Searching for the QCD Critical Point with Correlation and Fluctuation Measurements in PHENIX

4th Workshop on Particle Correlations and Femtoscopy – 9/12/08
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Outline

- **Multiplicity Fluctuations**
- **$\langle p_T \rangle$ Fluctuations**
- **$\langle K/\pi \rangle$ Fluctuations**
- **Correlation Length from Multiplicity Fluctuations**
- **Azimuthal Correlations at Low p_T**

Divergent Quantities at the Critical Point

Near the critical point, several properties of a system diverge. The rate of the divergence can be described by a set of critical exponents. For systems in the same universality class, all critical exponent values are identical.

- The critical exponent for compressibility, γ: $$k_T \propto \left(\frac{T - T_c}{T_C}\right)^{-\gamma}$$
- The critical exponent for heat capacity, α: $$C_V \propto \left(\frac{T - T_c}{T_C}\right)^{-\alpha}$$
- The critical exponent for correlation length, ξ: $$\xi \propto \left(\frac{T - T_c}{T_C}\right)^{-\nu}$$
- The critical exponent for correlation functions, η: $$C(R) \propto R^{-(d-2+\eta)}$$ (d=3)

Susceptibilities at the Critical Point

Consider quark number susceptibility, χ_q at the critical point.

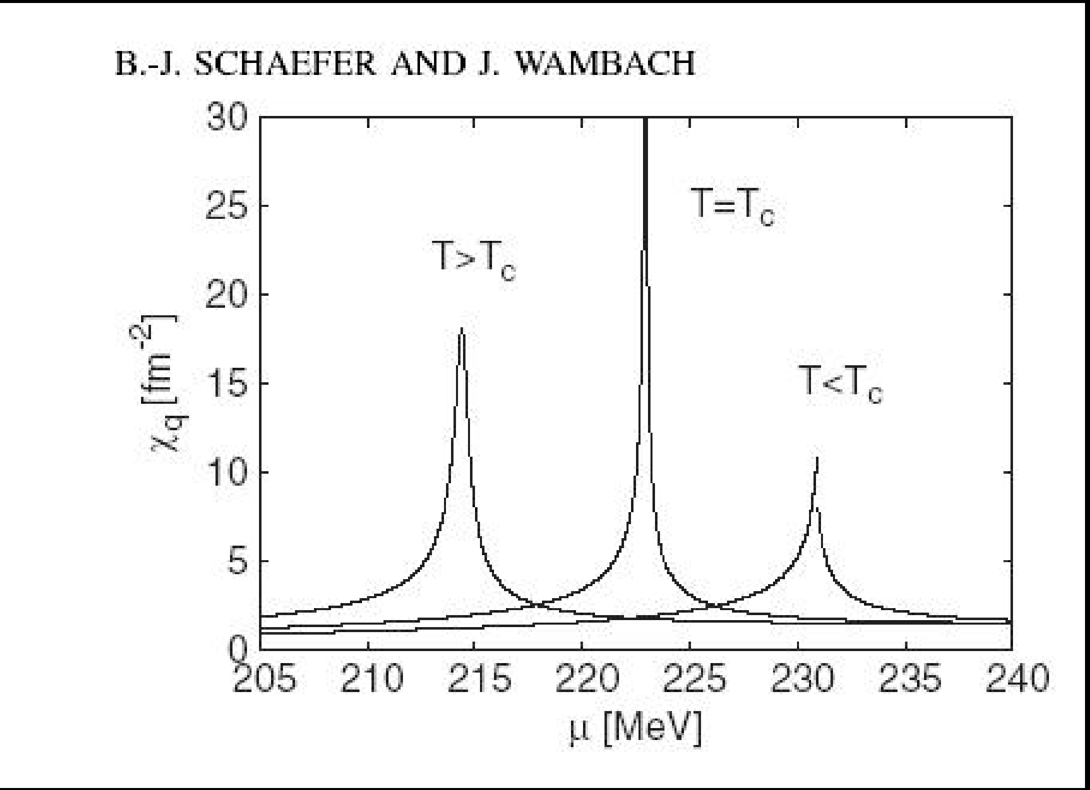
$\chi_q = \langle q^\dagger q \rangle = \partial n(T,\mu)/\partial\mu$

This is related to the isothermal compressibility:

$k_T = \chi_q(T,\mu)/n^2(T,\mu)$

In a continuous phase transition, k_T diverges at the critical point…

$$k_T \propto \left(\frac{T - T_c}{T_C}\right)^{-\gamma}$$



B.-J. Schaefer and J. Wambach, Phys. Rev. D75 (2007) 085015.

Multiplicity Fluctuations

• Multiplicity fluctuations may be sensitive to divergences in the compressibility of the system near the critical point.

Grand Canonical Ensemble

$$\left(\sigma^2/\mu\right) = \omega_N = \mu/k_{NBD} + 1 = k_B T \left(\frac{\mu}{V}\right) k_T$$

ω_N → "Scaled Variance"

μ = N = Mean

• Multiplicity fluctuations have been measured in the following systems:

- 200 GeV Au+Au
- 62.4 GeV Au+Au
- 200 GeV Cu+Cu
- 62.4 GeV Cu+Cu
- 22.5 GeV Cu+Cu
- 200 GeV p+p (baseline)

• Survey completed as a function of centrality and p_T

Measuring Multiplicity Fluctuations with Negative Binomial Distributions

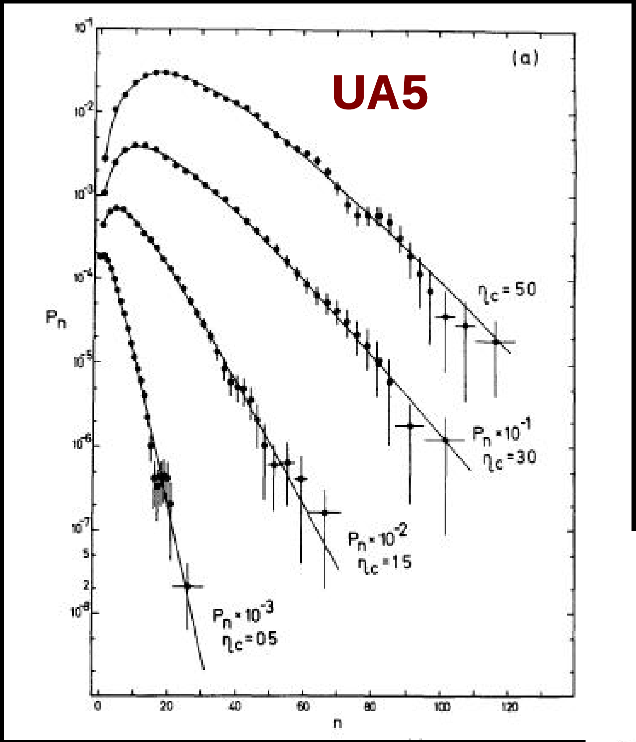
Multiplicity distributions in hadronic and nuclear collisions can be well described by the Negative Binomial Distribution.

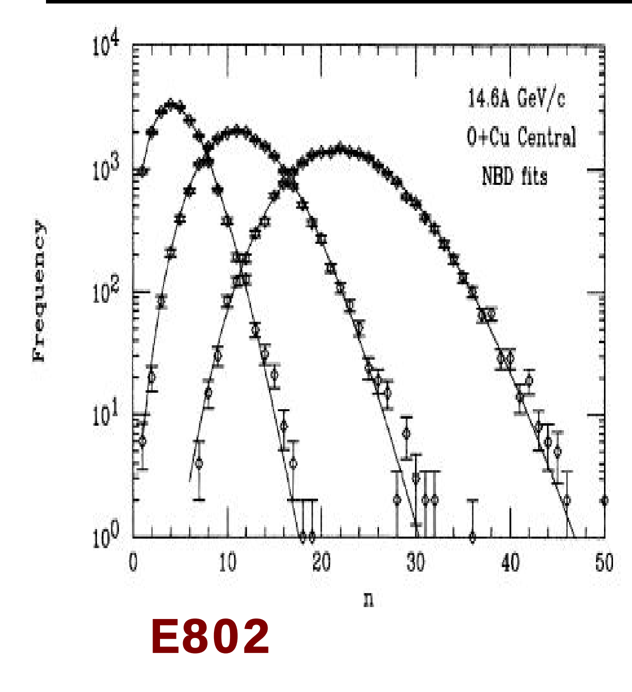
UA5: sqrt(s)=546 GeV p-pbar, Phys. Rep. 154 (1987) 247.

E802: 14.6A GeV/c O+Cu, Phys. Rev. C52 (1995) 2663.

$$P(m) = \frac{(m+k-1)!}{m!(k-1)!} \frac{\left(\mu/k\right)^m}{\left(1+\mu/k\right)^{m+k}}$$

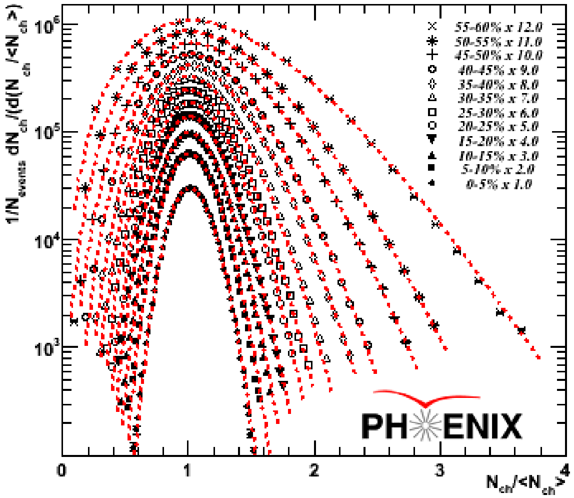
$$\frac{1}{k} = \frac{\sigma^2}{\mu^2} - \frac{1}{\mu}$$



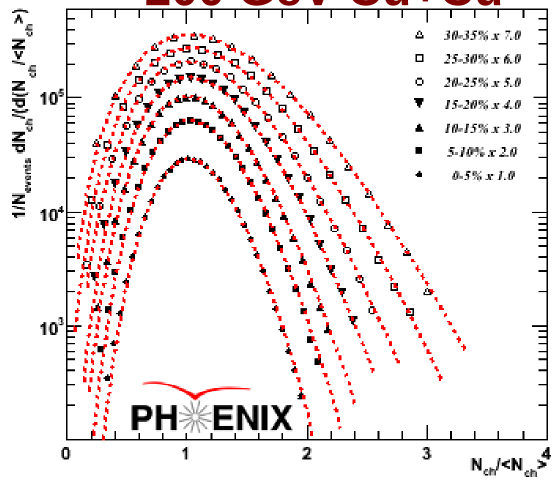


Au+Au, Cu+Cu, p+p NBD Distributions

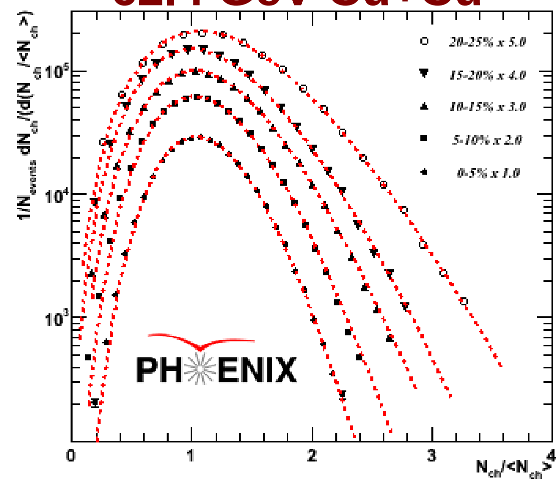
200 GeV Au+Au



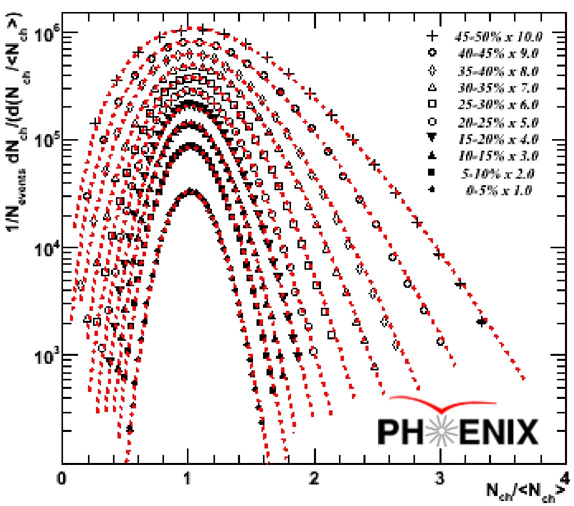
200 GeV Cu+Cu



62.4 GeV Cu+Cu

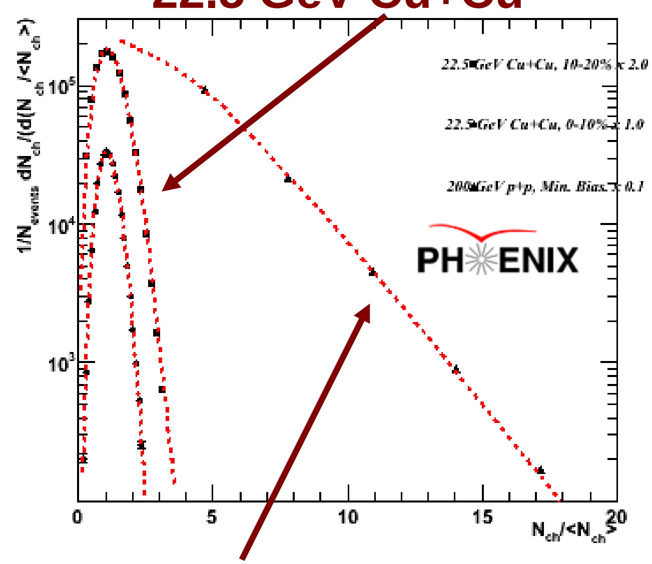


62.4 GeV Au+Au



Red lines represent the NBD fits. The distributions have been normalized to the mean and scaled for visualization. Distributions measured for $0.2<p_T<2.0$ GeV/c

22.5 GeV Cu+Cu



200 GeV p+p

Multiplicity Fluctuations: Participant Superposition Model

- In a Participant Superposition Model, multiplicity fluctuations are given by:

$$\omega_N = \omega_n + <N>\omega_{Np}$$

where $\omega = \sigma^2/\mu$. *ω_N = total fluctuation, ω_n = fluctuation of each source (e.g. hadron-hadron collision), ω_{Np} = fluctuation in number of sources (participants).*

- After correcting for fluctuations due to impact parameter, $\omega_N = \omega_n$ independent of centrality.
- Multiplicity fluctuations are also dependent on acceptance:

$$\omega_n = 1 - f + f\omega_n$$

where $f = N_{accepted}/N_{total}$. ω_n = fluctuations from each source in 4π

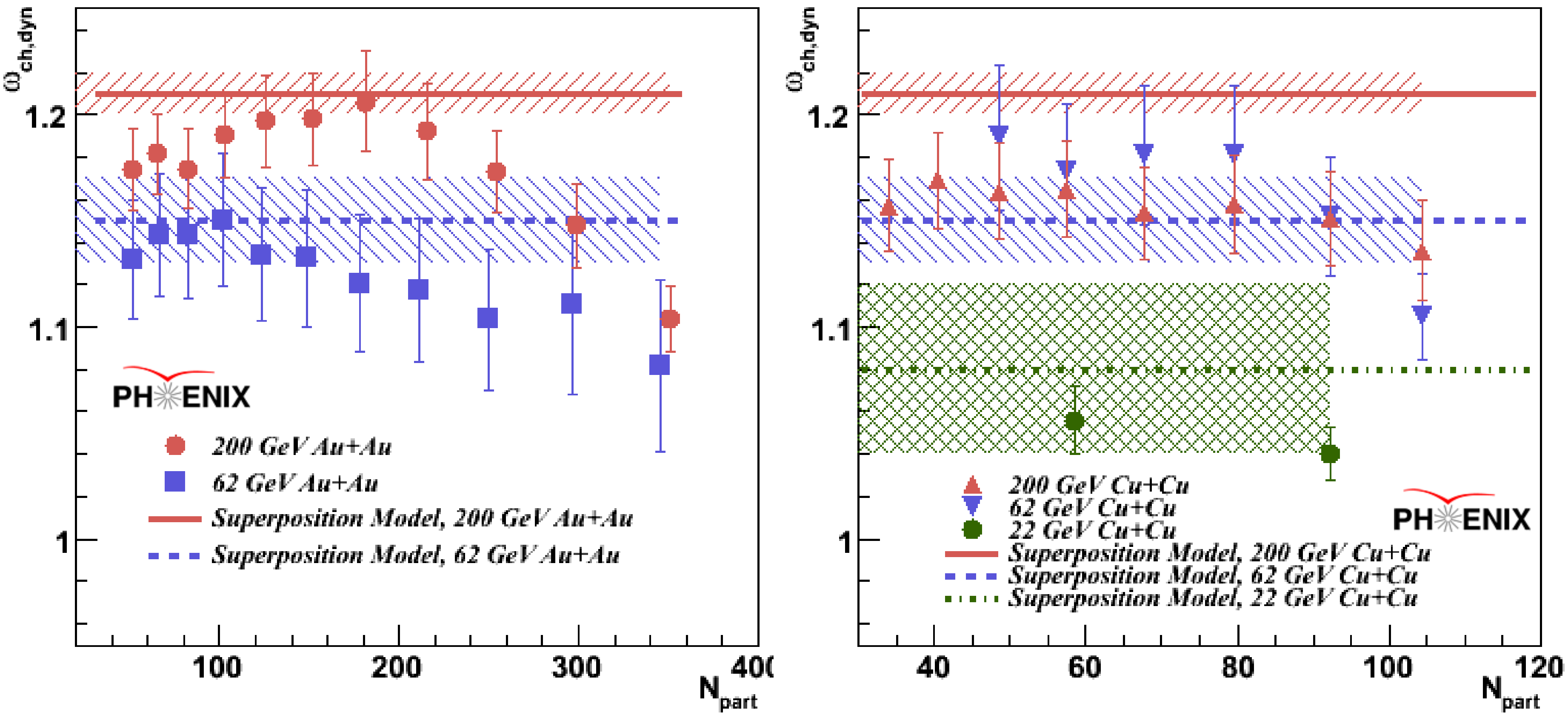
Superposition model at 200 GeV taken from PHENIX measurements of 200 GeV p+p. The results agree with UA5 measurements in PHENIX's pseudorapidity window.

Superposition model at 22 GeV taken from NA22 measurements in PHENIX's pseudorapidity window.

Superposition model at 62 GeV taken from interpolation of UA5 results in PHENIX's pseudorapidity window.

Multiplicity Fluctuation Results

Bottom line: Near the critical point, the multiplicity fluctuations should exceed the superposition model expectation → No significant evidence for critical behavior is observed.

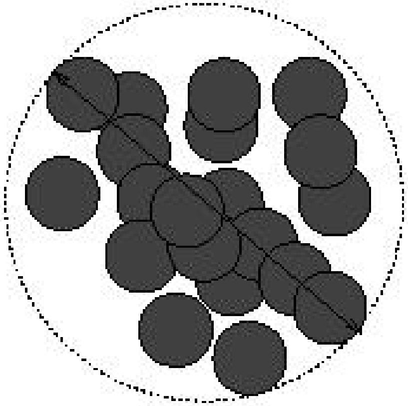


String Percolation Model

- So we try to introduce a phase transition (≡QGP?)

(N. Armesto et al., PRL77 (96); J.Dias de Deus et al., PLB491 (00); M. Nardi and H. Satz).

- How?: Strings fuse forming clusters. At a certain critical density η_c (central PbPb at SPS, central AgAg at RHIC, central SS at LHC) a macroscopic cluster appears which marks the percolation phase transition (second order, non thermal).



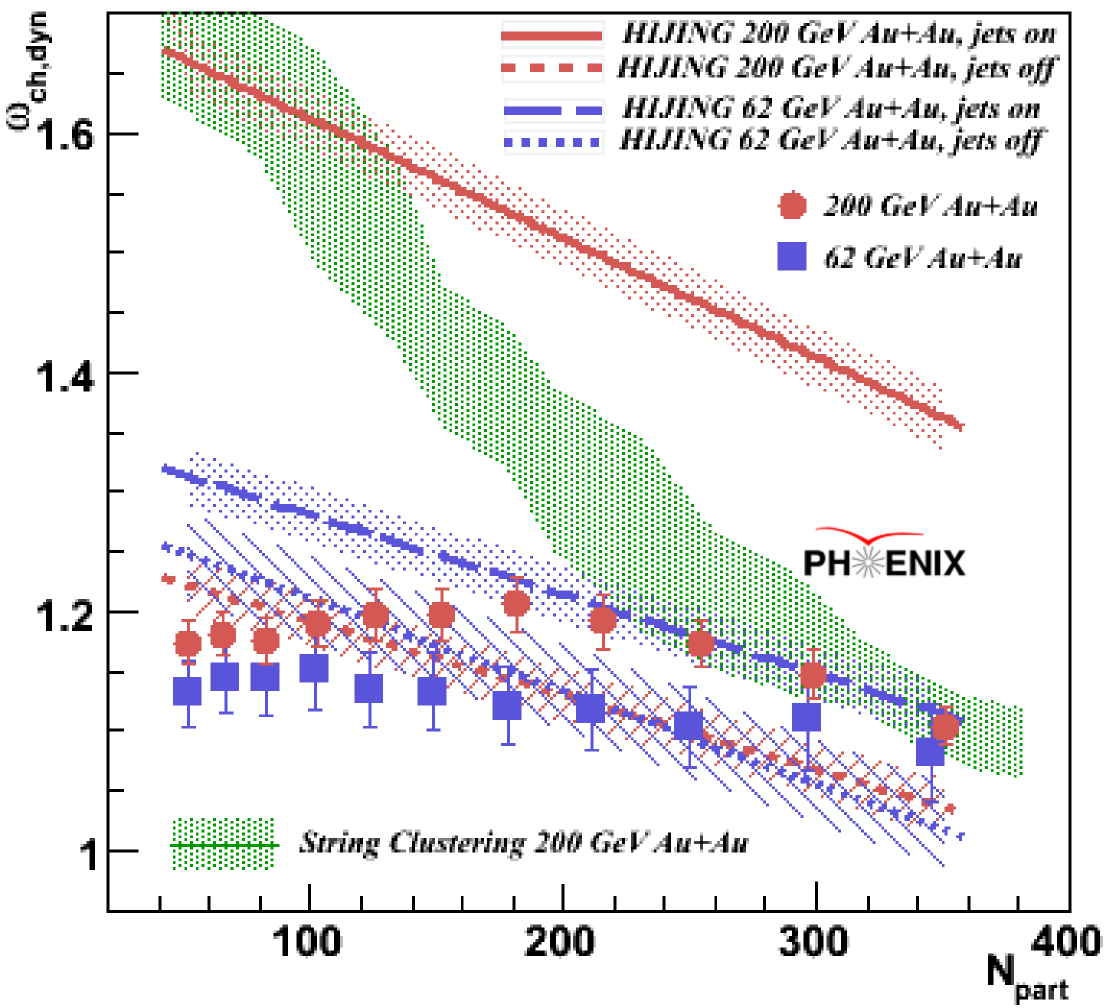
Slide by C. Pajares

$\eta = N_{st}\frac{S_1}{S_A}$, $S_1 = \pi r_0^2$, $r_0 = 0.2$ fm, $\eta_c = 1.1 \div 1.2$.

- Hypothesis: clusters of overlapping strings are the sources of particle production, and central multiplicities and transverse momentum distributions are little affected by rescattering.

Scaled Variance: String Percolation Model

L. Cunqueiro et al., Phys. Rev. C72 (2005) 024907.

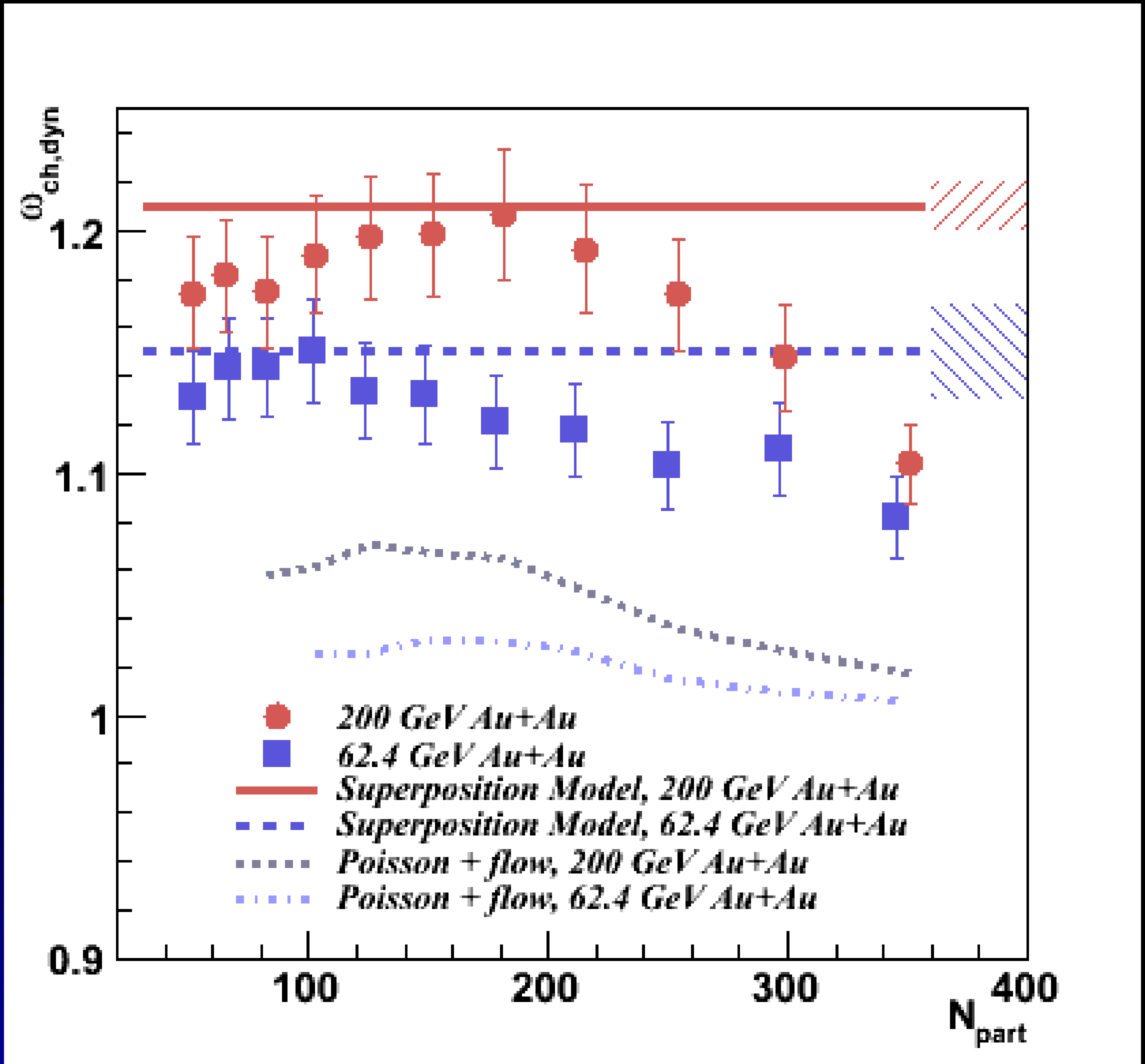


String percolation provides a possible explanation for the decrease in the scaled variance with increasing centrality.

Shown in green are the direct predictions of the string percolation model for 200 GeV Au+Au, scaled down to the PHENIX acceptance.

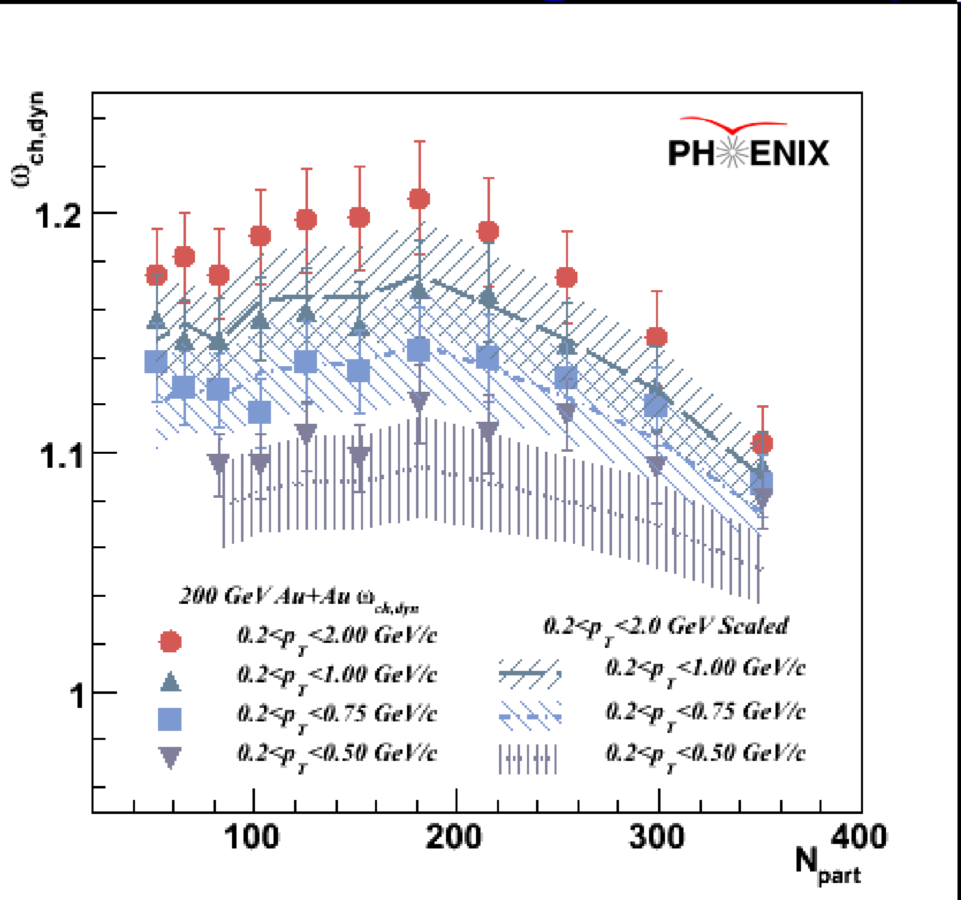
Percolation still does not explain the plateau in the most peripheral Au+Au collisions.

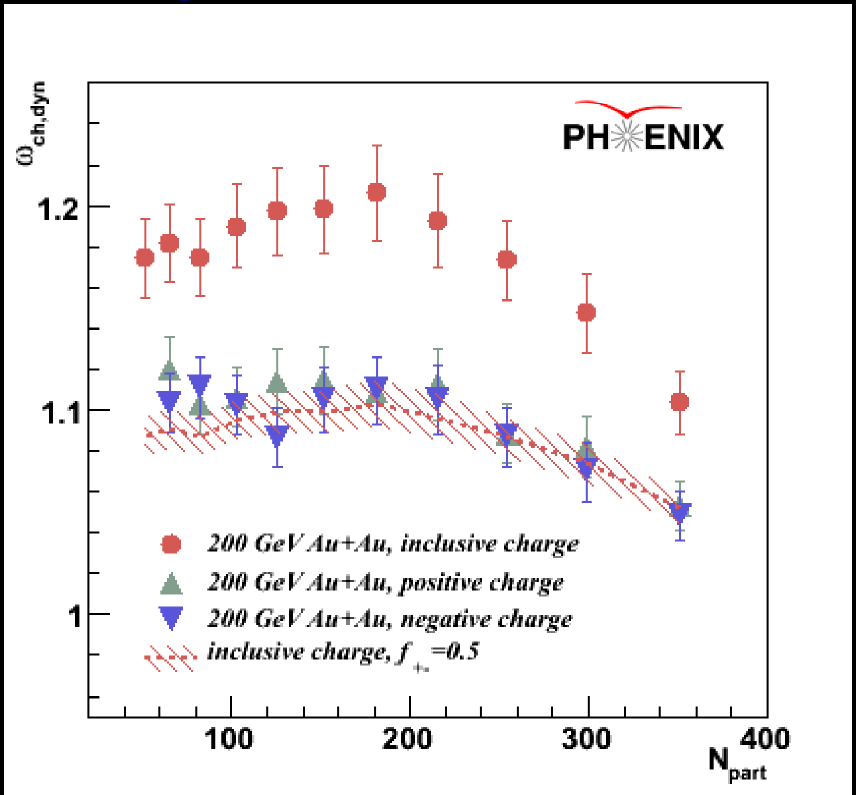
Multiplicity Fluctuations: Elliptic Flow



- The elliptic flow contribution estimated using a simple model as follows:
- For each event, a random reaction plane angle is generated.
- A particle azimuthal distribution is sampled using the PHENIX measured values of v_2 at the mean p_T of each bin.
- The multiplicity within the PHENIX acceptance is recorded for each event and the fluctuations are determined.
- The resulting contributions can be as large as 20% and can explain the centrality-dependence of the fluctuations.

Charge and p_T-Dependence





If the p_T-dependence is random, the scaled variance should scale with <N> in the same manner as acceptance:

$$\omega_{pT} = 1 - f + f\omega_{pT,max}$$

As with acceptance, with no charge-dependent correlation, the scaled variance will scale:

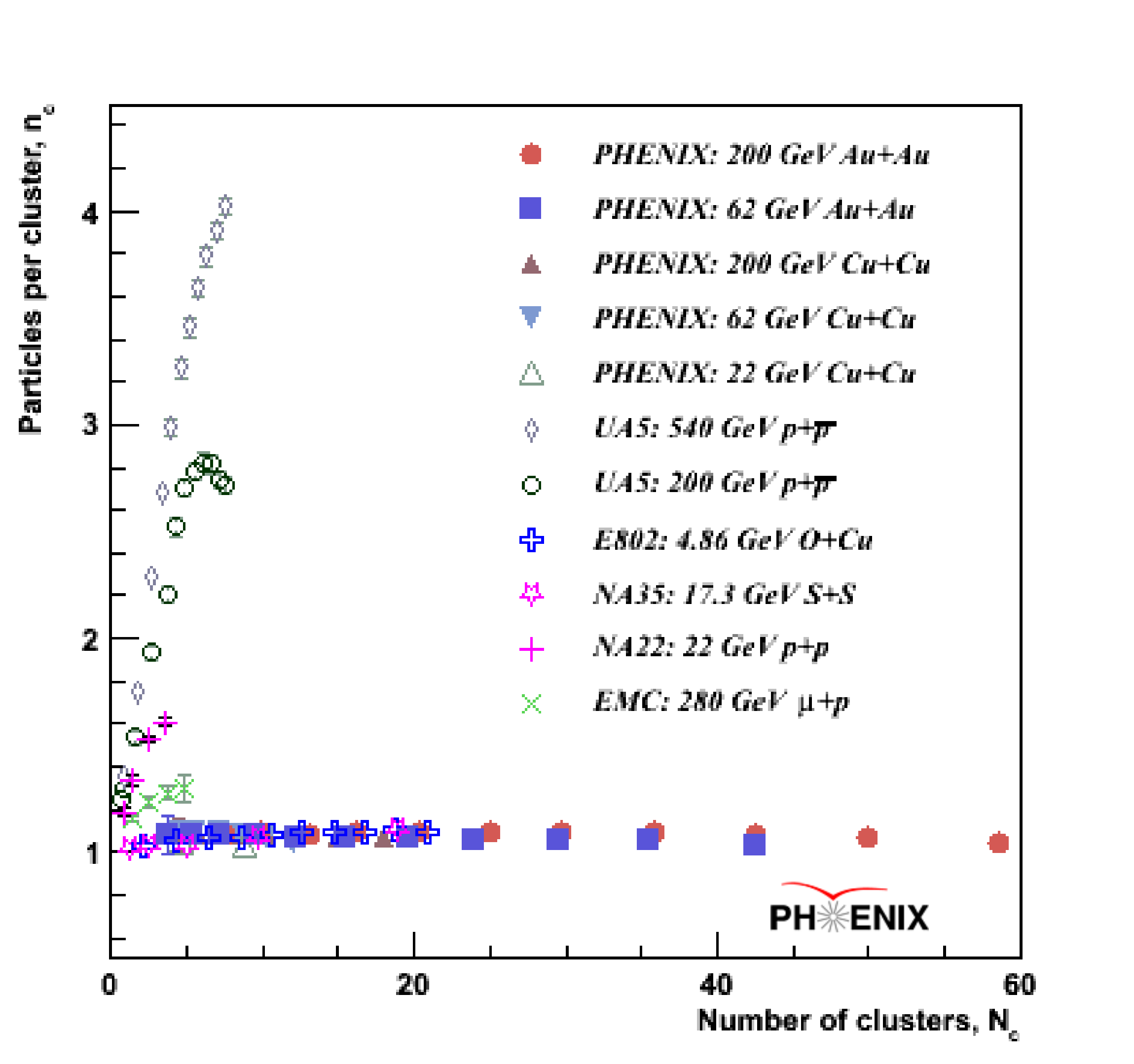
$$\omega_{+-} = 1 - f + f\omega_{inclusive}$$

where f=0.5.

Within errors, no charge dependence of the fluctuations is seen for 200 GeV Au+Au.

CLAN Model

A. Giovannini et al., Z. Phys. C30 (1986) 391.



The CLAN model was developed to attempt to explain the reason that p+p multiplicities are described by NBD rather than Poisson distributions.

Hadron production is modeled as independent emission of a number of hadron clusters, N_c, each with a mean number of hadrons, n_c. These parameters can be related to the NBD parameters:

$N_c = k_{NBD} \log(1 + \mu_{ch}/k_{NBD})$ and
$\langle n_c \rangle = (\mu_{ch}/k_{NBD})/\log(1 + \mu_{ch}/k_{NBD})$.

A+A collsions exhibit weak clustering characteristics, independent of collision energy.

Event-by-Event Mean p_T Fluctuations

- <p_T> fluctuations may be sensitive to divergences in the heat capacity of the system near the critical point.

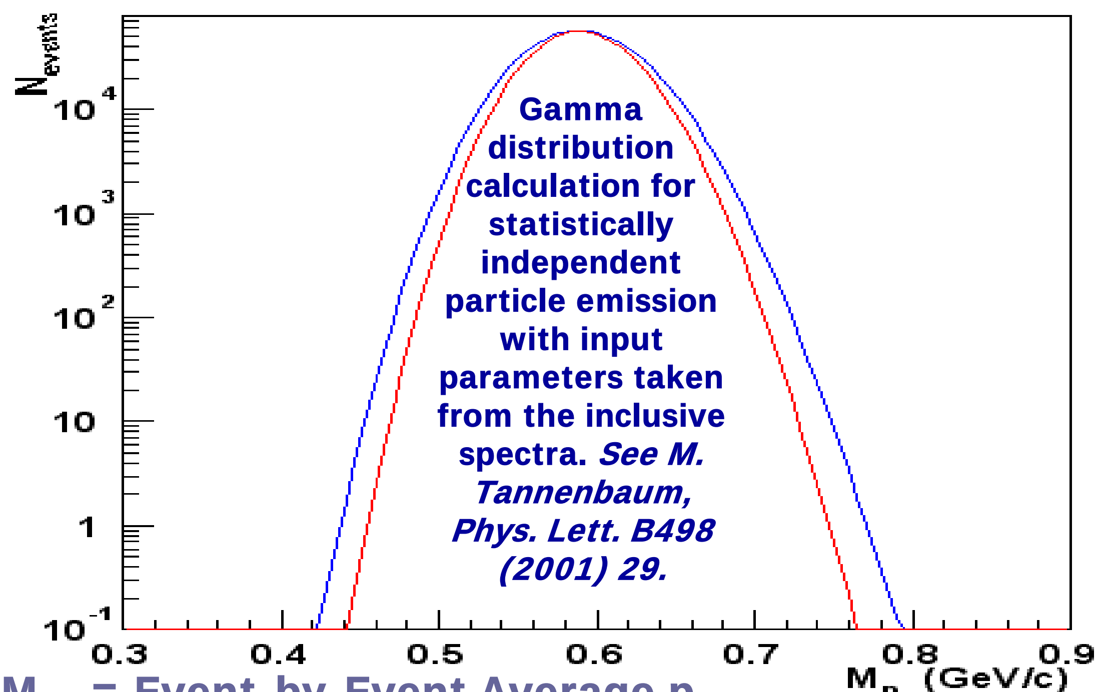
  - <p_T> fluctuations have been measured in the following systems:
    - 200 GeV Au+Au
    - 62.4 GeV Au+Au
    - 200 GeV Cu+Cu
    - 62.4 GeV Cu+Cu
    - 22.5 GeV Cu+Cu

$$C_V \propto \left(\frac{T - T_c}{T_C}\right)^{-\alpha}$$

  - Survey completed as a function of centrality and p_T

Measuring $<p_T>$ Fluctuations

- Σ_{pT} = (event-by-event p_T variance) – [(inclusive p_T variance)/(mean multiplicity per event)], normalized by the inclusive mean p_T. Random = 0.0.
- Σ_{pT} is the mean of the covariance of all particle pairs in an event normalized by the inclusive mean p_T.
- Σ_{pT} can be related to the inverse of the heat capacity.



M_{pT} = Event-by-Event Average p_T

Red: Random Expectation (Γ distribution)

Blue: STAR acceptance fluctuation of:

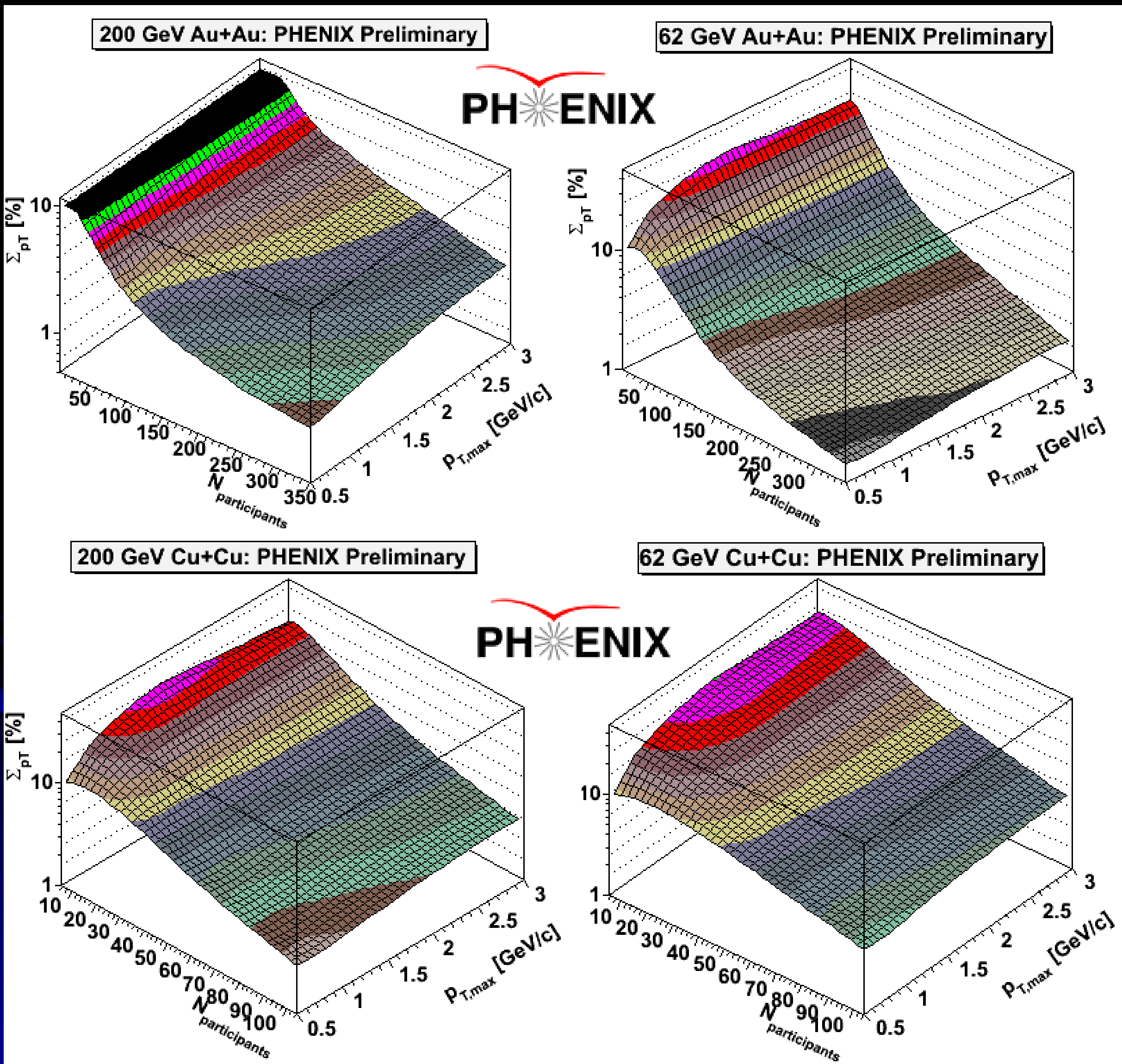
ϕ_{pT} = 52.6 MeV,

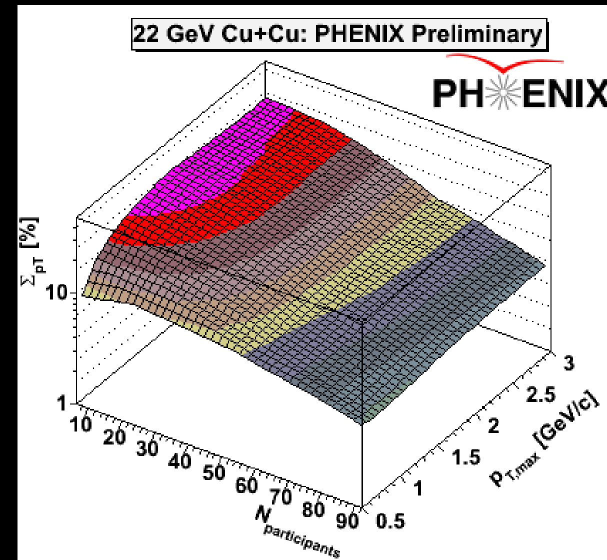
F_{pT} = 14%,

$\sigma^2_{pT,dyn}$ = 52.3 (MeV/c^2),

Σ_{pT} = 9.8%

<p_T> Fluctuations Survey



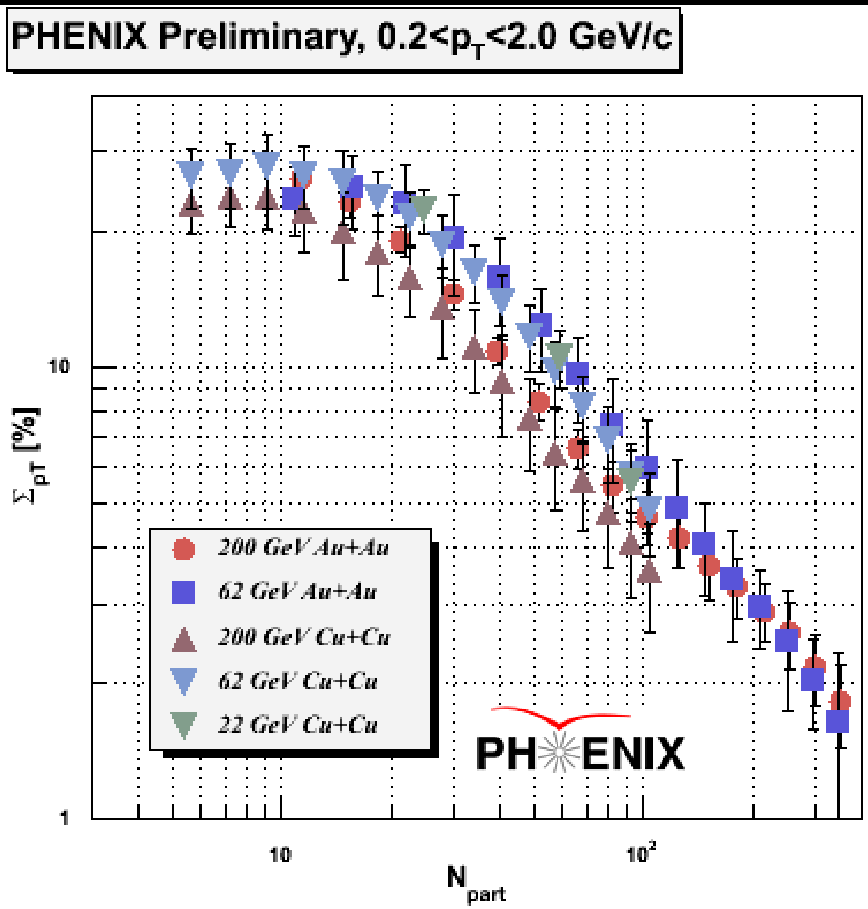


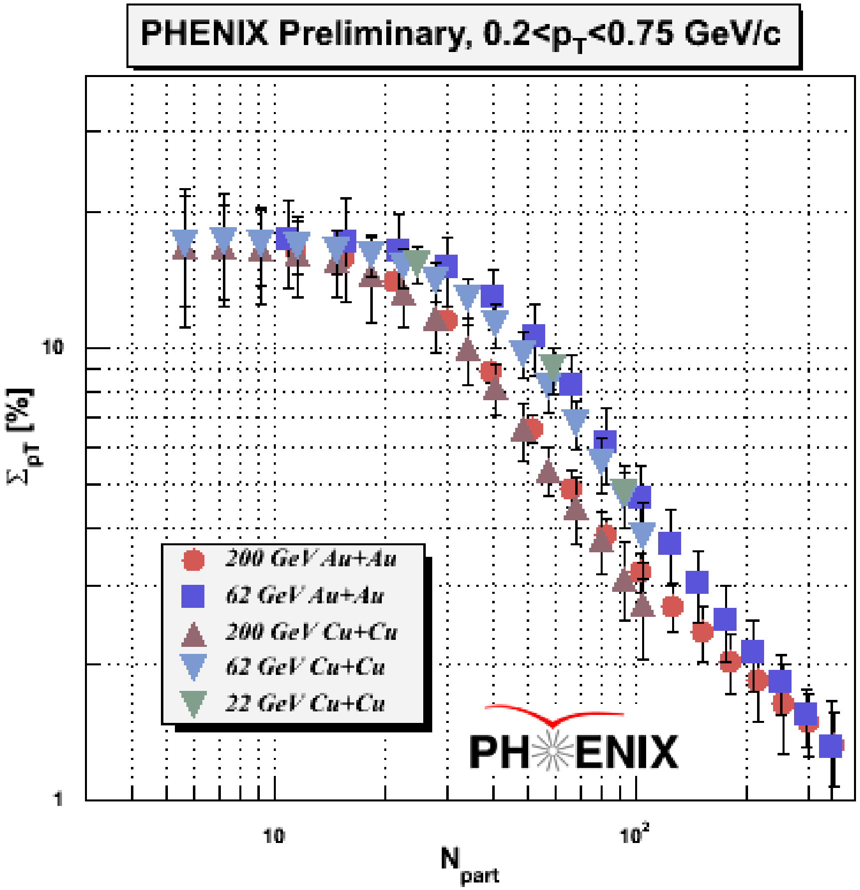
<u>Features:</u> Σ_{pT} increases with decreasing centrality. Similar trend to multiplicity fluctuations (σ^2/μ^2). Increases with increasing p_T. Same behavior for all species, including 22 GeV Cu+Cu.

NOTE: Random fluctuations, Σ_{pT}=0.0.

$\langle p_T \rangle$ Fluctuations vs. Centrality

The magnitude of Σ_{pT} varies little as a function of sqrt(s_{NN}) and species. In a simple model that embeds PYTHIA hard scattering events into inclusively parametrized events, the jet fraction necessary to reproduce the fluctuations does not scale with the jet cross section.

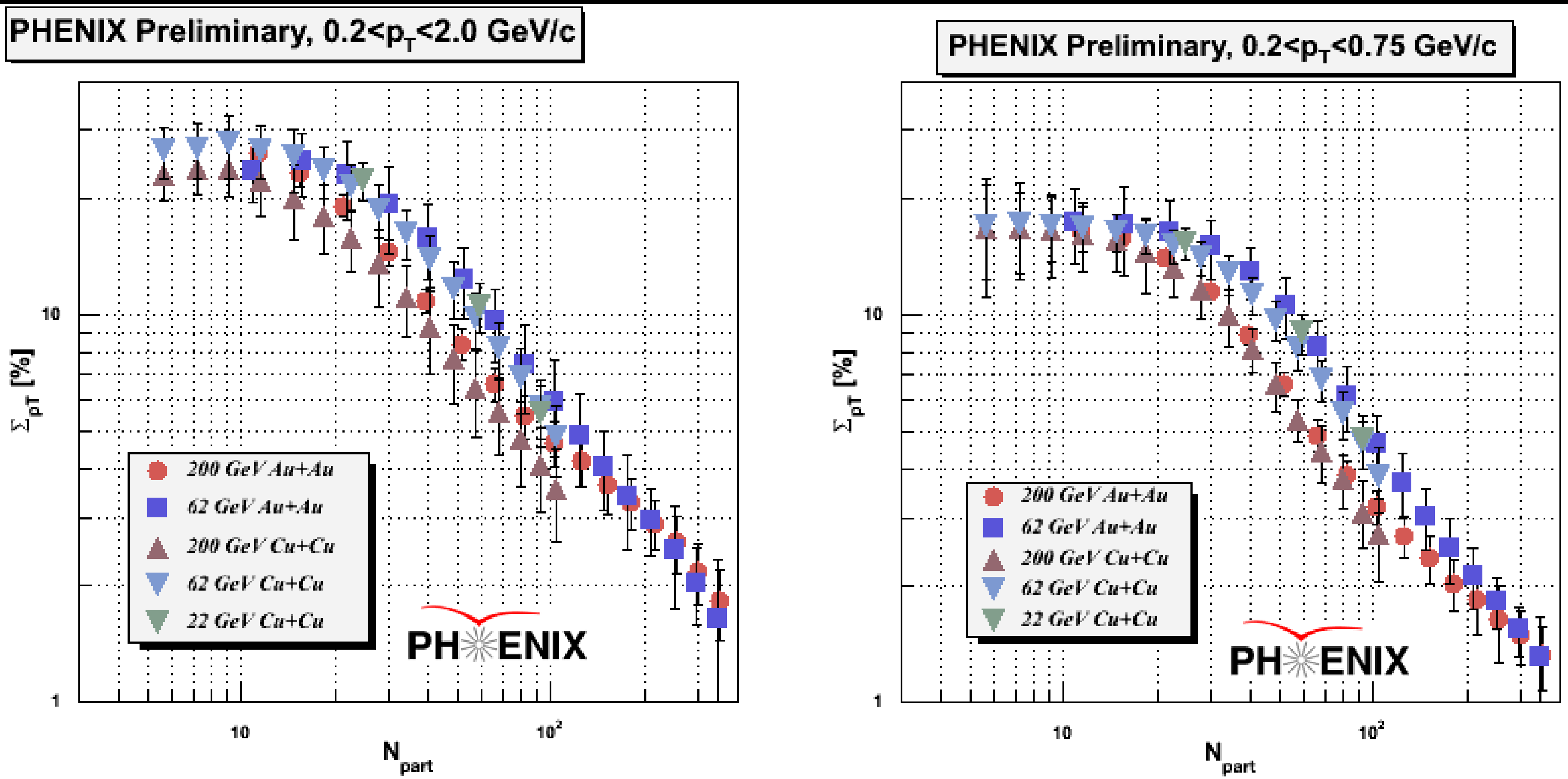




$<p_T>$ Fluctuations vs. Centrality

Above N_{part}~30, the data can be described by a power law in N_{part}, independent of the p_T range down to $0.2<p_T<0.5$ GeV/c:

$$\Sigma_{p_T} \propto N_{part}^{-1.02\pm0.10}$$



Meson-meson (strangeness) and baryon-meson fluctuations

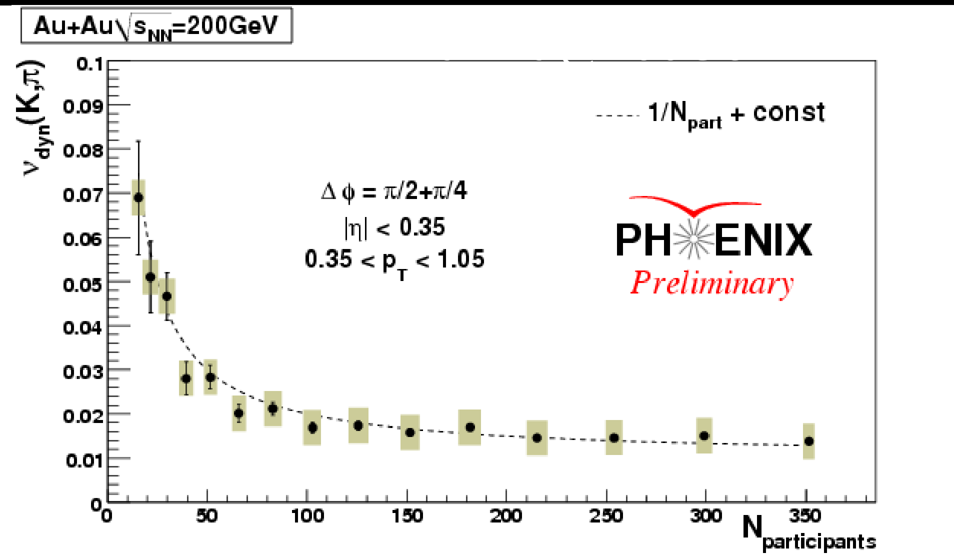
$$\nu_{dyn}(K,\pi) = \frac{\langle \pi(\pi-1)\rangle}{\langle\pi\rangle^2} + \frac{\langle K(K-1)\rangle}{\langle K\rangle^2} - 2\frac{\langle \pi\, K\rangle}{\langle\pi\rangle\langle K\rangle}$$

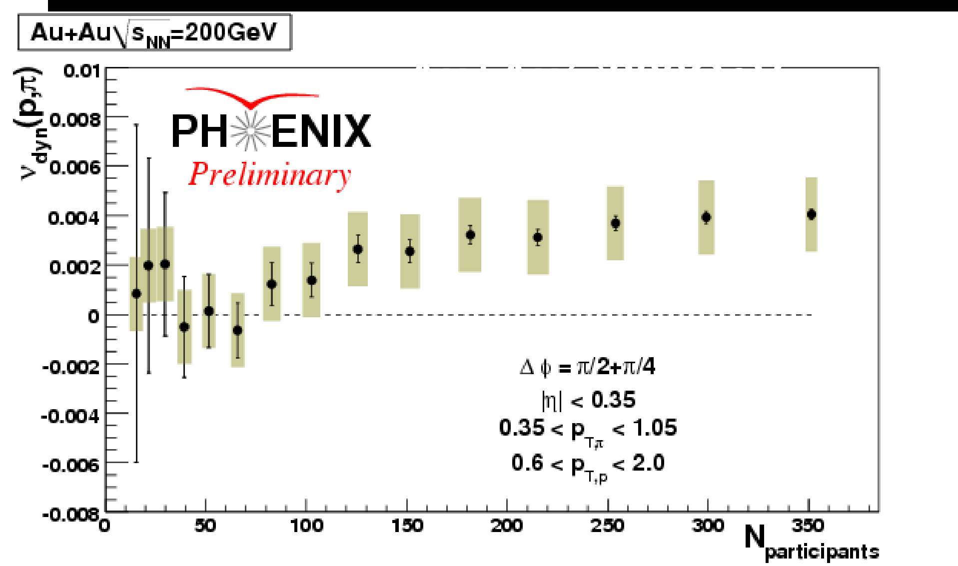
$\nu_{dyn} = 0 \rightarrow$ No dynamical fluctuations.
Independent of acceptance.

K to π fluctuations display a clear $1/N_{part}$ dependence with the addition of a constant term, while p to π fluctuations appear flat and has lower absolute values

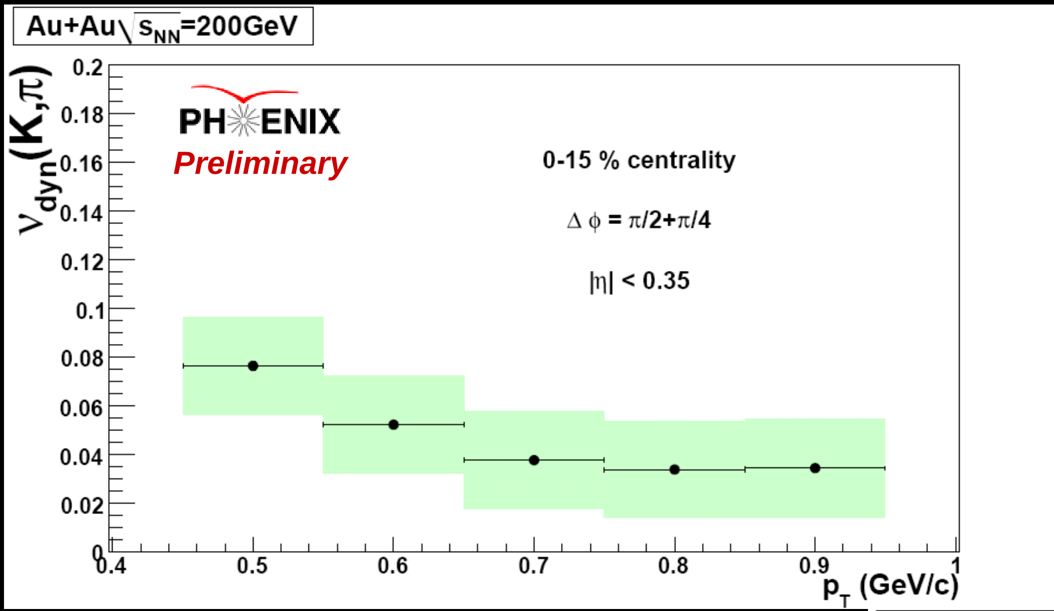
$$\nu_{dyn}(p,\pi) = \frac{\langle \pi(\pi-1)\rangle}{\langle\pi\rangle^2} + \frac{\langle p(p-1)\rangle}{\langle p\rangle^2} - 2\frac{\langle \pi\, p\rangle}{\langle\pi\rangle\langle p\rangle}$$

Measuring particle ratio fluctuations should cancel the contributions due to volume fluctuations.

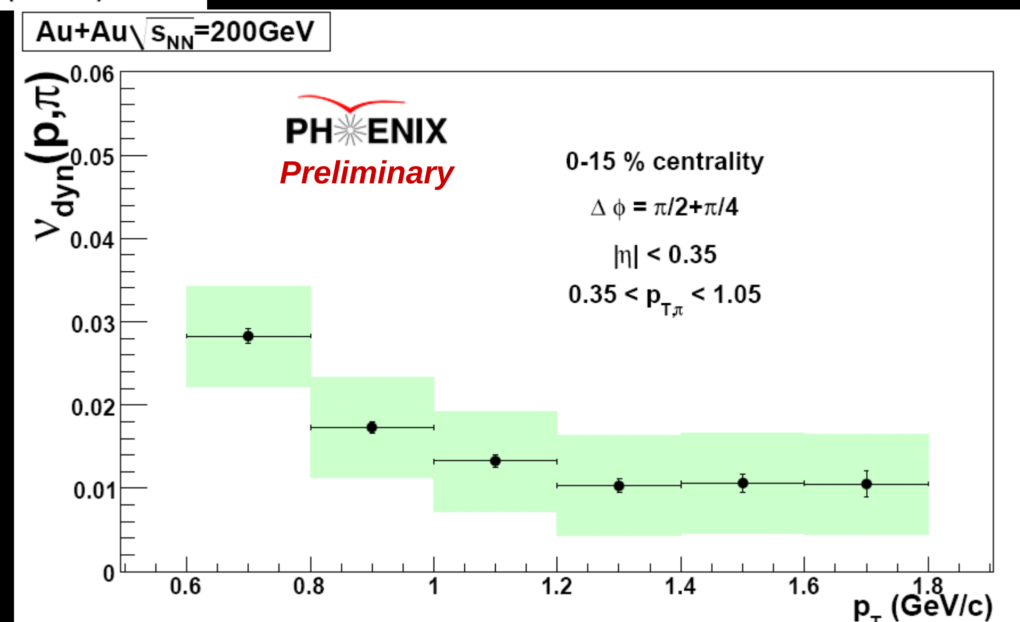




p_T-Dependence

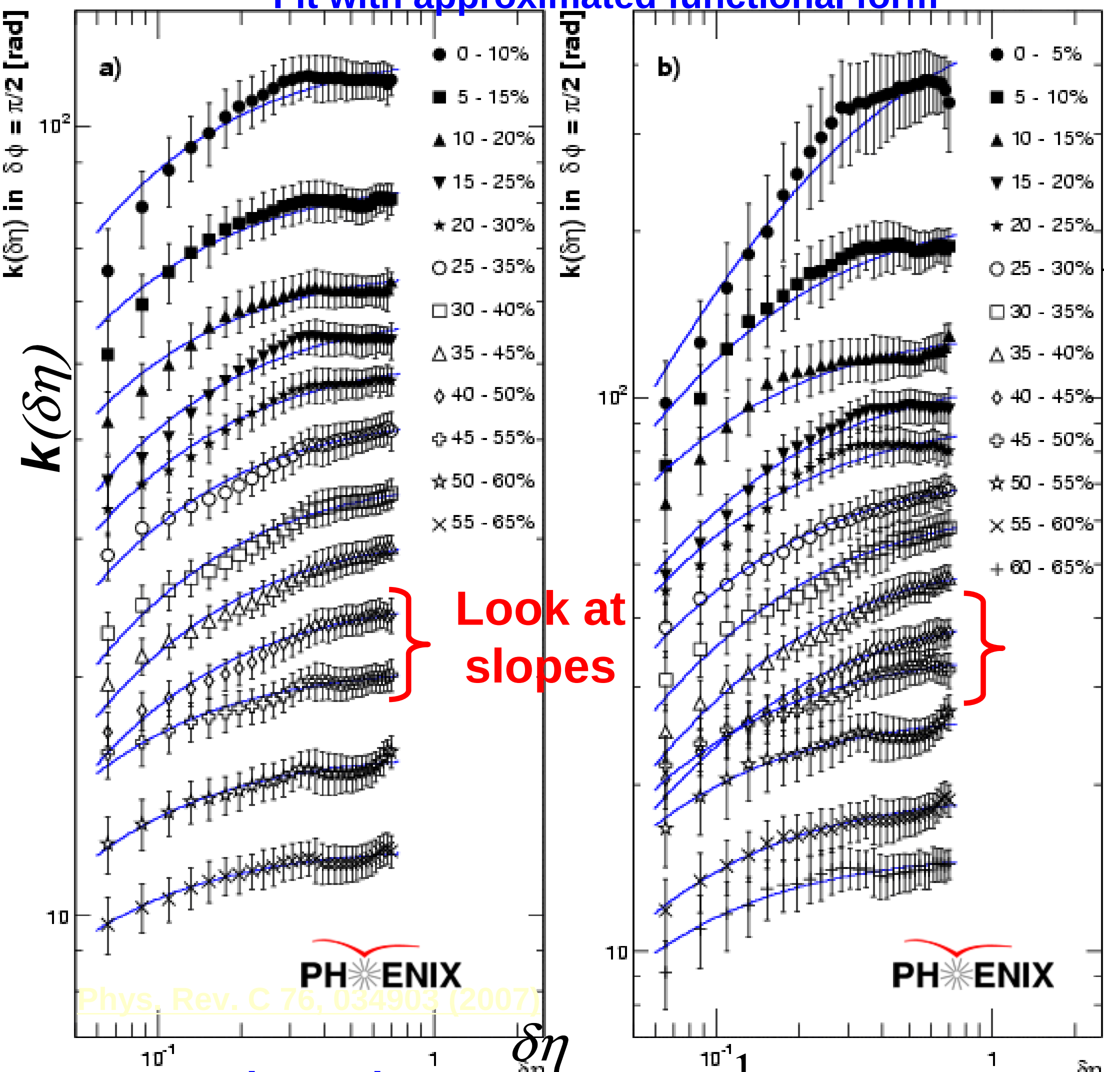


The dependence on transverse momentum
is similar for K to π (top) and p to π (bottom),
weakly increasing with decreasing momentum,
but there is a large difference in absolute value.



Extraction of αξ with multiplicity fluctuations

Fit with approximated functional form



Approximated functional form

$$k(\delta\eta) = \frac{1}{2\alpha\xi/\delta\eta + \beta} \quad (\xi << \delta\eta)$$

Parametrization of two particle correlation

$$C_2(\eta_1,\eta_2) \equiv \rho_2(\eta_1,\eta_2) - \rho_1(\eta_1)\rho_1(\eta_2)$$

$$\frac{C_2(\eta_1,\eta_2)}{\bar{\rho}_1^2} = \alpha e^{-\delta\eta/\xi} + \beta$$

β absorbs rapidity independent bias such as centrality bin width

Exact relation with NBD *k*

$$k^{-1}(\delta\eta) = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2} - 1$$

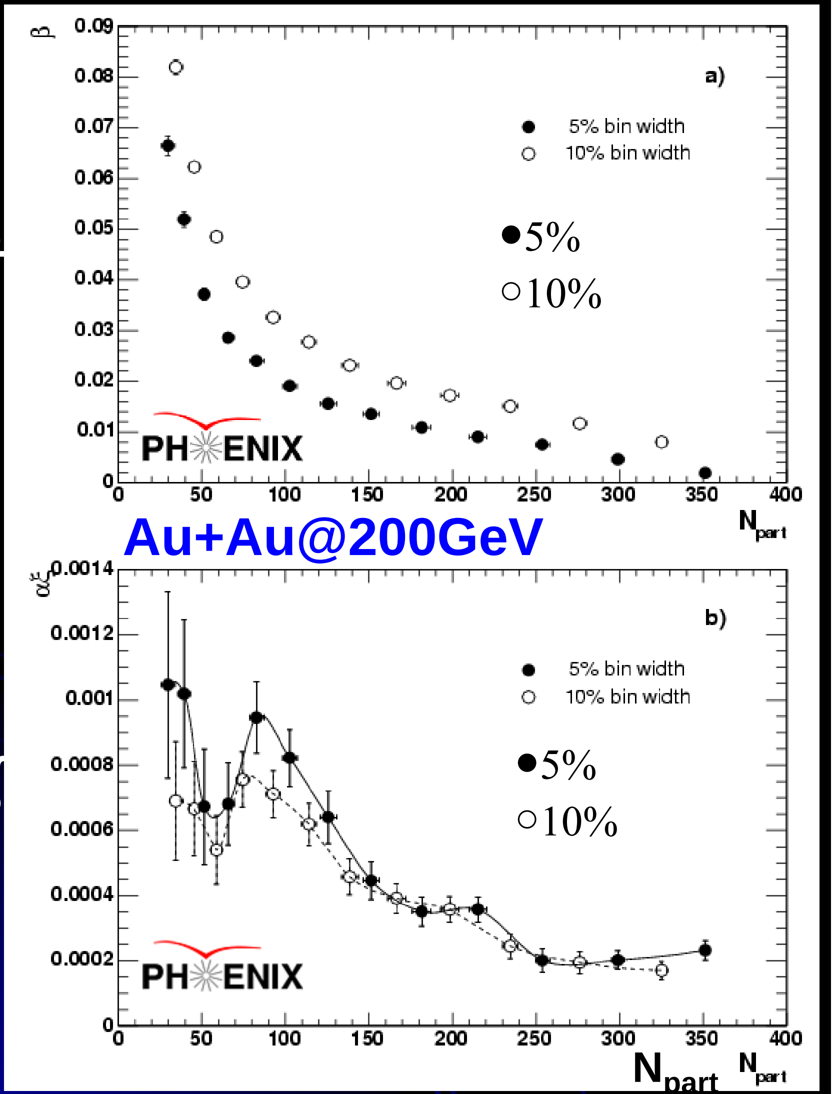
$$= \frac{\int_0^{\delta\eta}\int_0^{\delta\eta} C_2(\eta_1,\eta_2)\,d\eta_1 d\eta_2}{\delta\eta^2\bar{\rho}_1^2}$$

$$= \frac{2\alpha\xi^2(\delta\eta/\xi - 1 + e^{-\delta\eta/\xi})}{\delta\eta^2} + \beta$$

$\alpha\xi$, β vs. N_{part}

β

αξ



Phys. Rev. C 76, 034903 (2007)

β is systematically shift to lower values as the centrality bin width becomes smaller from 10% to 5%. This is understood as fluctuations of Npart for given bin widths

αξ product, which is monotonically related with $\chi_{k=0}$ indicates the non-monotonic behavior around Npart ~ 90.

$$\alpha\xi = \chi_{k=0} T / \bar{\rho}_1^{\,2} \propto \bar{\rho}_1^{\,-2} \frac{T}{|T - T_C|}$$

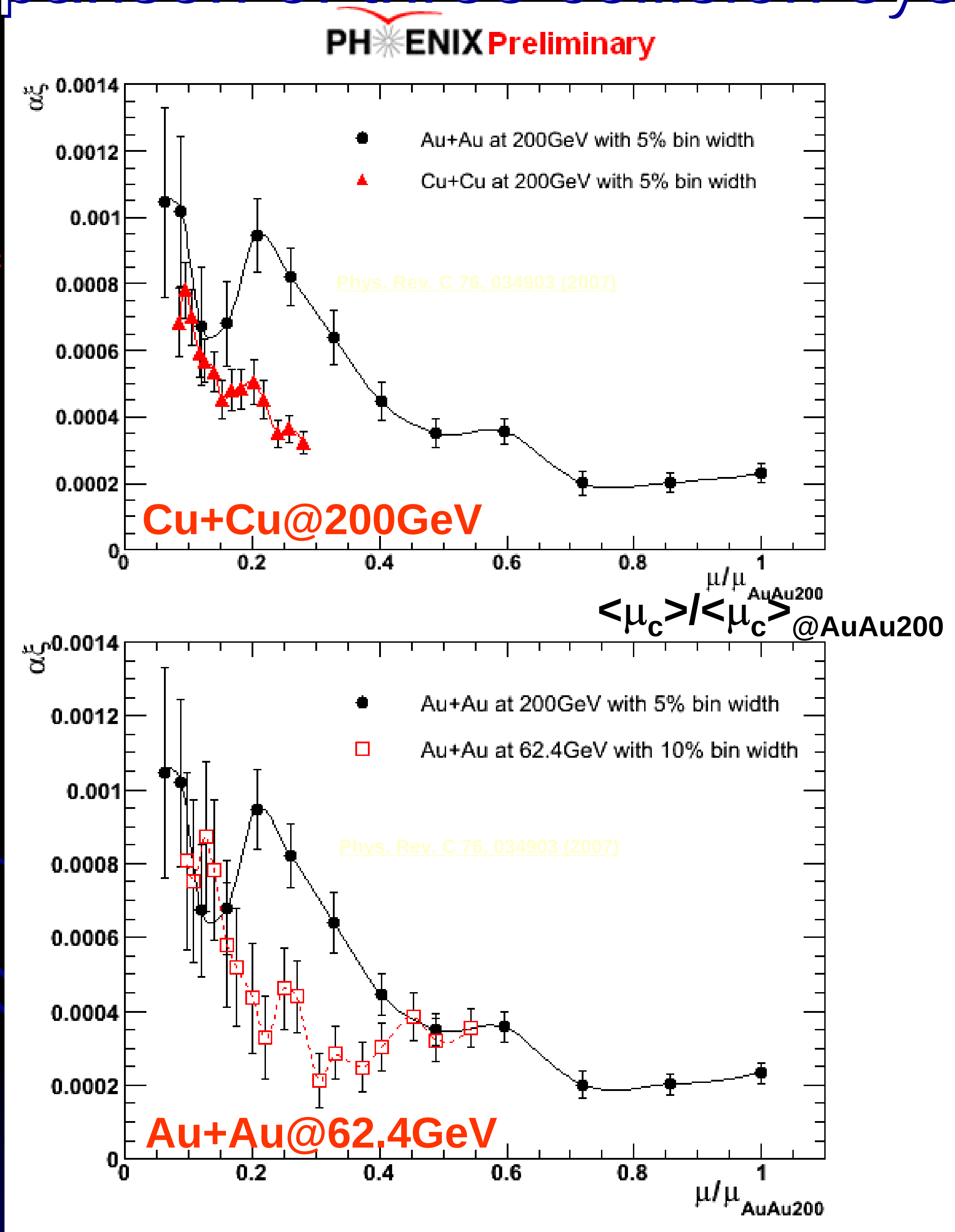
Significance with Power + Gaussian:
3.98 σ (5%), 3.21 σ (10%)
Significance with Line + Gaussian:
1.24 σ (5%), 1.69 σ (10%)

Comparison of three collision systems

Npart~90 in AuAu@200GeV

$\varepsilon_{BJ}\tau$~2.4GeV/fm^2/c

$\alpha\xi$



$<\mu_c>/<\mu_c>_{@AuAu200}$

Normalized mean multiplicity to that of top 5% in Au+Au at 200 GeV

Azimuthal Correlations at Low p_T

- **This study will quote correlation amplitudes in a given centrality, p_T, and $\Delta\phi$ bin with no trigger particle determined using the mixed event method via:**

$C(\Delta\phi) = (dN/d\phi_{data} / dN/d\phi_{mixed}) * (N_{events,mixed} / N_{events,data})$

- **There is no trigger particle. All particle pairs are included in the correlation function calculation.**
- **Red dashed lines are fits to the following equation:**

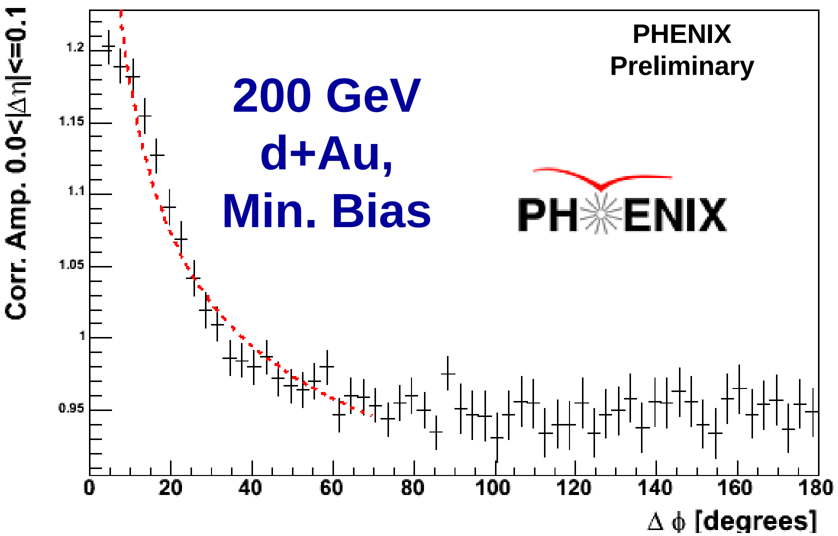
$$C(\Delta\phi) \propto \Delta\phi^{-(1+\eta)}$$

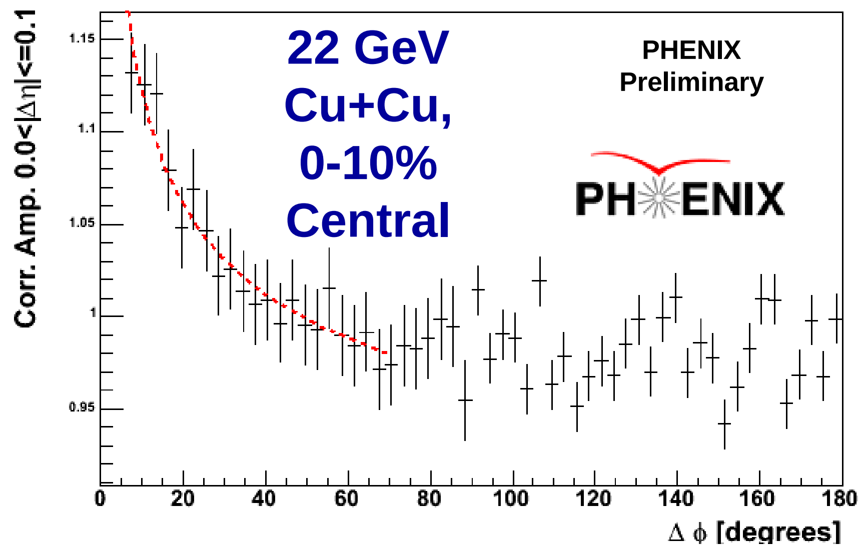
- **Shown are results for the following systems:**
  - **200 GeV Au+Au**
  - **62.4 GeV Au+Au**
  - **200 GeV Cu+Cu**
  - **62.4 GeV Cu+Cu**
  - **22.5 GeV Cu+Cu**
  - **200 GeV d+Au**
  - **200 GeV p+p**

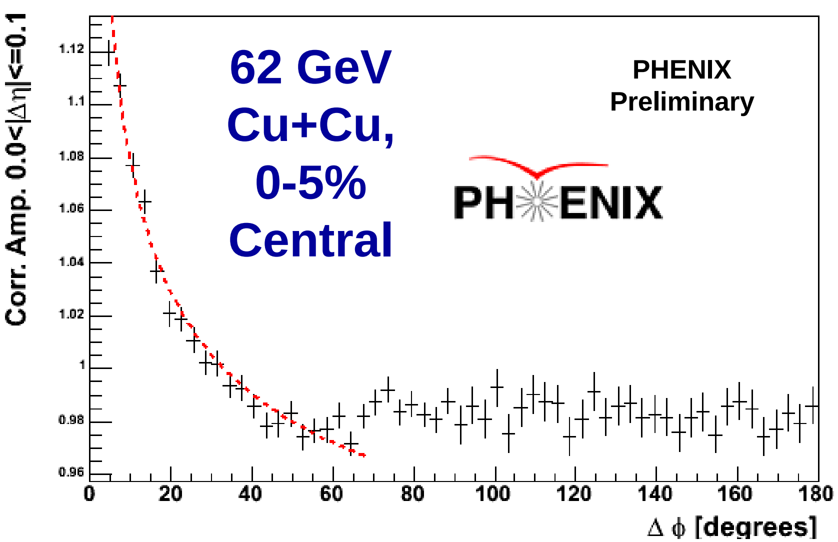
Assuming that QCD belongs in the same universality class as the (d=3) 3-D Ising model, the expected value of η is 0.5 (Reiger, Phys. Rev. B52 (1995) 6659 .

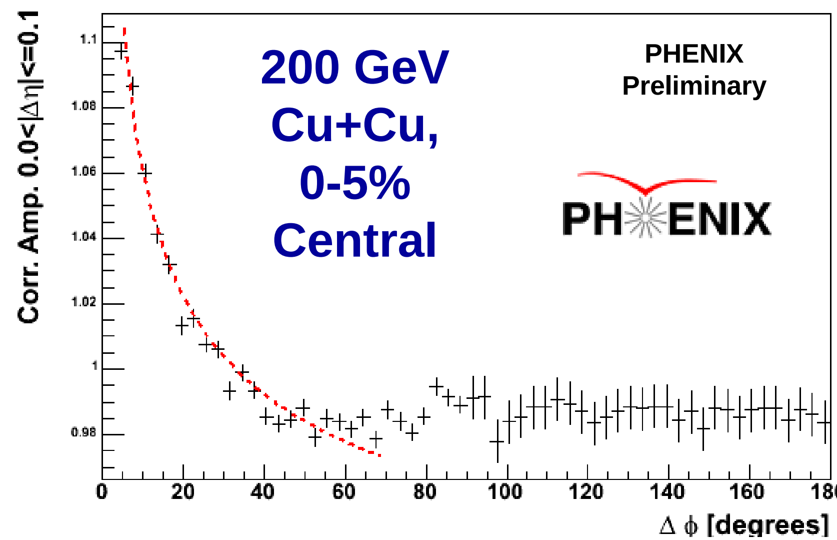
Like-Sign Pair Azimuthal Correlations: d+Au, Cu+Cu

$0.2 < p_{T,1} < 0.4$ GeV/c, $0.2 < p_{T,2} < 0.4$ GeV/c, $|\Delta\eta|<0.1$



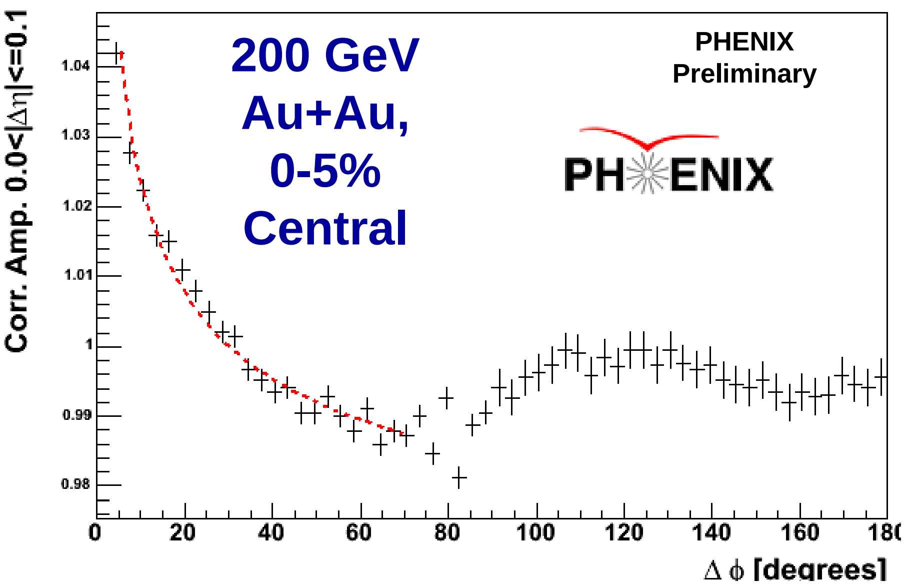


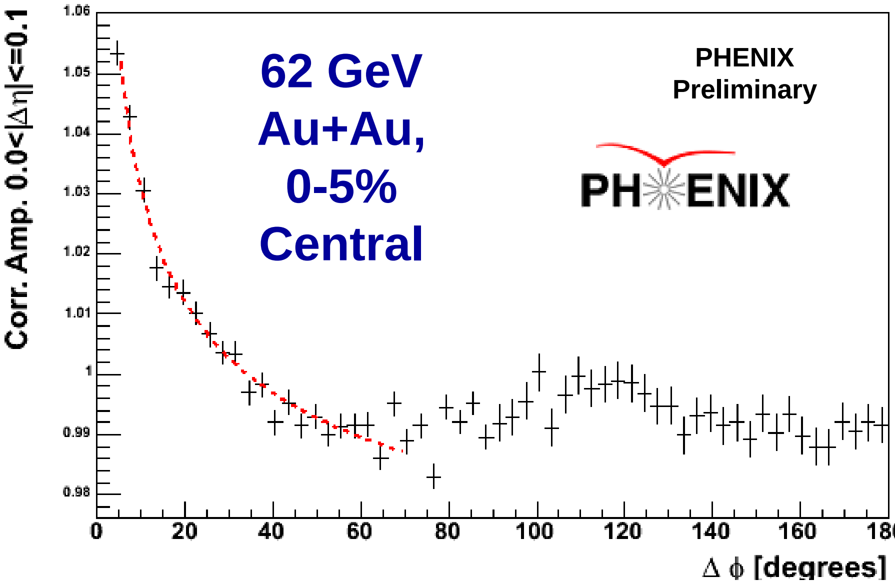




Like-Sign Pair Azimuthal Correlations: Au+Au

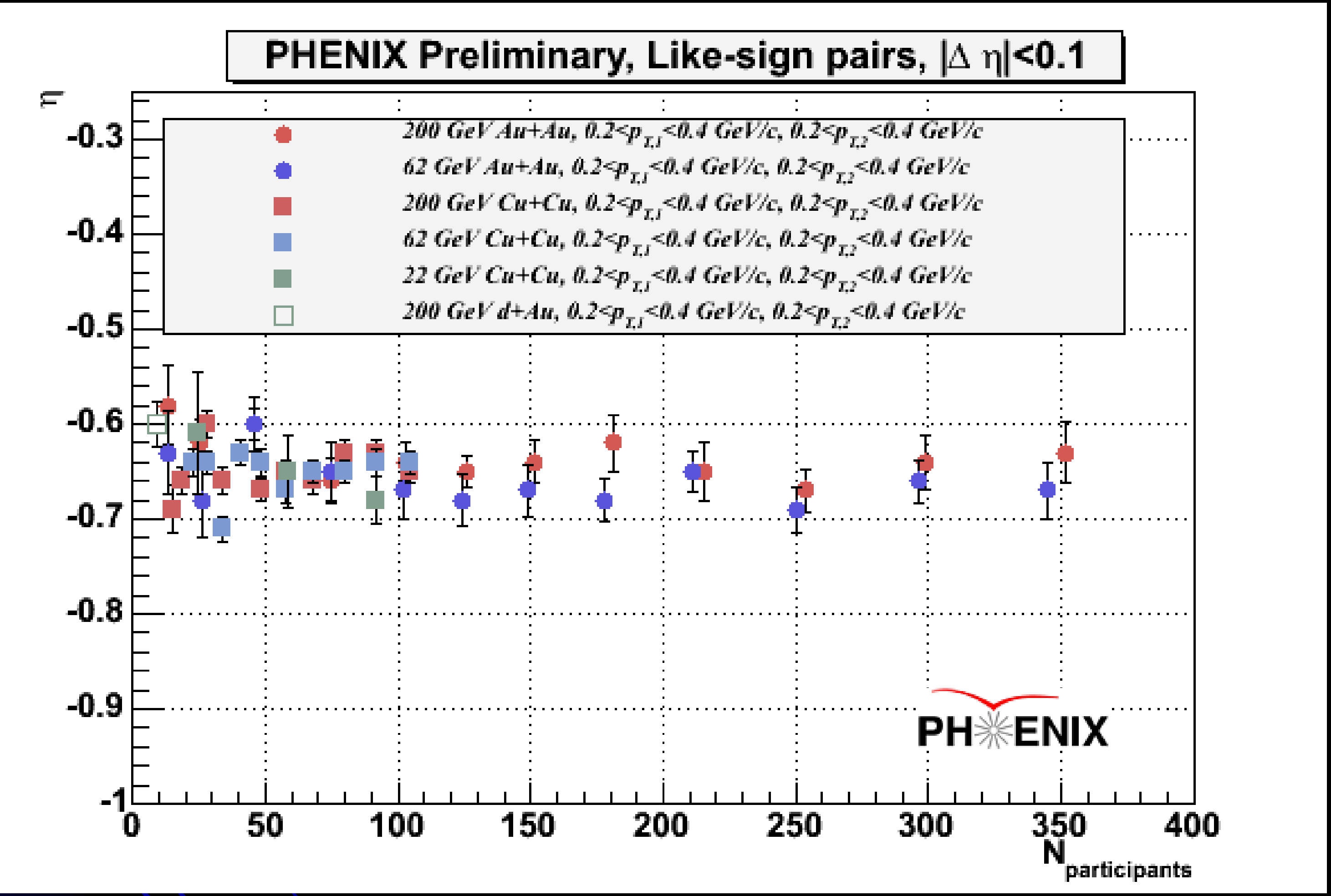
$0.2 < p_{T,1} < 0.4$ GeV/c, $0.2 < p_{T,2} < 0.4$ GeV/c, $|\Delta\eta|<0.1$





- The power law function fits the data well for all species and centralities.
- A displaced away-side peak is observed in the Au+Au correlation functions.

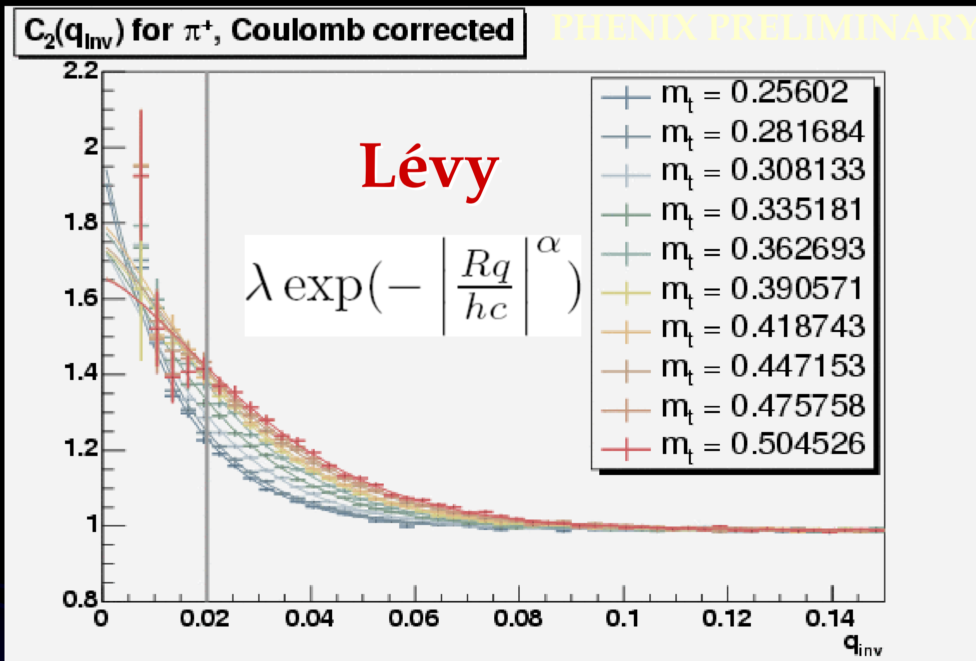
Exponent η vs. Centrality

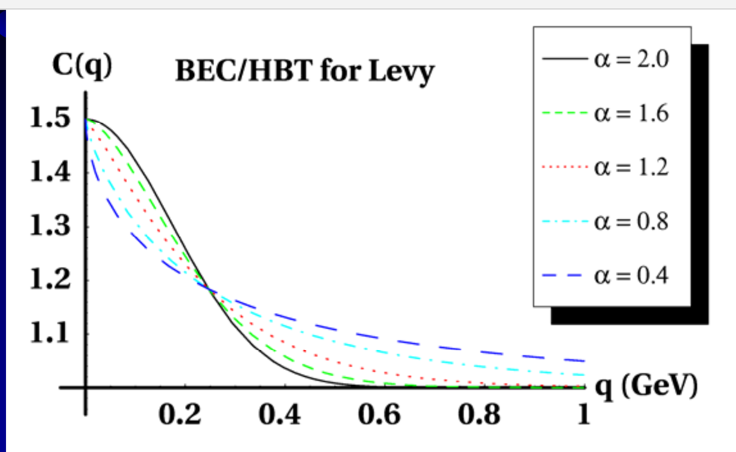


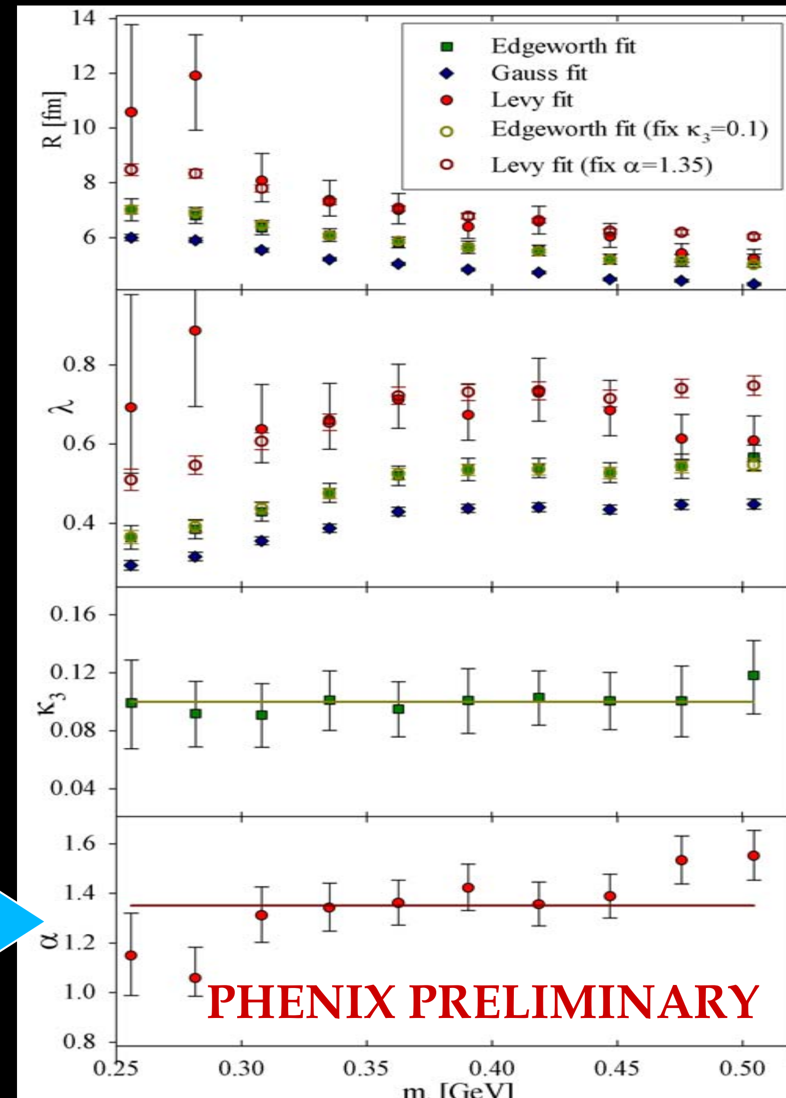
The exponent η is independent of species, centrality, and collision energy.

The value of η is inconsistent with the d=3 expectation at the critical point.

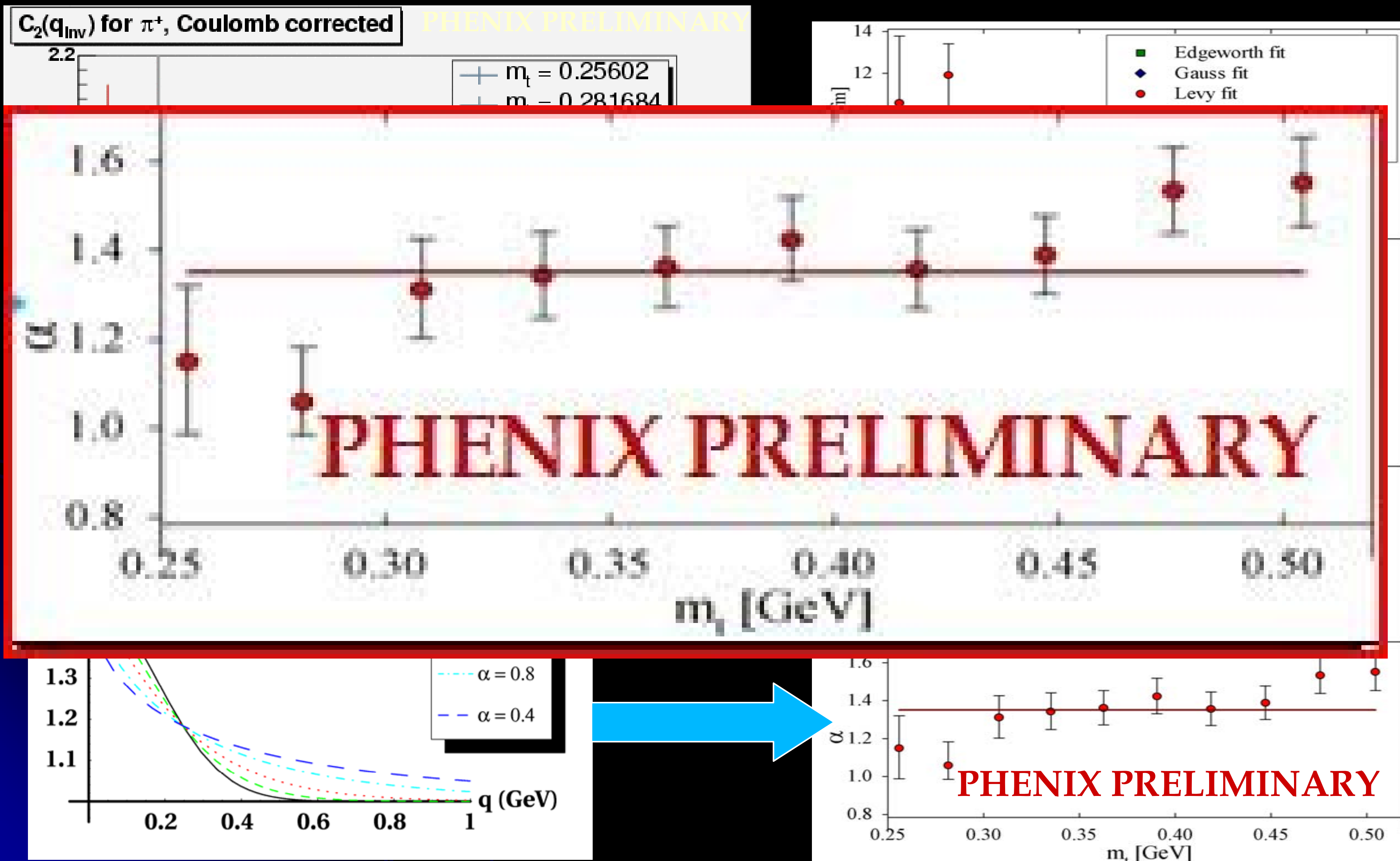
Lévy fits to q_{inv}
Central 200 GeV Au+Au





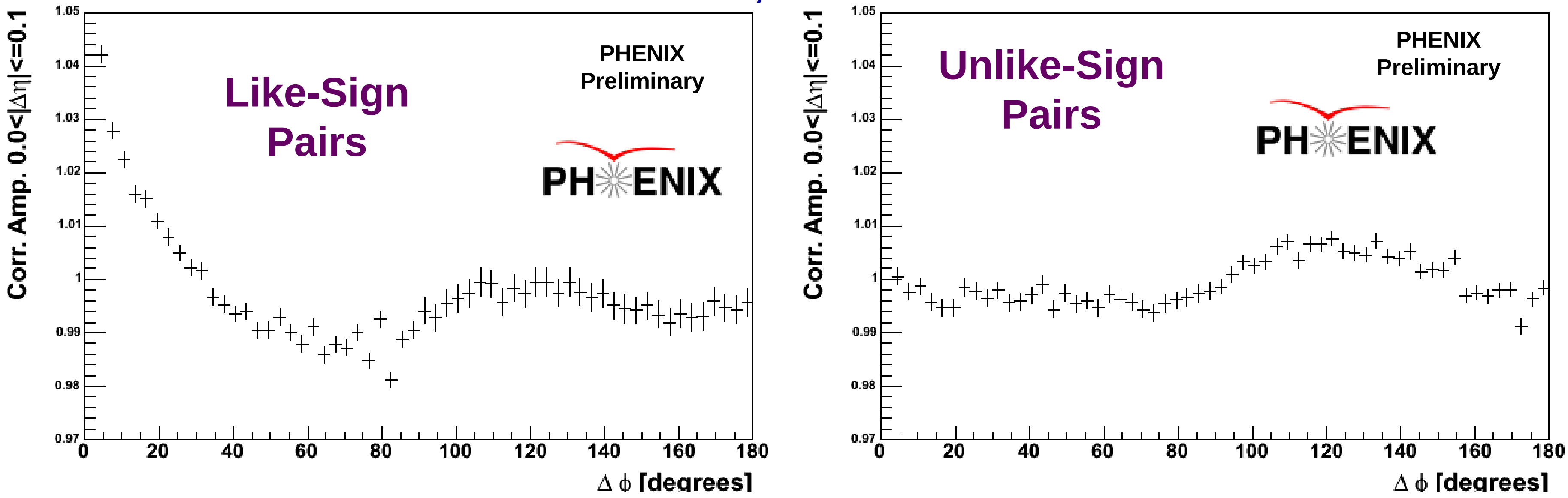


Lévy fits to q_{inv}
Central 200 GeV Au+Au



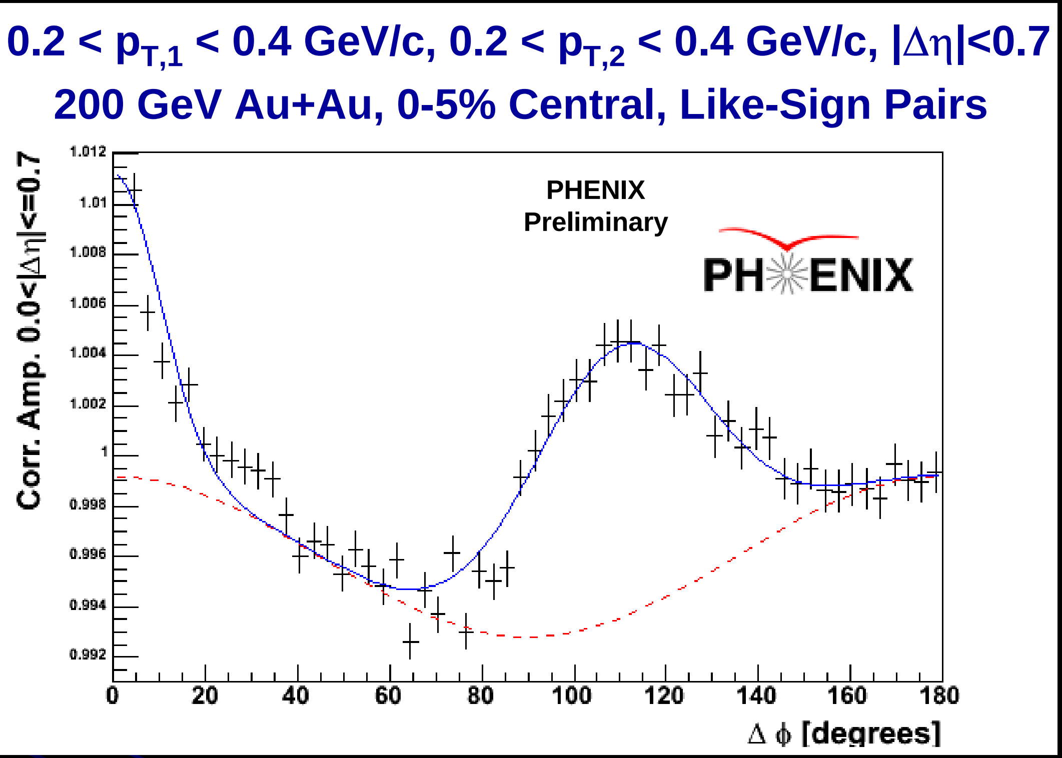
Controlling HBT: LS vs. US Pairs

$0.2 < p_{T,1} < 0.4$ GeV/c, $0.2 < p_{T,2} < 0.4$ GeV/c, $|\Delta\eta|<0.1$
200 GeV Au+Au, 0-5% Central



- The HBT peak apparent in like-sign pair correlations disappears in unlike-sign pair correlations.
- The displaced away-side peak persists both like-sign and unlike-sign pair correlations.
- The displaced away-side peak extends across the PHENIX acceptance in $\Delta\eta$.

Extracting the properties of the correlations

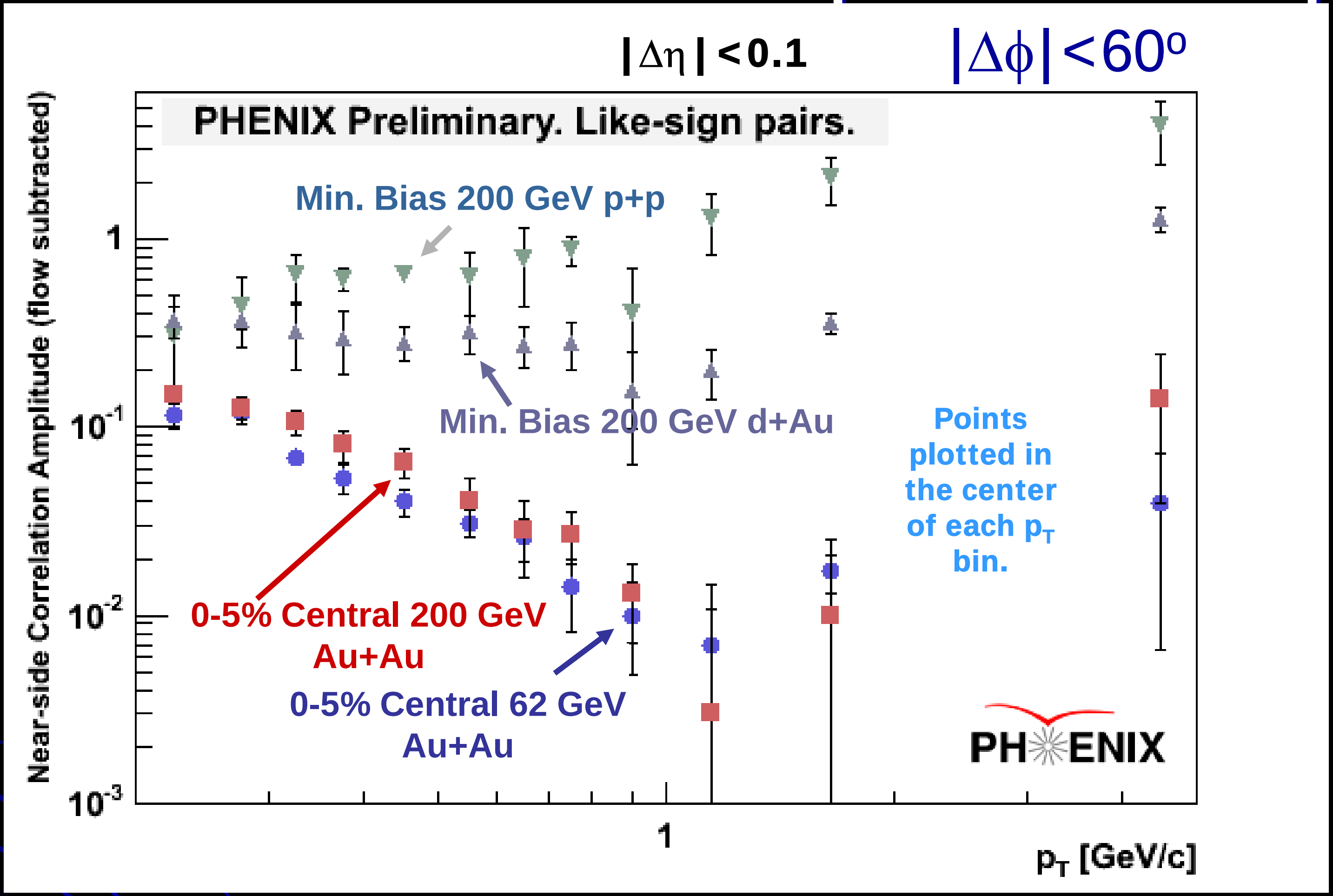


- The blue line is a fit to a function with a v_2 component, a near-side Gaussian at $\Delta\phi=0$ and an away-side Gaussian at $\Delta\phi=\pi$-D
- The dashed red line is the v_2 component.

$$C(\Delta\varphi) = B(1 + 2c_2\cos(2\Delta\varphi)) + Gauss_{Near,1}(\Delta\varphi;\sigma_{Near}) + AJ(\Delta\varphi - \pi)$$

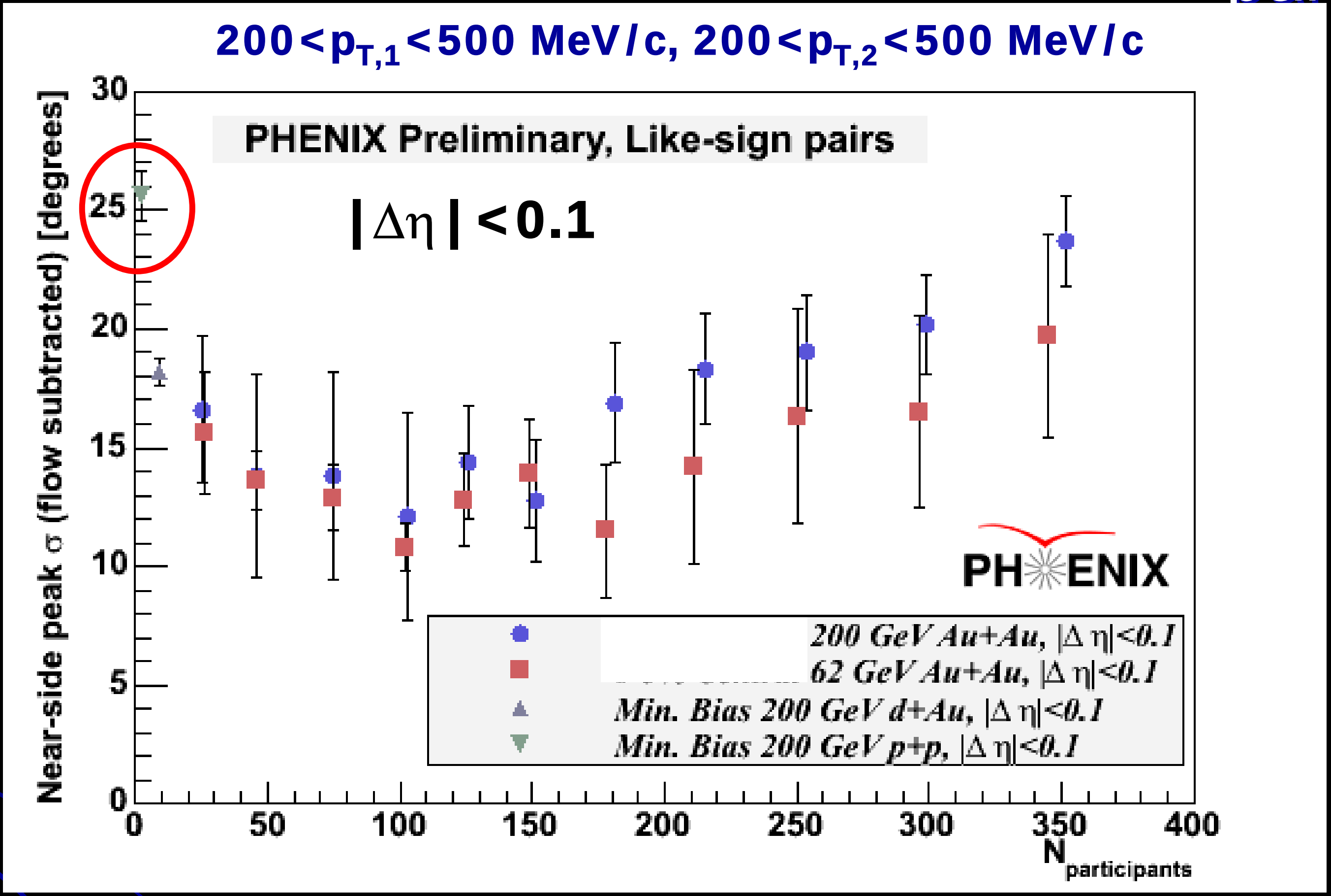
$$AJ(\Delta\varphi - \pi) \equiv \frac{S_A}{\sqrt{2\pi}\sigma_A}\left[\exp\left\{-\frac{(\Delta\varphi - \pi - D)^2}{2\sigma_A^2}\right\} + \exp\left\{-\frac{(\Delta\varphi - \pi + D)^2}{2\sigma_A^2}\right\}\right]$$

Near-Side Peak Amplitude vs. p_T



- The p_T bins have been chosen so that there are equal numbers of particles per event in each bin to offset the effects of statistical dilution of the correlation amplitudes.
- The Au+Au amplitudes for p_T<1 GeV/c show a power law decrease with p_T not seen in p+p or d+Au.
- The increase in amplitudes for p_T>1 GeV/c are due to the onset of the jet peak.

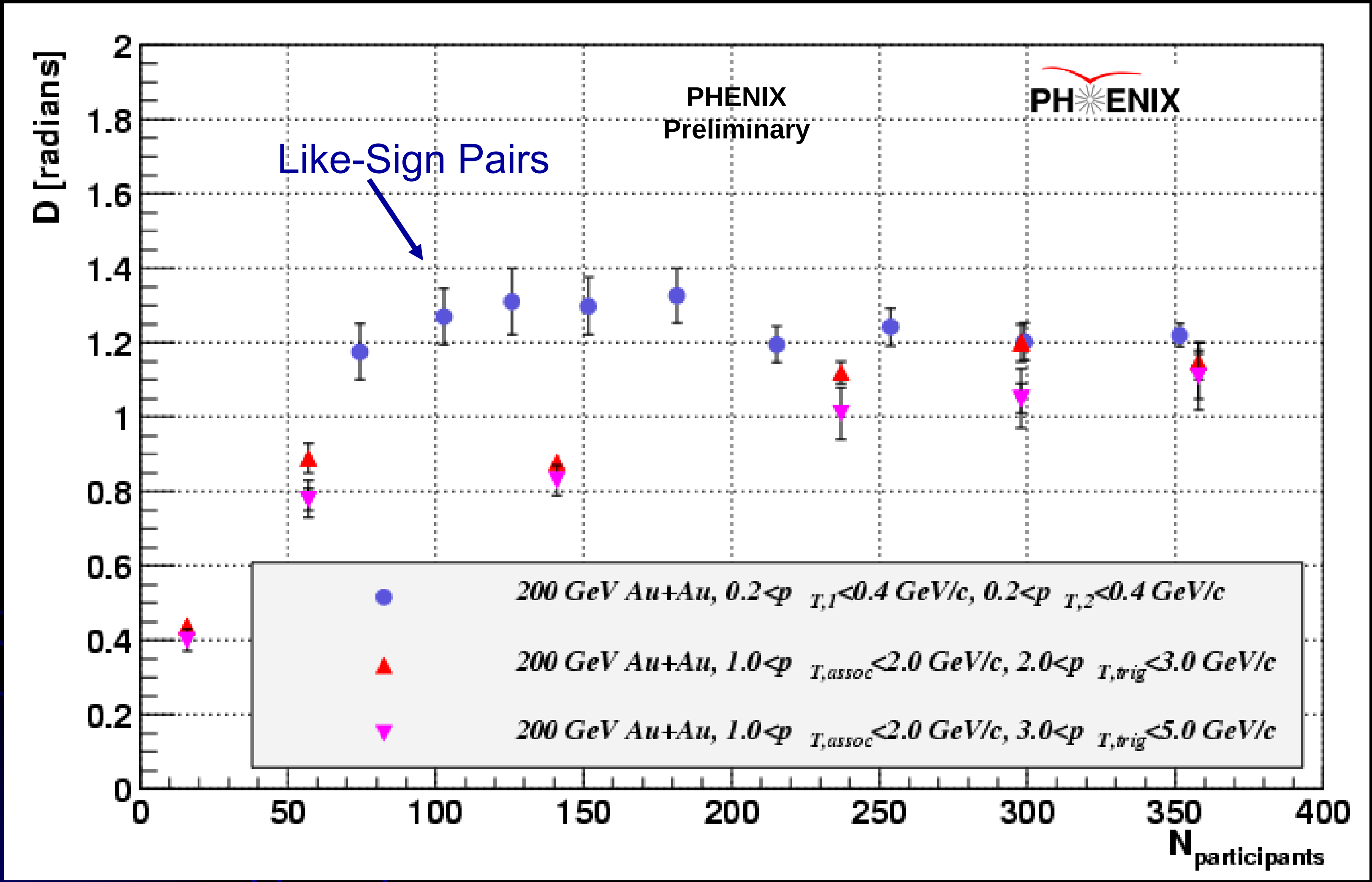
Near-Side Peak Width vs. N_{part}



Weak centrality dependence on the near-side peak widths.

d+Au and Au+Au widths are narrower than p+p.

Location of the Displaced Away-Side Peak

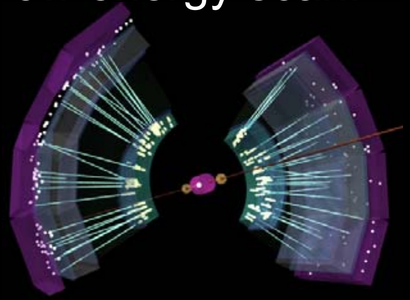


The location of the displaced peak at low p_T shows little centrality dependence. The location deviates from that at high p_T in more peripheral collisions.

Summary

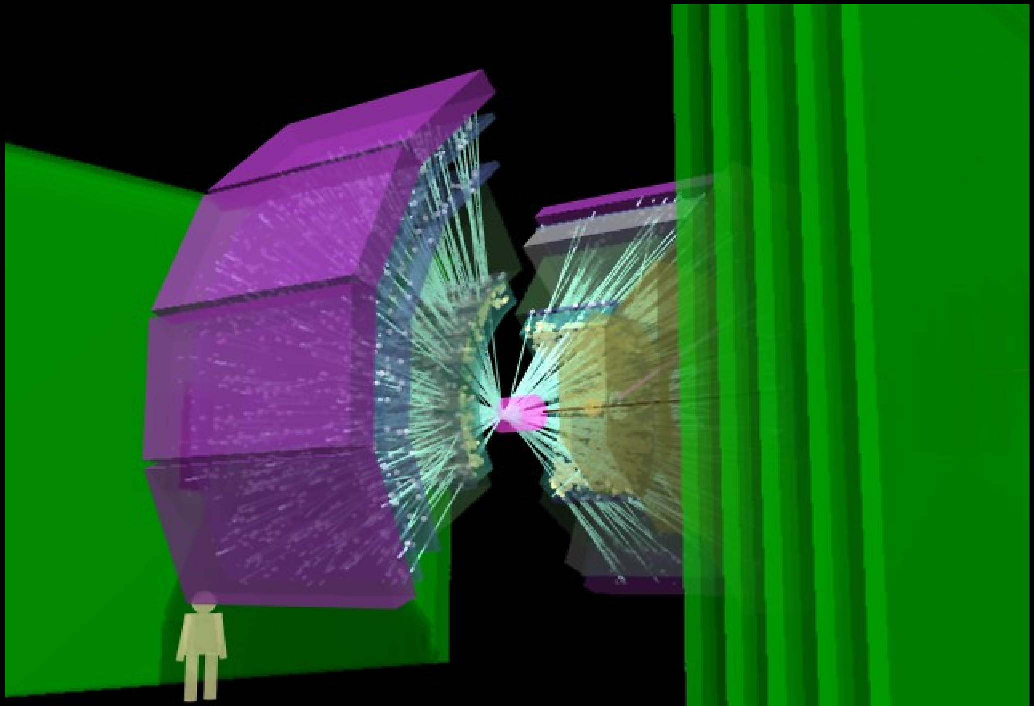
- **Multiplicity fluctuations**:
  - Consistent with or below the expectation of a participant superposition model based upon p+p data. No evidence for critical behavior seen.
- **$\langle p_T \rangle$ fluctuations:**
  - Exhibit a universal power law scaling as a function of N_{part} in central collisions.
  - The magnitude of $\langle p_T \rangle$ fluctuations as a function of sqrt(s_{NN}) do not scale with the jet production cross section.
- **Baryon-baryon and Meson-meson Fluctuations**
  - **$\langle K/\pi \rangle$ fluctuations ~1/N_{part}, $\langle p/\pi \rangle$ fluctuations relatively flat with N_{part}**
- **Extraction of $\alpha\xi$ with Multiplicity Fluctuations at low p_T**
  - Possible non-monotonic behavior at Npart~90
- **Low-p_T Correlations**:
  - The exponent η extracted from the HBT peak is identical for all collision species. No evidence of critical behavior is seen.
  - A displaced away-side peak is observed in azimuthal correlations at low p_T in Au+Au collisions.
  - Further studies of this phenomenon are underway.

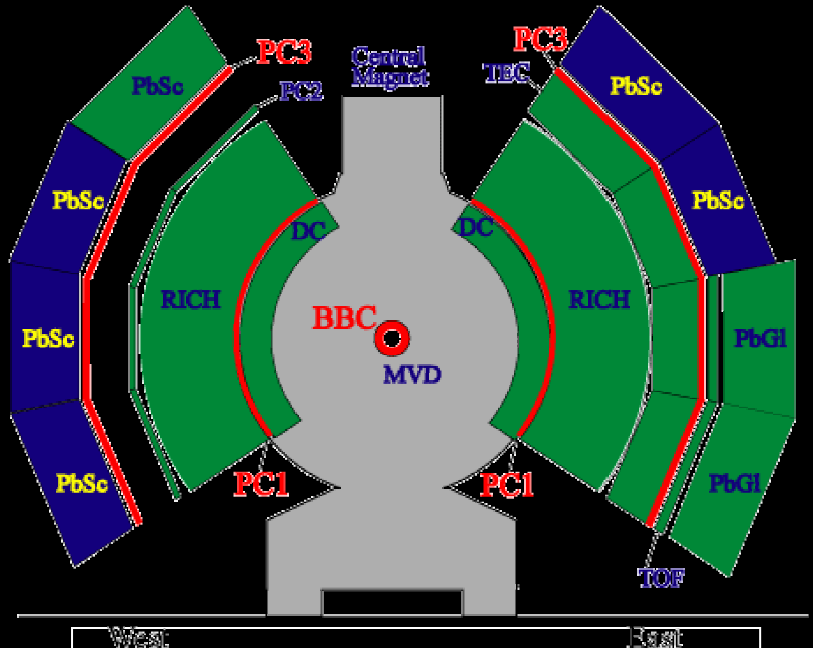
The analysis framework for measuring several critical exponents in RHIC collisions is in place → Bring on a RHIC low energy scan!



Auxiliary Slides

The PHENIX Detector





Acceptance:

$|\eta| \sim 0.35$, $|\Delta\phi| \sim \pi$

Two "central arm" spectrometers anchored by drift chambers and pad chambers for 3-D track reconstruction within a focusing magnetic field.

Although the PHENIX acceptance is traditionally considered small for event-by-event measurements, the acceptance is large enough to provide a competitive sensitivity to most observables.

p_T Fluctuations: Updating the Measure

- The consensus to quantify dynamical p_T fluctuations
  - Define the quantity $<\Delta p_{T,1}\Delta p_{T,2}>$.
  - It is a covariance and an integral of 2-particle correlations.
  - It equals zero in the absence of dynamical fluctuations
  - Defined to be positive for correlation and negative for anti-correlation.

$$\langle \Delta p_{t,1} \Delta p_{t,2} \rangle = \frac{1}{N_{event}} \sum_{k=1}^{N_{event}} \frac{C_k}{N_k (N_k - 1)}$$

N_{event} = number of events

$\langle p_t \rangle_i$ = average p_t for i^{th} event

N_k = number of tracks for k^{th} event

$p_{t,i} = p_t$ for i^{th} track in event

where

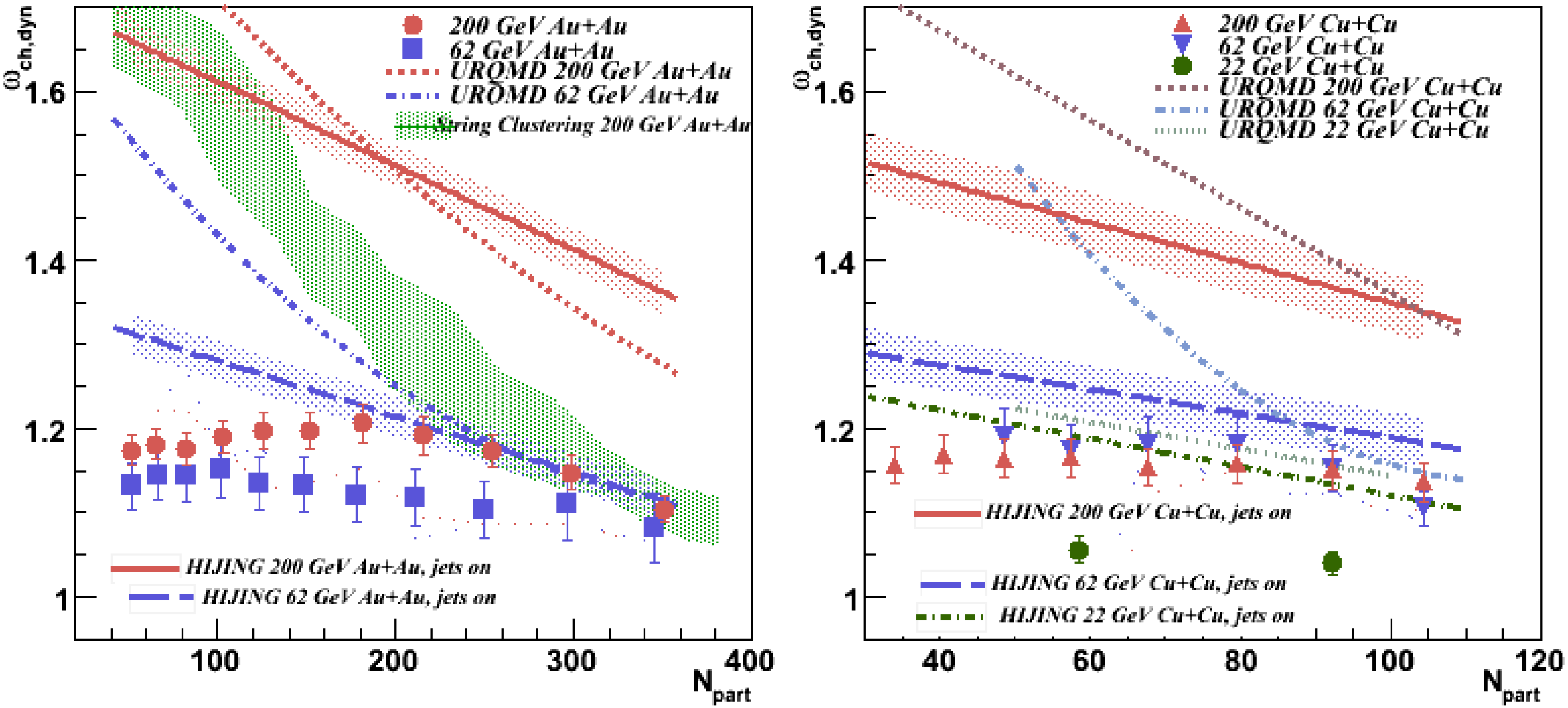
$$C_k = \sum_{i=1}^{N_k} \sum_{j=1, i \neq j}^{N_k} \left(p_{t,i} - \langle\langle p_t \rangle\rangle\right)\left(p_{t,j} - \langle\langle p_t \rangle\rangle\right)$$

and $\langle\langle p_t \rangle\rangle = \left(\sum_{k=1}^{N_{event}} \langle p_t \rangle_k \right) / N_{event}$ and $\langle p_t \rangle_k = \left(\sum_{i=1}^{N_k} p_{t,i} \right) / N_k$

Then normalize as follows for a dimensionless quantity:

$$\Sigma_{pT} = \sqrt{\langle \Delta p_{T,1} \Delta p_{T,2} \rangle} \Big/ \langle\langle p_T \rangle\rangle$$

Scaled Variance vs. URQMD



URQMD gives similar results to HIJING → Scaled variance decreases with centrality. Correction factors differ from HIJING by at most 10%. URQMD does not reproduce multiplicity as a function of centrality.

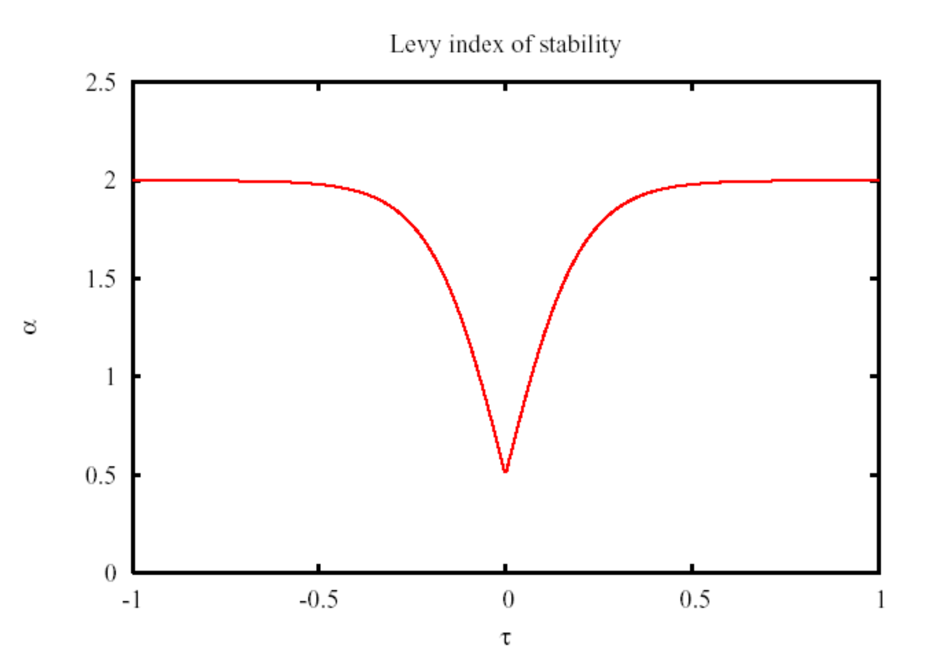
Correlation signal of the CEP

- If the source distribution
- at CEP is a Lévy, it decays as:

$$\rho(R) \propto R^{-(1+\alpha)}$$

- at CEP, the tail decreases as:

$$\rho(R) \propto R^{-(d-2+\eta)}$$



- hence: & excitation of α as a function of $\tau = | T - T_c| / T_c$

$$\alpha(\text{Lévy}) = \eta(\text{3d Ising}) = 0.50 \pm 0.05$$

T. Cs, S. Hegyi, T. Novák, W.A.Zajc, Acta Phys. Pol. B36 (2005) 329-337

Slide by T. Csorgo