Initial conditions in A+A, p+A, d+A and p+p collisions

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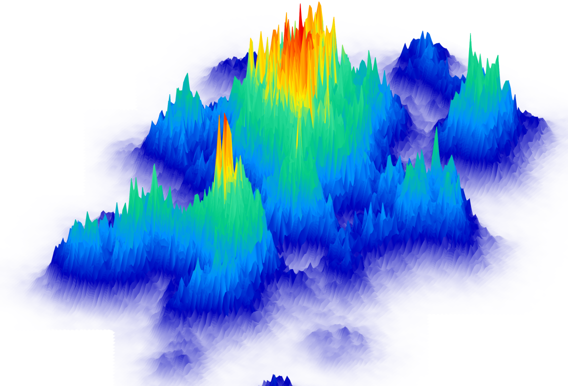
Jet Quenching at RHIC vs LHC in Light of Recent dAu and pPb Controls

BNL, Upton, NY

BROOKHAVEN NATIONAL LABORATORY

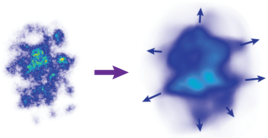


Introduction - Initial state fluctuations and flow

- Large elliptic flow has indicated fluid behavior of matter created at RHIC in early 2000's BNL announces "perfect liquid" in 2005 press release
- The importance of fluctuations in the initial state was realized later and analysis of odd flow harmonics began in 2010
- Measured anisotropic flow is largely due to the response to the initial fluctuating geometry
- Geometry of the medium is relevant for jet evolution
- I will
  - present different models for the initial state in A+A
  - show some results of harmonic flow from these models
  - apply the models to p+p, p+A and d+Au collisions

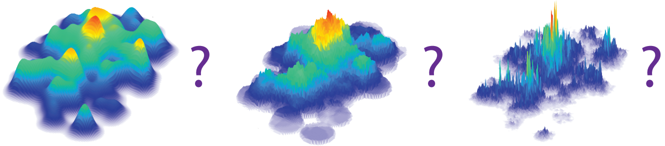
Modeling the initial state

Flow is driven by the initial geometry
Final result depends on what we start with



Hard partons are affected by the geometry of the background

We need a rigorous understanding
of the initial state and its fluctuations



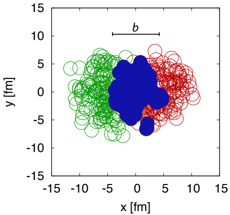
MC-Glauber MC-KLN IP-Glasma

MC-Glauber

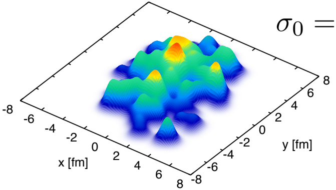
- Sample (un-)correlated nucleons in nucleus A and B, then overlap the two distributions with impact parameter b
- Interaction when the distance d between a nucleon from nucleus A and one from nucleus B fulfills (black disk):

$$d \leq \sqrt{\sigma_{\rm inel}^{NN}/\pi}$$

(alternatively use a Gaussian overlap function)



- Add Gaussian energy density with width σ_0 for every wounded nucleon, binary collision, or combination

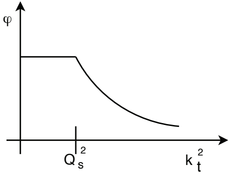


$$\sigma_0 = 0.4\,\mathrm{fm}$$

MC-KLN

- Sample nucleon positions from Woods-Saxon distribution
- Local nucleon density $T_{A,B}(\mathbf{x}_\perp)$ from counting nucleons in tube of transverse area $S = \sigma_{\rm inel}^{NN}$
- Kharzeev-Levin-Nardi (KLN) model for uGDF ϕ:

$$\phi_{A,B}(x, k_\perp^2, \mathbf{r}_\perp) \sim \frac{1}{\alpha_s(Q_s^2)} \frac{Q_s^2}{\max(Q_s^2, k_\perp^2)}$$



with $Q_{s,A}^2(x, \mathbf{r}_\perp) = 2\,\mathrm{GeV}^2 \frac{T_A(\mathbf{x}_\perp)}{1.53\,\mathrm{fm}^{-2}} \left(\frac{0.01}{x}\right)^\lambda, \lambda = 0.28$

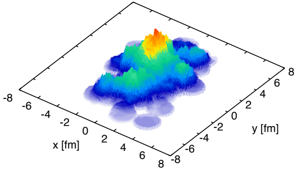
- Determine gluon production using k_T-factorization:

cannot be derived for dense-dense systems
$\rightarrow$ assume it works

$$\frac{dN}{dr_\perp^2\, dy} \sim \int \frac{d^2p_\perp}{p_\perp^2} \int d^2k_\perp \alpha_s \phi_A(x_1, k_\perp^2) \phi_B(x_2, (p_\perp - k_\perp)^2)$$

- Energy density analogously

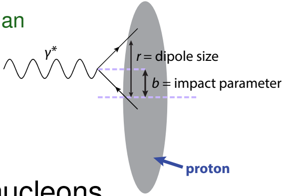
Drescher, Nara, Phys.Rev. C75 (2007) 034905



IP-Glasma: before the collision

- Energy and impact parameter b dependence of $Q_s(x, \mathbf{b})$ can be modeled in the IP-Sat model Kowalski, Teaney, Phys.Rev. D68 (2003) 114005

- Parametrize cross sections for DIS on protons and fit to HERA diffractive data
  A. Rezaeian, M. Siddikov, M. Van de Klundert, R. Venugopalan Phys.Rev. D87 (2013) 034002



- Sample nucleon positions from Woods-Saxon distribution

- Sum Gaussian thickness functions T_p of A nucleons

$$\frac{\mathrm{d}\sigma^{\mathrm{p}}_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}_\perp}(\mathbf{r}_\perp, x, \mathbf{b}_\perp) = 2\,\mathcal{N}(\mathbf{r}_\perp, x, \mathbf{b}_\perp) = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c}\mathbf{r}_\perp{}^2\alpha_s(Q^2)xg(x, Q^2)\sum_{i=1}^{A} T_p(\mathbf{b}_\perp - \mathbf{b}_T^i)\right)\right]$$

then determine local $Q_s(\mathbf{x}_\perp)$ $(\mathcal{N}(1/Q_s(x, \mathbf{x}_\perp), x, \mathbf{x}_\perp) = 1 - e^{-1/2})$

- Color charge density $g\mu(\mathbf{x}_\perp)$ is proportional to $Q_s(\mathbf{x}_\perp)$

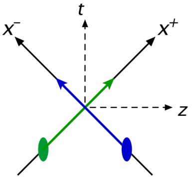
IP-Glasma: before the collision

- Self-consistently solve $x = Q_s(\mathbf{x}_\perp, x)/\sqrt{s}$
- Run the coupling with $Q_s(\mathbf{x}_\perp)$
- Sample color charges ρ^a from local Gaussian

$$\langle \rho^a(\mathbf{x}_\perp)\rho^b(\mathbf{y}_\perp)\rangle = \delta^{ab}\delta^2(\mathbf{x}_\perp - \mathbf{y}_\perp)g^2\mu^2(\mathbf{x}_\perp)$$

- Color charges determine incoming color currents:

$$J_1^\nu = \delta^{\mu+}\rho_1(x^-, \mathbf{x}_\perp)$$
$$[D_\mu, F^{\mu\nu}] = J_1^\nu$$

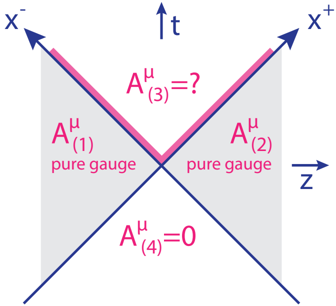


$$J_2^\nu = \delta^{\mu-}\rho_2(x^+, \mathbf{x}_\perp)$$
$$[D_\mu, F^{\mu\nu}] = J_2^\nu$$

- Solve Yang-Mills equations for the gauge fields
  $A^+(x^-, \mathbf{x}_\perp) = -\frac{g\rho(x^-,\mathbf{x}_\perp)}{\nabla_\perp^2 + m^2}$

IP-Glasma: after the collision

Initial condition on the lightcone:



Configuration in Schwinger gauge $A^\tau = 0$

Solution:

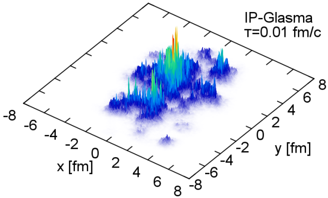
Kovner, McLerran, Weigert, Phys. Rev. D52, 3809 (1995)

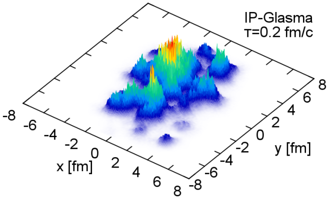
$$A^i_{(3)}|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

$$A^\eta_{(3)}|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

Numerical solution

Krasnitz, Venugopalan, Nucl.Phys. B557 (1999) 237

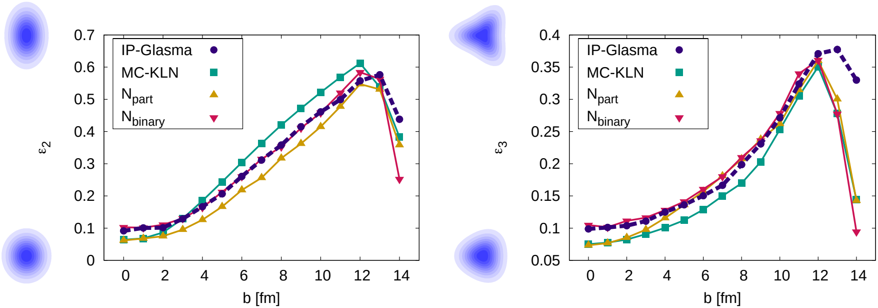




Eccentricities
B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.Lett. 108, 252301 (2012)

Characterize the initial distribution by its ellipticity, triangularity, etc...

$$\varepsilon_n = \sqrt{\langle r^n \cos(n\phi)\rangle^2 + \langle r^n \sin(n\phi)\rangle^2}/\langle r^n \rangle$$

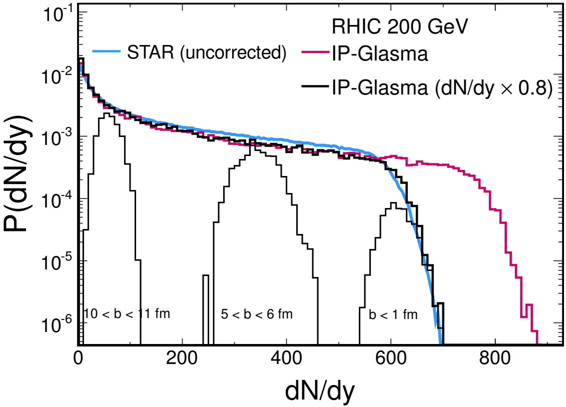


- ε_2 smaller in IP-Glasma than MC-KLN for (for $b > 3\,\mathrm{fm}$)
- ε_2 and ε_3 very similar between IP-Glasma and MC-Glauber with binary collision scaling

Multiplicity B.Schenke, P.Tribedy, R.Venugopalan, Phys. Rev. C86, 034908 (2012)

$P(dN_{\rm g}/dy)$ at time $\tau = 0.4\,{\rm fm}$ with $P(b)$ from a Glauber model

Experimental data: STAR, Phys. Rev. C79, 034909 (2009)



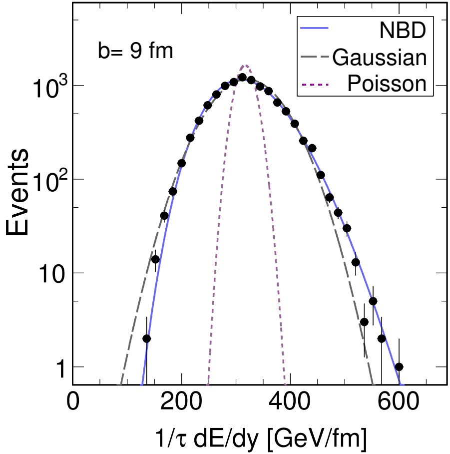
Glasma model gives a convolution of negative binomial distributions

No need to put them in by hand

Negative binomial fluctuations

B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.Lett.108, 252301 (2012)

Fluctuations in the total energy per unit rapidity produce negative binomial distribution (NBD).



$$P(n) = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}}$$

Good, since multiplicity in pp collisions can be described well with NBD.

In AA, convolution of NBDs at all impact parameters describes data well too.

P. Tribedy and R. Venugopalan
Nucl.Phys. A850 (2011) 136-156

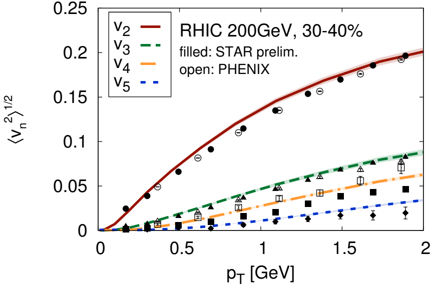
MC-KLN does not do that - these fluctuations need to be added by hand.

see Dumitru and Nara, Phys.Rev. C85 (2012) 034907

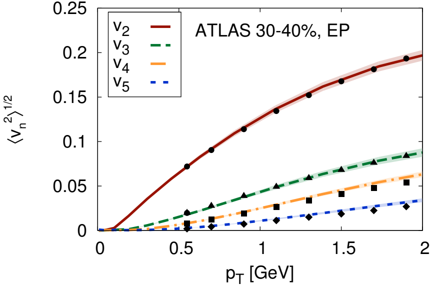
Viscous flow at RHIC and LHC

C. Gale, S. Jeon, B.Schenke, P.Tribedy, R.Venugopalan, PRL110, 012302 (2013)

RHIC $\eta/s = 0.12$



LHC $\eta/s = 0.2$



Initial state fluctuations + flow work to describe v_n in A+A

Lower effective η/s at RHIC than at LHC needed to describe data

Hints at increasing η/s with increasing temperature

Experimental data:

A. Adare et al. (PHENIX Collaboration), Phys.Rev.Lett. 107, 252301 (2011)

Y. Pandit (STAR Collaboration), Quark Matter 2012, (2012)

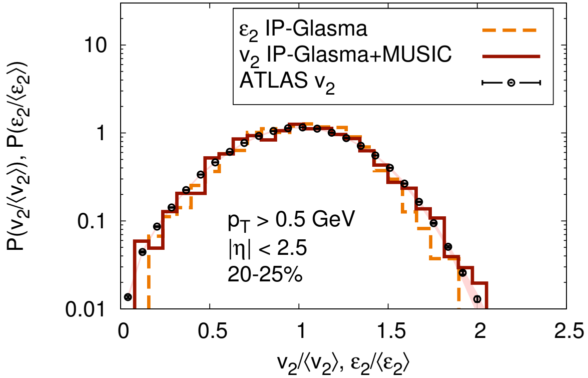
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)

Event-by-event distributions of v_n

Experimenatal data:

ATLAS collaboration https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF-2012-114/

J. Jia, S. Mohapatra, arXiv:1304.1471



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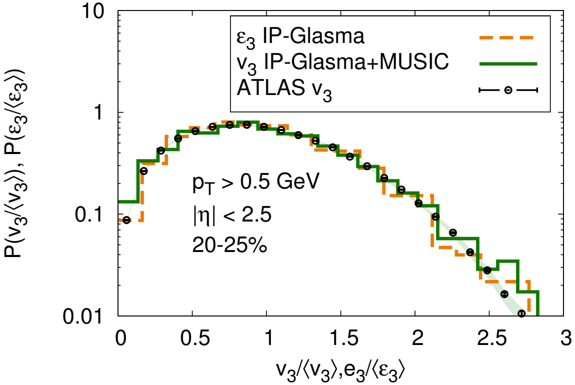

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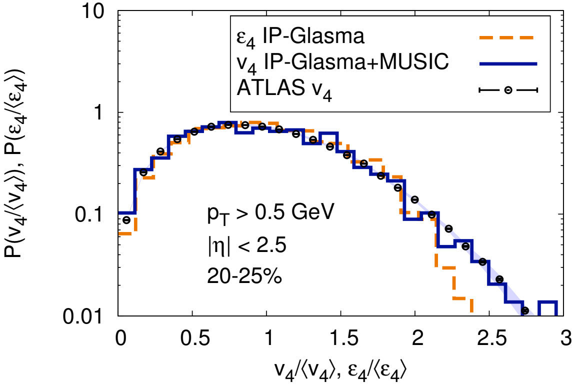

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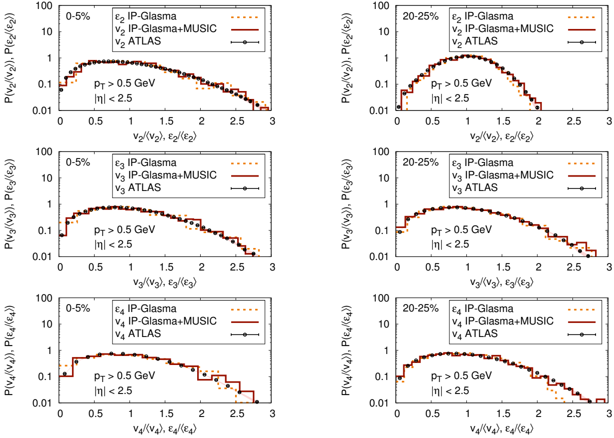
J. Jia, S. Mohapatra, arXiv:1304.1471

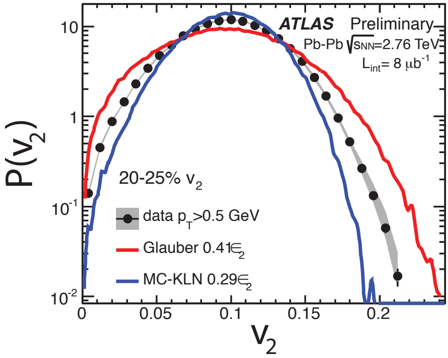
0-5% 20-25%

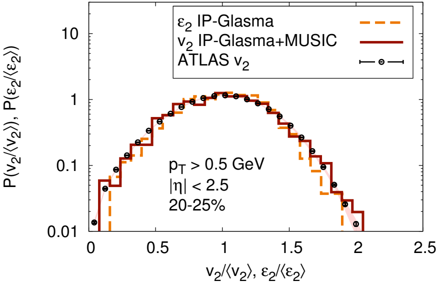


C. Gale, S. Jeon, B.Schenke, P.Tribedy, R.Venugopalan, PRL110, 012302 (2013)

Event-by-event distributions of v_n - other models

Showing eccentricity distributions (yellow on the right)





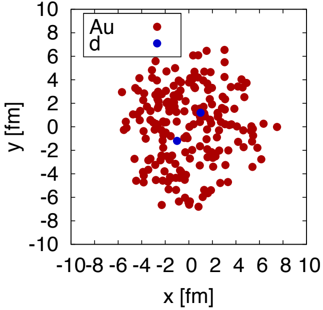
Event-by-event distributions can potentially distinguish between different initial state models

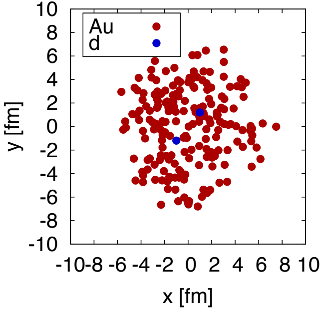
Experimental data: `https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF-2012-114/`

d+Au collisions A. Bzdak, B. Schenke, P. Tribedy, R. Venugopalan, arXiv:1304.3403

In small systems the different models lead to very different intial states

Energy density for the same nucleon positions:





In MC-Glauber all nucleons that are barely 'touched' contribute fully to the energy density

an MC-Glauber implementation is used in e.g. P. Bozek, Phys.Rev. C85 (2012) 014911

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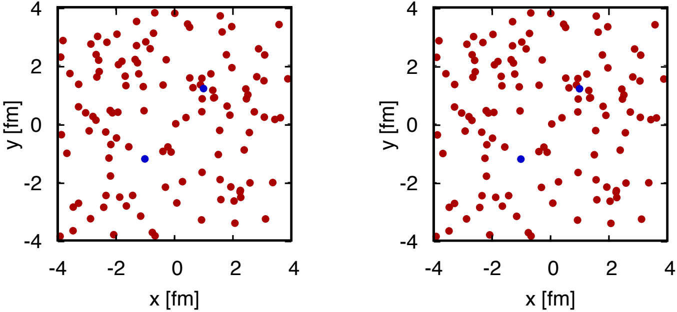

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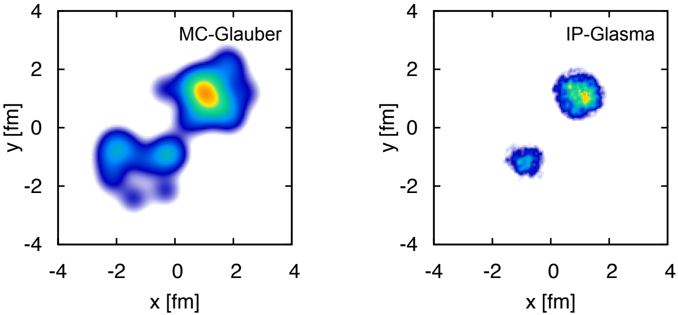

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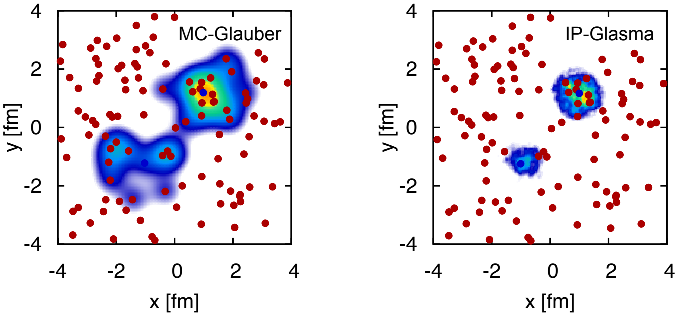

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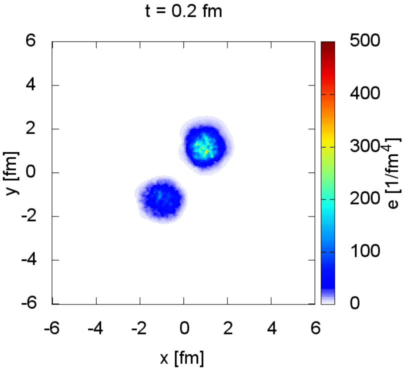
Evolution in d+Au

Energy density and initial flow velocity from $u_\mu T^{\mu\nu} = \varepsilon u^\nu$ as input for hydrodynamic simulation



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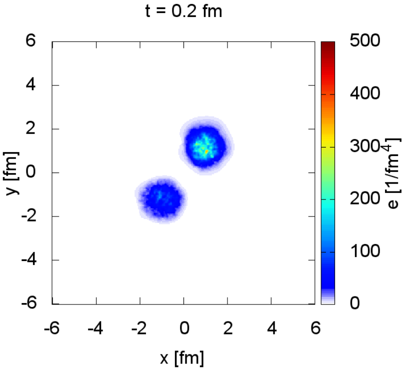
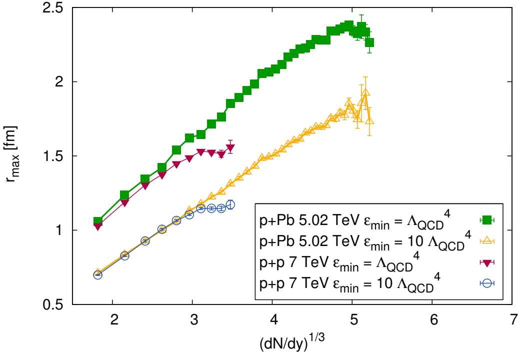


p+p and p+Pb collisions A. Bzdak, B. Schenke, P. Tribedy, R. Venugopalan, arXiv:1304.3403

Initial system size (vs dN/dy) in p+p and p+Pb collisions is very similar in the IP-Glasma model

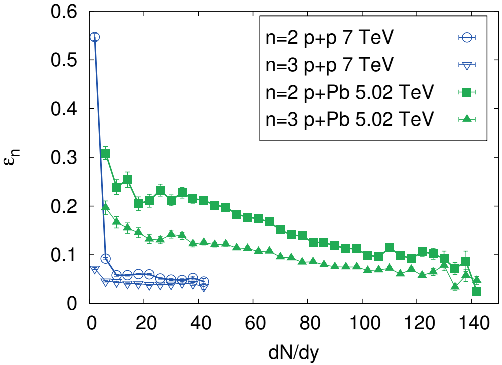
$r_{\max} =$ maximal radius at which energy density is above $\sim \Lambda^4_{\mathrm{QCD}}$



If there is no flow in either system, HBT radii will be similar

p+p and p+Pb collisions A. Bzdak, B. Schenke, P. Tribedy, R. Venugopalan, arXiv:1304.3403

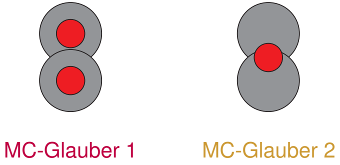
Initial eccentricities (vs dN/dy) in p+p and p+Pb collisions are quite different in the IP-Glasma model

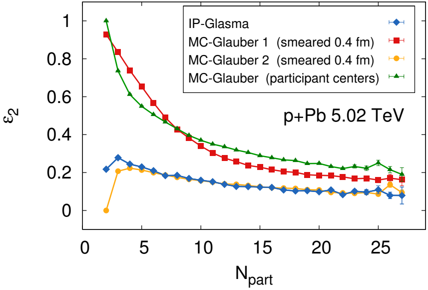


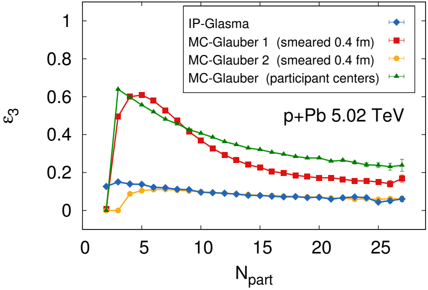
How well do eccentricities represent flow in p+p and p+Pb collisions?

p+Pb collisions A. Bzdak, B. Schenke, P. Tribedy, R. Venugopalan, arXiv:1304.3403

Eccentricities from different models can differ significantly





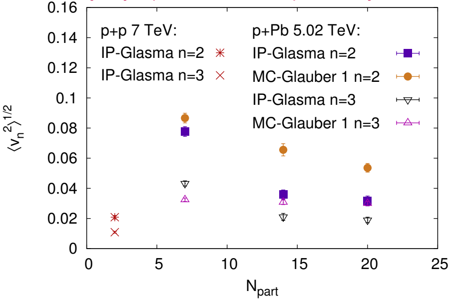


Lesson: Do not divide v_n by ε_n in small systems

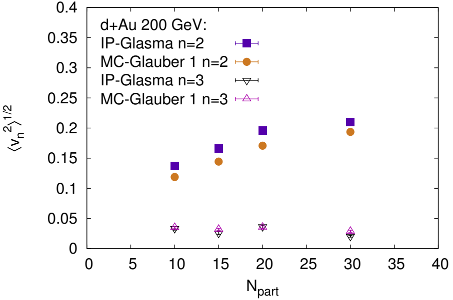
Flow in p+p, p+Pb and d+Au collisions

Only qualitative scaling between flow and eccentricities

p+p (at $b = 0\,\mathrm{fm}$) and p+Pb



d+Au



p+Pb: Ellipitc flow decreases with N_{part}

d+Au: Elliptic flow increases with N_{part}

p+p: Elliptic flow small, but not as small as expected from eccentricity

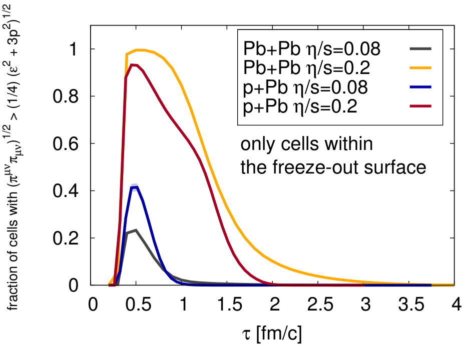
Need sophisticated centrality selection to compare with experiments

A. Bzdak, B. Schenke, P. Tribedy, R. Venugopalan, arXiv:1304.3403

p+A collisions - is viscous hydro valid?

How “crazy” is it to use hydrodynamics in p+A?

Initial $\pi_0^{\mu\nu} = 0$, $b = 0$ fm, IP-Glasma. Cells within f.o. surface that have $> 25\%$ viscous correction in p+Pb and Pb+Pb:

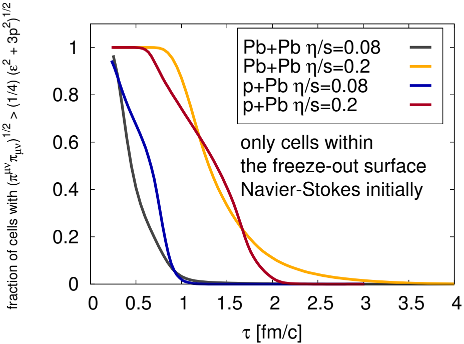


Important: Lifetime in Pb+Pb is about 6 times longer than in p+Pb

Also see Dumitru, Molnar, Nara, Phys.Rev. C76 (2007) 024910

p+A collisions - is viscous hydro valid?

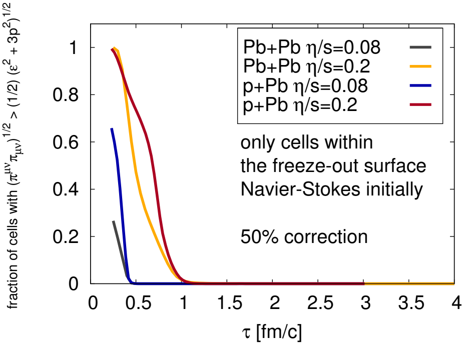
same with Navier-Stokes $\pi_0^{\mu\nu}$, count cells within f.o. surface that have more than a 25% viscous correction in p+Pb and Pb+Pb:



Important: Lifetime in Pb+Pb is about 6 times longer than in p+Pb

p+A collisions - is viscous hydro valid?

Initial Navier-Stokes $\pi_0^{\mu\nu}$, count cells within f.o. surface that have more than a 50% viscous correction in p+Pb and Pb+Pb:



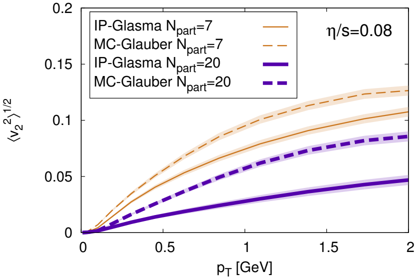
Important: Lifetime in Pb+Pb is about 6 times longer than in p+Pb

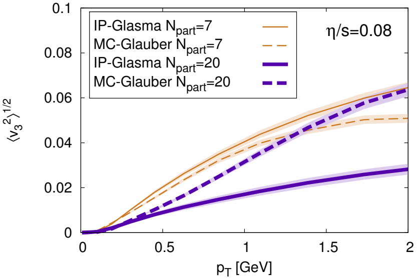
Summary and conclusions

- Different initial state models produce different geometries
- Differences are not very large in A+A, details still relevant for flow and jet energy loss studies (e.g. jet v_2)
- Differences are much larger in small systems (p+p, p+A, d+A)
- Eccentricites not the only relevant measure/predictor of flow
- Correct centrality selection in the model important
- Viscous corrections: Large at early times - affect larger fraction of total evolution time in p+A than A+A

BACKUP

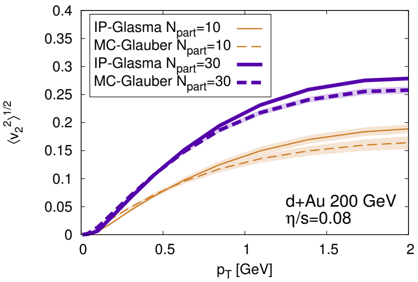
Flow in p+Pb

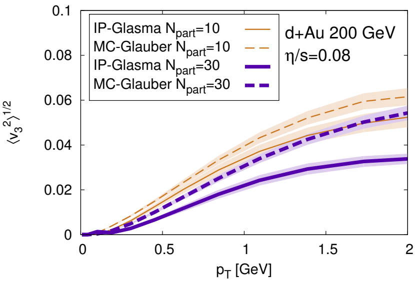




A. Bzdak, B. Schenke, P. Tribedy, R. Venugopalan, arXiv:1304.3403

Flow in d+Au

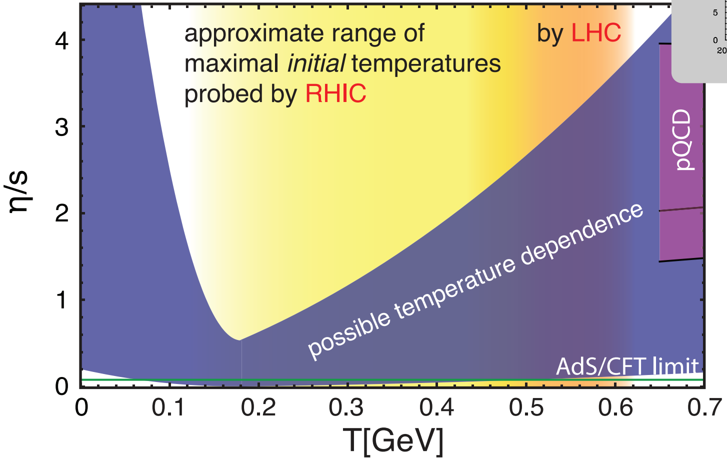




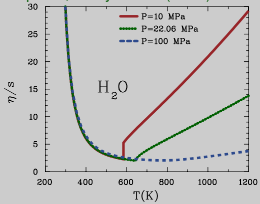
A. Bzdak, B. Schenke, P. Tribedy, R. Venugopalan, arXiv:1304.3403

Learning about QCD

Example: extraction of $(\eta/s)(T)$



Kapusta, J.Phys. G34 (2007) S295

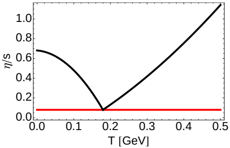


Temperature dependent η/s

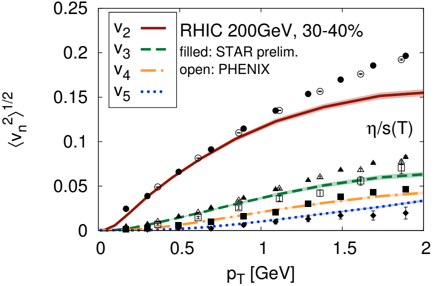
C. Gale, S. Jeon, B.Schenke,
P.Tribedy, R.Venugopalan, PRL110, 012302 (2013)

Use $\eta/s(T)$ as in Niemi et al., Phys.Rev.Lett. 106 (2011) 212302

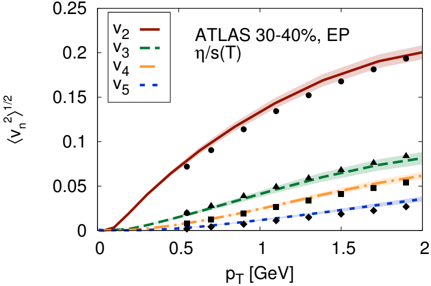
Experimental data: A. Adare et al. (PHENIX), Phys.Rev.Lett. 107, 252301 (2011)
Y. Pandit (STAR), Quark Matter 2012, (2012)
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)



RHIC



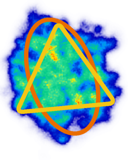
LHC



One $(\eta/s)(T)$ will be able to describe both RHIC and LHC data
Used parametrization not yet perfect: no surprise
More detailed study needed - include different RHIC energies and LHC

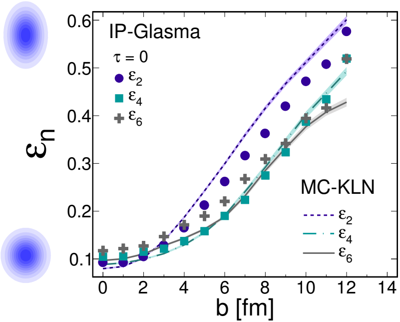
Eccentricities

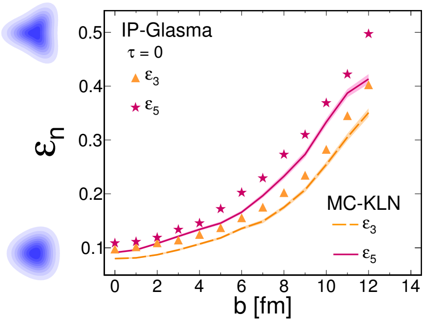
B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.C86, 034908 (2012)



Characterize the initial distribution by its ellipticity, triangularity, etc...

$$\varepsilon_n = \sqrt{\langle r^n \cos(n\phi)\rangle^2 + \langle r^n \sin(n\phi)\rangle^2} / \langle r^n \rangle$$

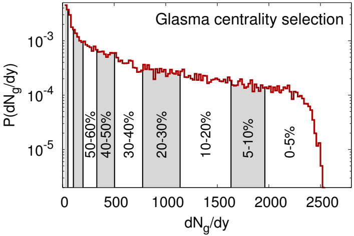




- ε_n larger in Glasma model for odd n
- ε_n smaller in Glasma model for $n = 2$ (for $b > 3\,\mathrm{fm}$)
  about equal for $n = 4$, larger for $n = 6$

Viscous flow at LHC

C. Gale, S. Jeon, B.Schenke, P.Tribedy, R.Venugopalan, PRL110, 012302 (2013)



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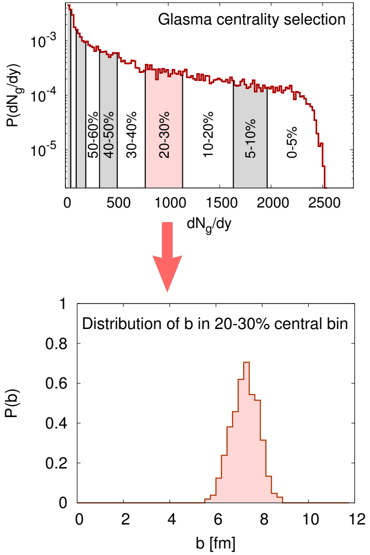

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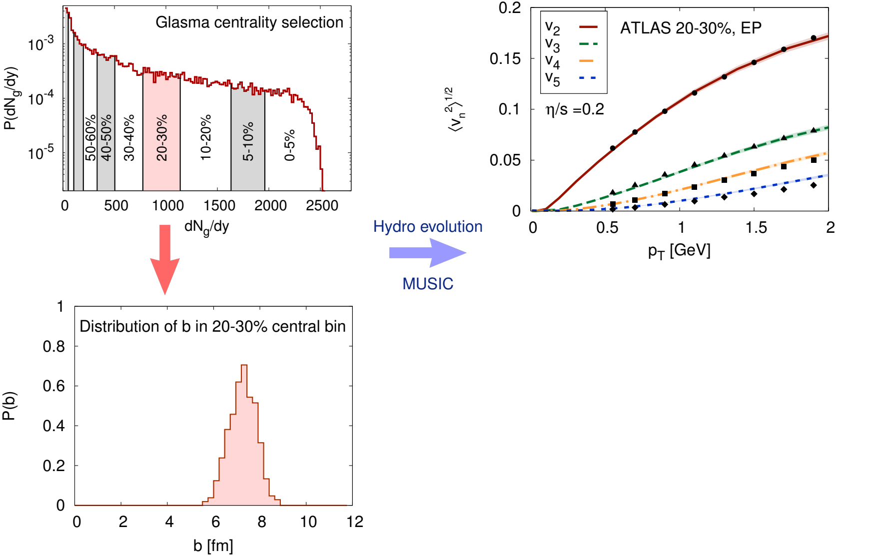

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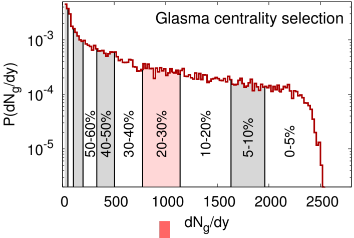
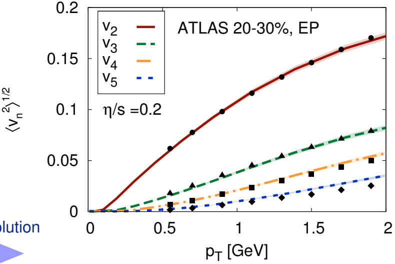
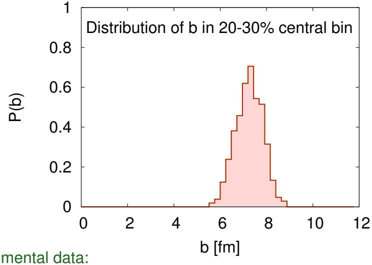
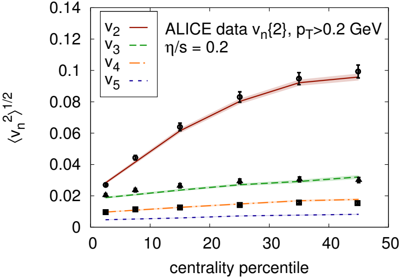


Hydro evolution

MUSIC







Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)
ALICE collaboration, Phys. Rev. Lett. 107, 032301 (2011)

Relativistic fluid-dynamics

Bulk of the matter produced in heavy-ion collisions
is well described by fluid-dynamics...

... and that is just energy and momentum conservation
in a system with small mean free path (compared to the system size)

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu} + \pi^{\mu\nu}$$

need additional equation to close the set:

Equation of state: $P = P(\epsilon)$

(comes e.g. from lattice QCD / hadron gas model)

$\pi^{\mu\nu}$ contains dissipative effects

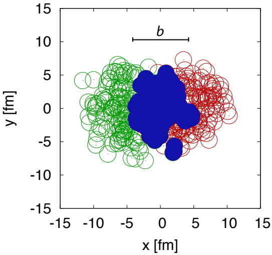
Event-by-event fluctuations

- Fluctuations are important
  Affect all harmonics v_1, v_2, v_3, etc.

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + \sum_n 2v_n \cos(n(\phi - \psi_n))\right)$$

  In particular: odd harmonics are not zero
  Mishra et al., Phys.Rev. C77, 064902 (2008), Takahashi et al., Phys. Rev. Lett. 103, 242301 (2009)
  Alver and Roland, Phys. Rev. C81, 054905 (2010)

- Axes and eccentricities determined by fluctuating geometry



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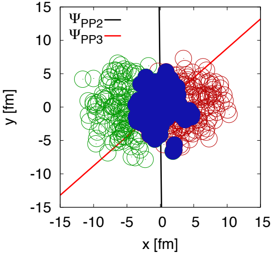

Participant plane angle (spatial):

$$\psi_{PPn} = \frac{1}{n}\arctan\frac{\langle r^n \sin(n\phi)\rangle}{\langle r^n \cos(n\phi)\rangle}$$

average over nucleon positions

Event plane angle (momentum):

$$\psi_{EPn} = \frac{1}{n}\arctan\frac{\langle p_T^\alpha \sin(n\psi)\rangle}{\langle p_T^\alpha \cos(n\psi)\rangle}$$

average over produced particles, $\alpha = 0, 1, \ldots$

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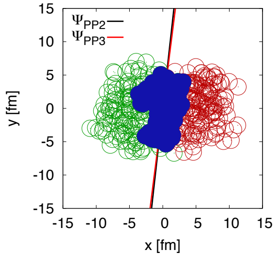

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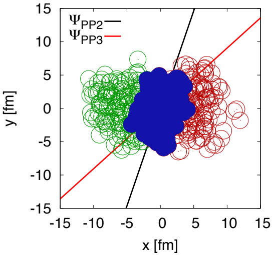

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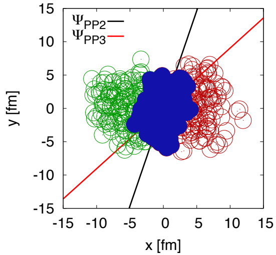


Eccentricities:

$$\varepsilon_n = \frac{\sqrt{\langle r^n \cos(n\phi)\rangle^2 + \langle r^n \sin(n\phi)\rangle^2}}{\langle r^n \rangle}$$

All $\varepsilon_n = 0 \rightarrow$ perfect circle
$\varepsilon_n \neq 0$: shape modulation in the n^{th} harmonic

3+1D event-by-event viscous fluid-dynamics

The first relativistic hydrodynamic simulation to include viscosity, fluctuations and 3+1 dimensions:

MUSIC: MUScl for Ion Collisions

MUSCL = Monotonic Upstream Centered Scheme for Conservation Laws

B. Schenke, S. Jeon, and C. Gale, Phys.Rev.Lett.106, 042301 (2011)

- 3+1 dimensions
- Kurganov-Tadmor algorithm
  A. Kurganov, E. Tadmor, Journal of Computational Physics **160**, 241-282 (2000)
- expanding geometry
- viscous effects (2^{nd} order Israel-Stewart formalism)
- fluctuating initial conditions
- equation of state from lattice QCD and hadron resonance gas

MUSIC: studying the effect of viscosity and fluctuations

energy density

- Setup:
  Wounded nucleons are assigned
  a Gaussian energy density distribution
  width σ_0 is a free parameter
- Evolution:
  Hydrodynamic evolution with shear viscous effects
  System expands, becomes dilute, freezes out
  Initial spatial anisotropy is transformed into momentum anisotropy
- Energy density $\rightarrow$ Particle spectra:
  Cooper-Frye formula:

$$E\frac{dN}{d^3p} = \int_\Sigma d\Sigma_\mu p^\mu f(T, p_\mu u^\mu, \pi^{\mu\nu})$$

followed by resonance decays

$\Sigma =$ freeze-out surface (surface of constant temperature)

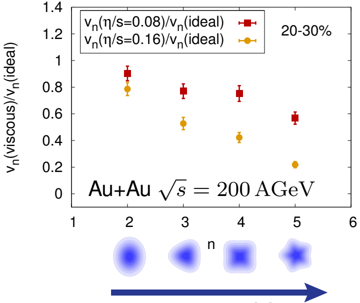
$f =$ particle distribution

Cooper and Frye, Phys.Rev.D10, 186 (1974)

Sensitivity of v_n on viscosity and fluctuations

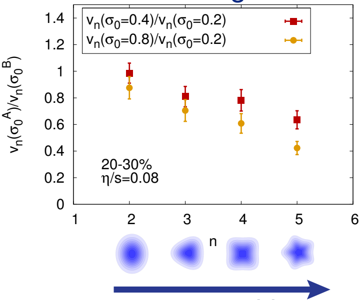
B. Schenke, S. Jeon, C. Gale, Phys.Rev.C85, 024901 (2012)

viscous to ideal results



more sensitive

smoother to more granular results



more sensitive

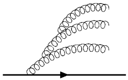
Viscosity decreases anisotropic flow (it's friction)
Smoother initial conditions decrease anisotropic flow

Sensitivity to viscosity and initial state structure increases with n

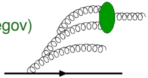
Nuclei at high energy: Gluon saturation

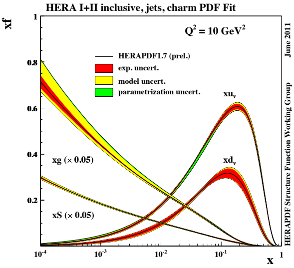
As we go to higher energy / smaller x, gluons split, number increases:

BFKL (Balitsky,Fadin,Kuraev,Lipatov) equation describes x-evolution
but violates unitarity: cross-sections grow without bound



JIMWLK (Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner) and BK (Balitsky, Kovchegov)
equations include non-linear evolution $\rightarrow$ saturation





- Transverse gluon density:
  $$\rho \sim \frac{xg_A(x,Q^2)}{S_\perp} \sim \frac{Axg(x,Q^2)}{A^{2/3}} \sim A^{1/3}xg(x,Q^2)$$
- Recombination cross section:
  $$\sigma_{gg\rightarrow g} \sim \frac{\alpha_s}{Q^2}$$
- Saturation when
  $$\rho\sigma_{gg\rightarrow g} \gtrsim 1$$

$\Rightarrow$ Saturation at scale $Q_s^2 = \alpha_s A^{1/3} xg(x, Q_s^2)$

x = longitudinal momentum fraction of partons in a hadron or nucleus

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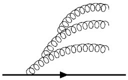

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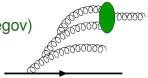


$\mathbf{p}_T \lesssim Q_s(x)$:

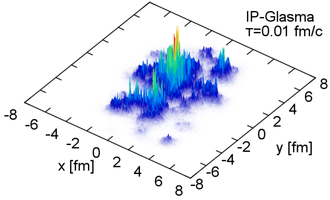
- strong saturated fields $A_\mu \sim 1/g$
- occupation numbers $\sim 1/\alpha_s$
- $\Rightarrow$ classical field approximation

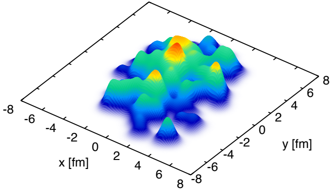
McLerran and Venugopalan, Phys.Rev. D49 (1994) 2233-2241

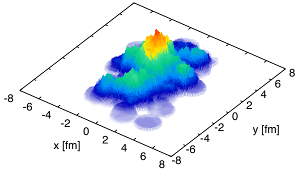
Energy density B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.Lett. 108, 252301 (2012)

Compute energy density in the fields at $\tau = 0$ and later times with CYM evolution

arbitrary units
same nucleon positions
in all events, b=4fm







MC-KLN: Drescher, Nara, nucl-th/0611017
mckln-3.52 from `http://physics.baruch.cuny.edu/files/CGC/CGC_IC.html` with defaults, energy density scaling

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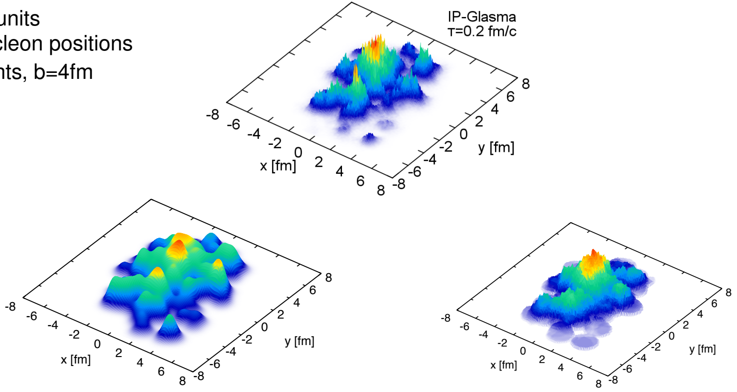

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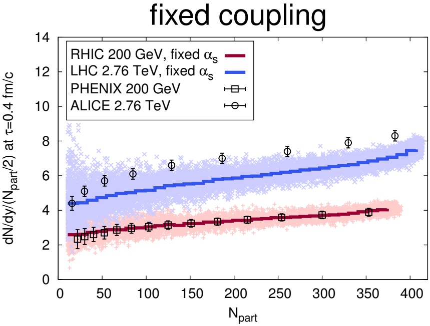
Multiplicity B.Schenke, P.Tribedy, R.Venugopalan, Phys. Rev. C86, 034908 (2012)

$dN_{\rm g}/dy$ in transverse Coulomb gauge $\partial_i A^i = 0$
$N_{\rm part}$ from MC-Glauber with $\sigma_{NN} = 42\,{\rm mb}$ and $64\,{\rm mb}$ respectively



Experimental data: PHENIX, Phys.Rev.C71 034908 (2004) and ALICE, Phys.Rev.Lett. 106, 032301 (2011)

Scaled by $2/3$ to compare to charged particles.
Original version. Nuclear 'oomph' added by hand.

Multiplicity B.Schenke, P.Tribedy, R.Venugopalan, Phys. Rev. C86, 034908 (2012)

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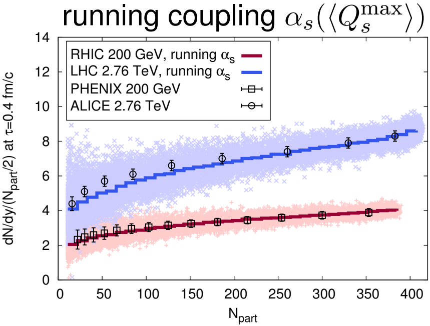

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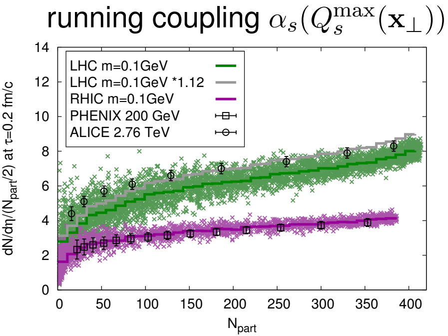
Multiplicity B.Schenke, P.Tribedy, R.Venugopalan, Phys. Rev. C86, 034908 (2012)

New version. Using IP-Sat for nuclei. No adjustments by hand anymore.



Experimental data: PHENIX, Phys.Rev.C71 034908 (2004) and ALICE, Phys.Rev.Lett. 106, 032301 (2011)

Expect more entropy production at LHC - need hydro results to check

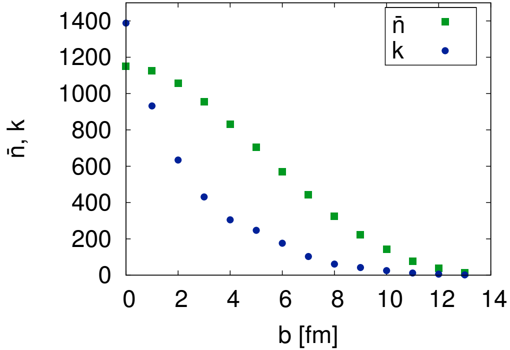
Negative binomial fluctuations

B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.C86, 034908 (2012)

Extract k and $\bar{n}$ using a fit with

$$P(n) = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}}$$

at fixed impact parameters



Ratio of $k/\bar{n}$ is > 1 for small b and becomes small ~ 0.14 for large b.
That is close to the value extracted for $p+p$ collisions: Dumitru and Nara arXiv:1201.6382

NBDs and Glasma flux tubes

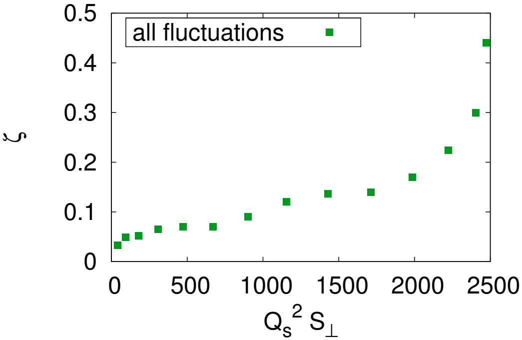
B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.C86, 034908 (2012)

Glasma flux tube picture:

$$k = \zeta \frac{N_c^2 - 1}{2\pi} Q_s^2 S_\perp$$

Gelis, Lappi, Mclerran, arXiv:0905.3234

Width of NBD is inversely proportional to the number of flux tubes $Q_s^2 S_\perp$.
$S_\perp$ = interaction area



ζ should be close to constant in the flux tube picture

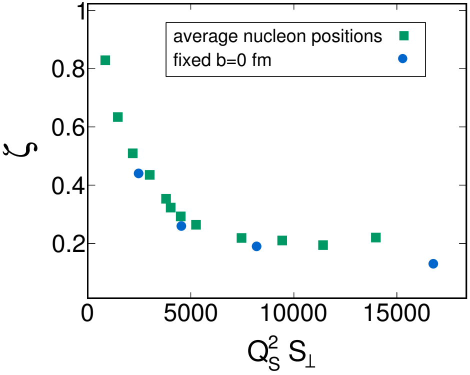
NBDs and Glasma flux tubes

B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.C86, 034908 (2012)

ζ is not constant because geometric fluctuations are very important
Were not considered in the derivation of

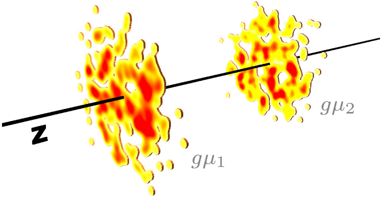
$$k = \zeta \frac{N_c^2 - 1}{2\pi} Q_s^2 S_\perp$$

Eliminate by using smooth nucleon distributions:



Color charge densities of incoming nuclei

- Sample positions of nucleons from Woods-Saxon distributions in nucleus A and B.
- IP-Sat provides $Q_s^2(x, \mathbf{b}_\perp)$ for each nucleon.
  The color charge density squared $g^2\mu^2$ is proportional to Q_s^2.
  (proportionality factor depends on details of calculation - see Lappi, arXiv:0711.3039)
- We add all $g^2\mu^2(\mathbf{x}_\perp)$ in each nucleus to obtain $g^2\mu_1^2(\mathbf{x}_\perp)$ and $g^2\mu_2^2(\mathbf{x}_\perp)$.



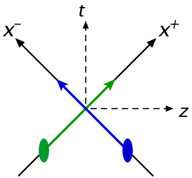
- Sample ρ^a from local Gaussian distribution for each nucleus

$$\langle \rho^a(\mathbf{x}_\perp)\rho^b(\mathbf{y}_\perp)\rangle = \delta^{ab}\delta^2(\mathbf{x}_\perp - \mathbf{y}_\perp)g^2\mu^2(\mathbf{x}_\perp)$$

Gauge fields before the collision

Color currents:

$$J_1^\nu = \delta^{\mu +}\rho_1(x^-,\mathbf{x}_\perp)$$
$$[D_\mu, F^{\mu\nu}] = J_1^\nu$$



$$J_2^\nu = \delta^{\mu -}\rho_2(x^+,\mathbf{x}_\perp)$$
$$[D_\mu, F^{\mu\nu}] = J_2^\nu$$

Solution in covariant gauge:

$$A^+_{\mathrm{cov}(1,2)}(x^-,\mathbf{x}_\perp) = -\frac{g\rho_{(1,2)}(x^-,\mathbf{x}_\perp)}{\nabla_\perp^2 + m^2}$$

with infrared cutoff m of order Λ_{QCD}.

Solution in light cone gauge:

$$A^+_{(1,2)}(\mathbf{x}_\perp) = A^-_{(1,2)}(\mathbf{x}_\perp) = 0$$
$$A^i_{(1,2)}(\mathbf{x}_\perp) = \frac{i}{g}V_{(1,2)}(\mathbf{x}_\perp)\partial_i V^\dagger_{(1,2)}(\mathbf{x}_\perp)$$

V is the path-ordered exponential of $A^+_{\mathrm{cov}(1,2)}$

Gauge fields before the collision

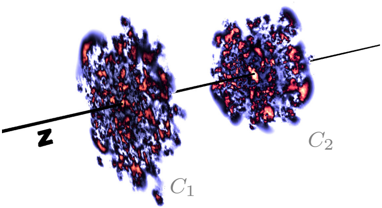
The correlator of the Wilson lines (same one we looked at before)

$$C_{(1,2)}(\mathbf{x}_\perp) = \frac{1}{N_c}\mathrm{Re}[\mathrm{tr}(V(1,2)^\dagger(0,0)V(1,2)(x,y))]$$

with

$$V_{(1,2)}(\mathbf{x}_\perp) = P\exp\left(-ig\int dx^- \frac{\rho_{(1,2)}(x^-,\mathbf{x}_\perp)}{\nabla_\perp^2 + m^2}\right)$$

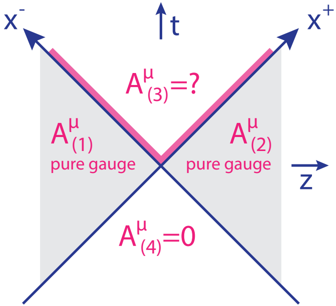
shows the degree of correlations and fluctuations in the gluon fields.



The length scale of fluctuations is $1/Q_s$. Not the nucleon size.

IP-Glasma: Gauge fields after the collision (Glasma)

Initial condition on the lightcone:



Configuration in Schwinger gauge $A^\tau = 0$

Solution:

$$A^i_{(3)}|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

$$A^\eta_{(3)}|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

We solve for the gauge fields numerically
Krasnitz, Venugopalan, Nucl.Phys. B557 (1999) 237

Time evolution follows Yang-Mills equations

Modeling x and b dependence: IP-Sat model

Want to determine color charge distribution in a nucleus. Proton first:
Use IP-Sat model to parametrize

- x-dependence
- Impact parameter dependence (IP)

  Kowalski, Teaney, Phys.Rev. D68 (2003) 114005

x-evolution can be computed using JIMWLK,
but parametrization is easier for now

Modeling x and b dependence: IP-Sat model

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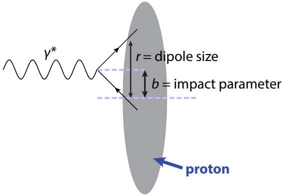


x-evolution can be computed using JIMWLK, but parametrization is easier for now

IP-Sat proton dipole cross section in deeply inealstic scattering (DIS):

$$\frac{\mathrm{d}\sigma^{\mathrm{p}}_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}_\perp}(\mathbf{r}_\perp, x, \mathbf{b}_\perp) = 2\,\mathcal{N}(\mathbf{r}_\perp, x, \mathbf{b}_\perp) = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c}\mathbf{r}_\perp{}^2\alpha_s(Q^2)xg(x,Q^2)T_p(\mathbf{b}_\perp)\right)\right]$$

gluon density

Gaussian shape

with Q^2 an energy scale related to the dipole size $\mathbf{r}_\perp$

Parameters fit to HERA diffractive data

Q_s is defined by the scale r where $\mathcal{N}$ reaches the saturated regime

$\mathcal{N}(R_s, x, \mathbf{b}_\perp) = 1 - e^{-1/2}$, with $Q_s^2 = \frac{2}{R_s^2}$

Gauge fields after the collision (Glasma)

Initial condition on the lightcone: require that fields match smoothly on the lightcone.

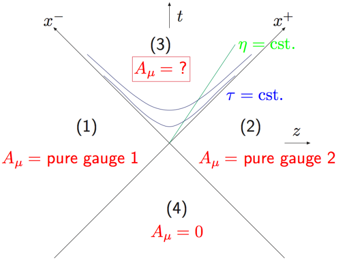


figure from Lappi, arXiv:1003.1852

Solution:

$$A^i_{(3)}|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

$$A^\eta_{(3)}|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

On the lattice the Wilson lines in the future lightcone are obtained from the condition:

$$\mathrm{tr}\left\{t^a\left[\left(U^i_{(1)} + U^i_{(2)}\right)\left(1 + U^{i\dagger}_{(3)}\right) - \left(1 + U^i_{(3)}\right)\left(U^{i\dagger}_{(1)} + U^{i\dagger}_{(2)}\right)\right]\right\} = 0$$

where t^a are the generators of $SU(N_c)$ in the fundamental representation. Solve iteratively.

Krasnitz, Venugopalan, Nucl.Phys. B557 (1999) 237

$$U^i_{(1,2),j} = V_{(1,2),j} V^\dagger_{(1,2),j+\hat{e_i}}$$ (gauge transform of $\mathbf{1}$: pure gauge)

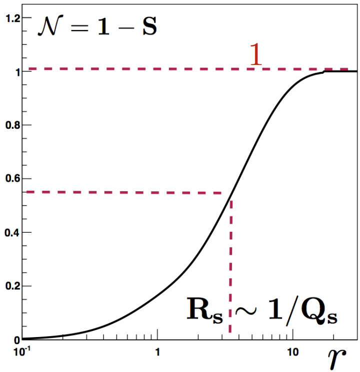
E_η can be obtained from $U^i_{(1,2,3)}$

Modeling x and b dependence of Q_s: IP-Sat model

After fitting parameters to HERA DIS data the model provides a distribution of $Q_s^2(x, \mathbf{b}_\perp)$ of the proton, which will be our input

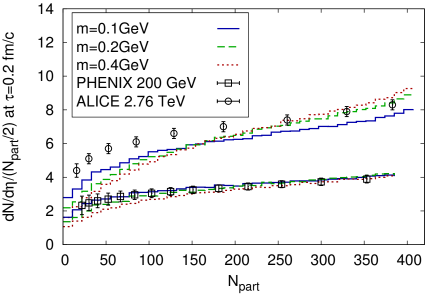
It is determined self consistently from the requirement that

$$\mathcal{N}(R_s, x, \mathbf{b}_\perp) = 1 - e^{-1/2}, \quad \text{with } Q_s^2 = \frac{2}{R_s^2}$$



Multiplicity B.Schenke, P.Tribedy, R.Venugopalan, Phys. Rev. C86, 034908 (2012)

$dN_{\rm g}/dy$ in transverse Coulomb gauge $\partial_i A^i = 0$
$N_{\rm part}$ from MC-Glauber with $\sigma_{NN} = 42\,{\rm mb}$ and $64\,{\rm mb}$ respectively



Experimental data: PHENIX, Phys.Rev.C71 034908 (2004) and ALICE, Phys.Rev.Lett. 106, 032301 (2011)

Scaled by $2/3$ to compare to charged particles.
New version. Using IP-Sat for nuclei. m-dependence.

Equations to be solved

Explicit version of $\partial_\mu T^{\mu\nu} = 0$ in an appropriate coordinate system that expands longitudinally
proper time $\tau = \sqrt{t^2 - z^2}$, space-time rapidity $\eta_s = \frac{1}{2}\ln\frac{t+z}{t-z}$

$$\partial_\tau(\tau T^{\tau\tau}) + \partial_v(\tau T^{v\tau}) + \partial_{\eta_s}(T^{\eta_s\tau}) + T^{\eta_s\eta_s}$$
$$+\partial_\tau(\tau\pi^{\tau\tau}) + \partial_v(\tau\pi^{v\tau}) + \partial_{\eta_s}(\pi^{\eta_s\tau}) + \pi^{\eta_s\eta_s} = 0$$

$$\partial_\tau(\tau T^{\tau\eta_s}) + \partial_v(\tau T^{v\eta_s}) + \partial_{\eta_s}(T^{\eta_s\eta_s}) + T^{\tau\eta_s}$$
$$+\partial_\tau(\tau\pi^{\tau\eta_s}) + \partial_v(\tau\pi^{v\eta_s}) + \partial_{\eta_s}(\pi^{\eta_s\eta_s}) + \pi^{\tau\eta_s} = 0$$

$$\partial_\tau(\tau T^{\tau v}) + \partial_w(\tau T^{wv}) + \partial_{\eta_s}(T^{\eta_s v})$$
$$+\partial_\tau(\tau\pi^{\tau v}) + \partial_w(\tau\pi^{wv}) + \partial_{\eta_s}(\pi^{\eta_s v}) = 0$$

spatial derivatives (computed using Kurganov-Tadmor method)
treated as sources

Equations to be solved

In addition, we have to solve the equation for $\pi^{\mu\nu}$
(2^{nd} order Israel-Stewart viscous fluid-dynamics)

$$
\begin{aligned}
\partial_c(u^c\pi^{ab}) = &- \frac{1}{2\tau}u^\tau\pi^{ab} + \frac{1}{\tau}\Delta^{a\eta}u^\eta\pi^{b\tau} - \frac{1}{\tau}\Delta^{a\tau}u^\eta\pi^{b\eta} \\
&- g_{cf}\pi^{cb}u^a Du^f - \frac{\pi^{ab}}{2\tau_\pi} - \frac{1}{6}\pi^{ab}\partial_c u^c \\
&+ \frac{\eta}{\tau_\pi}\Bigg(-\frac{1}{\tau}\Delta^{a\eta}g^{b\eta}u^\tau + \frac{1}{\tau}\Delta^{a\eta}g^{b\tau}u^\tau \\
&\qquad + g^{ac}\partial_c u^b - u^a Du^b - \frac{1}{3}\Delta^{ab}\partial_c u^c\Bigg) \\
&+ (a \leftrightarrow b)\,,
\end{aligned}
$$

relaxation time τ_π
$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$
$D = u^\mu\partial_\mu$
$\tau_\pi = 3\eta/(\varepsilon + P)$: shear relaxation time

Yang-Mills evolution

The equations of motion are formulated on the lattice in 2+1D ($\phi = A_\eta$)

$$\dot{U}_i = i\frac{g^2}{\tau}E^i U_i \text{ (no sum over } i\text{)}$$

$$\dot{\phi} = \tau E^\eta$$

$$\dot{E}^1 = \frac{i\tau}{2g^2}[U_{1,2} + U_{1,-2} - U^\dagger_{1,2} - U^\dagger_{1,-2} - T_1] + \frac{i}{\tau}[\tilde{\phi}_1, \phi]$$

$$\dot{E}^2 = \frac{i\tau}{2g^2}[U_{2,1} + U_{2,-1} - U^\dagger_{2,1} - U^\dagger_{2,-1} - T_2] + \frac{i}{\tau}[\tilde{\phi}_1, \phi]$$

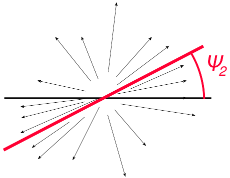
$$\dot{E}^\eta = \frac{1}{\tau}\sum_i \left[\tilde{\phi}_i + \tilde{\phi}_{-i} - 2\phi\right]$$

where $T_1 = \frac{1}{N_c}\mathrm{tr}[U_{1,2} + U_{1,-2} - U^\dagger_{1,2} - U^\dagger_{1,-2}]\,\mathbf{1}$, and
$T_2 = \frac{1}{N_c}\mathrm{tr}[[U_{2,1} + U_{2,-1} - U^\dagger_{2,1} - U^\dagger_{2,-1}]\,\mathbf{1}$ with the $N_c \times N_c$ unit matrix $\mathbf{1}$
$\tilde{\phi}^j_i = U^i_j \phi_{j+\hat{e}_i} U^{i\dagger}_j$ and $U^j_{1,2} = U^1_j\, U^2_{j+\hat{e}_1}\, U^{1\dagger}_{j+\hat{e}_2}\, U^{2\dagger}_j$

Event plane

To get non-zero odd moments, we rotate the event plane in each event. Event plane is defined by the angle:

$$\psi_n = \frac{1}{n} \arctan \frac{\langle w \sin(n\phi) \rangle}{\langle w \cos(n\phi) \rangle}$$



using particle momenta.
($w = p_T$ (first results) or 1 (later results))

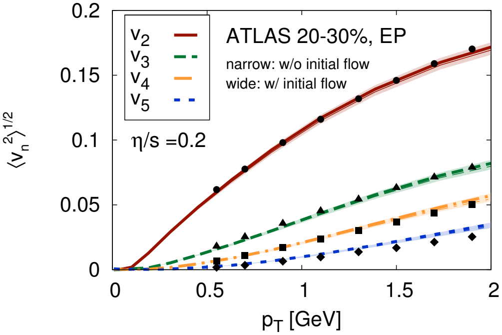
A.Poskanzer and S.Voloshin, Phys.Rev.C58:1671-1678 (1998)

$$v_n = \langle \cos(n(\phi - \psi_n)) \rangle$$

... different angle for every flow coefficient.

In the simulation we know the true event-plane (no correction factor).

Effect of initial flow



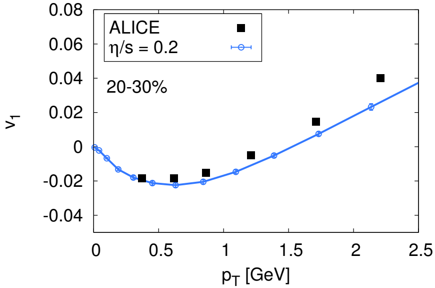
Weak effect of initial flow on hadron $v_n(p_T)$

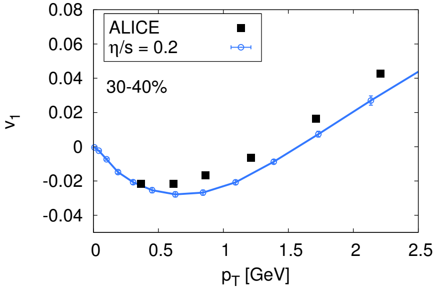
Expect stronger effect for photon v_n:
Photons are mostly produced early at high temperatures

Effect of different switching time $0.4\,\mathrm{fm}/c$ is very weak

Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)

Directed flow v_1



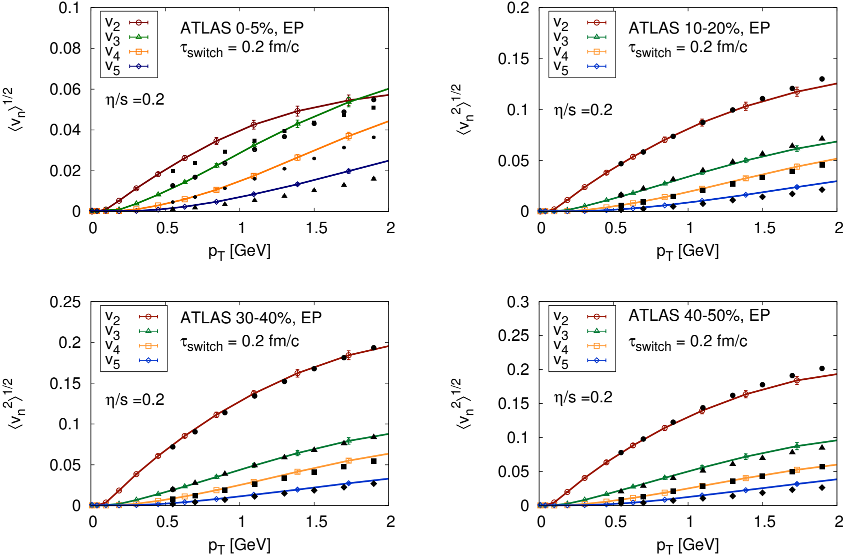


Experimental data:

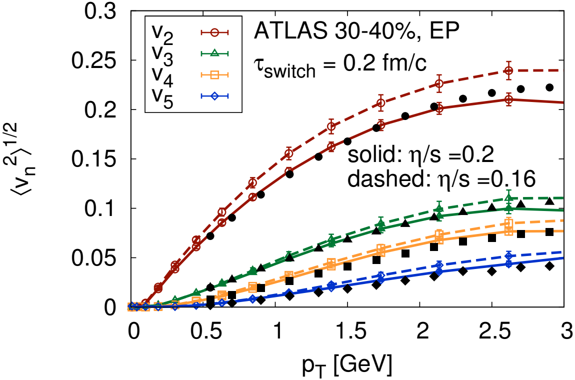
extracted in Retinskaya et al., Phys.Rev.Lett. 108 (2012) 252302

from ALICE data in K. Aamodt et al., Phys. Lett. B 708, 249 (2012)

More centrality classes: IP-Glasma + MUSIC



Smaller average η/s



Using $\eta/s = 0.16$ overestimates all v_n

Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)