Notes for Quantum Mechanics

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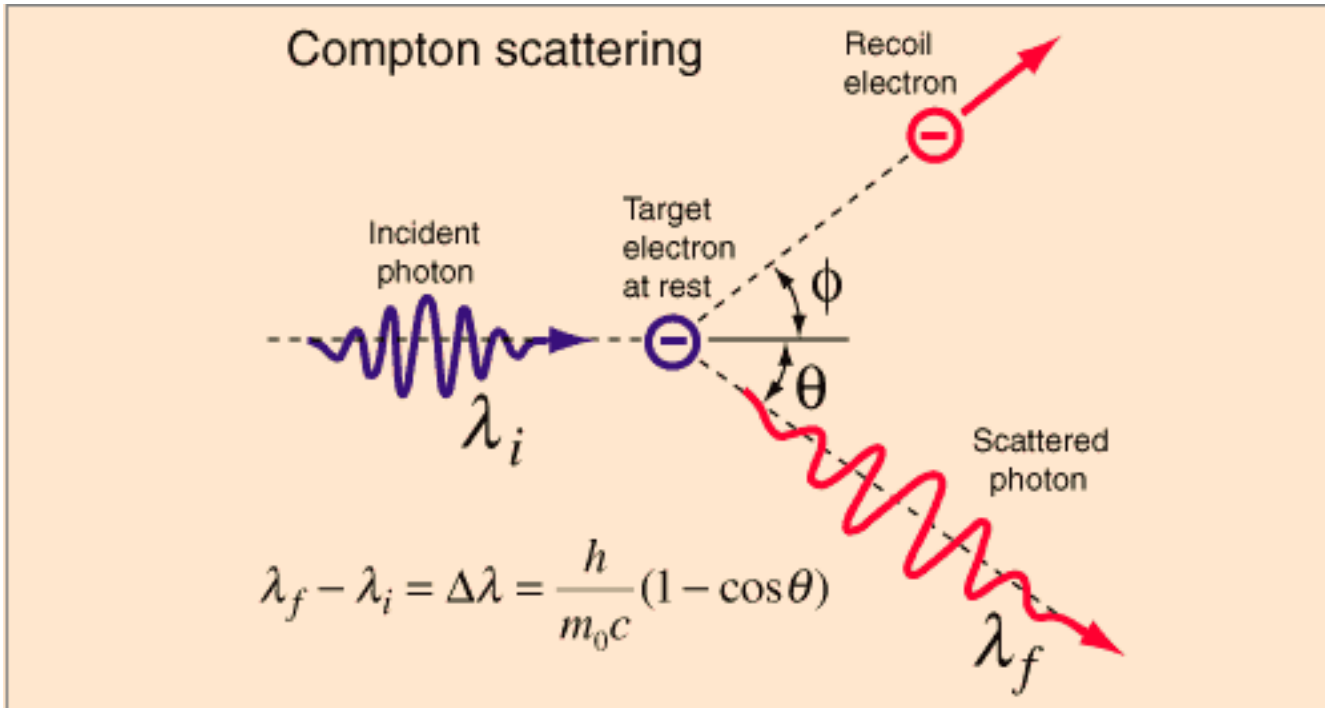
Liboff 2.28 - compton scattering



$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

The scattering of photons from charged particles is called Compton scattering after Arthur Compton who was the first to measure photon-electron scattering in 1922. When the incoming photon gives part of its energy to the electron, then the scattere photon has lower energy and according to the Planck relationship has lower frequency and longer wavelength. The wavelength change in such scattering depen only upon the angle of scattering for a given target particle. The constant in the Compton formula above can be written

$$\frac{h}{m_e c} = \frac{hc}{m_e c^2} = \frac{1240\ \ eV\ \ nm}{0.511\ \ MeV} = .00243\ \ nm$$ *Compton wavelength for the electron*

and is called the Compton wavelength for the electron. The formula presumes that scattering occurs in the rest frame of the electron.

This problem is an algebraic pain. I presume that everyone understands the initial equations given in the book - i.e. energy and momentum conservation. I will start with these (using the notation in the book where the final state is indicated by a prime)

$h\nu + mc^2 = h\nu' + m\gamma c^2$ energy conservation (1)

$\frac{h}{\lambda} = m\gamma\beta c \cos\phi + \left(\frac{h}{\lambda'}\right) \cos\theta$ momentum conservation in the x direction (2)

$0 = -m\gamma\beta c \sin\phi + \left(\frac{h}{\lambda'}\right) \sin\theta$ momentum conservation in the y direction (3)

also $\nu = \frac{c}{\lambda}$ (4)

rewrite (2)

$h\nu = mc^2 \beta\gamma\cos\phi + h\nu' \cos\theta$ (5)

rewrite (3)

$0 = -mc^2 \beta\gamma\sin\phi + h\nu' \sin\theta$ (6)

Solve (6) for $\sin\phi$

$\sin\phi = \frac{h\nu' \sin\theta}{mc^2 \beta\gamma}$ so $mc^2 \beta\gamma\cos\phi = \sqrt{m^2 c^4 \beta^2 \gamma^2 - h^2 \nu'^2 \sin^2\theta}$ (7)

Substutute this in to (5)

$h\nu = \sqrt{m^2 c^4 \beta^2 \gamma^2 - h^2 \nu'^2 \sin^2\theta} + h\nu' \cos\theta$ (8)

So then^2 $\nu^2 - 2 h^2 \nu\nu' \cos\theta = m^2 c^4 \beta^2 \gamma^2 - h^2 \nu'^2$

$[h\nu - h\nu' \cos\theta]^2 = m^2 c^4 \beta^2 \gamma^2 - h^2 \nu'^2 \sin^2\theta$

$h^2 \nu^2 - 2 h^2 \nu\nu' \cos\theta + h^2 \nu'^2 \cos^2\theta = m^2 c^4 \beta^2 \gamma^2 - h^2 \nu'^2 \sin^2\theta$

put the $\sin^2$ and $\cos^2$ together to get (9)

$h^2 \nu^2 - 2 h^2 \nu\nu' \cos\theta = m^2 c^4 \beta^2 \gamma^2 - h^2 \nu'^2$

$-2 h^2 \nu\nu' \cos\theta = -h^2 \nu'^2 - h^2 \nu^2 + m^2 c^4 \beta^2 \gamma^2$

$-2 h^2 \nu\nu' \cos\theta = -2 h^2 \nu\nu' - h^2 \nu'^2 + 2 h^2 \nu\nu' - h^2 \nu^2 + m^2 c^4 \beta^2 \gamma^2$

$-2 h^2 \nu\nu' \cos\theta = -2 h^2 \nu\nu' - (h\nu - h\nu')^2 + m^2 c^4 \beta^2 \gamma^2$ (10)

now rewrite (1)

$h\nu - h\nu' = m\gamma c^2 - mc^2 = mc^2 (\gamma - 1)$ (11)

then use (11) in (10)

$$-2\,h^2\,\nu\nu'\,\cos\theta = -2\,h^2\,\nu\nu' - (m\gamma c^2 - mc^2)^2 + m^2\,c^4\,\beta^2\,\gamma^2$$

$$-2\,h^2\,\nu\nu'\,\cos\theta = -2\,h^2\,\nu\nu' - m^2\,\gamma^2\,c^4 + 2\,m^2\,c^4\,\gamma - m^2\,c^4 + m^2\,c^4\,\beta^2\,\gamma^2$$

$$-2\,h^2\,\nu\nu'\,\cos\theta = -2\,h^2\,\nu\nu' + 2\,m^2\,c^4\,\gamma - m^2\,c^4 + m^2\,c^4\,\gamma^2\,(\beta^2 - 1)$$

now $\beta^2 - 1 = \frac{-1}{\gamma^2}$ so $\gamma^2\,(\beta^2 - 1) = -1$

$$-2\,h^2\,\nu\nu'\,\cos\theta = -2\,h^2\,\nu\nu' + 2\,m^2\,c^4\,\gamma - m^2\,c^4 - m^2\,c^4$$

$$-2\,h^2\,\nu\nu'\,\cos\theta = -2\,h^2\,\nu\nu' + 2\,m^2\,c^4\,\gamma - 2\,m^2\,c^4$$

$$-2\,h^2\,\nu\nu'\,\cos\theta = -2\,h^2\,\nu\nu' + 2\,mc^2\,mc^2\,(\gamma - 1) \tag{12}$$

using (11)

$$-2\,h^2\,\nu\nu'\,\cos\theta = -2\,h^2\,\nu\nu' + 2\,mc^2\,(h\nu - h\nu')$$

$$-2\,h^2\,\nu\nu'\,\cos\theta = -2\,h^2\,\nu\nu' + 2\,mc^2\,h\nu - 2\,mc^2\,h\nu'$$

$$-\cos\theta = -1 + \frac{2\,mc^2\,h\nu}{-2\,h^2\,\nu\nu'} - \frac{2\,mc^2\,h\nu'}{-2\,h^2\,\nu\nu'}$$

$$1 - \cos\theta = \frac{mc^2}{h\nu'} - \frac{mc^2}{h\nu}$$

$$\frac{h}{mc}\,(1 - \cos\theta) = \frac{c}{\nu'} - \frac{c}{\nu}$$

now $\lambda_c = \frac{h}{mc}$ (13)

$$\lambda_c\,(1 - \cos\theta) = \lambda' - \lambda \tag{14}$$