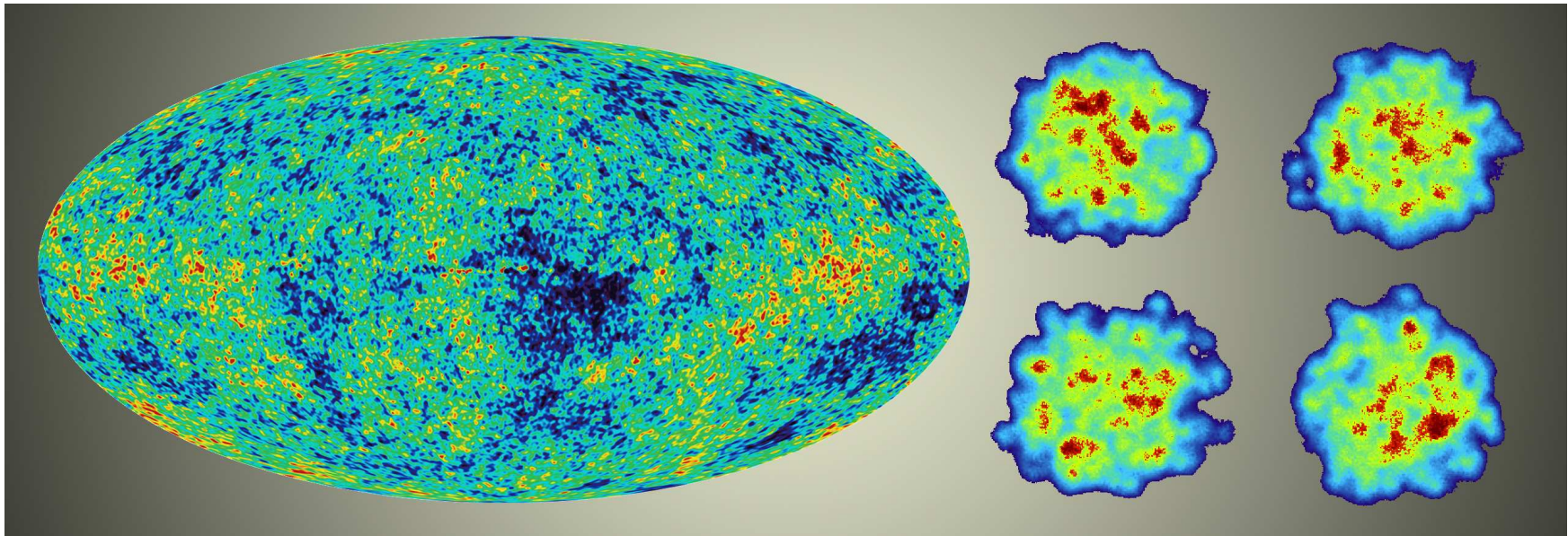


Fluctuations, flow and viscosity in the Little Bang*

Ulrich Heinz (The Ohio State University)



presented at:

Jet quenching at RHIC vs. LHC in Light of Recent dAu vs. pPb Controls
Brookhaven National Laboratory, 15-17 April 2013



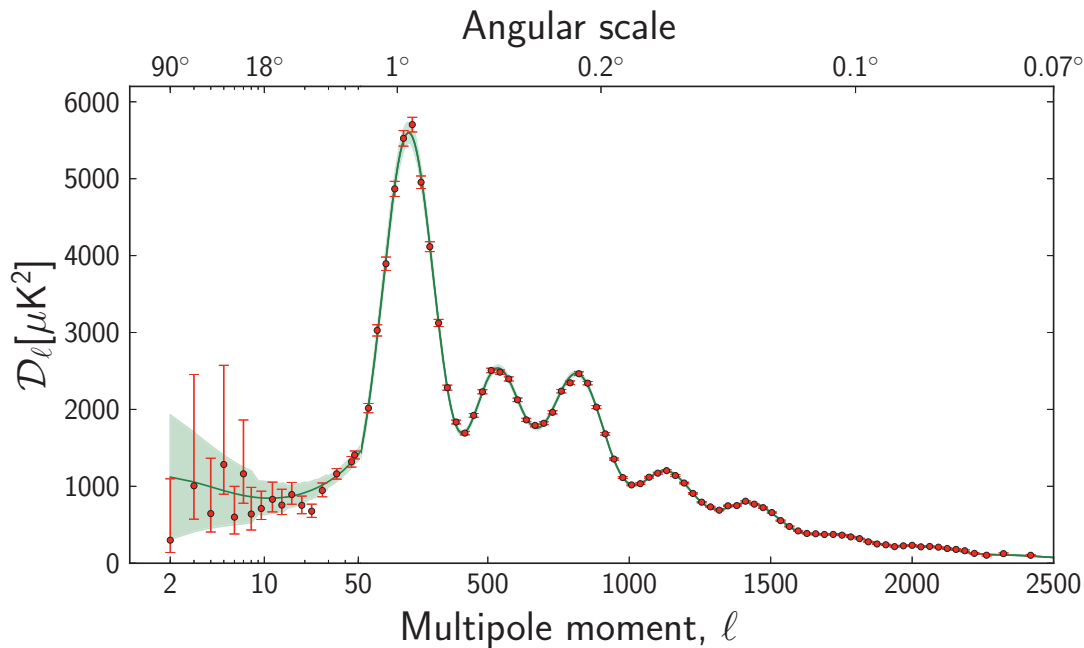
*Supported by the U.S. Department of Energy



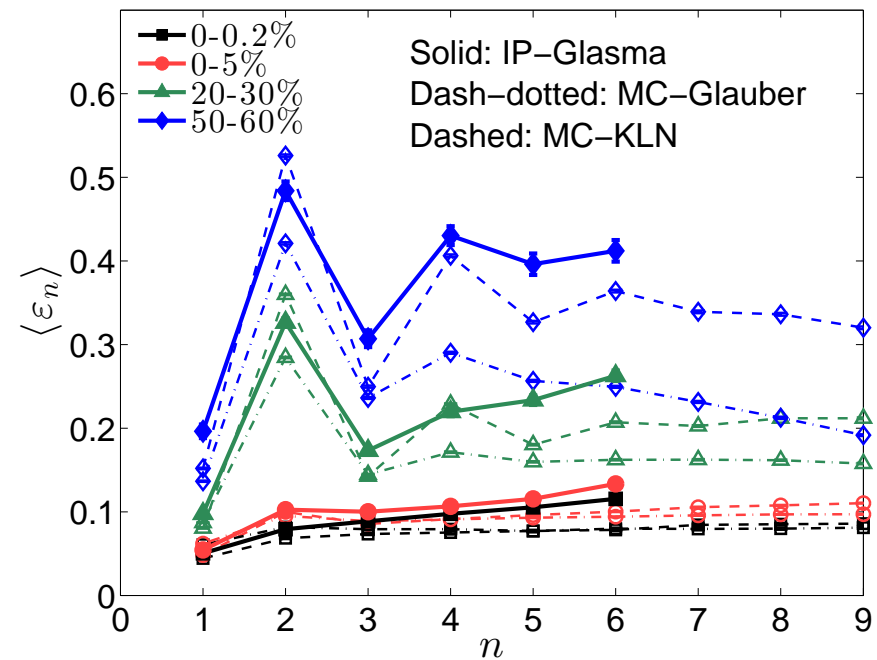
References: Heinz, Qiu, Shen, PRC 87 (2013) 034913
Qiu, Heinz, PLB 717 (2012) 261

Big Bang vs. Little Bang: The fluctuation power spectrum

Planck 2013 CMB temperature power spectrum



Little Bang density power spectra 2013



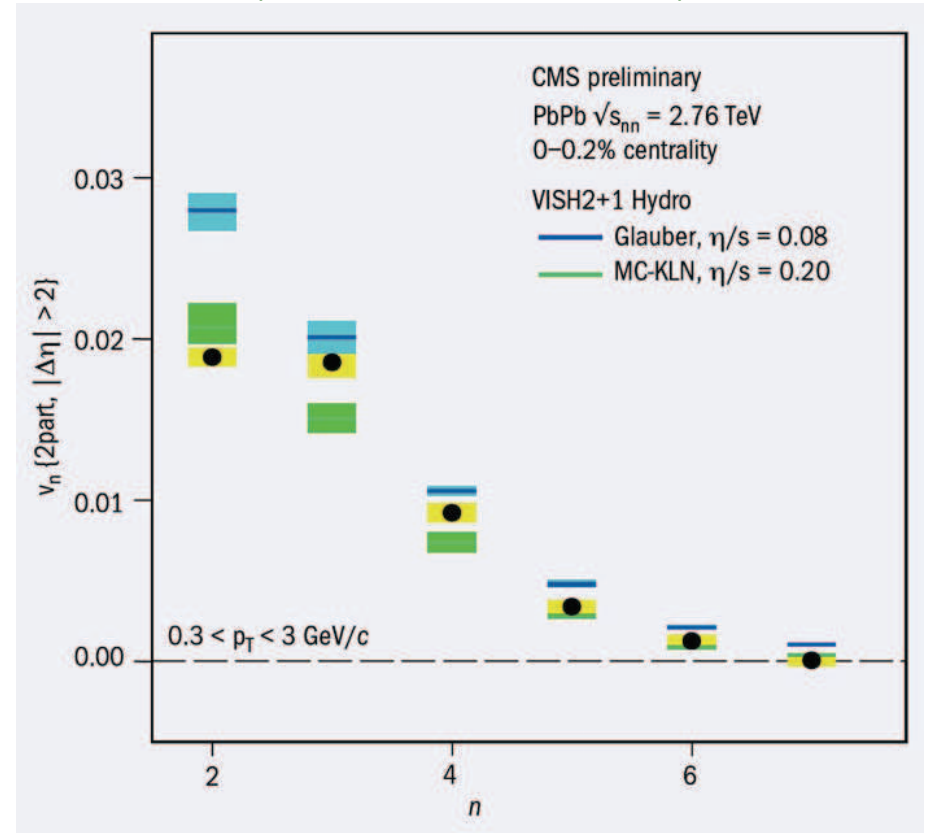
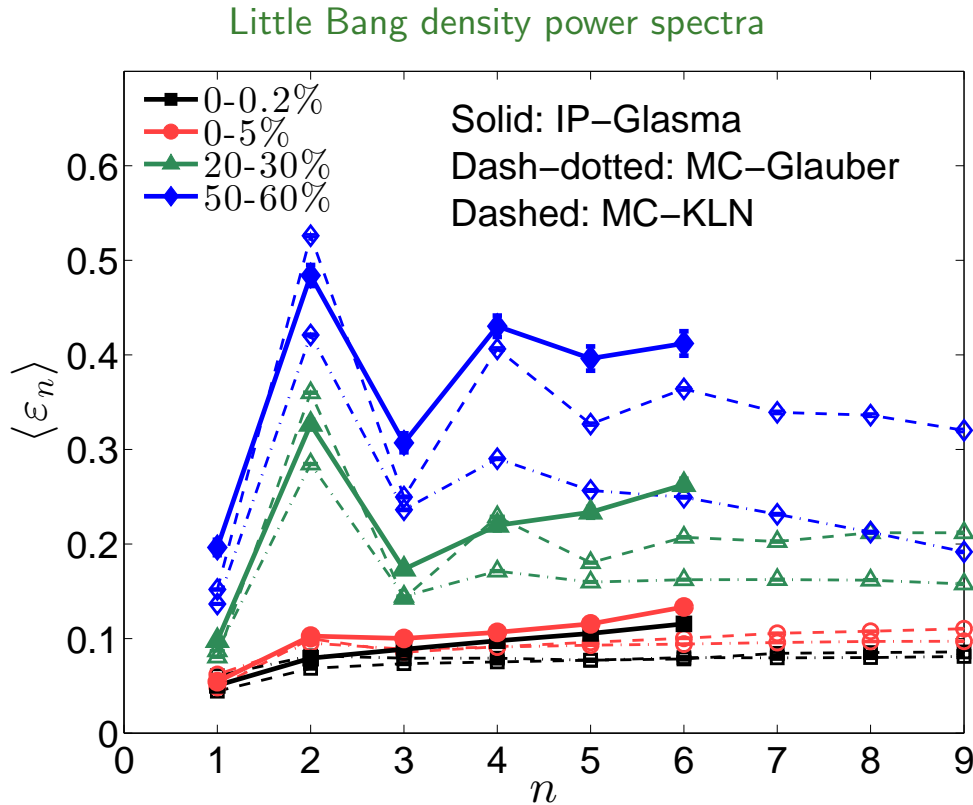
Initial eccentricities ϵ_n and associated participant-plane angles Φ_n of the Little Bang:

$$\epsilon_1 e^{i\Phi_1} \equiv \frac{\int r dr d\varphi r^3 e^{i\varphi} e(r, \varphi)}{\int r dr d\varphi r^3 e(r, \varphi)}, \quad \epsilon_n e^{in\Phi_n} \equiv \frac{\int r dr d\varphi r^n e^{in\varphi} e(r, \varphi)}{\int r dr d\varphi r^n e(r, \varphi)} \quad (n > 1)$$

A detailed study of fluctuations is a powerful discriminator between models!

The fluctuation power spectrum: initial vs. final

Flow power spectrum for ultracentral pPb Little Bangs
(CMS, Quark Matter 2012)



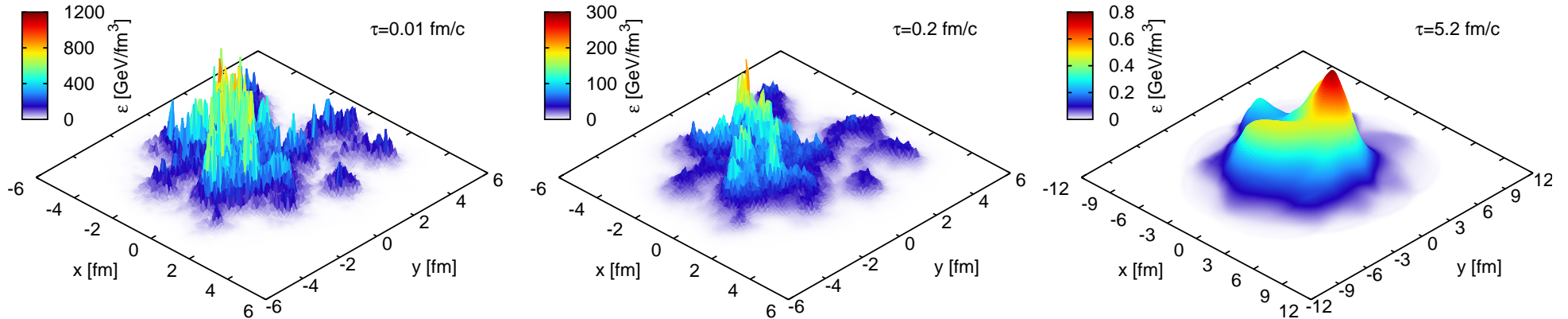
Higher flow harmonics get suppressed by shear viscosity

A detailed study of fluctuations is a powerful discriminator between models!

Each Little Bang evolves differently!

Density evolution of a single $b = 8$ fm Au+Au collision at RHIC, with IP-Glasma initial conditions, Glasma evolution to $\tau = 0.2$ fm/c followed by (3+1)-d viscous hydrodynamic evolution with MUSIC using $\eta/s = 0.12 = 1.5/(4\pi)$

Schenke, Tribedy, Venugopalan, PRL 108 (2012) 252301:



Single event anisotropic flow coefficients

In a single event, the specific initial density profile results in a set of complex, y - and p_T -dependent flow coefficients (we'll suppress the y -dependence):

$$V_n = v_n e^{in\Psi_n} := \frac{\int p_T dp_T d\phi e^{in\phi} \frac{dN}{dy p_T dp_T d\phi}}{\int p_T dp_T d\phi \frac{dN}{dy p_T dp_T d\phi}} \equiv \{e^{in\phi}\},$$

$$V_n(p_T) = v_n(p_T) e^{in\Psi_n(p_T)} := \frac{\int d\phi e^{in\phi} \frac{dN}{dy p_T dp_T d\phi}}{\int d\phi \frac{dN}{dy p_T dp_T d\phi}} \equiv \{e^{in\phi}\}_{p_T}.$$

Together with the azimuthally averaged spectrum, these completely characterize the measurable single-particle information for that event:

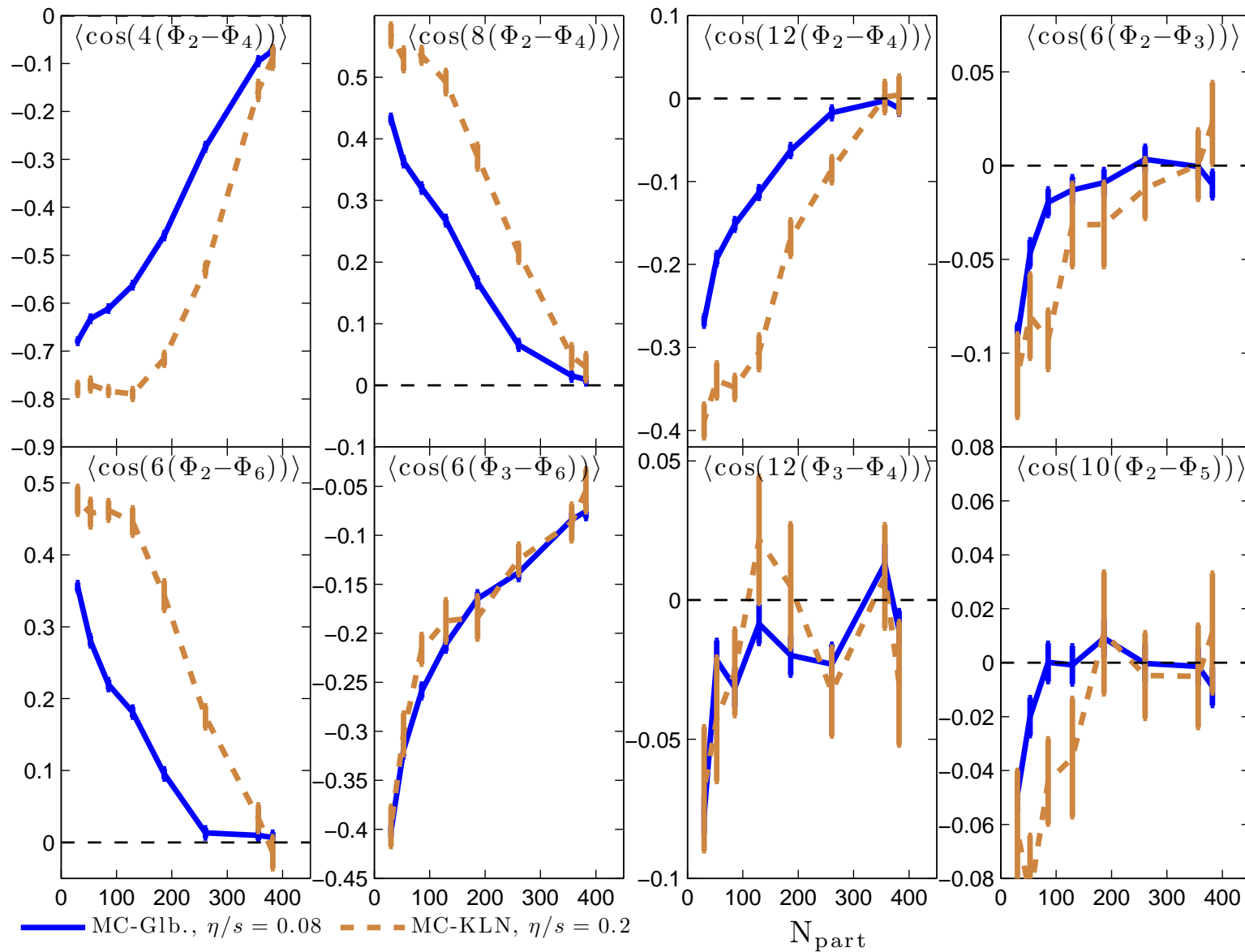
$$\frac{dN}{dy d\phi} = \frac{1}{2\pi} \frac{dN}{dy} \left(1 + 2 \sum_{n=1}^{\infty} v_n \cos[n(\phi - \Psi_n)] \right),$$

$$\frac{dN}{dy p_T dp_T d\phi} = \frac{1}{2\pi} \frac{dN}{dy p_T dp_T} \left(1 + 2 \sum_{n=1}^{\infty} v_n(p_T) \cos[n(\phi - \Psi_n(p_T))] \right).$$

- Both the magnitude v_n and the direction Ψ_n (“flow angle”) depend on p_T .
- $v_n, \Psi_n, v_n(p_T), \Psi_n(p_T)$ **all fluctuate from event to event.**
- $\Psi_n(p_T) - \Psi_n$ fluctuates from event to event.

Initial participant plane correlations in PbPb@LHC

Zhi Qiu, UH, PLB 717 (2012) 261

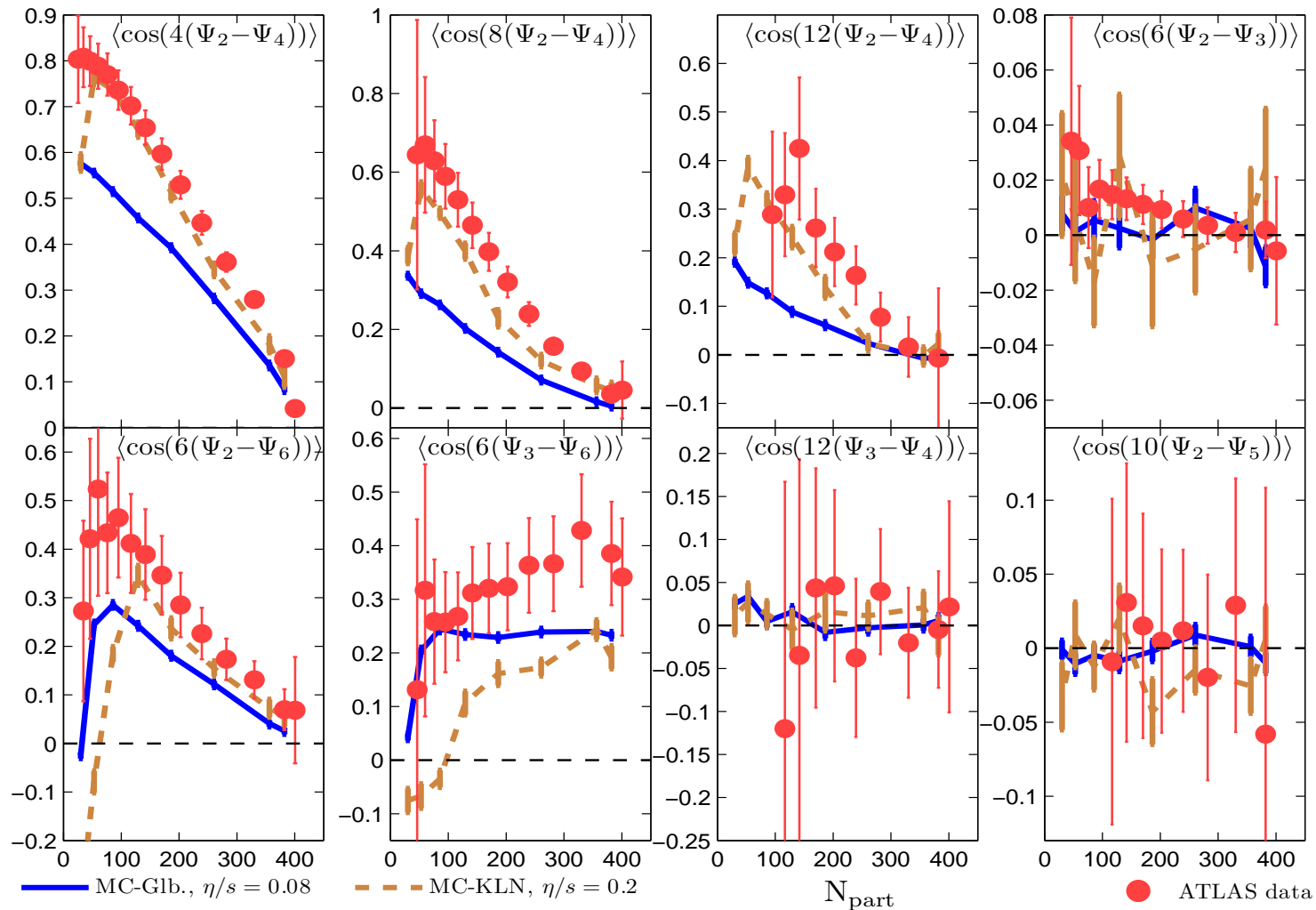


Qualitatively similar, but quantitative differences between models

Final flow angle correlations in PbPb@LHC

Data: ATLAS Coll., J. Jia et al., Hard Probes 2012

Event-by-event hydrodynamics: Zhi Qiu, UH, PLB 717 (2012) 261 (VISH2+1)



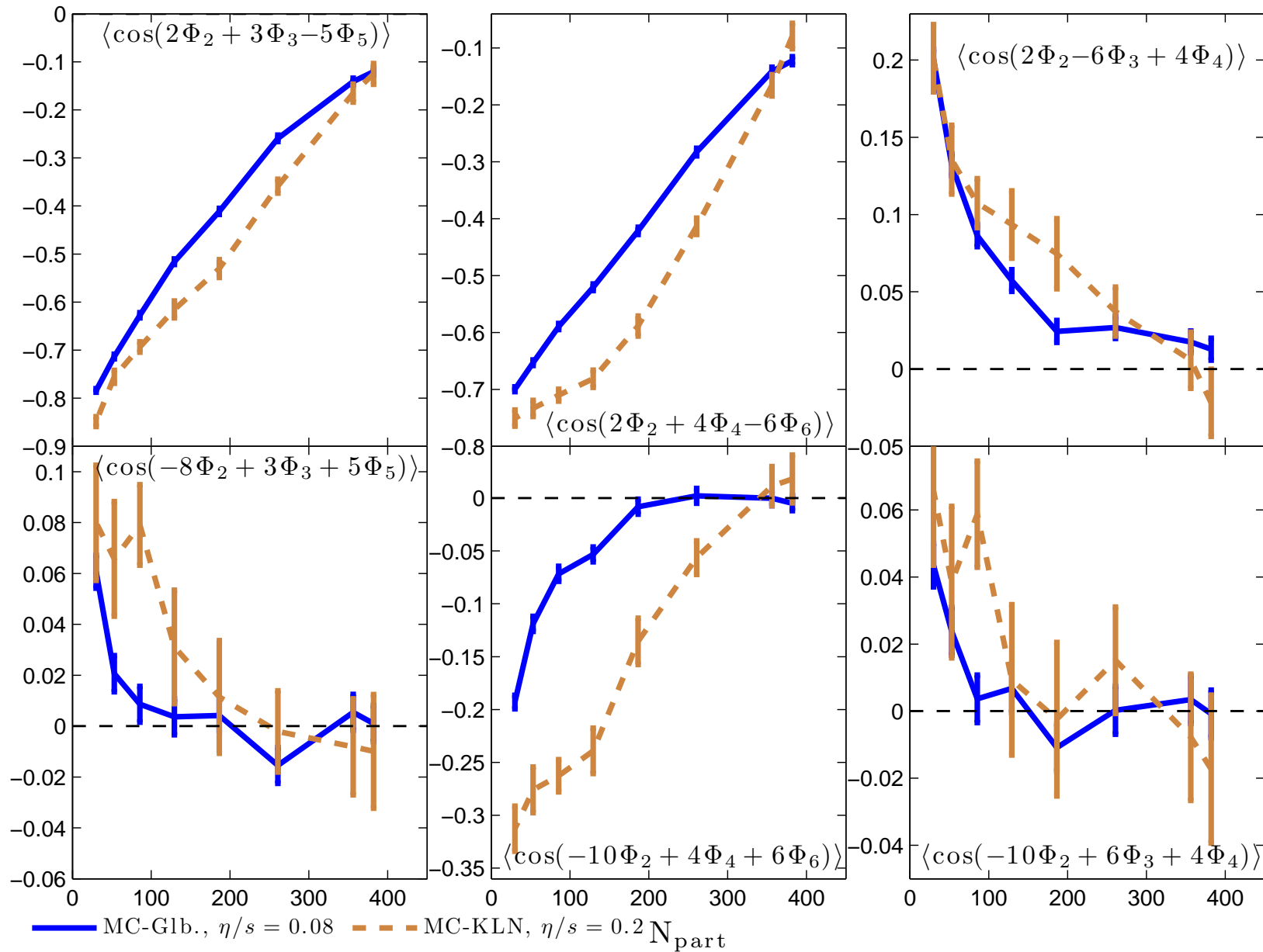
VISH2+1 reproduces qualitatively the centrality dependence of all measured event-plane correlations
 Initial part.-plane correlations disagree qualitatively with the measured final-state flow-plane correlations

⇒ **Nonlinear mode coupling through hydrodynamic evolution essential to describe the data!**

Larger viscosity appears to yield stronger flow-angle correlations

Initial three-plane correlations in PbPb@LHC

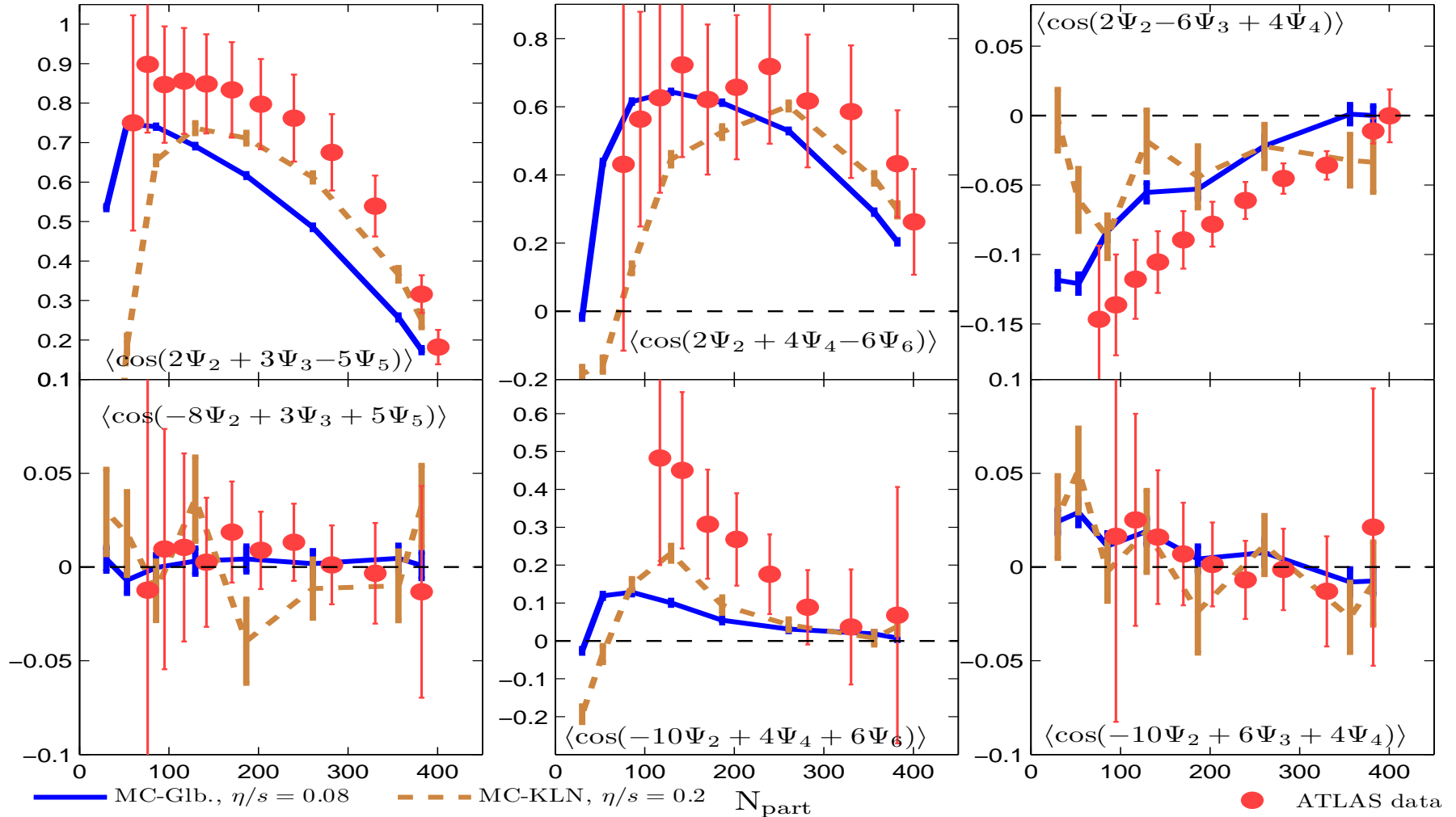
Zhi Qiu, UH, PLB 717 (2012) 261



Final three-plane flow correlations in PbPb@LHC

Data: ATLAS Coll., J. Jia et al., Hard Probes 2012

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Nonlinear mode coupling through hydrodynamic evolution essential to describe the data!

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In a single event, the specific initial density profile results in a set of complex, y - and p_T -dependent flow coefficients (we'll suppress the y -dependence):

$$V_n = v_n e^{in\Psi_n} := \frac{\int p_T dp_T d\phi e^{in\phi} \frac{dN}{dy p_T dp_T d\phi}}{\int p_T dp_T d\phi \frac{dN}{dy p_T dp_T d\phi}} \equiv \{e^{in\phi}\},$$

$$V_n(p_T) = v_n(p_T) e^{in\Psi_n(p_T)} := \frac{\int d\phi e^{in\phi} \frac{dN}{dy p_T dp_T d\phi}}{\int d\phi \frac{dN}{dy p_T dp_T d\phi}} \equiv \{e^{in\phi}\}_{p_T}.$$

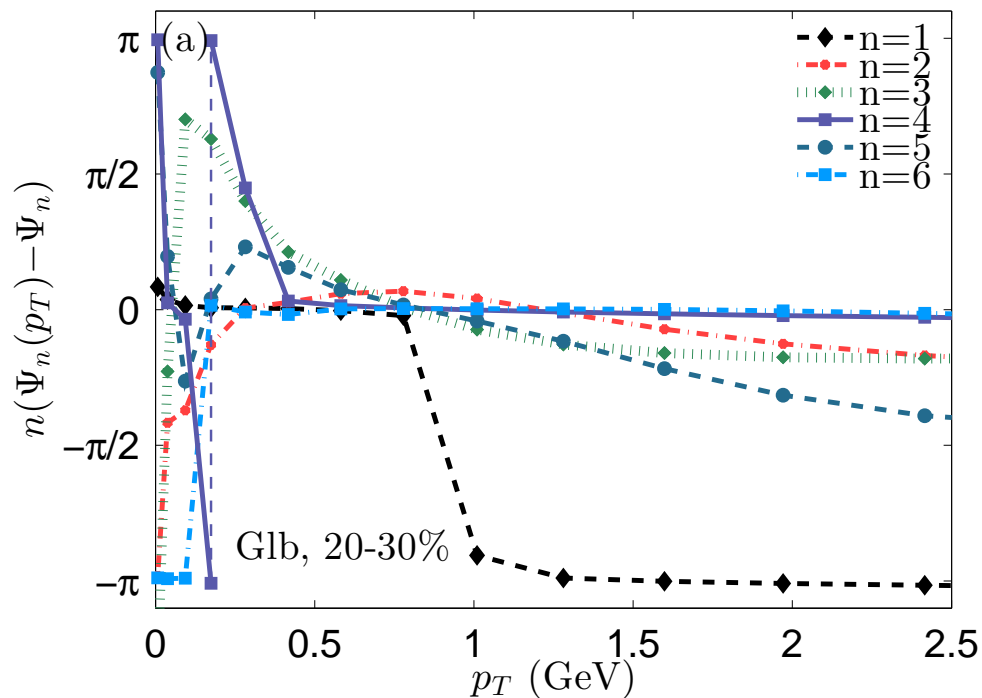
Together with the azimuthally averaged spectrum, these completely characterize the measurable single-particle information for that event:

$$\frac{dN}{dy d\phi} = \frac{1}{2\pi} \frac{dN}{dy} \left(1 + 2 \sum_{n=1}^{\infty} v_n \cos[n(\phi - \Psi_n)] \right),$$

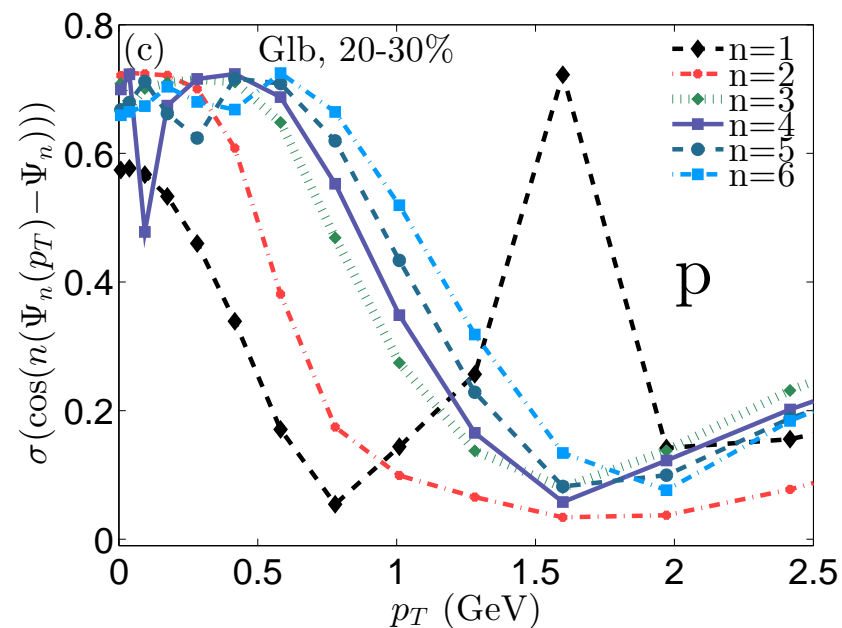
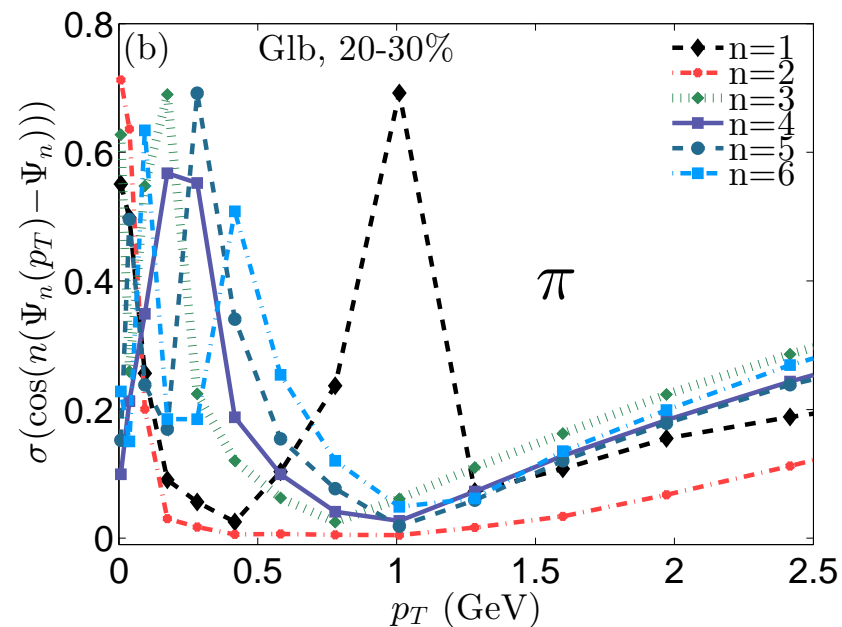
$$\frac{dN}{dy p_T dp_T d\phi} = \frac{1}{2\pi} \frac{dN}{dy p_T dp_T} \left(1 + 2 \sum_{n=1}^{\infty} v_n(p_T) \cos[n(\phi - \Psi_n(p_T))] \right).$$

- Both the magnitude v_n and the direction Ψ_n (“flow angle”) depend on p_T .
- $v_n, \Psi_n, v_n(p_T), \Psi_n(p_T)$ **all fluctuate from event to event.**
- $\Psi_n(p_T) - \Psi_n$ fluctuates from event to event.

p_T -dependent flow angles and their fluctuations



- Except for directed flow ($n=1$), $\Psi_n(p_T) - \Psi_n$ fluctuates most strongly at low p_T
- Directed flow angle $\Psi_1(p_T)$ flips by 180° at $p_T \sim 1$ GeV for charged hadrons (pions) and at $p_T \sim 1.5$ GeV for protons (momentum conservation)



Flow measures from two-particle correlations $\langle \{e^{in(\phi_1 - \phi_2)}\} \rangle$

“rms flow”:

$$v_n^2[2] := \langle \{e^{in\phi_1}\} \{e^{-in\phi_2}\} \rangle = \langle v_n^2 \rangle \equiv v_n\{2\};$$

$$v_n^2[2](p_T) := \langle \{e^{in\phi_1}\}_{p_T} \{e^{-in\phi_2}\}_{p_T} \rangle = \langle v_n^2(p_T) \rangle \quad (\neq v_n\{2\}(p_T)!).$$

“differential 2-particle cumulant flow”:

$$v_n\{2\}(p_T) := \langle \{e^{in\phi_1}\}_{p_T} \{e^{-in\phi_2}\} \rangle / v_n\{2\} = \left\langle v_n(p_T) v_n \cos[n(\Psi_n(p_T) - \Psi_n)] \right\rangle / v_n[2].$$

“event plane flow”:

$$v_n\{\text{EP}\}(p_T) := \left\langle \{e^{in\phi}\}_{p_T} e^{-in\Psi_n} \right\rangle = \left\langle v_n(p_T) \cos[n(\Psi_n(p_T) - \Psi_n)] \right\rangle.$$

“mean flow”:

$$\langle v_n(p_T) \rangle := \left\langle \left| \{e^{in\phi}\}_{p_T} e^{-in\Psi_n} \right| \right\rangle = \left\langle \sqrt{\{\cos(n\phi)\}_{p_T}^2 + \{\sin(n\phi)\}_{p_T}^2} \right\rangle.$$

“two-particle flows”:

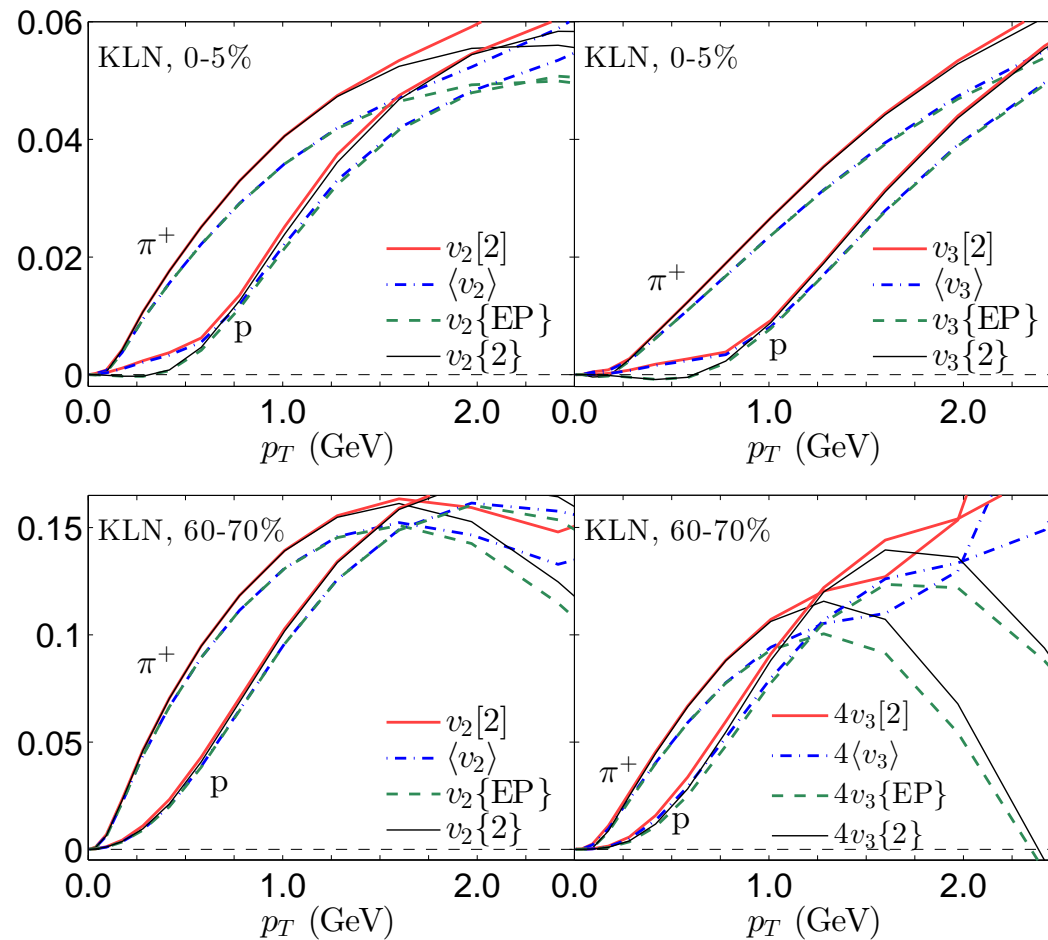
$$\tilde{V}_{n\Delta}(p_{T1}, p_{T2}) := \left\langle \{e^{in(\phi_1 - \phi_2)}\}_{p_{T1}p_{T2}} \right\rangle = \left\langle v_n(p_{T1}) v_n(p_{T2}) \cos[n(\Psi_n(p_{T1}) - \Psi_n(p_{T2}))] \right\rangle;$$

$$\left\langle v_n(p_{T1}) v_n(p_{T2}) \right\rangle := \left\langle \sqrt{\{\cos(n\Delta\phi)\}_{p_{T1}, p_{T2}}^2 + \{\sin(n\Delta\phi)\}_{p_{T1}, p_{T2}}^2} \right\rangle.$$

Here: both particles taken from same species (but this is not necessary).

Fluctuation effects related to finite number of particles in the observed final state are ignored.

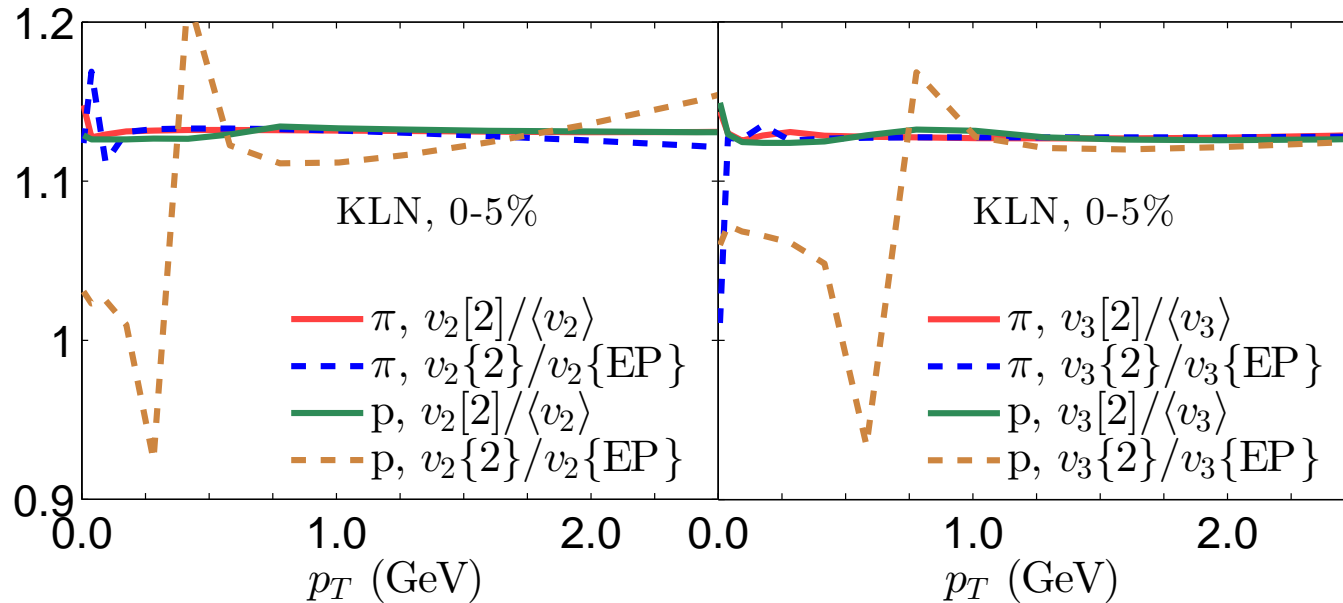
Elliptic and triangular flow comparison (I)



In central collisions, angular fluctuations suppress $v_n\{EP\}(p_T)$ and $v_n\{2\}(p_T)$ below the mean and rms flows at low p_T (clearly visible for protons)

This effect disappears in peripheral collisions, but a similar effect then takes over at higher p_T , for both pions and protons.

Elliptic and triangular flow comparison (II): v_n ratios



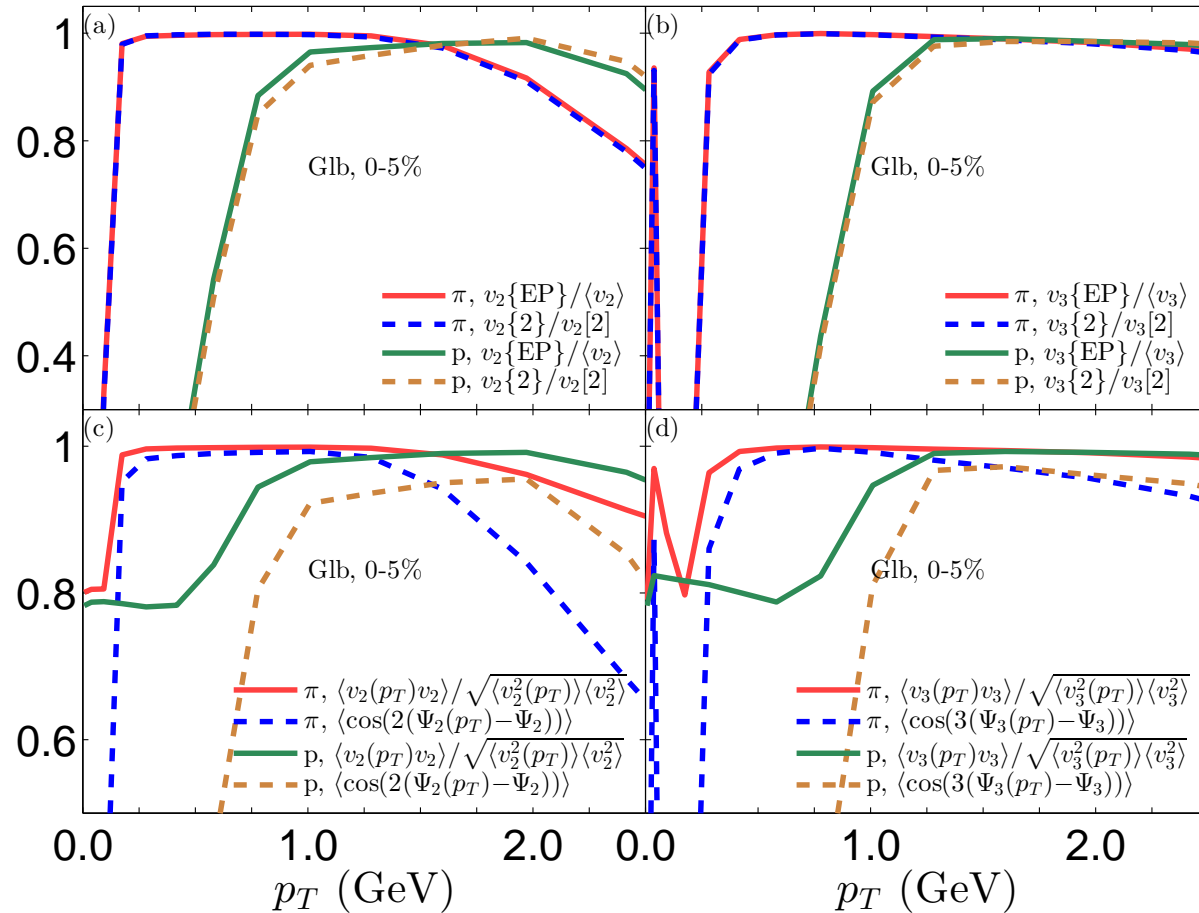
Except for where the numerator or denominator goes through zero, for central collisions these ratios are equal to $2/\sqrt{\pi} \approx 1.13$, independent of p_T . Expected if flow angles are randomly oriented (Bessel-Gaussian distribution for v_n , see [Voloshin et al., PLB 659, 537 \(2008\)](#)).

Not true in peripheral collisions, especially not for v_2 ([Gardim et al., 1209.2323](#))

That this works even for $v_n\{2\}/v_n\{EP\}$ suggests an approximate factorization of angular fluctuation effects!

Elliptic and triangular flow comparison (III): v_n ratios

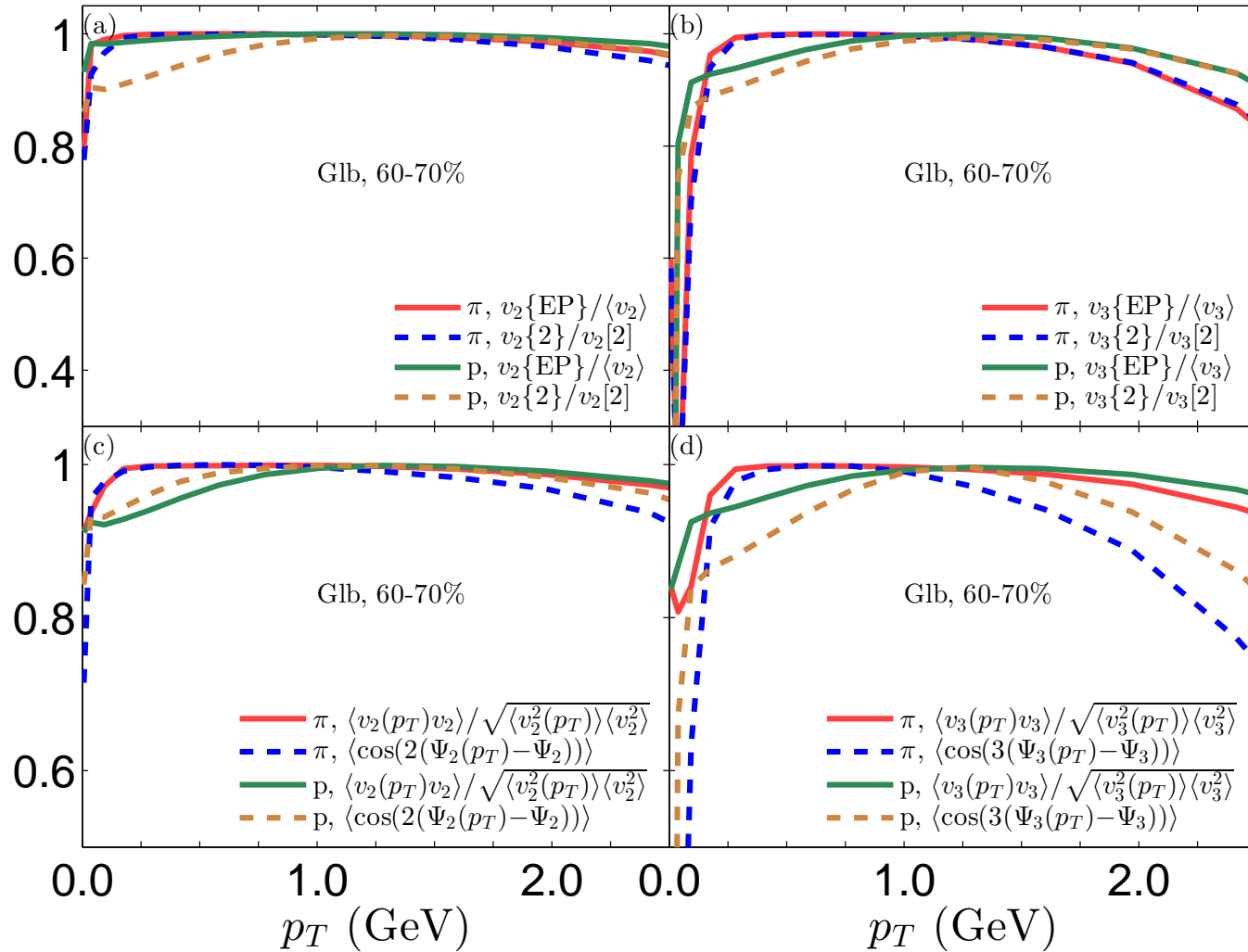
Central collisions:



- The angular fluctuation factor $\langle \cos[n(\Psi_n(p_T) - \Psi_n)] \rangle$ completely dominates the p_T -dependence of these ratios!
- Angular fluctuations have similar effect as poor event-plane resolution: they reduce v_n .
- Angular fluctuations are effective both at low and high p_T , but not at intermediate p_T .
- The window for seeing flow angle fluctuation effects at low p_T is smaller for pions than for protons.

Elliptic and triangular flow comparison (IV): v_n ratios

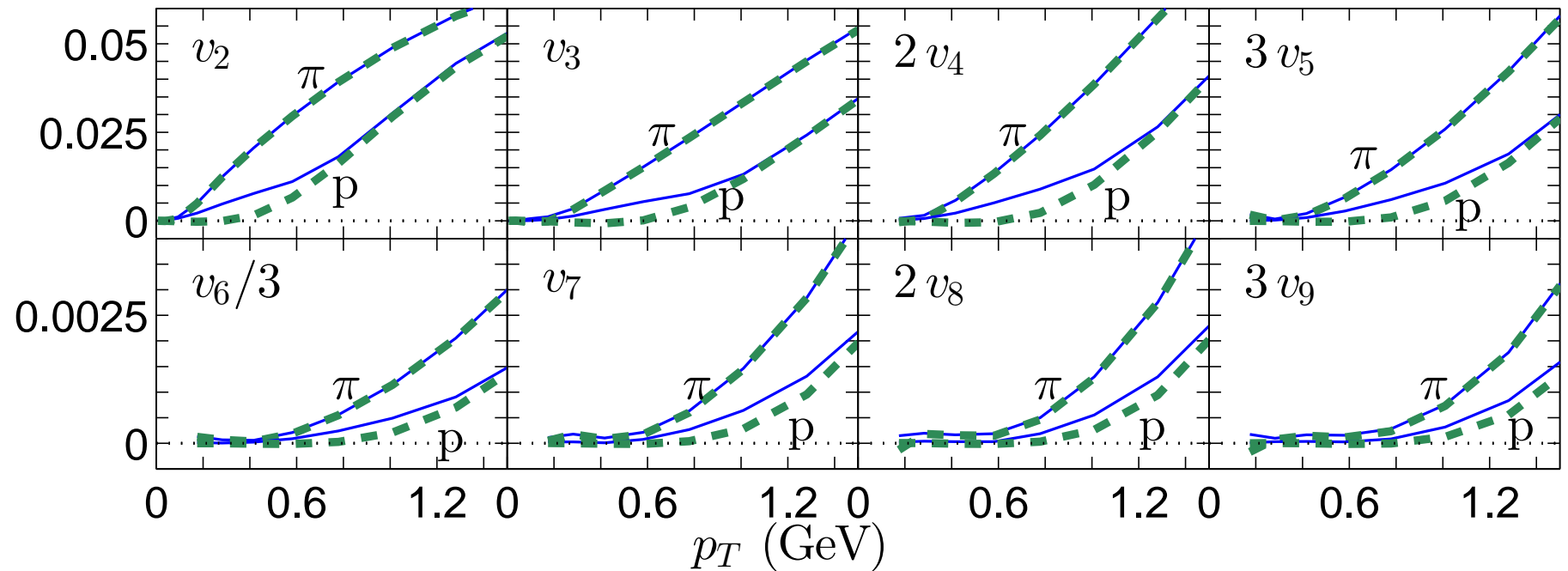
Peripheral collisions:



The window for seeing flow angle fluctuation effects at low p_T closes in peripheral collisions.

Flow angle fluctuation effects for higher order $v_n(p_T)$

Central collisions; solid: $\langle v_n(p_T) \rangle$; dashed: $v_n\{\text{EP}\}(p_T)$:



As harmonic order n increases, suppression of $v_n\{\text{EP}\}(p_T)$ (or $v_n\{2\}(p_T)$) from flow angle fluctuations for protons gets somewhat weaker but persists to larger p_T .

Test of factorization of two-particle spectra

Factorization $V_{n\Delta}(p_{T1}, p_{T2}) := \langle \{\cos[n(\phi_1 - \phi_2)]\}_{p_{T1}p_{T2}} \rangle \approx "v_n(p_{T1}) \times v_n(p_{T2})"$ was checked experimentally as a test of hydrodynamic behavior, and found to hold to good approximation.

Gardim et al. (1211.0989) pointed out that event-by-event fluctuations break this factorization even if 2-particle correlations are exclusively due to flow.

They proposed to study the following ratio:

$$r_n(p_{T1}, p_{T2}) := \frac{V_{n\Delta}(p_{T1}, p_{T2})}{\sqrt{V_{n\Delta}(p_{T1}, p_{T1})V_{n\Delta}(p_{T2}, p_{T2})}} = \frac{\langle v_n(p_{T1})v_n(p_{T2})\cos[n(\Psi_n(p_{T1}) - \Psi_n(p_{T2}))] \rangle}{v_n[2](p_{T1})v_n[2](p_{T2})}$$

Even in the absence of flow angle fluctuations, this ratio is < 1 due to v_n fluctuations (Schwarz inequality), except for $p_{T1} = p_{T2}$.

But it additionally depends on flow angle fluctuations.

To assess what share of the deviation from 1 is due to flow angle fluctuations, we can compare with

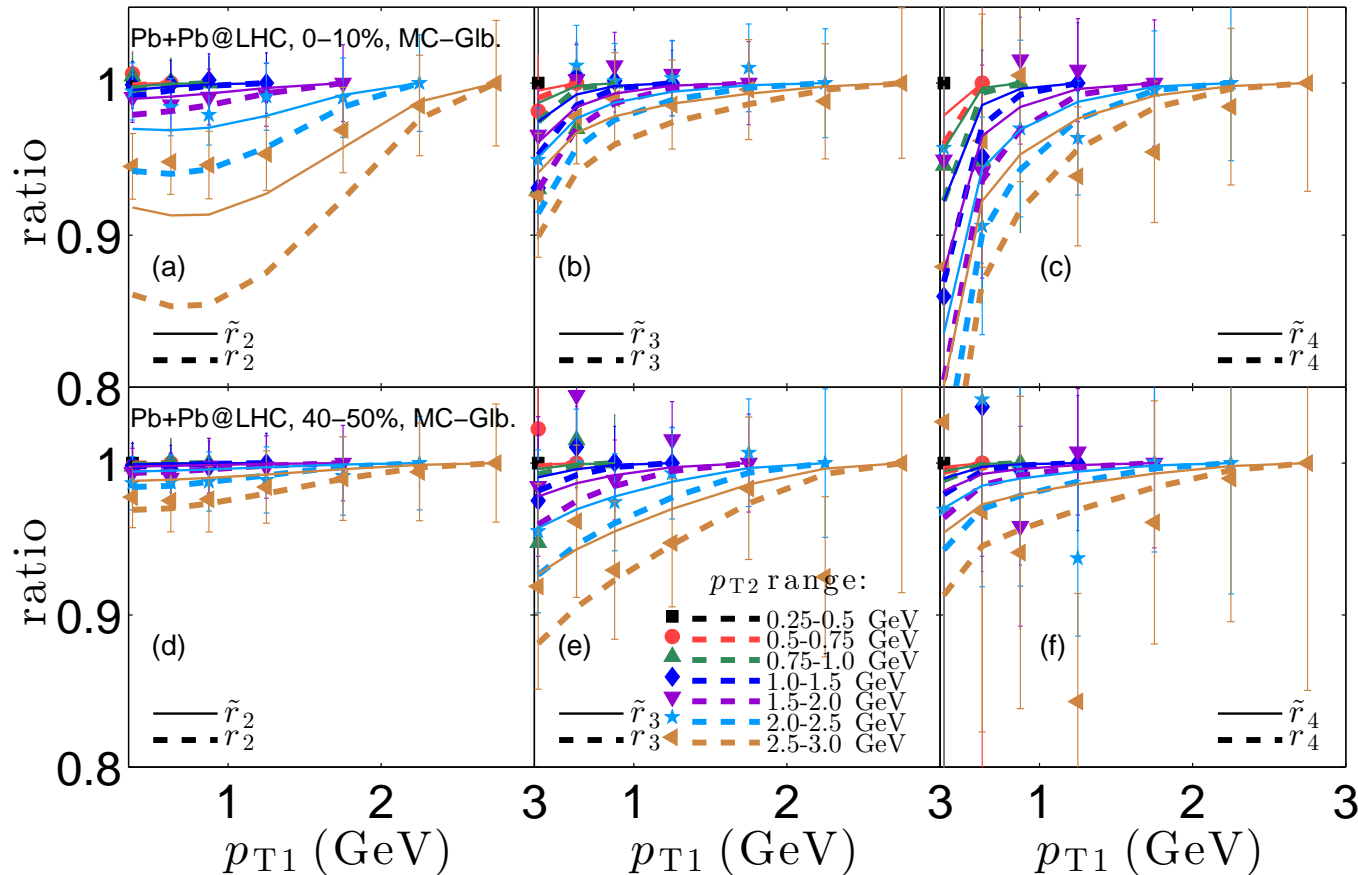
$$\tilde{r}_n(p_{T1}, p_{T2}) := \frac{\langle v_n(p_{T1})v_n(p_{T2})\cos[n(\Psi_n(p_{T1}) - \Psi_n(p_{T2}))] \rangle}{\langle v_n(p_{T1})v_n(p_{T2}) \rangle}$$

which deviates from 1 **only** due to flow angle fluctuations. Again, this ratio approaches 1 for $p_{T1} = p_{T2}$.

Gardim et al. studied r_n for ideal hydro; we have studied r_n and \tilde{r}_n for viscous hydro.

Breaking of factorization by e-by-e fluctuations (I)

Monte Carlo Glauber initial conditions, $\eta/s = 0.08 = 1/(4\pi)$:



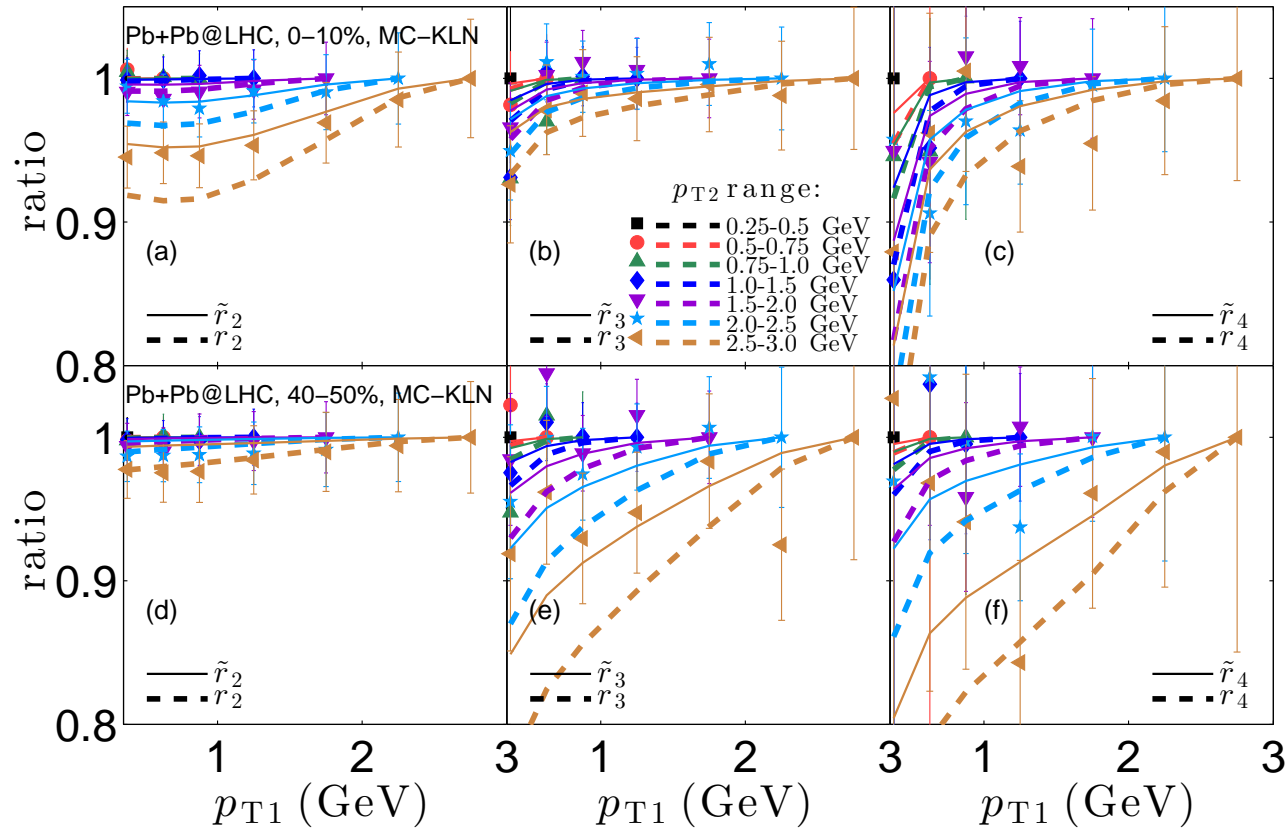
More than half of the factorization breaking effects are due to flow angle fluctuations.

In central collisions, $\eta/s = 0.08$ appears to overpredict the breaking of factorization (consistent with [Gardim et al.](#) who saw still larger effects for ideal hydro).

Factorization breaking effects appear to be larger for fluctuation-dominated flow harmonics.

Breaking of factorization by e-by-e fluctuations (II)

Monte Carlo KLN initial conditions, $\eta/s = 0.2 = 2.5/(4\pi)$:



In central collisions, factorization-breaking effects decrease with increasing η/s .

In peripheral collisions, larger η/s appears to cause a larger breaking of factorization, mostly due to flow angle fluctuations.

Data may indicate slight preference for larger η/s value, but more experimental precision and more detailed theoretical studies are needed to settle this. Analysis of ATLAS data in progress.

Conclusions

- Both the magnitudes v_n and the flow angles Ψ_n depend on p_T and fluctuate from event to event.
- In each event, the “ p_T -averaged” (total-event) flow angles Ψ_n are identical for all particle species, but their p_T distribution differs from species to species.
- The mean v_n values and their p_T -dependence at RHIC and LHC have already been shown to put useful constraints on the QGP shear viscosity and its temperature dependence (see next talk by B. Schenke)
- **The effects of v_n and Ψ_n fluctuations can be separated experimentally by studying different V_n measures based on two-particle correlations.**
- Flow angle correlations are a powerful test of the hydrodynamic paradigm and will help to further constrain the spectrum of initial-state fluctuations and QGP transport coefficients.
- Studying event-by-event fluctuations of the anisotropic flows v_n and their flow angles Ψ_n as functions of p_T , as well as the correlations between different harmonic flows (both their magnitudes and angles), provides a rich data base for identifying the **“Standard Model of the Little Bang”**, by pinning down its initial fluctuation spectrum and its transport coefficients.