QCD and Spin Physics

Lecture 2

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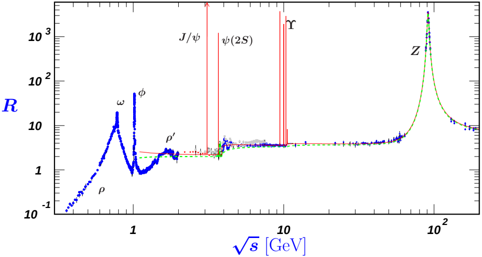
$e^+e^- \to$ hadrons

$$R = \frac{\sigma(e^+e^- \to X)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

for $\sqrt{s} \gg$ resonance masses

plot from PDG 2008



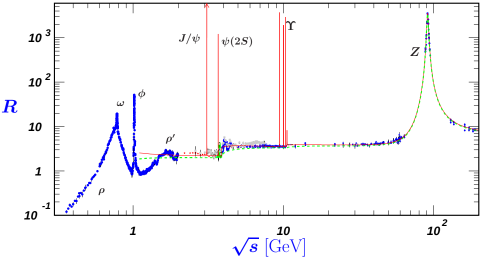
- removing electroweak part $\rightsquigarrow \sum_X |\mathcal{A}(\gamma^* \text{ or } Z \to X)|^2$
- among simplest applications of perturbative QCD
  - fully inclusive final state
  - no hadrons in initial state
- closely related theory description for

$$R_\tau = \frac{\Gamma(\tau \to \nu_\tau + X)}{\Gamma(\tau \to \nu_\tau + e\nu_e)} \quad \rightsquigarrow \quad \sum_X |\mathcal{A}(W \to X)|^2$$

at lowest order in α_s:

$$R = N_c \sum_q e_q^2$$

from $\gamma^* \to q\bar{q}$ with $m_q = 0$



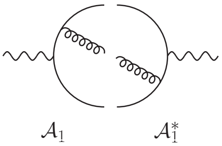
- expansion known up to $R = R_0 \left[1 + \frac{1}{\pi}\alpha_s + C_2\alpha_s^2 + C_3\alpha_s^3\right]$
  - quark mass corrections and α_s^4 terms also partly known
  - same for τ decays (α_s^4 terms fully known)
  - suitable observables for α_s determination
- underlying concept: parton-hadron duality:

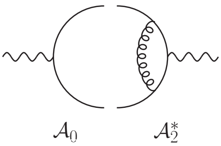
$$\sum_{X \in \text{partons}} |\mathcal{A}(\gamma^* \to X)|^2 = \sum_{X \in \text{hadrons}} |\mathcal{A}(\gamma^* \to X)|^2$$

  - $\gamma^* \to$ partons valid description for short space-time $\sim 1/\sqrt{s}$
  - subsequent dynamics changes final state, but not inclusive rate

A closer look at the $\mathcal{O}(\alpha_s)$ corrections

- expand $\mathcal{A}(q\bar{q}g) = g\mathcal{A}_1 + \ldots$ and $\mathcal{A}(q\bar{q}) = \mathcal{A}_0 + g^2\mathcal{A}_2 + \ldots$





real corrections: extra partons in final state
virtual corrections: loops in $\mathcal{A}$ or $\mathcal{A}^*$

- virtual corrections have UV divergences
  $\longrightarrow$ standard renormalization procedure
- real and virtual corrections: soft and collinear divergences
  - regions where gluon momentum $\rightarrow 0$ or $\propto$ momentum of q or $\bar{q}$
  - cancel in sum over all graphs
    otherwise could not use parton-hadron duality

more detail $\rightsquigarrow$ blackboard

A closer look at soft and collinear divergences

- have soft (= IR) div. because of massless gluons
  same phenomenon in QED: soft photons → "IR catastrophe"
- have collinear (= mass) div. if set quark masses to zero

  could formally keep $m_q \neq 0$, but perturbative results not trustworthy for configurations with virtualities $\sim \mathrm{MeV}^2$
- divergences cancel → result dominated by large virtualities

  technically: calculate in $D = 4 + 2\epsilon$ dim's, sum over graphs, take $\epsilon \to 0$

Theorems about cancellation of divergences:

- Block Nordsieck: QED (with finite fermion mass)
  IR div. cancel if sum over soft (unobs.) photons in final state
- KLN (Kinoshita, Lee, Nauenberg): IR and coll. div. cancel if sum over degenerate final and initial states

  for $\gamma^* \to$ partons need only sum in final state

Beyond inclusive final states: jets

- extend idea of parton-hadron duality: dynamics leading from partons (times $\sim 1/Q$) to final-state hadrons (times $\to \infty$) approx. conserves momentum (hadronization effects $\sim \mathrm{GeV}$)
- to use perturbative QCD beyond tree level need observables (jet algorithms, event shapes, ...) that can evaluate both for hadrons (measurement) and for massless partons (calculation)

  $\rightsquigarrow$ IR and collinear safe quantities

  do not change if replace $q \to (q +$ soft $g)$, $g \to$ (coll. $q\bar{q}$), etc.
- after cancellation of singularities retain for each power of α_s
  - single logarithms $\log \frac{Q}{\mu_c}$ from collinear sing.
  - double logarithms $\log \frac{Q}{\mu_s} \log \frac{Q}{\mu_c}$ from soft sing.

    $\rightsquigarrow$ Sudakov factors

A useful tool: the optical theorem

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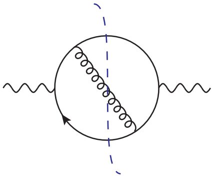
A useful tool: the optical theorem

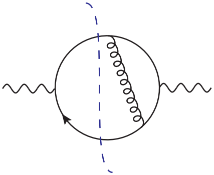
$$\frac{1}{i}\big(\mathcal{T}_{i\to f} - \mathcal{T}^*_{f\to i}\big) = \sum_X \mathcal{T}_{i\to X}\, \mathcal{T}^*_{f\to X}$$

- for $i = f$ get $2\,\mathrm{Im}\,\mathcal{T}_{i\to i} = \sum_X |\mathcal{T}_{i\to X}|^2 \propto \sigma_{\text{tot}}(i \to X)$
- l.h.s. $= 2\,\mathrm{Im}\,\mathcal{T}_{i\to f}$ if $\mathcal{T}_{i\to f} = \mathcal{T}_{f\to i}$ (happens for simple cases)
  otherwise l.h.s. often called "imaginary part" of $\mathcal{T}_{i\to f}$ nevertheless
  also called "absorptive part" or "discontinuity"
- applies to individual Feynman graphs: Cutkosky/cutting rules:
  to obtain "imaginary part" of graph $G_{i\to f}$
  - cut graph into two graphs $G_{i\to X}$ and $G_{f\to X}$
  - for cut propagators replace $\frac{i}{k^2-m^2+i\epsilon} \to 2\pi\delta(k^2-m^2)\,\theta(k^0)$
    plus appropriate replacements for spinors and pol. vectors
  - integrate $G_{i\to X}\, G^*_{f\to X}$ over phase space of X
    sum over all possible cuts

A new view on $\gamma^* \to X$

- vacuum polarization $\mathcal{A}(\gamma^* \to \gamma^*)$ at large Q^2 dominated by short distances $\rightsquigarrow$ calculate at quark-gluon level
- opt. theorem holds both in perturbation theory ($\gamma^* \to$ partons) and for full QCD ($\gamma^* \to$ hadrons)





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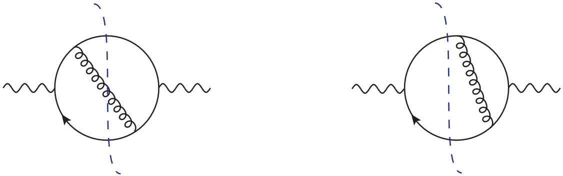

- real and virtual corrections $\to$ different cuts of same graphs for $l^\mu_{\text{gluon}} \to 0$ kinematics of cut graphs coincides

Excursion: time-reversal odd observables

borrowed from K Hagiwara et al., PRD 27 (1983) 84

- time-reversal: reverse signs of three-momenta and spins
  write $|i\rangle \to |Ti\rangle$
- T odd quantities: odd under $|i\rangle \to |Ti\rangle$, $|f\rangle \to |Tf\rangle$
  without interchange $|i\rangle \leftrightarrow |f\rangle$
- with opt. theorem find: $|\mathcal{T}_{i\to f}|^2 - |\mathcal{T}_{Ti\to Tf}|^2$ only nonzero if
  - time reversal invariance broken (e.g. by CKM mechanism)
  - or have absorptive part

  $\rightsquigarrow$ blackboard

Excursion continued: single spin asymmetries

- parity transformation: reverse three-momenta but not spins
  write $|i\rangle \to |Pi\rangle$
- combination $|PTi\rangle$ only reverses spins
- with opt. theorem have

$$|\mathcal{T}_{i\to f}|^2 - |\mathcal{T}_{PTi\to PTf}|^2 \\ = \big(|\mathcal{T}_{i\to f}|^2 - |\mathcal{T}_{Ti\to Tf}|^2\big) + \big(|\mathcal{T}_{Ti\to Tf}|^2 - |\mathcal{T}_{PTi\to PTf}|^2\big)$$

  only nonzero if
  - parity invariance broken (e.g. by weak currents)
  - or time reversal invariance broken
  - or have absorptive part
- obtain SSA from above by summing/averaging over all spins except one

Summary so far

- perturbative calculations beyond tree level
  only for quantities that are IR and collinear safe
  $\rightsquigarrow$ dominated by large virtualities
  - simplest examples: total cross sections/decay rates for colorless initial states
  - for jets in final state suitable (and unsuitable) observables exist
- opt. theorem:
  - can trade inclusive quantities $\leftrightarrow$ forward amplitudes
  - generalization to non-forward amplitudes
    with important consequences for asymmetries
  - valid for individual Feynman graphs $\to$ cutting rules