In[5]:=
```
Solve[{a*Exp[-I*k*x] + b*Exp[I*k*x] == c*Exp[-I*j*x] + d*Exp[I*j*x],
  c*Exp[I*j*x] + d*Exp[-I*j*x] == f*Exp[I*k*x],
  -a*k*Exp[-I*k*x] + b*k*Exp[I*k*x] == -c*j*Exp[-I*j*x] + d*j*Exp[I*j*x],
  c*j*Exp[I*j*x] - d*j*Exp[-I*j*x] == f*k*Exp[I*k*x], a == 1}, {a, b, c, d, f}]
```

Out[5]=
$$\left\{\left\{b \to \frac{e^{-2 i k x} \left(-1+e^{4 i j x}\right) \left(-j^2+k^2\right)}{-j^2+e^{4 i j x} j^2-2 j k-2 e^{4 i j x} j k-k^2+e^{4 i j x} k^2},\ f \to -\frac{4 e^{2 i j x-2 i k x} j k}{-j^2+e^{4 i j x} j^2-2 j k-2 e^{4 i j x} j k-k^2+e^{4 i j x} k^2},\ c \to -\frac{2 e^{i j x-i k x} k (j+k)}{-j^2+e^{4 i j x} j^2-2 j k-2 e^{4 i j x} j k-k^2+e^{4 i j x} k^2},\ d \to -\frac{2 e^{3 i j x-i k x} (j-k) k}{-j^2+e^{4 i j x} j^2-2 j k-2 e^{4 i j x} j k-k^2+e^{4 i j x} k^2},\ a \to 1\right\}\right\}$$

In[6]:=
```
FullSimplify[%]
```

Out[6]=
$$\left\{\left\{b \to \frac{e^{-2 i k x} \left(-1+e^{4 i j x}\right) \left(-j^2+k^2\right)}{e^{4 i j x} (j-k)^2-(j+k)^2},\ f \to -\frac{4 e^{2 i (j-k) x} j k}{e^{4 i j x} (j-k)^2-(j+k)^2},\ c \to -\frac{2 e^{i (j-k) x} k (j+k)}{e^{4 i j x} (j-k)^2-(j+k)^2},\ d \to -\frac{2 e^{i (3 j-k) x} (j-k) k}{e^{4 i j x} (j-k)^2-(j+k)^2},\ a \to 1\right\}\right\}$$

In[7]:=
$$\mathbf{ff} := -\frac{4 e^{2 i (j-k) x} j k}{e^{4 i j x} (j-k)^2-(j+k)^2}$$

In[8]:=
```
((Re[ff]^2 + Im[ff]^2))^-1
```

Out[8]=
$$\frac{1}{16\,\mathrm{Im}\left[\frac{e^{2 i (j-k) x} j k}{e^{4 i j x} (j-k)^2-(j+k)^2}\right]^2+16\,\mathrm{Re}\left[\frac{e^{2 i (j-k) x} j k}{e^{4 i j x} (j-k)^2-(j+k)^2}\right]^2}$$

In[9]:=
```
FullSimplify[Factor[ComplexExpand[%]]]
```

Out[9]=
$$\frac{j^4+6 j^2 k^2+k^4-\left(j^2-k^2\right)^2 \mathrm{Cos}[4 j x]}{8 j^2 k^2}$$

In[10]:=
```
% /. {k -> Sqrt[2*m*h], j -> Sqrt[2*m*(h - v)]}
```

Out[10]=
$$\frac{4 h^2 m^2+24 h m^2 (h-v)+4 m^2 (h-v)^2-\left(-2 h m+2 m (h-v)\right)^2 \mathrm{Cos}\left[4 \sqrt{2} \sqrt{m (h-v)}\, x\right]}{32 h m^2 (h-v)}$$

In[11]:=
```
fff = FullSimplify[Factor[%]]
```

Out[11]=
$$\frac{8 h^2-8 h v+v^2-v^2 \mathrm{Cos}\left[4 \sqrt{2} \sqrt{m (h-v)}\, x\right]}{8 h (h-v)}$$

In[12]:=
```
fff - 1
```

Out[12]=
$$-1+\frac{8 h^2-8 h v+v^2-v^2 \mathrm{Cos}\left[4 \sqrt{2} \sqrt{m (h-v)}\, x\right]}{8 h (h-v)}$$

In[13]:= **FullSimplify[%]**

Out[13]= $$\frac{v^2 \operatorname{Sin}\left[2\sqrt{2}\sqrt{m(h-v)}\,x\right]^2}{4h(h-v)}$$

This expression matches the one for $1/T-1$ in the book

In[14]:= $$b := \frac{e^{-2ikx}(-1+e^{4ijx})(k^2-j^2)}{e^{4ijx}(j-k)^2-(j+k)^2}$$

$$f := -\frac{4e^{2i(j-k)x}jk}{e^{4ijx}(j-k)^2-(j+k)^2}$$

$$c := -\frac{2e^{i(j-k)x}k(j+k)}{e^{4ijx}(j-k)^2-(j+k)^2}$$

$$d := \frac{2e^{i(3j-k)x}k(k-j)}{e^{4ijx}(j-k)^2-(j+k)^2}$$

In[18]:= **bb = *b* /. {*k* –> Sqrt[2 ∗ *m* ∗ *h*], *j* –> Sqrt[2 ∗ *m* ∗ (*h* – *v*)]}**

Out[18]= $$\frac{e^{-2i\sqrt{2}\sqrt{hm}\,x}\left(-1+e^{4i\sqrt{2}\sqrt{m(h-v)}\,x}\right)(2hm-2m(h-v))}{e^{4i\sqrt{2}\sqrt{m(h-v)}\,x}\left(-\sqrt{2}\sqrt{hm}+\sqrt{2}\sqrt{m(h-v)}\right)^2-\left(\sqrt{2}\sqrt{hm}+\sqrt{2}\sqrt{m(h-v)}\right)^2}$$

In[19]:= **cc = *c* /. {*k* –> Sqrt[2 ∗ *m* ∗ *h*], *j* –> Sqrt[2 ∗ *m* ∗ (*h* – *v*)]}**

Out[19]= $$-\frac{2\sqrt{2}\,e^{i\left(-\sqrt{2}\sqrt{hm}+\sqrt{2}\sqrt{m(h-v)}\right)x}\sqrt{hm}\left(\sqrt{2}\sqrt{hm}+\sqrt{2}\sqrt{m(h-v)}\right)}{e^{4i\sqrt{2}\sqrt{m(h-v)}\,x}\left(-\sqrt{2}\sqrt{hm}+\sqrt{2}\sqrt{m(h-v)}\right)^2-\left(\sqrt{2}\sqrt{hm}+\sqrt{2}\sqrt{m(h-v)}\right)^2}$$

In[20]:= **dd = *d* /. {*k* –> Sqrt[2 ∗ *m* ∗ *h*], *j* –> Sqrt[2 ∗ *m* ∗ (*h* – *v*)]}**

Out[20]= $$\frac{2\sqrt{2}\,e^{i\left(-\sqrt{2}\sqrt{hm}+3\sqrt{2}\sqrt{m(h-v)}\right)x}\sqrt{hm}\left(\sqrt{2}\sqrt{hm}-\sqrt{2}\sqrt{m(h-v)}\right)}{e^{4i\sqrt{2}\sqrt{m(h-v)}\,x}\left(-\sqrt{2}\sqrt{hm}+\sqrt{2}\sqrt{m(h-v)}\right)^2-\left(\sqrt{2}\sqrt{hm}+\sqrt{2}\sqrt{m(h-v)}\right)^2}$$

In[21]:= **ff = *f* /. {*k* –> Sqrt[2 ∗ *m* ∗ *h*], *j* –> Sqrt[2 ∗ *m* ∗ (*h* – *v*)]}**

Out[21]= $$-\frac{8\,e^{2i\left(-\sqrt{2}\sqrt{hm}+\sqrt{2}\sqrt{m(h-v)}\right)x}\sqrt{hm}\sqrt{m(h-v)}}{e^{4i\sqrt{2}\sqrt{m(h-v)}\,x}\left(-\sqrt{2}\sqrt{hm}+\sqrt{2}\sqrt{m(h-v)}\right)^2-\left(\sqrt{2}\sqrt{hm}+\sqrt{2}\sqrt{m(h-v)}\right)^2}$$

Now lets find some typical numbers for the coeeficients. The units are such that things are the mass for 1 electron $k \sim 1/$ angstroms energies are in ev. *m* is set to 5 to make the units work

In[22]:= **Norm[*N*[bb /. {*m* → 5, *h* → .8, *v* → 1, *x* → 1}]]**

Out[22]= 0.995527

In[23]:= **Norm[*N*[cc /. {*m* → 5, *h* → .8, *v* → 1, *x* → 1}]]**

Out[23]= 0.434473

In[24]:= **Norm[*N*[dd /. {*m* → 5, *h* → .8, *v* → 1, *x* → 1}]]**

Out[24]= 0.0256799

In[25]:= **Norm[*N*[ff /. {*m* → 5, *h* → .8, *v* → 1, *x* → 1}]]**

Out[25]= 0.0944762

In[26]:= **bbb[x_] := Norm[*N*[bb /. {*m* → 5, *h* → .8, *v* → 1}]]**

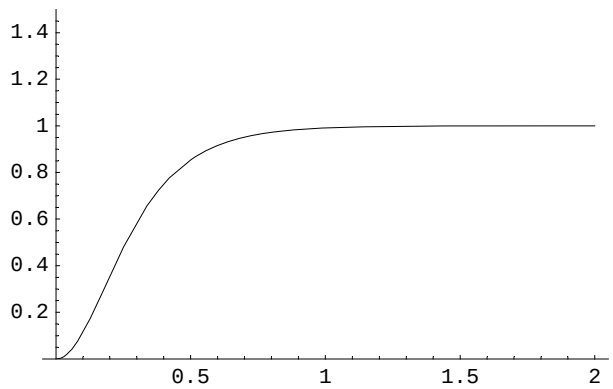
In[27]:= **ccc[x_] := Norm[*N*[cc /. {*m* → 5, *h* → .8, *v* → 1}]]**

In[28]:= **ddd[x_] := Norm[*N*[dd /. {*m* → 5, *h* → .8, *v* → 1}]]**

In[29]:= **fffff[x_] := Norm[*N*[ff /. {*m* → 5, *h* → .8, *v* → 1}]]**

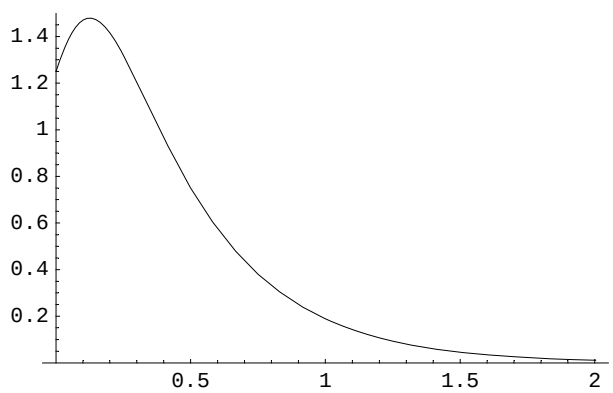
Now lets plot these coefficents vs the width of the barrier

In[30]:= **Plot[bbb[*x*]^2, {*x*, 0, 2}, PlotRange → {0, 1.5}]**



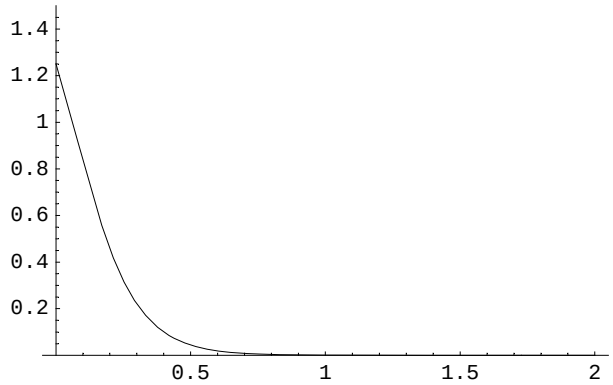
Out[30]= - Graphics -

In[31]:= **Plot[ccc[*x*]^2, {*x*, 0, 2}, PlotRange → {0, 1.5}]**



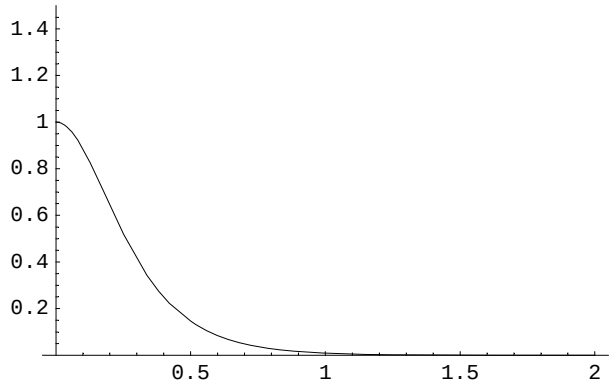
Out[31]= - Graphics -

In[32]:= **Plot[ddd[x]^2, {x, 0, 2}, PlotRange → {0, 1.5}]**



Out[32]= - Graphics -

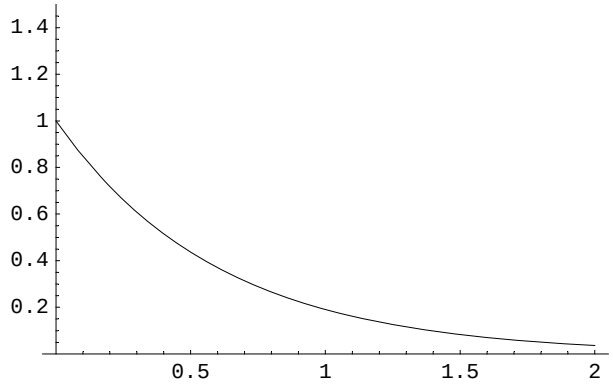
In[33]:= **Plot[fffff[x]^2, {x, 0, 2}, PlotRange → {0, 1.5}]**



Out[33]= - Graphics -

check that the term growing in the barrier does not dominate, *i.e.* plot growing term / falling term vs width

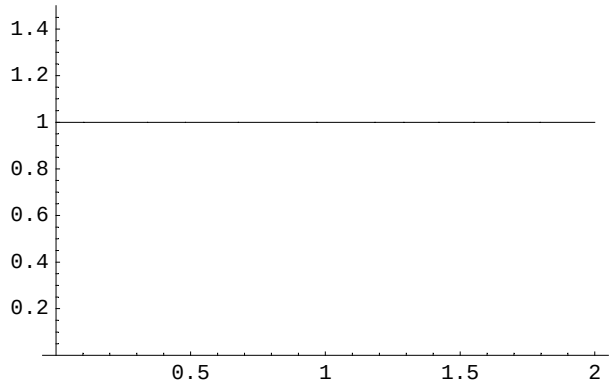
In[34]:= **Plot[(ddd[x] ∗ Exp[x])^2 / (ccc[x] ∗ Exp[−x])^2, {x, 0, 2}, PlotRange → {0, 1.5}]**



Out[34]= - Graphics -

Check that $T + R = 1$

In[35]:= **Plot[bbb[x]^2 + fffff[x]^2, {x, 0, 2}, PlotRange → {0, 1.5}]**

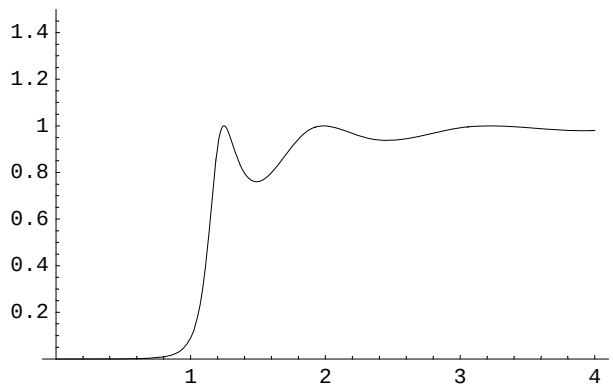


Out[35]= - Graphics -

Plot T vs E where $V = 1$

In[36]:= **ffff[h_] := Norm[N[ff /. {m → 5, v → 1, x → 1}]]**

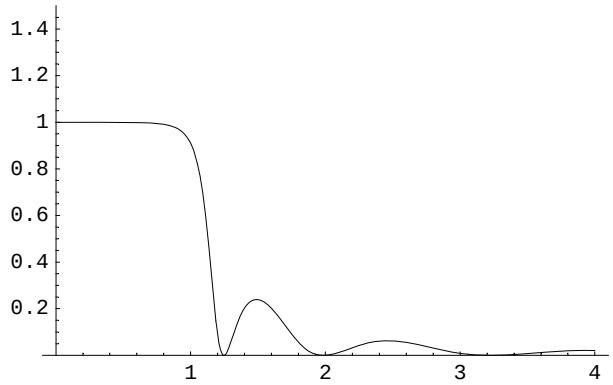
In[37]:= **Plot[ffff[h]^2, {h, 0, 4}, PlotRange → {0, 1.5}]**



Out[37]= - Graphics -

In[38]:= **pb[h_] := Norm[N[bb /. {m → 5, x → 1, v → 1}]]**

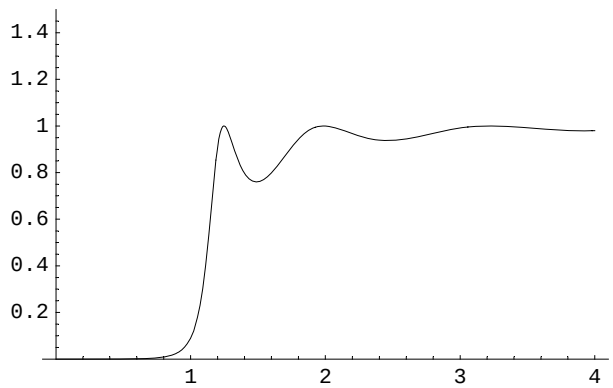
In[39]:= **Plot[pb[h]^2, {h, 0, 4}, PlotRange → {0, 1.5}]**



Out[39]= - Graphics -

In[40]:= **pf[h_] := Norm[N[ff /. {m → 5, x → 1, v → 1}]]**

In[41]:= **Plot[pf[h]^2, {h, 0, 4}, PlotRange → {0, 1.5}]**

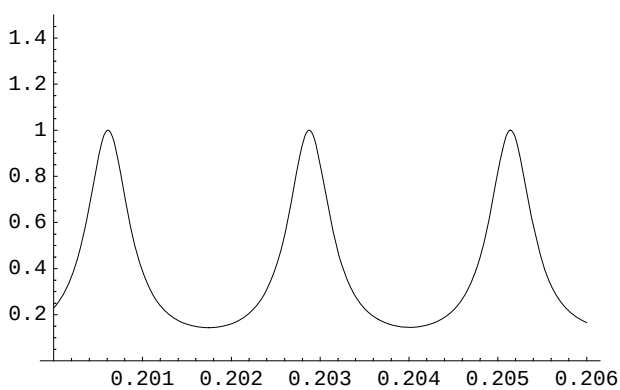


Out[41]= - Graphics -

Now lets go to the case where $V < 0$ *i.e.* the Ramsauer effect

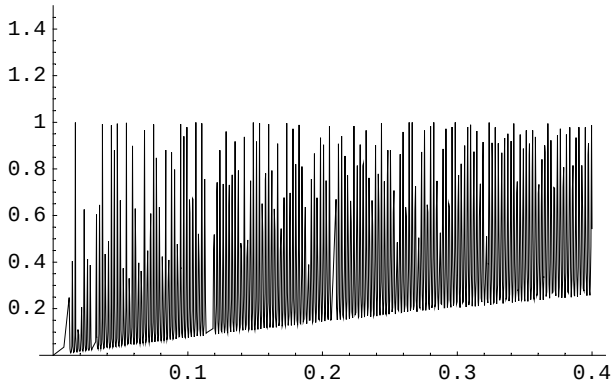
In[42]:= **ffff[h_] := Norm[N[ff /. {m → 5, v → −5, x → 1000}]]**

In[43]:= **Plot[ffff[h]^2, {h, .2, .206}, PlotRange → {0, 1.5}]**



Out[43]= - Graphics -

In[44]:= **Plot[ffff[h]^2, {h, 0, .4}, PlotRange → {0, 1.5}]**

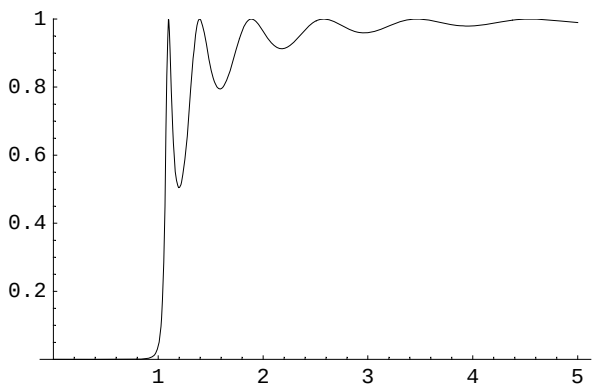


Out[44]= - Graphics -

Now lets go to the case where $E < V$, *i.e* tunneling. This should give the same sorts of pictures as above

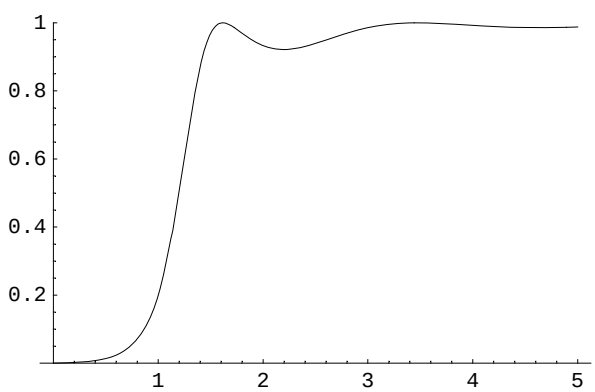
In[45]:= **z[v_, e_, a_] := (1. + .25 ∗ v ∗ v / e / (v − e) ∗ Sinh[2 ∗ Sqrt[v − e] ∗ 5 ∗ a] ∗ Sinh[2 ∗ Sqrt[v − e] ∗ 5 ∗ a])$^{-1}$**

In[46]:= **Plot[z[1., e, 1.], {e, 0, 5}, PlotRange → {0, 1}]**



Out[46]= - Graphics -

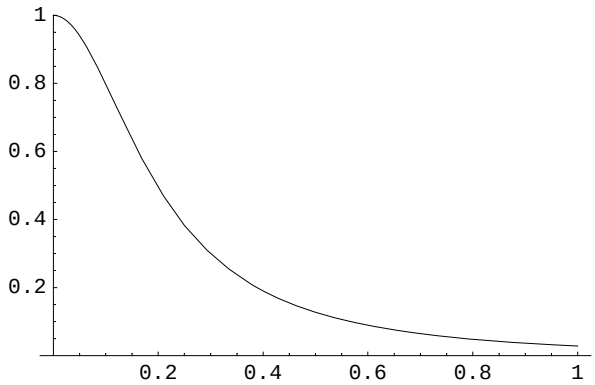
In[47]:= **Plot[z[1., e, .4], {e, 0, 5}, PlotRange → {0, 1}]**



Out[47]= - Graphics -

Plot T vs the width of the barrier. Like for an STM

In[48]:= **Plot[z[1., 0.99, a], {a, 0, 1}, PlotRange → {0, 1}]**



Out[48]= - Graphics -

Now lets look at the wave function norms for tunneling

In[49]:=
```
phi1[y_] := Exp[I * k * y] + (e^(-2 i k x) (-1 + e^(4 i j x)) (k^2 - j^2))/(e^(4 i j x) (j - k)^2 - (j + k)^2) * Exp[-I * k * y]

phi2[y_] := -(2 e^(i (j-k) x) k (j + k))/(e^(4 i j x) (j - k)^2 - (j + k)^2) * Exp[I * j * y] + (2 e^(i (3 j-k) x) k (k - j))/(e^(4 i j x) (j - k)^2 - (j + k)^2) * Exp[-I * j * y]

phi3[y_] := (4 e^(2 i (j-k) x) j k)/(e^(4 i j x) (j - k)^2 - (j + k)^2) * Exp[I * k * y]
```

In[53]:=
$$b := \frac{e^{-2ikx}(-1+e^{4ijx})(k^2-j^2)}{e^{4ijx}(j-k)^2-(j+k)^2}$$
$$f := -\frac{4e^{2i(j-k)x}jk}{e^{4ijx}(j-k)^2-(j+k)^2}$$
$$c := -\frac{2e^{i(j-k)x}k(j+k)}{e^{4ijx}(j-k)^2-(j+k)^2}$$
$$d := \frac{2e^{i(3j-k)x}k(k-j)}{e^{4ijx}(j-k)^2-(j+k)^2}$$

In[57]:= **kphi1[y_, h_, v_, m_, x_] = FullSimplify[phi1[y] /. {k –> Sqrt[2 * m * h], j –> Sqrt[2 * m * (h – v)]}]**

Out[57]=
$$e^{i\sqrt{2}\sqrt{hm}\,y} - \frac{e^{-i\sqrt{2}\sqrt{hm}\,(2x+y)}\,m\,v}{m\,(-2h+v) - 2i\sqrt{hm}\sqrt{m\,(h-v)}\,\mathrm{Cot}\left[2\sqrt{2}\sqrt{m\,(h-v)}\,x\right]}$$

In[58]:= **kphi2[y_, h_, v_, m_, x_] = FullSimplify[phi2[y] /. {k –> Sqrt[2 * m * h], j –> Sqrt[2 * m * (h – v)]}]**

Out[58]=
$$\Big(2\,e^{-i\sqrt{2}\left(\left(\sqrt{hm}-\sqrt{m\,(h-v)}\right)x-\sqrt{m\,(h-v)}\,y\right)}\left(-\left(-1+e^{2i\sqrt{2}\sqrt{m\,(h-v)}\,(x-y)}\right)hm+\left(1+e^{2i\sqrt{2}\sqrt{m\,(h-v)}\,(x-y)}\right)\sqrt{hm}\sqrt{m\,(h-v)}\right)\Big)\Big/$$
$$\left(-2\left(-1+e^{4i\sqrt{2}\sqrt{m\,(h-v)}\,x}\right)hm+2\left(1+e^{4i\sqrt{2}\sqrt{m\,(h-v)}\,x}\right)\sqrt{hm}\sqrt{m\,(h-v)}+\left(-1+e^{4i\sqrt{2}\sqrt{m\,(h-v)}\,x}\right)m\,v\right)$$

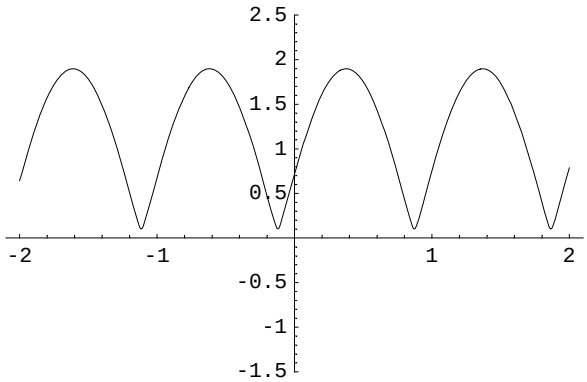
In[59]:= **kphi3[y_, h_, v_, m_, x_] = FullSimplify[phi3[*y*] /. {*k* –> Sqrt[2 ∗ *m* ∗ *h*], *j* –> Sqrt[2 ∗ *m* ∗ (*h* – *v*)]}]**

Out[59]=
$$-\frac{2\, e^{-i\sqrt{2}\sqrt{h\,m}\,(2x-y)}\sqrt{h\,m}\,\sqrt{m\,(h-v)}}{2\sqrt{h\,m}\,\sqrt{m\,(h-v)}\,\mathrm{Cos}\left[2\sqrt{2}\,\sqrt{m\,(h-v)}\,x\right]-i\,m\,(2\,h-v)\,\mathrm{Sin}\left[2\sqrt{2}\,\sqrt{m\,(h-v)}\,x\right]}$$

In[60]:= **Re[kphi1[3, 1, 2, 5, 1]]**

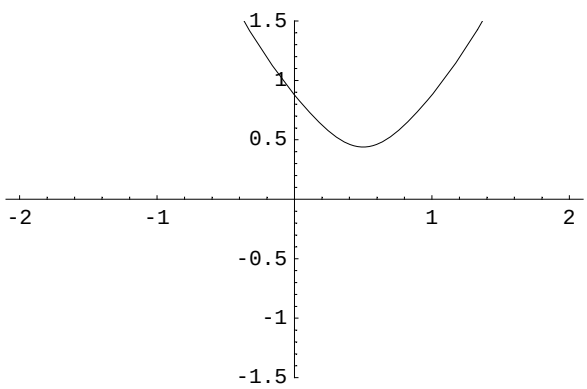
Out[60]= $\mathrm{Cos}\left[3\sqrt{10}\right]-\mathrm{Sin}\left[5\sqrt{10}\right]\mathrm{Tanh}\left[2\sqrt{10}\right]$

In[61]:= **p1 = Plot[Norm[kphi1[*y*, 1, 1.1, 5, .5]], {*y*, –2, 2}, PlotRange → {–1.5, 2.5}]**



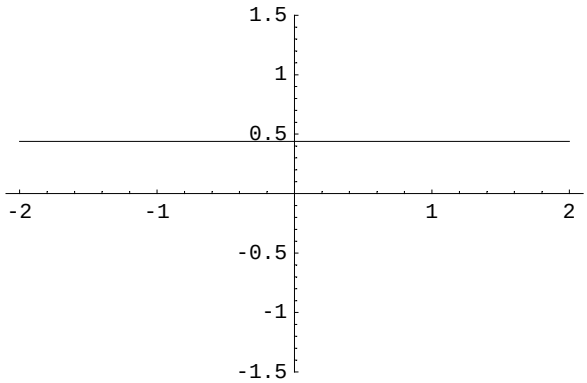
Out[61]= - Graphics -

In[62]:= **p2 = Plot[Norm[kphi2[*y*, 1, 1.1, 5, .5]], {*y*, –2, 2}, PlotRange → {–1.5, 1.5}]**



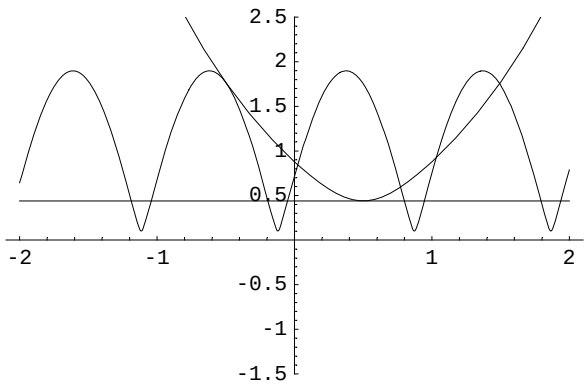
Out[62]= - Graphics -

In[63]:= **p3 = Plot[Norm[kphi3[*y*, 1, 1.1, 5, .5]], {*y*, −2, 2}, PlotRange → {−1.5, 1.5}]**



Out[63]= - Graphics -

In[64]:= **Show[p1, p2, p3]**



Out[64]= - Graphics -