General Doppler Formula (Serway convention)

$$f_L = \frac{v + v_L}{v - v_S} f_S$$

f_s= frequency emitted by source v_S=Velocity of source

f_L= frequency received by listener v_L=Velocity of listener

positive direction is when one is moving toward the other

V_S (positive) $\longrightarrow$

V_L (positive) $\longleftarrow$

Source

Listener

Example

$$f_L = \frac{v + v_L}{v - v_S} f_S$$

f_s= frequency emitted by source v_S=Velocity of source

f_L= frequency received by listener v_L=Velocity of listener

positive direction is when one is moving toward the other

Example: Police car w/ siren and passenger car approaching each other; both moving.

$f_{\text{siren}} = 250\ \text{Hz} \quad |v_s| = 27\ \text{m/s (60 mph)} \quad |v_L| = 27\ \text{m/s}$

both moving toward each other v_L and v_s positive

$$f' = (250)\left(\frac{330 + 27}{330 - 27}\right) = 295\ \text{Hz}$$

Once they pass each other and are moving away

both moving away from each other v_L and v_s both negative

$$f' = (250)\left(\frac{330 - 27}{330 + 27}\right) = 213\ \text{Hz}$$

Ultrasound

10 wks

f=3Hz λ_{water}~0.5 mm

Chapter 21

Superposition and standing waves

http://www.phy.ntnu.edu.tw/java/waveSuperposition/waveSuperposition.html

When waves combine

Two waves propagate in the same medium
Their amplitudes are not large
Then the ***total*** perturbation is the ***sum*** of the ones for each wave

On a string:
Wave 1: y_1
Wave 2: y_2
Total wave: y_1+y_2

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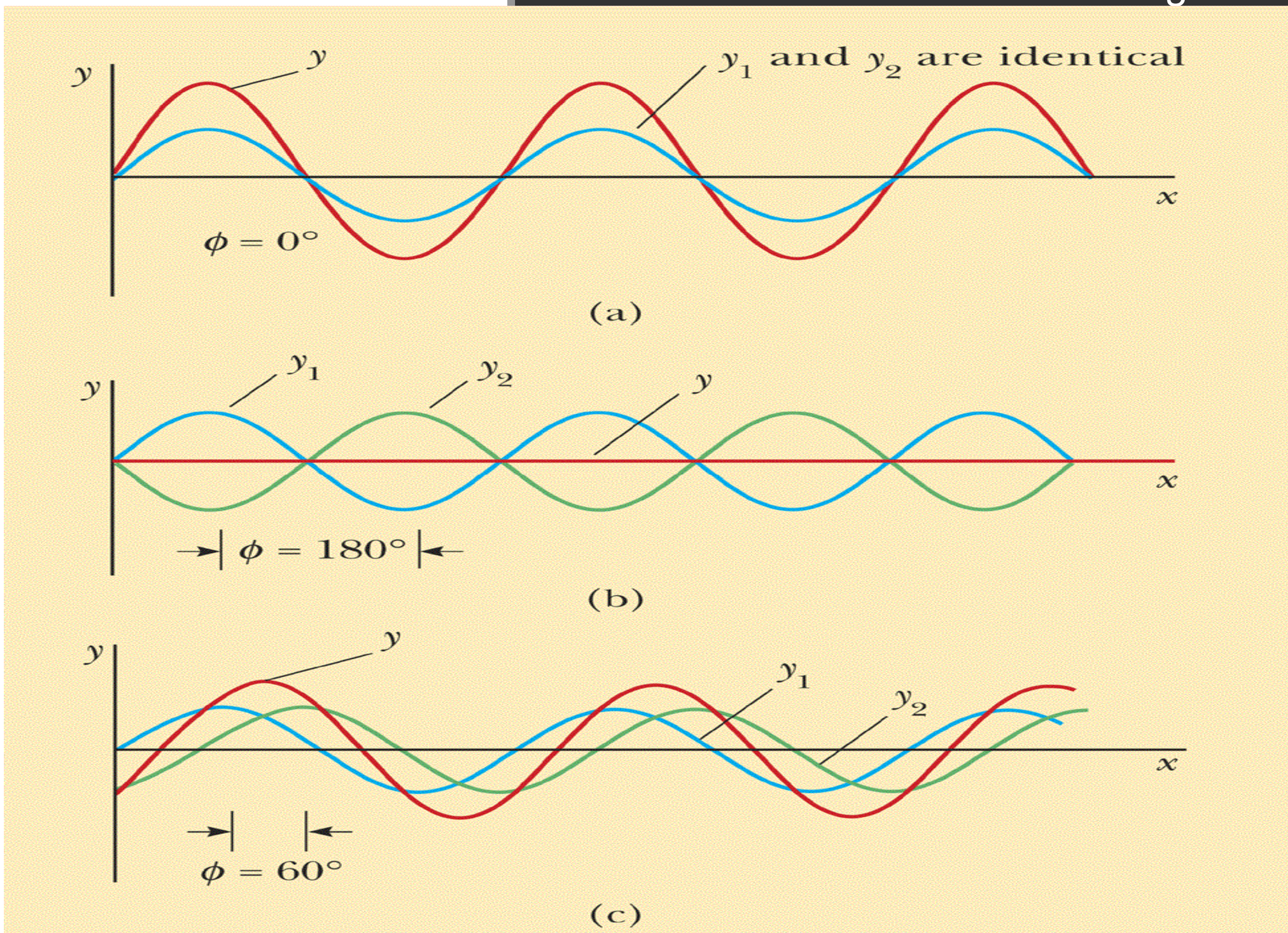
The combination of waves can give very



to get

$$y = y1 + y2 = 2A \cos(\Delta k\ x - \Delta\omega\ t + \Delta\phi) \sin(kx - \omega t + \phi)$$

Assume

$\Delta k = 0 \ \Delta\omega = 0$

$$y = [2A \cos(\Delta\phi)] \sin(kx - \omega t + \phi)$$

constructive interference

When $\Delta\phi = 0$

$$y = [2A] \sin(kx - \omega t + \phi)$$

The waves add up

destructive interference

When $\Delta\phi = \pi/2$ or

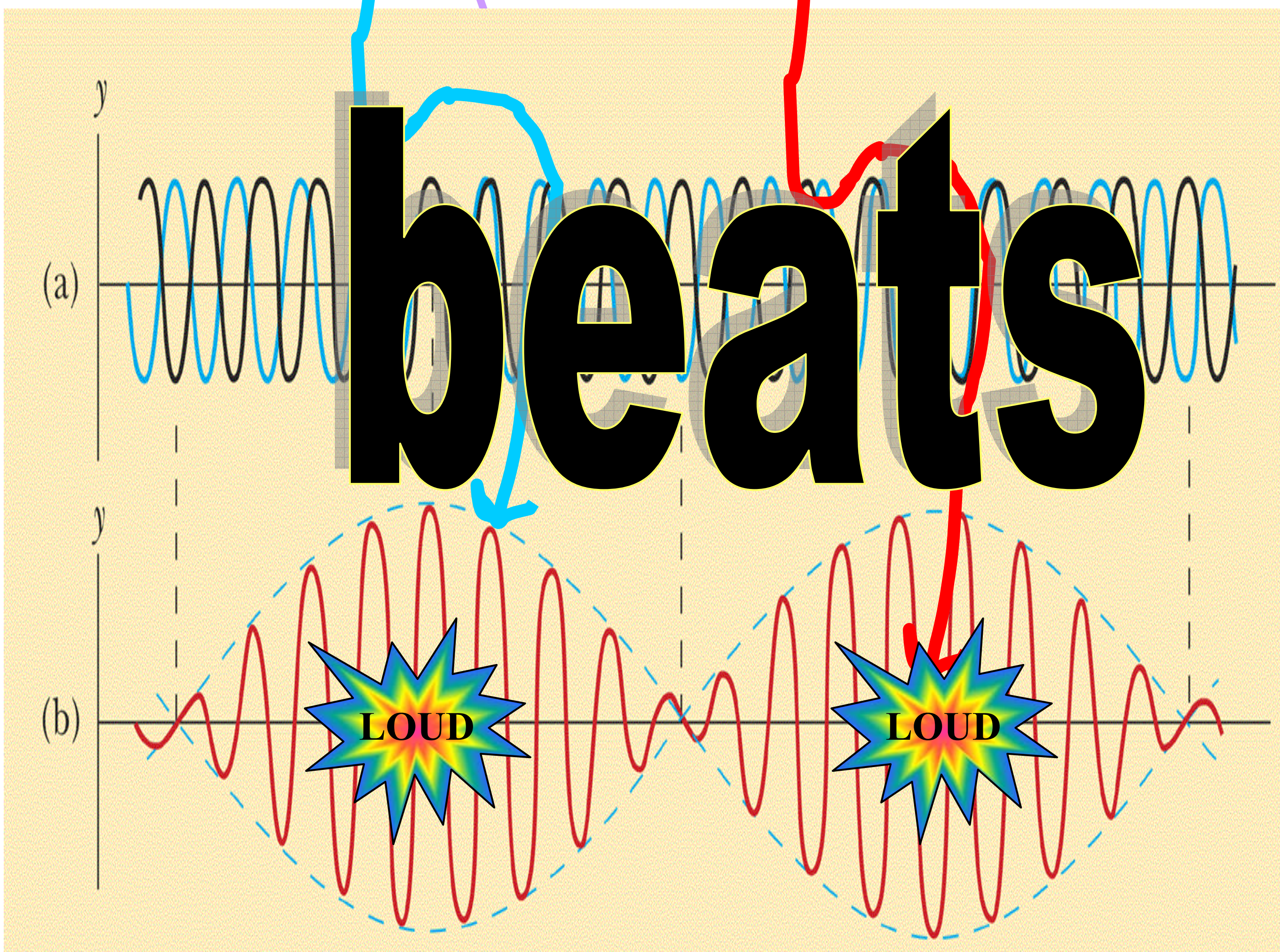
$\phi_2 - \phi_2 = 2\Delta\phi = \pi$

$$y = 0$$

The waves cancel out

More Generally

$$y = [2A\cos(\Delta k\, x - \Delta\omega\, t + \Delta\phi)]\,\sin(kx - \omega t + \phi)$$



Two waves are observed to interfere constructively. Their phase difference, in radians, could be

a. $\pi/2$.

b. π.

c. 3π.

d. 7π.

e. 6π.

Standing waves

A pulse on a string with both ends attached travels back and forth between the ends

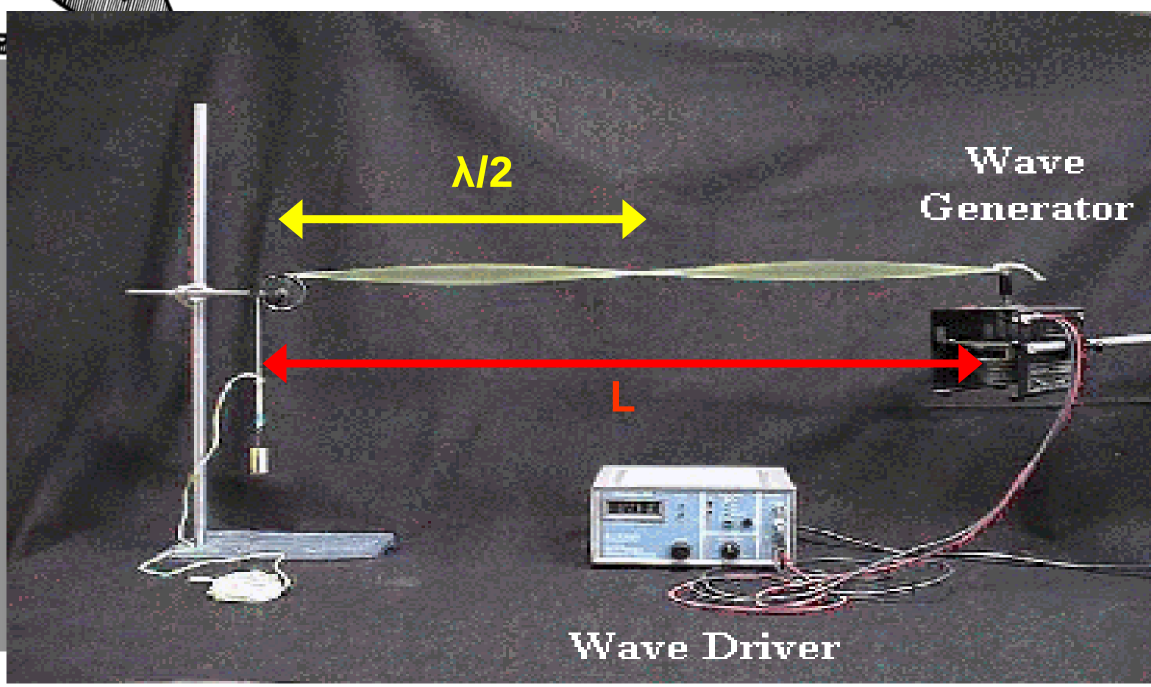
If we have two waves moving in opposite directions:
Wave 1: $y_1 = A \sin(kx - \omega t)$
Wave 2: $y_2 = A \sin(kx + \omega t)$

$$y = y1 + y2 = 2A \sin(k x) \cos(\omega t)$$

For example, if $\lambda = L/2$

rvive for an ***attached*** string of length L
$y(x=0) = y(x=L) = 0$

$\sin(k L) = 0$



applet

A pulse on a string with both ends attached travels back and forth between the ends

Standing wave:

y = 2A sin(k x) cos(ω t)

λ=2L/n, n=1,2,3,…

$$f = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \qquad n = 1,2,3,\ldots$$

Smallest *f* is the <u>fundamental frequency</u>
The others are multiples of it or, <u>harmonics</u>

$$f_{\text{fund}} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

14.3

Two pulses are traveling on the same string are described by

$$y_1 = \frac{5}{(3x-4t)^2+2}$$

$$y_2 = -\frac{5}{(3x+4t-6)^2+2}$$

a) In which direction does each pulse travel?

b) At what time do the two cancel everywhere?

c) At what point do the two waves always cancel?

Exercise 14.3

A wave moving right looks like *f(x-vt)*
A wave moving left looks like *f(x+vt)*
I need the superposition principle

(a)

$$y_1 = \frac{5}{(3x-4t)^2+2} \rightarrow \text{right moving}$$

$$y_2 = \frac{5}{(3x+4t-6)^2+2} \rightarrow \text{left moving}$$

Cancellation :

$$\frac{5}{(3x-4t)^2+2} - \frac{5}{(3x+4t-6)^2+2} = 0$$

$$\Rightarrow (3x-4t)^2 = (3x+4t-6)^2$$

$$\Rightarrow (3x-4t) = \pm(3x+4t-6)$$

(b) Cancel for all x: choose the upper sign

$$3x-4t = 3x+4t-6 \Rightarrow t = 3/4$$

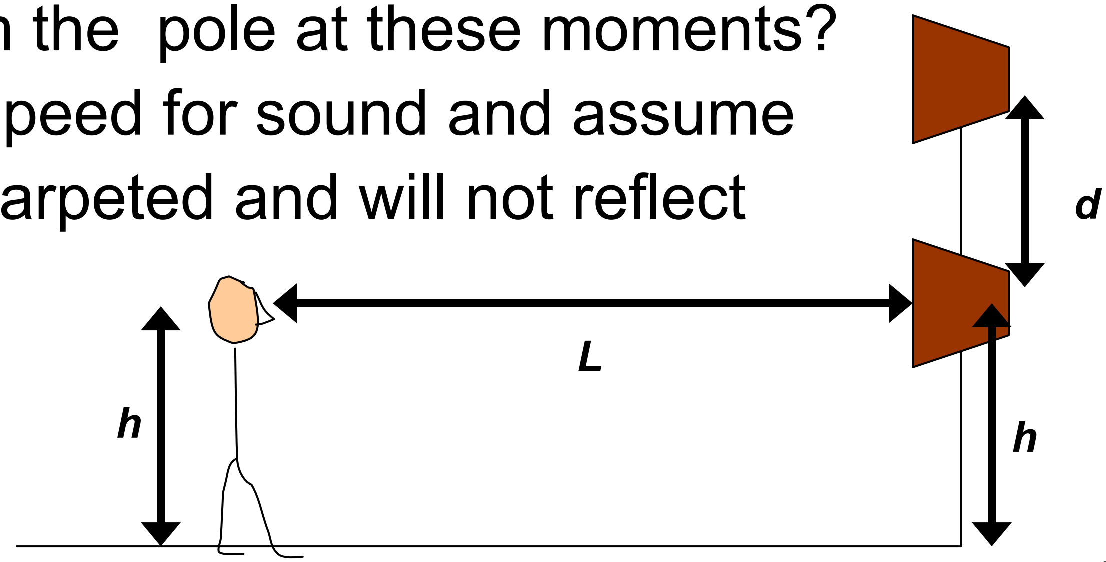
(c) Cancel for all time : choose lower sign

$$(3x-4t) = -(3x+4t-6) \Rightarrow x = -1$$

14.9

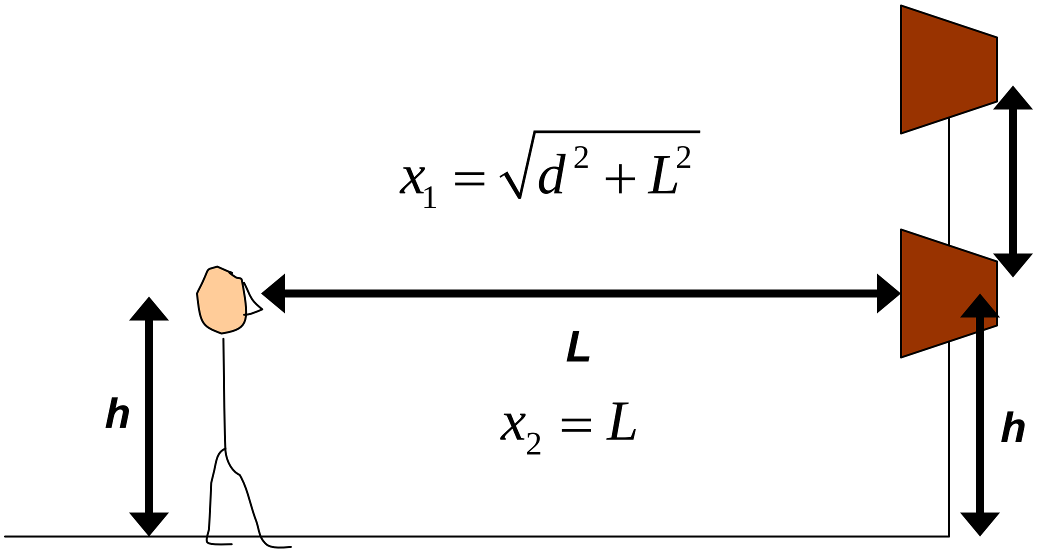
- Two speakers are driven in phase by the same oscillator of frequency f. They are located a distance d from each other on a vertical pole as shown in the figure. A man walks towards the speakers in a direction perpendicular to the pole.
- a) How many times will he hear a minimum in the sound intensity
- b) How far is he from the pole at these moments?

Let v represent the speed for sound and assume that the ground is carpeted and will not reflect the sound



Exercise 14.9

This is a case of interference
I will need Pythagoras theorem



$$x_1 = \sqrt{d^2 + L^2}$$

$$x_2 = L$$

$$x = x_1 - x_2 \qquad X = \frac{x_1 + x_2}{2}$$

$$x_1 = X + x/2 \qquad x_2 = X - x/2$$

$$2\cos x \cos y = \cos(x+y) + \cos(x-y)$$

Cancellations when
kx=… -5p, -3p, -p, p, 3p, 5p …

$$\begin{aligned} y &= A\cos(kx_1 - \omega t + \phi) + A\cos(kx_2 - \omega t + \phi) \\ &= A\cos[k(X - x/2) - \omega t + \phi] + A\cos[k(X + x/2 - \omega t + \phi] \\ &= 2A\cos(kX - \omega t + \phi)\cos(k\,x/2) \end{aligned}$$

$$m = odd \qquad x = \frac{m\pi}{k} = \frac{m\pi}{2\pi}\lambda = \frac{m}{2}\lambda = \left(n + \frac{1}{2}\right)\lambda \quad n = \text{integer}$$

$$x = \sqrt{d^2 + L^2} - L = \left(n + \frac{1}{2}\right)\lambda \quad n = \text{integer}$$

$$\sqrt{d^2 + L^2} - L = (n + 1/2)\lambda, \quad n = 0, \pm 1, \pm 2 \cdots$$

$$\Rightarrow L = \frac{d^2 - \left[(n + 1/2)\lambda\right]^2}{2(n + 1/2)\lambda}$$

$$\Rightarrow \frac{L}{\lambda} = \frac{(d/\lambda)^2 - (n + 1/2)^2}{2n + 1}$$

Allowed n:
all such that the right hand side is positive

$\lambda = v/f$

- Destructive interference occurs when the path difference is

A. $\lambda/2$

B. λ

C. 2λ

D. 3λ

E. 4λ

14.13

- Two sinusoidal waves combining in a medium are described by the wave functions

$y_1=(3cm)\sin\pi(x+0.6t)$

$y_2=(3cm)\sin\pi(x-0.6t)$

where x is in cm an t in seconds. Describe the maximum displacement of the motion at

a) x=0.25 cm b) x=0.5 cm c) x=1.5 cm

d) Fine the three smallest (most negative) values of x corresponding to antinodes

nodes when x=0 at all times

Exercise 14.13

Superposition problem

sin(*a*+b) + sin(*a*-b)=2 sin(*a*) cos(b)

$$y = 3\sin(\pi\, x + 0.6\pi\, t) + 3\sin(\pi\, x - 0.6\pi\, t)$$
$$= 6\sin(\pi\, x)\cos(0.6\pi\, t)$$

$$(a) \quad x = 0.25: \quad y = 6\sin(\pi/4)\cos(0.6\pi\ t) \le 6\sin(\pi/4) = 4.24\text{m}$$

$$(b) \quad x = 0.5: \quad y = 6\sin(\pi/2)\cos(0.6\pi\, t) \le 6\text{m}$$

$$(c) \quad x = 1.5: \quad y = 6\sin(3\pi/2)\cos(0.6\pi\, t) \le 6\text{m}$$

$$(d) \quad \text{Antinodes}: \sin(\pi\, x) = -1$$
$$\Rightarrow x = 0\text{cm}, 1\text{cm}, 2\text{cm}, ,3\text{cm} \cdots$$

Sources of Musical Sound

- Oscillating strings (guitar, piano, violin)
- Oscillating membranes (drums)
- Oscillating air columns (flute, oboe, organ pipe)
- Oscillating wooden/steel blocks (xylophone, mari

- Standing Waves-Reflections & Superposition
- Dimensions restrict allowed wavelengths-Resonant Frequencies
- Initial disturbance excites various resonant frequencies

http://www.falstad.com/membrane/index.html

Standing Wave Patterns for Air Columns

1. Pipe Closed at Both Ends
   - Displacement Nodes
   - Pressure Anti-nodes
   - Same as String

http://www.physics.gatech.edu/academics/Classes/2211/main/demos/standing_wwaves/stlwaves.htm

$$\lambda = \frac{2L}{n} \qquad f_n = n\frac{v}{2L} \qquad \text{all n}$$

2. Pipe Open at Both Ends
   - Displacement Anti-nodes
   - Pressure Nodes (Atmospheric)
   - Same as String

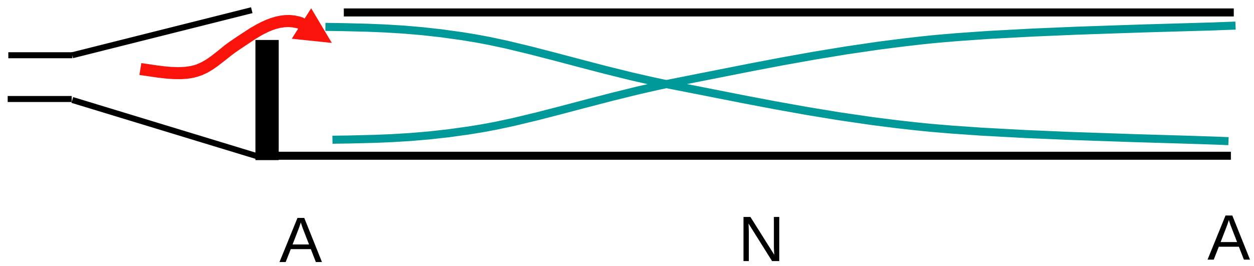
$$\lambda = \frac{2L}{n} \qquad f_n = n\frac{v}{4L} \qquad \text{n odd}$$

3. Open One End-Closed Other End

$$\lambda = \frac{4L}{n} \qquad f_n = n\frac{v}{2L} \qquad \text{all n}$$

beats demo

Organ Pipe-Open Both Ends



$n = 1$

$\lambda = 2L$

Turbulent Air flow excites large number of harmonics

"Timbre"