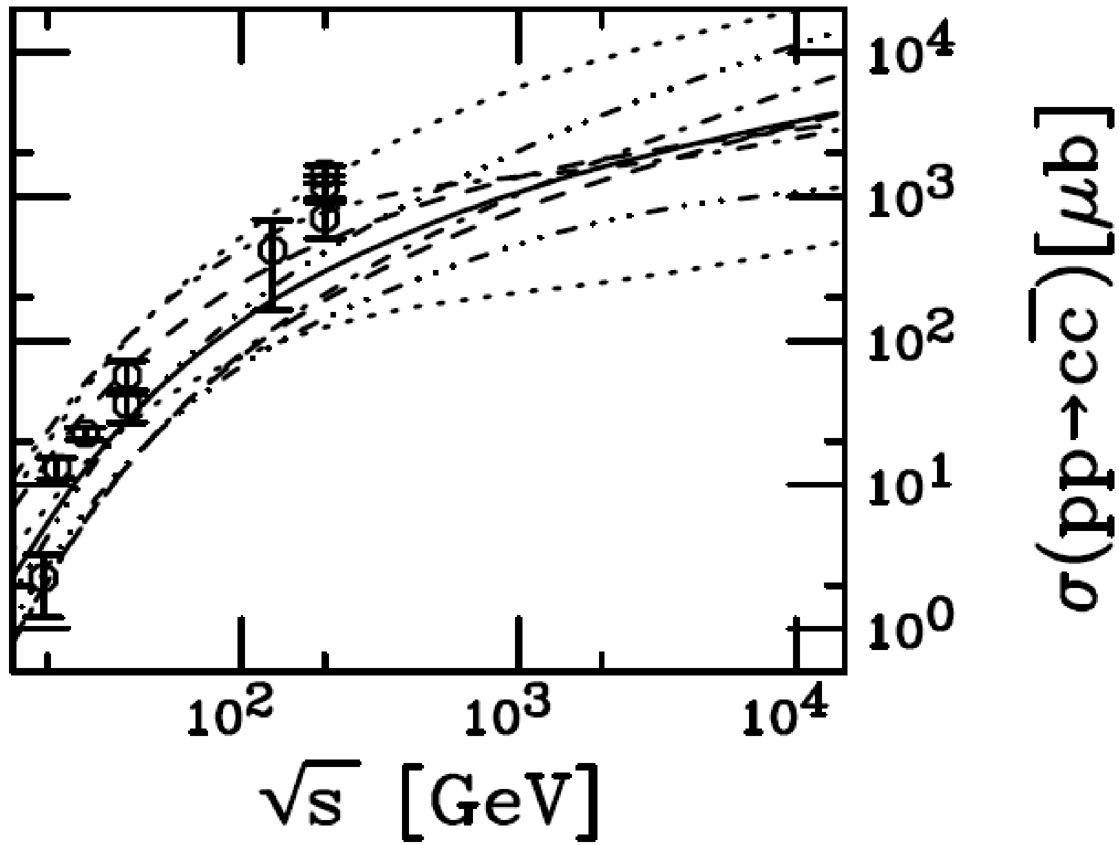
σ: 178 > 120 > 75 μb

$0.5 < \mu_{R,F}/\mu_0 < 2$ && $0.5 < \mu_R/\mu_F < 2$

....which becomes a ±40% one when going off-diagonal!!!

Example of uncertainties: how accurately can we predict charm?

Plot by R. Vogt:
variations of fact/ren scales and charm mass



Not that well.......

Order of magnitude theoretical uncertainty on total cross section, due of course to the large value of the strong coupling at the charm scale and phase space mass effects

On top of this, there will also be non-perturbative corrections of course...

Real observables

Total cross sections are rarely really measured. Often only a limited phase space region is avalable. These data can then be extrapolated to the full phace space.

This, of course, means adding the bias of a theoretical prejudice to an experimental measuremens

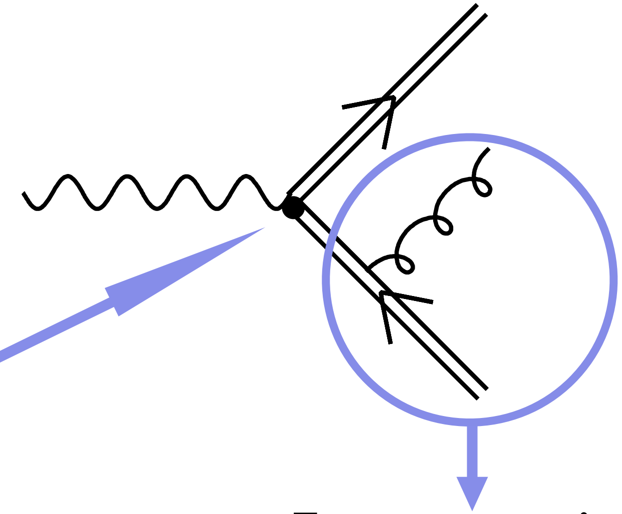
When the theoretical calculations are known to be accurate and precise and/or the extrapolation is small, there is no problem in doing so. Otherwise, we might end up with something closer to a theoretical estimate than to an experimental measurements.

==> use differential observable quantities for comparing to theoretical predictions (e.g. electron spectra at RHIC)

Unfortunately, theoretical predictions for differential observables are **harder**:

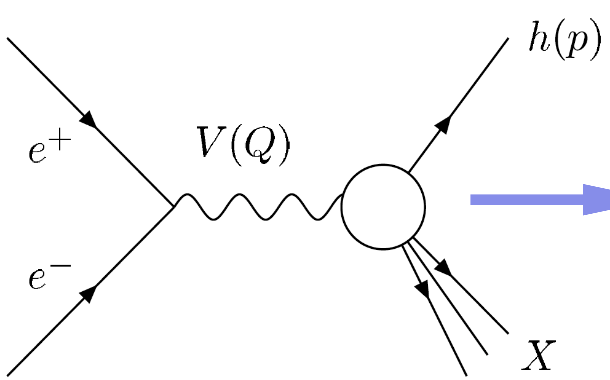
- the problem becomes a multi-scale one => **resummations** are often necessary
- **non-perturbative** ingredients will usually need to be included

Heavy Quarks in e+e-

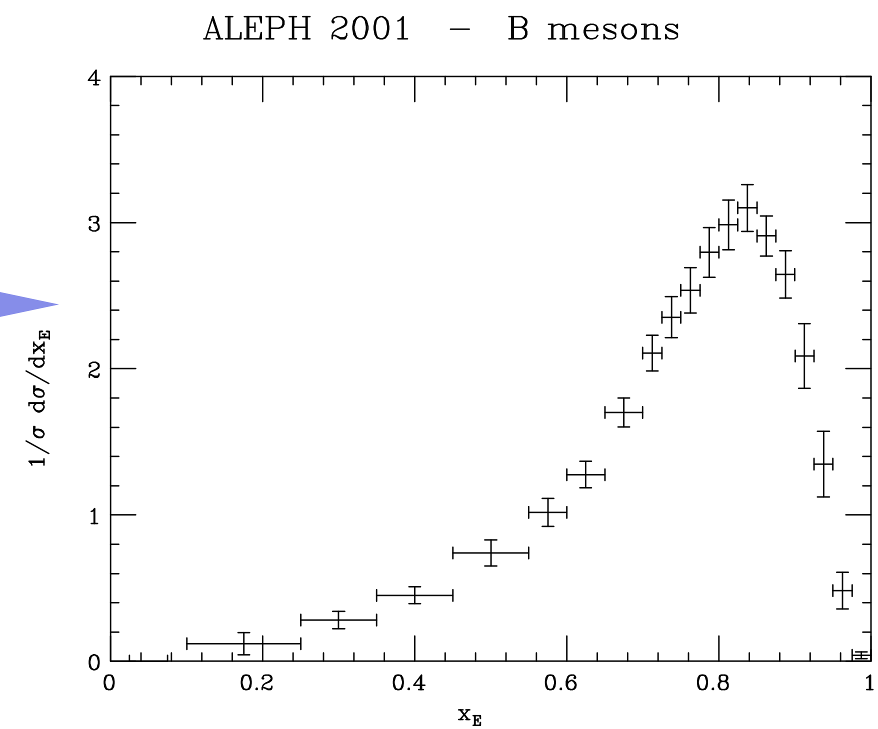


The Born level total production is of course an **electroweak** process

Fragmentation
is, instead, a QCD issue



$$x_E \equiv \frac{E_h}{E_{beam}}$$



At Born level x = 1. The momentum degradation of the heavy hadrons is a combination of **perturbative** and **non-perturbative** QCD effects

Perturbative Fragmentation

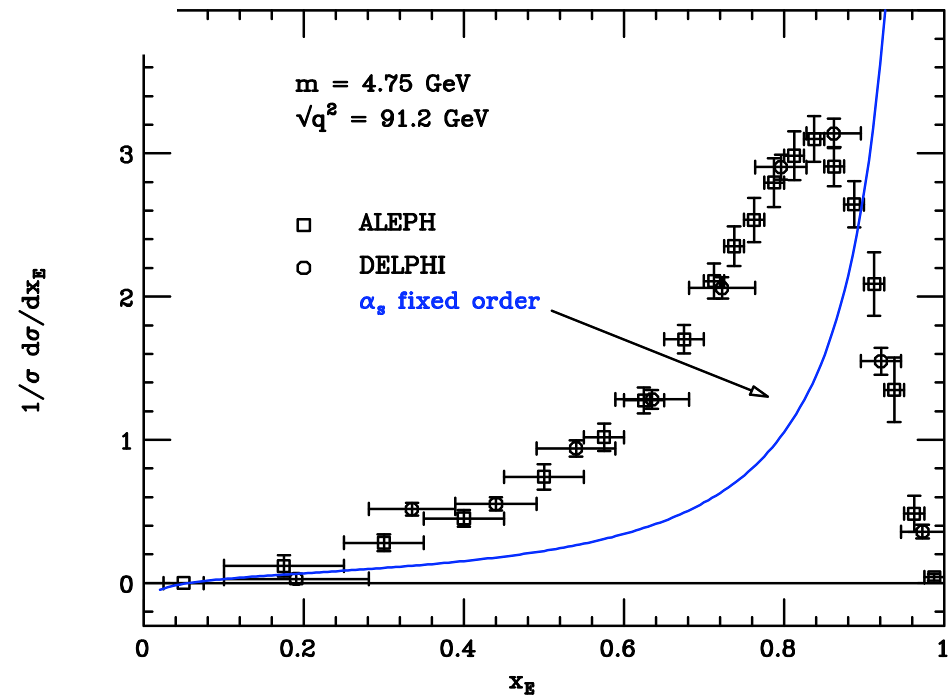
Collinear log

$$\frac{1}{\sigma}\frac{d\sigma}{dx} = \delta(1-x)+\frac{\alpha_s(Q^2)}{2\pi}\left\{C_F+C_F\left[\ln\frac{Q^2}{m^2}\left(\frac{1+x^2}{1-x}\right)_+\right.\right.$$
$$+\quad 2\frac{1+x^2}{1-x}\log x-\left(\frac{\ln(1-x)}{1-x}\right)_+(1+x^2)+\frac{1}{2}\left(\frac{1}{1-x}\right)_+(x^2-6x-2)$$
$$\left.\left.+\quad \left(\frac{2}{3}\pi^2-\frac{5}{2}\right)\delta(1-x)\right]\right\} + \mathcal{O}\left(\frac{m}{Q}\right)$$

Soft logs

The LO calculation is a textbook example in QCD.
NB. Mass effects suppressed by the cms energy => negligible at LEP

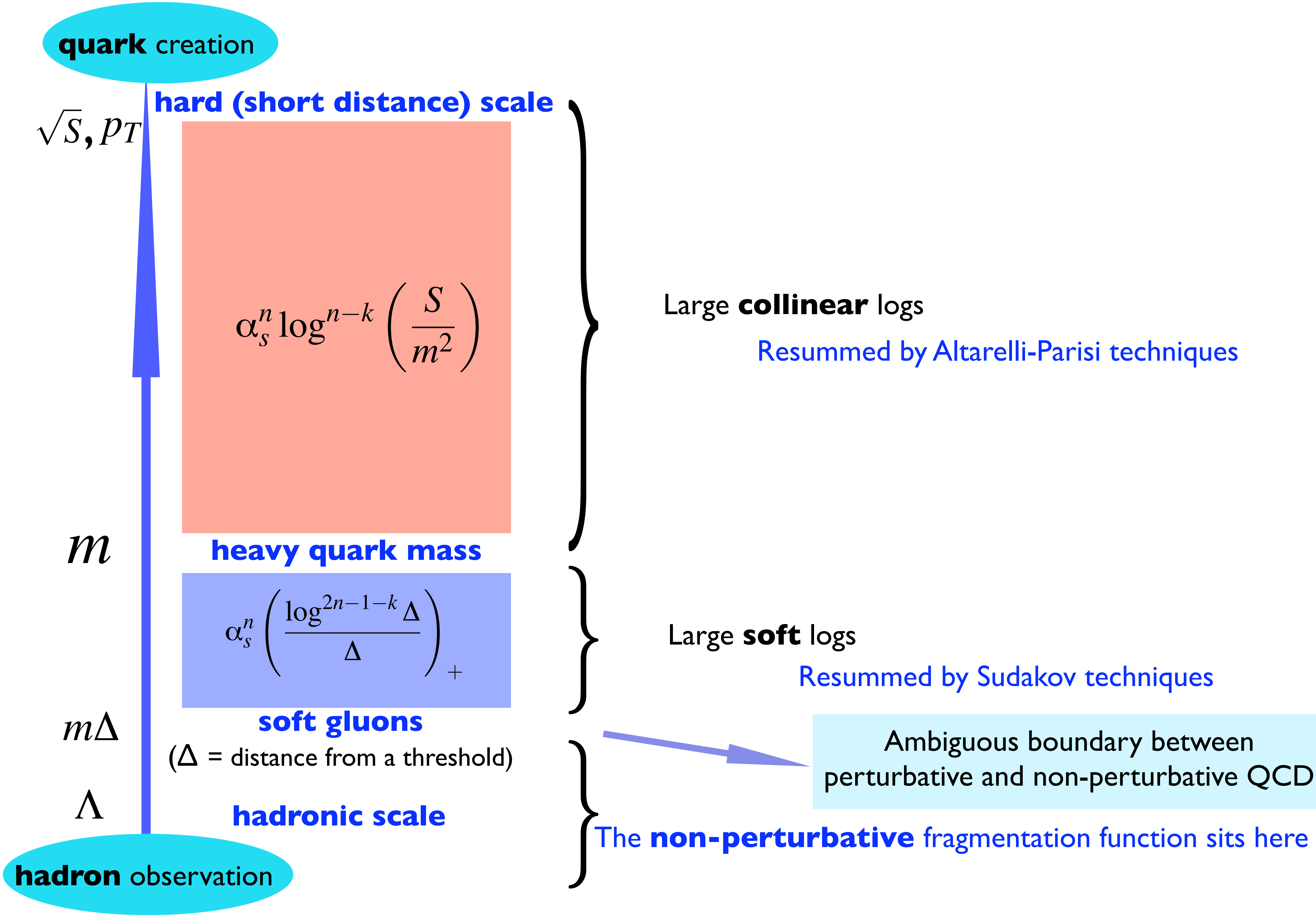
Comparison with experimental data is of course less than thrilling:



Two reasons for this:

- large **perturbative logs** spoil the convergence of the perturbative series
- significantly large **non-perturbative** contributions are missing

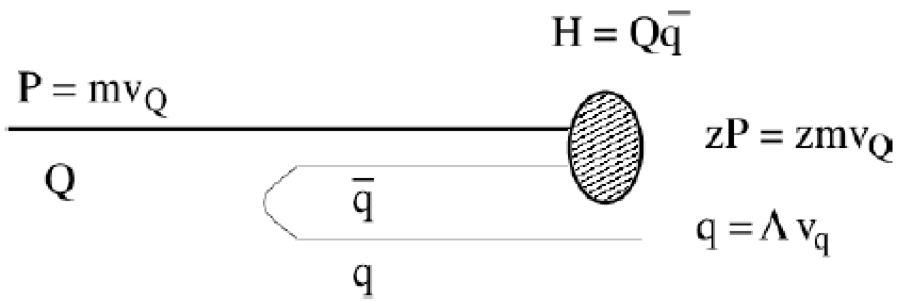
The many scales in heavy quark production



Non-perturbative Fragmentation

A heavy quark produced in a high energy event will lose a fraction of its momentum when picking up a light quark from the vacuum in order to hadronize into a heavy meson or baryon.

How much exactly we cannot tell (it's a non-perturbative process), but we can try to estimate it (more rigorous derivations are available):



The heavy quark has momentum $P = mv_Q$, the light quarks have momentum $q = \Lambda v_q$, with Λ a hadronic mass scale

For the binding we need $v_Q \simeq v_q = v$. We have then

$$P = zP + q \qquad\qquad mv = zmv + \Lambda v$$

and therefore

$$\langle z \rangle \simeq 1 - \frac{\Lambda}{m}$$

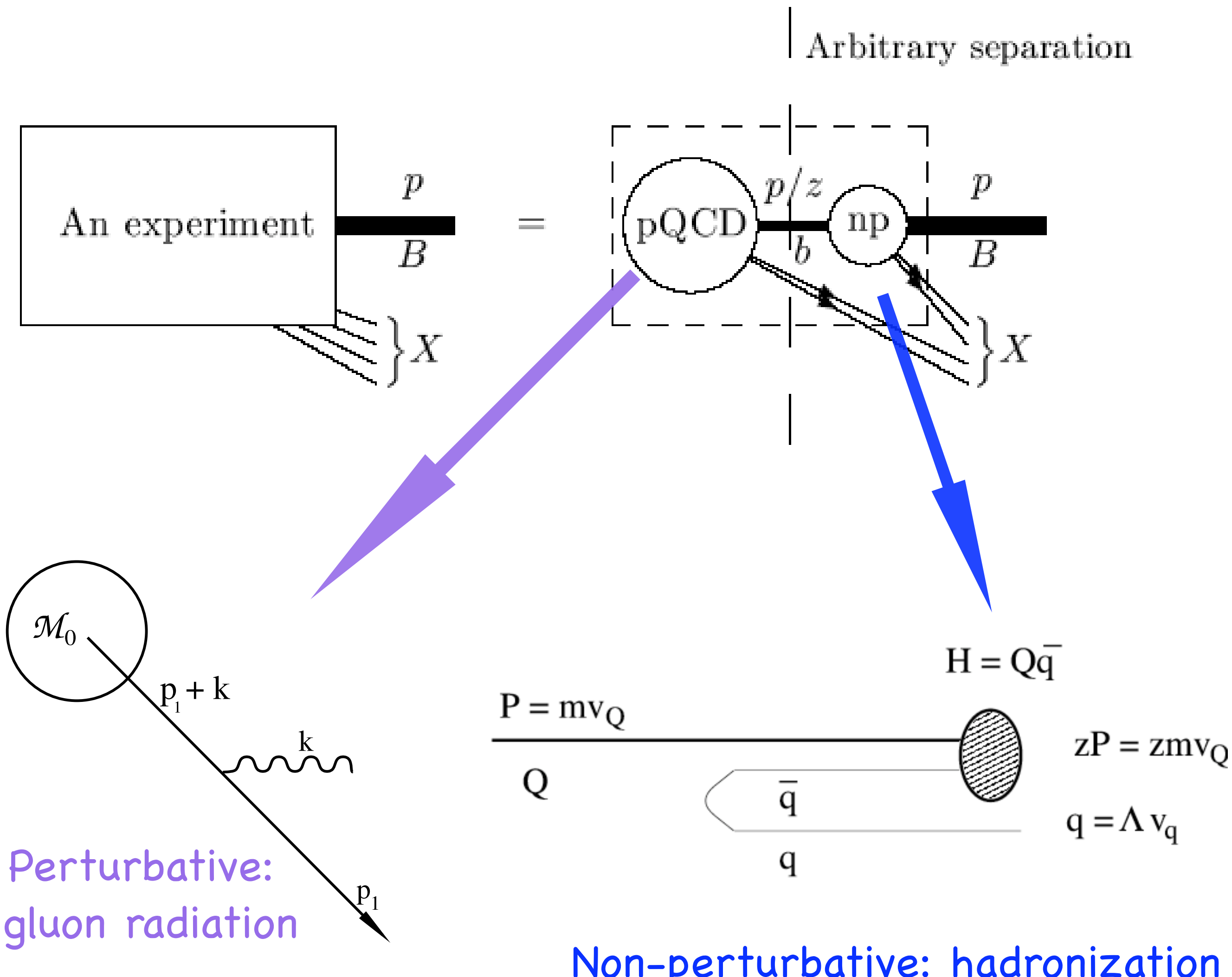
QCD predicts that the non-perturbative fragmentation of a heavy quark is very hard: the heavy quark loses a limited fraction of momentum (light hadron mass/heavy quark mass) when hadronising

This effect is far from negligible compared to the perturbative one. It usually parametrized by a **non-perturbative fragmentation function**, usually extracted from expt. data

No sensible phenomenology is possible without including non-perturbative contributions

Non-perturbative Fragmentation

When extracting the **heavy quark → heavy meson non-perturbative fragmentation function** from data, one must consider the **unavoidable interplay with perturbative physics**



Not being the c/b quark a physical particle, **the non-perturbative fragmentation function cannot be a physical observable**: its details depend on the perturbative calculation it is interfaced with.

A single fragmentation function cannot do for all calculations

Extraction of the non-perturbative component

Three issues are important:

1. The perturbative description (and its parameters) used in extracting the FF must match the one used in calculating predictions using the FF

2. Try to extract an as universal as possible non-perturbative FFs. Resumming the perturbative collinear logarithms (large at LEP: log(√S/m)) helps doing precisely this

3. Because of the steep slope of transverse momentum distributions in hadron-hadron collisions, higher Mellin moments of the FF are actually more important than its x-space shape:

Assuming $\frac{d\sigma}{d\hat{p}_T} \sim \frac{1}{\hat{p}_T^N}$ we get $\frac{d\sigma}{dp_T} \sim \int \frac{dz}{z} (\frac{z}{\hat{p}_T})^N f(z) = f_N \frac{d\sigma}{d\hat{p}_T}$

Heavy **quark** spectrum, N typically ~ 4,5

Heavy **meson** spectrum

Mellin **moment** of the fragmentation function

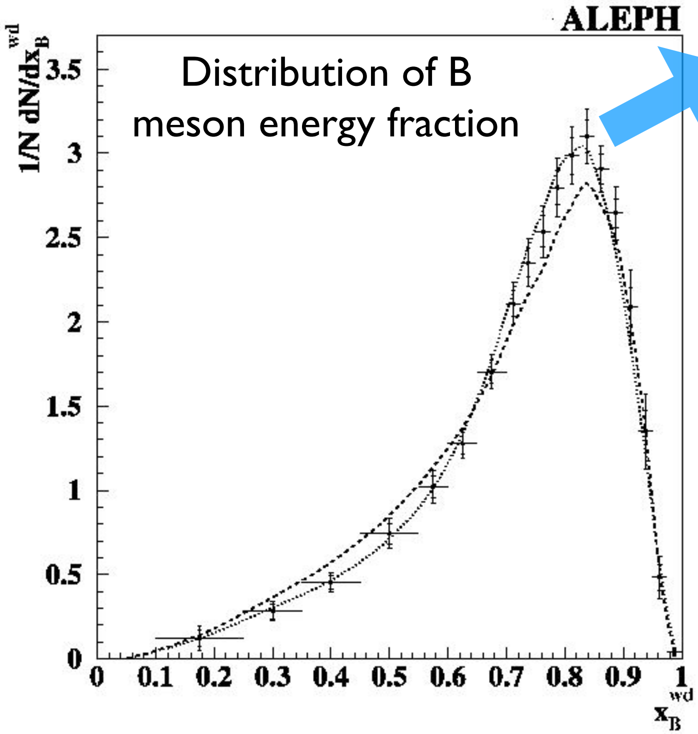
Heavy **quark** spectrum

Fitting well the proper moments (N ~ 4-5) is therefore more important then describing the whole fragmentation spectrum in e+e- collisions, if the fragmentation function is then to be used for making predictions in hadronic collisions

[This third step, is a bit exotheric, but numerically fairly important. It's the one which explaines why the usual Peterson FF in conjunction with a NLO calculation does not give a good description of heavy quark fragmentation: FF's extracted from moments are quite harder!]

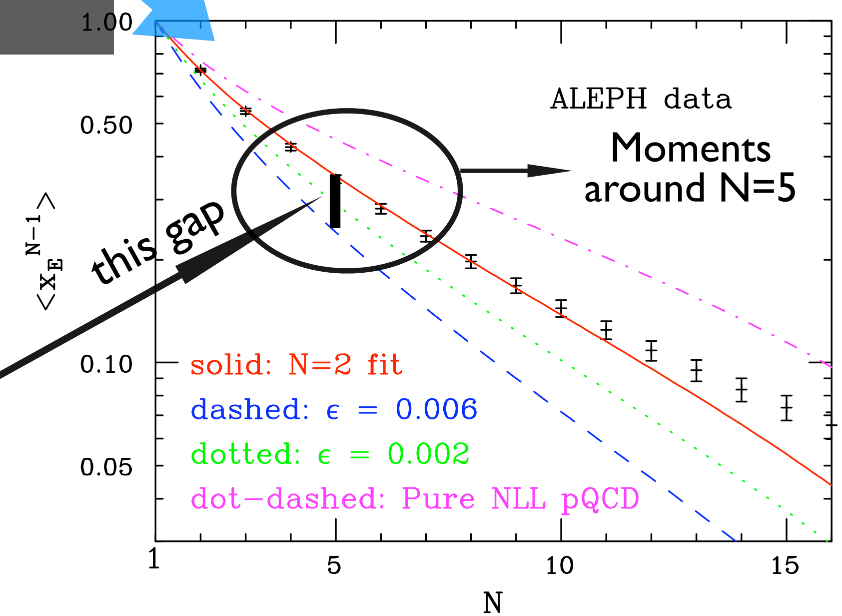
Extraction of the non-perturbative component II

Fit **moments** of LEP fragmentation data:



$$\langle x_E^{N-1}\rangle = \int_0^1 x_E^{N-1} f(x_E)\, dx_E$$

Note that Peterson with ϵ_b = 0.006 underestimates the moments around N=5. Its use will consequently underestimate the hadronic B cross section



Don’t fit this……

…but rather this

The extracted fragmentation functions are specific to the FONLL framework.
For a comparison, they **roughly** correspond to Peterson et al. FF’s with $\boldsymbol{\epsilon_c \approx 0.005}$ and $\boldsymbol{\epsilon_b \approx 0.0005}$

⇒ quite harder than ‘usual’ values $\epsilon_c \approx 0.06$ and $\epsilon_b \approx 0.006$

⇒ hadronic cross sections will be larger

One more word of caution

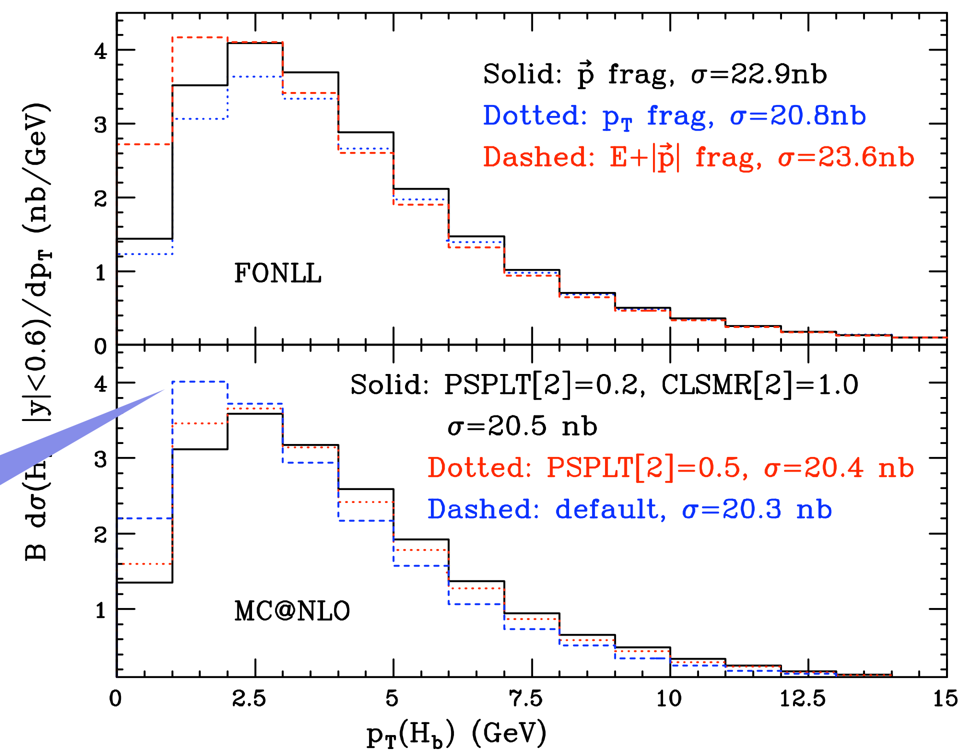
The non-perturbative FF is usually employed in hadronic collisions by writing

$$E_H \frac{d^3\sigma_H(p_H)}{dp_H^3} = E_Q \frac{d^3\sigma_Q(p_Q)}{dp_Q^3} \otimes D^{np}_{Q\to H}$$

Bear in mind that when the transverse momentum is small two things happen:

1. The "independent fragmentation" picture fails, as factorization-breaking higher twists grow large. So, whatever the result of the convolution above, there will be further uncertainties looming over it

2. Scaling a massive particle's 4-momentum is an ambiguous operation. One can scale the transverse momentum at constant rapidity, the 3-momentum at constant angle in a given frame, etc.



Different fragmentation choices

Putting things together

A modern tool for producing phenomenological predictions for heavy quark at the differential level will

1- properly **resum** (say to next-to-leading log accuracy) the large logarithms

2- **match** the resummation to a full NLO fixed order calculation

3- properly **extract** from data (and **use** for predictions) **non-perturbative** fragmentation functions describing the hadronization of the heavy quarks

NB. Whether you need all this or not depends, of course, on your accuracy goal. If you are happy with a factor of two uncertainties or more none of this is probably necessary: take the 15 years old NLO calculation and go ahead (but then don't come to me complaining of discrepancies with QCD!)

On the other hand, if you aim at a few 10% accuracy then you need this stuff.

FONLL (MC, M. Greco, S. Frixione, P. Nason)

(Fixed Order plus Next-to-Leading Logarithms)

http://home.cern.ch/cacciari/FONLL

FONLL is a code for calculating double-differential, **single inclusive** heavy quark production cross sections in pp(bar) and (electro)photoproduction.
(and - preliminary, not yet public implementation - also in pA and AA collisions via the EKS shadowing corrections. No quenching yet.....)

(NB. Not a Monte Carlo. No exclusive events)

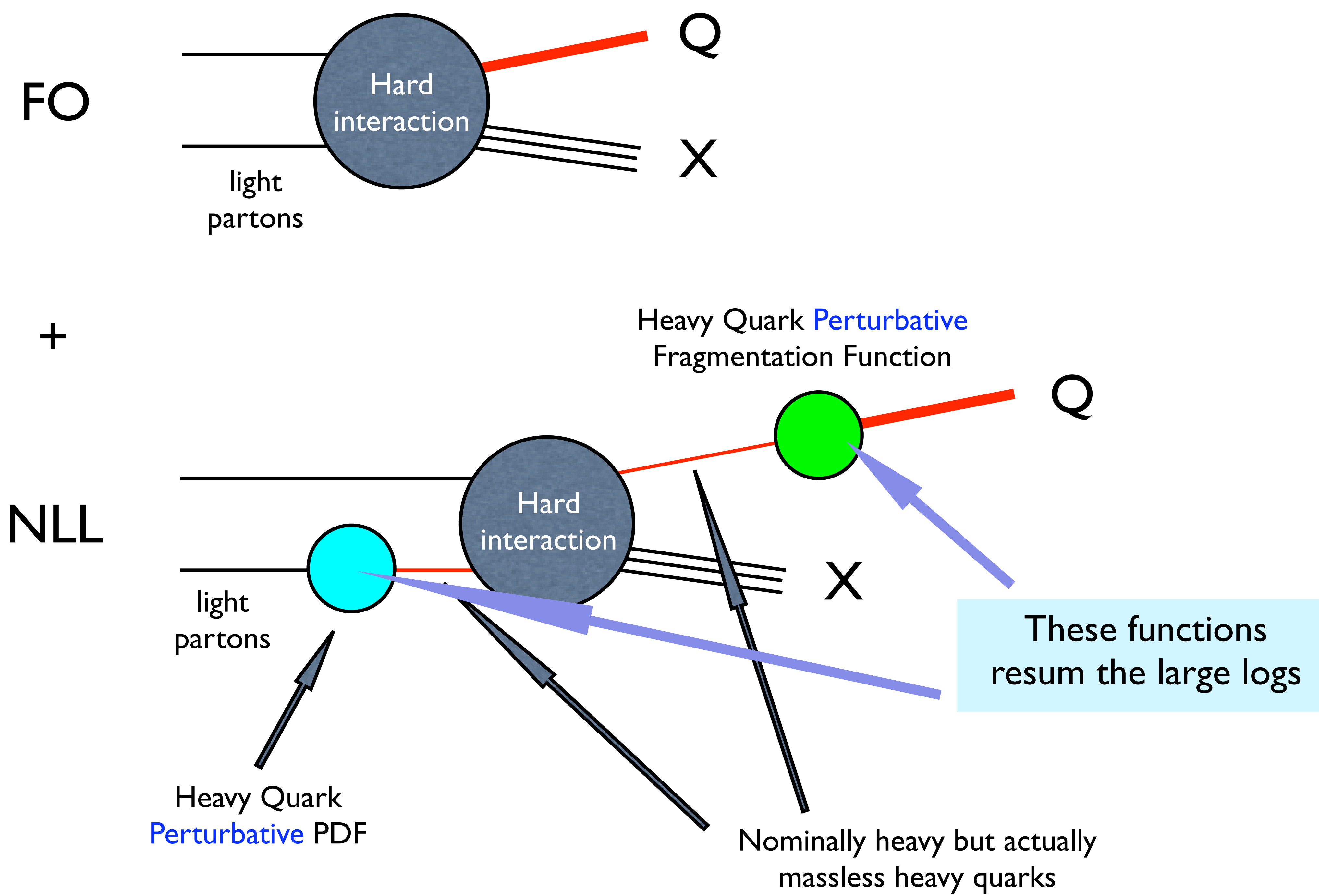
FONLL merges the **massive NLO calculation** with the **NLL resummation** of terms of collinear origin, $\alpha_S \log(p_T/m)$, which become large when $p_T >> m$

Two **advantages** of an FONLL-like framework:

1- the perturbative uncertainty does not increase when $p_T >> m$

2- non-perturbative input describing the **quark-to-meson hadronization** can be extracted from e+e- data and included in predictions for hadronic collisions in a self-consistent way

FONLL: How does it work?



MC@NLO

As for FONLL, the general structure is

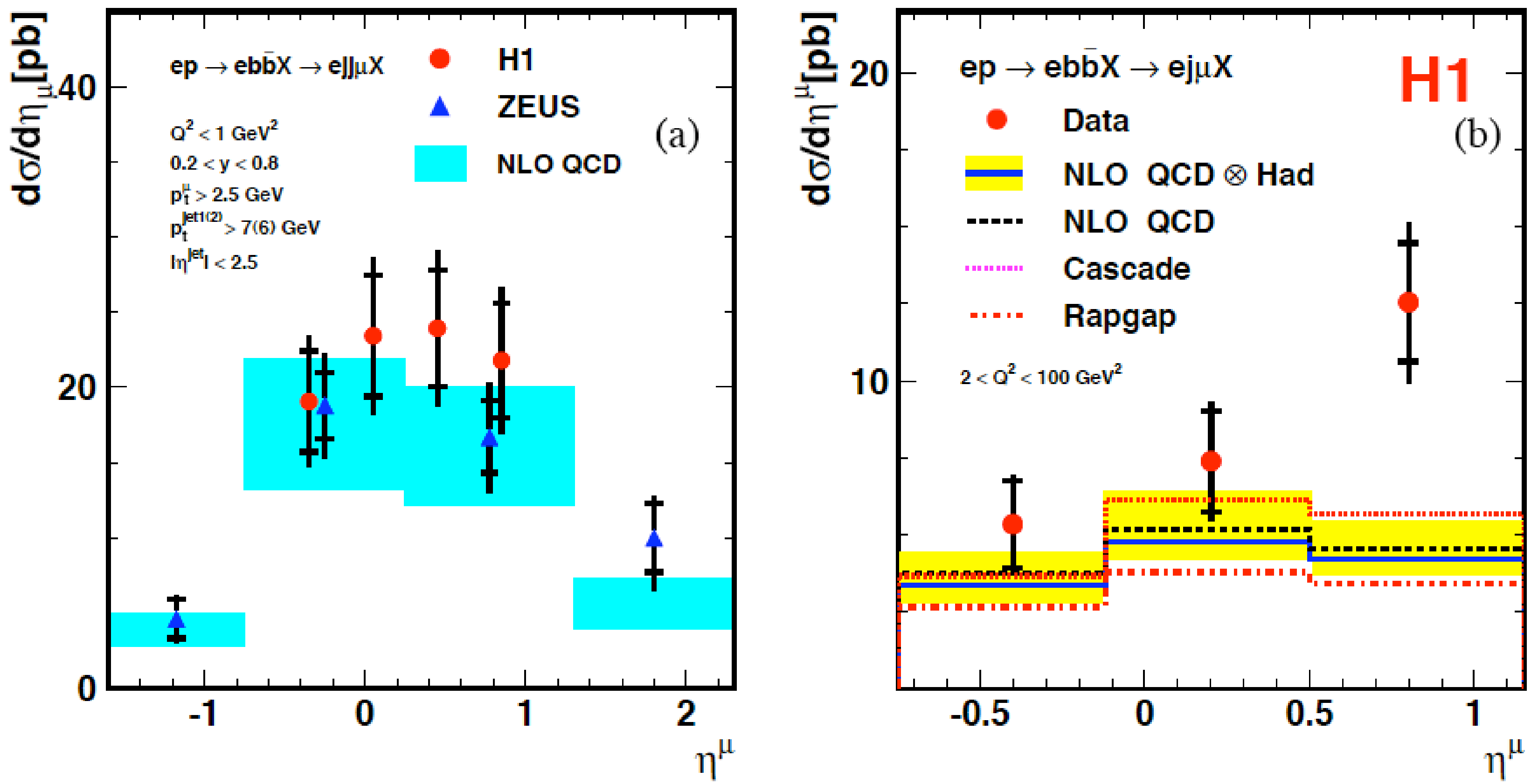
Fixed Order + Resummed - Double Counting

- Total rates are accurate to NLO
- Hard emissions are treated as in NLO computations
- Soft/collinear emissions are treated as in MC => resummation (LL, usually)
- NLO results are recovered upon expansion of MC@NLO results in α_S.
  In other words: there is no double counting in MC@NLO
- The matching between hard- and soft/collinear-emission regions is smooth
- The output is a set of events, which are fully exclusive
- MC hadronization models are adopted

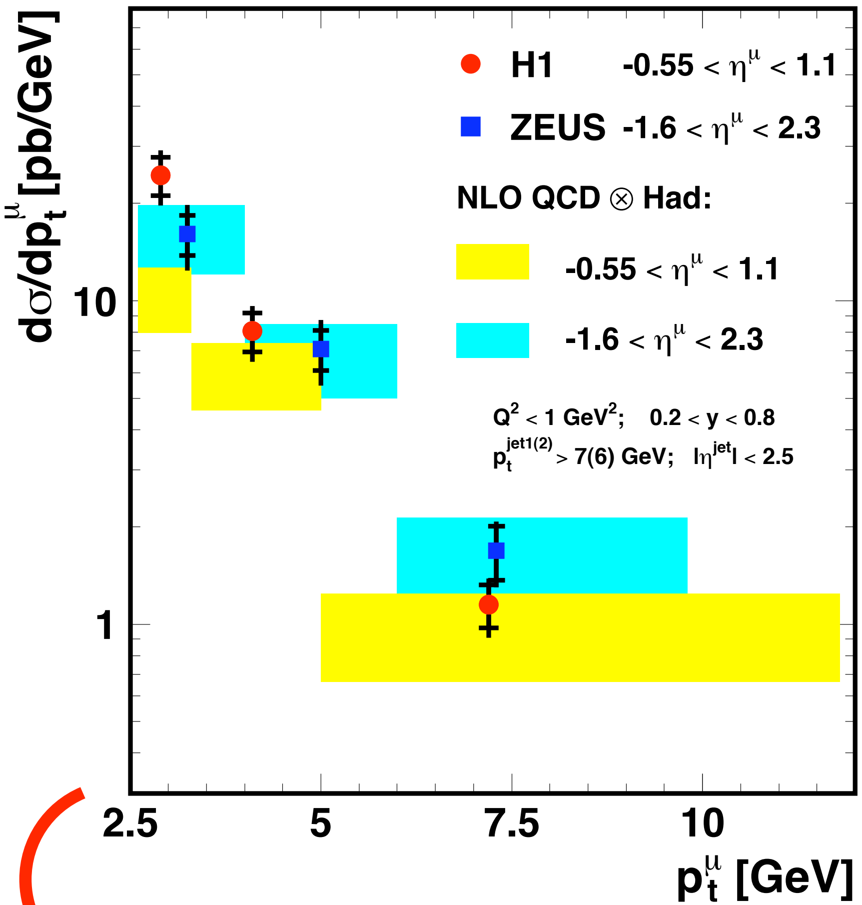
`http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO`

Comparisons of phenomenological QCD predictions and experimental data

Does it work? Bottom photoproduction @ HERA

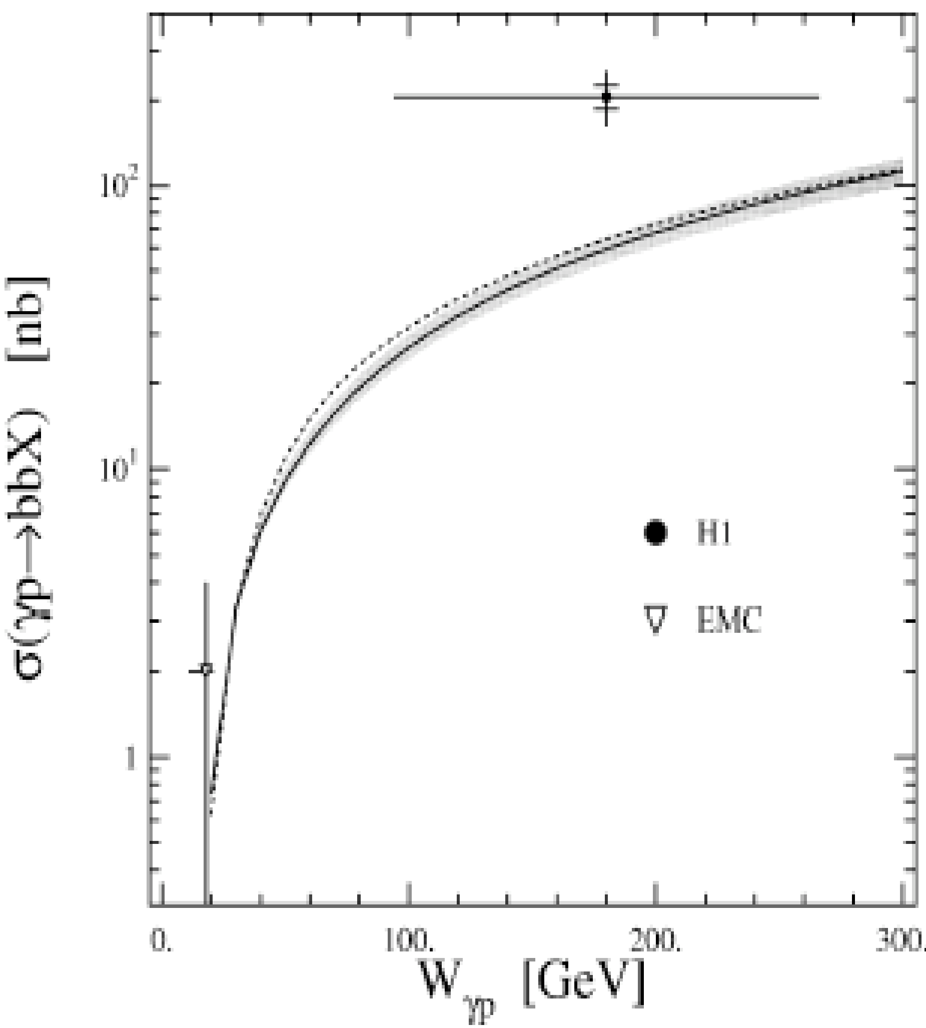


Does it work? Bottom photoproduction @ HERA

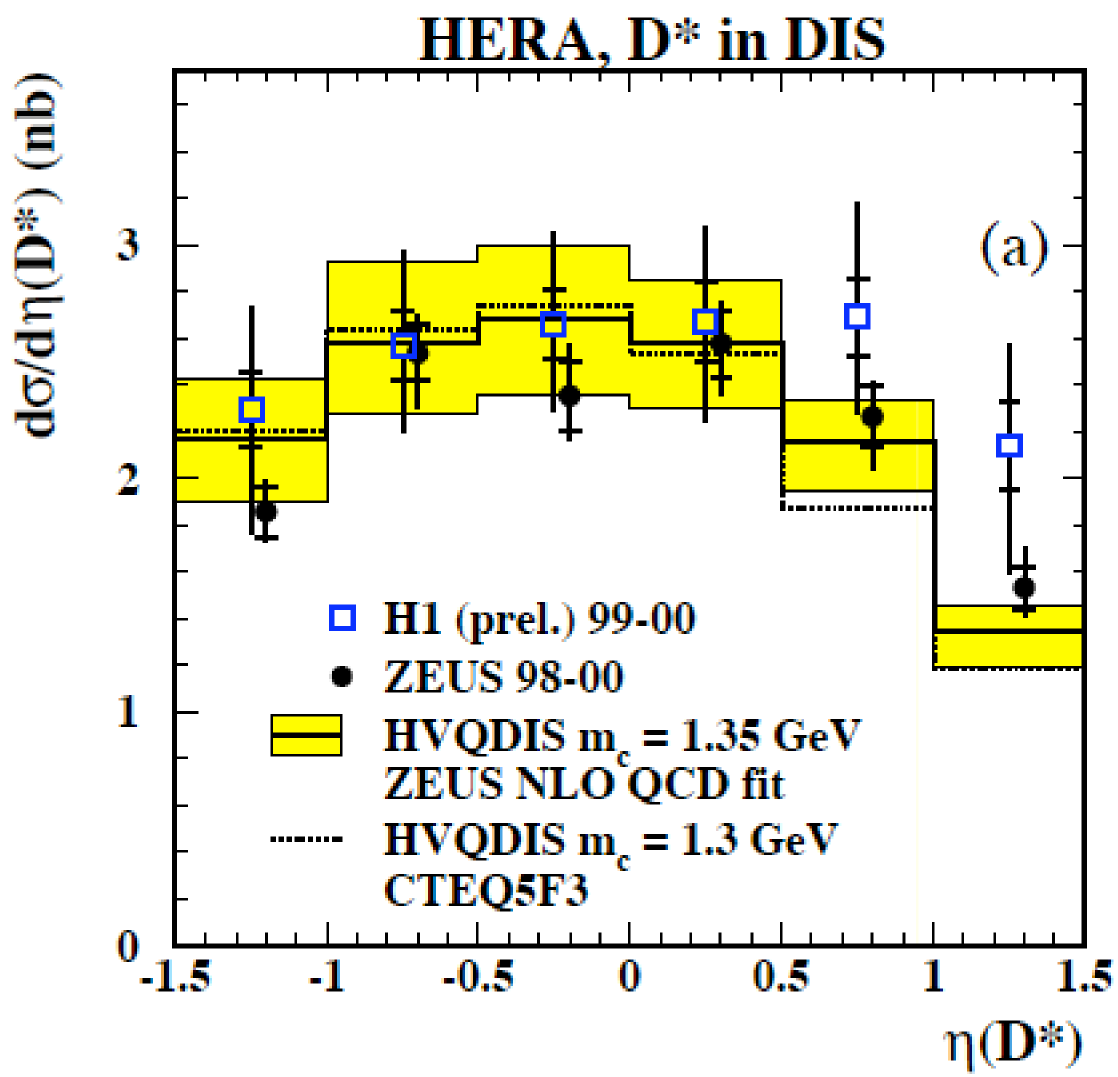


ZEUS: No excess at low p_t^μ

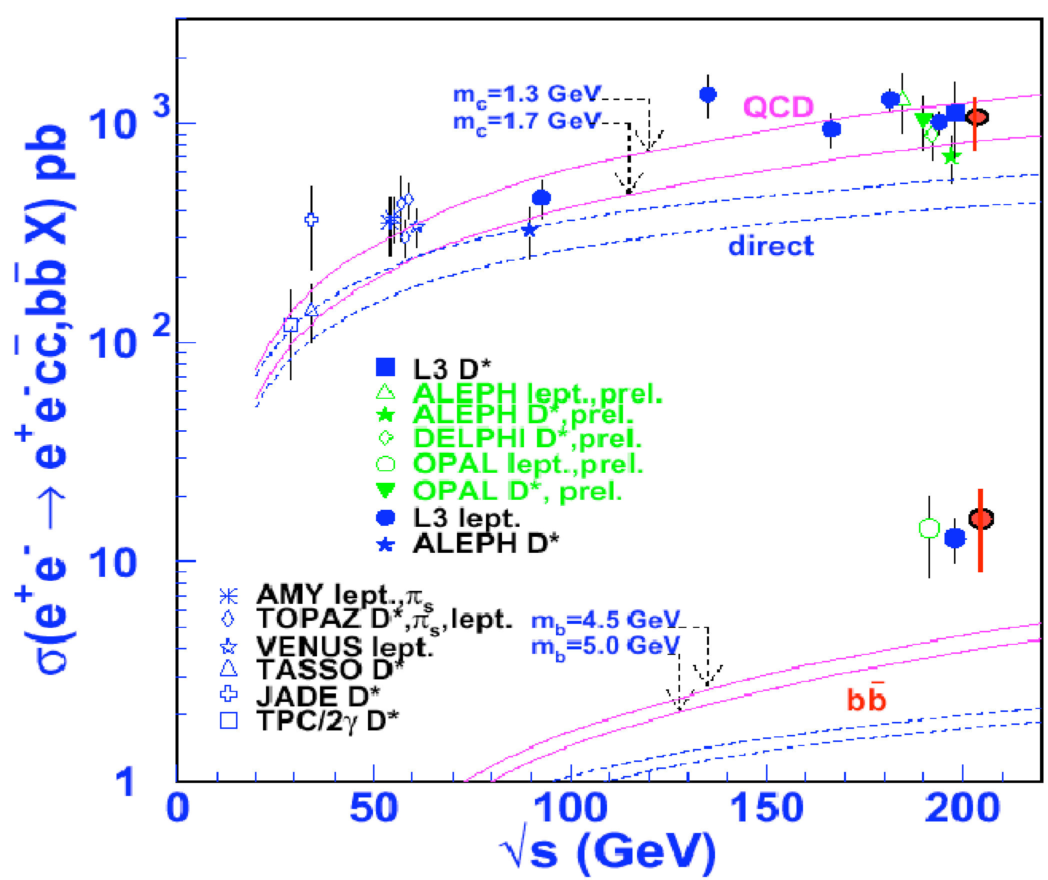
Compare to old extrapolated result. First HERA bottom measurement



Does it work? Charm production in DIS @ HERA



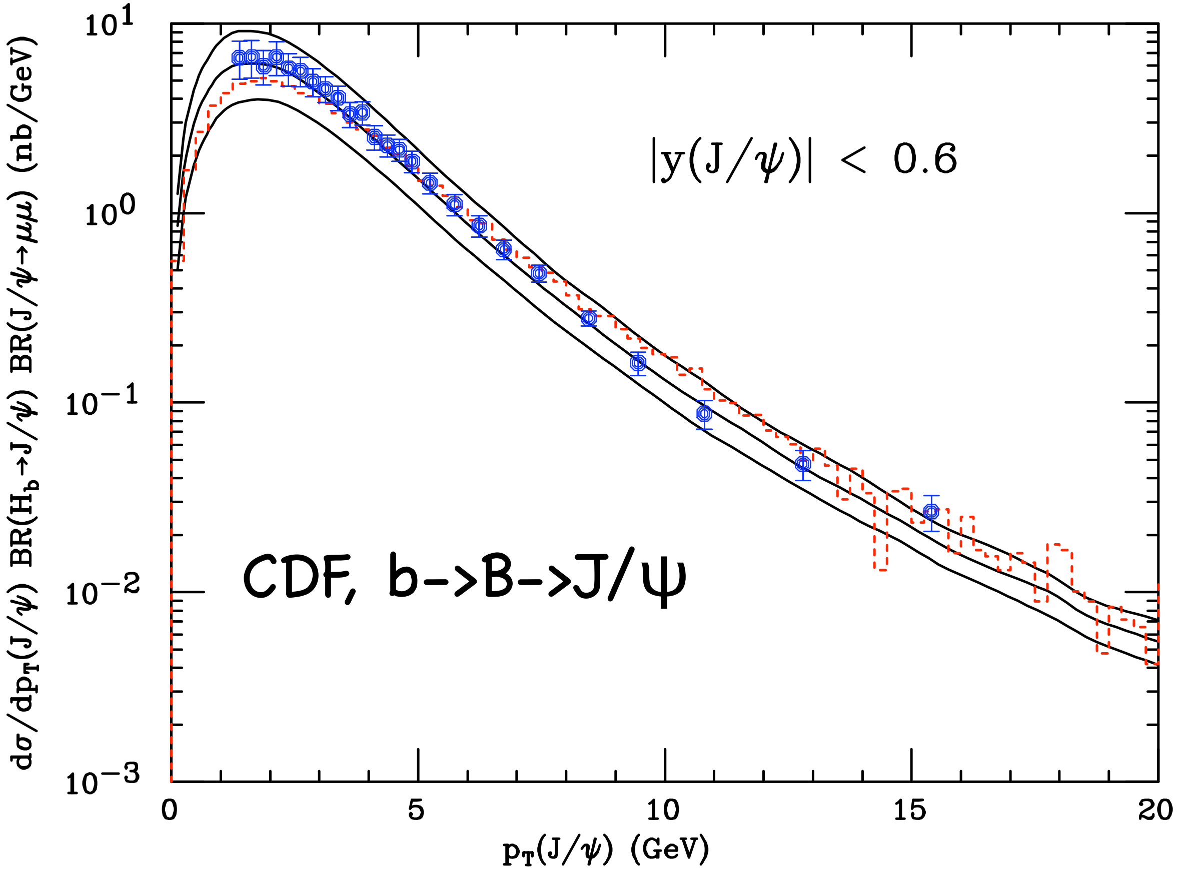
Does it work? Bottom production @ LEP 2



But.... data extrapolated to total cross section.

Not a real differential observable quantity

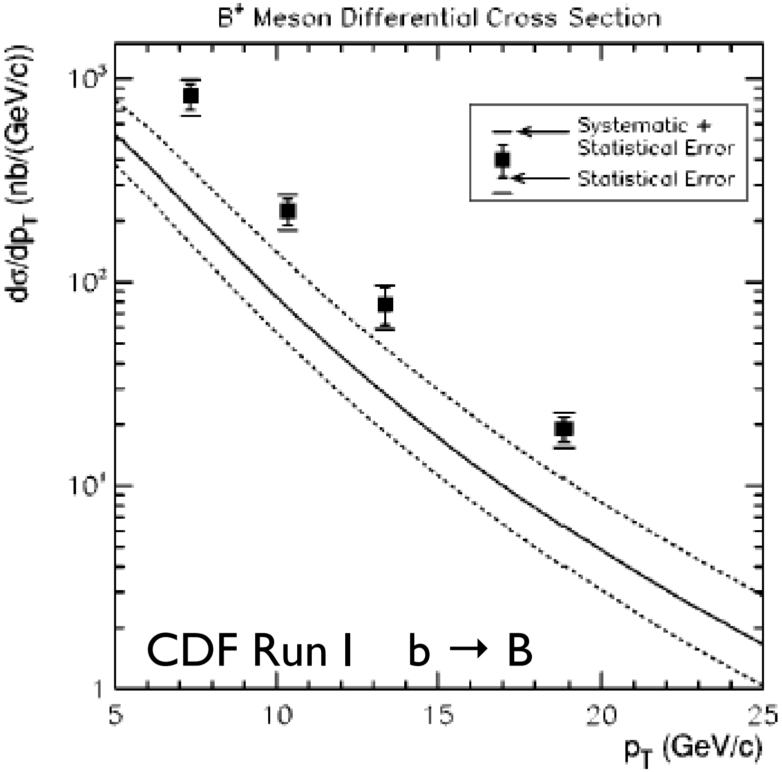
Does it work? Bottom production @ Tevatron Run 2



Lines: FONLL

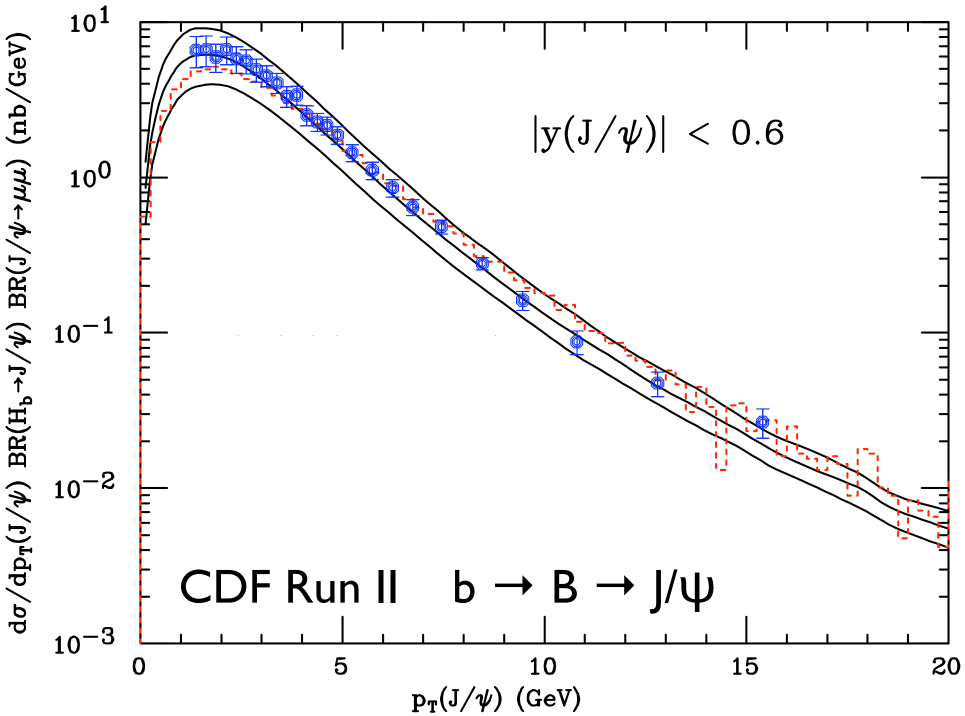
Histograms: MC@NLO

‘Before’



‘Factor of 3’ excess

‘After’



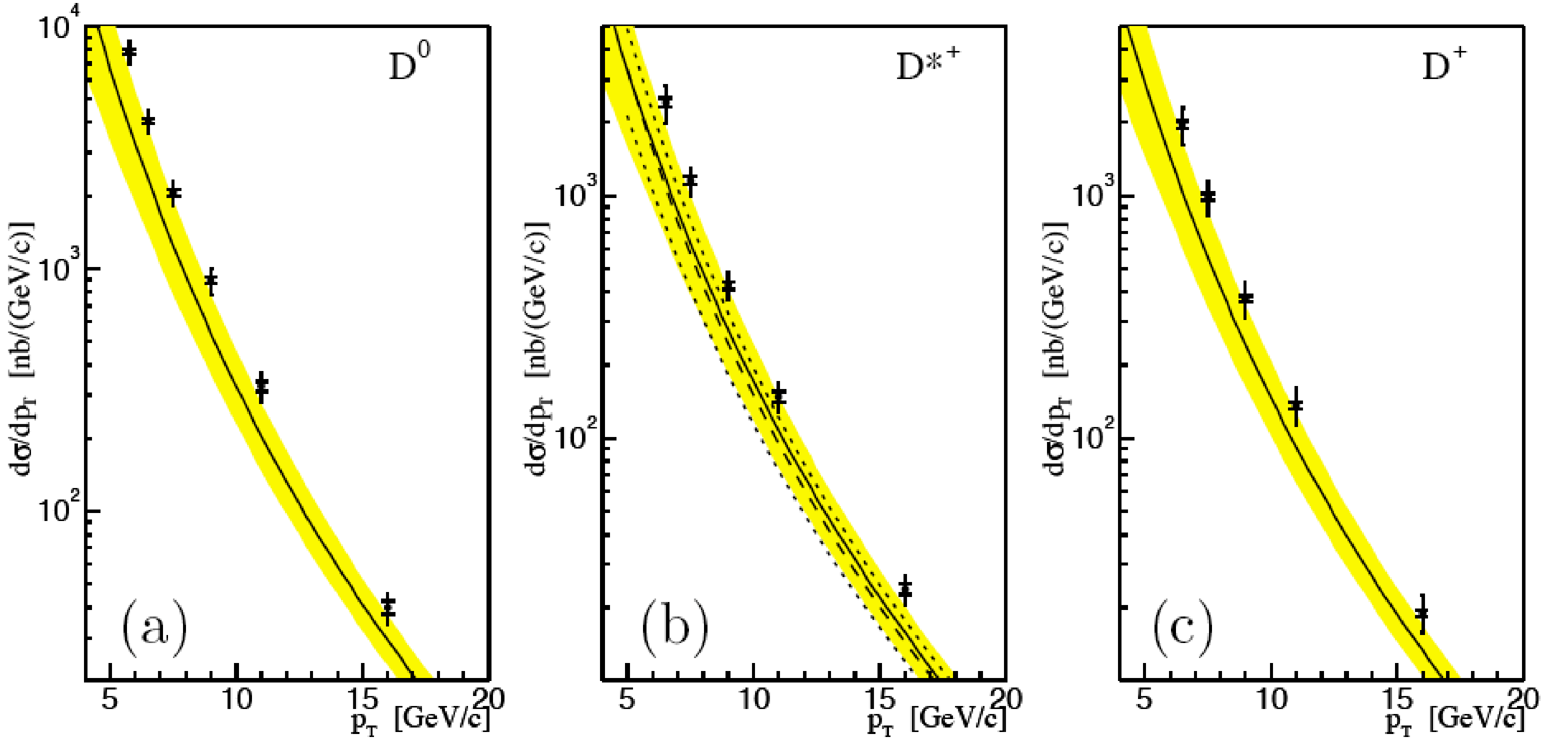
Perfect agreement

Key improvement:
b → B non-perturbative fragmentation properly extracted from LEP data within the FONLL framework

Does it work? Charm production @ Tevatron Run 2

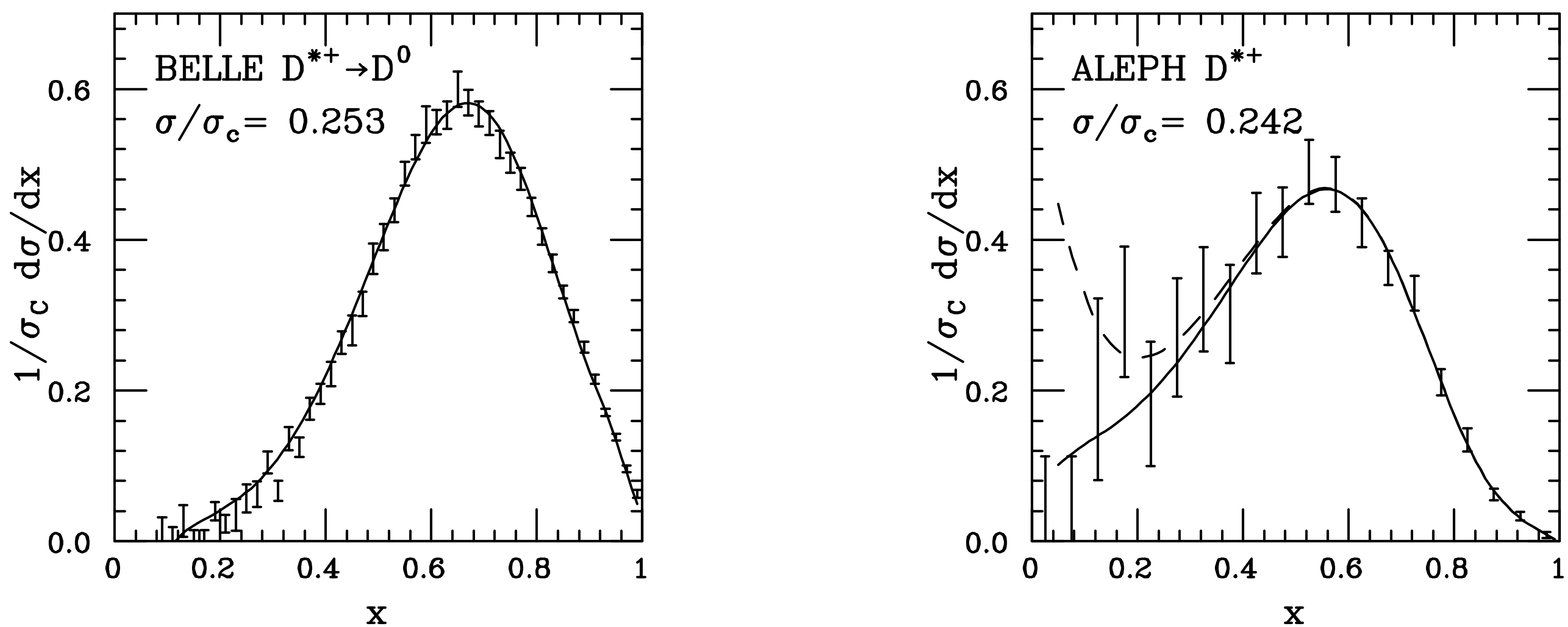
The same FONLL framework also provides a good description of charm production at the Tevatron:

CDF Run II   c->D data   [PRL 91:241804,2003]



The non-perturbative charm fragmentation needed to describe the c → D hadronization has been extracted from moments of ALEPH data at LEP. Now even better fragmentation data from CLEO and BELLE are available:

D* fragmentation from BELLE and ALEPH



These curves are made of two contributions:
a perturbative one convoluted with a non-perturbative term

The experimentally observed fragmentation is different at the two machines (of course!), but thanks to collinear resummation the extracted non-perturbative contributions coincide to within about 10%

==> not much uncertainty on the hadronic production rates

Did experimentalists and theorists converge?

Not much room to wiggle around:

The NLO calculation has been around for 15 years. With the addition of the NLL resummation, its perturbative uncertainty at large transverse momentum is not larger than a few 10%

The uncertainty from the PDFs should be fairly constrained. Say 10-15%

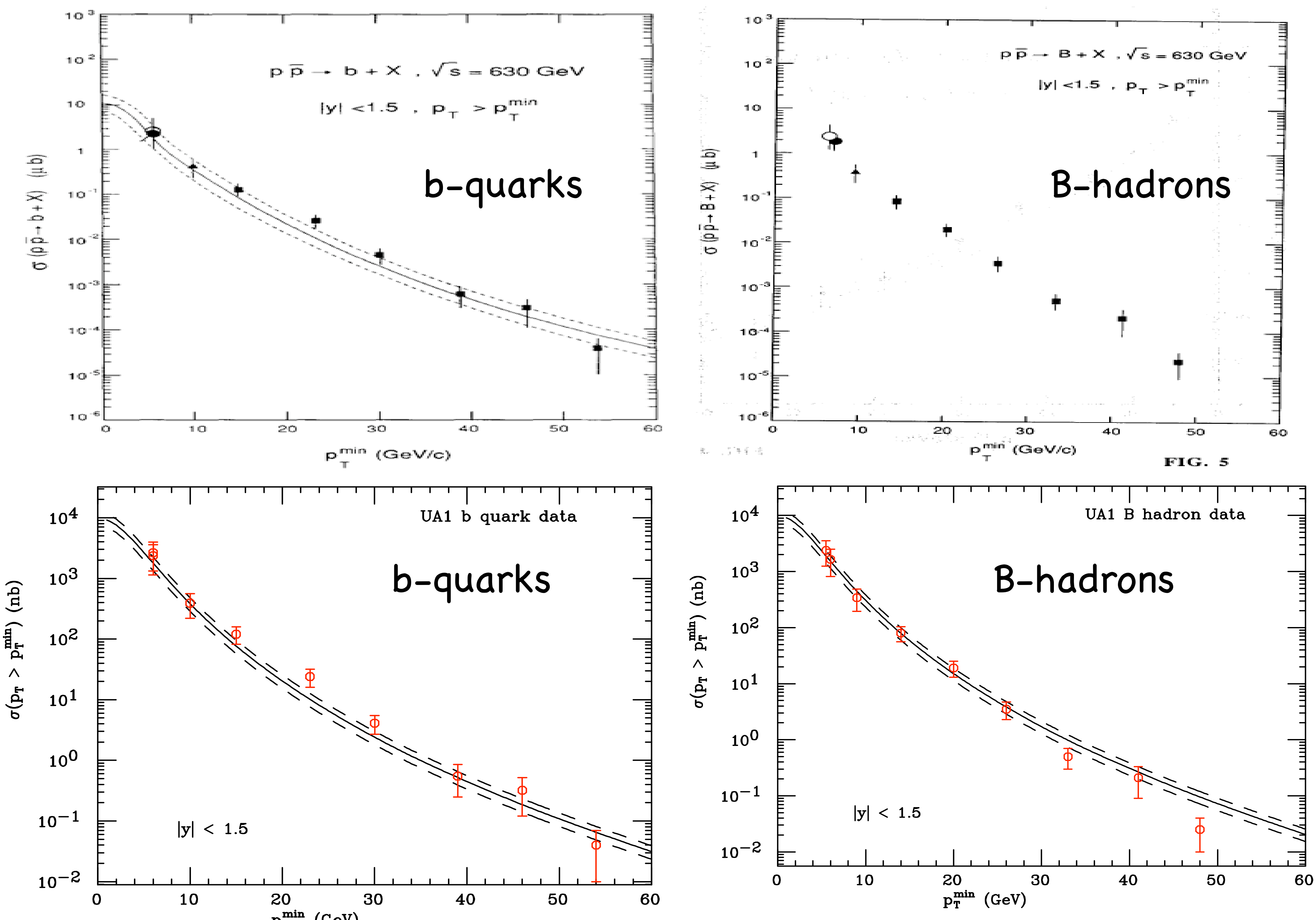
The non-perturbative fragmentation contribution is tightly constrained by e+e- data. It is definitely known to better than 10%

So, at large transverse momenta, where the theoretical framework is better under control, the overall uncertainty of the theoretical prediction should be smaller than 40-50%.

==> No room for factors of three discrepancies

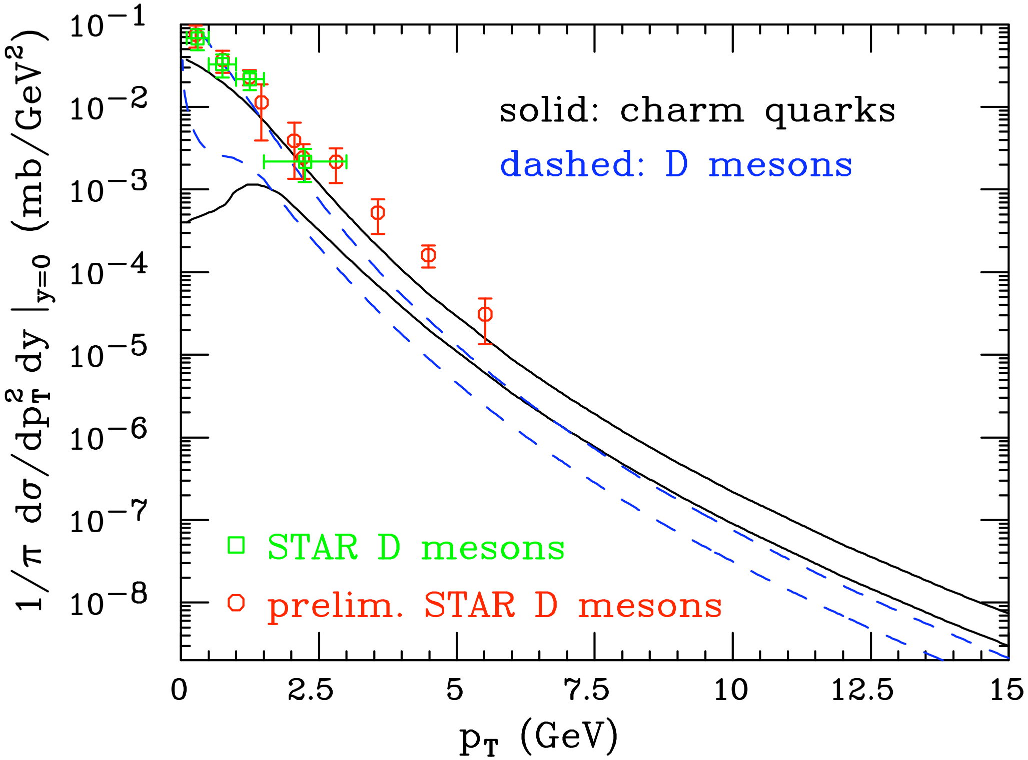
(BTW: the expt. accuracy is actually often better than the theoretical one!)

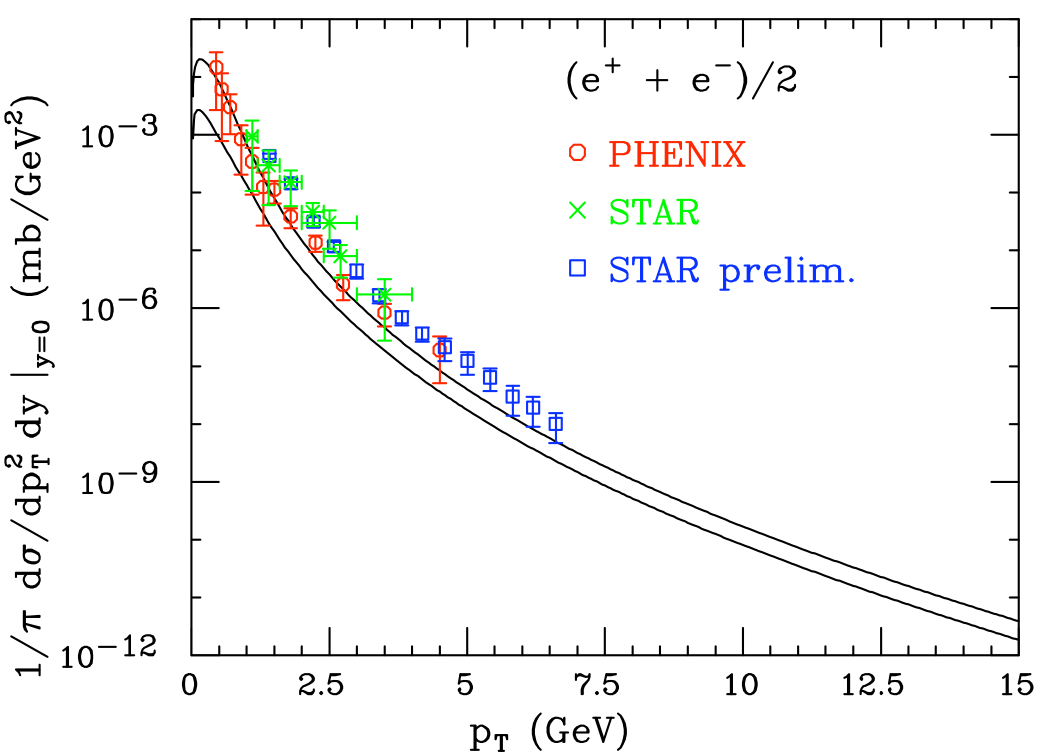
What about the old UA1 data?



Does it work? Charm and bottom production @ RHIC

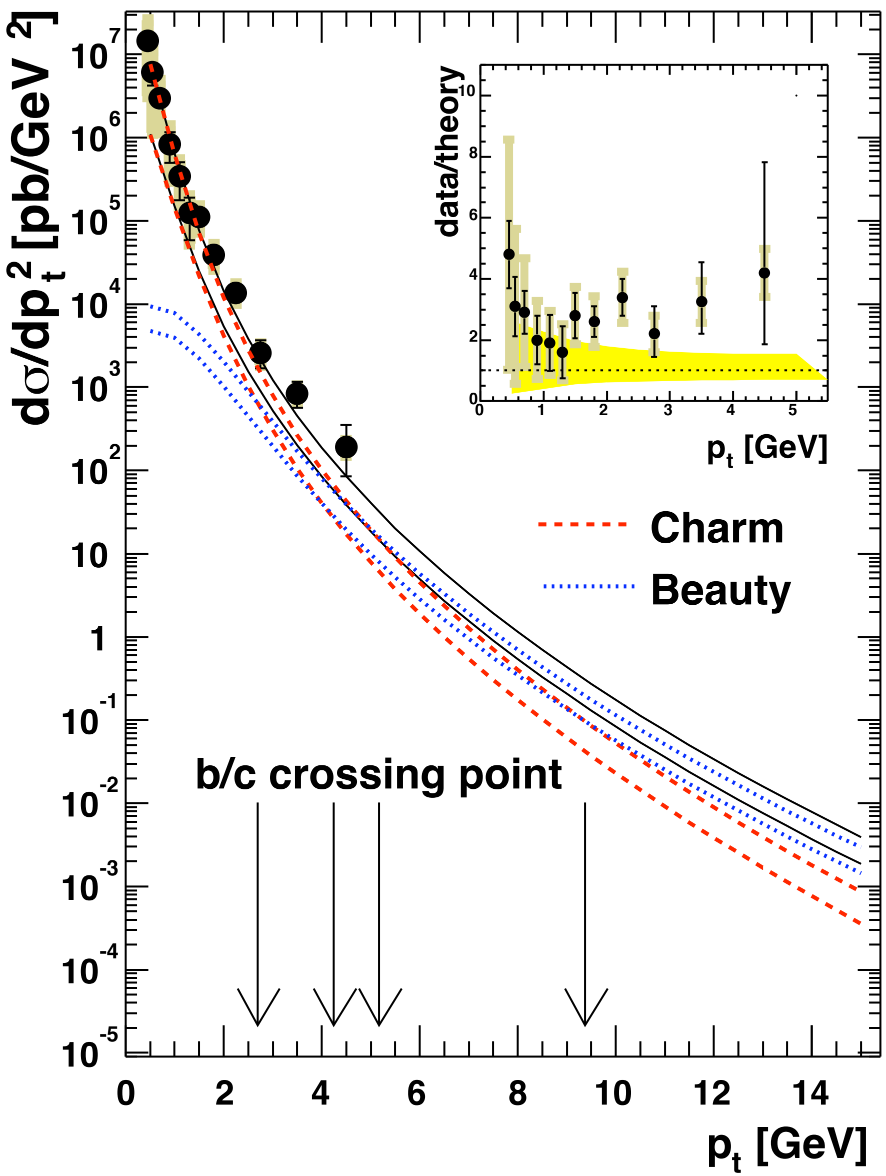
The very same FONLL formalism + NP FF can be applied at RHIC





Fair agreement. Note, though, the large uncertainties at low transverse momentum, especially for charm => we cannot claim to really have charm under control at NLO

Charm and bottom production @ RHIC

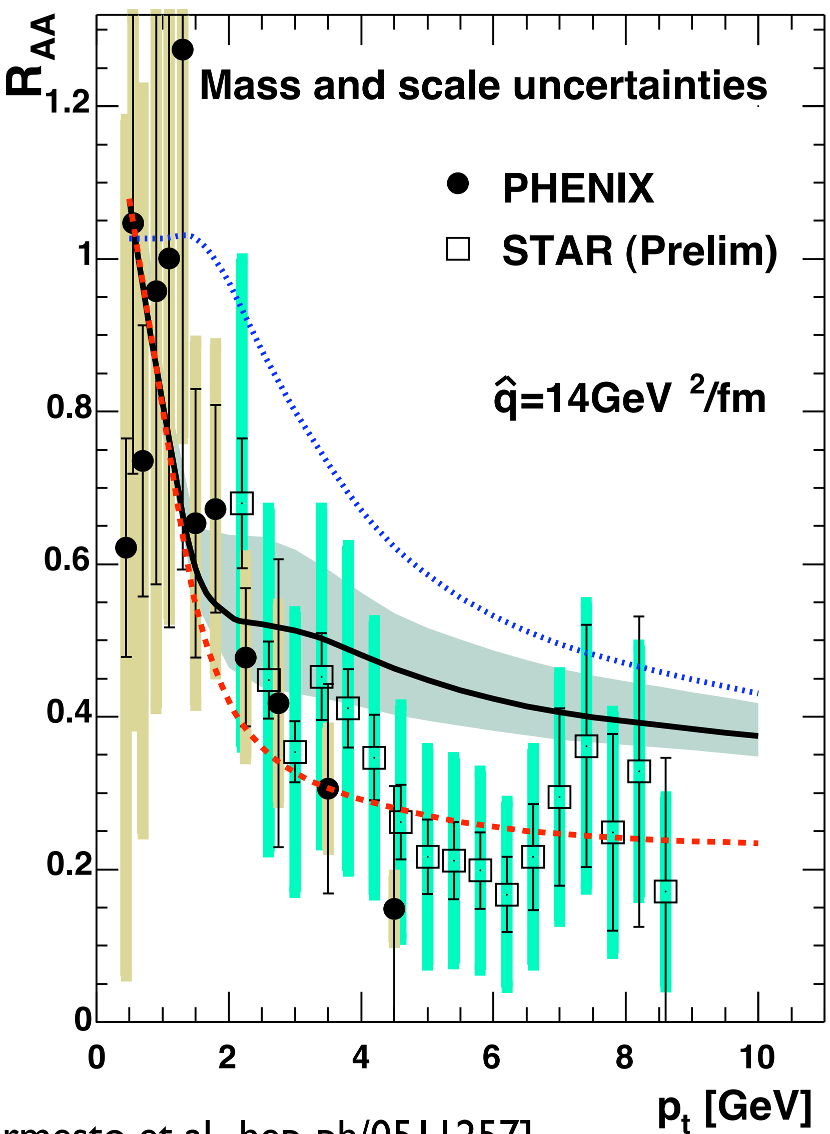


The slope of the charm and bottom contribution is fairly similar: the crossing point easily moves, though the relative contributions are less affected by uncertainties

NB. Especially for bottom the transverse momentum is small: all the further uncertainties previously mentioned can apply

R_AA for Charm and bottom @ RHIC

The charm and bottom spectra easily translate into R_AA via the application of quenching weights



[Armesto et al., hep-ph/0511257]

The uncertainty on the charm and bottom relative contribution reflects on an uncertainty of order 0.1 on R_AA

R_AA looks too high. However, remember the very large perturbative uncertainty on charm: the NNLO prediction could be quite larger. Observation: if you normalize charm to the data R_AA comes out about right

Conclusions

- Heavy quark phenomenology is mature and has the tools to produce predictions in many realistic situations. These predictions can include all the available knowledge for calculating heavy quark production in QCD. Since they are implemented in a rigorous framework, it is usually possible to also provide a (more or less reliable) estimate of the theoretical uncertainty

- Most predictions seem to agree well with Tevatron and HERA data for charm and bottom production. For Heavy Ions collisions they should provide a solid benchmark against which to compare in the search for nuclear effects

- Final note: given the size of intrinsic pQCD uncertainty, it is very unlikely that effects of the order of a few (tens of) percent will ever be visible just by comparing to the absolute value of the cross sections