Searching for a New Source(s) of T-Violation in Spin Dependent Total Cross Section Measurements

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Abstract

We first re-prove with a more complete method that the the minimum standard model, with the inclusion of the CKM-matrix, requires the T-odd/P-odd total cross section of two spin-1/2 particles to vanish in all orders [1]. The influences of stronger interactions to the result are studied by treating stronger interactions exactly and weak interaction perturbatively to keep the effects of stronger interactions in control. Then we study the contribution to T-odd/P-odd total scattering cross sections from various channels within the Higgs sector, and optimize conditions for possible experimental measurements of these effects. These studies show that such contributions can appear at tree level, and that the spin dependent cross section asymmetry is measurable with current technlogies and knowledge about the lightest Higgs paarticle, *e.g.* $m_H = 125.35\, GeV$, if suitable reaction channels and beam energies and luminosities are chosen.

1 Introduction

The minimal standard model [2] with the Cabibbo-Kobayashi-Maskawa mixing matrix [3], MSMCKM, explains CP-violations in heavy quark decays. However, it is natural to wonder if other CP-violations are possible. Various attempts to explain the baryon asymmetry of the universe require much larger CP-violation [4] than suggested by the MSMCKM, which may indicate additional CP or T violating mechanisms. Indeed, the possibility of CP (or T) violation due to the Higgs sector has been independently studied by the authors in refs. [6] and [7], and models proposed that introduce CP-violation through both neutral and charged Higgs-boson exchange [8].

Recently, it was shown [1] that while the MSMCKM gives a null result to all orders for an experimental test of time-reversal symmetry $(1/2 + 1/2 \rightarrow 1/2 + 1/2)$ suggested by ref. [9], this is not necessarily true if the Higgs sector also contributes to T (or CP) violation. Thus ref. [1] indicates that such a measurement is a null test of extensions to the MSMCKM which could include CP (or T)-violation contributed by the Higgs sector. However, for most of the choices of beams and targets, e.g. hadrons 1 collides with hadrons 2, one would deal the situations that there are interactions other than weak interaction in a system and the effects due to weak interaction could be influenced or even simulated by stronger interactions, e.g. see ref. [5]. Therefore, to carry out an accurate tests of T (or CP) violation in $1/2{+}1/2 \rightarrow 1/2{+}1/2$ scattering, it is very necessary to get rid of the influences due to stronger interactions or have them under control. Furthermore, from an experimental view point, one must know whether a test is feasible, and what precision would be required. We would like to answer these questions in this note.

In section 2, we re-prove the theorem that the T-odd/P-odd total cross section in the MSMCKM vanishes to all orders [1] by a different, and more complete, method. In section 3, we study the influence to the T-dd/P-odd total cross sections due to stronger interactions, e.g. electric and strong interactions, by treating the stronger interactions exactly and the weak interaction purterbatively [15]. We conclude that, while stronger interactions could influence the effect due to weak interaction for individual channels, it would not have an effect for total cross section measurements. In section 4, various possible channels in a model extended to include the Higgs sector contributing to T (or CP) violation, is studied as a function of the beam energy, in order to optimize the conditions for an experimental test. Finally, conclusions are given in section 5.

2 Proof of Null T-odd/P-odd $A_{x,y}$ within the MSM-CKM

This theorem was originally proven in ref. [1]. Now consider the reaction of two spin-1/2 particles ($12 + 12 \to 12 + 12$). The forward-scattering matrix element is written as [9]

$$\begin{aligned} M(0) &= a_{0,0} + a_{0,z}\sigma_0\sigma_z + a_{z,0}\sigma_z\sigma_0 + a_{x,x}(\sigma_x\sigma_x + \sigma_y\sigma_y) + \\ &\quad a_{z,z}\sigma_z\sigma_z + a_{x,y}(\sigma_x\sigma_y - \sigma_y\sigma_x)\,, \end{aligned} \tag{1}$$

where $\sigma_x \equiv \sigma \cdot \mathbf{x}$, etc., and we have chosen a coordinate system by defining the unit vectors;

$$\begin{aligned} \mathbf{e_z} &= \mathbf{k_z}/k_z \\ \mathbf{e_y} &= \mathbf{k} \times \mathbf{k}'/|\mathbf{k} \times \mathbf{k}'| \\ \mathbf{e_x} &= \mathbf{e_y} \times \mathbf{e_z}. \end{aligned} \tag{2}$$

Here $\mathbf{k}$ and $\mathbf{k}'$ are the incident and scattered momenta of the particles, respectively. The conditions for parity-non-conserving (PNC) and time-reversal-volated (TRV) amplitudes have the properties;

$$PNC \;\; (TRV) \;\; if \;\; n_x + n_z \;\; (n_x) \;\; is \;\; odd. \tag{3}$$

In this equation $n_x(n_z)$ is the number of $x(z)$ subscripts. Thus the only TRV amplitude in eq. (1) is a_{xy}, which is also PNC, *i.e.* T-odd/P-odd.

Using the optical theorem, which relates the total cross section to the imaginary part of the forward-scattering amplitude, the total spin-correlation coefficient $A_{x,y}$ for $p_1 = p_x$ and $p_2 = \pm p_y$ is given by

$$A_{x,y} = Ima_{x,y}/Ima_{0,0}\,. \tag{4}$$

Therefore, this spin-correlation coefficient $A_{x,y}$ is both time (T) and parity (P) odd. A non-zero $A_{x,y}$ indicates not only that the reaction violates T and P, but also the T-odd/P-odd total cross section is not zero. This is a null test, and provides a framework to precisely investigate TRV processes.

The above derivation is obtained from the two-component spinor description, but the conclusions obtained using eq. (1) to eq. (4) can be applied in a four-component relativistic description, provided the center-of-mass frame (CMS) is used. This is because ([10]) (1) the four-component relativistic scattering matrix can be reduced to the two-component formalism in the center-of-mass frame (CMS); (2) the reduced Pauli scattering matrix has the same transformation properties as the non-relativistic scattering matrix under spatial reflections and time reversal; and (3) the spin vector in the non-relativistic treatment can be equated to the relativistic spin vector when the latter is measured in the particle rest-frame related to the CMS by a Lorentz transformation.

Ref. [1] proved that $A_{x,y}$ of polarized scattering $1/2 + 1/2 \to 1/2 + 1/2$ in the MSMCKM is identically zero. For a more complete proof, we note that in the MSMCKM, CP(T)-violation is caused by mixing among the three generations of quarks. A complex phase, δ, in the CKM-matrix provides a natural mechanism for the small, but nonzero violation, of CP conservation. The T (or CP) violating components of the Lagrangian are contained in the expression,

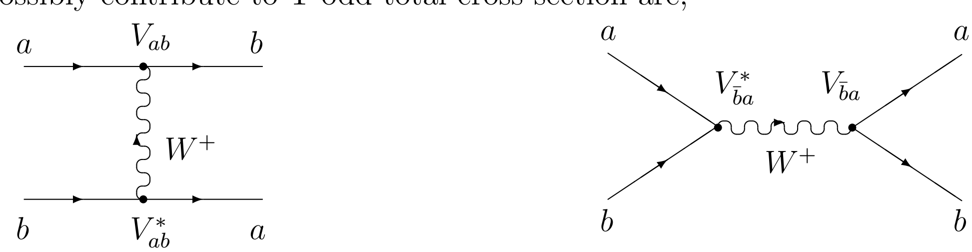
$$\mathcal{L} = \frac{g}{\sqrt{2}}(J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu}) \ , \tag{5}$$

where g is the coupling constant, $W_\mu^\pm$ are the charged vector bosons, and $J_\mu^\pm$ are the $SU(2)$ fermionic currents. These have the values;

$$\begin{aligned} g &= e\sin\theta_W \ , \qquad W_\mu^\pm = (A_\mu^1 \mp iA_\mu^2)/\sqrt{2} \ , \\ J_\mu^+ &= J_\mu^1 + iJ_\mu^2 = \frac{1}{2}\overline{U}\gamma_\mu(1-\gamma^5)VD \ , \qquad J_\mu^- = J_\mu^{+\dagger} \ . \end{aligned} \tag{6}$$

Here V is the CKM mixing matrix, θ_W is the Weinberg angle, and U and D are quark triplets (u, c, t) and (d, s, b), respectively. Based on eqs. (5) and (6), only the scatterings of quarks can possibly introduce T (or CP) vioation components.

At tree level, the Feynman Diagrams of the forward scattering amplitude which could possibly contribute to T-odd total cross section are,



(i) (ii)

Fig. 1: The forward scattering amplitudes $\mathcal{M}(ab \longrightarrow ab)$ at tree level that could possibly contribute to T-odd total cross section.

The forward scattering amplitude for Fig. 1(i) ($\mathcal{M}_{1i}$) and Fig. 1(ii) ($\mathcal{M}_{1ii}$) in

Feynman-'t Hooft gauge are given by,

$$\begin{aligned}
\mathcal{M}_{1i} &= \frac{-g^2}{8}\frac{1}{k_i^2 - M_W^2 + i\epsilon} V_{ab}V_{ab}^* \cdot [\overline{u}_b\,\gamma^\mu(1-\gamma^5)\,u_a][\overline{u}_a\,\gamma_\mu(1-\gamma^5)\,u_b]\,; \\
\mathcal{M}_{1ii} &= \frac{-g^2}{8}\frac{1}{k_{ii}^2 - M_W^2 + i\epsilon} V_{\bar{b}a}^* V_{\bar{b}a} \cdot [\overline{u}_{\bar{b}}\,\gamma^\mu(1-\gamma^5)\,u_a][\overline{u}_a\,\gamma_\mu(1-\gamma^5)\,u_{\bar{b}}]\,; \qquad (7)
\end{aligned}$$

with $k_i^2 = (p_a - p_b)^2$ and $k_{ii}^2 = (p_a + p_b)^2$.

From the T-even condition, $\mathcal{M}^T = \mathcal{M}^\dagger$, one can obtain the T-even and T-odd amplitudes as;

$$\begin{aligned}
\mathcal{M}_{1i}^{even} &= \frac{-g^2}{8}\frac{1}{k_a^2 - M_W^2 + i\epsilon} Re(V_{ab}V_{ab}^*)[\overline{u}_b\,\gamma^\mu(1-\gamma^5)\,u_a][\overline{u}_a\,\gamma_\mu(1-\gamma^5)\,u_b]\,; \\
\mathcal{M}_{1ii}^{even} &= \frac{-g^2}{8}\frac{1}{k_b^2 - M_W^2 + i\epsilon} Re(V_{\bar{b}a}^* V_{\bar{b}a})[\overline{u}_{\bar{b}}\,\gamma^\mu(1-\gamma^5)\,u_a][\overline{u}_a\,\gamma_\mu(1-\gamma^5)\,u_{\bar{b}}]\,; \\
\mathcal{M}_{1i}^{odd} &= \frac{-g^2}{8}\frac{1}{k_a^2 - M_W^2 + i\epsilon} iIm(V_{ab}V_{ab}^*)[\overline{u}_b\,\gamma^\mu(1-\gamma^5)\,u_a][\overline{u}_a\,\gamma_\mu(1-\gamma^5)\,u_b]\,; \\
\mathcal{M}_{1ii}^{odd} &= \frac{-g^2}{8}\frac{1}{k_b^2 - M_W^2 + i\epsilon} iIm(V_{\bar{b}a}^* V_{\bar{b}a})[\overline{u}_{\bar{b}}\,\gamma^\mu(1-\gamma^5)\,u_a][\overline{u}_a\,\gamma_\mu(1-\gamma^5)\,u_{\bar{b}}]\,.
\end{aligned} \qquad (8)$$

Since $Im(V_{ab}V_{ab}^*) = 0$ and $Im(V_{\bar{b}a}^* V_{\bar{b}a}) = 0$, the T-odd amplitudes are zero at tree level.

At one-loop level, the Feynman diagrams of the forward scattering amplitudes which could possibly contribute to T-odd total cross section are the following:

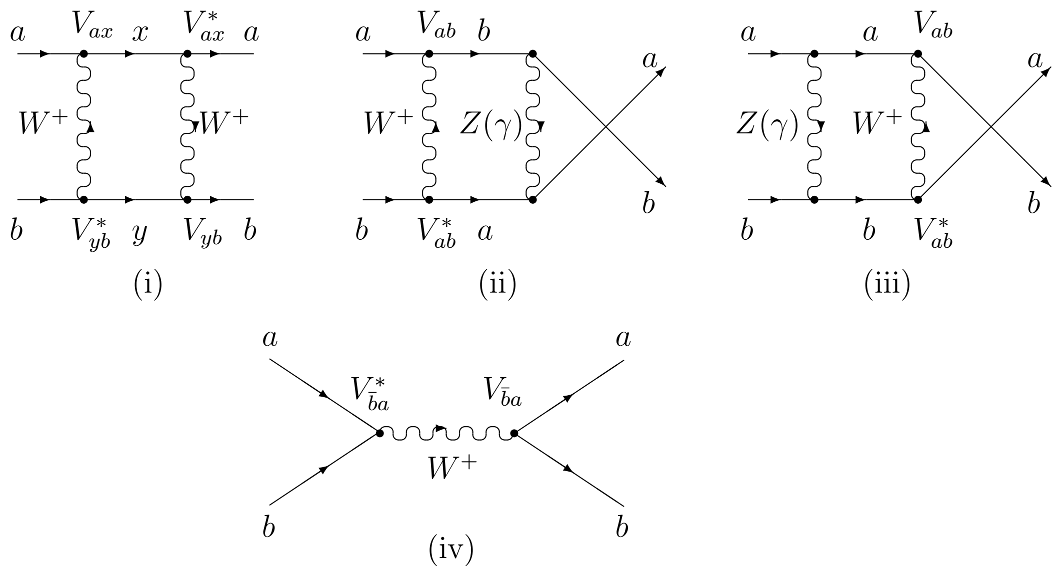


Fig. 2: The forward scattering amplitudes $\mathcal{M}(ab \longrightarrow ab)$ at one-loop level that could possibly contribute to T-odd total cross section.

Since

$$[(\bar{u}_c\gamma^\mu(1-\gamma^5)u_a)(\bar{u}_d\gamma^\mu(1-\gamma^5)u_b)]^T = [(\bar{u}_c\gamma^\mu(1-\gamma^5)u_a)(\bar{u}_d\gamma^\mu(1-\gamma^5)u_b)]^\dagger \tag{9}$$

the only factor that determines T-odd or T-even is the CKM-matrix elements in the amplitudes. The possible T-odd factors in Fig. 2(ii, iii) are the same as the factors in Fig. 1 and give zero T-odd amplitudes, *i.e.* the vertices that do not include CKM-matrix elements do not introduce T-odd factors. For Fig. 2(i), the multiplication of the matrix elements is given by

$$V_{xa}V^*_{xa}V^*_{by}V_{by} = |V_{xa}V_{by}|^2 = real \;\; and \;\; therefore \;\; Im(V_{xa}V^*_{xa}V^*_{by}V_{by}) = 0\,. \tag{10}$$

For Fig. 2(iv), the multiplication of the matrix elements is given by

$$V_{\bar{b}a}V^*_{\bar{y}x}V_{\bar{y}x}V^*_{\bar{b}a} = |V_{\bar{b}a}V_{\bar{y}x}|^2 = real \;\; and \;\; therefore \;\; Im(V_{\bar{b}a}V^*_{\bar{y}x}V_{\bar{y}x}V^*_{\bar{b}a}) = 0\,. \tag{11}$$

Therefore, at one-loop level, T-odd amplitude is zero.

For arbitrary n-th order, the forward scattering $a + b \to a + b$ would go through combinations of the following processes as shown in Figs. 3 and Fig. 4, depending on the particles a and b. [11]

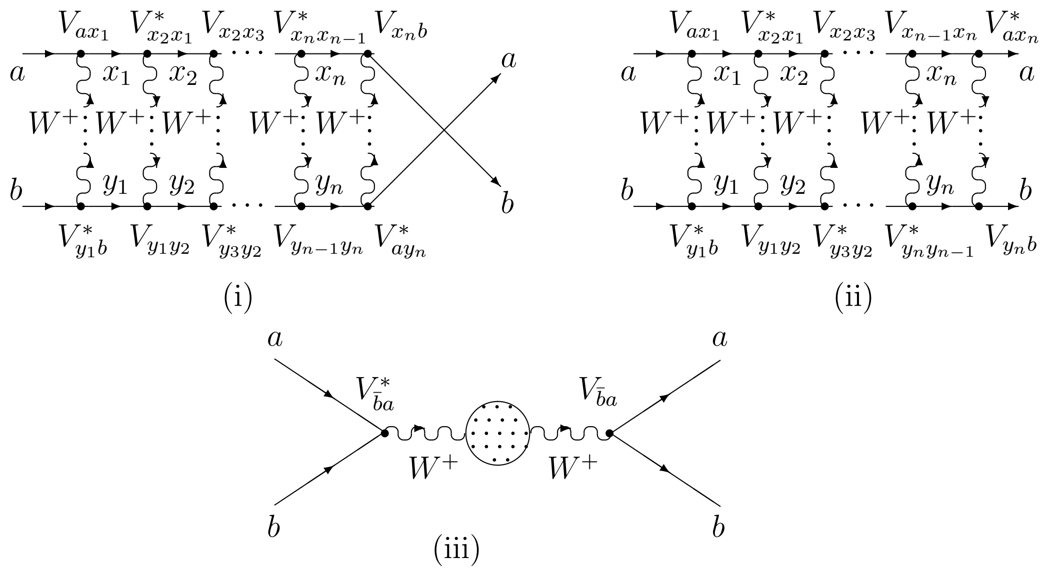


Fig. 3: The forward elastic scattering amplitudes $\mathcal{M}(ab \longrightarrow ab)$ at n-th order that could possibly contribute to T-odd total cross section.

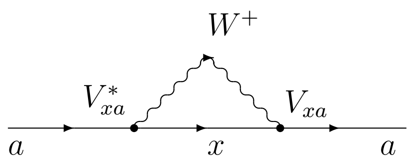


Fig. 4: Quark self-mass diagram.

If a and b belong to the $U(D)$ and $D(U)$ sectors respectively, the forward scattering goes through the processes shown in Figs. 3(a) and (b), but if a and b belong to $U(\bar{D})$ and $\bar{D}(U)$ sectors respectively or $\bar{U}(D)$ and $D(\bar{U})$ sectors respectively, the forward scattering goes through the processes shown in Figs. 3(c).

We prove in the following that the multiplication of CKM matrix elements in each diagram in Figs. 3 and 4 is real; and since an arbitrary T-odd amplitude must be a combination of Figs. 3(i, ii, iii) and 4, the T-odd forward scattering amplitude for an arbitrary order is zero.

Fig. 4 is T-even as is obvious since the CKM-matrix contribution $V_{xa}V^*_{xa}$ is real. Without losing generality, let a be in the D sector and b in the U sector. The n-th order forward scattering amplitude could go through Figs. 3(i) if n is an odd number and Figs. 3(ii) if n is an even number.

The amplitude of Fig. 3(i) has the form

$$\begin{aligned}
\mathcal{M}_{3i}(0) &\propto \mathcal{M}_{3i,1}(0)\mathcal{M}_{3i,2}(0) \quad with \\
\mathcal{M}_{3i,1}(0) &\propto (V_{x_n b}\bar{u}_b\gamma^{\mu_{n+1}}(1-\gamma^5)u_{x_n}W^+_{\mu_{n+1}})(V^*_{x_n x_{n-1}}\bar{u}_{x_n}\gamma^{\mu_n}(1-\gamma^5)u_{x_{n-1}}W^{+\dagger}_{\mu_n}) \\
&\quad (V_{x_{n-2}x_{n-1}}\bar{u}_{x_{n-1}}\gamma^{\mu_{n-1}}(1-\gamma^5)u_{x_{n-2}}W^+\mu_{n-1})\cdots \\
&\quad (V_{x_2x_3}\bar{u}_{x_3}\gamma^{\mu_3}(1-\gamma^5)u_{x_2}W^+_{\mu_3})(V^*_{x_2x_1}\bar{u}_{x_2}\gamma^{\mu_2}(1-\gamma^5)u_{x_1}W^{+\dagger}_{\mu_2}) \\
&\quad (V_{ax_1}\bar{u}_{x_1}\gamma^{\mu_1}(1-\gamma^5)u_aW^+_{\mu_1}) \\
\mathcal{M}_{3i,2}(0) &\propto (V^*_{ay_n}\bar{u}_a\gamma^{\mu_{n+1}}(1-\gamma^5)u_{y_n}W^{+\dagger}_{\mu_{n+1}})(V_{y_{n-1}y_n}\bar{u}_{y_n}\gamma^{\mu_n}(1-\gamma^5)u_{y_{n-1}}W^+_{\mu_n}) \\
&\quad (V_{y_{n-1}y^*_{n-2}}\bar{u}_{y_{n-1}}\gamma^{\mu_{n-1}}(1-\gamma^5)u_{y_{n-2}}W^{+\dagger}_{\mu_{n-1}})\cdots \\
&\quad (V^*_{y_3y_2}\bar{u}_{y_3}\gamma^{\mu_3}(1-\gamma^5)u_{y_2}W^{+\dagger}_{\mu_3})(V_{y_1y_2}\bar{u}_{y_2}\gamma^{\mu_2}(1-\gamma^5)u_{y_1}W^+_{\mu_2}) \\
&\quad (V^*_{y_1b}\bar{u}_{y_1}\gamma^{\mu_1}(1-\gamma^5)u_bW^{+\dagger}_{\mu_1})\,, \qquad (12)
\end{aligned}$$

where the repeated indices x_i and y_i should be summed over the particles in the corresponding quark sectors and the corresponding momenta of the particles should be integrated based on conservation of momenta.

Based on the MSMCKM Lagrangian, x_i and y_{n-i+1} should be in the same quark sector.[11] It is obvious that

$$\mathcal{M}_{3i,1}(0) = \mathcal{M}^\dagger_{3i,2}(0) = \mathcal{M}^*_{3i,2}(0) \text{ and } \mathcal{M}_{3i,1}(0)\mathcal{M}_{3i,2}(0) = \text{real}\,. \qquad (13)$$

Therefore, the T-odd amplitude from Fig. 3(i) is zero.

The amplitude of Fig. 3(ii) has the form

$$\begin{aligned}
\mathcal{M}_{3ii}(0) &\propto \mathcal{M}_{3ii,1}(0)\mathcal{M}_{3ii,2}(0) \quad with \\
\mathcal{M}_{3ii,1}(0) &\propto (V^*_{ax_n}\bar{u}_a\gamma^{\mu_{n+1}}(1-\gamma^5)u_{x_n}W^{+\dagger}_{\mu_{n+1}})(V_{x_{n-1}x_n}\bar{u}_{x_n}\gamma^{\mu_n}(1-\gamma^5)u_{x_{n-1}}W^+_{\mu_n}) \\
&\quad (V^*_{x_{n-1}x_{n-2}}\bar{u}_{x_{n-1}}\gamma^{\mu_{n-1}}(1-\gamma^5)u_{x_{n-2}}W^{+\dagger}_{\mu_{n-1}})\cdots \\
&\quad (V_{x_2x_3}\bar{u}_{x_3}\gamma^{\mu_3}(1-\gamma^5)u_{x_2}W^+_{\mu_3})(V^*_{x_2x_1}\bar{u}_2\gamma^{\mu_2}(1-\gamma^5)u_{x_1}W^{+\dagger}_{\mu_2}) \\
&\quad (V_{ax_1}\bar{u}_1\gamma^{\mu_1}(1-\gamma^5)u_aW^+_{\mu_1}) \\
\mathcal{M}_{3ii,2}(0) &\propto (V_{y_nb}\bar{u}_b\gamma^{\mu_{n+1}}(1-\gamma^5)u_{y_n}W^+_{\mu_{n+1}})(V^*_{y_ny_{n-1}}\bar{u}_{y_n}\gamma^{\mu_n}(1-\gamma^5)u_{y_{n-1}}W^{+\dagger}_{\mu_n}) \\
&\quad (V_{y_{n-2}y_{n-1}}\bar{u}_{y_{n-1}}\gamma^{\mu_{n-1}}(1-\gamma^5)u_{y_{n-2}}W^+_{\mu_{n-1}})\cdots \\
&\quad (V^*_{y_3y_2}\bar{u}_{y_3}\gamma^{\mu_3}(1-\gamma^5)u_{y_2}W^{+\dagger}_{\mu_3})(V_{y_1y_2}\bar{u}_{y_2}\gamma^{\mu_2}(1-\gamma^5)u_{y_1}W^+_{\mu_2}) \\
&\quad (V^*_{y_1b}\bar{u}_{y_1}\gamma^{\mu_1}(1-\gamma^5)u_bW^{+\dagger}_{\mu_1})\,, \qquad (14)
\end{aligned}$$

where the repeated indices x_i and y_i should be summed over the particles in the corresponding quark sectors and the corresponding momenta of the particles should be integrated based on conservation of momenta.

Also, based on the MSMCKM Lagrangian, $x_i(y_i)$ and $x_{n-i+1}(y_{n-i+1})$ should be in the same quark sector[11] and one should have,

$$\mathcal{M}_{3ii,l}(0) = \mathcal{M}^{\dagger}_{3ii,l}(0) = \mathcal{M}^{*}_{3ii,l}(0) \ = real \ \ l = 1, 2 \,. \tag{15}$$

Therefore the T-odd amplitude from Fig. 3(ii) is zero.

In Fig. 3(iii), a W^+ propagator placed after the annihilation of incoming quarks a and b could create a quark pair in U (D) and $\bar{D}$ ($\bar{U}$) sectors, introducing a possible T-odd contribution. The U (D) and $\bar{D}$ ($\bar{U}$) pair thus created, will go through the processes of either Fig. 3(i) or Fig. 3(ii) before the final quark pair in the process is annihilated. This again creates a W^+ propagator. [12] Therefore, Fig. 3(iii) can be broken down to a combination of Fig. 3(i) or Fig. 3(ii), a smaller part of the form of Fig. 3(iii), and Fig. 4. The smaller part of Fig. 3(iii) can be continuously divided into a combination of the smaller parts of Fig. 3(i) or Fig. 3(ii), and an even smaller part of the form of Fig. 3(iii), and Fig. 4. If this division is continued, one can eventually break Fig. 3(iii) into a combination of several components of Fig. 3(i) or Fig. 3(ii), Fig. 2(iv), and Fig. 4. As shown above, all these contributions from Fig. 3(i), Fig. 3(ii), Fig. 2(iv), and Fig. 4 do not contribute to T-odd amplitudes. Therefore, the T-odd amplitude from Fig. 3(iii) is zero.

Since an arbitrary possible T-odd forward scattering amplitude can be obtained from combinations of Fig. 3(i) or Fig. 3(ii), Fig. 3(iii) and Fig. 4, we conclude that T-odd forward scattering amplitude of a polarized reaction $1/2 + 1/2 \to 1/2 + 1/2$ within the MSMCKM is identically zero to all orders. This implies that the T-odd total cross section of a polarized reaction $1/2+1/2 \to 1/2+1/2$ is zero to all orders, by the optical theorem.

Because both T-odd/P-even and T-odd/P-odd amplitudes have the same CKM-matrix factors, this indicates that both T-odd/P-even and T-odd/P-odd amplitudes should vanish to all order. We also note that T-odd/P-even amplitude should be zero based on eq. 2, and the zero T-odd/P-odd amplitude is due to the fact that the source of T-odd amplitude in the MSMCKM is introduced by the phase in the CKM matrix. The proof is therefore completed.

Two points need to be re-stated.

(1) The above conclusion shows that a non-zero T-odd total cross section of a polarized reaction $1/2 + 1/2 \to 1/2 + 1/2$ indicates the existance of additional T (or CP) violation source(s) besides the phase in the CKM matrix. Furthermore, it is a null test in which a high experimental accuracy can be achieved.

(2) A zero T-odd total cross section does not indicate T (or CP) conservation in the physical process, i.e. a T (or CP) violation in a physical process is a necessary but not sufficient condition for an existance of a T-odd total cross section.

Ref. [1] showed that the T-odd/P-odd total cross section of a $1/2 + 1/2 \to 1/2 + 1/2$ reaction could be none-zero if the Higgs sector contributes to T (or CP) violation. Thus, a measurement of a T-odd/P-odd total cross section, proportional to the forward scattering amplitude, would indicate an additional mechanism(s) of T (or CP) violation. However, there could be other stronger interactions in a reaction and we would like to study the effects on the T-odd/P-odd total cross section measurement due to strong and electric interactions next.

3 The Influences to T-odd, P-odd Total Cross Section Due to Strong and Electric Interactions

The discussion in sction 2 tells us that a non-zero tranversely polarized $1/2+1/2 \to 1/2 + 1/2$ forward scattering would indicate additional mechanisms of T (or CP) violation. Therefore, it is worthwhile to carry out this experiment in the near future. However, for most of the choices of beams and targets, e.g. hadrons 1 collides with hadrons 2, one would deal the situation that there are interactions other than weak interaction in a system and the effects due to weak interaction could be influenced or even simulated by stronger interactions, e.g. see [5]. Therefore, to carry out an accurate tests of T (or CP) violation in $1/2+1/2 \to 1/2+1/2$ scattering, one should get rid of the influences due to stronger interactions or have them under control. We would like to investigate the possibility to get rid of the effects due to stronger interactions or have them under control in this section.

Our followng discussion references to [15]. We will treat stronger interactions exactly without worrying about the so-called perturbative or non-perturbative effects. The weak interaction can be treated perturbatively.

For a collision between two hadrons, the interaction between two hadrons is:

$$H_{int} = H_w + H_{st}, \tag{16}$$

where H_w is the weak interaction and H_{st} includes both electric and strong interactions.

As we know today, hadrons are not elementary particles but are bound states of quarks. Let the state vector before the collision is ψ_i and the state vector after the collision is ψ_f, where ψ_i represents the initial bound hadron 1 and bound hadron 2, and ψ_f represents the bound hadrons in the final state. ψ_i and ψ_f should satisfy the equations

$$(K + V_i)|\psi_i\rangle = E_i|\psi_i\rangle\,, \;\; (K + V_f)|\psi_f\rangle = E_f|\psi_f\rangle\,, \tag{17}$$

where K may be the kinetic energy or the kinetic energy plus the potential energy between a pair of particles, and the contributions to V_i and V_f should be from H_{st} although we do not know their exact forms.

The complete state vector of the system, $\Psi_i^{(\epsilon)}$, can be seen from the physical boundary conditions to be the solution of the integral equation

$$|\Psi_i^{(\epsilon)}\rangle = |\psi_i\rangle + \frac{1}{E_i - K - V_i + i\epsilon}(H_w + H_{st} + V_f)|\Psi_i^{(\epsilon)}\rangle \,. \tag{18}$$

By algebraic manipulation, (eq. 18) can be rewritten as

$$\begin{aligned} |\Psi_i^{(\epsilon)}\rangle &= |\psi_i\rangle + \frac{1}{E_i - K - V_i - V_f - H_w - H_{st} + i\epsilon}(H_w + H_{st} + V_f)|\psi_i\rangle \\ &= \frac{1}{E_i - K - V_f + i\epsilon}(H_{st} + H_w + V_i)|\Psi_i^{(\epsilon)}\rangle. \end{aligned} \tag{19}$$

The rate of transition into the final state ψ_f is given by

$$N_i \dot{w}_{fi} = \frac{\partial}{\partial t}|\langle\psi_f| \exp\left[i(E_f - K - V_i - V_f - H_w - H_{st})t\right]|\Psi^{(\epsilon)}\rangle|^2, \tag{20}$$

where

$$N_i = \langle\psi_i(t)|\psi_i(t)\rangle \tag{21}$$

For the transition rate we need only compute $N_i\dot{w}_{fi}$ at $t = 0$. We find that

$$N_i\dot{w}_{fi}(t=0) = -i\langle\psi_f|(V_i + H_w + H_{st})|\Psi^{(\epsilon)}\rangle\langle\psi_f|\Psi^{(\epsilon)}\rangle^* + c.c., \tag{22}$$

where we have used (eq. 17).

Using (eq. 19), one has:

$$
\begin{aligned}
& \langle\psi_f|\Psi^{(\epsilon)}\rangle \\
= & \langle\psi_f|\psi_i\rangle + \langle\psi_f|\frac{1}{E_i - K - V_i - V_f - H_{st} - H_w + i\epsilon}(H_{st} + H_w + V_f)|\psi_i\rangle \\
= & \langle\psi_f|\psi_i\rangle + \langle\psi_f|\frac{1}{E_i - K - V_f + i\epsilon}(H_{st} + H_w + V_f)|\psi_i\rangle + \\
& \langle\psi_f|\frac{1}{E_i - K - V_f + i\epsilon}(H_{st} + H_w + V_i)\frac{1}{E_i - K - V_i - V_f - H_{st} - H_w + i\epsilon}(\\
& H_{st} + H_w + V_f)|\psi_i\rangle \\
= & \langle\psi_f|\psi_i\rangle + \frac{1}{E_i - E_f + i\epsilon}\langle\psi_f|(H_{st} + H_w + V_f)|\psi_i\rangle + \frac{1}{E_i - E_f + i\epsilon}\langle\psi_f|(H_{st} + \\
& + H_w + V_i)\frac{1}{E_i - K - V_i - V_f - H_{st} - H_w + i\epsilon}(H_{st} + H_w + V_f)|\psi_i\rangle \\
= & \langle\psi_f|\psi_i\rangle + \frac{1}{E_i - E_f + i\epsilon}\langle\psi_f|(H_{st} + \\
& H_w + V_f)|\psi_i\rangle + \frac{1}{E_i - E_f + i\epsilon}\langle\psi_f|(H_{st} + H_w + V_i)(|\Psi_i^{(\epsilon)}\rangle - |\psi_i\rangle) \\
= & \langle\psi_f|\psi_i\rangle + \frac{1}{E_i - E_f + i\epsilon}\langle\psi_f|(V_f - V_i)|\psi_i\rangle + \\
& \frac{1}{E_i - E_f + i\epsilon}\langle\psi_f|(H_{st} + H_w + V_i)|\Psi_i^{(\epsilon)}\rangle. \qquad (23)
\end{aligned}
$$

Using (eq. 17), one has:

$$\langle\psi_f|(V_f - V_i)|\psi_i\rangle = (E_f - E_i)\langle\psi_f|\psi_i\rangle + -(\langle K\psi_f|\psi_i\rangle - \langle|K\psi_i\rangle). \qquad (24)$$

The first term in (eq. 23) cancels out due to energy conservation and the second term in (eq. 23) reduces to a surface integral which vanishes in the limit of infinite quantization volume. Thus

$$\langle\psi_f|\Psi_i^{(\epsilon)}\rangle = \frac{1}{E_i - E_f + i\epsilon}\langle\psi_f|(H_{st} + H_w + V_i)|\Psi_i^{(\epsilon)}\rangle. \qquad (25)$$

where we have ignored the term $\langle\psi_f|\psi_i\rangle = 0$, which is the term without scatterings.

Substituting (eq. 25) back to (eq. 22)), we find that

$$N_i\dot{w}_{fi} = 2\pi\delta(E_i - E_f)|\langle\psi_f|(H_{st} + H_w + V_i)|\Psi_i^{(\epsilon)}\rangle|^2 \qquad (26)$$

Let us introduce state vectors $|\chi_f^{(-)}\rangle$ which are the solutions of $|\psi_f\rangle$ with $H_w = 0$ defined by

$$|\chi_f^{(-)}\rangle = |\psi_f\rangle + \frac{1}{E_f - K - V_f - i\epsilon}(H_{st} + V_i)|\chi_f^{(-)}\rangle, \tag{27}$$

i.e. $|\chi_f^{(-)}\rangle$ is the scattering due to stronger interactions only.

With (eq. 27), one can have:

$$\begin{aligned} & \langle\psi_f|(H_{st} + H_w + V_i)|\Psi_i^{(\epsilon)}\rangle \\ = & \langle\chi_f^{(-)}|(H_{st} + H_w + V_i)|\Psi_i^{(\epsilon)}\rangle - \\ & \langle\chi_f^{(-)}|(H_{st} + V_i)\frac{1}{E_f - K - V_f + i\epsilon}(H_{st} + H_w + V_i)|\Psi_i^{(\epsilon)}\rangle \\ = & \langle\chi_f^{(-)}|(H_{st} + H_w + V_i)|\Psi_i^{(\epsilon)}\rangle - \langle\chi_f^{(-)}|(H_{st} + V_i)|\Psi_i^{(\epsilon)}\rangle \\ = & \langle\chi_f^{(-)}|H_w|\Psi_i^{(\epsilon)}\rangle. \end{aligned} \tag{28}$$

With (eq. 28), (eq. 26) can be written as:

$$N_i\dot{w}_{fi} = 2\pi\delta(E_i - E_f)|\langle\chi_f^{(-)}|H_w|\Psi_i^{(\epsilon)}\rangle|^2 \tag{29}$$

If we define $|\chi_i^{(+)}\rangle$ as

$$|\chi_i^{(+)}\rangle = |\psi_i\rangle + \frac{1}{E_i - K - V_i + i\epsilon}(H_{st} + V_f)|\chi_i^{(+)}\rangle, \tag{30}$$

i.e. $|\chi_i^{(+)}\rangle$ is the scattering due to strong interactions only, one would have

$$|\Psi_i^{(\epsilon)}\rangle = |\chi_i^{(+)}\rangle + \frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon}H_w|\Psi_i^{(\epsilon)}\rangle \tag{31}$$

Using the convenient Möller wave operators, $\Omega^{(\pm)}$, (eq. 27) can be wriiten as

$$|\chi_f^{(-)}\rangle = \Omega_1^{(-)}(H_{st}, V_i, V_f)|\psi_f\rangle, \tag{32}$$

with $\Omega_1^{-}(H_{st}, V_i, V_f)$ satisfies the operator integral equation

$$\Omega_1^{(-)}(H_{st}, V_i, V_f) = 1 + \frac{1}{E_f - K - V_f - i\epsilon}(H_{st} + V_i)\Omega_1^{(-)}(H_{st}, V_i, V_f); \tag{33}$$

(Eq. 30) can be written as

$$|\chi_i^{(+)}\rangle = \Omega_2^{(+)}(H_{st}, V_i, V_f)|\psi_f\rangle \tag{34}$$

with $\Omega_2^{(+)}(H_{st}, V_i, V_f)$

$$\Omega_2^{(+)}(H_{st}, V_i, V_f) = 1 + \frac{1}{E_i - K - V_i + i\epsilon}(H_{st} + V_f)\Omega_2^{(+)}(H_{st}, V_i, V_f); \tag{35}$$

and (eq. 31) can be written as

$$|\Psi_i^{(\epsilon)}\rangle = \Omega^{(+)}(H_{st}, H_w, V_i, V_f)|\chi_i^{(+)}\rangle \tag{36}$$

with $\Omega^{(+)}(H_{st}, H_w, V_i, V_f)$ satisfies the operator integral equation

$$\begin{aligned} & \Omega^{(+)}(H_{st}, H_w, V_i, V_f) \\ = \; & 1 + \frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon} H_w \Omega^{(+)}(H_{st}, H_w, V_i, V_f). \end{aligned} \tag{37}$$

Since weak interaction H_w is weak, (eq. 37) would have the iterated solution

$$\begin{aligned} \Omega^{(+)}(H_{st}, H_w, V_i, V_f) = 1 + \frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon} H_w + \\ \frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon} H_w \frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon} H_w + \cdots, \end{aligned} \tag{38}$$

which is the usual perturbative expansion for weak interaction.

Using the Möller wave operators, (eq. 29) can be rewritten as:

$$\begin{aligned} N_i \dot{w}_{fi} = \; & 2\pi\delta(E_i - E_f)|\langle\psi_f|\Omega_1^{(-)\dagger}(H_{st}, V_i, V_f) H_w \Omega^{(+)}(H_{st}, H_w, V_i, V_f)\Omega_2^{(+)}(\\ & H_{st}, V_i, V_f)|\psi_i\rangle|^2 \end{aligned} \tag{39}$$

From (eq. 39), the scattering amplitute is given by

$$\begin{aligned} & R_{fi} \\ = \; & \langle\psi_f|\Omega_1^{(-)\dagger}(H_{st}, V_i, V_f) H_w \Omega^{(+)}(H_{st}, H_w, V_i, V_f)\Omega_2^{(+)}(H_{st}, V_i, V_f)|\psi_i\rangle \\ = \; & \langle\psi_f|\Omega_1^{(-)\dagger}(H_{st}, V_i, V_f) H_w \Omega_2^{(+)}(H_{st}, V_i, V_f)|\psi_i\rangle + \\ & \langle\psi_f|\Omega_1^{(-)\dagger}(H_{st}, V_i, V_f) H_w \frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon} H_w \Omega_2^{(+)}(H_{st}, V_i, V_f)|\psi_i\rangle + \\ & \langle\psi_f|\Omega_1^{(-)\dagger}(H_{st}, V_i, V_f) H_w \frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon} H_w \\ & \frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon} H_w \Omega_2^{(+)}(H_{st}, V_i, V_f)|\psi_i\rangle + \cdots. \end{aligned} \tag{40}$$

Expanding $|\psi_i\rangle$ and $|\psi_f\rangle$ in terms of the complete set of states $|JMw\rangle$ with total angular momentum J, a definite z component of J, $J_z = M$, a definite parity w, we obtain

$$
\begin{aligned}
|\chi_f^{(-)}\rangle &= \Omega_1^{(-)}(H_{st}, V_i, V_f)|\psi_f\rangle \\
&= \sum_{J_f, M_f, w_f} \exp\left(-i\delta_{J_f w_f}\right)|J_f M_f w_f\rangle\langle J_f M_f w_f|\psi\rangle \\
|\chi_i^{(+)}\rangle &= \Omega_2^{(+)}(H_{st}, V_i, V_f)|\psi_i\rangle \\
&= \sum_{J_i, M_i, w_i} \exp\left(i\delta_{J_i w_i}\right)|J_i M_i w_i\rangle\langle J_i M_i w_i|\psi\rangle,
\end{aligned}
\tag{41}
$$

where we have used the equations

$$
\begin{aligned}
\Omega_1^{(-)}(H_{st}, V_i, V_f)|J_f M_f w_f\rangle &= \exp\left(-i\delta_{J_f w_f}\right)|J_f M_f w_f\rangle \\
\Omega_2^{(+)}(H_{st}, V_i, V_f)|J_i M_i w_i\rangle &= \exp\left(-i\delta'_{J_i w_i}\right)|J_i M_i w_i\rangle,
\end{aligned}
\tag{42}
$$

which is a consequence of $\Omega_1^{(-)}$ and $\Omega_2^{(+)}$ being unitary, rotationally invariant, and J, M, w are good quantum numbers of stronger interactions.

We would like to point out that $\delta_{J,w}$ and $\delta'_{J,w}$ are different angles. However, for elastic forward scattering, one would have $V_i = V_f$ and $\Omega_1^{(+)}(H_{st}, V_i, V_f) = \Omega_2^{(+)}(H_{st}, V_i, V_f)$. Therefore, $\delta_{J,w} = \delta'_{J,w}$. Since we mainly study elastic forward scattering here, we will not distinguish $\delta_{J,w}$ and $\delta'_{J,w}$ in the following.

Substituting (eq. 41) into (eq. 40), one has

$$
\begin{aligned}
R_{fi} = &\sum_{J_f, M_f, w_f, J_i, M_i, w_i} \exp\left[i(\delta_{J_f w_f} - \delta_{J_i w_i})\right]\langle\psi_f|J_f M_f w_f\rangle\langle J_f M_f w_f|H_w(1 + \\
&\frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon} + \cdots)H_w|J_i M_i w_i\rangle\langle J_i M_i w_i|\psi_i\rangle.
\end{aligned}
\tag{43}
$$

Because of the term $\exp\left[i(\delta_{J_f w_f} - \delta_{J_i w_i})\right]$ originated from stronger interactions, the amplitude for T-odd elastic forward scattering would consist of contributions from $\cos(\delta_{J_f w_f} - \delta_{J_i w_i})R_{ii}^{T-odd,P-odd}$ and $\sin(\delta_{J_f w_f} - \delta_{J_i w_i})R_{ii}^{T_even,P-odd}$, i.e. the $A_{x,y}$ in the previous section is given by

$$
A_{x,y} \propto \cos(\delta_{J_f w_f} - \delta_{J_i w_i})R_{ii}^{T-odd,P-odd} + \sin(\delta_{J_f w_f} - \delta_{J_i w_i})R_{ii}^{Teven,P-odd},
\tag{44}
$$

In doing this, we have used the fact that $V_i = V_f$ and $\Omega_1^{(-)\dagger} = \Omega_2^{(+)}$ for forward elastic scattering. However, we would like to show that, while this could be an issue for reactions of individual channels, it is not a problem for total cross section measurements.

The optical theorm tells us that, [15]

$$
\begin{aligned}
\sigma_{tot} = \sum_{f\neq i}\sigma_{fi} = -\frac{2}{v} \lim_{\substack{\epsilon \longrightarrow 0^+ \\ L \longrightarrow \infty}} ImR_{ii}
\end{aligned}
$$

$$
\begin{aligned}
= & -\frac{2}{v} \lim_{\substack{\epsilon \longrightarrow 0^+ \\ L \longrightarrow \infty}} L^3 \sum_{\substack{J_f, M_f, w_f \\ J_i, M_i, w_i}} \{ \\
& -i\cos(\delta_{J_f w_f} - \delta_{J_i w_i})[\langle\psi_i|J_f M_f w_f\rangle\langle J_f M_f w_f|H_w(1 + \\
& \frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon} H_w \cdots)|J_i M_i w_i\rangle\langle J_i M_i w_i|\psi_i\rangle - \\
& \langle\psi_i|J_f M_f w_f\rangle\langle J_f M_f w_f|H_w(1 + \\
& \frac{1}{E_i - K - H_{st} - V_i - V_f - i\epsilon} H_w \cdots)|J_i M_i w_i\rangle\langle J_i M_i w_i|\psi_i\rangle + \\
& \sin(\delta_{J_f w_f} - \delta_{J_i w_i})[\langle\psi_i|J_f M_f w_f\rangle\langle J_f M_f w_f|H_w(\\
& \frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon} H_w + \cdots)|J_i M_i w_i\rangle\langle J_i M_i w_i|\psi_i\rangle + \\
& \langle\psi_i|J_i M_i w_i\rangle\langle J_i M_i w_i|H_w(1 + \\
& \frac{1}{E_i - K - H_{st} - V_i - V_f - i\epsilon} H_w + \cdots)|J_f M_f w_f\rangle\langle J_f M_f w_f|\psi_i\rangle]\} \\
= & -\frac{2}{v} \lim_{\substack{\epsilon \longrightarrow 0^+ \\ L \longrightarrow \infty}} L^3 \sum_{\substack{J_f, M_f, w_f \\ J_i, M_i, w_i}} \{ \\
& -i\cos(\delta_{J_f w_f} - \delta_{J_i w_i})[\langle\psi_i|J_f M_f w_f\rangle\langle J_f M_f w_f|H_w(1 + \\
& \frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon} H_w + \cdots)|J_i M_i w_i\rangle\langle J_i M_i w_i|\psi_i\rangle - \\
& \langle\psi_i|J_f M_f w_f\rangle\langle J_f M_f w_f|H_w(1 + \\
& \frac{1}{E_i - K - H_{st} - V_i - V_f - i\epsilon} H_w + \cdots)|J_i M_i w_i\rangle\langle J_i M_i w_i|\psi_i\rangle] + \\
& \sin(\delta_{J_f w_f} - \delta_{J_i w_i})[\langle\psi_i|J_f M_f w_f\rangle\langle J_f M_f w_f|H_w(1 + \\
& \frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon} H_w + \cdots)|J_i M_i w_i\rangle\langle J_i M_i w_i|\psi_i\rangle - \\
& \langle\psi_i|J_f M_f w_f\rangle\langle J_f M_f w_f|H_w(1 + \\
& \frac{1}{E_i - K - H_{st} - V_i - V_f - i\epsilon} H_w + \cdots)|J_i M_i w_i\rangle\langle J_i M_i w_i|\psi_i\rangle]\}, \qquad (45)
\end{aligned}
$$

where we have used the fact that $V_i = V_f$ for forward scattering amplitude R_{ii}.

As one can see from the above equation, both terms after $\cos(\delta_{J_f w_f} - \delta_{J_i w_i})$ and $\sin(\delta_{J_f w_f} - \delta_{J_i w_i})$ are equal. This term cannot be real and imaginary at the same time. Therefore, they cannot appear in the total cross section at the same time.

To find the spin correlation coefficient $A_{x,y}$ defined in Section 2 and in ref. [1], one has:

$$\begin{aligned}
A_{x,y} &= \frac{\sigma_{tot}(s_x, s_y) - \sigma_{tot}(s_x, -s_y)}{\sigma_{tot}(s_x, s_y) + \sigma_{tot}(s_x, -s_y)} \\
&\propto \sum_{\substack{J_f, M_f, w_f \\ J_i, M_i, w_i}} \{-i \cos(\delta_{J_f w_f} - \delta_{J_i w_i})[\langle \psi_i | J_f M_f w_f \rangle \langle J_f M_f w_f | H_w (\\
&\quad \frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon} - \frac{1}{E_i - K - H_{st} - V_i - V_f - i\epsilon} + \cdots \\
&\quad) H_w) | J_i M_i w_i \rangle \langle J_i M_i w_i | \psi_i \rangle | J_i M_i w_i \rangle \langle J_i M_i w_i | \psi_i \rangle]^{T-odd, P-odd} + \\
&\quad \sin(\delta_{J_f w_f} - \delta_{J_i w_i})[\langle \psi_i | J_f M_f w_f \rangle \langle J_f M_f w_f | H_w (\\
&\quad \frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon} - \frac{1}{E_i - K - H_{st} - V_i - V_f - i\epsilon} + \cdots \\
&\quad) H_w | J_i M_i w_i \rangle \langle J_i M_i w_i | \psi_i \rangle | J_i M_i w_i \rangle \langle J_i M_i w_i | \psi_i \rangle]^{Teven, R-odd} \}.
\end{aligned} \tag{46}$$

As it has been shown in ref. [1] that the T-odd, P-odd contribution would be zero if T-violation is only due to the phase in CMK matrix but not be zero if T-violation is due to spontaneous symmetry breaking. Therefore, if one of the mechanisms of T-violation is due to spontaneous symmetry breaking, the term with $\sin(\delta_{J_f w_f} - \delta_{J_i w_i})$ will be zero and stronger interactions will not have an effect on the measurement of $A_{x,y}$. The $A_{x,y}$ is given by

$$\begin{aligned}
A_{x,y} &= \frac{\sigma_{tot}(s_x, s_y) - \sigma_{tot}(s_x, -s_y)}{\sigma_{tot}(s_x, s_y) + \sigma_{tot}(s_x, -s_y)} \\
&\propto \sum_{\substack{J_f, M_f, w_f \\ J_i, M_i, w_i}} -i \cos(\delta_{J_f w_f} - \delta_{J_i w_i})[\langle \psi_i | J_f M_f w_f \rangle \langle J_f M_f w_f | H_w (\\
&\quad \frac{1}{E_i - K - H_{st} - V_i - V_f + i\epsilon} - \frac{1}{E_i - K - H_{st} - V_i - V_f - i\epsilon} + \cdots \\
&\quad) H_w | J_i M_i w_i \rangle \langle J_i M_i w_i | \psi_i \rangle]
\end{aligned} \tag{47}$$

4 Several T-Odd/P-Odd Processes

Based on the discussions in Sections 2 and 3, the total cross section of transversally polarized spin-1/2 particles gives a sensitive tests of mechanisms of T-violation. The next questions are how large the signals one could expect and what the optimize conditions for possible experimental measurements are. We would like to investigate this in this section.

The neutral scalar-quark interactive Lagrangian is given by,

$$\begin{aligned}\mathcal{L}_\phi &= -\frac{1}{\sqrt{2}|\lambda_1|}\overline{D}m_D D\Phi_1 + \frac{i|\lambda_2|}{\sqrt{2}|\lambda_1|\sqrt{|\lambda_1|^2+|\lambda_2|^2}}\overline{D}m_D\gamma^5 D\Phi_3 \\ &\quad -\frac{1}{\sqrt{2}|\lambda_2|}\overline{U}m_U U\Phi_2 + \frac{i|\lambda_1|}{\sqrt{2}|\lambda_2|\sqrt{|\lambda_1|^2+|\lambda_2|^2}}\overline{U}m_U\gamma^5 U\Phi_3 + h.c. \,. \qquad (48)\end{aligned}$$

At tree level, there could be three possible T-odd/P-odd forward scattering amplitudes for $a(1/2)+b(1/2)\to a(1/2)+b(1/2)$, shown in Fig. 5.

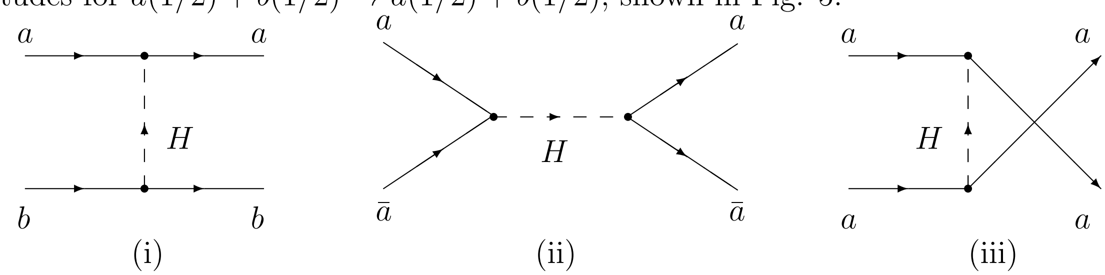


Fig. 5: The Higgs contributions to forward scattering amplitudes of $a+b\to a+b$

Here Fig. 5(i) is for the forward scattering of two arbitrary spin-1/2 particles, Fig. 5(ii) is the forward scattering for a spin-1/2 particle and its anti-particle, and Fig. 5(iii) is for the forward scattering of two identical, spin-1/2 particles. Based on the optical theorem, the T-odd/P-odd total cross sections of the processes shown in Fig, 5 are given by

$$\begin{aligned}\sigma^{TP}_{t,5i} &= 0\,, \\ \sigma^{TP}_{t,5ii} &= \frac{-1}{v_{rel}}\frac{m_a^2|\lambda_i|v_j v_3}{|\lambda_j|^2\sqrt{|\lambda_1|^2+|\lambda_2|^2}}\frac{p_{az}}{p_{a0}}\frac{m_H\Gamma}{(4p_{a0}^2-m_H^2)^2+(m_H\Gamma)^2}s_{ax}s_{\bar{a}y}\,, \\ \sigma^{TP}_{t,5iii} &= \frac{-1}{v_{rel}}\frac{m_a^2|\lambda_i|v_j v_3}{|\lambda_j|^2\sqrt{|\lambda_1|^2+|\lambda_2|^2}}\frac{p_{az}}{p_{a0}}\frac{m_H\Gamma}{(4|\vec{p}_a|^2+m_H^2)^2+(m_H\Gamma)^2}s_{ax}s_{\bar{a}y}\,, \qquad (49)\end{aligned}$$

where v_{rel} is the relative velocity of two incoming particles, and $i=1(2),\ j=2(1)$ if the particle a is in the $U(D)$ sector. The effect of scalar exchange is assumed to be dominated by the lightest neutral-scalar particle of mass m_H, i.e.

$$< \Phi_i \Phi_j >_k \simeq \frac{v_i v_j}{k^2 - m_H^2 + i\epsilon} \ . \tag{50}$$

Note from eq. (48) that $\sigma^{TP}_{t,5ii}$ reaches its maximums at $p_{a0} = 0.5 m_H$, which corresponds to the resonance region of the scattering, and that $\sigma^{TP}_{t,5iii}$ reaches its maximums when p_{a0} is slightly larger than 0.[13] Since there is no resonance in Fig. 3(iii), the maximum of $\sigma^{TP}_{t,5iii}$ can be orders of magnitude smaller than the maximum of $\sigma^{TP}_{t,5ii}$. Obviously, it is experimentally more favorable to choose scattering channels and incoming particle momenta which produce maximum T-odd/P-odd total cross sections. Further investigation of the magnitude of the T-odd/P-odd total cross sections will require knowledges of the Higgs sector and the masses of the quarks. The following assumptions are adopted.

(1) There is no preference coupling the Higgs to quarks in the U and D sectors. This leads to

$$|\lambda_1| \sim |\lambda_2| \sim (\sqrt{2} G_F)^{-1/2} = 246 \, GeV \ . \tag{51}$$

(2) The order of magnitude of $v_a v_b$ in eqs. (48) and (49) is approximately 1.

(3) The u and d quark masses are approximately $5 \, MeV$.

Due to the small quark masses and large vacuum expectation values, the couplings between quarks and the Higgs particle are very small, and small T-odd/P-odd total cross sections are expected. Threfore one should attempt to find the largest open channels. We consider several processes in the following.

4.1 $p\bar{p}$ scattering

We consider several factors in the T-odd/P-odd total cross section for $p\bar{p}$ [14].

(1) Since the valence quark composition of a proton is uud and the valence quark composition of an anti-proton is $\bar{u}\bar{u}\bar{d}$, the dominant contributions to T-odd/P-odd total cross section are from $\sigma^{TP}_{t,5ii}$ in eq. (17).

(2) Vallence quarks in a proton only contribute about 30% of the proton spin. Thus it is assumed that each valence quark contributes about 10% of the total spin.

(3) Both vallence and sea quarks in a proton only contribute about 50% of the proton total momentum. The most probable momentum for a valence quark in a protone is about at $x = 0.15$.

Then the maximum total cross section of $p\bar{p}$ is roughly given by

$$\begin{aligned}\sigma^{TP}_{p\bar{p}} &\simeq (\frac{2}{10})\sigma^{TP}_{t,5ii}(u\bar{u}\to u\bar{u}) + (\frac{1}{10})\sigma^{TP}_{t,5ii}(d\bar{d}\to d\bar{d}) \\ &= (\frac{2}{10\times 10})\frac{1}{v_{rel}}\frac{m_u^2|\lambda_1|v_2v_3}{|\lambda_2|^2\sqrt{|\lambda_1|^2+|\lambda_2|^2}}\frac{p_{uz}}{p_{u0}}\frac{1}{m_H\Gamma}s_{ux}s_{\bar{u}y} \\ &\quad +(\frac{1}{10\times 10})\frac{1}{v_{rel}}\frac{m_d^2|\lambda_2|v_1v_3}{|\lambda_1|^2\sqrt{|\lambda_1|^2+|\lambda_2|^2}}\frac{p_{dz}}{p_{d0}}\frac{1}{m_H\Gamma}s_{dx}s_{\bar{d}y}\,. \end{aligned} \tag{52}$$

Using m_H and Γ given in ref. [16], and assuming $\sigma_{t,p\bar{p}} \simeq 50\,mb$, an estimate of $\sigma^{TP}_{p\bar{p}}$ and A_{xy} is given in **Table 1**.

Table 1

$m_H\,(GeV)$	$\Gamma_H\,(GeV)$	$\sigma^{TP}_{p\bar{p}}\,(mb)$	A_{xy}	beam energy (GeV)
125.53	4.6×10^{-3}	2×10^{-12}	4×10^{-14}	418

Note that the beam energy in **Table 1** is collider energy. For a fixed target experiment, the beam energy must be properly adjusted.

One can see from **Table 1** that it requires high accuracy and sensitivity to measure such small cross sections.

Modern superconducting technology can measure current changes as low as $10^{-8}\sim 10^{-9}\,A$. [17] If the luminosity of the beam and target is reasonably large, the small cross section in **Table 1** should be measurable.

4.2 pp scattering

To estimate T-odd/P-odd total cross section of pp, a few factors will be considered. [14]

(1) The sea quarks in a proton are mainly found in the small x region. Thus the major contributions to T-odd/P-odd forward scattering would most likely occur in valence quark collisions, *i.e.* $\sigma^{TP}_{t,5iii}$, for beam energies below or around $m_H/2$. However, if the beam energies were much beyond $m_H/2$, contributions from sea quarks, *i.e.* $\sigma^{TP}_{t,5ii}$, could also be important. Only beam energies below or around $m_H/2$ are considered. For beam energies much beyond $m_H/2$, one should refer to section 3.1.

(2) Parton model is assumed to be valid at these beam energies.

(3) As we only consider beam energies below or around $m_H/2$, points 2 and 3 in section 3.1 remain valid.

The total cross section of pp is roughly given by

$$\begin{aligned}\sigma_{pp}^{TP} &\simeq (\frac{2}{10})\sigma_{t,5iii}^{TP}(uu \to uu) + (\frac{1}{10})\sigma_{t,5iii}^{TP}(dd \to dd) \\ &\simeq (\frac{2}{10 \times 10})\frac{-1}{v_{rel}}\frac{m_u^2|\lambda_1|v_2v_3}{|\lambda_2|^2\sqrt{|\lambda_1|^2+|\lambda_2|^2}}\frac{p_{uz}}{p_{u0}}\frac{m_H\Gamma}{(4|\vec{p}_u|^2+m_H^2)^2+(m_H\Gamma)^2}s_{ux}s_{\bar{u}y} + \\ &\quad (\frac{1}{10 \times 10})\frac{-1}{v_{rel}}\frac{m_d^2|\lambda_2|v_1v_3}{|\lambda_1|^2\sqrt{|\lambda_1|^2+|\lambda_2|^2}}\frac{p_{dz}}{p_{d0}}\frac{m_H\Gamma}{(4|\vec{p}_d|^2+m_H^2)^2+(m_H\Gamma)^2}s_{dx}s_{\bar{d}y}\,. \quad (53)\end{aligned}$$

Considering $m_H = 125.35\, GeV$ and $\Gamma = 4.6 \times 10^{-3}\, GeV$[16] and assuming $\sigma_{pp} \simeq 50\, mb$, the estimated values of σ_{pp}^{TP} and A_{xy} are given in **Table 2**.

Table 2: $p\bar{p}$ **scattering vs beam energies for** $m_H = 125.35\, GeV$

beam energy (GeV)	σ_{pp}^{TP} (mb)	A_{xy}
5	2×10^{-20}	4×10^{-22}
10	2×10^{-20}	4×10^{-22}
50	8×10^{-21}	2×10^{-23}
100	2×10^{-21}	4×10^{-23}

Note that the beam energies in **Table 2** are also collider energies.

As one can see from the results in **Table 2**, the T-odd/P-odd total cross sections for pp are much smalller than for $p\bar{p}$. This is understandable as there are no resonances in this channel.

One aslo notices that the variations of σ_{pp}^{TP} and A_{xy} versus beam energies are small. This is due to the large mass of the Higgs particle, as compared to the incoming particle energies.

In general, although both $\sigma_{p\bar{p}}^{TP}$ and σ_{pp}^{TP} are very small, $\sigma_{p\bar{p}}^{TP}$ should be measurable with current technologies. However, as the above estimates are based on a neutral scalar boson, other possible source(s) of T (or CP) violation could be larger than these estimates. A careful comparisons between pp and $p\bar{p}$ T-odd/P-odd total cross sections could provide us more information of T (or CP) violation mechanisms.

4.3 $l\bar{l}$ and ll scatterings

If the coupling between the lepton sectors and the Higgs sector is similar to the coupling between quark sectors and the Higgs sector, polarized $l\bar{l}$ and ll scatterings can also have T-odd/P-odd total cross sections. We consider the following points.

(1) Leptons are elementary particles and one does not need to consider the unpolarized and polarized structure fountions. Therefore, one can directly uses the result in eq. (17) for the $l\bar{l}$ and ll T-odd/P-odd total cross sections.

(2) The total cross sections for $l\bar{l}$ and ll should be significantly smaller than the $p\bar{p}$ and pp cross sections since only electro-weak interactions are involved. For beam energies which are not in the Z resonance region, $\sigma_{l\bar{l}} \sim \sigma_{ll} \sim 10\,\mu b$ is assumed for simplicity [18]

The estimated $\sigma_{l\bar{l}}(\sigma_{ll})$ and A_{xy} are given in **Tables 3-6**.

Table 3: Maximum $\sigma_{e\bar{e}}$ for $m_a = 125.35 GeV H$

$m_H\,(GeV)$	$\Gamma_H\,(GeV)$	$\sigma^{TP}_{e\bar{e}}\,(mb)$	A_{xy}	beam energy (GeV)
125.35	4.6×10^{-3}	1×10^{-12}	1×10^{-10}	63

Table 4: ee scattering vs beam energies for $m_H = 125.35\,GeV$

beam energy (GeV)	$\sigma^{TP}_{ee}\,(mb)$	A_{xy}
0.5	7×10^{-21}	7×10^{-19}
1.0	7×10^{-21}	7×10^{-19}
10.0	7×10^{-21}	7×10^{-19}
100.0	6×10^{-22}	6×10^{-20}

Table 5: Maximum $\sigma^{TP}_{\mu\bar{\mu}}$ for $m_H = 125.35 GeV$

$m_H\,(GeV)$	$\Gamma_H\,(GeV)$	$\sigma^{TP}_{\mu\bar{\mu}}\,(mb)$	A_{xy}	beam energy (GeV)
125.35	4.6×10^{-3}	1×10^{-8}	6×10^{-6}	63

Table 6: $\mu\mu$ scattering vs beam energies for $m_H = 125.35\,GeV$

beam energy (GeV)	$\sigma^{TP}_{\mu\mu}\,(mb)$	A_{xy}
0.5	8×10^{-17}	8×10^{-15}
1.0	8×10^{-17}	8×10^{-15}
10.0	7×10^{-17}	7×10^{-15}
100.0	6×10^{-18}	6×10^{-16}

Similar to the results of baryon collisions, $l\bar{l}$ collisions have larger T-odd/P-odd total cross sections if the Higgs sector is one of the T-violation sources and if the coupling between the lepton sectors and the Higgs sector is similar to the one between the quark sectors and the Higgs sector. Especially for the $\mu\bar{\mu}$ channel, its

T-odd/P-odd total cross section is orders of magnitude larger than the corresponding $p\bar{p}$ channel due to the larger masses of μ and $\bar{\mu}$. If high energy polarized μ and $\bar{\mu}$ beams were available, the measurements of T-odd/P-odd total cross sections of μ and $\bar{\mu}$ collisions would provide a sensitive test.

On the other hand, the T-odd/P-odd total cross section of ll collisions is an order(s) of magnitude smaller than the corresponding $l\bar{l}$ total cross section. These estimates are based on the assumption that the neutral Higgs particle contributes to T (or CP) violation. If a small T-odd/P-odd cross section of ll collisions is measured, a careful comparison between ll and the corresponding $l\bar{l}$ T-odd/P-odd total cross sections would provide us additional information of T (or CP) violation.

5 Summary and Future Prospect

This note addresses the possibilities of seaching for additional sources of T (or CP) violation through T-odd/P-odd total cross section measurements. It shows the following. (1) A non-zero T-odd/P-odd total cross section in the null test $1/2 + 1/2 \rightarrow 1/2 + 1/2$ will indicate that there is(are) additional source(s) of T (or CP) violation besides the phase in CKM matrix. (2) Stronger interactions would not interfere with the total cross section measurements contributed from weak interaction. (3) The contributions to T-odd/P-odd total cross section from the Higgs sector can appear at tree level if the Higgs sector contribute to T (or CP) violation, and the channels with resonance can be measurable with modern technology if the lightest Higgs mass is $125.35GeV$, as we know today, and beam luminosities are reasonably large. (3) If the Higgs coupling to the leptons is similar to the coupling to quarks, A_{xy} in $l\bar{l}$ is larger than the one for $p\bar{p}$ due to the smaller $l\bar{l}$ total cross section and simpler structures of lepton, and $\mu\bar{\mu}$ provides the most sensitive channel due to the larger muon mass. (4) The present study only considers the coupling of neutral Higgs particle as the additional source of T (or CP) violation. Actual measurements could be larger if other mechnisms of T (or CP) violation occur. A careful comparison between channels with and without resonance could reveal mechanisms of T (or CP) violation beyond current ideas. (5) Measurement of $A(x, y)$ in the total cross section is a null test and as such has the possibility to give very accurate results. (6) The proposed measurements can provide information on possible extentions of MSMCKM and the properties of vacuum.

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[11] Here the $Z(\gamma)$ exchanges are excluded for they do not introduce additional T-odd factors.

[12] The only difference here from Fig. 3(i, ii) is that the initial and final quarks need to be summed over all components in the corresponding quark sector, but this difference does not change the conclusions of eqs. (13) and (15).

[13] The exact solution of $|\vec{p}_a|$ to have maximum σ^{TP}_{5iii} can be determined from this cubic equation $64|\vec{p}_a|^3 + 16(m_H^2 + 3m_a^2)|\vec{p}_a|^2 + 8m_a^2 m_H^2 |\vec{p}_a| + m_a^2 m_H^2 (m_H^2 + \Gamma^2) = 0$.

[14] This is just a rough estimate. A more accurate estimate needs calculations based on up-to-date quark structure functions and polarized quark structure functions. Sea quark contributions should also be considered.

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[18] For $l\bar{l}$ scattering, the maximum σ_t^{TP} is at the Higgs resonance region, but the total cross section is not expected to be dominant in the Higgs resonance region due to the small coupling constants.