NRQCD and Heavy-Quarkonium Production

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(For further details, see N. Brambilla *et al.*, arXiv:hep-ph/0412158.)

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Nonrelativistic QCD (NRQCD)

Heavy-Quarkonium: A Multi-Scale Problem

- Heavy quarkonium: a bound state of a heavy quark Q and heavy antiquark $\bar{Q}$ (charmonium, bottomonium).
- There are many important scales in in a heavy quarkonium:
  - m, the heavy-quark mass;
  - mv, the typical heavy-quark momentum;
  - mv^2, the typical heavy-quark kinetic energy and binding energy.
- $v \sim 1/R$ is the typical heavy-quark velocity in the quarkonium CM frame.
  - $v^2 \approx 0.3$ for charmonium.
  - $v^2 \approx 0.1$ for bottomonium.

- In theoretical analyses, it is useful to treat the physics at each of these scales separately.
  - $\alpha_s(m_c) \approx 0.25$ and $\alpha_s(m_b) \approx 0.18$, so we can treat physics at these scales perturbatively.
  - Approximate symmetries (*e.g.* heavy-quark spin symmetry) can be exploited at some scales.
  - Analytic calculations simplify when they involve only one scale at a time.
  - Lattice calculations can encompass only a limited range of scales, and so become more tractable after scale separation.
- Effective field theories provide a convenient way to separate scales.
  - Basic idea: construct an effective theory that describes the low-momentum degrees of freedom in the original (full) theory.
  - Do this by integrating out the high-momentum degrees of freedom in the original theory.
  - The high-momentum degrees of freedom are no longer manifest in the effective theory, but their effects on the low-momentum degrees of freedom are taken into account through the local interactions in the effective theory.

NRQCD

- In a nonrelativistic system, the scale m is distinct from mv and lower scales.
- Nonrelativistic QCD (NRQCD) separates scales of order m and higher from the other scales.
- Generalization of NRQED (W. E. Caswell, G. P. Lepage).
- The effective theory has a UV cutoff $\Lambda \sim m$.
- For processes with $p < \Lambda$, the effective theory reproduces full QCD.
- Processes with $p > \Lambda$ are not described in the effective theory, but they affect the coefficients of local interactions.

Construction of NRQCD

- In the path integrals for the amplitudes in QCD, integrate out:
  all light-quark and gluon modes with $|p_\mu| > \Lambda$,
  all the heavy-quark modes with $|E - m|$, $|p_i| > \Lambda$.
- Diagonalize the action in the heavy-quark and antiquark fields (Foldy-Wouthuysen tx.) and subtract m from the total energy.
- For the light-gluon–light-quark sector, the effective action is a cut-off version of the full action (e.g. lattice) plus "improvement" terms.
- Leading terms in $p/m = v$ in the heavy-quark sector are just the Schrödinger action.

$$\mathcal{L}_0 \;=\; \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2m} \right) \psi \;+\; \chi^\dagger \left(iD_t - \frac{\mathbf{D}^2}{2m} \right) \chi .$$

$$D_t = \partial_t + igA_0 . \qquad \mathbf{D} = \boldsymbol{\partial} - ig\mathbf{A} .$$

  - ψ is the Pauli spinor field that annihilates Q.
  - χ is the Pauli spinor field that creates $\bar{Q}$.

- To reproduce QCD completely, we would need an infinite number of interactions. For example, at next-to-leading order in v^2 we have

$$\begin{aligned}\delta\mathcal{L}_{\text{bilinear}} &= \frac{c_1}{8m^3}\left[\psi^\dagger(\mathbf{D}^2)^2\psi \;-\; \chi^\dagger(\mathbf{D}^2)^2\chi\right] \\ &+ \frac{c_2}{8m^2}\left[\psi^\dagger(\mathbf{D}\cdot g\mathbf{E}-g\mathbf{E}\cdot\mathbf{D})\psi+\chi^\dagger(\mathbf{D}\cdot g\mathbf{E}-g\mathbf{E}\cdot\mathbf{D})\chi\right] \\ &+ \frac{c_3}{8m^2}\left[\psi^\dagger(i\mathbf{D}\times g\mathbf{E}-g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\psi+\chi^\dagger(i\mathbf{D}\times g\mathbf{E}-g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\chi\right] \\ &+ \frac{c_4}{2m}\left[\psi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\psi \;-\; \chi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\chi\right].\end{aligned}$$

- In practice, work to a given precision in v.
- The c_i are called short-distance coefficients.
  - They can be computed in perturbation theory by matching amplitudes in full QCD and NRQCD.
  - By design, all of the low-scale physics is contained in the explicit NRQCD interactions.
  - The c_i's contain the effects from momenta $> \Lambda$.
- Λ plays the rôle of a factorization scale between the hard and soft physics.
- Determine the c_i's by matching amplitudes on shell.
  - Required because of the use of field re-definitions (equations of motion).
  - Convenient because it makes the matching gauge invariant.

The NRQCD Factorization Approach in Quarkonium Production (and Decays)

(GTB, E. Braaten, G. P. Lepage)

Factorization: a Separation of Scales

- In heavy-quarkonium hard-scattering production (and decays), large scales appear: Both the heavy-quark mass m and p_T are much larger than $\Lambda_{\rm QCD}$.
- Hope: Because of the large scales, asymptotic freedom will allow us to do perturbation theory.

$$\alpha_s(m_c) \approx 0.25; \qquad \alpha_s(m_b) \approx 0.18.$$

- But there are clearly low-momentum, nonperturbative effects in the heavy-quarkonium dynamics.
- We wish to separate the short-distance/high-momentum, perturbative effects from the long-distance/low-momentum, nonperturbative effects.
- This separation is known as “factorization.”

Factorization of the Inclusive Production Cross Section

Evolution of a $Q\bar{Q}$ Pair into a Quarkonium

- The probability for a $Q\bar{Q}$ pair to evolve into a heavy quarkonium can be calculated as a vacuum-matrix element in NRQCD. For example:

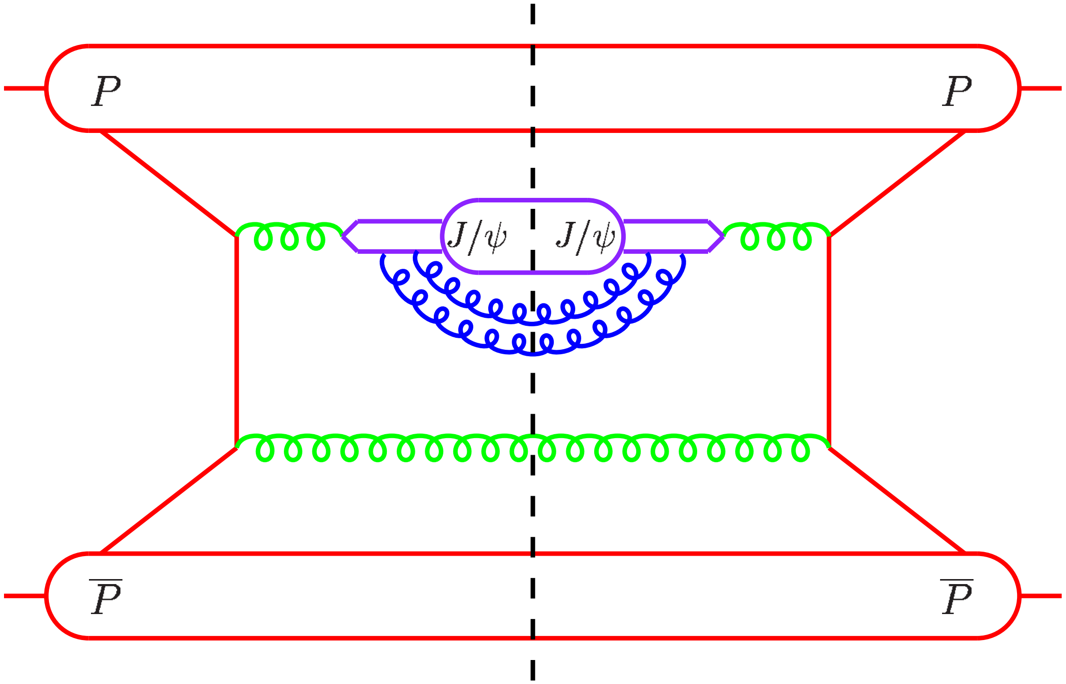
$$
\begin{aligned}
\mathcal{O}_1^H({}^1S_0) &= \langle 0|\chi^\dagger\psi\left(\sum_X |H+X\rangle\langle H+X|\right)\psi^\dagger\chi|0\rangle,\\
\mathcal{O}_1^H({}^3S_1) &= \langle 0|\chi^\dagger\boldsymbol{\sigma}\psi\cdot\left(\sum_X |H+X\rangle\langle H+X|\right)\psi^\dagger\boldsymbol{\sigma}\chi|0\rangle,\\
\mathcal{O}_8^H({}^1S_0) &= \langle 0|\chi^\dagger T^a\psi\cdot\left(\sum_X |H+X\rangle\langle H+X|\right)\psi^\dagger T^a\chi|0\rangle,\\
\mathcal{O}_8^H({}^3S_1) &= \langle 0|\chi^\dagger\boldsymbol{\sigma}T^a\psi\cdot\left(\sum_X |H+X\rangle\langle H+X|\right)\psi^\dagger\boldsymbol{\sigma}T^a\chi|0\rangle.
\end{aligned}
$$

- These are the matrix element of a four-fermion operator, but with a projection onto an intermediate state of the quarkonium H plus anything.
- The quarkonium evolves from color-octet, as well as color-singlet $Q\bar{Q}$ states.

Factorization Conjecture

- Conjecture (GTB, Braaten, Lepage): The inclusive cross section for producing quarkonium at large momentum transfer (p_T) can be written as hard-scattering cross section convolved with an NRQCD matrix element.

$$\sigma(H) = \sum_n \frac{F_n(\Lambda)}{m^{d_n-4}} \langle 0 | \mathcal{O}_n^H(\Lambda) | 0 \rangle .$$



- The “short-distance” coefficients $F_n(\Lambda)$ are essentially the process-dependent partonic cross sections to make a $Q\bar{Q}$ pair convolved with the parton distributions.

- Asymptotic freedom: The short-distance coefficients have an expansion in powers of α_s.
- They are insensitive to changes in the $Q\bar{Q}$ momentum of order m.
  - Implies that the $Q\bar{Q}$ bilinears in the matrix elements are at the same point (to within $\sim 1/m$).
  - Corrections to this are taken into account by including operators of higher order in v.
- The operator matrix elements are universal (process independent).
  - This gives NRQCD factorization much of its predictive power.
- The matrix elements have a known scaling with v.
- At leading orders in v, there are simplifying relations between operator matrix elements:
  - heavy-quark spin symmetry (order-v^2 corrections)
  - vacuum-saturation approximation (order-v^4 corrections).
- The NRQCD factorization formula for production is a double expansion in powers of α_s and v.
  - In practice, one truncates the series at a given level of precision.

Status of a Proof of Factorization

- A proof of factorization would involve a demonstration that
  - all soft singularities cancel or can be absorbed into NRQCD matrix elements,
  - all collinear singularities and spectator interactions can be absorbed into parton distributions.
- Nayak, Qiu, Sterman: The NRQCD matrix elements must be modified by the inclusion of eikonal lines to make them gauge invariant.
  - The eikonal lines are path integrals of the gauge field running from the annihilation points to infinity.
  - Essential at two-loop order to allow certain soft contributions to be absorbed into the matrix elements.
  - Does not affect existing phenomenology, which is at tree order or one-loop order.
- Factorization of the inclusive cross section beyond one-loop order is still an open question.
- Factorization at low p_T or for the cross section integrated over p_T is doubtful.
- If factorization holds at large p_T, then corrections are probably of order
  - m^2/p_T^2 for unpolarized cross sections,
  - m/p_T for polarized cross sections.

- Plausible arguments have been given for the correctness of a similar factorization formula for quarkonium decays.
- The production matrix elements are the crossed versions of quarkonium decay matrix elements.
  - Only the color-singlet production and decay matrix elements are simply related.
- NRQCD factorization for production relies on
  - NRQCD,
  - hard-scattering factorization.
- Comparisons with experiment test both.

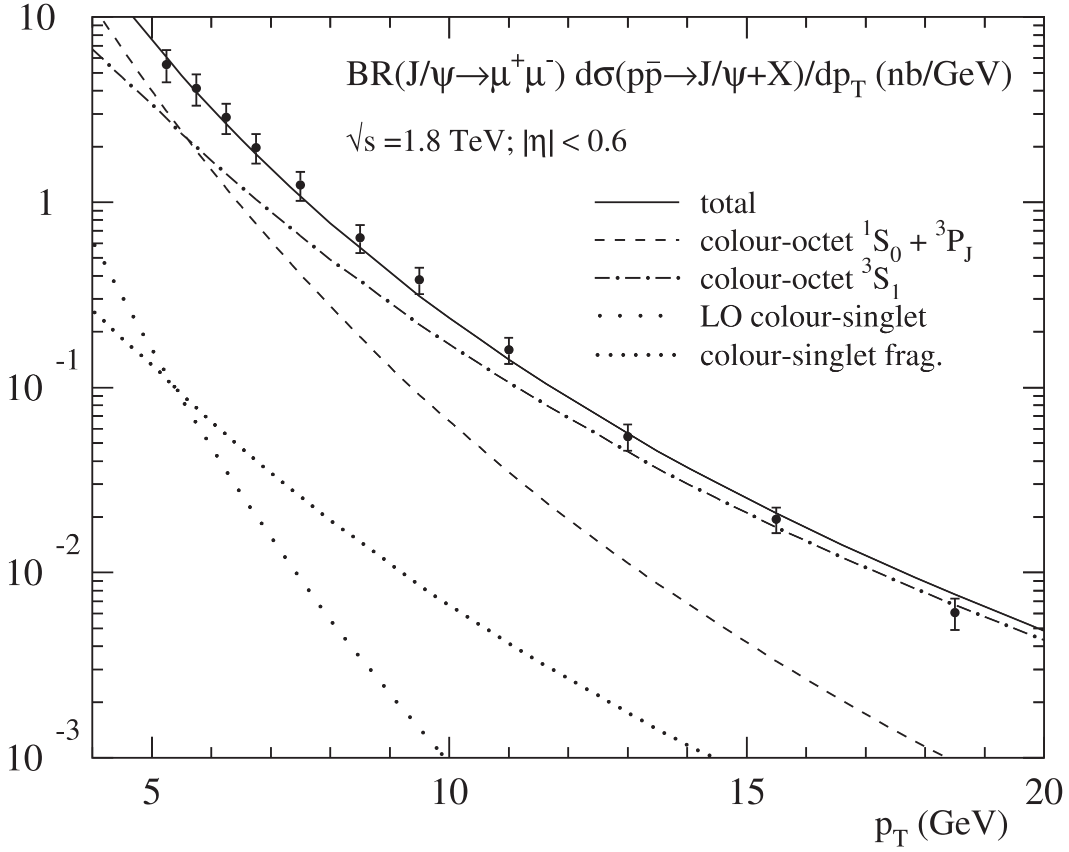
NRQCD Factorization and the Color-Singlet Model

- A key feature of NRQCD factorization:
  Quarkonium production can occur through color-octet, as well as color-singlet, $Q\bar{Q}$ states.
- If we drop all of the color-octet contributions and retain only the leading-in-v color-singlet contribution, then we have the color-singlet model (CSM).
  - Inconsistent for P-wave production: IR divergent.

Some Successes of the NRQCD Factorization Method

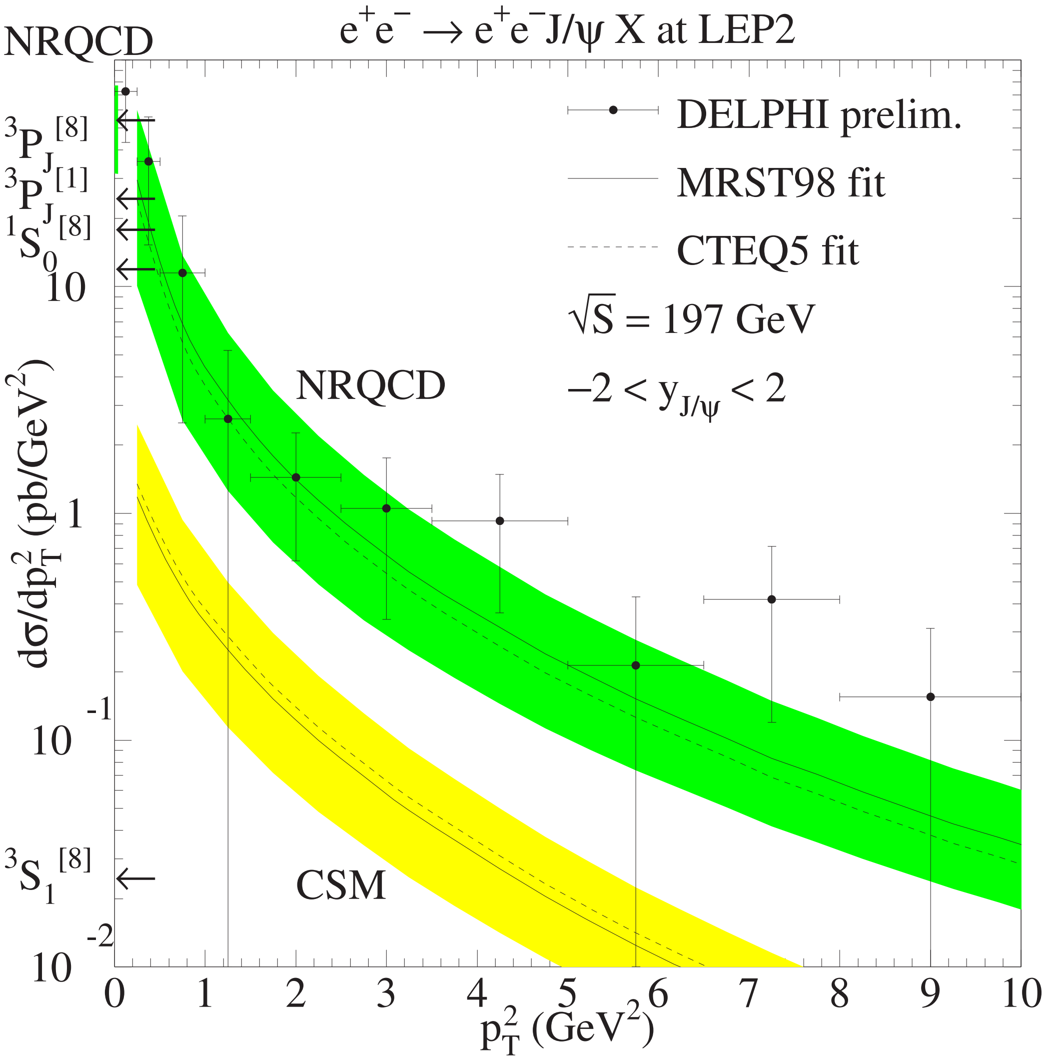
Quarkonium Production at the Tevatron

- Explanation (color-octet mechanism) of Tevatron data for J/ψ, ψ', Υ production.



- Data are more than an order of magnitude larger than the predictions of the color-singlet model.
- Color-octet matrix elements are determined from fits to the data.
- p_T distributions are consistent with NRQCD.

$\gamma\gamma \rightarrow J/\psi + X$ at LEP

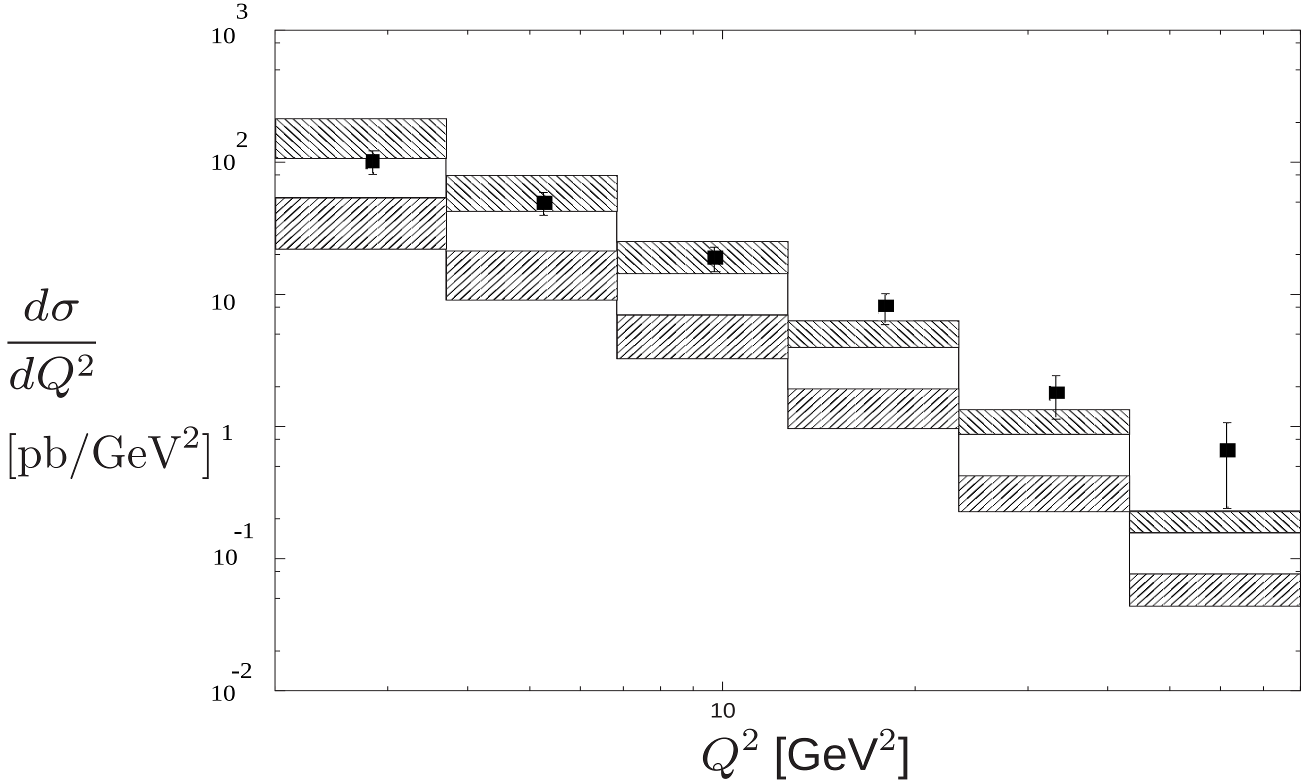


- Comparison of theory (Klasen, Kniehl, Mihaila, Steinhauser) with Delphi data clearly favors NRQCD over the color-singlet model.
- Theory uses Braaten-Kniehl-Lee matrix elements from Tevatron data and MRST98LO (solid) and CTEQ5L (dashed) PDF's.
- Theoretical uncertainties from
  - Renormalization and factorization scales (varied by a factor 2),
  - NRQCD color-octet matrix elements.
- Different linear combination of matrix elements than in Tevatron cross sections.

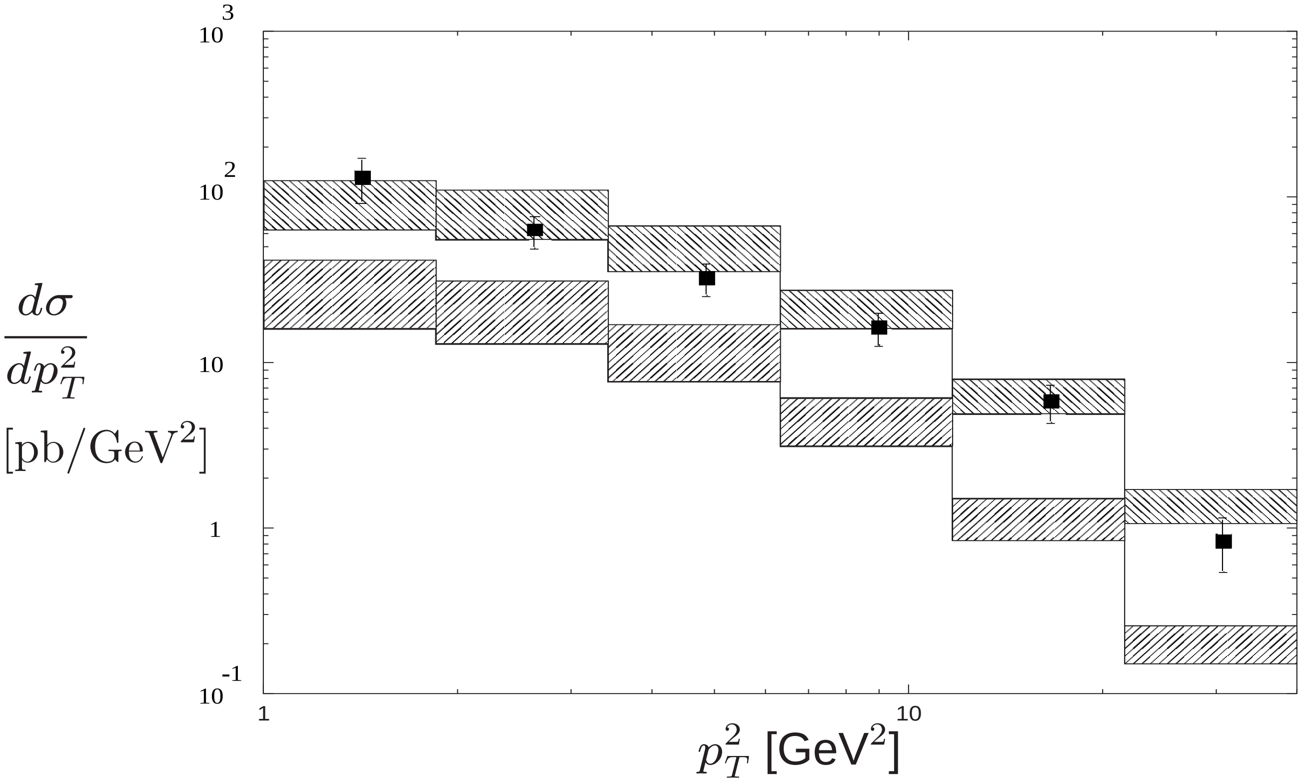
Quarkonium Production in DIS at HERA

- The NRQCD (Kniehl, Zwirner) prediction uses Braaten-Kniehl-Lee matrix elements extracted from the Tevatron data and MRST98LO and CTEQ5L PDF's.
- Theoretical uncertainties from
  - PDF's,
  - Renormalization and factorization scales (varied by a factor 2),
  - NRQCD color-octet matrix elements.
    - Different linear combination of matrix elements than in Tevatron cross sections.
- The calculation of Kniehl and Zwirner disagrees with a number of previous results. These disagreements have not yet been resolved fully.

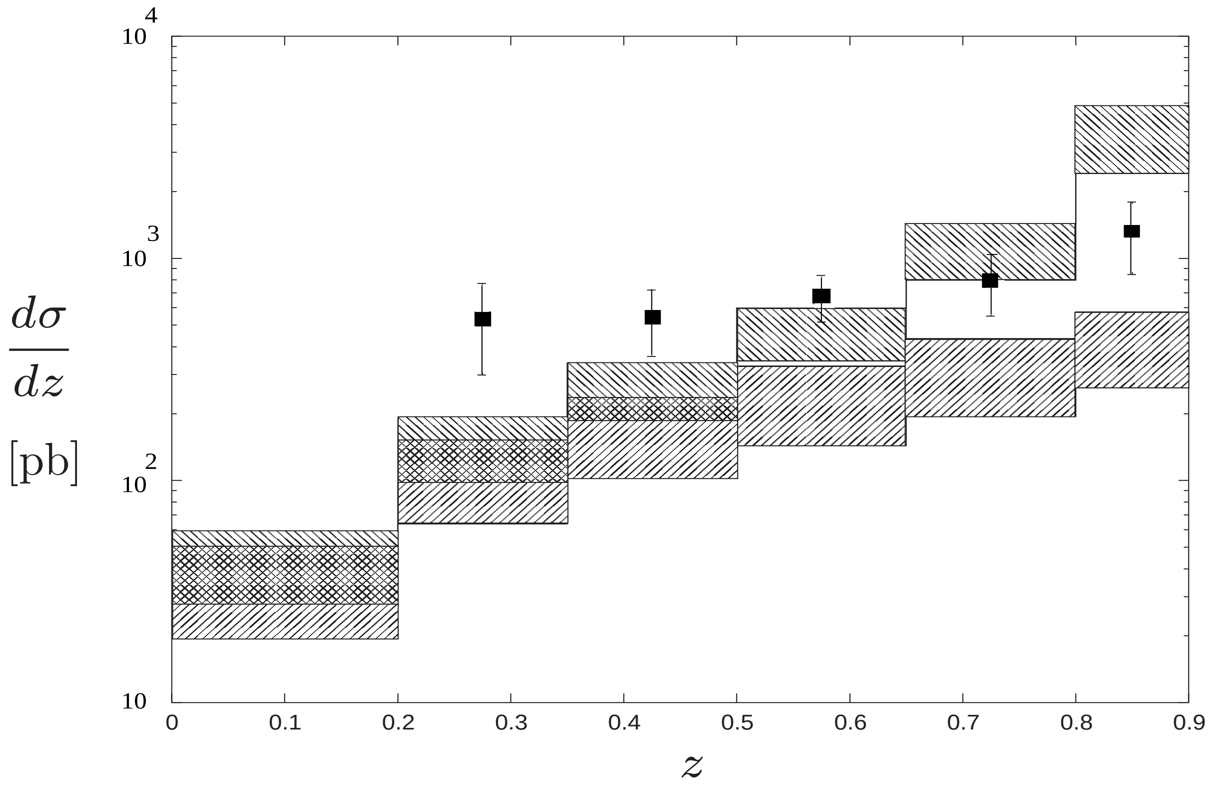
- The NRQCD prediction is favored over the color-singlet-model prediction by the H1 data when plotted vs. Q^2 and p_T^2, but not z.



H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).

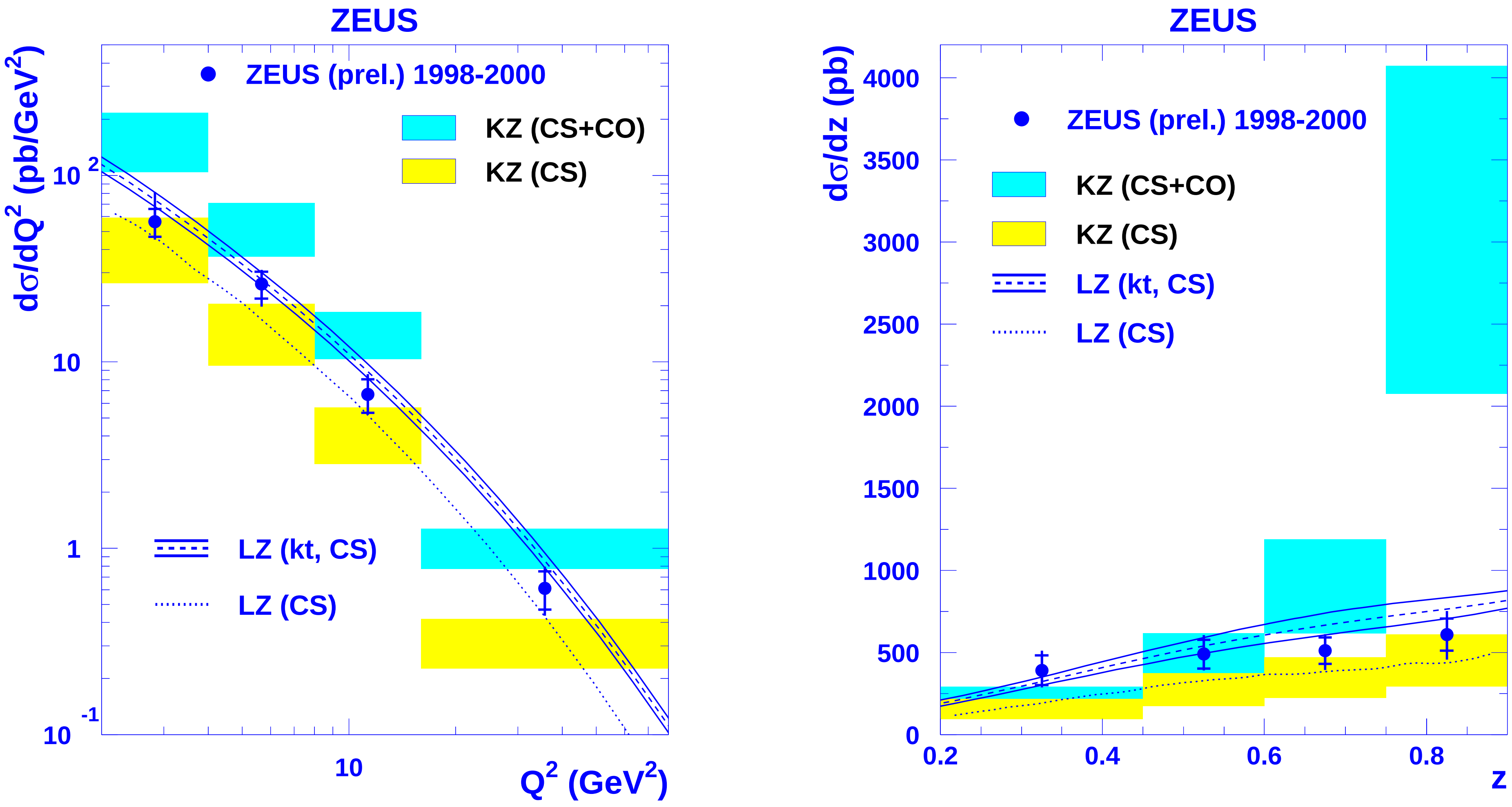


H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).



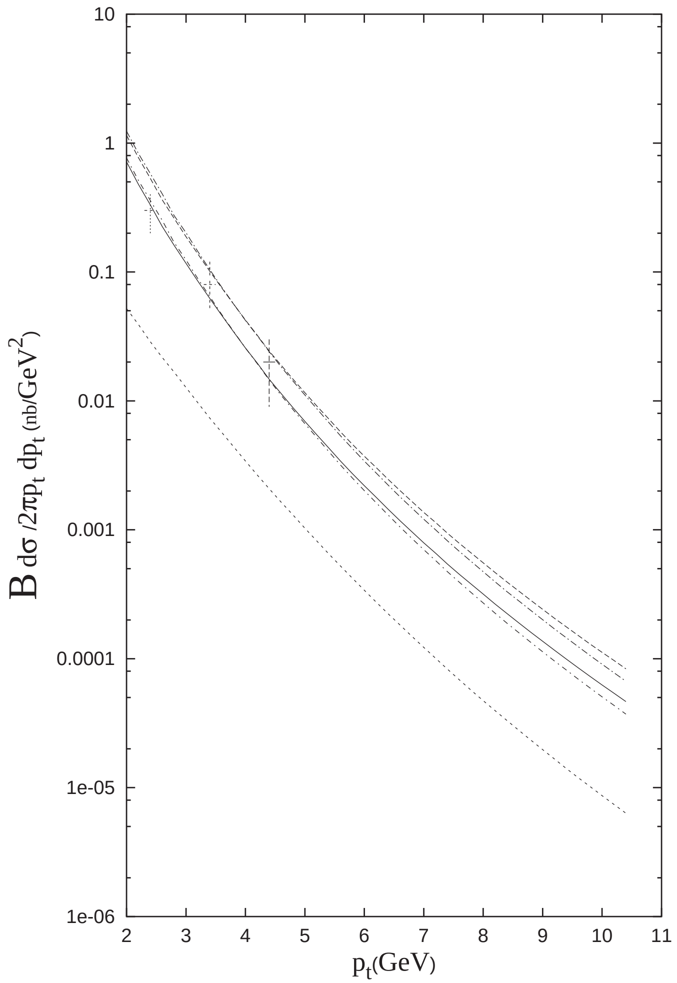
H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).

- On the other hand, the ZEUS data plotted as a function of Q^2 agree less well with the NRQCD prediction (but have larger error bars).



Curves from A.V. Lipatov and N.P. Zotov.

Quarkonium Production in pp Collisions at RHIC



- Solid line: color-octet, GRV98, large M.E.'s
- Upper dashed line: color-octet, GRV 98, small M.E.'s
- Upper dot-dashed line: color-octet, MRST98, large M.E.'s
- Lower dot-dashed line: color-octet, MRST98, small M.E.'s
- Lower dashed line: color-singlet

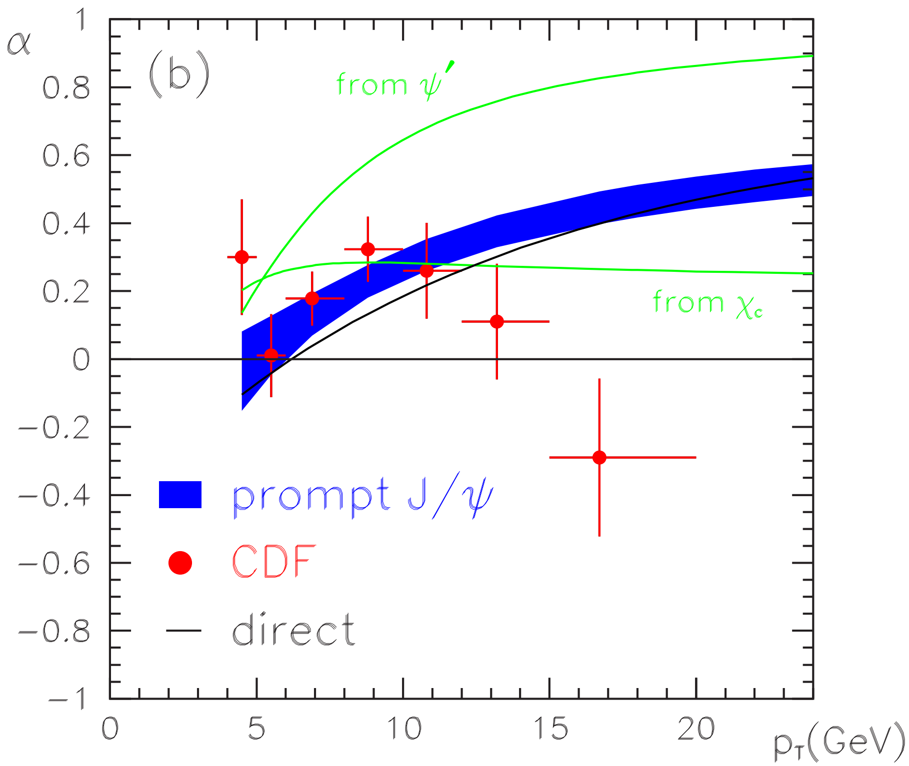
- PHENIX data for pp collisions at $\sqrt{s} = 200$ GeV.
- Theoretical calculation by Cooper, Liu, Nayak.
- Uses Cho-Leibovich NRQCD matrix elements extracted from Tevatron data.
- Low p_T and low statistics, but NRQCD factorization is clearly favored over the CSM.

Some Problematic Comparisons with Experiment

Polarization of Quarkonium at the Tevatron

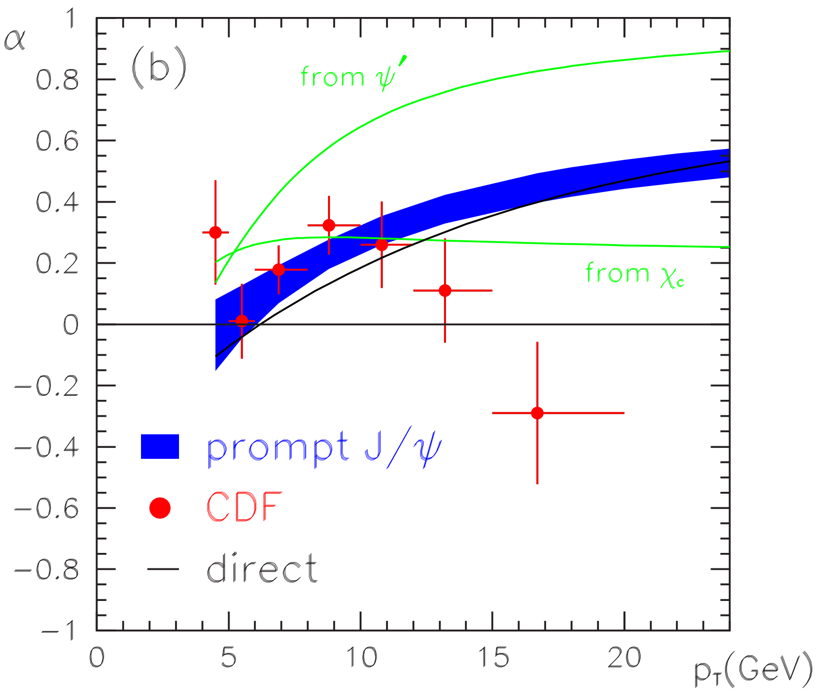
- Potentially a "smoking gun" for the color-octet mechanism.
- For large-p_T quarkonium production ($p_T \gtrsim 4m_c$ for J/ψ), gluon fragmentation via the color-octet mechanism dominates ($\langle\mathcal{O}_8(^3S_1)\rangle$).
- At large p_T, the gluon is nearly on mass shell, and, so, is transversely polarized.
- NRQCD predicts that spin-flip interactions are suppressed: Most of the gluon's polarization is transferred to the J/ψ. (Cho, Wise)
- Radiative corrections, color-singlet production dilute this. (Beneke, Rothstein; Beneke, Krämer)
- In the J/ψ case, feeddown is important, but has now been taken into account. (Braaten, Lee)
  - Feeddown from χ_c states is about 30% of the J/ψ sample and dilutes the polarization.
  - Feeddown from ψ' is about 10% of the J/ψ sample and is largely transversely polarized.

Run I data:

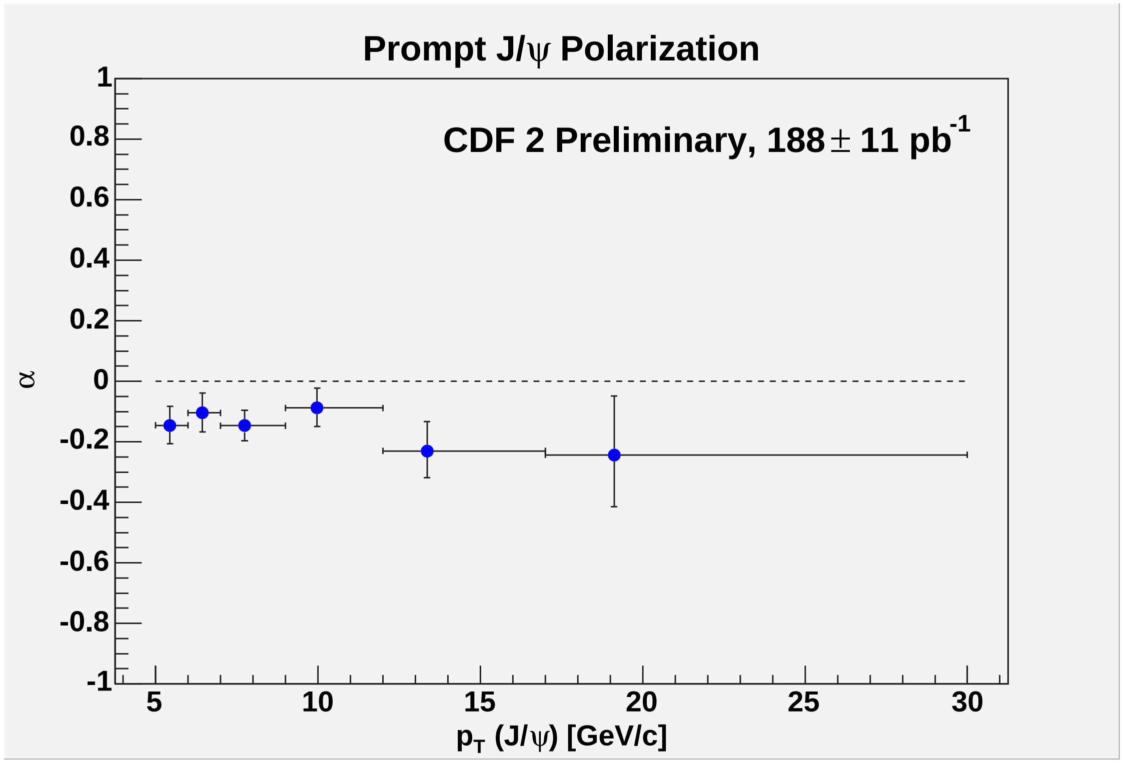


- $d\sigma/d(\cos\theta) \propto 1 + \alpha\cos^2\theta$.
  - $\alpha = 1$ is completely transverse;
  - $\alpha = -1$ is completely longitudinal.

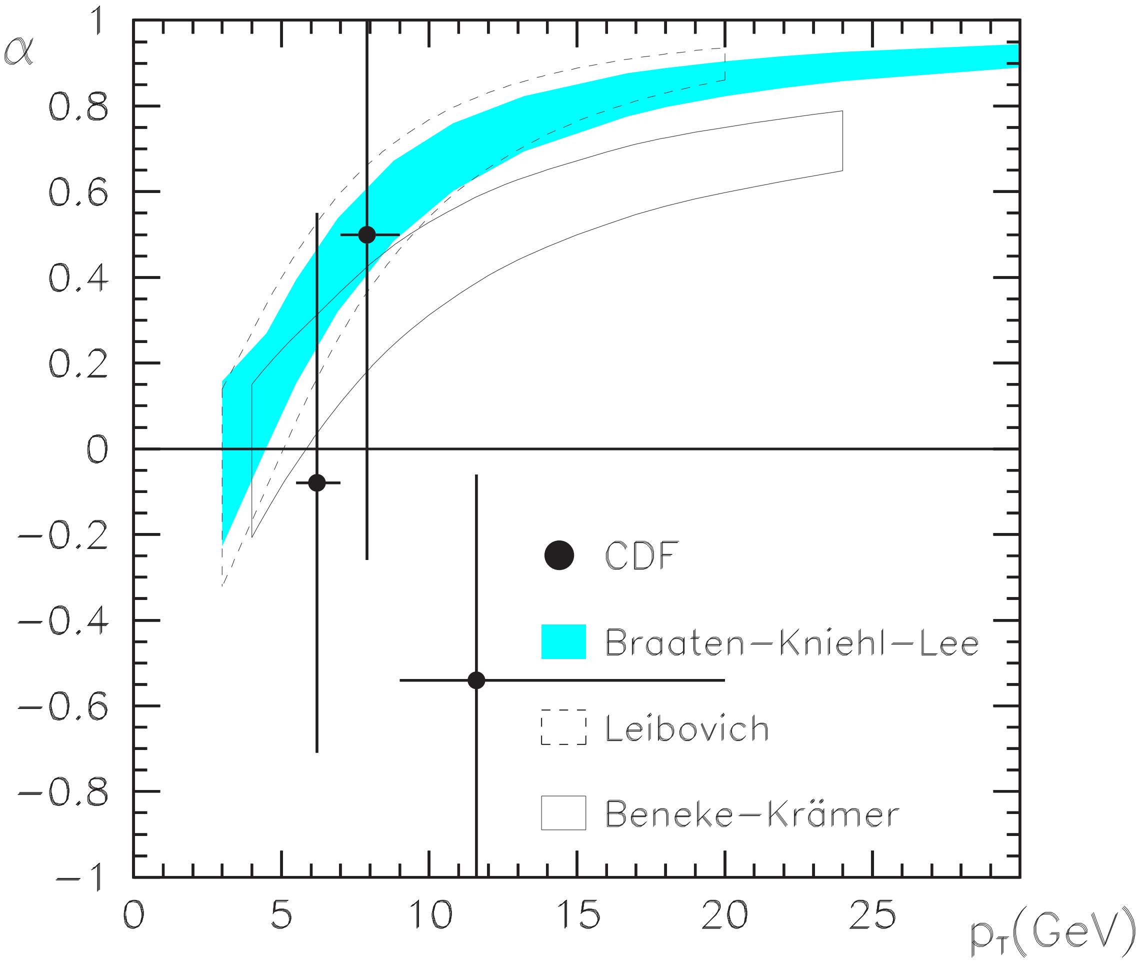
Run I data:



Run II data:



- In the ψ' case, feeddown is not important, but statistics are not as good.



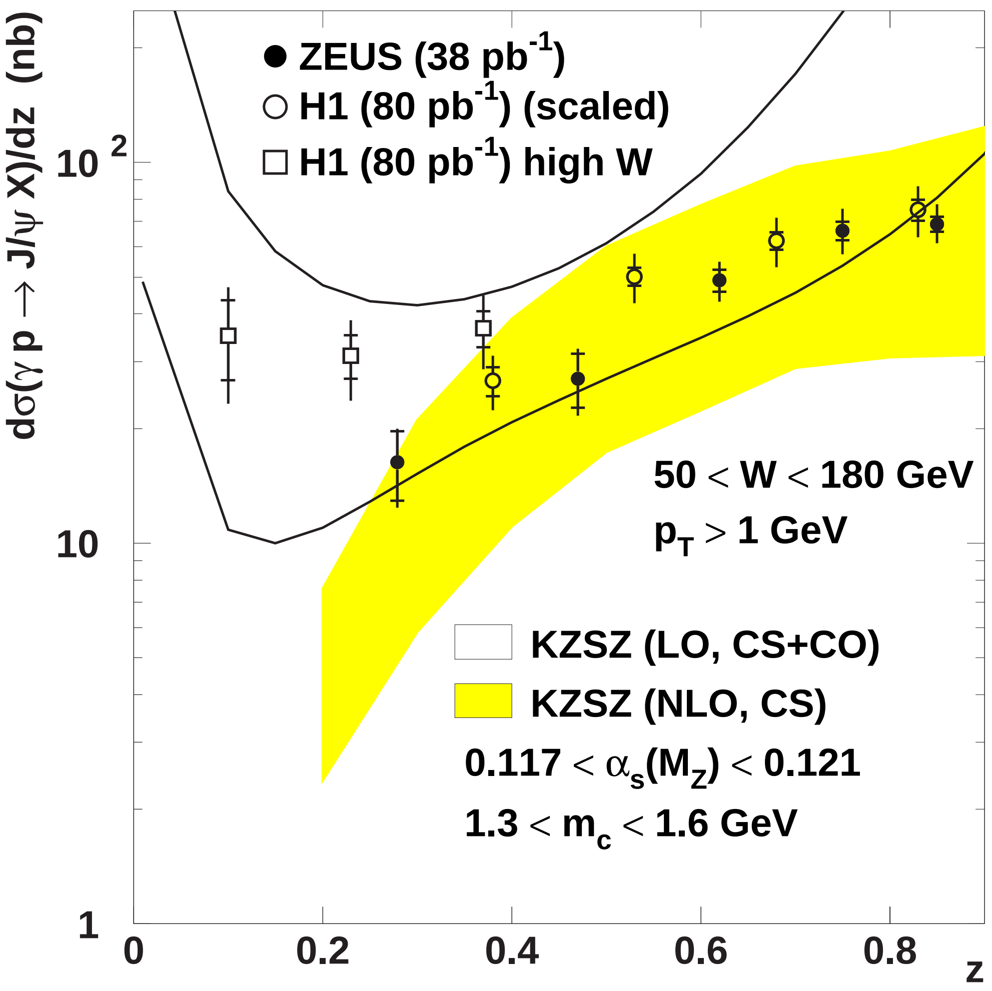
- The observed Run I J/ψ and ψ' polarizations are smaller than the predictions at large p_T and seem to decrease with p_T, but the error bars are large.
  - Only the highest-p_T data points are incompatible with the theory.
- The Run II data show no hint of the expected transverse polarization.
  - Many points are incompatible with the theory (and with Run I data).

There are many sources of theoretical uncertainty:

- Uncertainties in matrix elements (shown in plots)
- Contributions of higher order in α_s
  - Calculated for ${}^3S_1$ color-octet fragmentation
    (Braaten, Lee), which gives the bulk of the polarization.
  - Corrections to the non-fragmentation process could conceivably increase the unpolarized contribution by a factor 2.
- Multiple soft-gluon emission
  - Polarization depends on a ratio of processes.
  - Effects of multiple soft-gluon emission tend to cancel.
- Large order-v^2 corrections to gluon fragmentation to quarkonium. (GTB, Lee)
  - $+50\%$ for the color-singlet part.
    Yields a small correction to total the rate.
  - -40% for the color-octet part.
    Changes the normalization of the fitted matrix element, but not the rate.
  - Does the v expansion converge?

- Existing calculations assume that 100% of the $Q\bar{Q}$ polarization is transferred to the quarkonium.
  - Spin-flip corrections are suppressed only by v^2, not v^4, relative to the non-flip part. (GTB, Braaten, Lepage)
  - It could happen that the spin-flip corrections are anomalously large.
  - Do the velocity-scaling rules need to be modified?
    (Brambilla, Pineda, Soto, Vairo; Fleming, Rothstein, Leibovich)
  - A lattice calculation of color-octet decay matrix elements indicates that spin-flip processes are indeed suppressed by a factor v^2 or smaller. (GTB, Lee, Sinclair)
- It is difficult to see how there could not be substantial polarization in J/ψ or ψ' production for $p_T > 3m_c$.

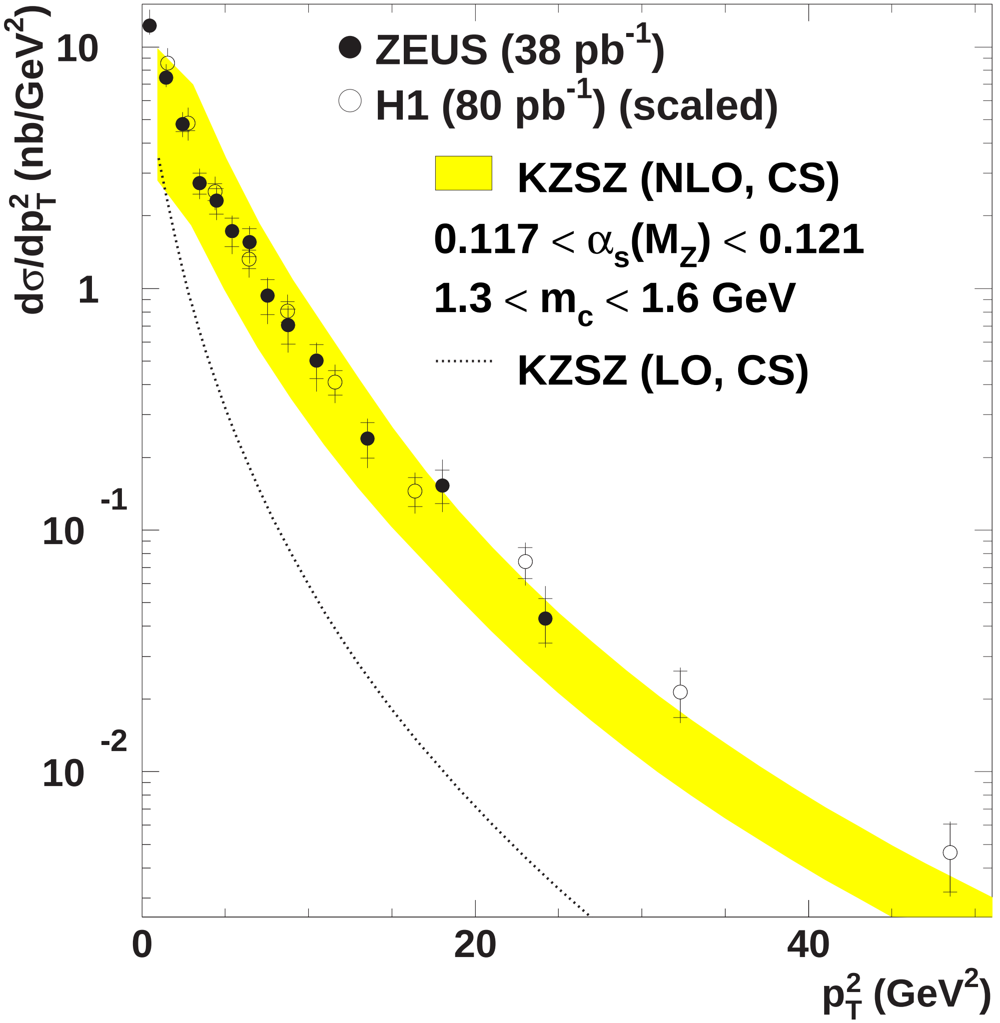
Inelastic Quarkonium Photoproduction at HERA



- NRQCD calculations by Cacciari, Krämer; Amundson, Fleming, Maksymyk; Ko, Lee, Song; Kniehl, Krämer.
- NLO CSM calculations by Krämer; Krämer, Zunft, Steegborn, Zerwas.

- There seems to be little room for the color-octet contribution in the photoproduction data.

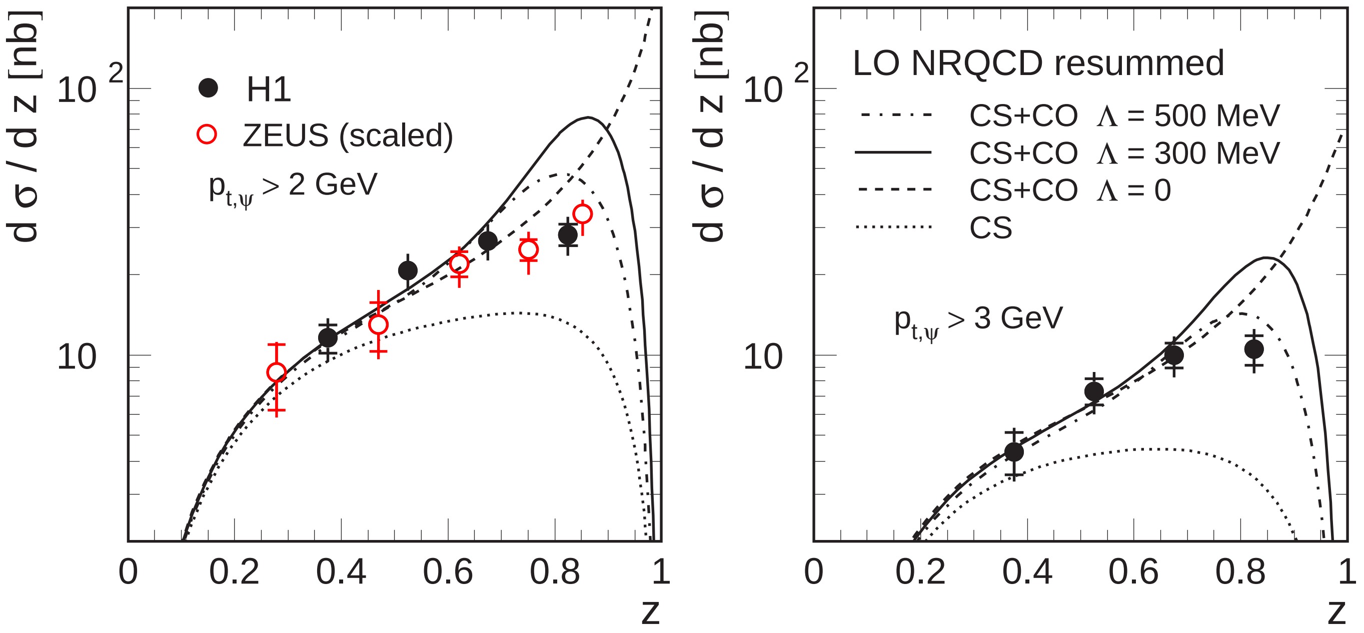
- $p_T > 1$GeV cut.
  Can question whether factorization is OK at such small p_T.
- However, the data differential in p_T are compatible with color-singlet production alone even at large p_T.



- NLO corrections increase the color-singlet piece substantially.
- They include $\gamma + g \longrightarrow (c\bar{c}) + gg$, which is dominated by t-channel gluon exchange.
- For large p_T, this process goes as $\alpha_s^3 m_c^2/p_T^6$, instead of $\alpha_s^2 m_c^4/p_T^8$.

- The data are fit well with no color-octet contribution.
  But. . .
- Large uncertainties in the color-singlet contribution (uncertainty in m_c) leave some room for a color-octet contribution.
- There are large uncertainties in the color-octet matrix elements.
  - Different linear combinations appear in photoproduction than appear in hadroproduction at the Tevatron.
  - Soft-gluon resummation decreases the sizes of the matrix elements extracted from the Tevatron data.
- The color-octet contribution is calculated only at leading order in α_s for photoproduction.
  - Resummation of multiple soft-gluon emission is needed near the $z = 1$.
    (Beneke, Schuler, Wolf)
- The v expansion breaks down near $z = 1$.
  - Resummation of the v expansion leads to a nonperturbative shape function.
    (Beneke, Rothstein, Wise)

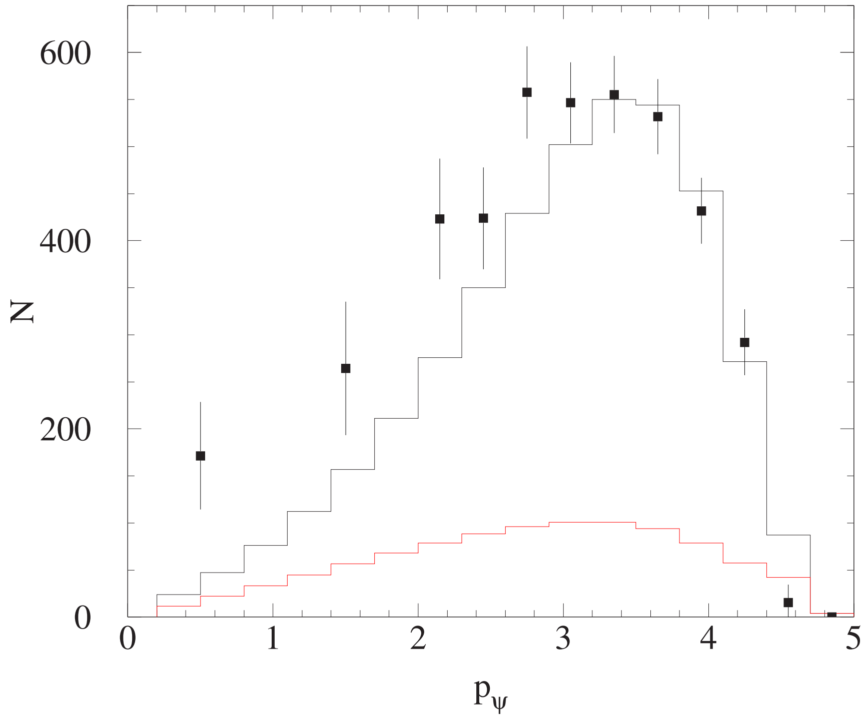
- Inclusion of a shape function with reasonable choices of parameters leads to an improved fit.



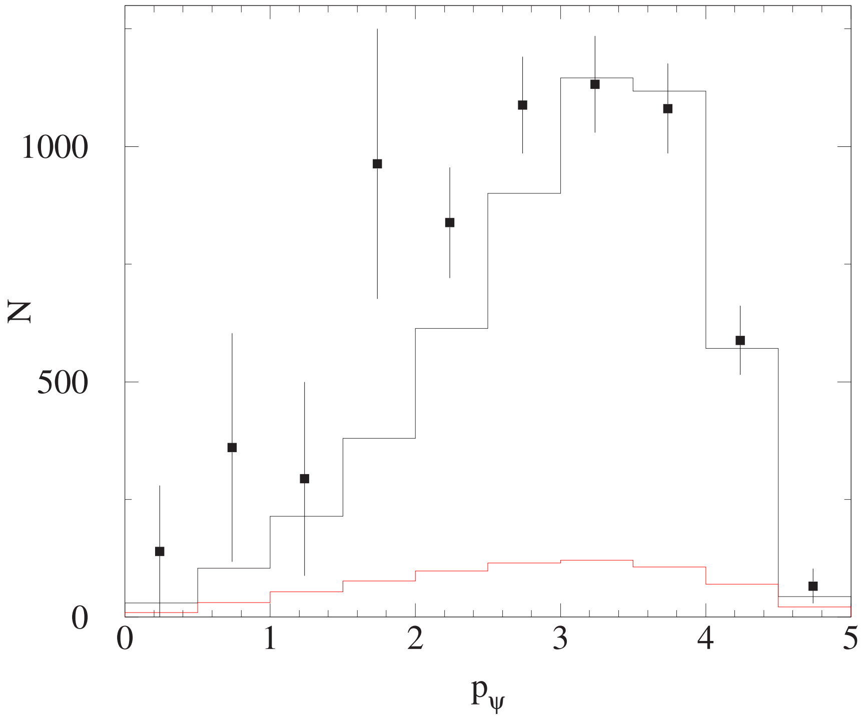
- New higher-p_T data are more compatible with a color-octet contribution.

- Soft-gluon-resummation and shape-function effects have been calculated for $e^+e^- \to J/\psi + X$ by Fleming, Leibovich, and Mehen.

Belle data:



BaBar data:



Red is color singlet. Black is color-octet plus color singlet.

- Strategy for future calculations:
  Use a shape function fitted to e^+e^- data plus soft-gluon resummation to make a firm prediction.

Double $c\bar{c}$ Production at Belle and BaBar

$e^+e^- \longrightarrow J/\psi + \eta_c$ (exclusive)

Situation in 2003

Belle: $\sigma(e^+e^- \longrightarrow J/\psi + \eta_c) \times B_{>4} = 33^{+7}_{-6} \pm 9$ fb.

NRQCD: $\sigma(e^+e^- \longrightarrow J/\psi + \eta_c) = 2.31 \pm 1.09$ fb.

- Order-of-magnitude discrepancy between theory and experiment.
- NRQCD factorization calculation by Braaten, Lee.
- The uncertainty from m_c is shown.
- There are also large uncertainties from corrections of higher order in α_s, v, and uncertainties in matrix elements.
- Exclusive process: the color-octet contribution is suppressed by v^4, so only color-singlet matrix elements are needed.

Present Situation

Belle: $\sigma(e^+e^- \to J/\psi + \eta_c) \times B_{>2} = 25.6 \pm 2.8 \pm 3.4$ fb.

BaBar: $\sigma(e^+e^- \to J/\psi + \eta_c) \times B_{>2} = 17.6 \pm 2.8 \pm 2.1$ fb.

NRQCD: $\sigma(e^+e^- \to J/\psi + \eta_c) = 3.78 \pm 1.26$ fb.

- Belle cross section has moved down.
- BaBar cross section is somewhat lower.
- Braaten and Lee corrected a sign error in the QED interference term, raising the prediction.
- QCD part confirmed by Liu, He, Chao: $\sigma(e^+e^- \to J/\psi + \eta_c) = 5.5$ fb. (Different choice of m_c, NRQCD matrix elements, α_s.)
- QCD calculation confirmed by Brodsky, Ji, and Lee in light-front QCD in the quarkonium nonrelativistic limit.
- Zhang, Gao, Chao: A new calculation of corrections at NLO in α_s shows that the K factor may be as large as 1.8.
  - Not sufficient to remove the discrepancy between theory and experiment by itself.

- A similar situation holds for production of J/ψ plus χ_{c0} or $\eta_c(2S)$:

	$\sigma(J/\psi + \eta_c)$ (fb)	$\sigma(J/\psi + \chi_{c0})$ (fb)	$\sigma(J/\psi + \eta_c(2S))$ (fb)
Belle ($\sigma \times B_{>2}$)	$25.6 \pm 2.8 \pm 3.4$	$6.4 \pm 1.7 \pm 1.0$	$16.5 \pm 3.0 \pm 2.4$
BaBar ($\sigma \times B_{>2}$)	$17.6 \pm 2.8 \pm 2.1$	$10.3 \pm 2.5 \pm 1.8$	$16.4 \pm 3.7 \pm 3.0$
Braaten and Lee	3.78 ± 1.26	2.40 ± 1.02	1.57 ± 0.52
Liu, He, and Chao	5.5	6.9	3.7

- The Belle angular distributions of $J/\psi + \eta_c$ and $J/\psi + \eta_c(2S)$ events disfavor NRQCD.

Some Possible Explanations

(GTB, Braaten, Lee): Some of the $J/\psi + \eta_c$ data sample may consist of $J/\psi + J/\psi$ events.

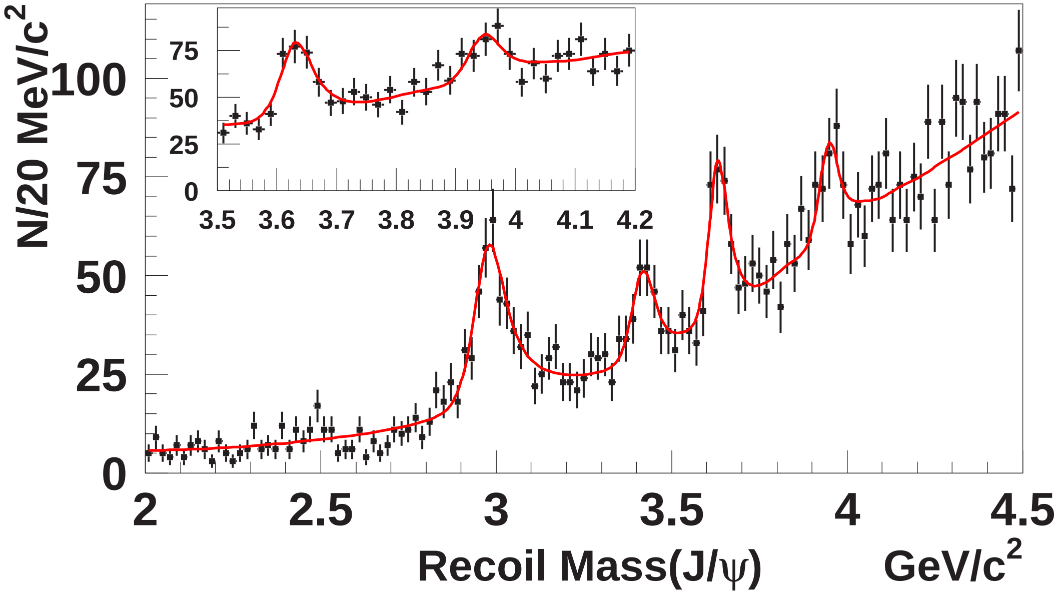
- Prediction:

$$\sigma(e^+e^- \to J/\psi + J/\psi) = 6.65 \pm 3.02 \text{ fb.}$$

  - Corrections of higher order in α and v may reduce this by a factor 3.
  - Comparable with the prediction

$$\sigma(e^+e^- \to J/\psi + \eta_c) = 3.78 \pm 1.26 \text{ fb.}$$

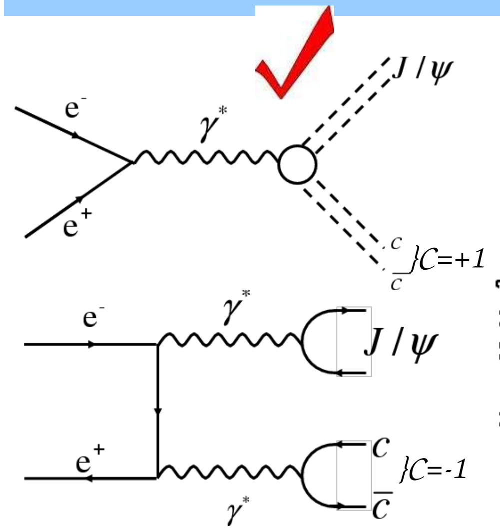
- New Belle result for spectrum recoiling against J/ψ:



- η_c, χ_{c0}, $\eta_c(2S)$, $X(3940)$ seen.
- No evidence for J/ψ, χ_{c1}, $\psi(2S)$.

Recoil Mass Spectrum

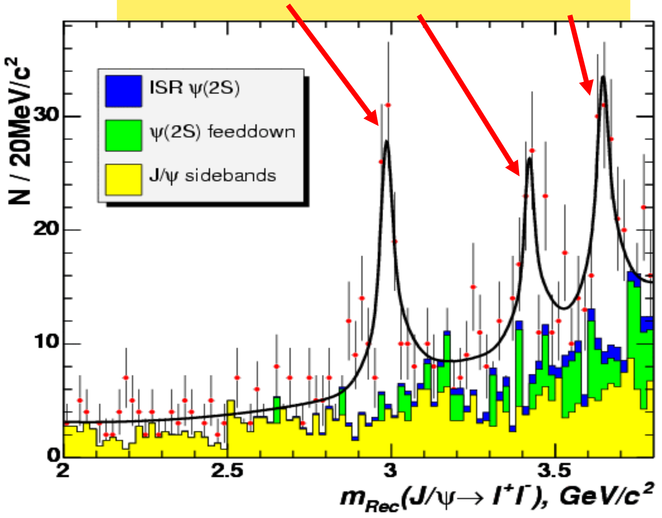
244 fb $^{-1}$



observe only C=+1 states

Observe

$\eta_c(1S)$ $\chi_c(1P)$ $\eta_c(2S)$



BaBar also finds no evidence for J/ψ, χ_{c1}, $\psi(2S)$.

From J. Coleman Moriond talk. Based on 124 fb^{-1}.

- Belle bound on $J/\psi + J/\psi$ cross section:

$$\sigma(e^+e^- \to J/\psi + J/\psi) \times B_{>2} < 9.1 \text{ fb}.$$

Compatible with the theory prediction.

Brodsky, Goldhaber, Lee: Some of the signal may be from $e^+e^- \to J/\psi +$ glueball.

- The Belle angular distributions of $J/\psi + \eta_c$ and $J/\psi + \eta_c(2S)$ disfavor production via a spin-0 glueball.
- The spin-2 glueball rate is suppressed relative to the spin-0 glueball rate by v^4.

Ma, Si; Bondar,Chernyak: The signal can be accounted for by a light-cone calculation using model wave functions.

- Claim: The large contribution comes from the finite width of the wave function.
- If so, large corrections should also appear in NRQCD in higher orders in v.
- It is not clear that the model wave functions accurately represent the true quarkonium wave functions.
- Work is in progress to reconcile the light-cone and NRQCD approaches (GTB, Lee, Kang).

There are large uncertainties in the color-singlet matrix elements.

- They are determined from $\eta_c \to \gamma\gamma$ and $J/\psi \to e^+e^-$.
- For $J/\psi \to e^+e^-$, it is known that the NNLO correction is large. (Beneke, Signer, Smirnov)

Conclusion: It is conceivable that corrections of higher order in α_s and v and more accurate NRQCD matrix elements could bring theory into agreement with experiment.

$e^+e^- \to J/\psi + c\bar{c}$ (inclusive)

- Belle result:

$$\begin{aligned} &\sigma(e^+e^- \to J/\psi + c\bar{c})/\sigma(e^+e^- \to J/\psi + X) \\ &\quad = 0.82 \pm 0.15 \pm 0.14 \\ &\quad > 0.48 \text{ (90\% confidence level)} \end{aligned}$$

- pQCD plus color-singlet model (Cho, Leibovich; Baek, Ko, Lee, Song; Yuan, Qiao, Chao):

$$\sigma(e^+e^- \to J/\psi + c\bar{c})/\sigma(e^+e^- \to J/\psi + X) \approx 0.1.$$

- Color-evaporation model Kang, J.-W. Lee, J. Lee, Kim, Ko:

$$\sigma(e^+e^- \to J/\psi + c\bar{c})/\sigma(e^+e^- \to J/\psi + X) \approx 0.049.$$

- The experimental and theoretical double-$c\bar{c}$ cross sections also disagree.
  - Belle: $\sigma(e^+e^- \to J/\psi + c\bar{c}) \approx 0.9$ pb.
  - Theory: $\sigma(e^+e^- \to J/\psi + c\bar{c}) =$ 0.10–0.15 pb.
- Work is in progress on corrections of higher order in α_s and v.

The discrepancies between theory and experiment in the double $c\bar{c}$ cross sections are significant challenges to the quantitative understanding of QCD.

- These are problems not just for NRQCD factorization, but for pQCD in general.
  - For $e^+e^- \to J/\psi+\eta_c$, one obtains the same result in the NRQCD and light-cone approaches in the non-relativistic limit.
  - The color-evaporation model gives an even smaller result for $\sigma(e^+e^- \to J/\psi + c\bar{c})/\sigma(e^+e^- \to J/\psi + X)$ than does NRQCD factorization.
- It is important for BaBar to check the Belle results for inclusive double $c\bar{c}$ production.
- If theory and experiment can't be reconciled, we may need to consider other possibilities:
  - new production mechanisms,
  - inapplicability of pQCD or NRQCD expansions,
  - failure of factorization,
  - new physics.

Summary

- The effective field theory NRQCD is a convenient formalism for separating physics at the scale of the heavy-quark mass from physics at the scale of quarkonium bound-state dynamics.
- The NRQCD factorization approach provides a systematic method for calculating quarkonium production (and decay) rates as a double expansion in powers of α_s and v.
- NRQCD factorization for production rates relies upon hard-scattering factorization and has not yet been established.
- NRQCD factorization has enjoyed a number of successes:
  - inclusive P-wave quarkonium decays,
  - quarkonium production at the Tevatron,
  - $\gamma\gamma \to J/\psi + X$ at LEP,
  - quarkonium production at DIS at HERA,
  - quarkonium production in pp collisions at RHIC.
- Other processes are (so far) more problematic:
  - quarkonium polarization at the Tevatron,
  - inelastic quarkonium photoproduction at HERA,
  - double $c\bar{c}$ production at Belle and BaBar.

- The Belle and BaBar results on exclusive double-$c\bar{c}$ production and the Belle results on inclusive double-$c\bar{c}$ production present a severe challenge to pQCD.
  - A check by BaBar of the inclusive results would be very useful.
- In many cases, inclusion of corrections of higher order in α_s and v and soft-gluon resummation should help.
- More precise theoretical predictions are hampered by uncertainties in the NRQCD matrix elements.
  - Lattice calculations can help to pin down the decay matrix elements.
  - It is not yet known how to formulate the calculation of production matrix elements on the lattice, except in the color-singlet case.
- There are many challenging problems in heavy-quarkonium physics that remain to be solved.