QCD and Spin Physics

Lecture 4

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A closer look at one-loop corrections

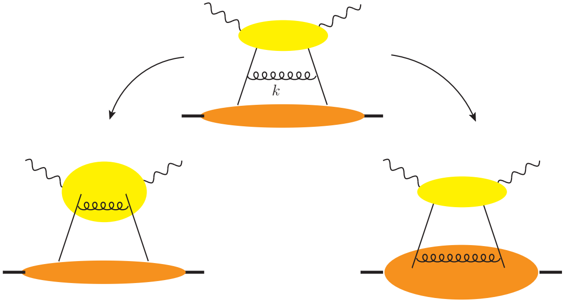
- example: DIS

- UV divergences removed by standard counterterms
- soft divergences cancel in sum over graphs
- collinear div. do not cancel, have integrals

$$\int_0 \frac{dk_T^2}{k_T^2}$$

what went wrong?

- hard graph should not contain internal collinear lines
  collinear graph should not contain hard lines
- must not double count $\rightsquigarrow$ factorization scale μ



- with cutoff: take $k_T > \mu$ take $k_T < \mu$
  $1/\mu \sim$ transverse resolution

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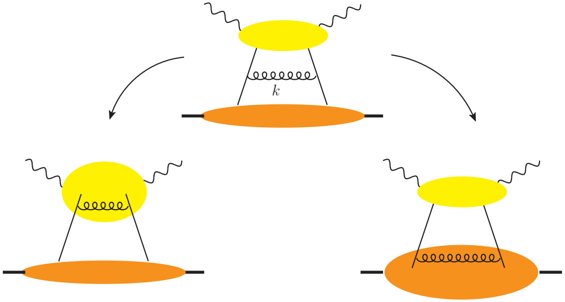
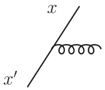


- with cutoff: take $k_T > \mu$ take $k_T < \mu$
  $1/\mu \sim$ transverse resolution
- in $D = 4 - 2\epsilon$:
  subtract collinear pole subtract ultraviolet pole

The DGLAP equations

- scale dependence in parton densities and in hard scattering
- DGLAP equations

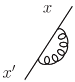
$$\frac{d}{d\log\mu^2}\, f(x,\mu) = \int_x^1 \frac{dx'}{x'}\, P\Big(\frac{x}{x'}\Big)\, f(x',\mu) \;=\; \big(P \otimes f(\mu)\big)(x)$$



- $P =$ splitting functions
  - have perturbative expansion

$$P(x) = \alpha_s(\mu)\, P^{(0)}(x) + \alpha_s^2(\mu)\, P^{(1)}(x) + \alpha_s^3(\mu)\, P^{(1)}(x) \ldots$$

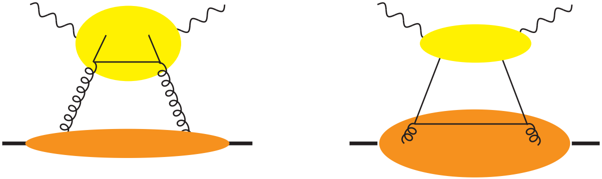
known to 3 loops (unpolarized)



  - contains terms $\propto \delta(1-x)$ from virtual corrections

Quark-gluon mixing

- quark and gluon densities mix under evolution:



- matrix evolution equation

$$\frac{d}{d\log\mu^2}\, f_i(x,\mu) = \sum_j \big(P_{ij}\otimes f_j(\mu)\big)(x) \qquad\qquad (i,j = q,\bar{q},g)$$

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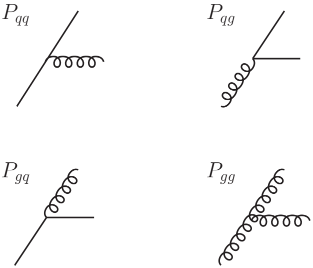


- at order α_s greatly simplifies
  - non-singlet combinations $q-\bar{q}$, $u-d$, ... evolve by themselves
  - 2×2 matrix equation for mixing of $g(x)$ with quark singlet $\Sigma(x) = \sum_q \big[q(x) + \bar{q}(x)\big]$
- analogous (with different kernels) for helicity distributions
- transversity: only quarks

Scale dependence of observables

$\rightsquigarrow$ blackboard

Evolution of moments

$\rightsquigarrow$ blackboard

$\Delta\Sigma$, Δg, and the axial anomaly

- for $\Delta g = \int dx\, \Delta g(x)$ and $\sum_q \int dx\, (\Delta q + \Delta\bar{q})$ have at LO

$$\frac{d}{d\log\mu^2}\,\Delta g = \frac{\alpha_s}{4\pi}\,\Big(4\Delta\Sigma + \beta_0\,\Delta g\Big)$$

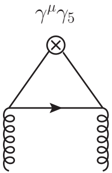
with $\beta_0 = 11 - \frac{2}{3}n_f$ and $\frac{d}{d\log\mu^2}\,\alpha_s = -\frac{1}{4\pi}\,\beta_0\,\alpha_s^2$

$$\Rightarrow \qquad \frac{d}{d\log\mu^2}\,(\alpha_s\Delta g) = \frac{\alpha_s^2}{\pi}\,\Delta\Sigma + \mathcal{O}(\alpha_s^3)$$

- evolution of $\Delta\Sigma$ $\rightsquigarrow$

The axial anomaly

- first moment $\int dx\,(\Delta q + \Delta\bar{q}) \;\leftrightarrow\;$ axial current $\bar{q}\gamma^{+}\gamma_5\, q$
- for massless quarks axial current conserved at level of Lagrangian (= Noether current of chiral symmetry)
- this symmetry broken by quantum corrections (loops)
  $\rightsquigarrow$ axial anomaly



- triangle graphs involving axial current have UV divergence $\rightsquigarrow$ must renormalize
- in regulated theory chiral symmetry not realized
  in $D \neq 4$ dim.: no γ_5 with $\{\gamma^\mu, \gamma_5\} = 0$ for all μ
- after UV subtraction find remnant of broken chiral symmetry when remove regulator

- flavor singlet axial current not conserved:

$$\partial^\mu \sum_q \bar{q}\gamma_\mu\gamma_5\, q = n_F\, \frac{\alpha_s}{4\pi}\, F^a_{\mu\nu}\, \widetilde{F}^{\mu\nu,a}$$

$$\frac{d}{d\log\mu^2}\,(\alpha_s\Delta g) = \frac{\alpha_s^2}{\pi}\,\Delta\Sigma + \mathcal{O}(\alpha_s^3)$$

- different schemes to define $\Delta q, \Delta\bar{q}$

  $\leftrightarrow$ renormalization schemes
- in $\overline{\text{MS}}$ scheme Σ scale dependent:

$$\frac{d}{d\log\mu^2}\,\Delta\Sigma = -n_F\,\frac{\alpha_s^2}{2\pi^2}\,\Delta\Sigma + \mathcal{O}(\alpha_s^3)$$

- several other schemes where

$$\Delta\Sigma' = \Delta\Sigma + n_F\,\frac{\alpha_s}{2\pi}\Delta g$$

  is scale independent (to all orders in α_s)

  chirally invariant schemes: Adler-Bardeen scheme; JET scheme; Altarelli-Ross scheme, off-shell scheme, ...

  have same $\Delta\Sigma$ but different $\Delta q(x), \Delta\bar{q}(x)$

  for overviews cf. e.g. Ball et al, hep-ph/9510449

  Leader et al, hep-ph/9807251

Summary of lecture 4

Evolution

- for consistency must in collinear factorization
  - remove collinear kinematic region in hard scattering
  - remove hard kinematic region in parton densities
    $\leftrightarrow$ UV renormalization

  procedure introduces factorization scale μ
  - separates "collinear" from "hard", "object" from "probe"
- scale dependence of parton densities (and hard scattering) given by evolution equations
  - for moments of parton densities get usual RGE
- special situation for first moments $\Delta\Sigma$ and Δg
  - due to axial anomaly
  - scheme choice exhibits limits of parton-model interpretation in full-fledged quantum field theory