QCD and Spin Physics

Lecture 1

M. Diehl

Deutsches Elektronen-Synchroton DESY

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Plan of lectures

- Introduction
- Renormalization and scale dependence
  choosing and varying the scale
- $e^+e^- \to$ hadrons
  some basics of applied perturbation theory
- Spin and spinors
  since this is a Spin Fest
- Collinear factorization
  parton densities and fragmentation functions in QCD
- Evolution
  scale dependence once more
- Transverse-momentum dependent factorization
  Boer-Mulders, Collins, Sivers, and friends

The main players

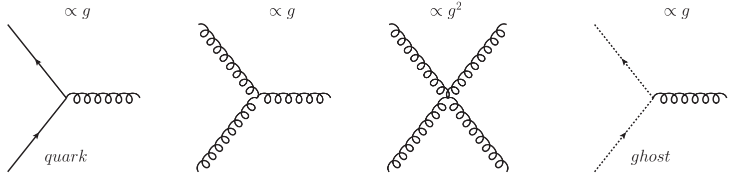
- degrees of freedom of the field theory: quarks and gluons
  observed degrees of freedom: hadrons
- wide range of quark masses:

  $m_u, m_d \sim$ a few MeV, $m_s \sim 100\,\mathrm{MeV}$,
  $m_c \approx 1.3\,\mathrm{GeV}, m_b \approx 4.2\,\mathrm{GeV}$, $m_t \approx 171\,\mathrm{GeV}$
- symmetries
  - gauge invariance: color SU(3)
  - Lorentz invariance
  - discrete symmetries: parity, charge conjugation, time reversal
  - approximate flavor symmetries: isospin (u, d), SU(3) (u, d, s)
- QCD is part of Standard Model: quarks couple to γ, W, Z

  coupling conveniently described in terms of currents:
  $j_V^\mu = \bar{q}\gamma^\mu q$ and $j_A^\mu = \bar{q}\gamma^\mu\gamma_5\, q$

Basics of perturbation theory

- the elementary vertices from the Lagrangian:

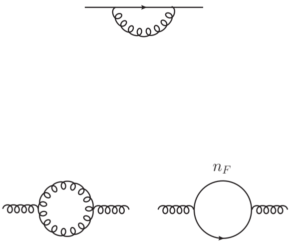


must fix gauge $\rightsquigarrow$ in general have ghost fields,
for physical amplitudes only appear in loops

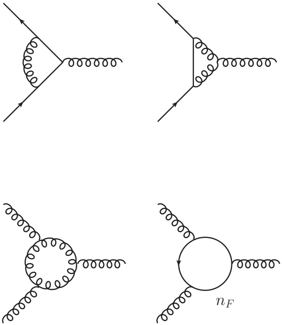
Loop corrections

- in loop corrections find ultraviolet divergences
- only appear in corrections to

propagators



elementary vertices



TREX (Truly Recommended EXercise): Draw the remaining one-loop graphs for all propagators and elementary vertices

- origin of UV divergences: region of ∞ly large loop momenta
  $\leftrightarrow$ quantum fluctuations at ∞ly small space-time distances
- idea: remove this when describe physics at some given scale μ
  $\rightsquigarrow$ renormalization
- technically:
  1. regularize: artificial change of theory
     such that divergent terms become finite
     - physically intuitive: momentum cutoff
     - in practice: dimensional reg.
  2. renormalize: absorb would-be infinities into
     - quark and gluon fields ("wave function renormalization")
     - quark masses $m_q(\mu)$
     - coupling constant $\alpha_s(\mu)$
  3. remove regulator: quantities are finite when expressed in terms of renormalized parameters and fields
- renormalization scheme: choice of which terms to absorb
  "∞" is as good as "$\infty + \log(4\pi)$"

Dimensional regularization in a nutshell

- choice of regulator $\approx$ choice between evils
- dim. reg.: little (any?) physics intuition, but keeps intact essential symmetries (gauge and Lorentz invariance)
- idea: integrals for Feynman graphs become UV finite in lower space-time dimension, e.g.

$$\int \frac{d^D k}{(2\pi)^D} \, \frac{1}{k^2 - m^2} \, \frac{1}{(k-p)^2 - m^2}$$

log. div. for $D = 4$
converg. for $D = 3, 2, 1$

- procedure:
  1. formulate theory in D dimensions (with D small enough)
  2. analytically continue results from integer to complex D
     original divergences appear as poles in $1/\epsilon$ $(D = 4 - 2\epsilon)$
  3. renormalize
  4. take $\epsilon \to 0$

Enter: a mass scale

- coupling in $4 - 2\epsilon$ dimensions is $\mu^{\epsilon} g$ with g dimensionless
- typical one-loop integral gives
  $\rightsquigarrow$ blackboard

Enter: a mass scale

- coupling in $4 - 2\epsilon$ dimensions is $\mu^{\epsilon} g$ with g dimensionless
- typical one-loop integral gives

$$\frac{1}{\epsilon} + \ln(4\pi) - \gamma + \ln \mu^2 + \text{fct. of external momenta}$$

  at higher orders get higher poles g^{2n}/ϵ^n
- MS (minimal subtraction): absorb only pole terms $\propto 1/\epsilon^n$
  leaves terms $\ln(4\pi) - \gamma \approx 1.95$ in final results
  artifacts of dim. reg.
- $\overline{\text{MS}}$ (modified min. subtr.): absorb these terms into the scale

$$\mu^2_{\overline{\text{MS}}} = 4\pi e^{-\gamma} \mu^2_{\text{MS}}$$

- any other regularization introduces some mass parameter
  renormalized quantities depend on a scale μ

Renormalization group equations (RGE)

- scale dependence of renormalized quantities described by differential equations

$$\frac{d}{d\log\mu^2}\,\alpha_s(\mu) = \beta\big(\alpha_s(\mu)\big)$$

$$\frac{d}{d\log\mu^2}\,m_q(\mu) = m_q(\mu)\,\gamma_m\big(\alpha_s(\mu)\big)$$

β, γ_m = perturbatively calculable functions
in region where $\alpha_s(\mu)$ is small enough

$$\beta = -b_0\,\alpha_s^2\,\big[1 + b_1\alpha_s + b_2\,\alpha_s^2 + b_3\,\alpha_s^3 + \ldots\big]$$

$$\gamma_m = -c_0\,\alpha_s\,\big[1 + c_1\alpha_s + c_2\,\alpha_s^2 + c_3\,\alpha_s^3 + \ldots\big]$$

coefficients known including b_3, c_3

$$b_0 = \tfrac{1}{4\pi}\big(11 - \tfrac{2}{3}n_F\big) \qquad\qquad c_0 = \tfrac{1}{\pi}$$

- $\beta_{\text{QCD}} < 0 \;\rightsquigarrow\;$ asymptotic freedom at large μ

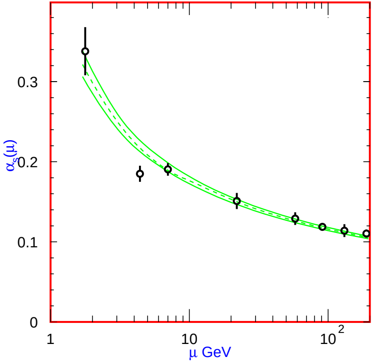
The running of α_s

- truncating $\beta = -b_0\, s\alpha_s^2(1+b_1\alpha_s)$ get

$$\alpha_s(\mu) = \frac{1}{b_0 L} - \frac{b_1 \log L}{(b_0 L)^2} + \mathcal{O}\Big(\frac{1}{L^3}\Big)$$

with $L = \log \dfrac{\mu^2}{\Lambda^2_{\mathsf{QCD}}}$

- more detail $\rightsquigarrow$ blackboard



plot from PDG 2008

Scale dependence of observables

- variation of μ in result including order α_s^n
  $\rightsquigarrow$ variation of observable corresponding to

$$\alpha_s^{n+1} \sum_{i=1}^{i_{\text{max}}} (\text{known coefficient}) \times \left[\log \frac{\mu^2}{Q^2} \right]^i + \mathcal{O}(\alpha_s^{n+2})$$

  but no information on α_s^{n+1} term without $\log$

consequences:

- when calculate higher orders, expect that scale dependence decreases
- scale variation estimates size of certain higher-order terms, but not of all
  - uncalculated higher orders often estimated by varying μ between $1/2$ and 2 times some central value
    is a conventional choice
  - but what to take for central value?

Renormalization scale choice

- prescriptions for scale choice aiming to minimize size of higher-order terms

  take NLO calc. of $C(\mu) = C_0 + \alpha_s(\mu) C_1 + \alpha_s^2(\mu) C_2(\mu) + \mathcal{O}(\alpha_s^3)$

  - $\mu =$ typical virtuality in hard-scattering graphs
    plausible guideline, but obviously not a well-defined quantity
  - principle of minimal sensitivity (PMS):
    $$\frac{d}{d\mu^2} \sum_{n=0}^{2} \alpha_s^n(\mu) C_n(\mu) = 0$$
  - fastest apparent convergence (FAC): $C_2(\mu) = 0$
  - Brodsky-Mackenzie-Lepage (BLM): $C_2(\mu)$ indep't of n_F
    recall: coefficients b_0, b_1, ... of β function depend on n_F
- how much these reduce higher orders depends on process
  cannot "predict" higher orders without calculating them

An example

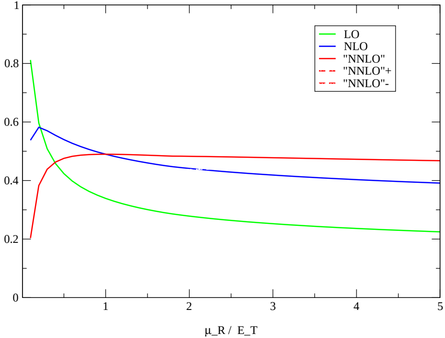
N. Glover, hep-ph/0211412

- single-jet production at Tevatron

$$\frac{d\sigma}{dE_T} = \alpha_s^2(\mu)\, C_2 + \alpha_s^3(\mu)\Big[C_3 + 2b_0 L C_2\Big] + \alpha_s^4(\mu)\Big[C_4 + 3b_0 L C_3 + (3b_0^2 L^2 + 2b_1 L) C_2\Big]$$

$$L = \log\frac{\mu^2}{E_T^2}$$

- C_4 not known, curves for "NNLO" and "NNLO"$\pm$ with $C_4 = 0$ and $C_4 = \pm C_3^2/C_2$
- weak μ dependence of cross section at "NNLO" but need C_4 to know its value



Summary of lecture 1

Renormalization

- beyond all technicalities reflects physical idea:
  eliminate details of physics at scales $\gg$ scale μ of problem
- dependence of observable on μ governed by RGE
  reflects (and estimates) particular higher-order corrections
  ... but not all
- prescriptions for scale choice = educated guesses