

# Testing Gauge–String Duality\* with Nuclear Collision Data from RHIC

Richard Seto  
University of California, Riverside

experimentalist

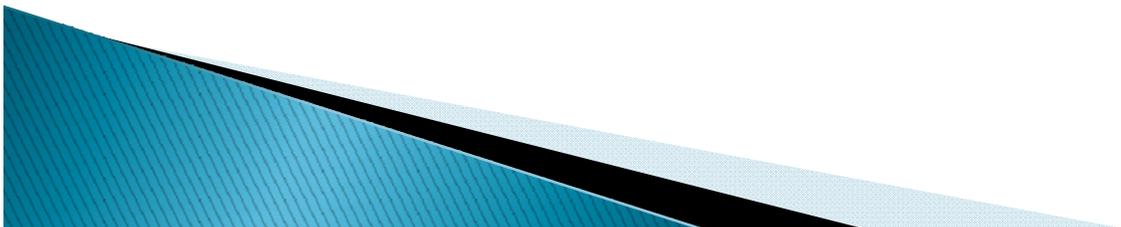
2010 APS April Meeting  
Thurgood Marshall East  
Feb 13, 2010

\*aka AdS5/CFT

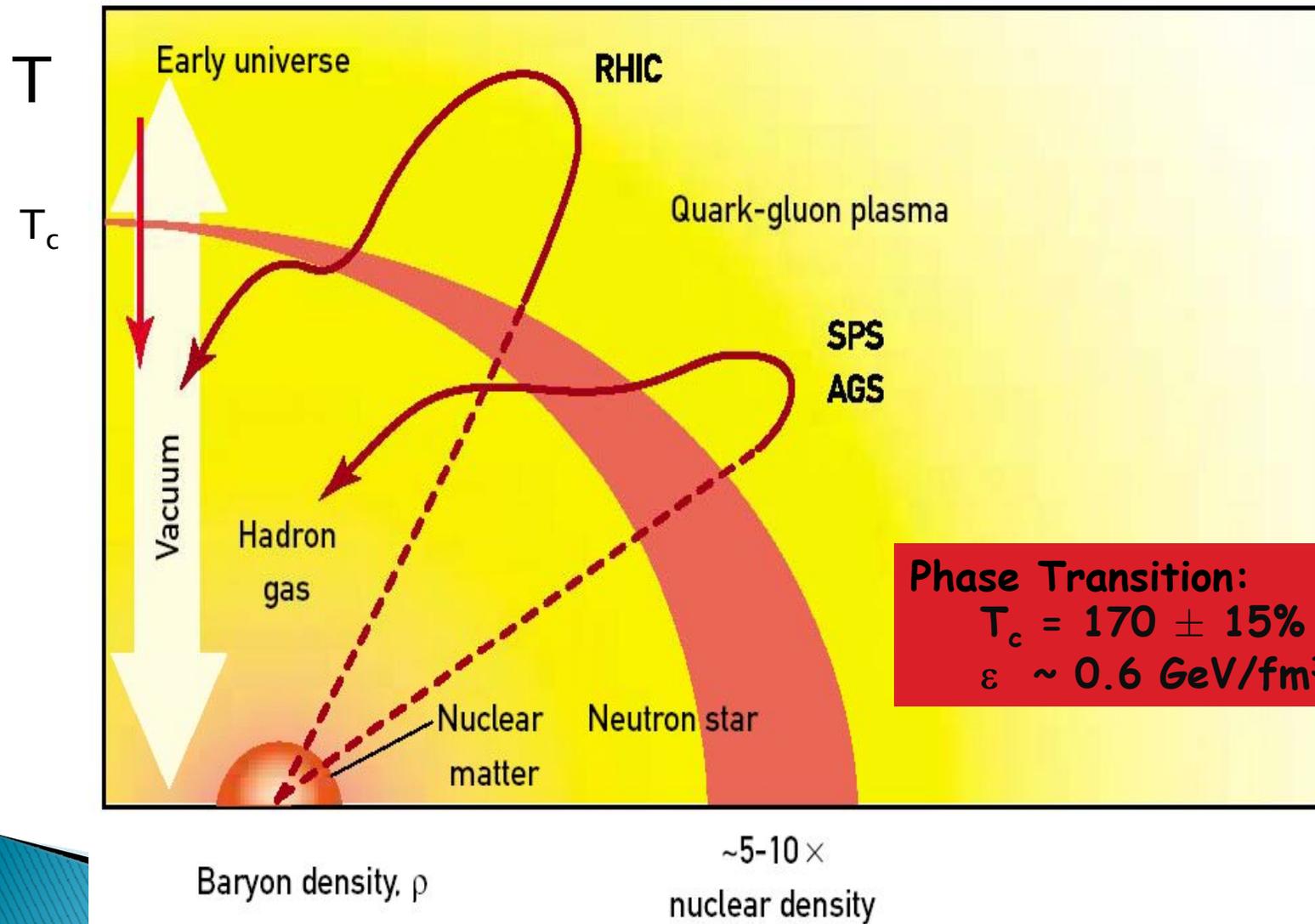
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# Outline

- ▶ Some thoughts about what we are looking for
- ▶ Introduction to heavy ions and RHIC
- ▶ Several Topics
  - Elliptic flow  $\Rightarrow \eta/s$
  - Jet (quark) energy loss  $\hat{q}$
  - Heavy quark diff coefficient
  - $\tau_0$  (thermalization time) and  $T_{\text{init}}$
  - Mention others
- ▶ Summarize (Scorecard) and conclude



# Phases of Nuclear Matter





BNL-RHIC  
Facility

Soon to Come - the LHC

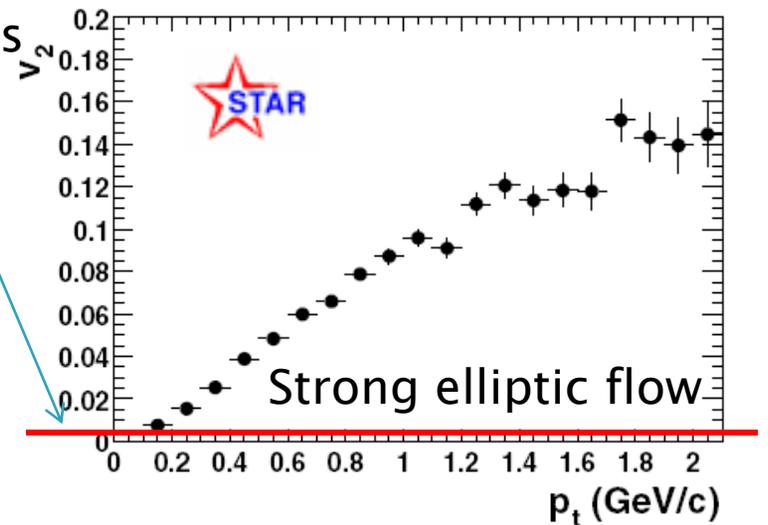
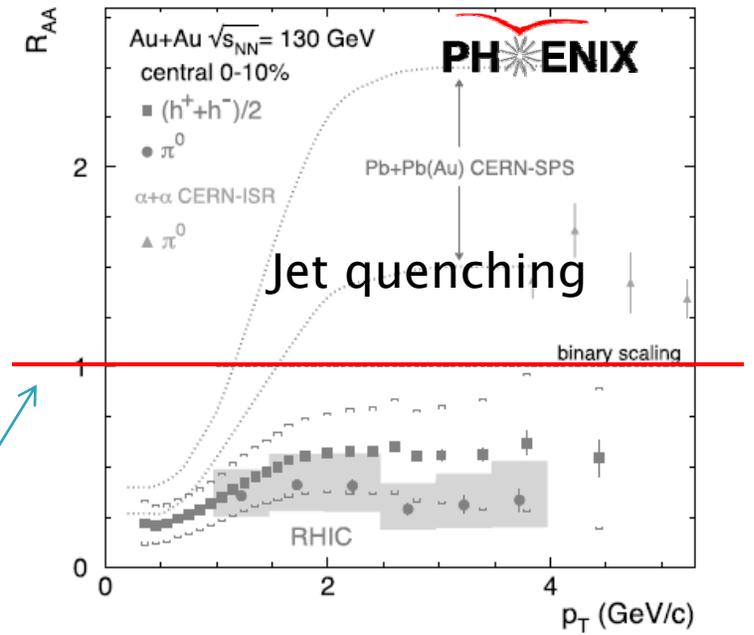
**Collide Au + Au ions for maximum volume**  
 **$\sqrt{s} = 200 \text{ GeV/nucleon pair, p+p and d+A to compare}$**

# A bit of history

- ▶ Old paradigm
  - RHIC – “weakly” interacting gas– Quark gluon plasma
    - Everything “easy” to calculate using pQCD and stat mech.
- ▶ What we found
  - Strongly interacting fluid– the sQGP

Naïve expectations

Everything “hard” to calculate



# Gauge -String Duality

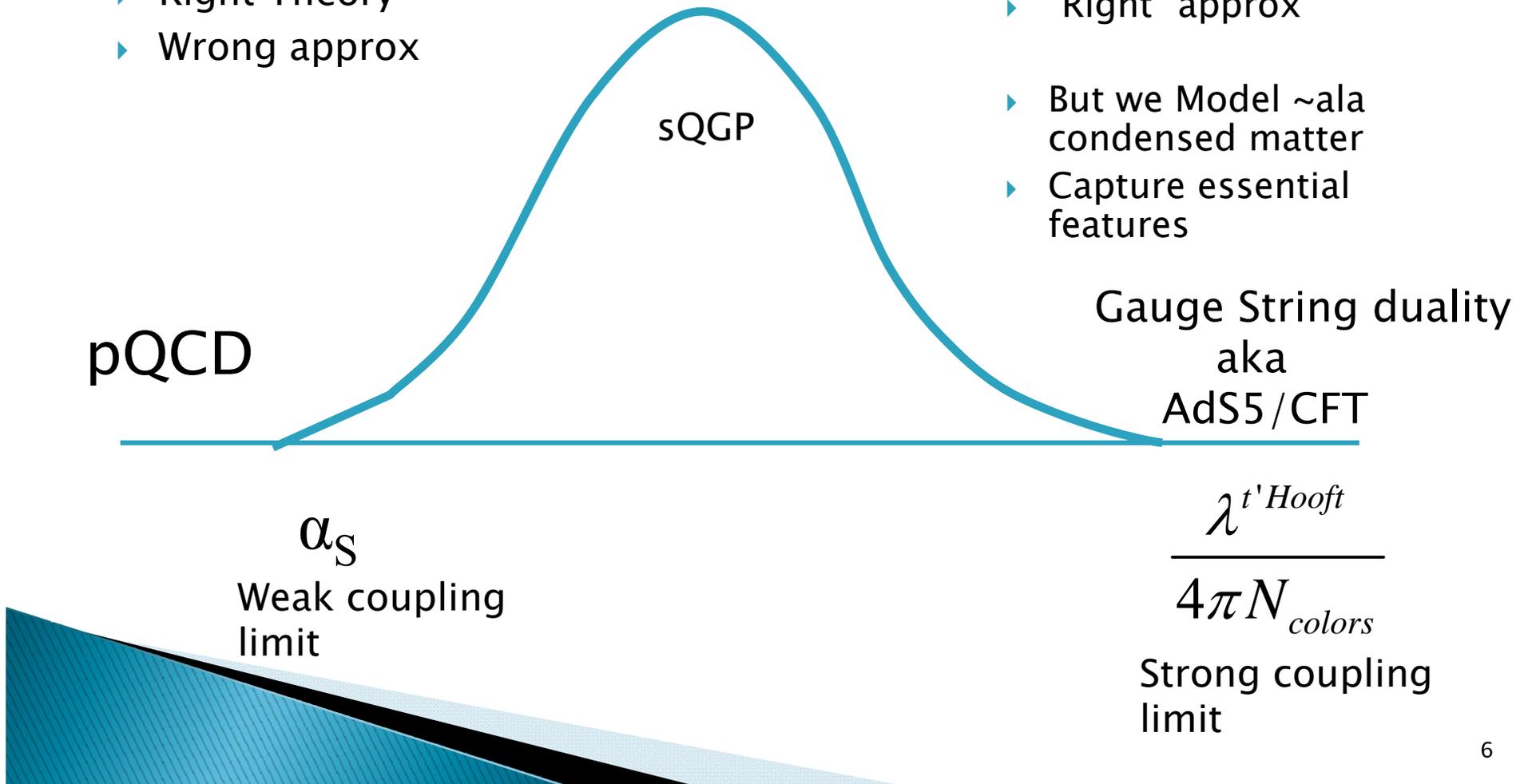
## ~~String Theory to the rescue ?~~

$\mathcal{N}=4$  Supersymmetry

- ▶ Right Theory
- ▶ Wrong approx

- ▶ Wrong Theory
- ▶ "Right" approx

- ▶ But we Model ~ala condensed matter
- ▶ Capture essential features



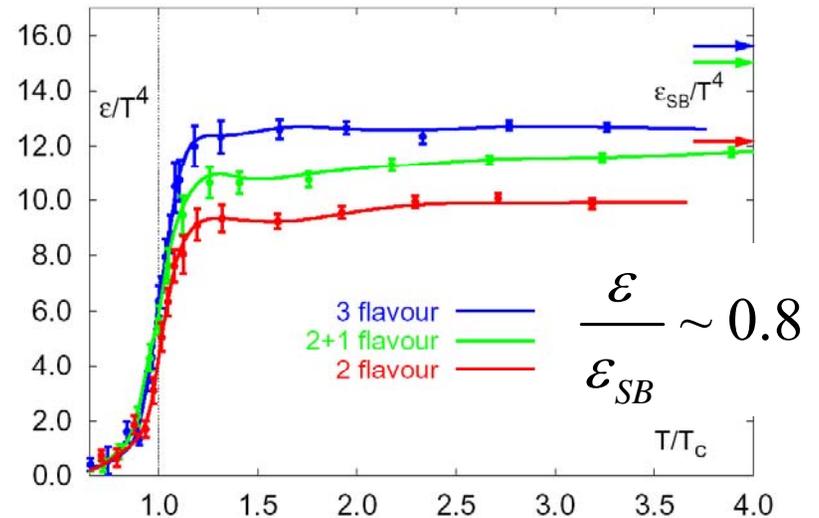
# Why do we think AdS/CFT will work?

▶ **Problems:**

- ▶ conformal theory (no running coupling)

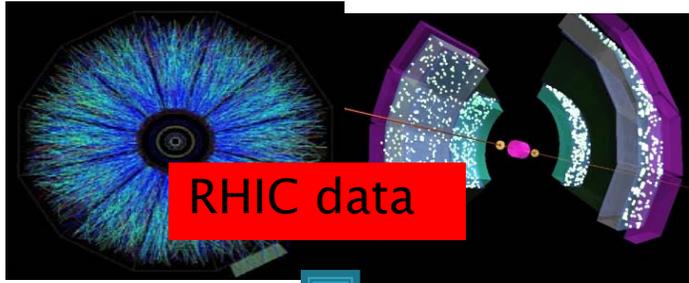
BUT!

- RHIC in a region  $T > T_c$  where  $\alpha_s$  is  $\sim$ constant ( $T$  is only scale)
- ▶ What about the coupling size?
  - expansion on the strong coupling side is in  $1/\lambda$ 
    - RHIC  $\alpha_s \sim 1/2 \Rightarrow 1/\lambda \sim 1/20 \quad \lambda = 4\pi N_{\text{colors}} \alpha_s \quad (N_C = 3 \sim \text{LARGE!})$
- ▶ What about all those extra super-symmetric particles, the fact that there are no quarks...
  - Find universal features
  - Pick parameters insensitive to differences
  - Make modifications to the simplest theory ( $\mathcal{N}=4$ ) to make it look more like QCD



Ultimately find a dual to QCD

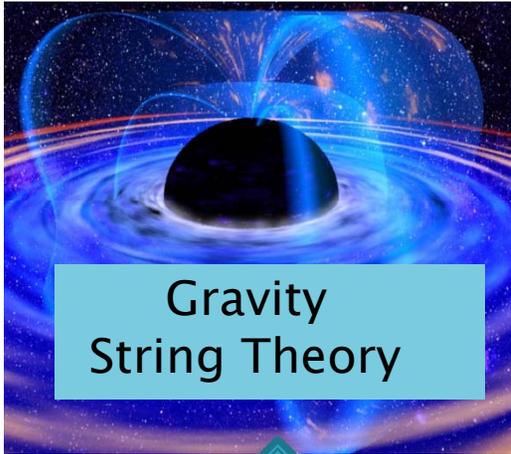
# The Task



Fits to data  
Hydro/QCD based model



Parameters



Gauge-String Duality aka AdS5/CFT



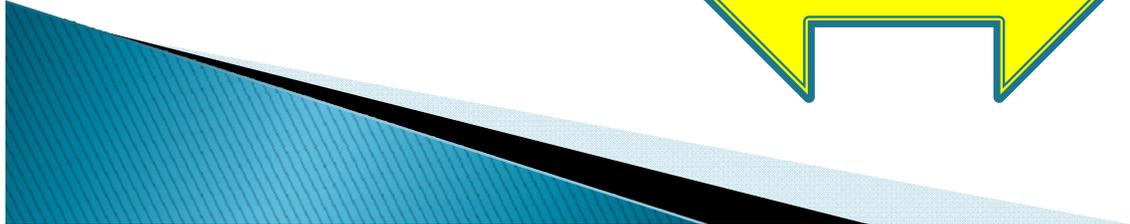
field theory e.g.  
 $\mathcal{N}=4$  Supersymmetry  
a "model"



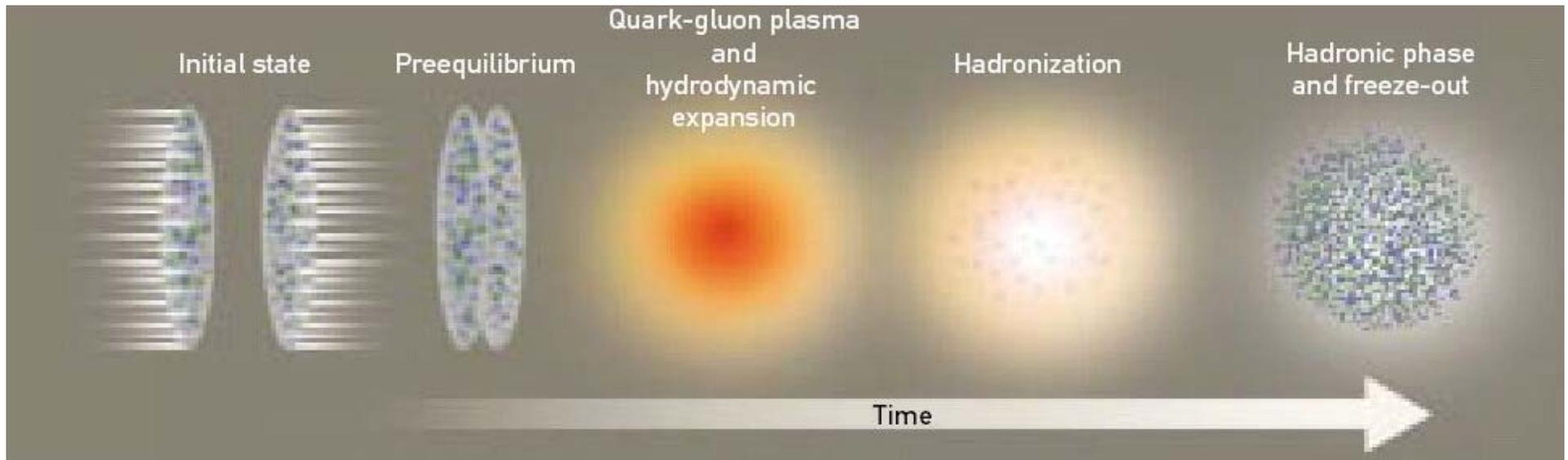
More modeling  
rescaling

Parameters

QCD

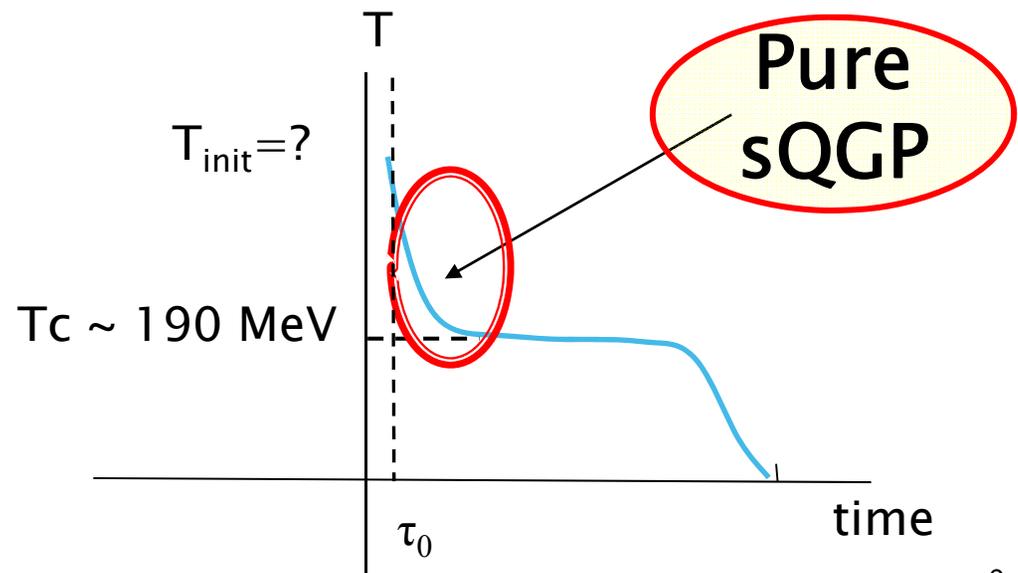


# Stages of the Collision



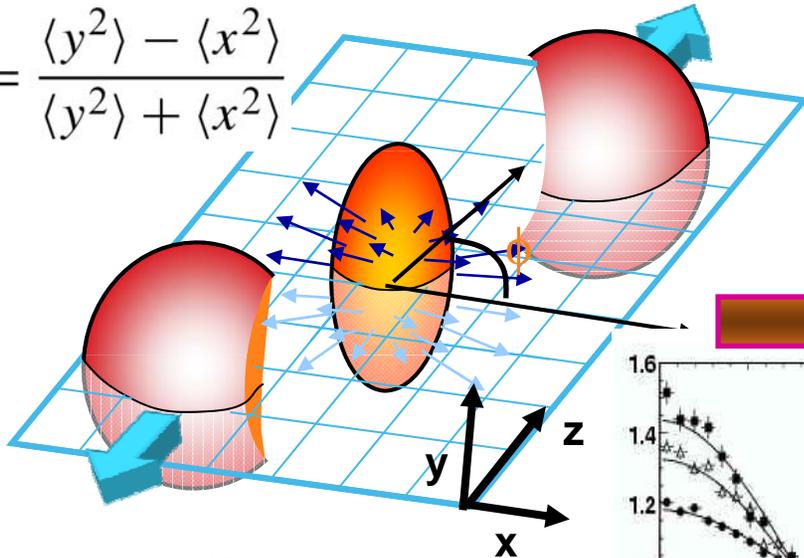
## Relativistic Heavy Ion Collisions

- Lorentz contracted pancakes
- Pre-equilibrium  $< \tau \sim 1 \text{ fm}/c$  ??
- QGP and hydrodynamic expansion  $\tau \sim \text{few fm}/c$  ??



# Flow: A collective effect

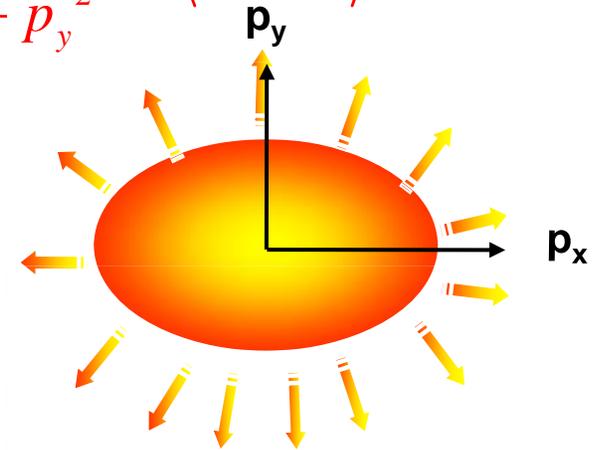
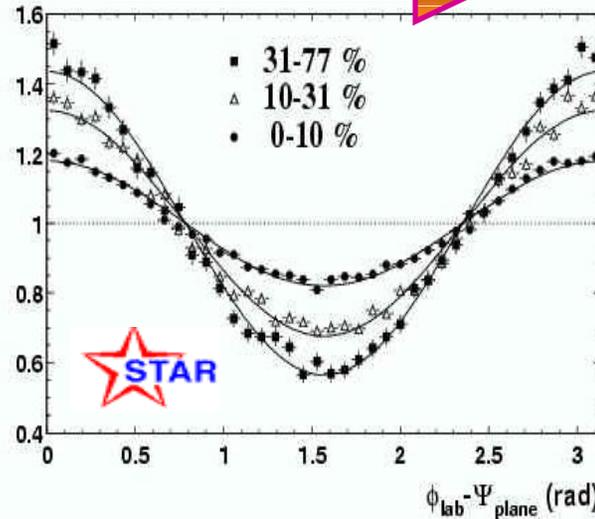
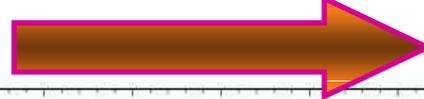
$$\varepsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle}$$



Coordinate space:  
initial asymmetry

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \langle \cos 2\phi \rangle$$

pressure



Momentum space:  
final asymmetry

$$dn/d\phi \sim 1 + 2 v_2(p_T) \cos(2\phi) + \dots$$

Initial spatial anisotropy converted into momentum anisotropy.

Efficiency of conversion depends on the properties of the medium. <sup>10</sup>

# Hydrodynamics

a critical tool to extract information from data

Hydrodynamic Equations

$$\partial_\mu T^{\mu\nu} = 0, \quad \text{Energy-momentum conservation}$$

$$\partial_\mu n_i^\mu = 0 \quad \text{Charge conservations (baryon, strangeness, etc...)}$$

For perfect fluids (neglecting viscosity!),

$$T^{\mu\nu} = (e + P)u^\mu u^\nu - P g^{\mu\nu}$$

Energy density

Pressure

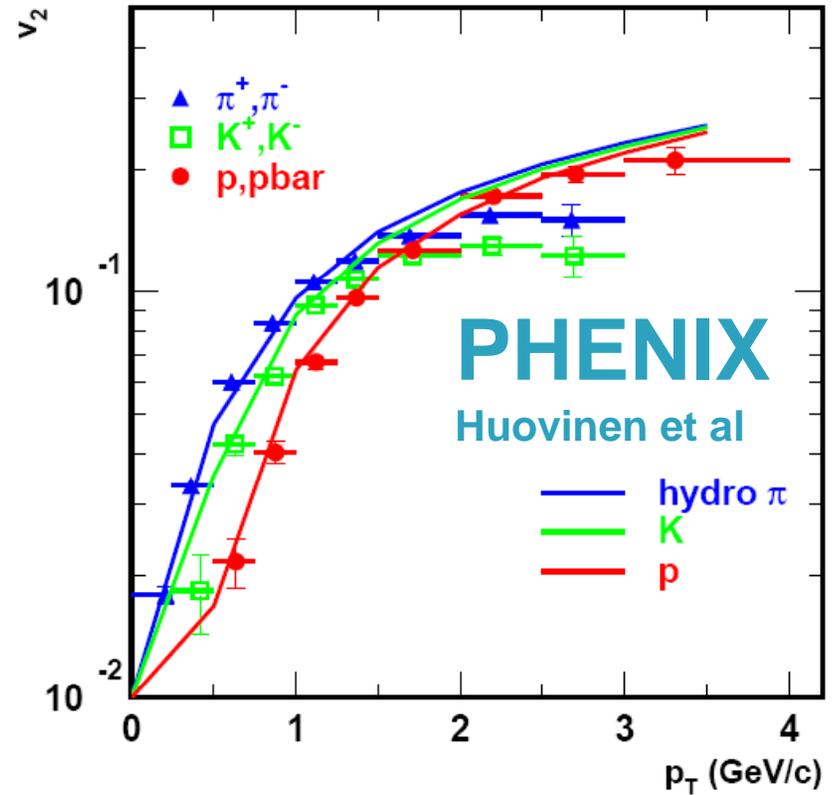
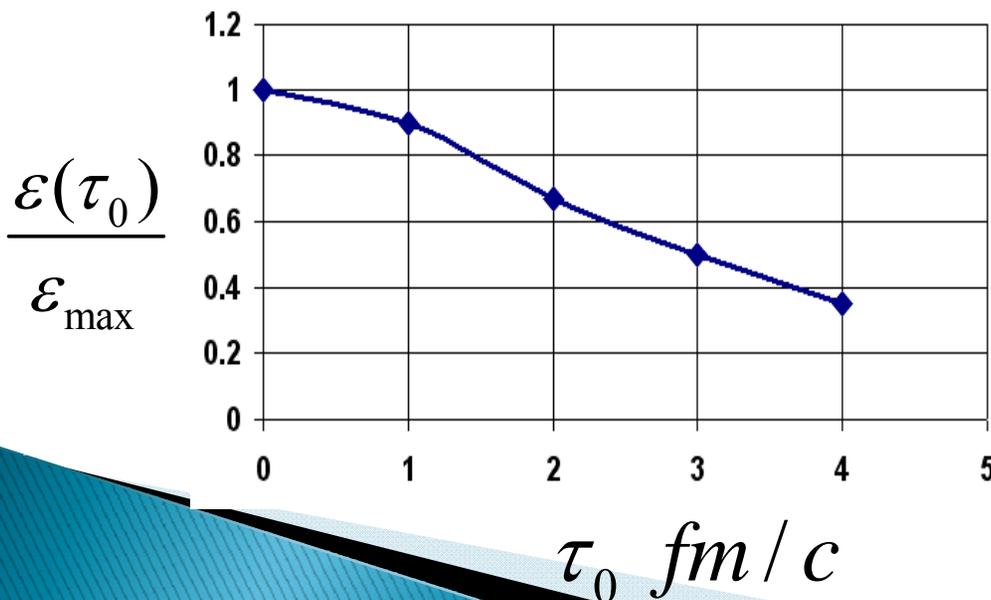
4-velocity

Within ideal hydrodynamics, pressure gradient  $dP/dx$  is the driving force of collective flow.

# Strong flow Implies *early* thermalization

- ▶ If system free streams
  - spatial anisotropy is lost
  - $v_2$  is not developed

$$\varepsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle}$$



detailed hydro calculations  
(QGP+mixed+RG, zero viscosity)

- $\tau_0 \sim 0.6 - 1.0$  fm/c
- $\varepsilon \sim 15 - 25$  GeV/fm<sup>3</sup>
- (ref: cold matter 0.16 GeV/fm<sup>3</sup>)

# Viscosity

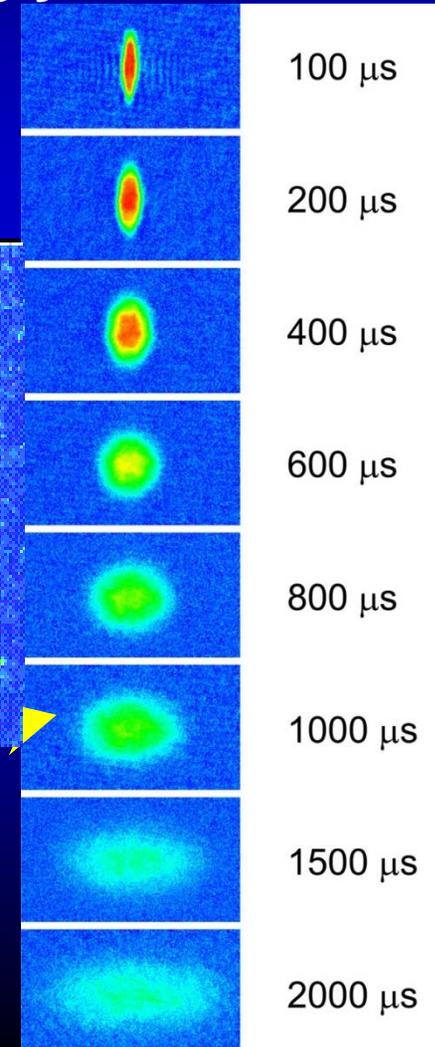
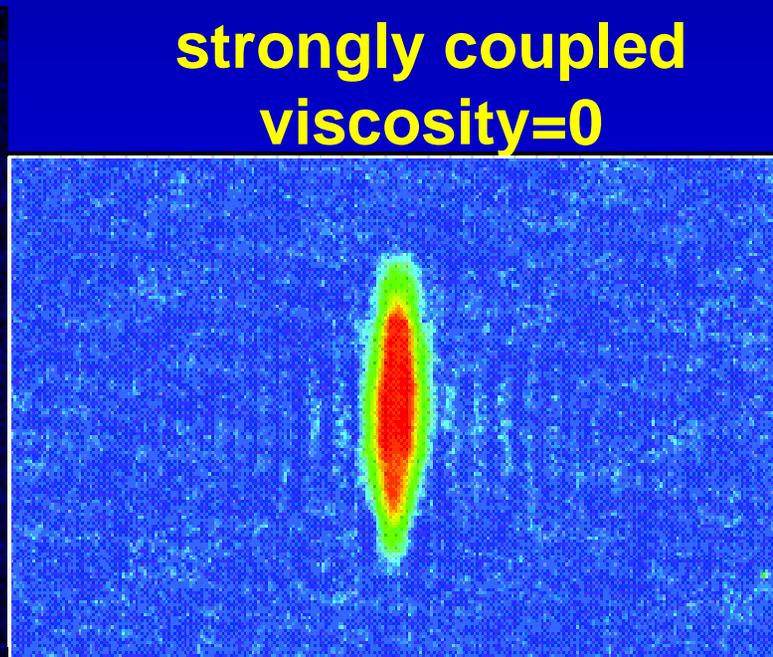
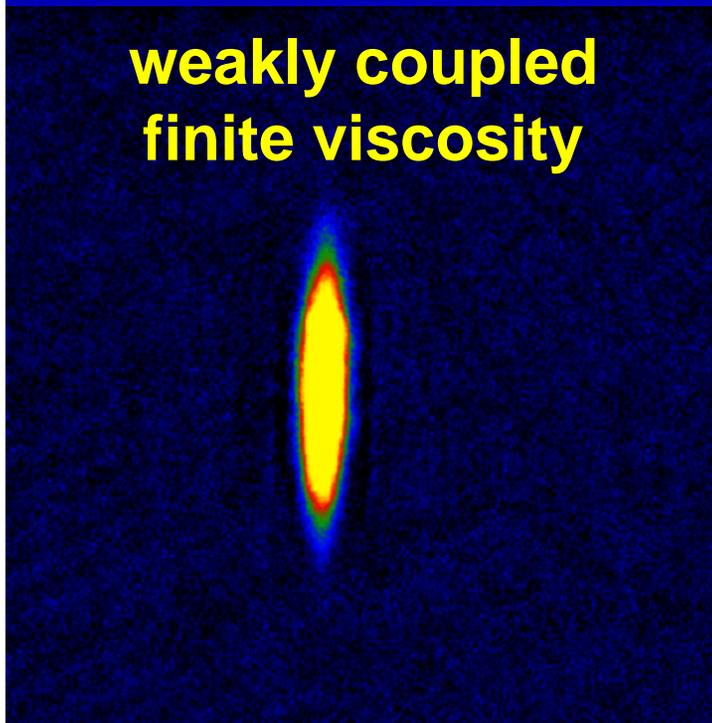
Strongly Coupled fluids have small viscosity!



# Anisotropic Flow

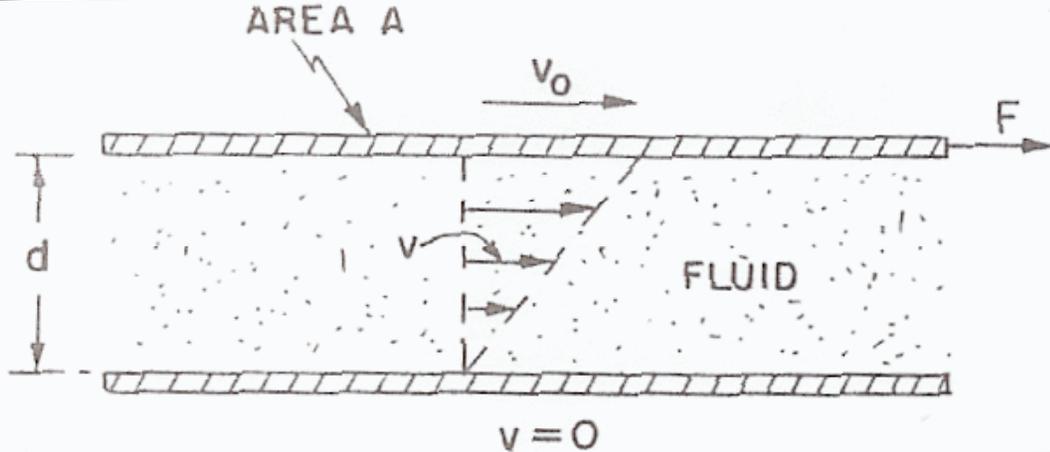
- Conversion of spatial anisotropy to momentum anisotropy depends on viscosity
- Same phenomena observed in gases of strongly interacting atoms (Li6)

M. Gehm, et al  
Science 298 2179 (2002)



**The RHIC fluid behaves like this,  
that is, viscosity~0**

# Calculating the viscosity



$$\frac{F}{A} = \eta \frac{v_0}{d}$$

$$\frac{\Delta F}{\Delta A} = \eta \frac{\Delta v_x}{\Delta y} = \eta \frac{\partial v_x}{\partial y}$$

the low-brow shear force(stress) is:

$$S_{xy} = \eta \left( \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right)$$

$$T_{ij} = \begin{pmatrix} \rho & \text{heat conduction} \\ \text{heat conduction} & \begin{matrix} \text{stress} \\ \text{Pressure} \\ \text{stress} \end{matrix} \end{pmatrix}$$

energy momentum  
stress tensor

$$T_{ij} = \delta_{ij}p - \eta \left( \partial_i u_j + \partial_j u_i - \frac{2}{3} \delta_{ij} \partial_k u_k \right)$$

$$\eta = \lim_{\omega \rightarrow 0} \frac{1}{2\omega} \int dt d\mathbf{x} e^{i\omega t} \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0, 0)] \rangle$$

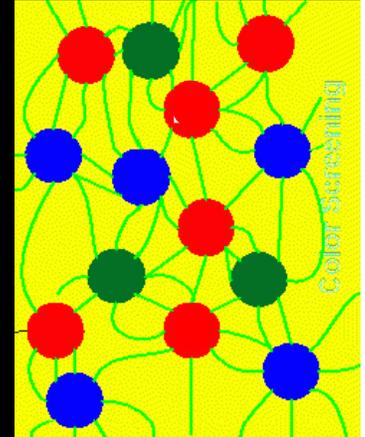
# using gauge-string duality



Gravity



$\mathcal{N}=4$  SYM  
"QCD"  
strong  
coupling



$$\sigma(\omega) = \frac{\kappa^2}{\omega} \int dt d\mathbf{x} e^{i\omega t} \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0, 0)] \rangle$$

Gravity  $\mathcal{N}=4$  Supersymmetry

$$\kappa = \sqrt{8\pi G}$$

Policastro, Son,  
Starinets hep-th 0104066

"The key observation... is that the right hand side of the Kubo formula is known to be proportional to the classical absorption cross section of gravitons by black three-branes."

$$\eta = \frac{1}{2\kappa^2} \sigma(0)$$

$\sigma(0)$ =area of horizon

$$\eta = \lim_{\omega \rightarrow 0} \frac{1}{2\omega} \int dt d\mathbf{x} e^{i\omega t} \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0, 0)] \rangle$$

# finishing it up

Entropy  
black hole "branes"



Entropy  
 $\mathcal{N}=4$  SYM  
"QCD"

Entropy  
black hole =  
Bekenstein, Hawking

Area of black  
hole horizon =  $\sigma(0)$

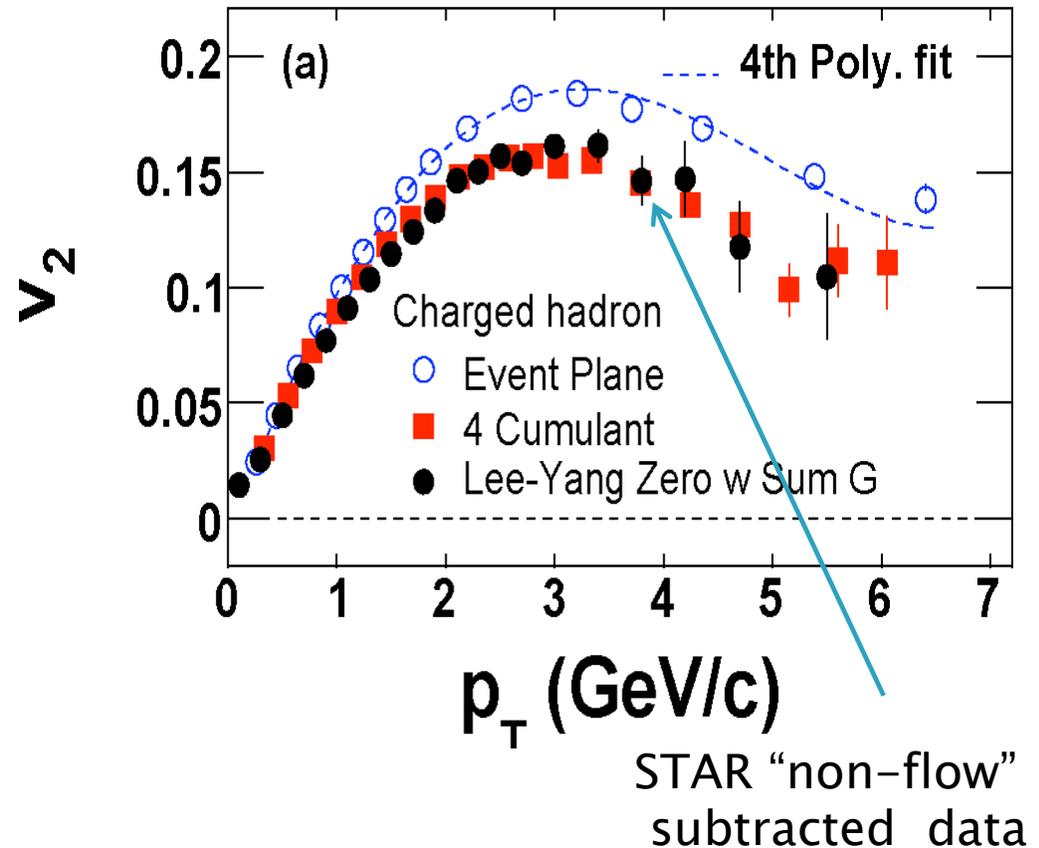
$$S_{\text{SYM "QCD"}} = \frac{\text{black hole area}}{4G}$$

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} = 6 \times 10^{-13} \text{ Ks}$$

This is believed to be a universal lower bound for a wide class of Gauge theories with a gravity dual

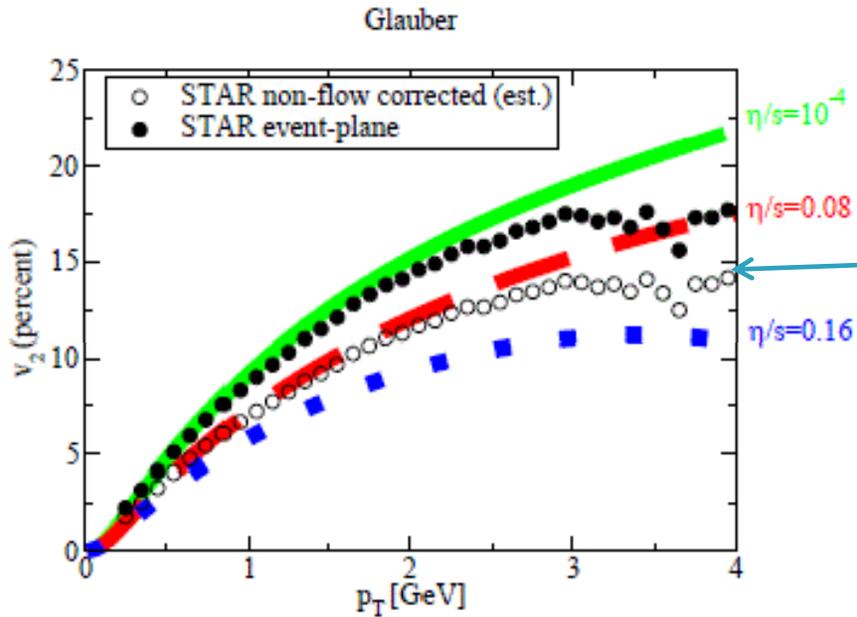
# What does the data say?

- ▶ Data from STAR
  - Note – “non-flow contribution subtraction (resonances, jets...)”
- ▶ Use Relativistic-Viscous hydrodynamics
- ▶ Fit data to extract  $\eta$ 
  - Will depend on initial conditions



Task was completed recently by Paul Romatschke after non-flow Subtraction was done on the data

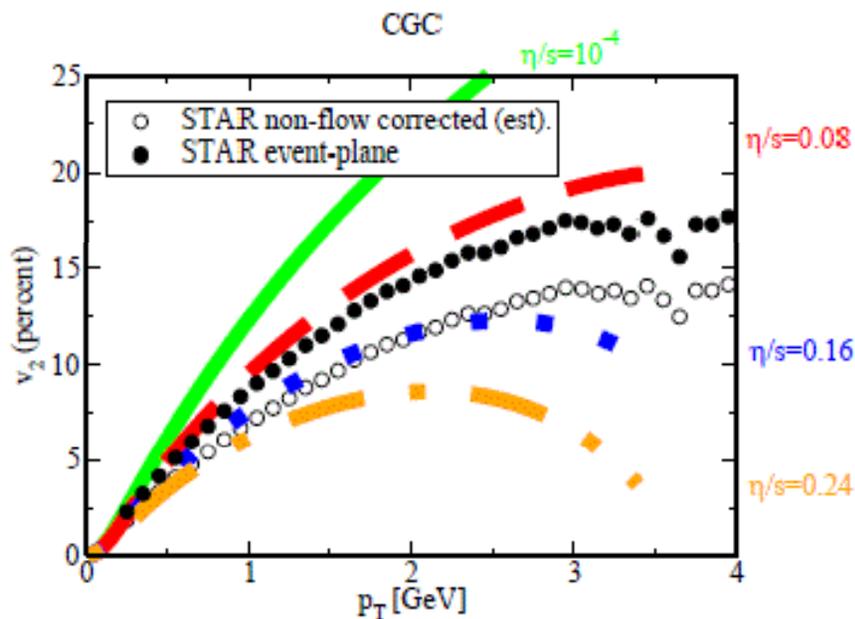
# Extracting $\eta/s$



STAR “non-flow” subtracted

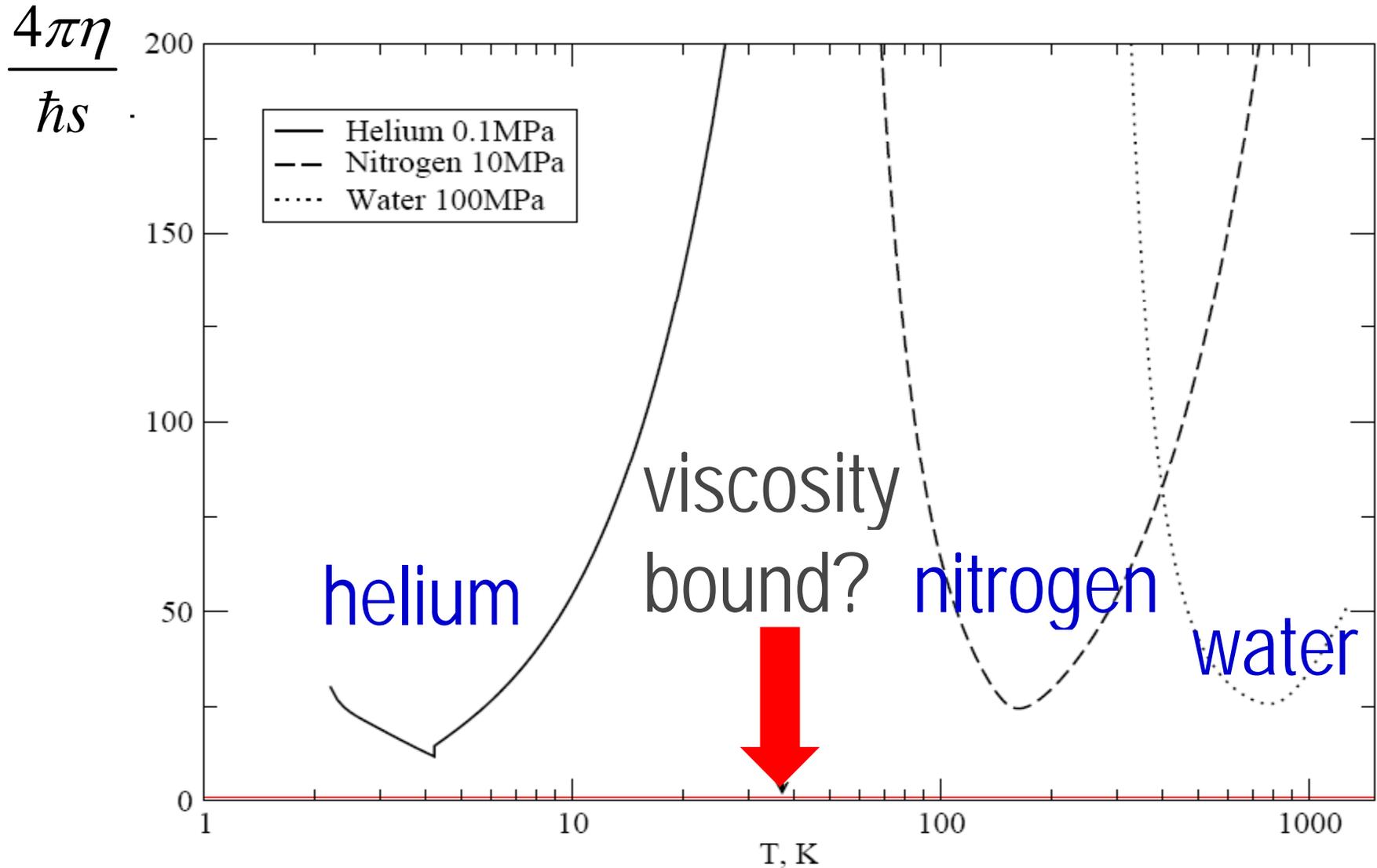
- ▶ Best fit is to  $\eta/s$   
 $\sim 0.08 = 1/4\pi$ 
  - With glauber initial conditions
- ▶ Using CGC (saturated gluon) initial conditions  
 $\eta/s \sim 0.16$

Phys.Rev.C78:034915 (2008)

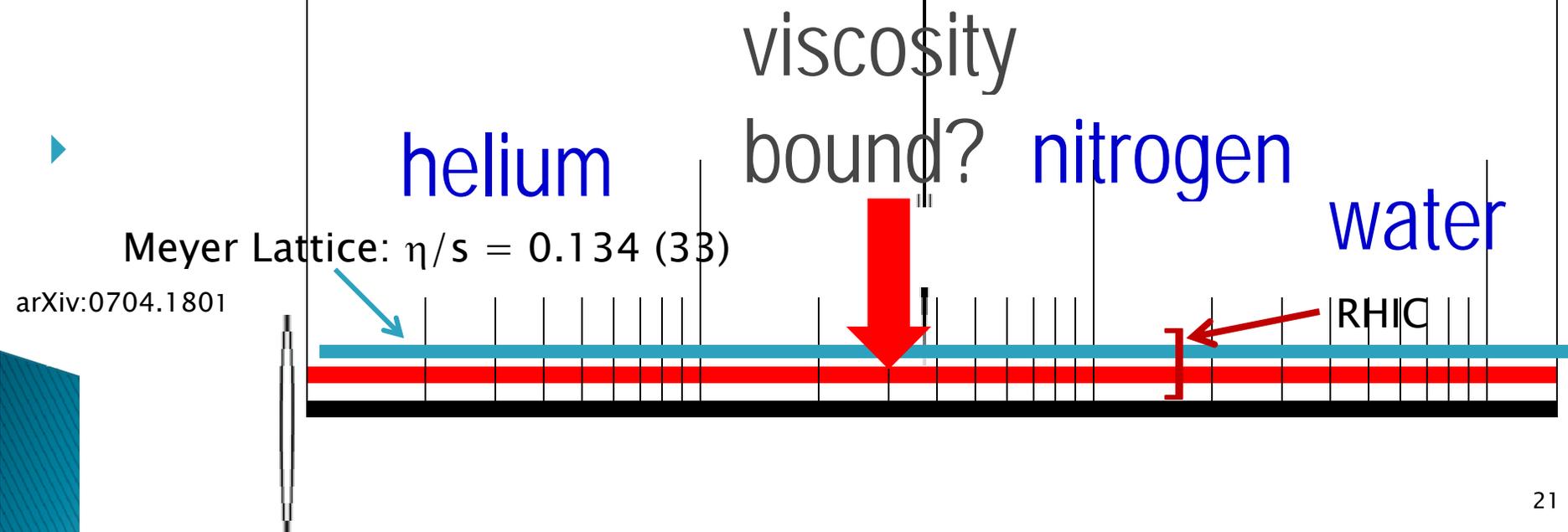


- ▶ Note: Demir and Bass
  - hadronic stages of the collision do not change the fact that the origin of the low viscosity matter is in the partonic phase (arXiv:0812.22422)

# sQGP – the most perfect fluid?



$$\frac{4\pi\eta}{\hbar s}$$

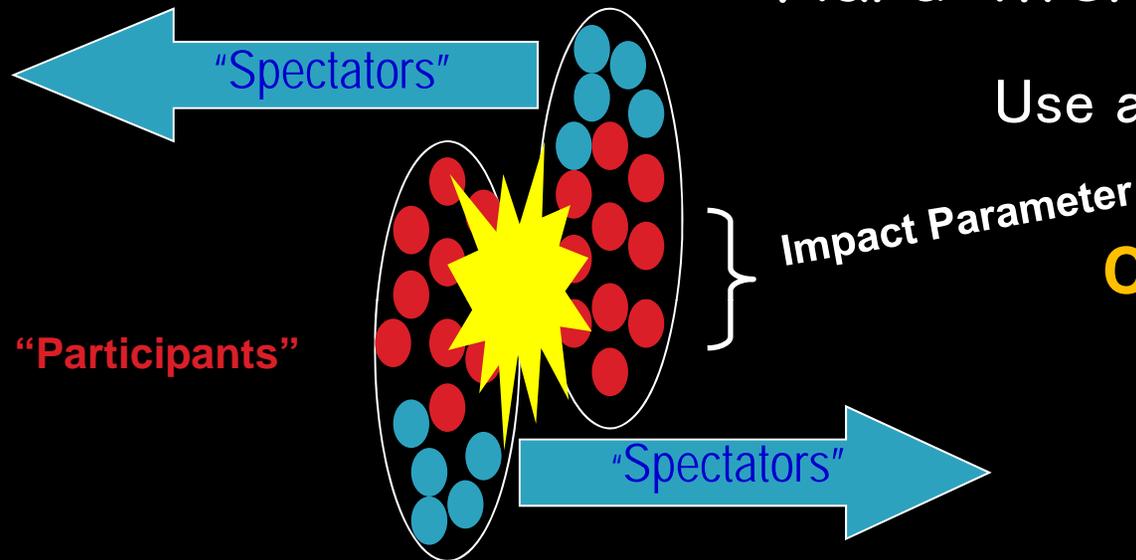


# Jet quenching

# Centrality and $R_{AA}$ : the suppression factor

- Hard interactions  $\sim N_{\text{collisions}}$

Use a Glauber model

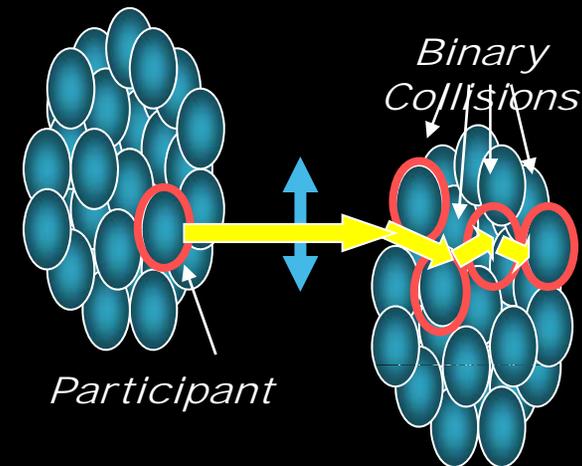


**Centrality def'n**  
**head on – central**  
**glancing - peripheral**

$$R_{AA} = \frac{\text{AuAu yield}}{N_{coll}^{AuAu} \cdot \frac{pp \text{ yield}}{N_{coll}^{pp}}}$$

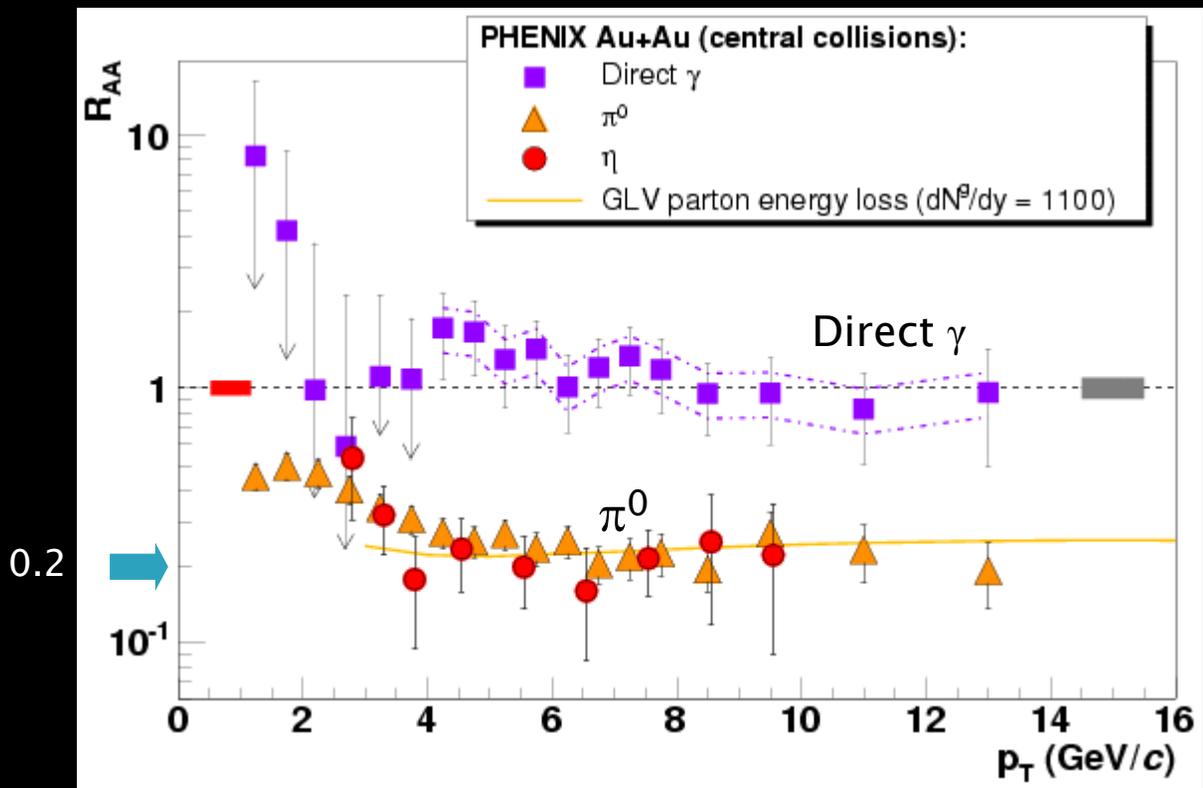
$$N_{coll}^{AuAu} = 1000$$

$$N_{coll}^{pp} = 1$$



# What is the energy density? “Jet quenching”

AuAu 200 GeV

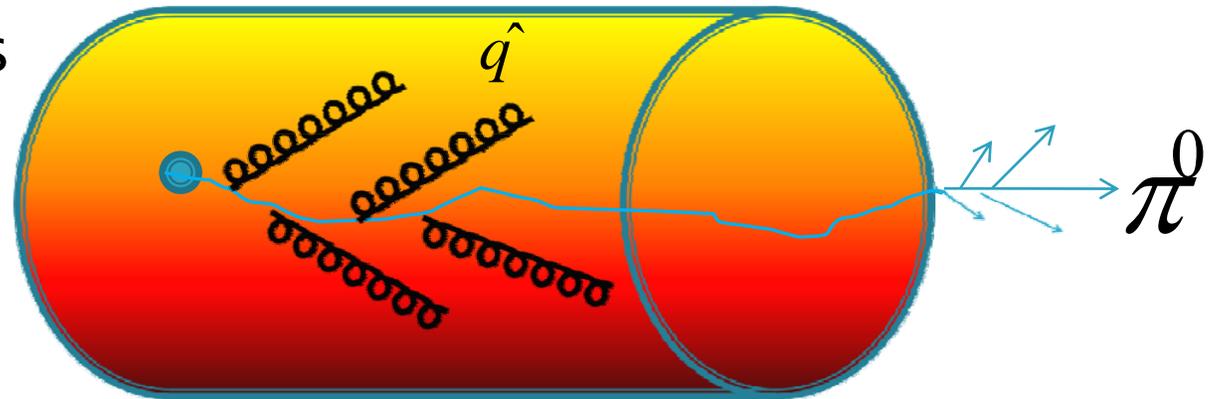


direct photons scale as  $N_{coll}$   
 $\pi^0$  suppressed by 5!

$$R_{AA} = \frac{1}{\langle n_{coll} \rangle} \frac{d^2 N_{AB} / dy dp_T}{d^2 N_{pp} / dy dp_T}$$

# Hard probes in QCD– the jet quenching parameter

- ▶ QCD – hard probes (jets)
- ▶ Assumption – Quark energy loss by radiation of gluons

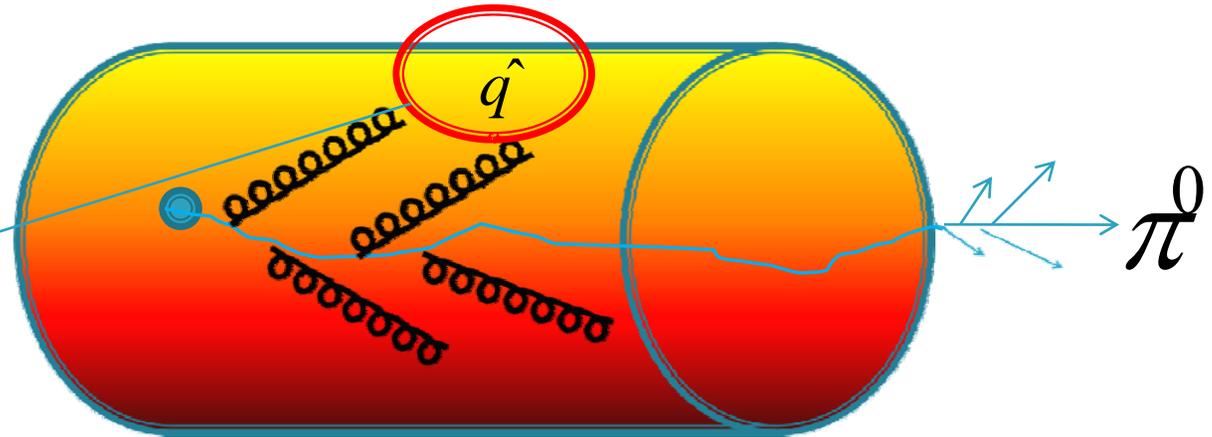


- ▶  $\mathcal{N}=4$  SYM– no running of coupling  $\Rightarrow$  no hard probes (a quark would not form a jet– just a soft glow of gluons)
- ▶ Various schemes
  - Introduce heavy quark
  - Start with jet as an initial state
  - Formulate the problem in pQCD
    - Non perturbative coupling to medium is embodied in one constant  $\hat{q}$  ( $\langle p_T^2 \rangle / \text{length}$ )
    - $\mathcal{N}=4$  SYM  $\Rightarrow \hat{q}$  (Liu, Rajagopal, Wiedemann)

# Hard probes in QCD– the jet quenching parameter

Formulate in terms of  
Wilson loop and calculate  
In N=4 SYM using Ads/CFT

$$\hat{q}_{SYM} = \frac{\pi^{3/2} \Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)} \sqrt{\lambda} T^3$$



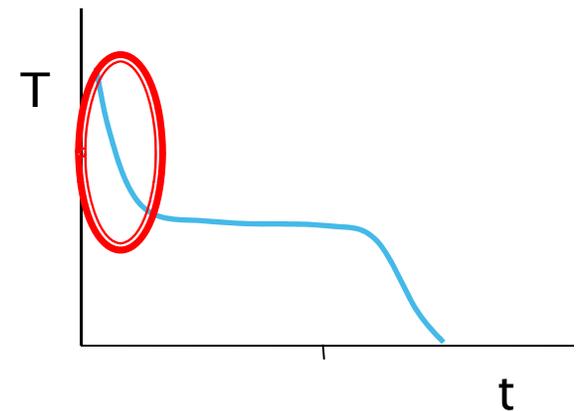
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- $\mathcal{N}=4$  SYM  $\Rightarrow \hat{q}$  (Liu, Rajagopal, Wiedemann)

# Plugging in numbers

- ▶ solution was for static medium at temperature  $T$

$T$	$q$
0.3 GeV	4.5 GeV <sup>2</sup> /fm
0.4 GeV	10.6
0.5 GeV	20.7



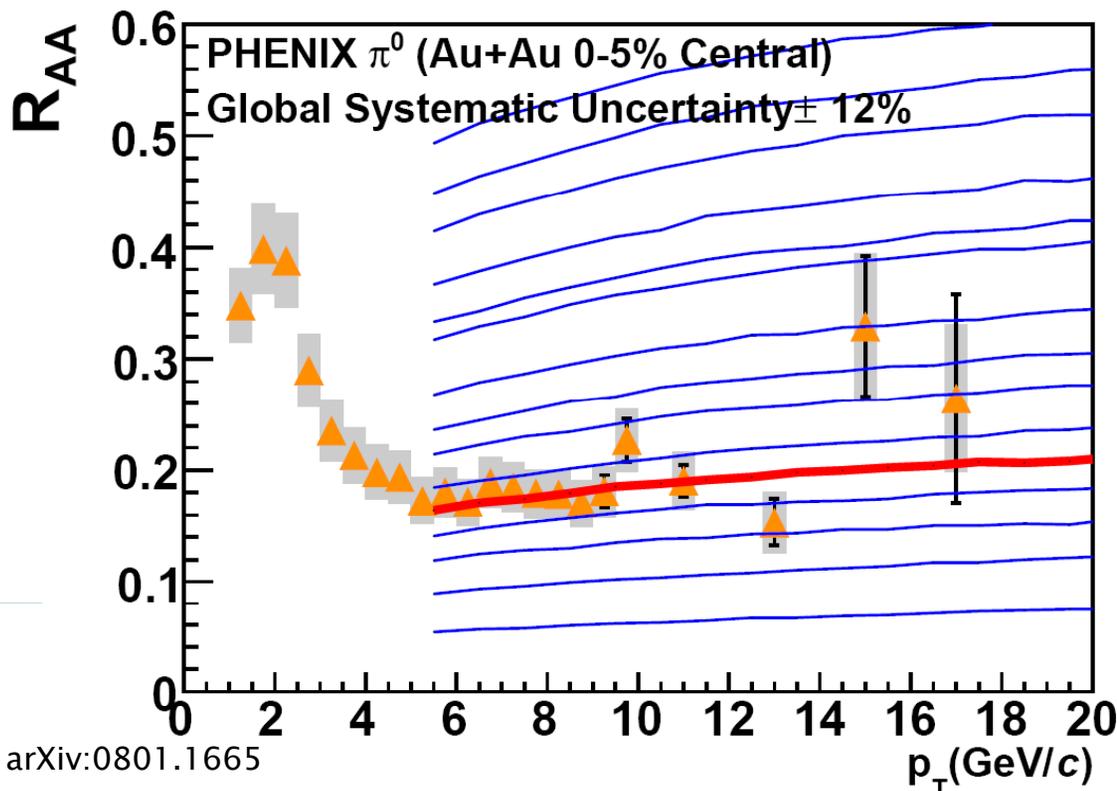
- ▶ But the temperature is falling at RHIC as a function of time.

$$\bar{q}(AdS) = 5 - 15 \text{ GeV}^2 / \text{fm}$$

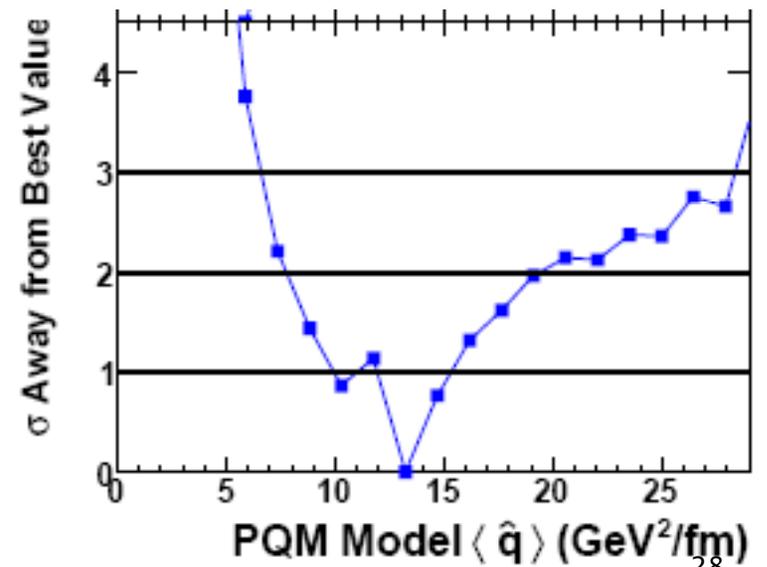
# What do the experiments get?

- ▶ Use PQM model (parton quenching model)
  - Realistic model of energy loss (BDMPS)  $\hat{q}$
  - Realistic geometry

cf  $\bar{\hat{q}} = 5 - 15 \text{ GeV}^2 / \text{fm}$



$$\hat{q} = 13.2^{+2.1}_{-3.1} \quad 2\sigma^{+6.3}_{-5.2}$$



# heavy quark diffusion

$m_u \sim 3 \text{ MeV}$   
 $m_d \sim 5 \text{ MeV}$   
 $m_s \sim 100 \text{ MeV}$   
 $m_c \sim 1,300 \text{ MeV}$   
 $m_b \sim 4,700 \text{ MeV}$

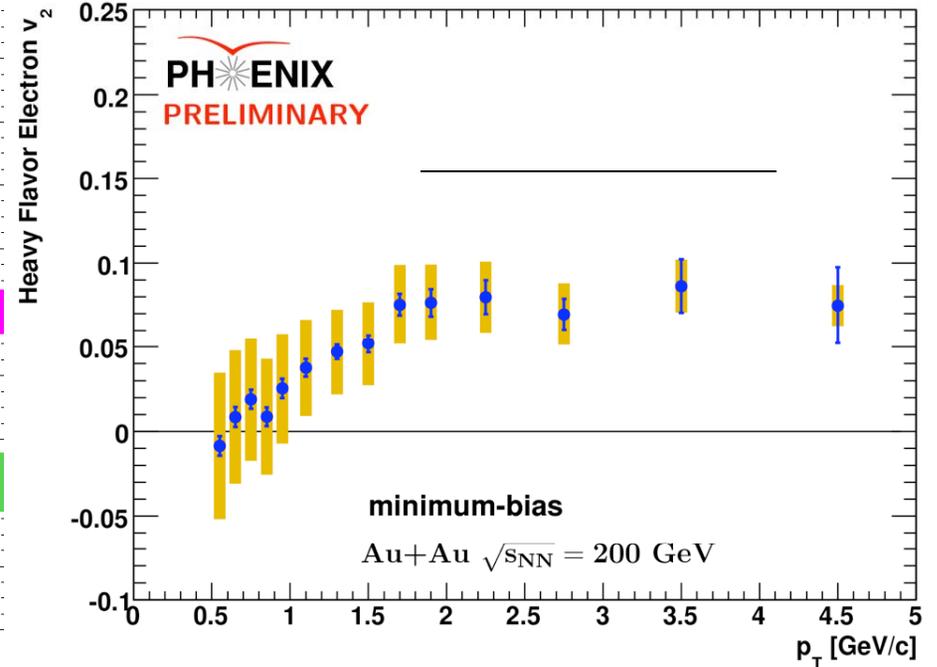
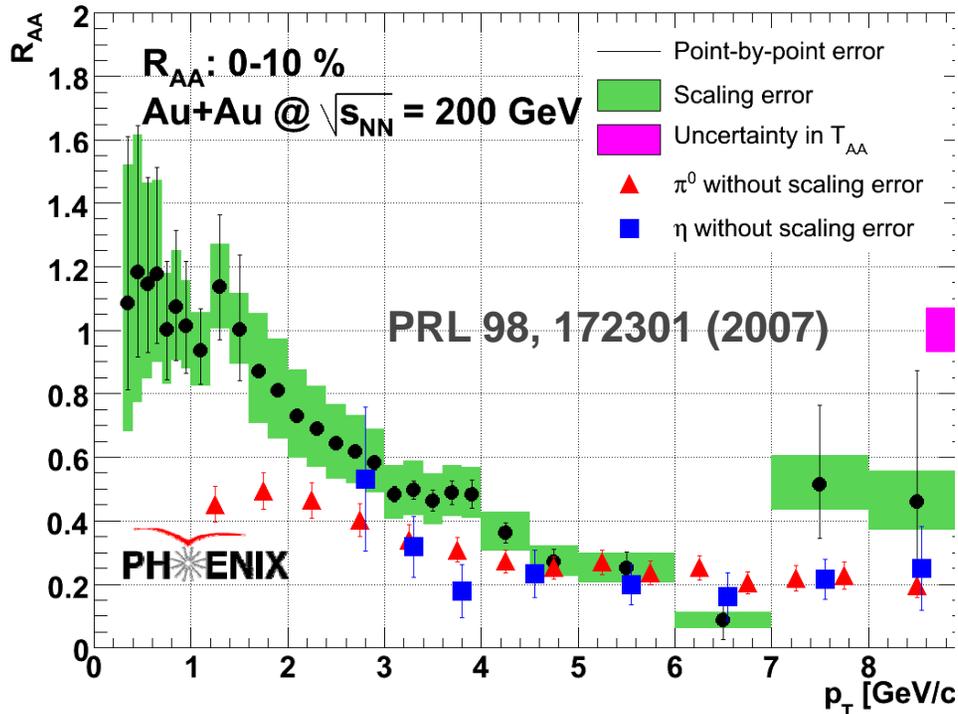
$\Lambda_{\text{QCD}} \sim 200 \text{ MeV}$

throw a pebble in the  
stream

see if it moves

# Non photonic electrons (aka charm+bottom)

00-10 %



- high  $p_T$  non-photonic  $e^\pm$  suppression increases with centrality
- similar to light hadron suppression at high  $p_T$

- non-photonic  $e^\pm$  Flow  
 → charm suppressed and thermalizes

Can we describe this in the context of the sQGP?

# AdS: $D_{\text{HQ}}$ the diffusion coefficient

- ▶ Make modification to  $\mathcal{N}=4$ 
  - Add heavy quark (adding a D7 Brane)
- ▶ Drag a heavy quark at a constant velocity and see how much force is needed – calculate a drag coefficient

- $M\mu = f/v$        $D = T / \mu M$        $D = \frac{2}{\pi\sqrt{\lambda T}} = \frac{0.15}{T}$

- ▶ Rescale to QCD

- Herzog:       $\frac{D_{QCD, \alpha_s=0.5}}{D_{QCD, \alpha_s \rightarrow 0}} \sim \frac{D_{SYM, \lambda=19}}{D_{SYM, \lambda \rightarrow 0}}$       Where  $\frac{D_{QCD, \alpha_s \rightarrow 0}}{D_{SYM, \lambda \rightarrow 0}} \sim 4$

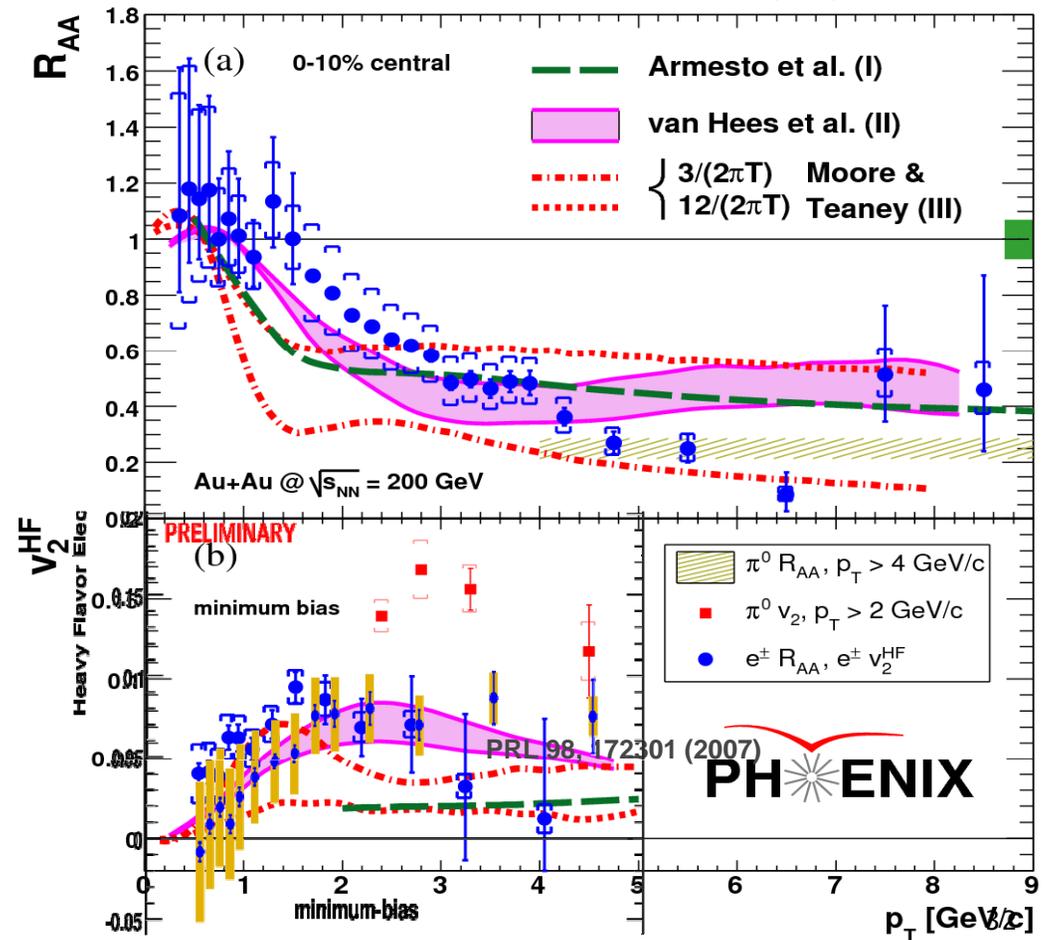
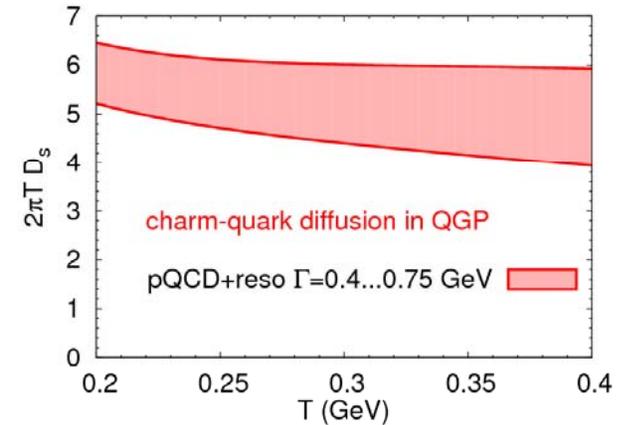
Gubser: alternate set of parameters ( $\lambda=5.5, 3^{1/4}T_{SYM}=T_{QCD}$ )

$D_{\text{HQ}}(\text{AdS}) = 0.6/T$

# Getting $D_{HQ}$ from data

- ▶ Rapp and vanHees
- ▶ model has “non-perturbative” features
  - resonant scattering
- ▶ fits flow and  $R_{AA}$
- ▶  $D_{HQ} = 4 - 6 / 2\pi T = 0.6 - 0.9 / T$

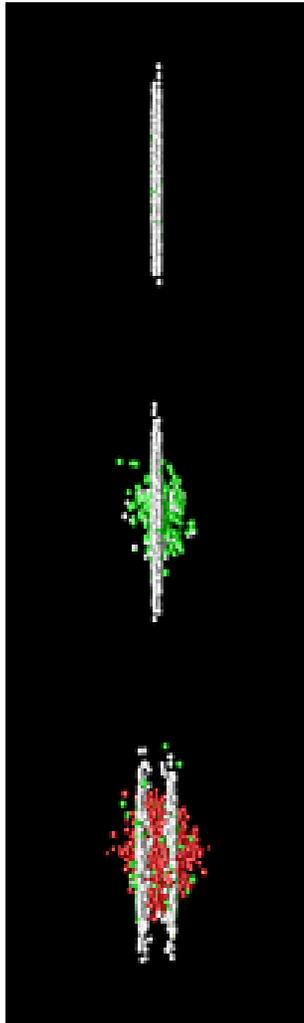
Cf  $D(\text{AdS}) = 0.6 / T$



# Equilibration and temperature

# Temperature and $\tau_0$

## ▶ Equilibration time ( $\tau_0$ ) from AdS



Rapidly expanding  
Spherical plasma Ball

dual

Black hole in AdS5

Perturb it

~ hydro-like modes  
~ non hydro-like

Study rate of  
equilibration  
to hydrodynamical  
description

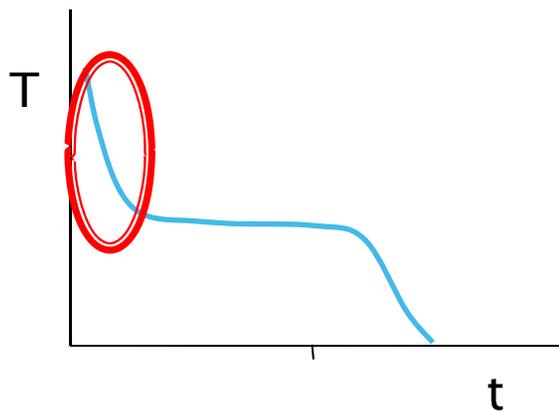
Find rate of decay  
Of non hydro-like  
modes

$$\tau_{\text{e-fold}} = 1 / 8.6 T_{\text{max}}$$

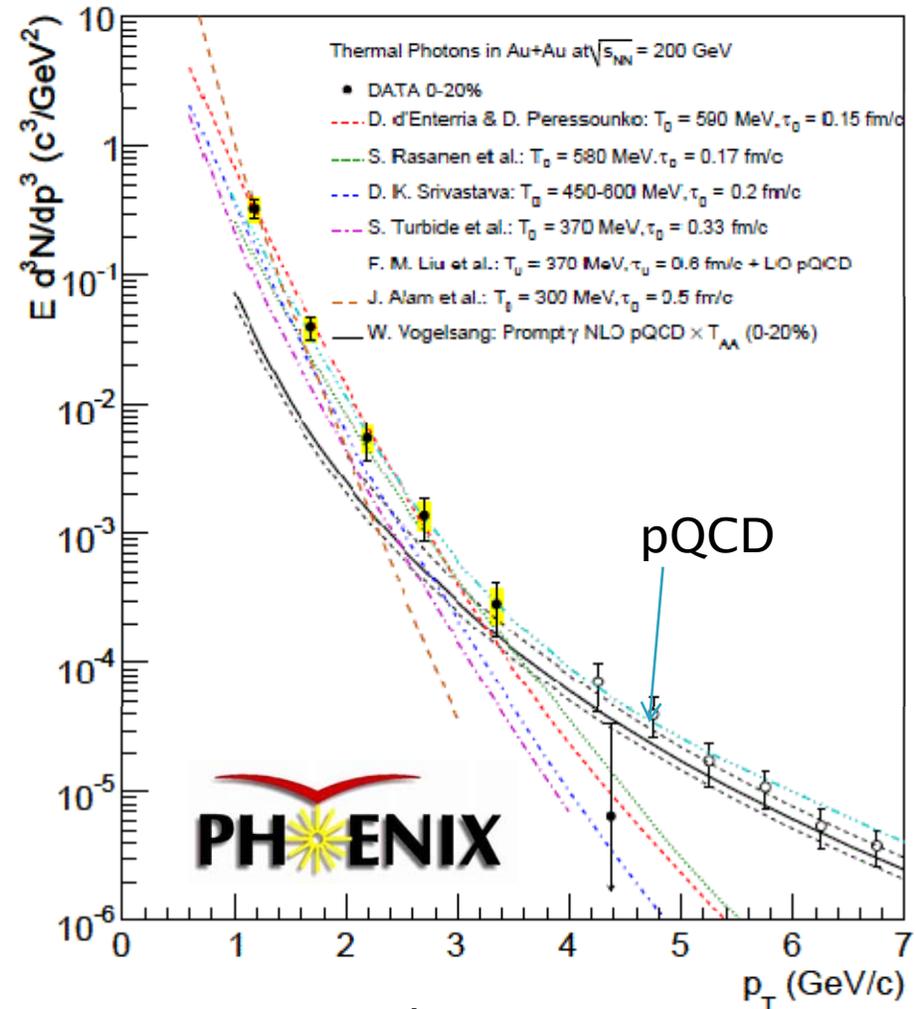
$$\tau_0(\text{AdS}) = 4 \tau_{\text{e-fold}}$$

# Thermal photons – Temperature from the data

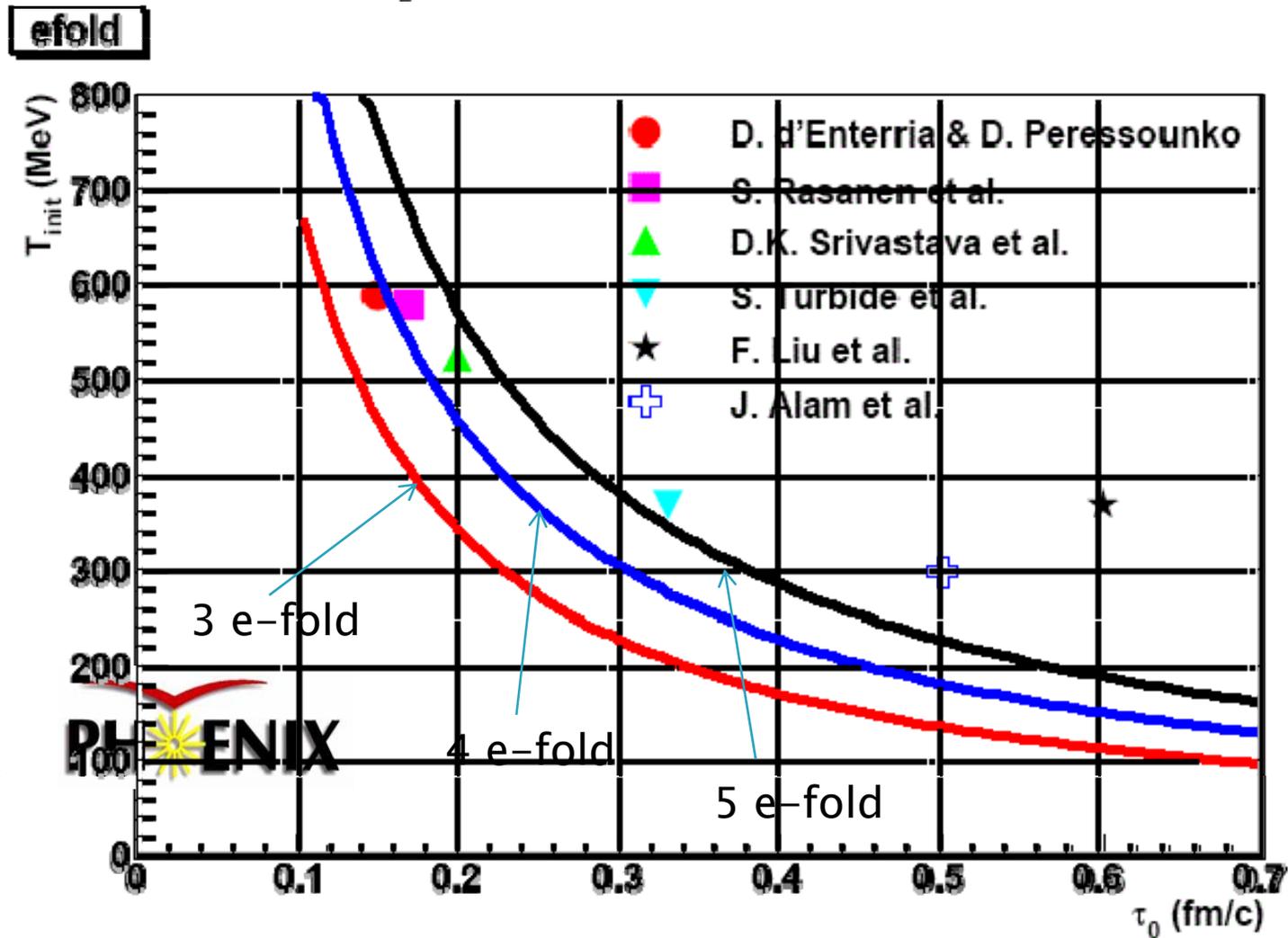
- ▶ Make a measure of low  $p_T$  photons (black body radiation)



Use a model of time dependence and fit data

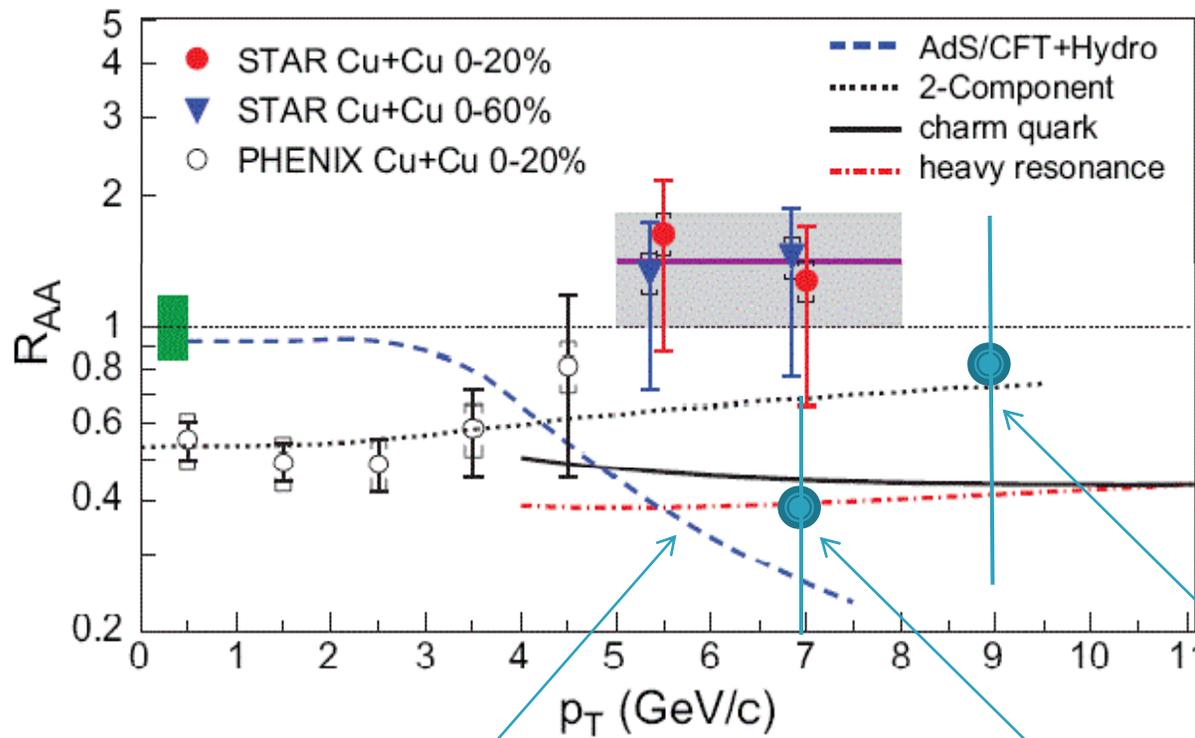


# Thermal photons



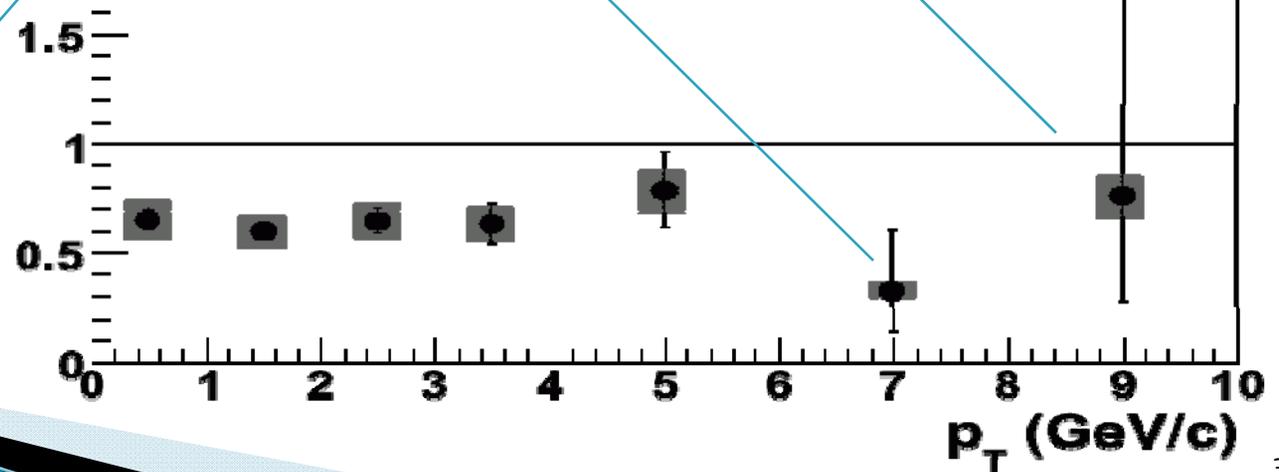
- ▶ Models follow same trend depend on  $\tau_0$

# J/psi high pt

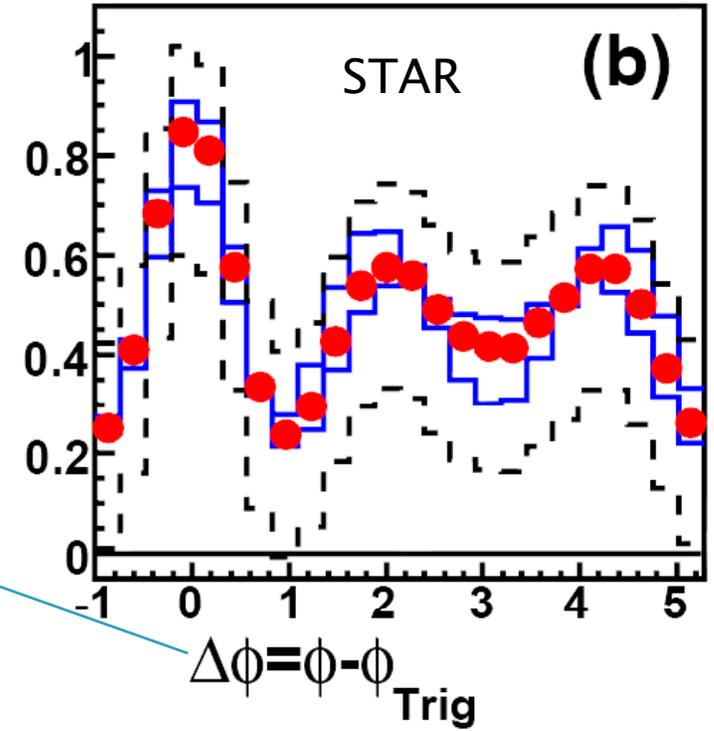
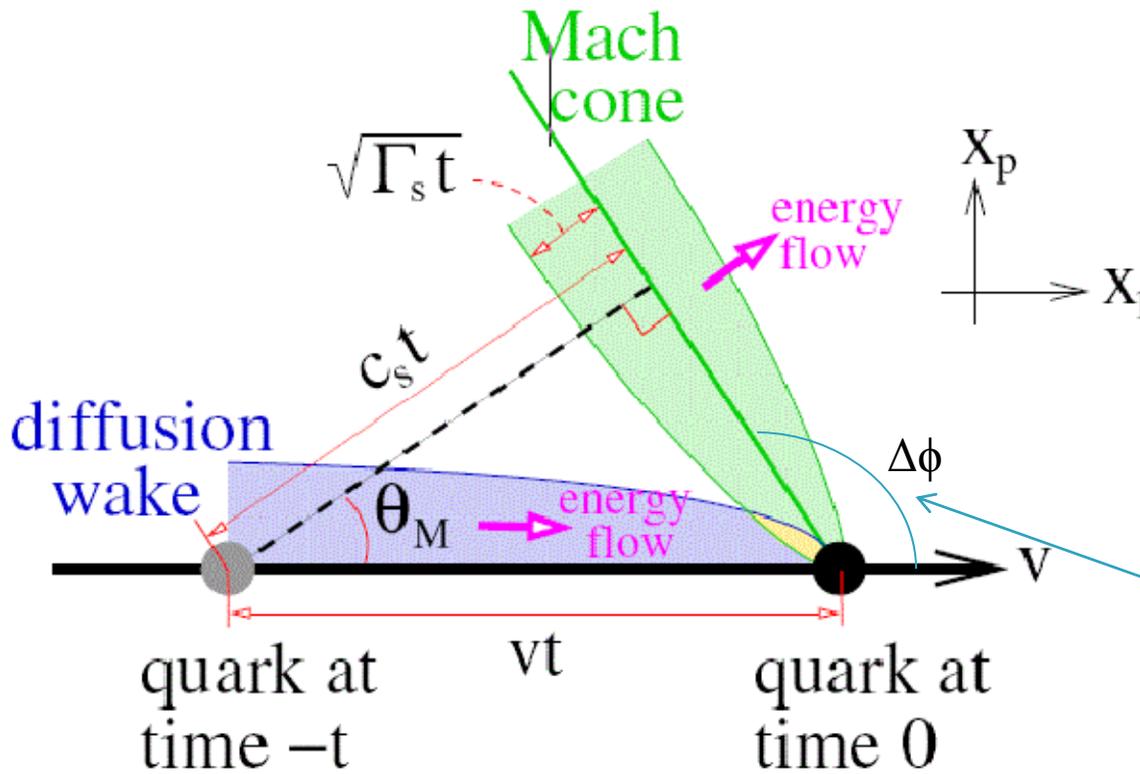


GeV Cu+Cu J/ψ  
 Run 5+6 pp ref  
**PHENIX Preliminary**  
 Centrality 0-94%  
 $\text{sys}_{\text{global}} \pm 16\%$

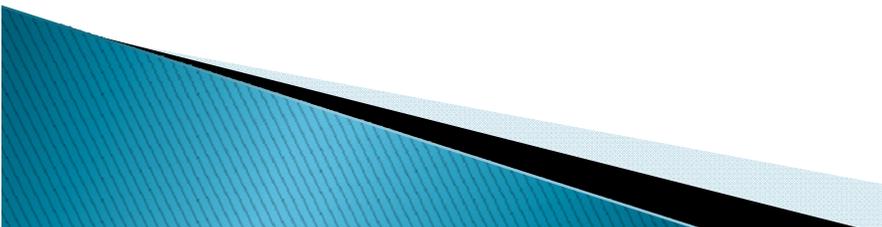
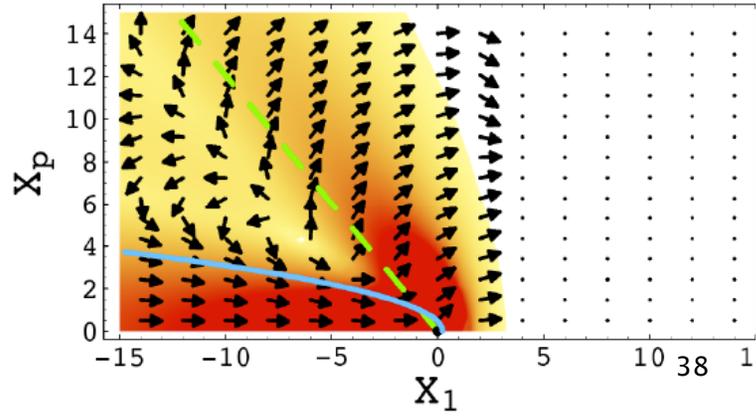
Liu, Rajagopal,  
 Wiedermann  
 "Hot Wind" +  
 Hydro



# Mach cones and diffusion wakes



S for  $v=0.75$



# SCORECARD

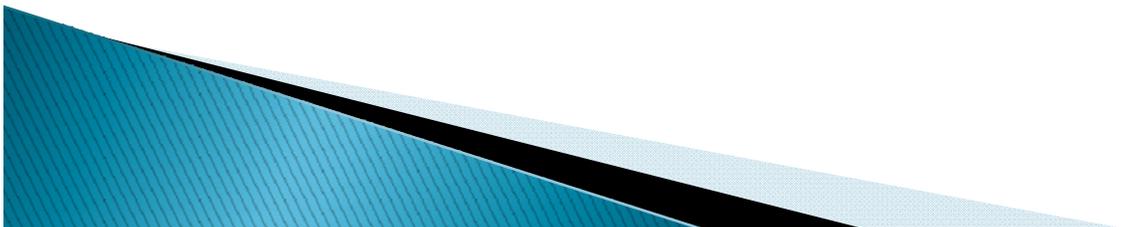
quantity	units	Theory	Experiment	data	method	Collab	comment
$s/s_{SB}$ [1]		0.77	~0.8			Lattice	1 <sup>st</sup> order correction
$\eta/s$ [2]		0.1	~0.08	Charge d flow	Viscous hydro[9]	STAR	Universal? 1 <sup>st</sup> order
$\hat{q}$ [3]	GeV <sup>2</sup> /fm	5-15	13	R_AA (pion)	Fit to PQM [10]	PHENIX	Average over time
$D_{HQ}(\tau_{HQ})$ @T=0.300 GeV [4]		0.6/T	0.6-0.9/T	HQ decay e-	Fit to[11] Rapp, vanHees	PHENIX	Rescaled theory
$T(\tau_0=3\text{fm})$ [5]	GeV	0.300 (4-e fold)	0.380		Fit to[12] models	PHENIX	
$\tau_0$ [6]	fm	0.3	0.6		Fit to flow [13]	STAR/ PHENIX	Init cond needed
J/ $\psi$ [7]	stronger suppression pt>5			Some inconsistency between experiments		wait for more data [14]	
Mach cones, [8] diffusion wakes	exists		Something is there			Need more studies [15]	

# A word of caution

- ▶ There are complications and assumptions that I skipped over e.g.
  - for Hydro
    - need to use Israel–Stewart as not to violate causality
    - need EOS (from lattice – at the moment various ones are used for the fits)
  - The electrons from experiment are actually  $\sim 1/2$  B mesons at high  $p_T$ . For the AdS side we (I) typically assumed  $m=1.4$ 
    - And the list goes on

# Conclusion

- ▶ The Gauge–String Duality (AdS/CFT) is a powerful new tool
- ▶ The various theoretical together are giving us a deeper understanding of the sQGP
  - Can work hand in hand to with the experimentalists
- ▶ pQCD, Gauge String Dualities, Lattice, Viscous Hydrodynamics, Cascade simulations, ...
  - Bring different strengths and weakness
  - ALL are needed and must work together



# References for scorecard

- ▶ [1]hep-th/9805156
- ▶ [2]hep-th 0405231
- ▶ [3]hep-ph/0605178
- ▶ [4]hep-th/0605158, hep-th/0605191
- ▶ [5]arXiv:hep-th/0611005
- ▶ [6]arXiv:0705.1234
- ▶ [7]hep-ph/0607062
- ▶ [8]arXiv:0706.4307
- ▶ [9]Phys.Rev.C78:034915 (2008)
- ▶ [10]arXiv:0801.1665
- ▶ [11]Phys. Rev. Lett. 98, 172301 (2007)
- ▶ [12]arXiv:0804.4168
- ▶ [13]nucl-th/0407067
- ▶ [14]Phys. Rev. C 80 (2009) 41902
- ▶ [15]arXiv:0805.0622

# Plugging in numbers

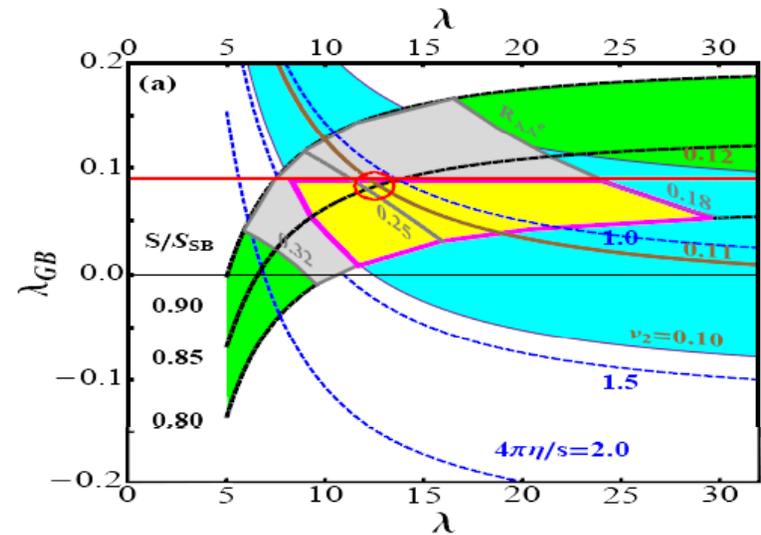
$$\alpha_S^{sQGP} \sim 0.5 \quad (0.35)$$

$$\lambda^{Supersym} = 4\pi N_{colors} \alpha_S \sim 19 \quad (13)$$

$$\frac{1}{\lambda} = 0.05 \quad (0.08)$$

$$\frac{\eta}{s} = \frac{1}{4\pi} \left( 1 + 15 \frac{\zeta(3)}{\lambda^{3/2}} + \dots \right)$$

$$\frac{S}{S_{SB}} = \frac{3}{4} \left( 1 + \frac{15}{8} \frac{\zeta(3)}{\lambda^{3/2}} + \dots \right)$$

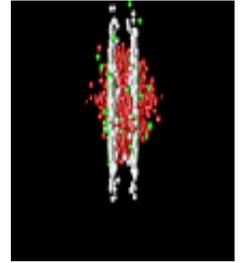


1<sup>st</sup> order correction 20%

1<sup>st</sup> order correction 3%

$\hat{q}$

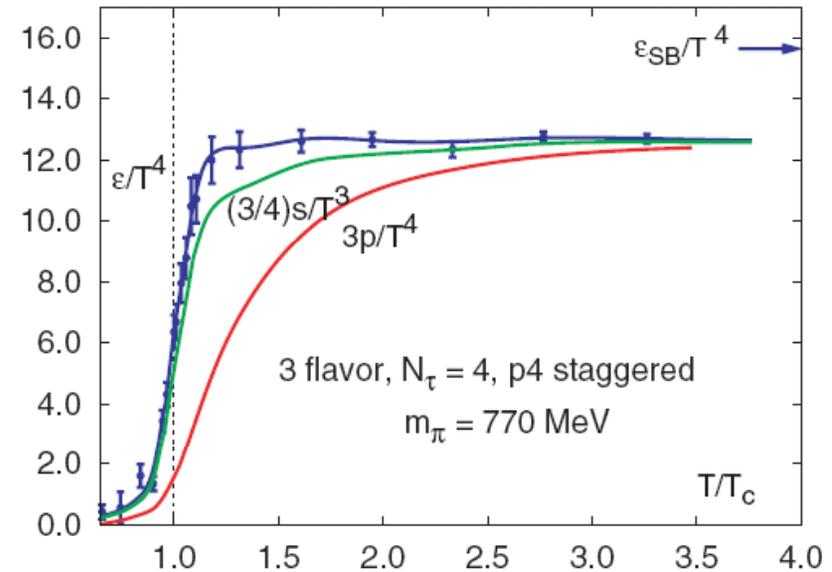
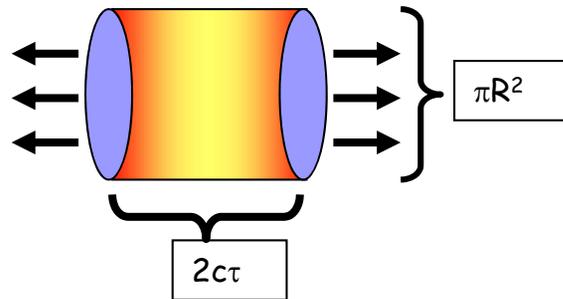
# Energy Density



Energy density far above transition value predicted by lattice.

$$\epsilon_{Bj} = \frac{1}{\pi R^2} \frac{1}{2c\tau} \left( 2 \frac{dE_T}{dy} \right)$$

$R \sim 7 \text{ fm}$



Lattice:  $T_C \sim 190 \text{ MeV}$  ( $\epsilon_C \sim 1 \text{ GeV/fm}^3$ )

Phase transition – fast cross over  
to experimentalist: 1<sup>st</sup> order

PHENIX: Central Au-Au yields

$$\left\langle \frac{dE_T}{d\eta} \right\rangle_{\eta=0} = 503 \pm 2 \text{ GeV}$$

$$\epsilon \sim 15 \frac{\text{GeV}}{\text{fm}^3} \quad @ \tau = 0.6 \text{ fm}/c \text{ (thermalization)}$$