Violation of TMD Factorization in Hadronic Collisions

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RHIC Spin Discussion
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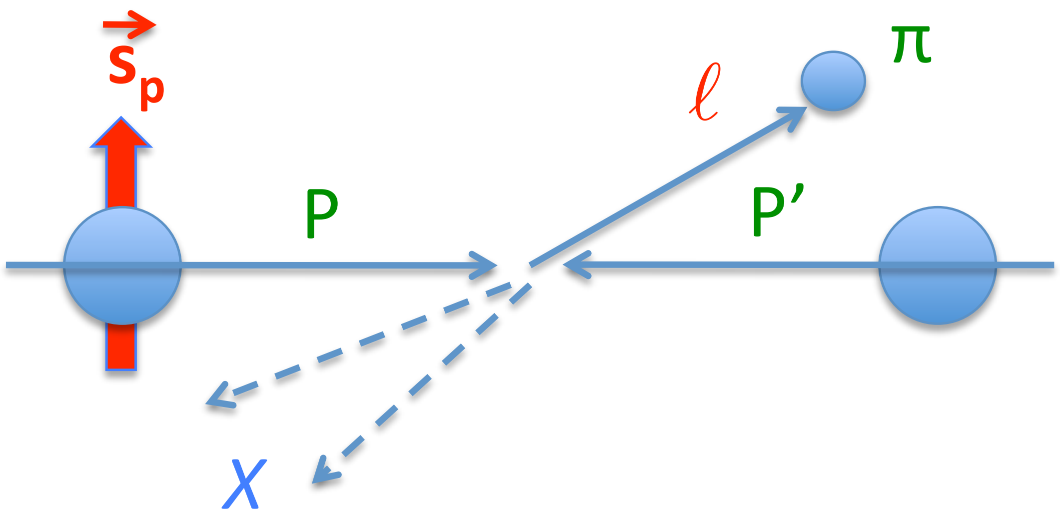
Start from conclusion

- Note: TMD and Collinear factorization apply in different kinematic domain

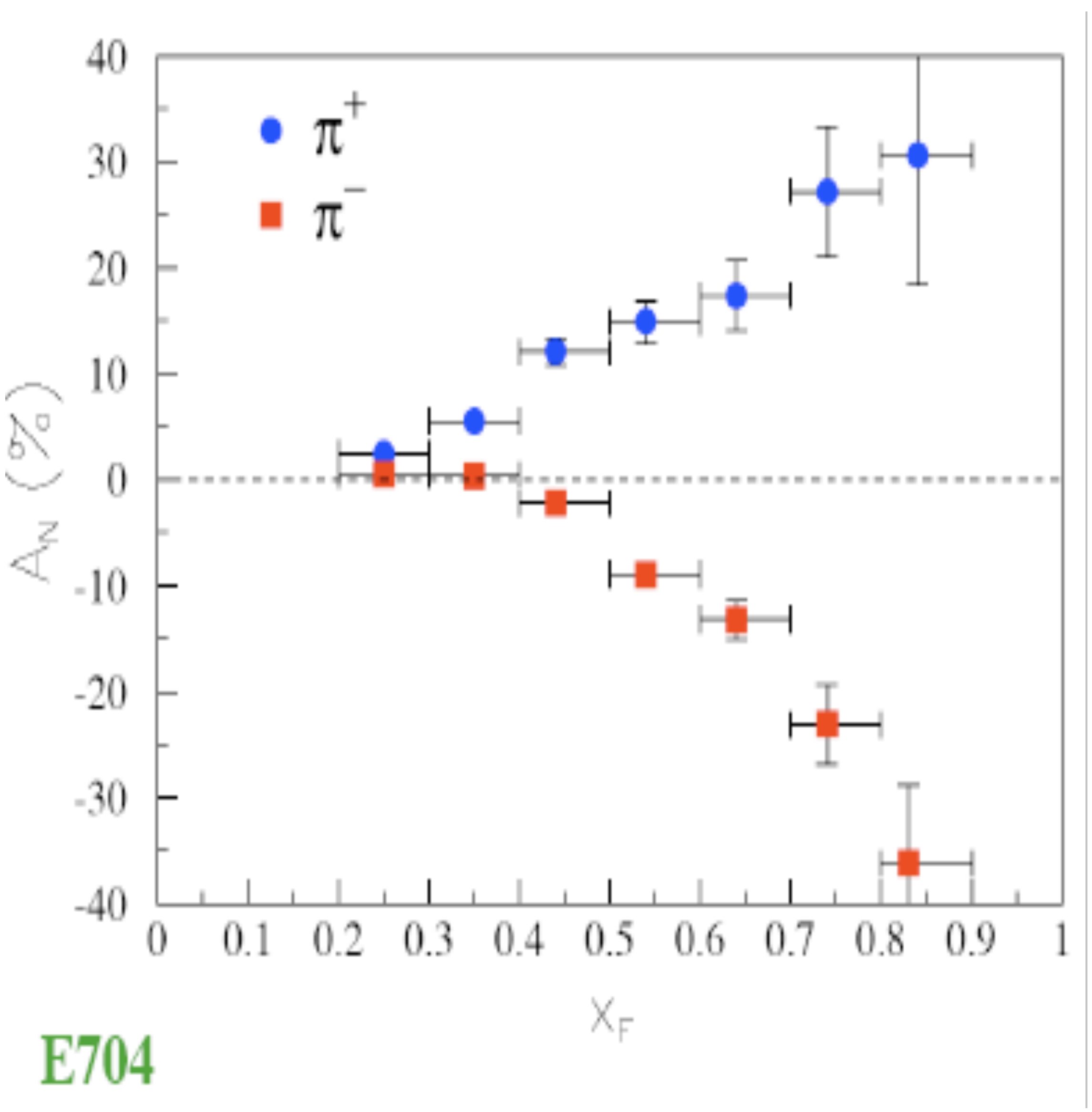
Process	TMD Factorization back-to-back	Collinear Factorization large separation
$\ell + p \to \ell + \pi + X$ $p + p \to \ell^+\ell^- + X$ $e^+e^- \to H_1 + H_2 + X$	✓	✓
$p + p \to H_1 + H_2 + X$ $p + p \to J_1 + J_2 + X$ $p + p \to \gamma + J + X$	✗	✓

History: when one puts a spin

- Large single transverse spin asymmetry observed at fixed-target experiments came as a surprise



$$p^\uparrow p \to \pi X$$



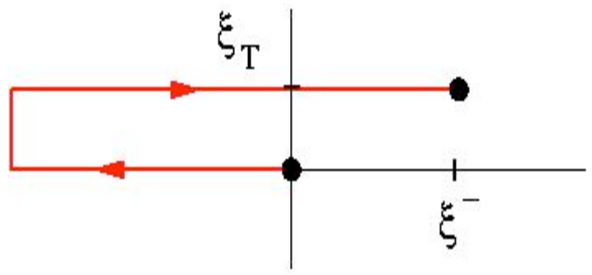
History: continuous and intensive theoretical studies (1)

- 1990: Sivers function PRD41, 83 (1990); PRD43, 261 (1991)
  - introduce kt dependence of PDFs, generate the SSA through a correlation between the hadron spin and the parton kt
- 1993: Collins function NPB396, 161 (1993)
  - introduce kt in TMD fragmentation function, generate the SSA through a correlation with the quark spin and the parton kt
  - show Sivers function vanishes due to time-reversal invariance
- 2002: S. J. Brodsky, D. S. Hwang, I. Schmidt PLB530, 99 (2002)
  - Explicit model calculation show the existence of the Sivers function in SIDIS
- 2002: J. Collins PLB536, 43 (2002)
  - Original proof missed the gauge link (needed to properly define gauge-invariant distribution)
  - Add gauge link: Sivers function in SIDIS = (-1) * Sivers function in DY
- 2002: S. J. Brodsky, D. S. Hwang, I. Schmidt NPB642, 344 (2002)
  - Verified the sign change through model calculation in DY

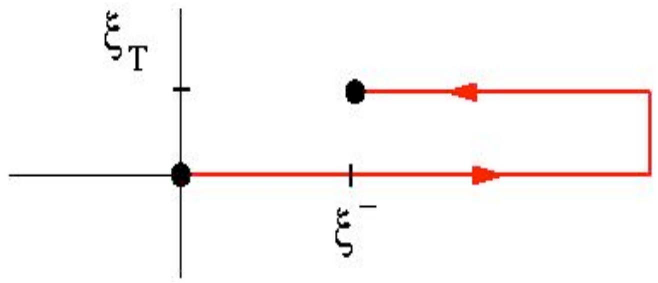
History: continuous and intensive theoretical studies (2)

- 2002: X. Ji, F. Yuan, A. V. Belitsky PLB543, 66 (2002); NPB656, 165 (2003)
  - the results by S. Brodsky, et.al is equivalent to introduce a transverse gauge link in the TMD distribution to make it fully gauge invariant
- 2003: Boer, Mulders, Pijlman NPB667, 201 (2003)
  - Use Feynman diagram approach to derive the gauge links
    - Resum collinear gluons => gauge links along the light-cone
    - Resum transverse gluons => transverse gauge links

DY

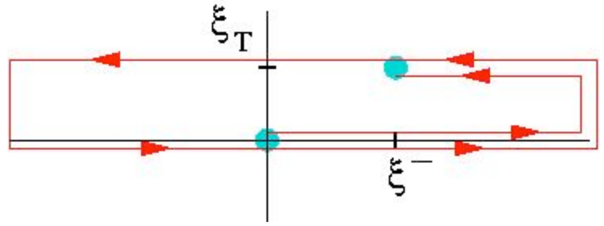


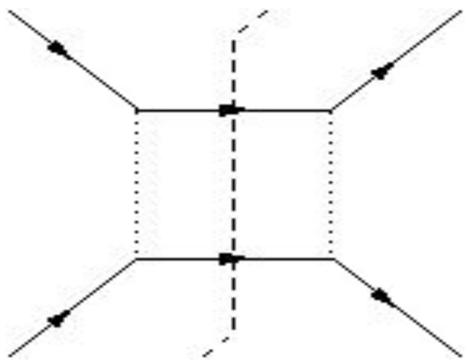
SIDIS



History: continuous and intensive theoretical studies (3)

- 2004: Bomhof, Mulders, Pijlman — PLB596, 277 (2004)
  - Apply same Feynman diagram approach to more complicated hard process 2->2
  - Very non-trivial gauge links were found
  - qq'->qq' by exchanging a photon





- 2005: Bacchetta, Bomhof, Mulders, Pijlman — PRD72, 034030 (2005)
  - calculate the gauge link for all QCD 2->2 subprocesses
  - conjectured a generalized TMD factorization formalism
  - Using weighted asymmetry, complicated gauge link structure reduces to simple pieces which are related to those in DY or SIDIS, thus have predicative power
- 2007: Bacchetta, Bomhof, D'Alesio, Mulders, Murgia — PRL99, 212002 (2007)
  - photon+Jet: the complicated gauge link leads to different (and opposite) SSA compared with the naive parton model without the effect of gauge links

History: continuous and intensive theoretical studies (4)

- 2007: Qiu, Collins PRD75, 114014 (2007)
  - Explicit example to show the process dependence of the gauge link, which indicates the breakdown of standard TMD factorization in these processes
- 2007: Collins, Vogelsang, Yuan arXiv: 0708.4410, PRD76, 094013 (2007)
  - Confirm the breakdown of standard TMD factorization at NLO level
- 2010: Rogers, Mulders arXiv: 1001.2977
  - There are pieces in the Feynman diagram expansion which cannot be incorporated to the conjectured TMD factorization formalism, further modification leads to inconsistency
  - No generalized TMD factorization for such processes

Why do we need perturbative QCD factorization?

- Cross sections with identified hadrons are infrared sensitive and non-perturbative
  - Scale of hadron wave function: $1/R \sim 1/\text{fm} \sim \Lambda_{QCD} \sim 200\text{MeV}$
  - Scale of hard partonic collisions: $Q > 2\text{GeV} >> \Lambda_{QCD}$

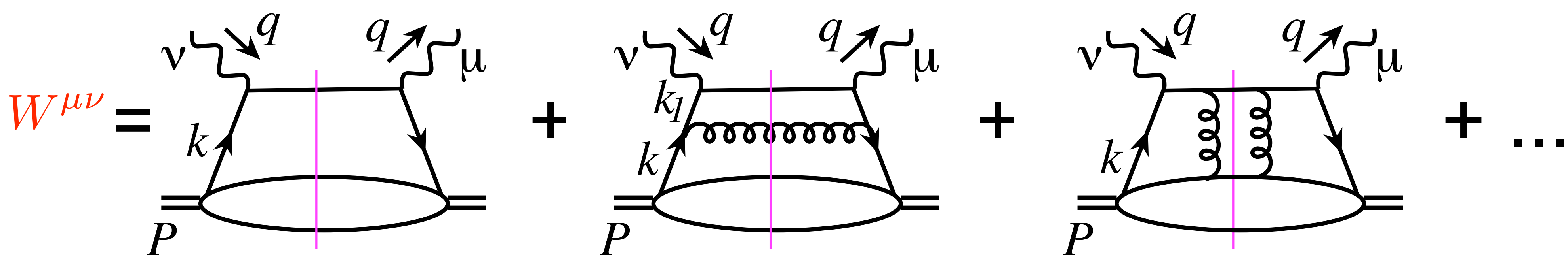
$\Rightarrow$ pQCD works at $\alpha_s(Q)$, but not at $\alpha_s(\Lambda_{QCD})$

- A way out: Factorization theorems
  - Quantum interference (correlation) between perturbative and non-perturbative scales can be neglected (power suppressed)

$$\sigma_{\text{Hadron}}(Q) = \phi_{\text{parton/Hadron}}(\Lambda_{QCD}) \otimes \hat{\sigma}_{\text{parton}}(Q) + \mathcal{O}(\Lambda_{QCD}/Q)$$

Collinear factorization: DIS as an example

- Deep inelastic scattering:



- Collinear factorizations:
  - parton momentum expansion: $k^\mu = xp^\mu + \dfrac{k^2+k_\perp^2}{2xp\cdot n}n^\mu + k_\perp^\mu$
  - If all observed physical scales: $Q \sim xp \gg k_\perp, \sqrt{k^2}$

$$W^{\mu\nu} = \int d^4k\ H(Q,k)\left(\frac{1}{k^2+i\epsilon}\right)\left(\frac{1}{k^2-i\epsilon}\right)T(k,\frac{1}{r_0})$$

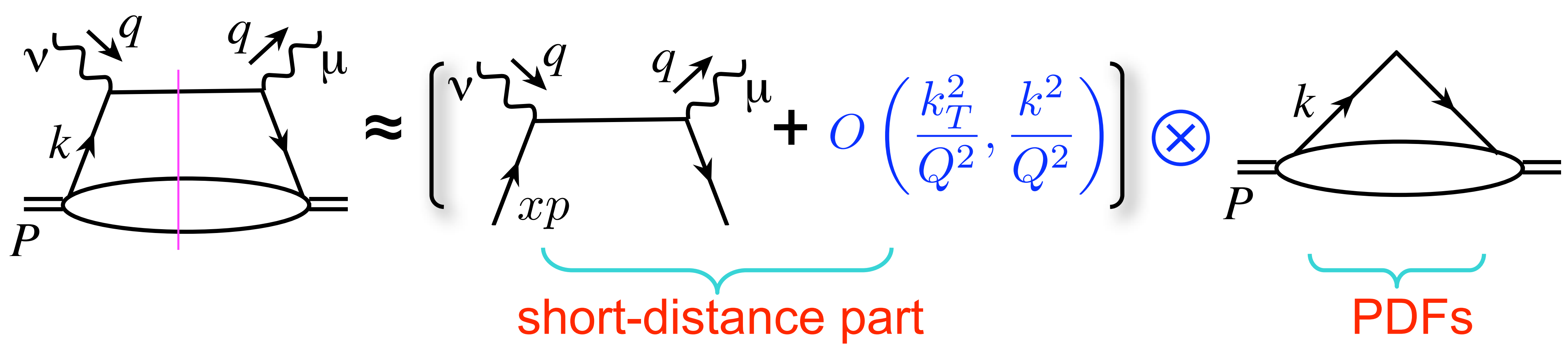
$$\approx \int \frac{dx}{x}\ H(Q,k=xp)\boxed{\int dk^2 d^2k_T\left(\frac{1}{k^2+i\epsilon}\right)\left(\frac{1}{k^2-i\epsilon}\right)T(k,\frac{1}{r_0})}$$

Nonperturbative matrix element (PDFs)

Short-distance

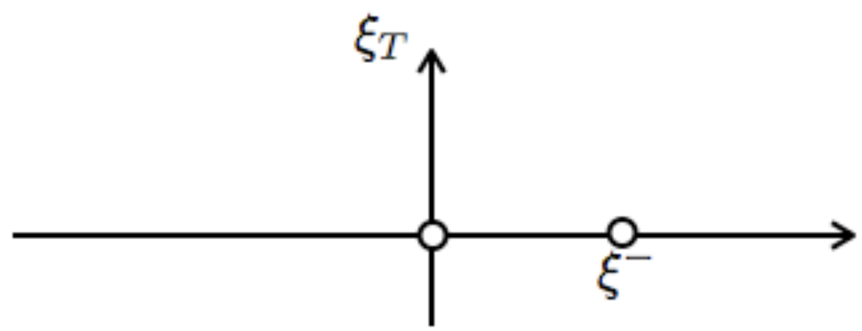
Gauge invariant PDFs: gauge link

- Leading order:



- A gauge-invariant PDFs need gauge links

$$\Phi(x) = \int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \langle p|\bar{\psi}(0) U_{[0,\xi]} \psi(\xi)|ps\rangle$$

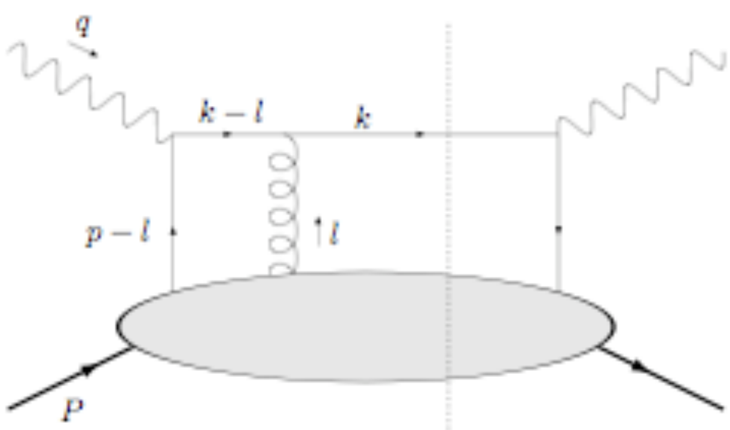


$$U_{[a,b]} = \mathcal{P}\exp\left[-ig\int_a^b d\eta^\mu A_\mu(\eta)\right]$$

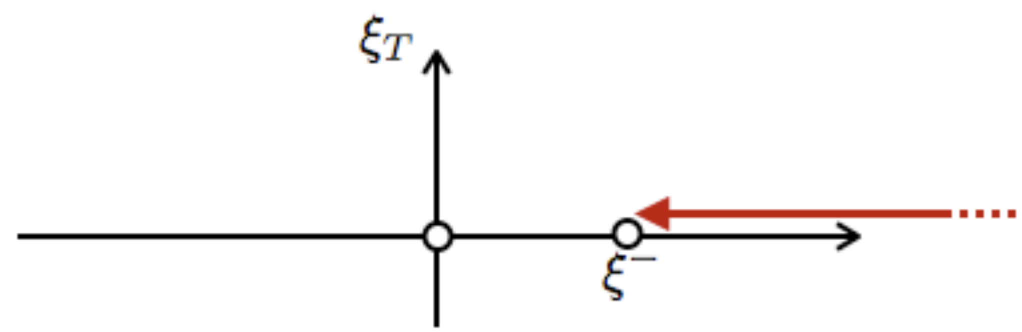
Birth of Gauge links:

- Gauge link along the light-cone: from resummation of all collinear gluons

$$(-ig)\frac{1}{-\ell^+ + i\epsilon}t_a$$

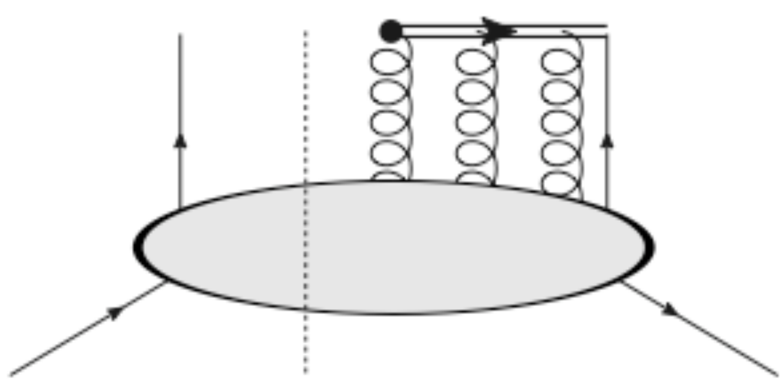


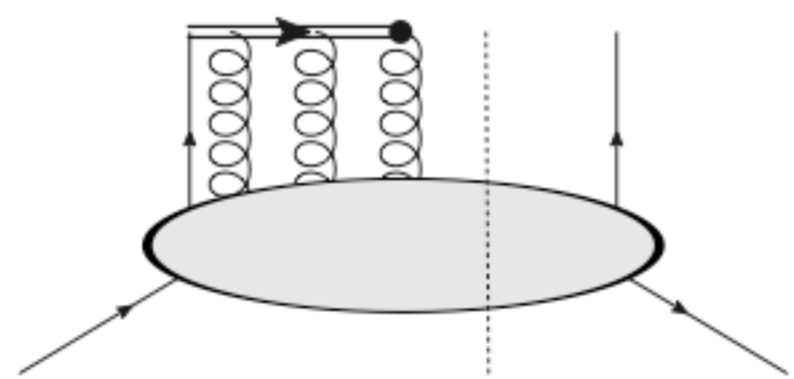
$$\Phi^{(a)}(x,S) \sim \langle P,S|\,\bar\psi(0)\,(-ig)\int_{\infty^-}^{\xi^-} \mathrm{d}\eta^-\; A^+(\eta)\,\psi(\xi)\,|P,S\rangle$$

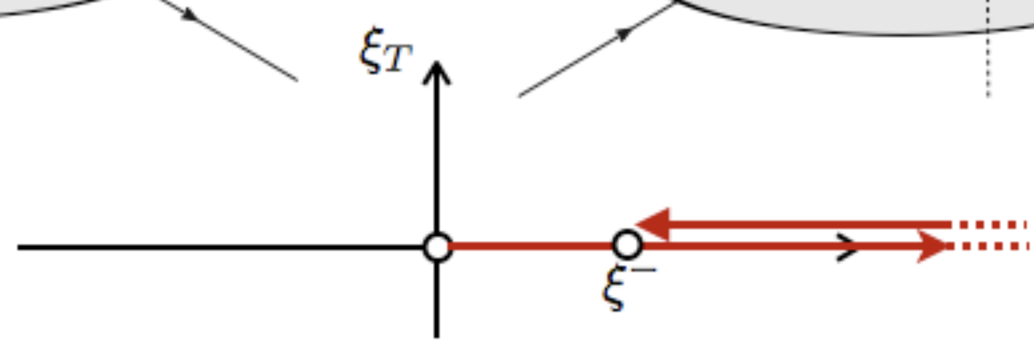


- To all orders

$$\Phi(x,S) \sim \langle P,S|\,\bar\psi(0)\,U_{[0,\infty^-]}\,U_{[\infty^-,\xi^-]}\psi(\xi)\,|P,S\rangle$$

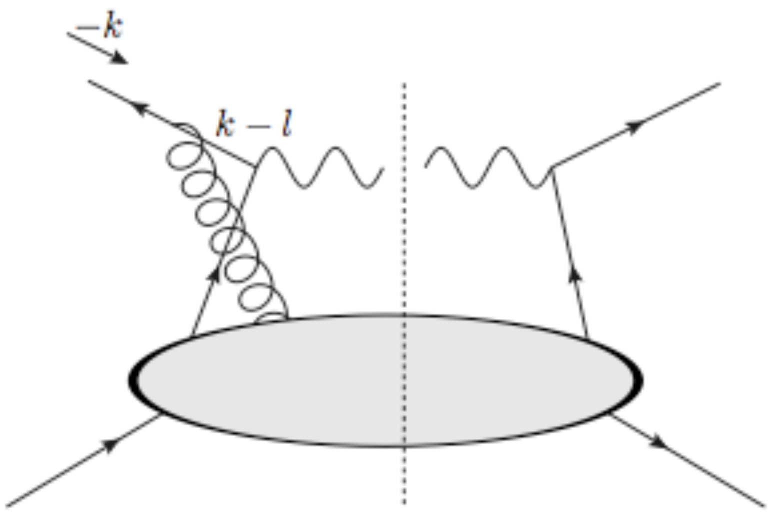


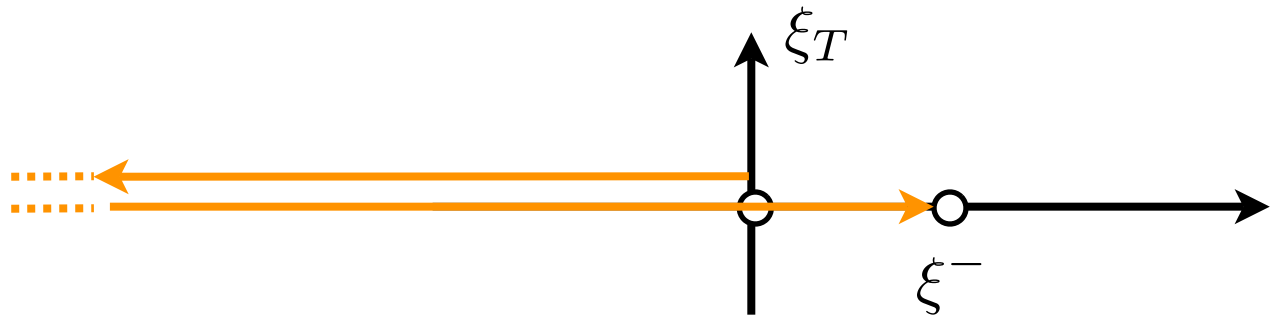




Gauge link for DY process in collinear case

- Gauge link along the light-cone in DY is same as those in DIS





TMD Factorization (k_T factorization)

- Momentum of the "long-lived" parton is not necessarily collinear to the hadron momentum

$$k^\mu = xp^\mu + \frac{k^2 + k_\perp^2}{2xp\cdot n} n^\mu + k_\perp^\mu$$

- Physical processes with two observed scales:

$$Q_1 \gg Q_2 \begin{cases} Q_1 & \text{necessary for pQCD factorization to have a chance} \\ Q_2 & \text{sensitive to parton's transverse momentum} \end{cases}$$

- TMD parton distributions: gauge links along light-cone plus transverse

$$f_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) = \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p, \vec{S}|\overline{\psi}(0^-, \mathbf{0}_\perp)\ \text{Gauge link}\ \frac{\gamma^+}{2}\psi(y^-, \mathbf{y}_\perp)|p, \vec{S}\rangle$$

Transverse Gauge Link: SIDIS and DY

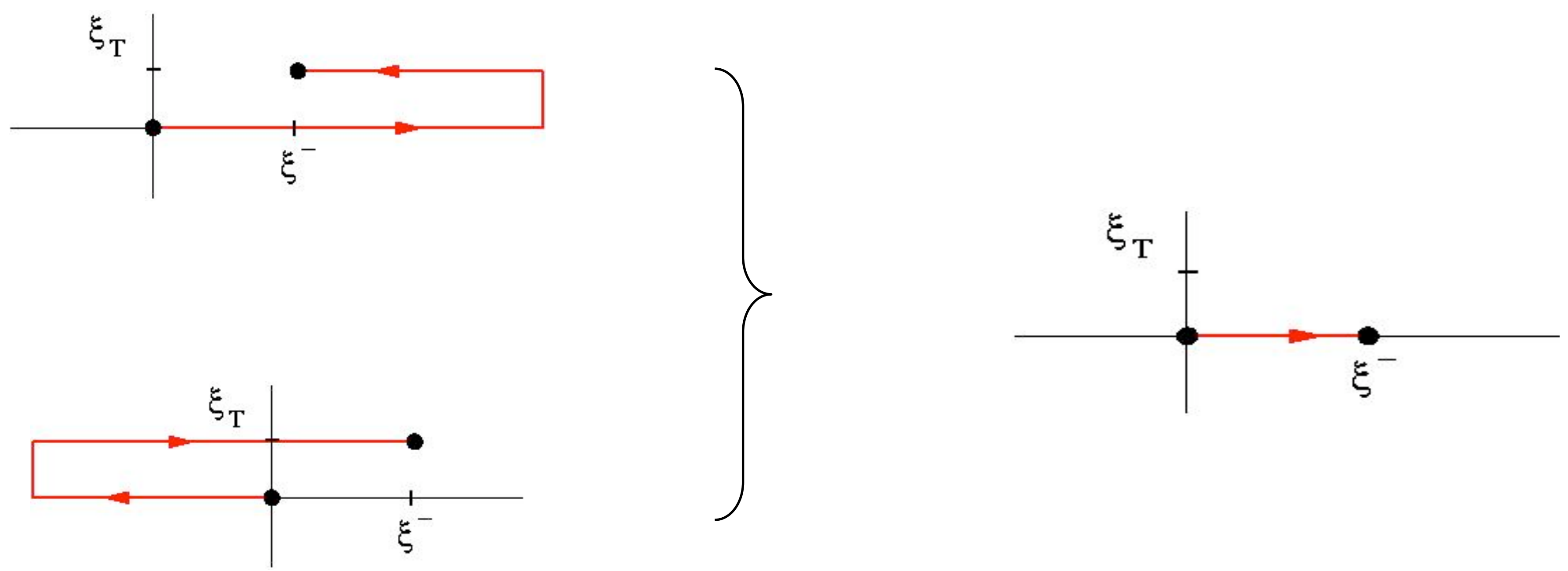
- Different TMD gauge link, but same collinear gauge link

$$\Phi^{[\pm]}(x,p_T)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{ip.\xi}\langle P|\psi^\dagger(0)U^n_{[0,\pm\infty]}U^T_{[0_T,\xi_T]}U^n_{[\pm\infty,\xi]}\psi(\xi)|P\rangle_{\xi.n=0}$$

$$\Phi^{[\pm]}(x)=\int\frac{d(\xi.P)}{(2\pi)}e^{ip.\xi}\langle P|\psi^\dagger(0)U^n_{[0,\xi]}\psi(\xi)|P\rangle_{\xi.n=\xi_T=0}$$

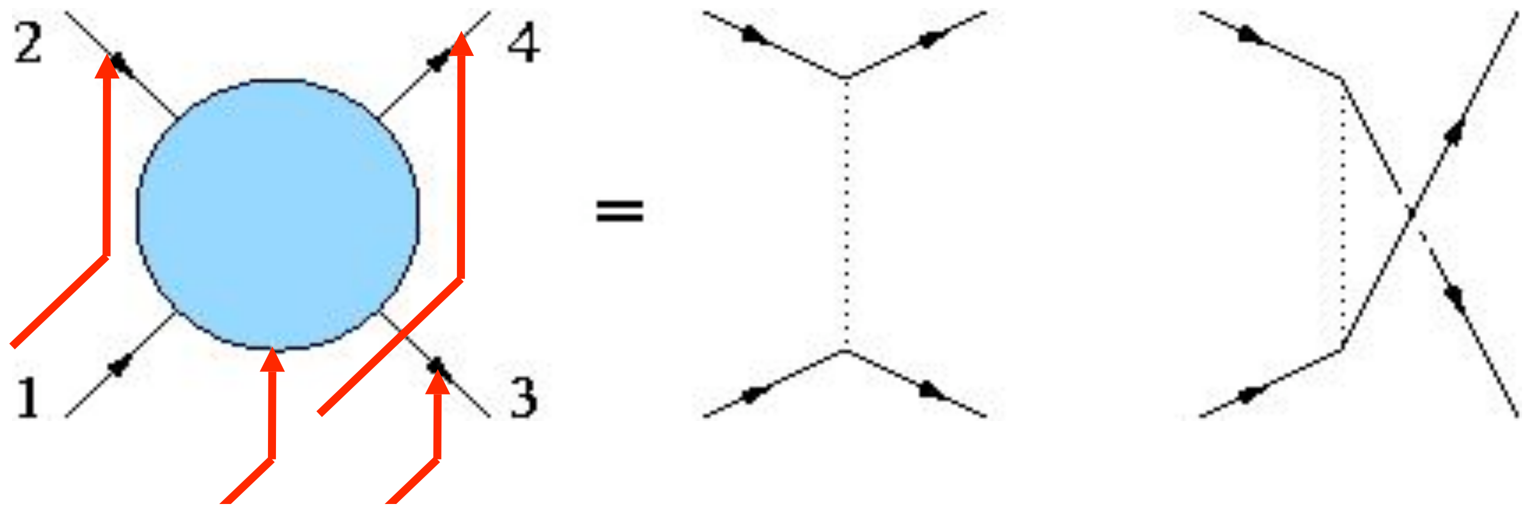
SIDIS

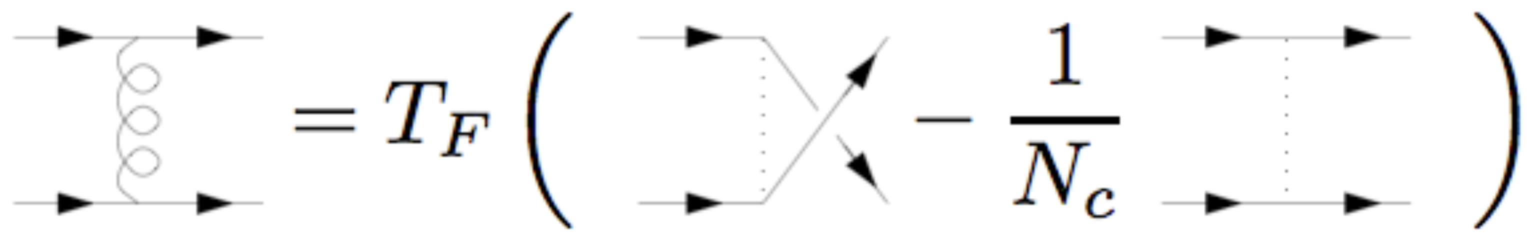
DY

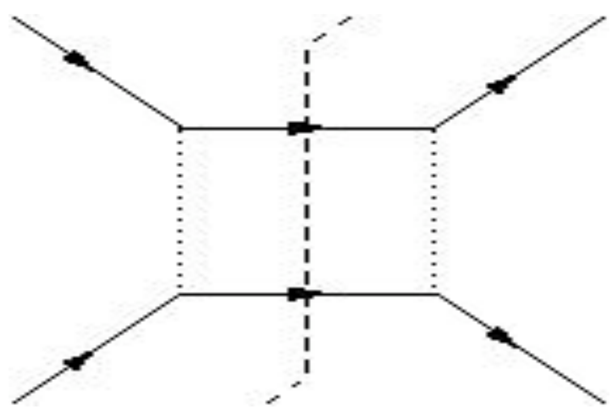


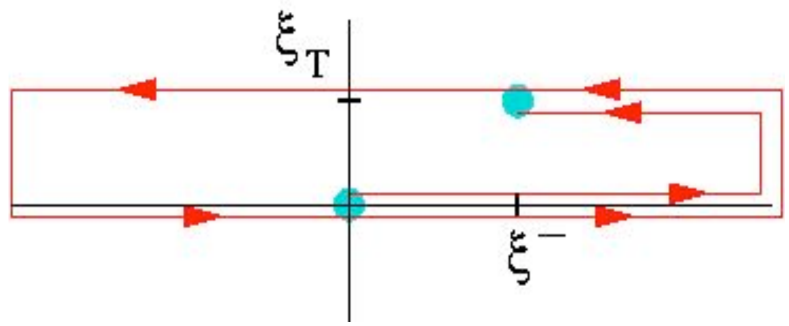
When more partons are involved in hard processes

- Example: qq->qq



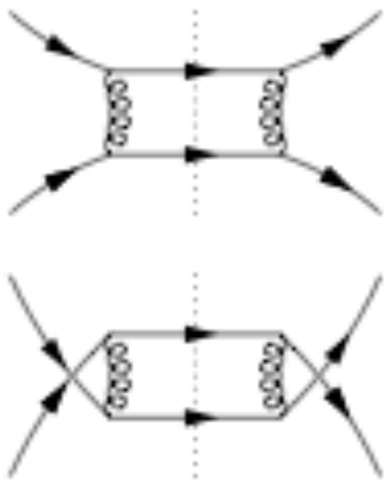
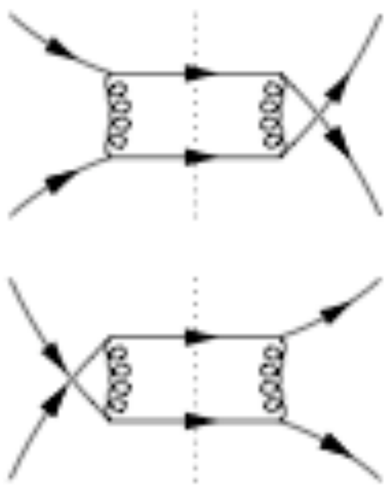






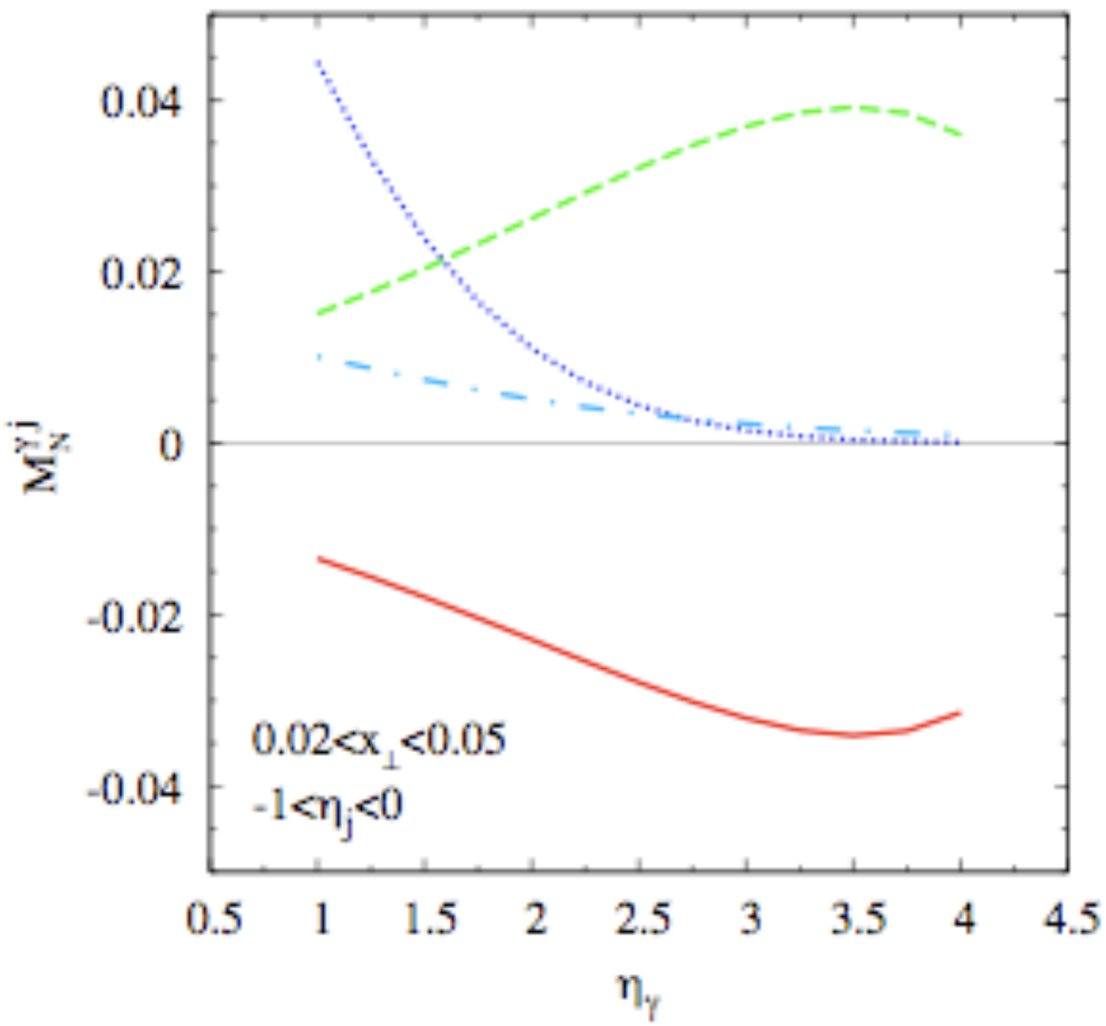
Different gauge link for different Feynman diagrams

- Different gauge link even for the crossing diagram for qq->qq

	$\Phi_q \propto \langle\, \overline{\psi}(0) \left\{ \frac{N_c^2+1}{N_c^2-1} \frac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]}]}{N_c} \mathcal{U}^{[+]} - \frac{2}{N_c^2-1} \mathcal{U}^{[\Box]}\mathcal{U}^{[+]} \right\} \psi(\xi) \,\rangle$ $\Delta_q \propto \langle\, \overline{\psi}(\xi) \left\{ \frac{N_c^2+1}{N_c^2-1} \frac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]}]}{N_c} \mathcal{U}^{[-]\dagger} - \frac{2}{N_c^2-1} \mathcal{U}^{[\Box]}\mathcal{U}^{[-]\dagger} \right\} \psi(0) \,\rangle$
	$\Phi_q \propto \langle\, \overline{\psi}(0) \left\{ \frac{2N_c^2}{N_c^2-1} \frac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]}]}{N_c} \mathcal{U}^{[+]} - \frac{N_c^2+1}{N_c^2-1} \mathcal{U}^{[\Box]}\mathcal{U}^{[+]} \right\} \psi(\xi) \,\rangle$ $\Delta_q \propto \langle\, \overline{\psi}(\xi) \left\{ \frac{2N_c^2}{N_c^2-1} \frac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]}]}{N_c} \mathcal{U}^{[-]\dagger} - \frac{N_c^2+1}{N_c^2-1} \mathcal{U}^{[\Box]}\mathcal{U}^{[-]\dagger} \right\} \psi(0) \,\rangle$

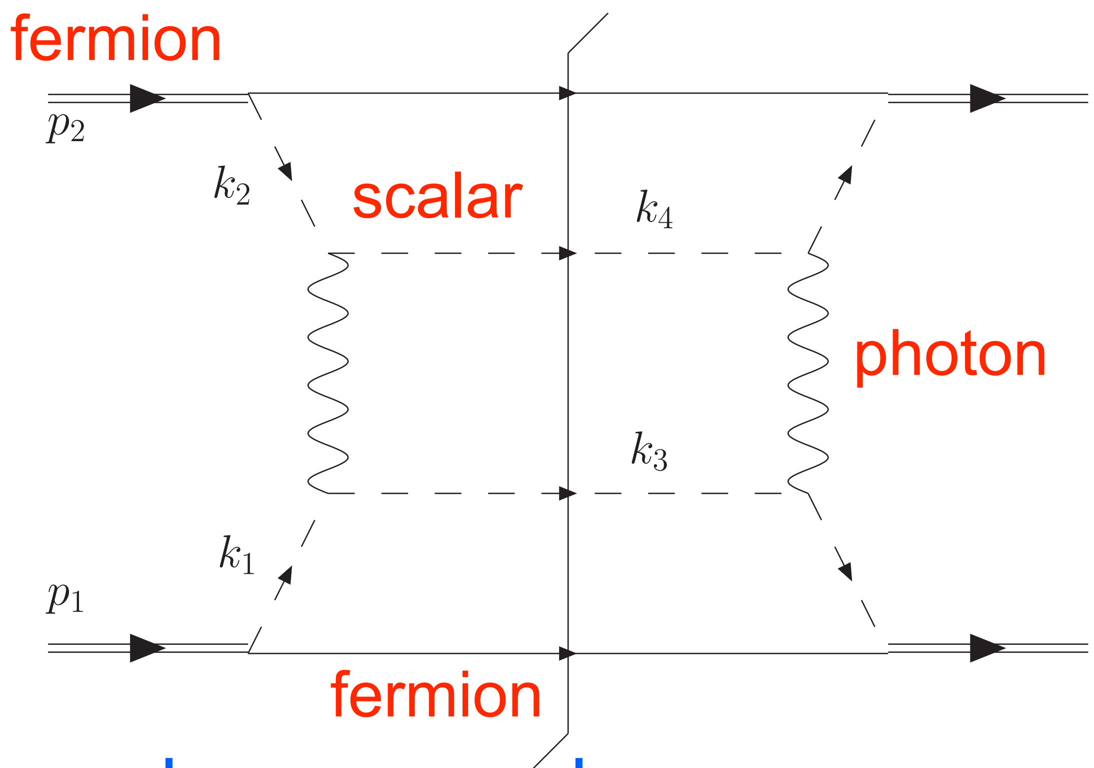
Conjectured generalized TMD formalism

- If one is equipped with the complicated gauge links for different subprocess, one can write down a formally "TMD factorized" formalism
- Weighted smartly, the complication of the gauge link disappears
  - photon+Jet:

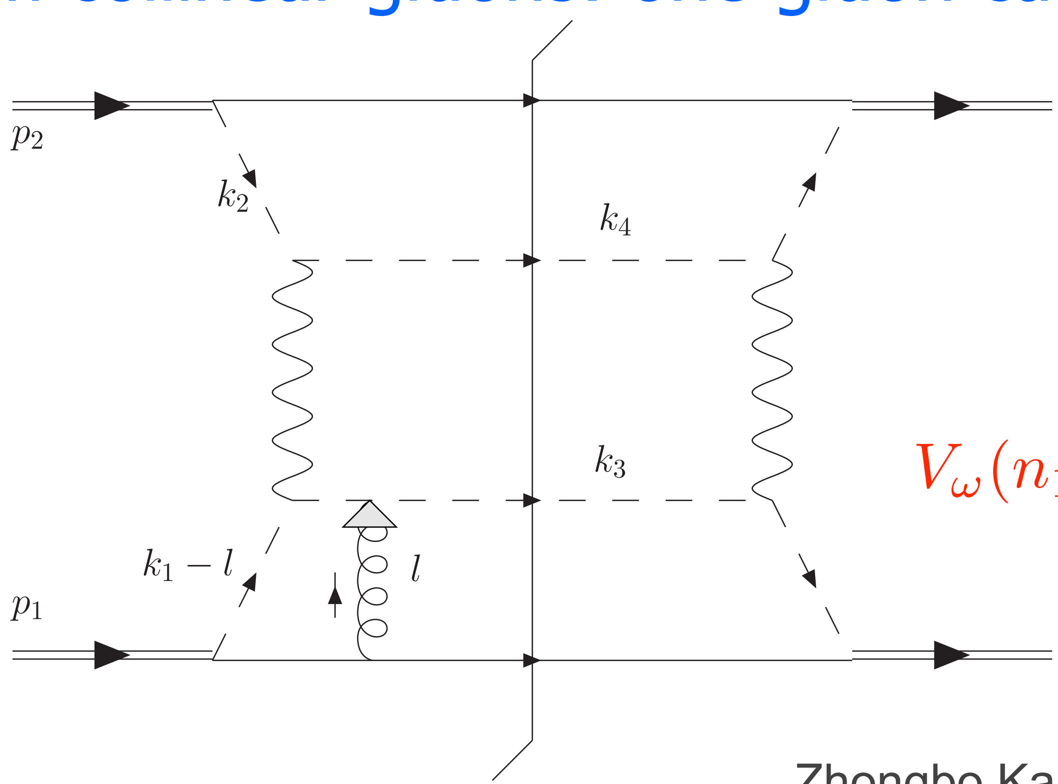


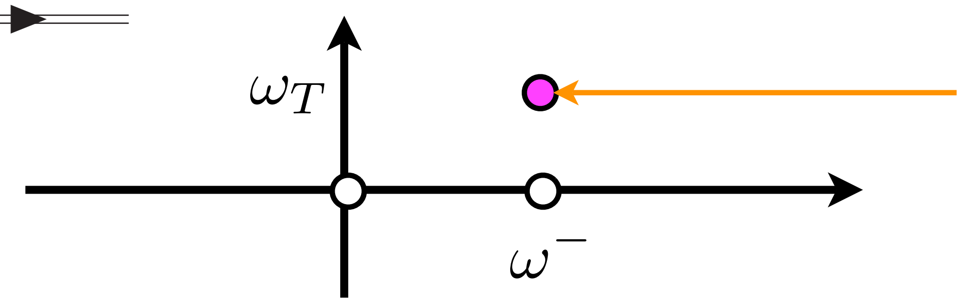
Non-trivial gauge link: breakdown of TMD factorizations

- Model:



- Sum collinear gluons: one gluon case



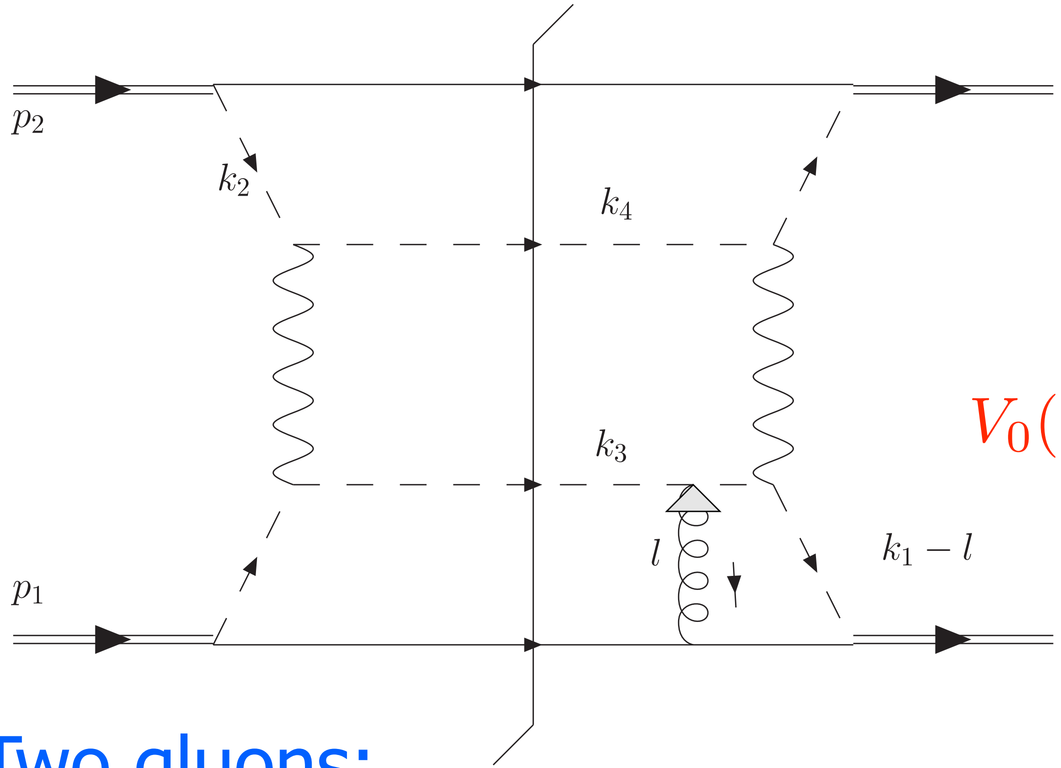


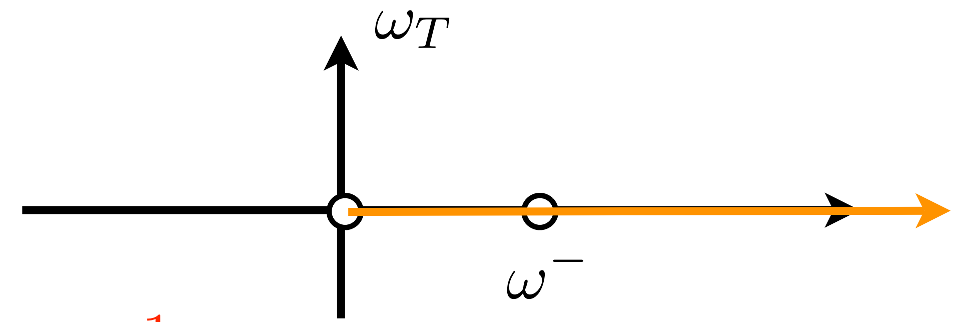
$$(-ig)\frac{1}{-\ell^+ + i\epsilon}t_a$$

$$V_\omega(n_1)^\dagger = P\exp\left(-igt^a\int_\infty^{\omega^-} d\lambda n_1\cdot A^a(\lambda n_1 + \omega_T)\right)$$

Verify the standard gauge link

- One gluon case: gluon to the right

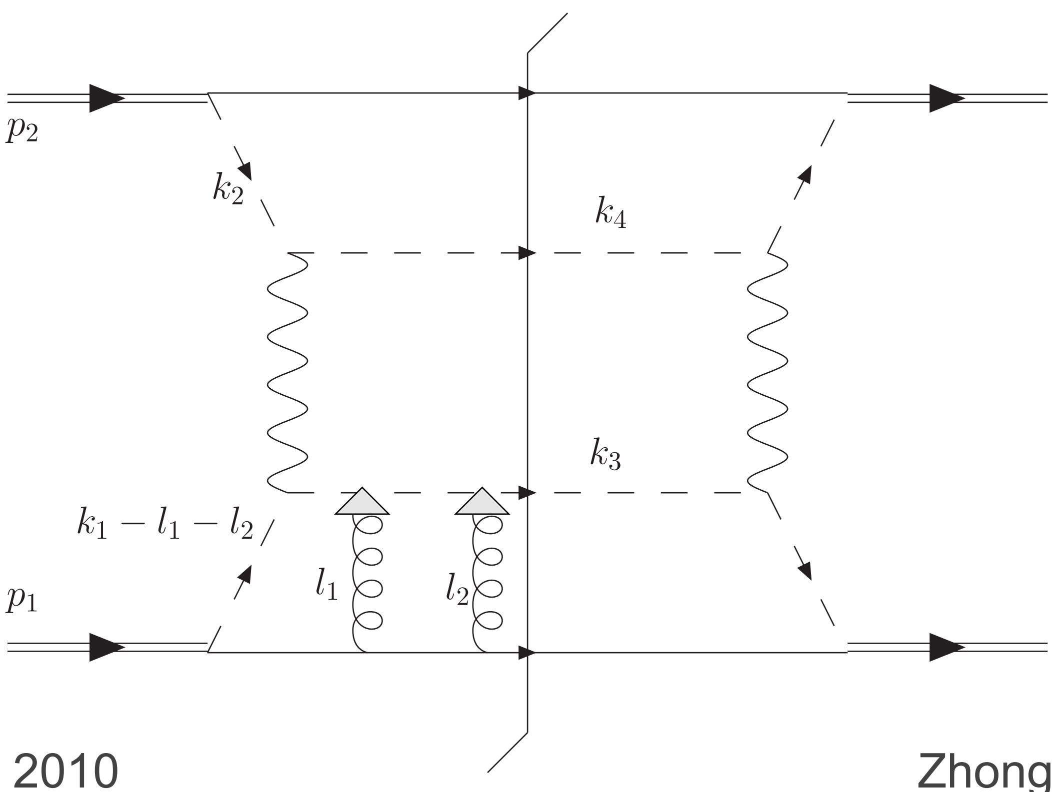


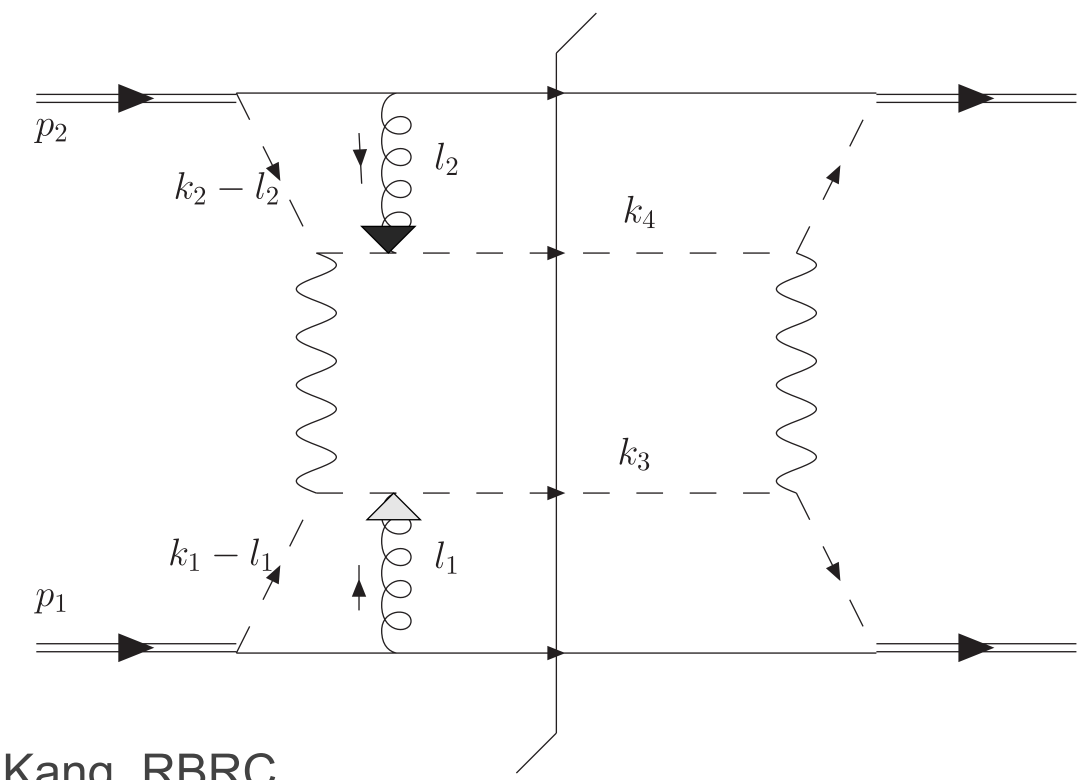


$$(-ig)\frac{1}{-\ell^+ - i\epsilon} t_a$$

$$V_0(n_1) = P\exp\left(-igt^a \int_0^\infty d\lambda n_1 \cdot A^a(\lambda n_1)\right)$$

- Two gluons:

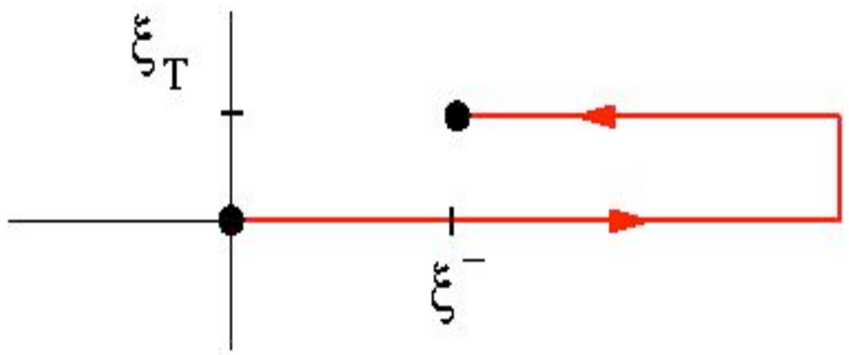




Standard gauge link

- Standard gauge link: same as SIDIS (expected)

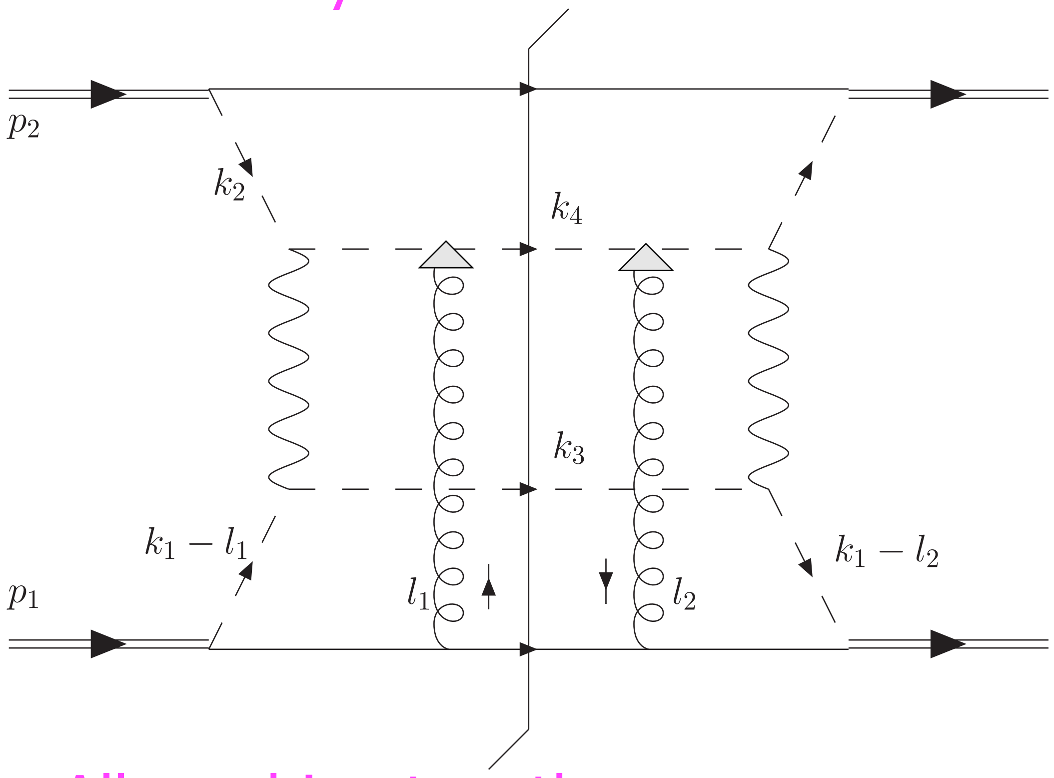
$$U_{jk}^{[n_1]}[0,w] = \left[V_w^\dagger(n_1)\right]_{jj'} [I(n_1)]_{j'k'} [V_0(n_1)]_{k'k}$$



- This tells us that the standard gauge link only represents the gluon attachments we discussed so far. However, there are certainly other gluon attachments. If these new gluon attachments do NOT vanish, then we have a violation of standard TMD factorization.

Violation of Standard TMD factorization

- Two gluons case:
  - Note: they can also be attached to k2, four combinations



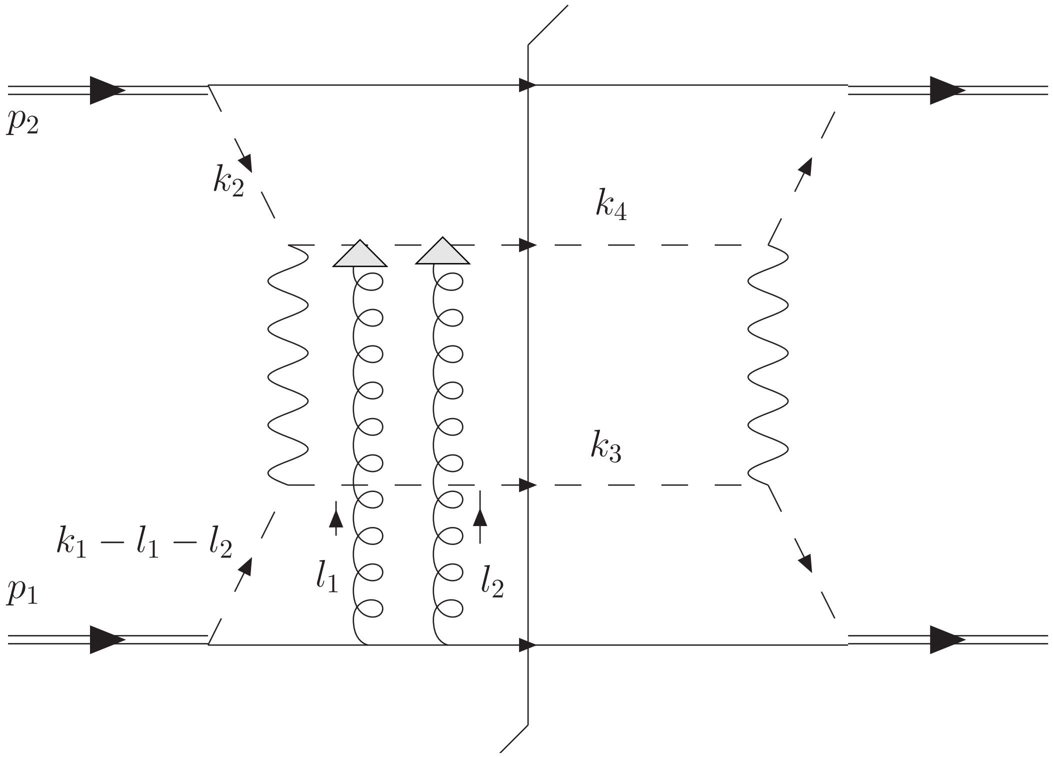
$$(-ig)\frac{1}{-\ell_1^+ + i\epsilon}(-ig)\frac{1}{-\ell_2^+ - i\epsilon}\mathrm{Tr}[t_a t_b]$$

  - All combine together

$$\mathrm{Tr_C}\left[t^a t^b\right] g^2 n_1^\mu n_1^\nu \left(\frac{1}{-l_1^+ + i\epsilon} + \frac{1}{l_1^+ + i\epsilon}\right) \times$$
$$\times \left(\frac{1}{-l_2^+ - i\epsilon} + \frac{1}{l_2^+ - i\epsilon}\right) =$$
$$4\pi^2 g^2 n_1^\mu n_1^\nu \mathrm{Tr_C}\left[t^a t^b\right] \delta(l_1^+)\delta(l_2^+)$$

Other two-gluon case

- Two gluons on the same side:



$$
\mathrm{Tr_C}\left[t^a t^b\right] g^2 n_1^\mu n_1^\nu \left\{ \left(\frac{1}{-l_1^+ + i\epsilon}\right)\left(\frac{1}{-l_2^+ + i\epsilon}\right) + \left(\frac{1}{l_1^+ + i\epsilon}\right)\left(\frac{1}{-l_2^+ + i\epsilon}\right) + \right.
$$

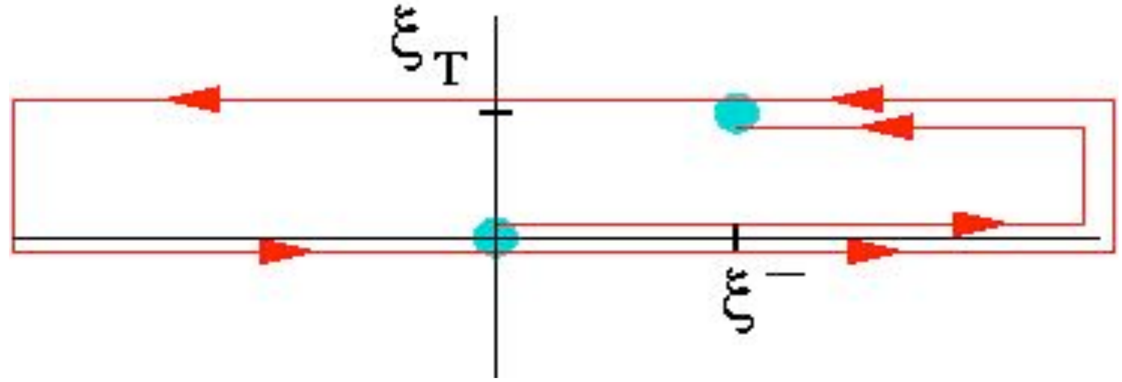
$$
\left. + \left(\frac{1}{-l_1^+ + i\epsilon}\right)\left(\frac{1}{l_2^+ + i\epsilon}\right) + \left(\frac{1}{l_1^+ + i\epsilon}\right)\left(\frac{1}{l_2^+ + i\epsilon}\right) \right\}
$$

$$
= \mathrm{Tr_C}\left[t^a t^b\right] g^2 n_1^\mu n_1^\nu \left(\frac{1}{l_1^+ + i\epsilon} + \frac{1}{-l_1^+ + i\epsilon}\right)\left(\frac{1}{l_2^+ + i\epsilon} + \frac{1}{-l_2^+ + i\epsilon}\right) = -4\pi^2 g^2 n_1^\mu n_1^\nu \mathrm{Tr_C}\left[t^a t^b\right]\delta(l_1^+)\delta(l_2^+)
$$

The contribution is not zero for a TMD case

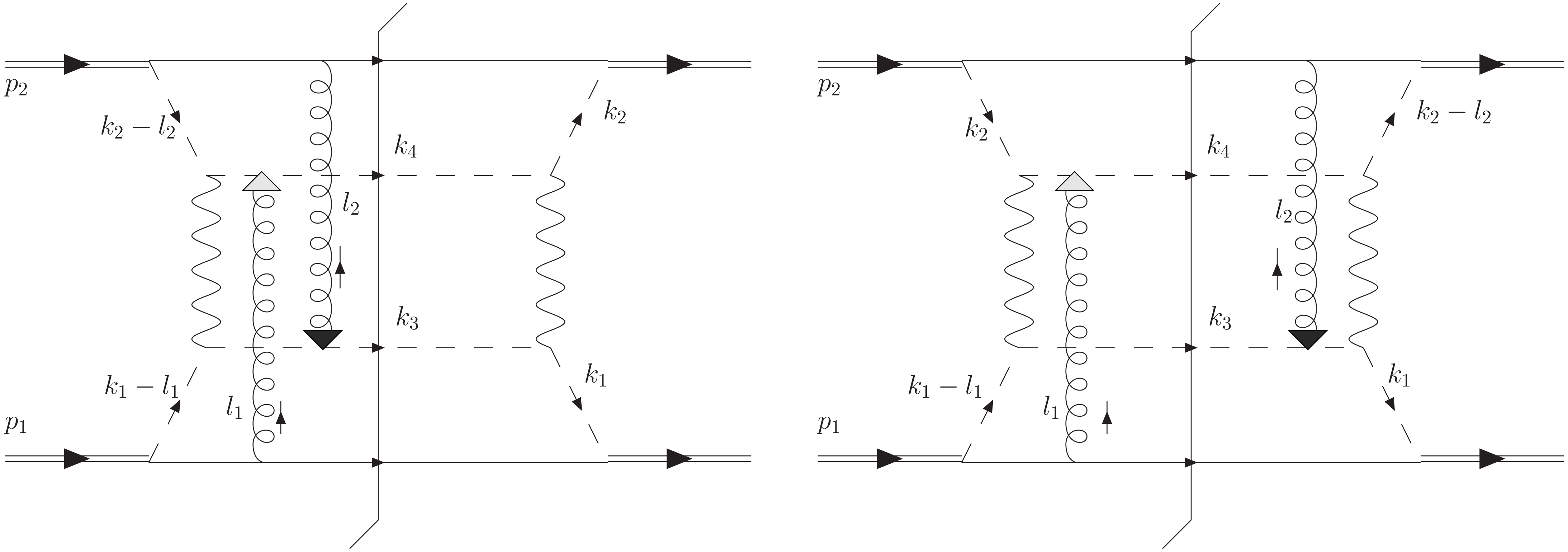
- Though the eikonal propagator parts seem cancel each other, the denominator parts are different
- Since this part contribution cannot be generated from standard gauge links, their nonvanishing represents the violation of standard TMD factorization
- However, they can be generated from the following gauge link, which has been conjectured before

$$\Phi_{H_1}^{[n_1,(\Box)]}(x_1,k_{1T}) = x_1 p_1^+ \int \frac{dw^- d^2\mathbf{w}_t}{(2\pi)^3} e^{-ix_1p_1^+w^- + i\mathbf{k}_t\cdot\mathbf{w}_t} \times$$
$$\times \langle H_1, s_1 | \phi_{1,r}^\dagger(0, w^-, \mathbf{w}_t)\, U_{rs}^{n_1}[0,w] U_{(\Box)}^{n_1} \phi_{1,s}(0) | H_1, s_1\rangle$$



Generalized gauge link indicate new TMD factorization?

- There are more ways to attach gluons. If they are not zero, then it will signal a violation of the generalized TMD factorization we juts proposed.



- Even the eikonal part does not cancel each other (adds together)

No TMD Factorization: a bad thing?

- New opportunity: Qiu, Collins

Troublesome though it may be for phenomenology, breaking of factorization should be viewed not as some kind of failure, but as an opportunity. Examination of the distribution of high-transverse-momentum hadrons in hadron-hadron collisions will lead to interesting nontrivial phenomena.

- Still interesting: Rogers, Mulders

I feel that the breakdown of generalized factorization is in itself a very interesting result.
It is my point of view that the question of factorization is a very deep one about the relationship between different scales in QCD.
Therefore, I think comparisons of data with results obtained using a generalized TMD factorization approach are still very interesting, since an observed contradiction with experiment would also be important.

New ideas: Collinear factorization still works

- Collinear factorization still works, and it works when all the observed scales >> Λ_{QCD}
- For dijet or photon+jet case - if they are not back-to-back!

$$H_1(p_A)+H_2(p_B)\Rightarrow Jet(p_1)+Jet(p_2)+X \qquad p_1=\frac{P}{2}+q \qquad p_2=\frac{P}{2}-q$$

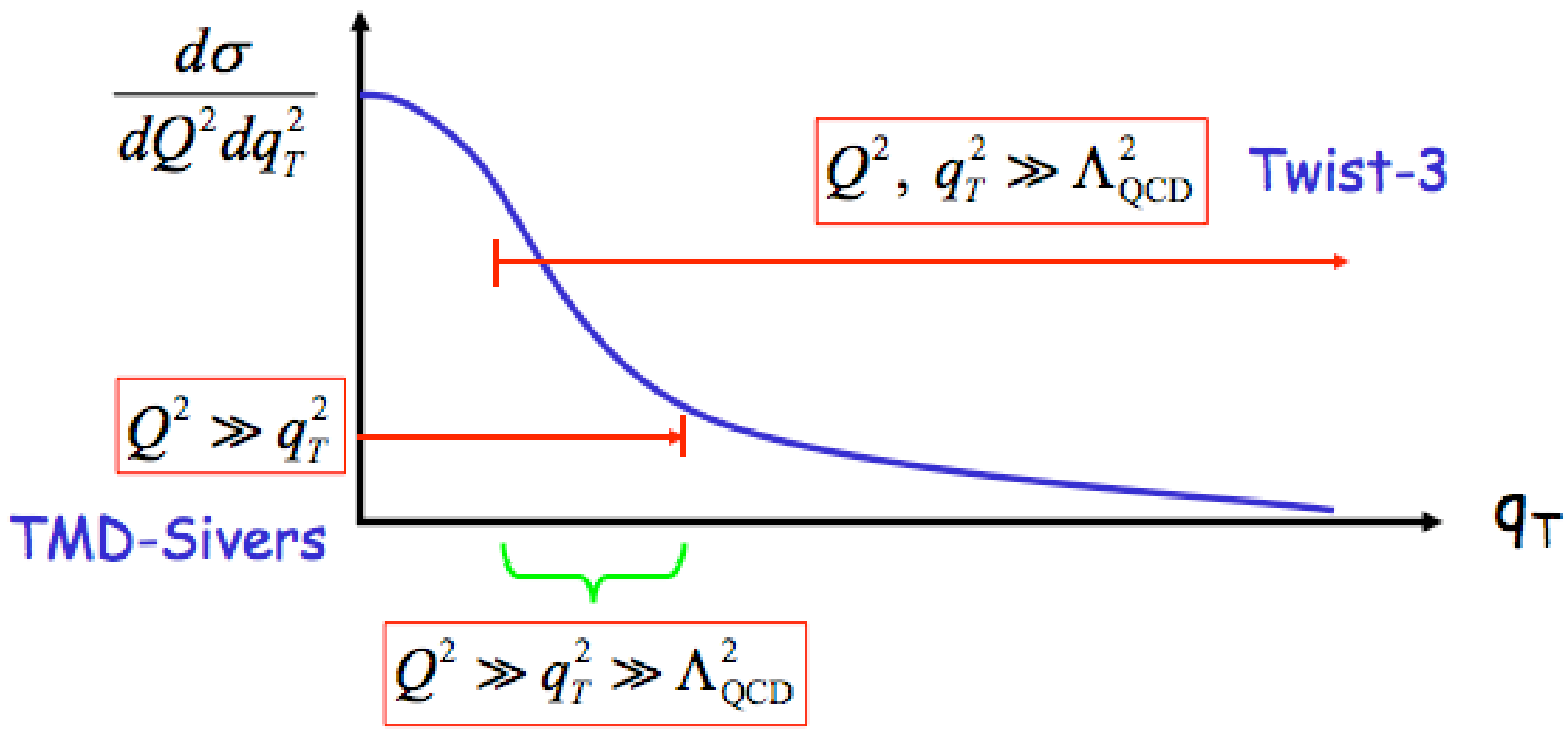
  - relevant kinematic region for collinear approach

$$P_T \gtrsim q_T \gg \Lambda_{\rm QCD}$$

- TMD factorization breaks down, while the collinear factorization works outside the back-to-back region. Measuring SSA of di-jet or photon +jet production as a function of momentum imbalance probes the transition to the nonfactorized regime

Transition to the non-factorized regime

- $Q = P_T$



Conclusion

- The gauge link are very important to ensure the gauge invariance of the distribution functions - the consequence of color
- Complicated gauge link structures appear for TMD distribution functions
- Process dependence of the gauge link leads to the breakdown of TMD factorization
- Failure of TMD factorization opens new opportunities
- RHIC measurements on SSA of di-jet or photon+jet production as a function of momentum imbalance probes the transition to the nonfactorized regime
- Without the TMD factorization, the role of the photon+jet in testing the sign change of Sivers functions is problematic - we need DY/Z measurements where TMD factorization is valid

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Thank You