# QCD and Spin Physics

## Lecture 5

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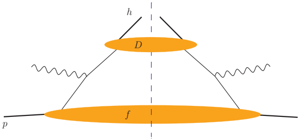
## Measured transverse momentum




- consider
  - Drell-Yan with measured small $q_T$ of $\gamma^*$
  - SIDIS with measured small $q_T$ of hadron
  - $e^+e^- \to h_1 h_2 + X$ with $h_1$, $h_2$ approx. opposite momenta and small relative $q_T$
- $k_T \sim m$ from collinear graphs matters in final state
  - can still neglect parton $k_T$ in hard scattering
  - but do not $\int d^2k_T$ in parton densities and fragm. fcts.
    $\rightsquigarrow$ $k_T$ dependent/unintegrated PDFs
    a.k.a. TMDs (transverse-momentum distributions)
- theoretical framework: $k_T$ factorization
  - a.k.a. TMD factorization
  - different from $k_T$ factorization in small-$x$/BFKL physics
  - similar to but in detail different from $k_T$ factorization for exclusive quantities

## $k_T$ dependent parton densities

$k_T$ integrated:

$$f_1(x) = \int \frac{dz^-}{4\pi}\, e^{iz^- p^+ x} \langle p, s \big| \bar{q}(0) \gamma^+ W(0,\infty) W(\infty, z^-)\, q(z^-) \big| p, s \rangle \Big|_{z^+=0,\, z_T=0}$$

$k_T$ dependent:

$$\int \frac{dz^-}{4\pi} \frac{d^2 z_T}{(2\pi)^2}\, e^{iz^- p^+ x}\, e^{-i\boldsymbol{k}_T \boldsymbol{z}_T} \langle p, s \big| \bar{q}(0) \gamma^+ W(0,\infty) W(\infty, z^-, z_T)\, q(z^-, z_T) \big| p, s \rangle \Big|_{z^+=0}$$

- fields at different transv. positions
  implications on Wilson lines $\longrightarrow$ later

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$$= f_1(x, k_T^2) - \frac{\epsilon^{ij} k_T^i s_T^j}{m}\, f_{1T}^{\perp}(x, k_T^2)$$

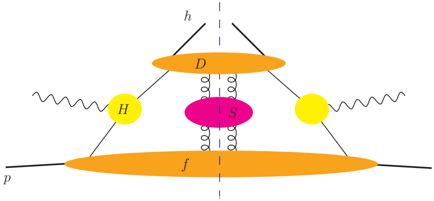
$$\epsilon^{12} = -\epsilon^{21} = 1 \qquad \epsilon^{11} = \epsilon^{22} = 0$$

- fields at different transv. positions
  implications on Wilson lines → later
- correlations between spins and transv. momentum
  e.g. Sivers function $f_{1T}^{\perp}$

- collinear twist 2 densities:
  - $f_1$ unpol. quark in unpol. proton
  - $g_1$ correlate $s_L$ of quark with $S_L$ of proton
  - $h_1$ correlate $s_T$ of quark with $S_T$ of proton
- $k_T$ dependent twist 2 densities:
  - $f_1, g_1, h_1$ as above
  - $f_{1T}^{\perp}$ correlate $k_T$ of quark with $S_T$ of proton (Sivers)
  - $h_1^{\perp}$ correlate $k_T$ and $s_T$ of quark (Boer-Mulders)
  - $g_{1T}, h_{1T}^{\perp}, h_{1L}^{\perp}$ three more densities
- analogous for fragmentation functions:
  - $f_1 \leftrightarrow D_1$ unpolarized
  - $h_1^{\perp} \leftrightarrow H_1^{\perp}$ Collins fragm. fct.

## $k_T$ dependent factorization (SIDIS as example)

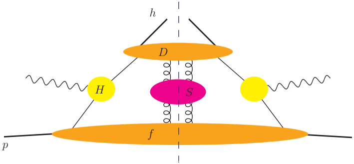
- take $Q$ large and $q_T$ small ($\sim m$ for power counting purposes)




- transverse-momentum dep't distribution and fragmentation fcts.
- soft gluon exchange does not cancel in sum over hadronic final state at leading-power accuracy gives soft factor in factorization formula (universal non-perturbative function)
- only virtual corrections to hard subgraph no radiation of high-$p_T$ partons allowed

original formulation: Collins, Soper '81

## SIDIS at low $q_T$

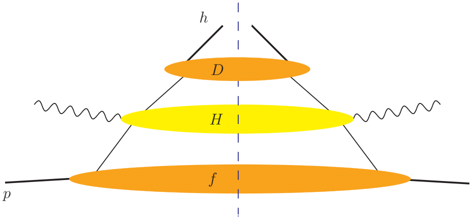



- factorization formula

$$\frac{d\sigma_{\gamma^* p}}{dz\, dq_T^2} = (\text{kin. fact.}) \times |H|^2 \int d^2p_T\, d^2k_T\, d^2l_T\, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T + \boldsymbol{q}_T) \times \sum_{i=q,\bar{q}} e_i^2\, f^i(x, p_T^2, \zeta)\, D^i(z, k_T^2, \zeta_h)\, S(l_T^2)$$

- hard factor: $|H|^2 = 1 + \mathcal{O}(\alpha_s)$
- no $\int d^2k_T$ in parton densities $\rightsquigarrow$ no DGLAP type evolution
- evolution in rapidity parameters $\zeta$, $\zeta_h$ with $\sqrt{\zeta\zeta_h} = Q^2 z/x$
  $\rightsquigarrow$ Collins-Soper equation $\rightsquigarrow$ Sudakov factor
- various azimuthal and spin asymmetries
- come close to full factorization proof for leading power in $1/Q$
  recent work by Ji, Ma, Yuan; Boer et al; Collins, Rogers, Staśto

# Compare with collinear factorization for large $q_T$




$$\frac{d\sigma_{\gamma^* p}}{dz\, dq_T^2} = \text{(kin. fact.)} \times \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}}\, \delta\bigg( \frac{{q_T}^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}} \bigg)$$

$$\times \sum_{i,j=q,\bar{q},g} f_i\Big(\frac{x}{\hat{x}}, \mu^2\Big) D_j\Big(\frac{z}{\hat{z}}, \mu^2\Big)\, C_{ij}\Big(\hat{x}, \hat{z}, \ln\frac{\mu^2}{Q^2}\Big)$$

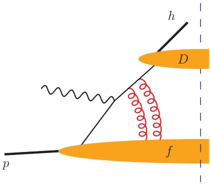
- $C_{ij}$ start at $\mathcal{O}(\alpha_s)$, emit partons recoiling against $\boldsymbol{q}_T$
- convolution in momentum fractions

# Wilson lines

$\rightsquigarrow$ blackboard

# More complicated cases: beyond leading power

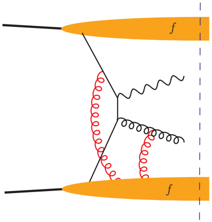
- certain asymmetries in SIDIS, Drell-Yan, $e^+e^-$ are suppressed by $1/Q$ (twist three)
  - full calculations at tree-level — Mulders et al. '97-'07
  - $A^+$ gluons between collinear and hard subgraphs $\rightsquigarrow$ Wilson lines
  - twist-three tree-level factorization formula as for twist-two case without $S$ and $|H|^2$
  - soft gluon exchange not analyzed so far $\rightsquigarrow$ factorization not assured



## More complicated processes

- examples: $pp \to \gamma + \text{jet} + X$, $pp \to \pi + \text{jet} + X$
- more partons in initial and final state
  - $\rightsquigarrow$ more complicated Wilson lines
  - $\rightsquigarrow$ more parton densities and fragm. functions

Bomhof, Mulders, Pijlman '04-'07



## Relation between high-$q_T$ and low-$q_T$ descriptions

- collinear fact. requires $q_T \gg m$
  $k_T$ fact. requires $q_T \ll Q$
  $\rightsquigarrow$ in region $m \ll q_T \ll Q$ both approaches are valid
- for $q_T \gg m$ calc. $k_T$ dependent densities from coll. ones:


$$f_1^i(x, k_T^2, \zeta) = \frac{1}{k_T^2} \sum_{j=q,\bar{q},g} \int_x^1 \frac{dx'}{x'} K^{ij}\Big(\frac{x}{x'}, \ln\frac{k_T^2}{\zeta}\Big) f_1^j(x')$$

$K$ closely related with DGLAP splitting functions $P$

## Comparison between high-$q_T$ and low-$q_T$ descriptions

- compare $q_T \gg m$ limit of $k_T$ fact. result
  with $q_T \ll Q$ limit of coll. fact. result
  $\rightsquigarrow$ full agreement for unpol. cross section

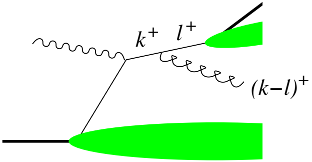
  Collins, Soper, Sterman '85; Bacchetta et al. '08

- detailed comparison also for various spin asymmetries
  e.g. Sivers asy. in SIDIS or Drell-Yan at low $q_T$ (Sivers fct.)
  and high $q_T$ (Qiu-Sterman fct.)

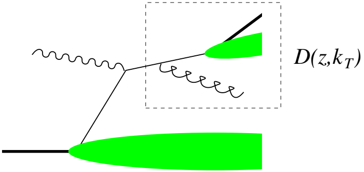
  Ji, Qiu, Vogelsang, Yuan '06; Koike, Vogelsang, Yuan '07

# Correspondence at level of graphs

high-$q_T$ calculation




low-$q_T$ calculation with $q_T \gg m$



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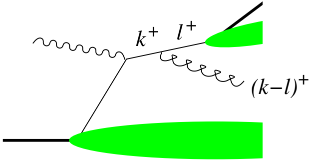
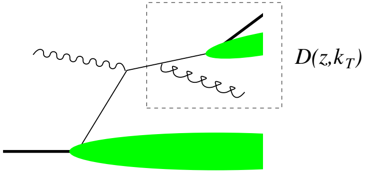
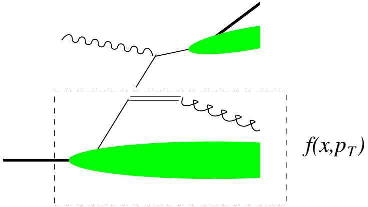



low-$q_T$ calculation with $q_T \gg m$



## Summary of lecture 5

- transverse-momentum dependent factorization for measured small transv. mom. (or small transv. mom. differences)
- more structure in parton densities and fragm. fcts.
  - $k_T$ dependence
  - correlations between $\boldsymbol{k}_T$ and spin of parton and/or hadron
- factorization more complicated
  - soft factor
  - Sudakov logarithms (but in turn no DGLAP evolution)
- nontrivial dynamics (including $T$ odd effects) from rescattering
  - incorporated into Wilson lines
  - best understood for $e^+e^-$, SIDIS, Drell-Yan
- $k_T$ factorization related with collinear factorization for high $q_T$